SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – I – Power System Analysis – SEE1302

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POWER SYSTEM MODELING

Need for system Analysis in planning and operation of power system – per phase analysis of symmetrical three – phase system. General aspects relating to power flow, short circuit and stability analysis – Modeling of generator, load, Shunt capacitor, transmission line, shunt reactor for short circuit, power flow and stability studies – per unit representation – bus admittance by analytical method and direct inspection method

Introduction

- **A Typical Power System Consists of a 3 – Phase grid to which all generating stations feeds energy and from which all substations taps energy**
- **A grid is either 3- phase single circuit or 3 phase two circuit transmission line, running throughout the length and breadth of a country or a state**

Components of Power System

Generators

Power Transformers

Transmission lines

Substation Transformers

Distribution Transformers

Loads

Single Line Diagram

- **It is a diagrammatic representation of power system in which the components are represented by their symbols and the interconnection between them are shown by a single line diagram (even though the system is 3 phase system)**
- **The ratings and the impedances of the components are also marked on the single line diagram**

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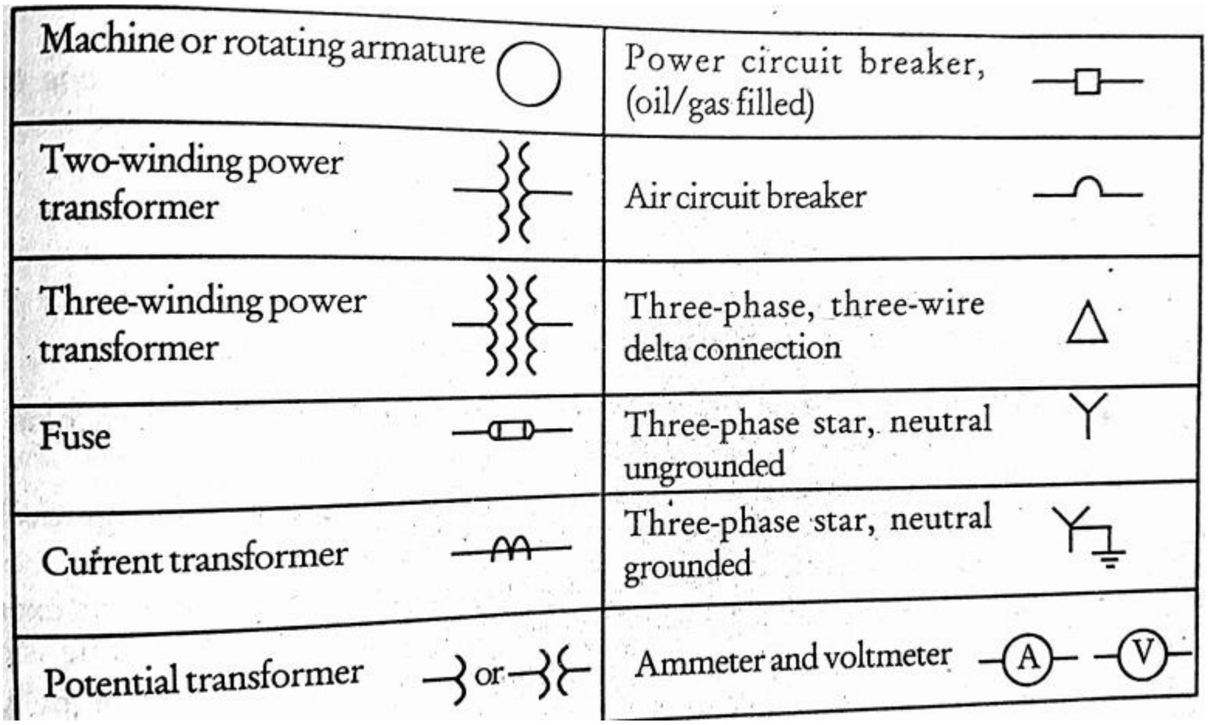
Per Unit Value

$$PerUnitValue = \frac{ActualValue}{BaseValue}$$

$$\%PerUnitValue = \frac{ActualValue}{BaseValue} * 100$$

Symbols used in Single line diagram



Machine or rotating armature	Power circuit breaker, (oil/gas filled)
Two-winding power transformer	Air circuit breaker
Three-winding power transformer	Three-phase, three-wire delta connection
Fuse	Three-phase star, neutral ungrounded
Current transformer	Three-phase star, neutral grounded
Potential transformer or	Ammeter and voltmeter

Table: 1.1

Single line Diagram

- **The various components of power system components of Power system like alternators, motors, transformers etc., have their voltage, power, current and impedance ratings in KV,KVA,KA and Ω**

- **The components or various sections of power system may operate at different voltage and power levels**
- **It will be convenient for analysis of power system if the voltage, power, current and impedance ratings of components of power system are expressed with reference to a common value called base value**

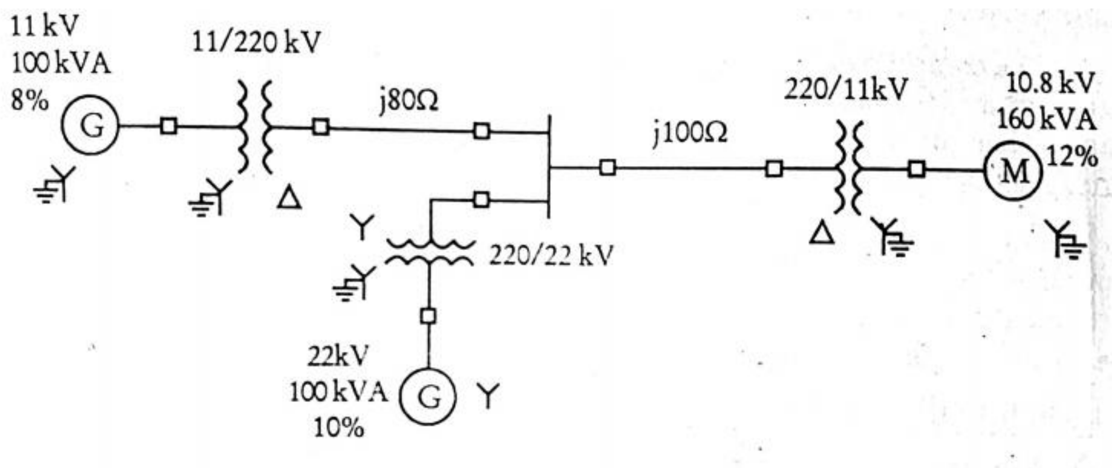


Figure:1.1

- **Hence the analysis purpose a base value is chosen for voltage, power, current and impedance**
- **The power system requires the base values of four quantities and they are Voltage, Power, Current and Impedance.**
- **Selection of base values for any two of them determines the base values of the remaining two**

Formula for finding base Value

Single Phase System

Let KVA_b = Base KVA

KV_b=Base voltage in KV

I_b=Base current in A

Z_b=Base impedance in Ω

$$I_b = \frac{KVA_b}{KV_b} in amps$$

$$Z_b = \frac{KV_b * 1000}{I_b} in \Omega$$

$$Z_b = \frac{KV_b * 1000}{\frac{KVA_b}{KV_b}} = \frac{(KV_b)^2}{\frac{KVA_b}{1000}} = \frac{(KV_b)^2}{MVA_b}$$

- **The same formula holds good for Three phase system also both for star connected and Delta connected**
- **In 3 phase system, the KV_b is a line value and MVA_b is a 3 phase MVA**

The Impedance value is always expressed as Phase Value

1. A three Phase generator with rating 1000KVA, 33KV has its armature resistance and synchronous reactance as 20Ω/Phase and 70Ω/Phase. Calculate P.U. impedance of the generator.

Solution:

KV_b=33KV

KVA_b=1000KVA

$$Z_b = \frac{(KV_b)^2}{MVA_b} = \frac{(33)^2}{\frac{1000}{1000}} = 1089\Omega$$

Z=(20+j70)Ω/ Phase

$$\therefore Z_{pu} = \frac{Actua\lim pedance}{Baseimpedance} = \frac{Z}{Z_b} = \frac{20 + j70}{1089} = 0.018 + j0.064 p.u$$

2. A three phase, Δ/Y transformer with rating 100KVA, 11KV/400V has its primary and secondary leakage reactance as 12Ω/Phase and 0.05Ω/Phase respectively. Calculate the p.u reactance of the transformer.

Solution:

Case(i)

High Voltage winding (Primary) is chosen as base values.

KV_b=11KV

KVA_b=100KVA

$$Z_b = \frac{(KV_b)^2}{MVA_b} = \frac{(11)^2}{\frac{100}{1000}} = 1210\Omega$$

Transformer line voltage ratio,

$$K = \frac{400}{11000} = 0.0364$$

Total leakage reactance referred to primary

$$X_{01} = X_1 + X_2^{'} = X_1 + \frac{X_2}{K^2} = 12 + \frac{0.05}{(0.0364)^2} = 12 + 37.737 = 49.737\Omega / Phase$$

X_{pu}=Total leakage reactance/ Base impedance

$$= \frac{X_{01}}{Z_b} = \frac{49,737}{1210} = 0.0411 p.u.$$

Case(ii)

Low Voltage winding (Secondary) is chosen as base values.

KV_b=400/1000=0.4KV

KVA_b=100KVA

$$Z_b = \frac{(KV_b)^2}{MVA_b} = \frac{(0.4)^2}{\frac{100}{1000}} = 1.6\Omega$$

Transformer line voltage ratio,

$$K = \frac{400}{11000} = 0.0364$$

Total leakage reactance referred to Secondary

$$X_{02} = X_1^{'} + X_2 = K^2 X_1 + X_2 = (0.0364)^2 * 12 + 0.05 = 0.0159 + 0.05 = 0.0659\Omega / Phase$$

X_{pu}=Total leakage reactance/ Base impedance

$$= \frac{X_{02}}{Z_b} = \frac{0.0659}{1.6} = 0.0411 p.u.$$

Note: 1. It is observed that P.U. reactance of a transformer referred to primary and secondary are same.

2. In three phase transformer if the voltage ratio K is obtained using line values then using this value of K ,

The phase impedance per phase of star side can be directly transferred to delta side or vice versa

Advantages of Per Unit Computations

1. **Manufactures usually specify the impedance of a device or machine in percent or per unit on the base of the name plate rating**

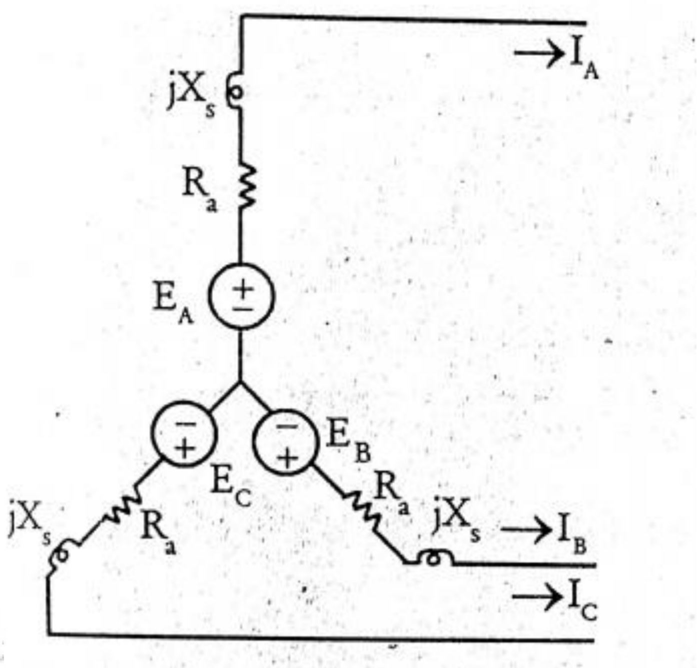
2. **The Per Unit impedances of a machines of the same type and widely different rating usually lie within a narrow range, although the ohmic Values differ widely for machines of different ratings**

3. **The Per Unit impedance of circuit element connected by transformers expressed on a proper base will be same if it is referred to either side of a transformer**

4. **The way in which the transformers are connected in a 3 phase circuits (Y/Δ) does not affect the per unit impedances of the equivalent circuit, although the transformer connection does determine the relation between the voltage bases on the two sides of the transformer**

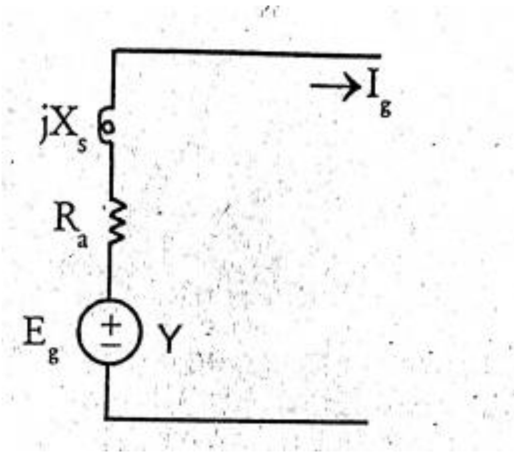
Equivalent Circuits of Components of Power System

Equivalent Circuit of Generator



3-Phase equivalent circuit

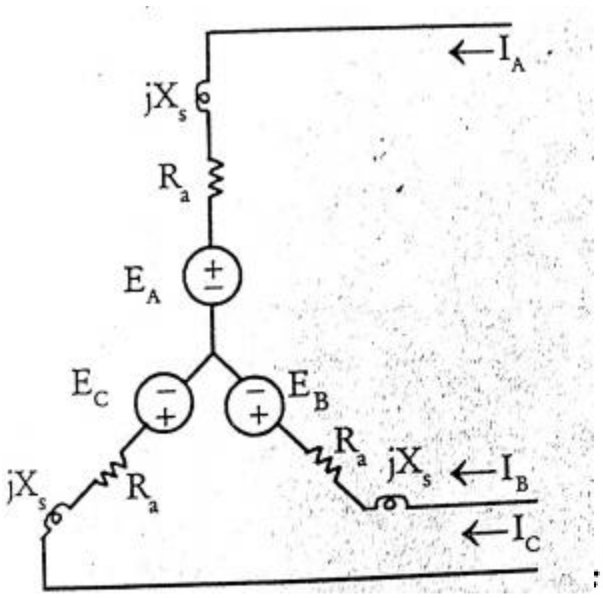
Figure 1.2



Single phase equivalent circuit

Figure 1.3

Equivalent Circuit of Synchronous motor



: 3-Phase equivalent circuit

Figure 1.4

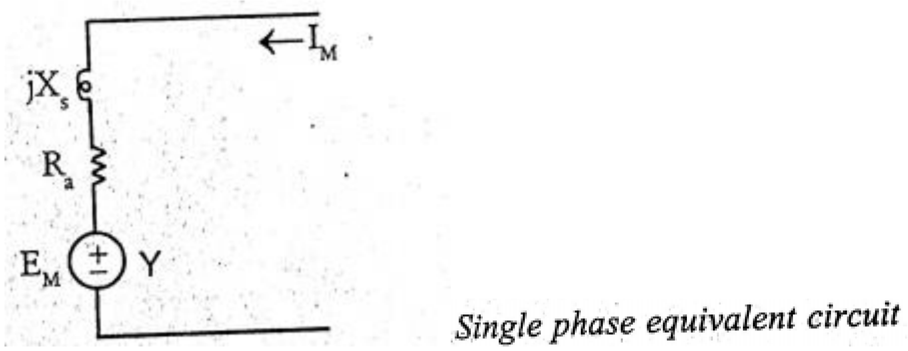


Figure 1.5

Equivalent Circuit of Transformer

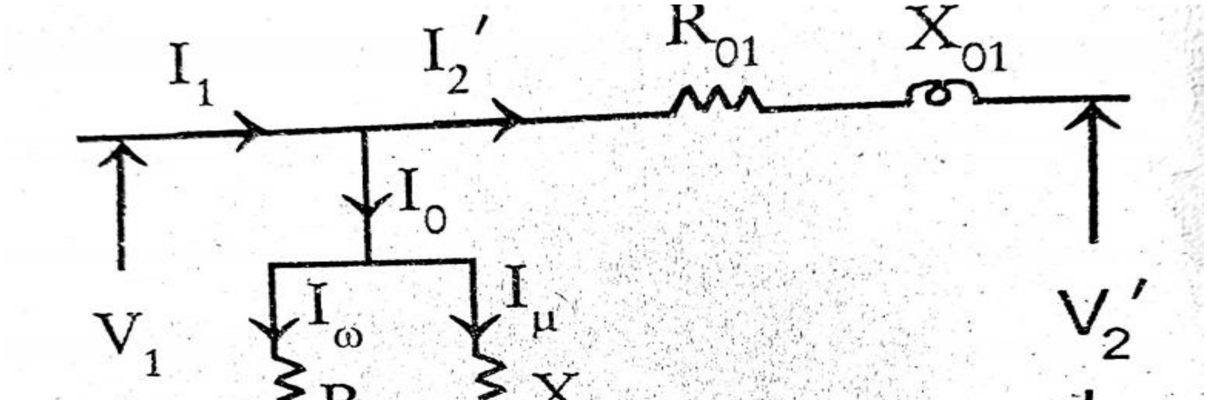


Figure 1.6

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

Equivalent Circuit of Induction Motor

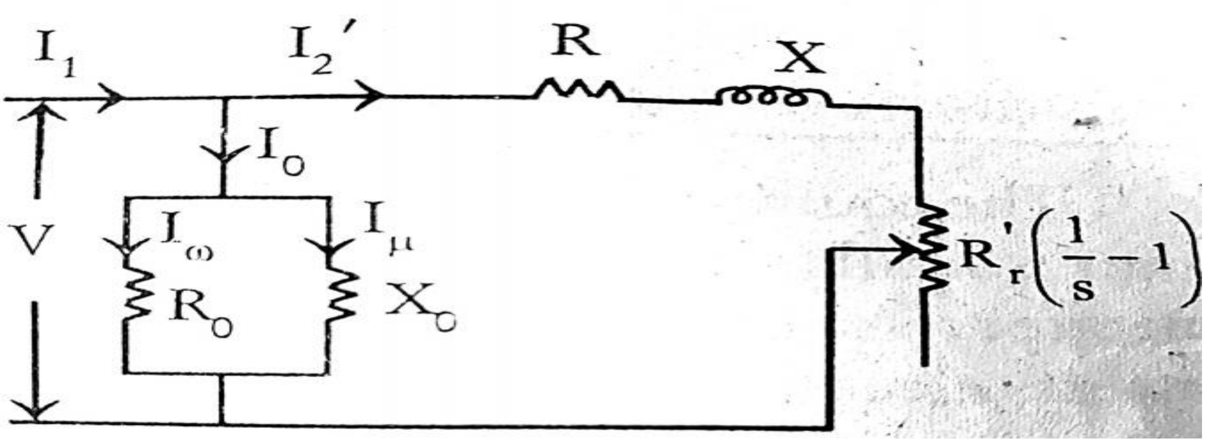


Figure 1.7

S= Slip

$R_r^{'}\left(\frac{1}{S}-1\right)$ **= Resistance representing load**

$R = R_s + R_r^{'}$ **= Equivalent resistance referred to stator**

$X = X_s + X_r^{'}$ **= Equivalent reactance referred to stator**

$R_{s,} X_s$ **= resistance and reactance of Stator**

$R_r^{'} X_r^{'}$ **= resistance and reactance of rotor**

Equivalent Circuit of Transmission line

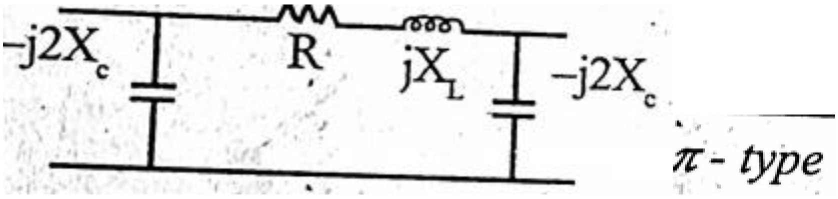


Figure :1.8

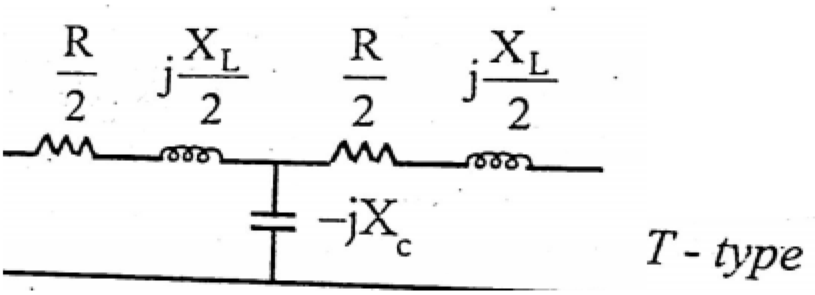


Figure:1.9

Representation of resistive and reactive loads

Single Phase Load

Constant Power representation

S=P+jQ

Constant Current representation

S=VI*

S*=V*I

S*=P-jQ

$$I = \frac{\sqrt{P^2+Q^2}\angle-\theta}{|V|\angle-\delta} = \frac{\sqrt{P^2+Q^2}}{|V|}\angle\delta-\theta = |I|\angle\delta-\theta$$

Where

$$\theta = \tan^{-1}\frac{Q}{P}$$

Constant Impedance representation

$$Z = \frac{|V|^2}{P-jQ}$$

$$Y = \frac{P-jQ}{|V|^2}$$

Three Phase Load (Balanced Star Connected load)

P= Three Phase active Power of star connected load in watts

Q= Three Phase reactive Power of star connected load in VARS

V, V_L = Phase & line voltage of load respectively

I, I_L= Phase & line current of load respectively

Constant Power representation

S=P+jQ

Constant Current representation

$$I = I_L = \frac{\sqrt{P^2 + Q^2}}{\sqrt{3}\,|V_L|} \angle\delta - \theta = |I_L| \angle\delta - \theta$$

Where

$$\theta = \tan^{-1}\frac{Q}{P}$$

Constant Impedance representation

$$Z = \frac{|V_L|^2}{P - jQ}$$

$$Y = \frac{P - jQ}{|V_L|^2}$$

Three Phase Load (Balanced Delta Connected load)

Constant Power representation

S=P+jQ

Constant Current representation

$$I_L = \frac{\sqrt{P^2+Q^2}}{\sqrt{3}|V_L|}\angle\delta-\theta = |I_L|\angle\delta-\theta \qquad I = \frac{\sqrt{P^2+Q^2}}{3|V_L|}\angle\delta-\theta = |I|\angle\delta-\theta$$

Where $\theta = \tan^{-1}\frac{Q}{P}$ *Where* $\theta = \tan^{-1}\frac{Q}{P}$

Constant Impedance representation

$$Z = \frac{3|V_L|^2}{P-jQ}$$

$$Y = \frac{P-jQ}{3|V_L|^2}$$

3. A 50Kw, three phase, Y connected load is fed by a 200 KVA transformer with voltage rating 11KV/400V through a feeder. The length of the feeder is 0.5km and the impedance of the feeder is 0.1+j0.2Ω/km. If the load p.f is 0.8, Calculate the p.u impedance of the load and feeder.

Solution:

Choose secondary winding rating of transformer as base values

Kv_b=400/1000 = 0.4KV

KVA_b=200KVA

$$Z_b = \frac{(KV_b)^2}{MVA_b} = \frac{(0.4)^2}{\frac{200}{1000}} = 0.8\Omega$$

$$Z_{fed} = (0.1 + j0.2)*0.5 = 0.05 + j0.1\Omega / Phase$$

$$Z_{pu,fed} = \frac{Z_{fed}}{Z_b} = \frac{0.05 + j0.1}{0.8} = 0.0625 + j0.125\,p.u$$

P=50Kw, pf=cosϕ=0.8

∴sinϕ=sin(cos^{-1}0.8)=0.6

$$Q = \frac{P}{\cos\phi} * \sin\phi = \frac{50}{0.8} * 0.6 = 37.5KVAR$$

Load impedance/Phase

$$Z_L = \frac{|V|^2}{P - jQ} = \frac{400^2}{(50 - j37.5)*10^3} = 2.56\angle 36.87^\circ = 2.048 + j1.536W / Phase$$

$$Z_{L,Pu} = \frac{Z_L}{Z_b} = \frac{2.048 + j1.536}{0.8} = 2.56 + j1.92\,pu$$

Impedance Diagram

- **The impedance diagram is the equivalent circuit of Power system in which the various components of power system are represented by their approximate or simplified equivalent circuits**
- **It is used for load flow studies**

Approximations made in Impedance Diagram

- **The neutral reactances are neglected**
- **The shunt branches in equivalent circuit of Transformers & induction motor are neglected**

Reactance Diagram

- **It is a simplified equivalent circuit of power system in which the various components are represented by their reactances**
- **It can be obtained from impedance diagram if all the resistive components are neglected**
- **It is used for fault calculations**

Approximations made in Reactance Diagram

1. **The neutral reactances are neglected**
2. **Shunt branches in the equivalent circuits of transformer are neglected**
3. **The resistances are neglected**
4. **All static loads and induction motors are neglected**
5. **The capacitance of the transmission lines are neglected**

Single Line Diagram

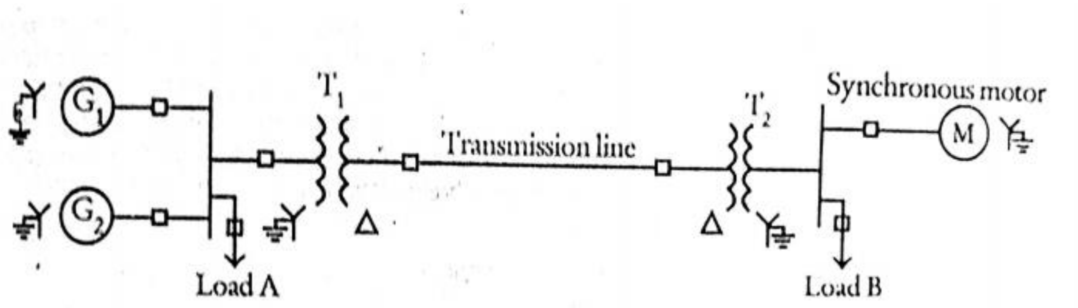


Figure:1.10

Impedance Diagram

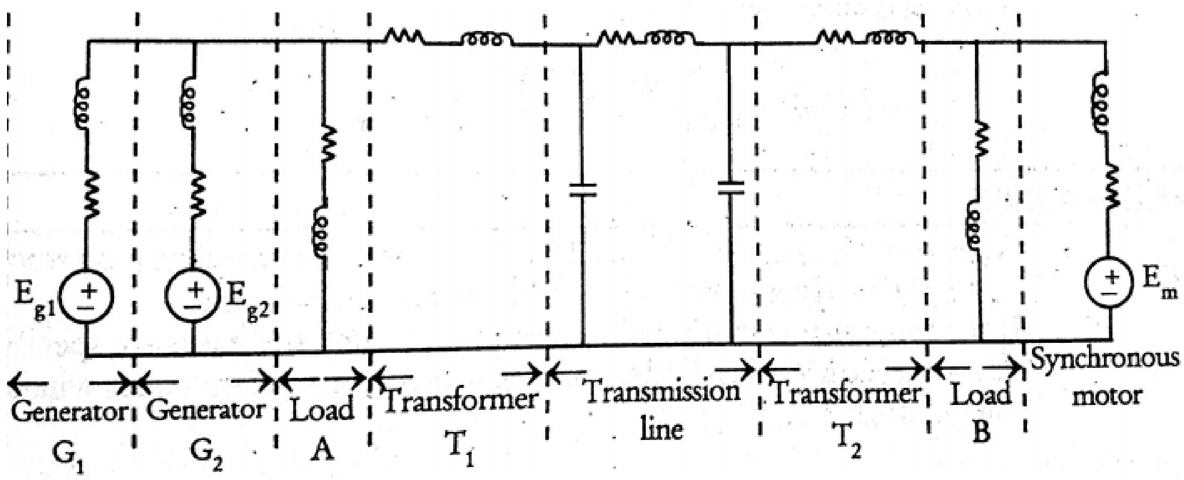


Figure 1.11

Reactance Diagram

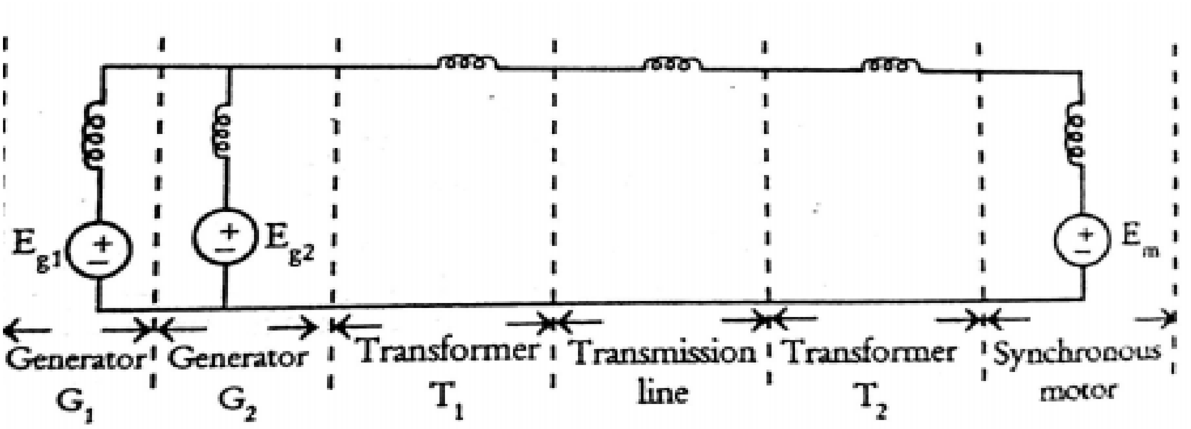


Figure:1.12

Equation for converting P.U impedance expressed in one base to another

$$Z_{pu,new} = Z_{pu,old} * \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

Equation for transforming base KV on LV side to HV side of transformer & Vice versa

Base KV on HT side = Base KV on LT side * (HT voltage rating / LT voltage rating)

Base KV on LT side = Base KV on HT side * (LT voltage rating / HT voltage rating)

4. A 300 MVA, 20KV, 3ϕ generator has a subtransient reactance of 20%. The generator supplies 2 synchronous motors through a 64Km transmission line having transformers at both ends as as shown in Fig. In this, T_1 is a 3ϕ transformer and T_2 is made of 3 single phase transformer of rating 100 MVA, 127/13.2KV, 10% reactance. Series reactance of the transmission line is 0.5Ω/Km. Draw the reactance diagram with all the reactances marked in p.u. Select the generator ratings as base values.

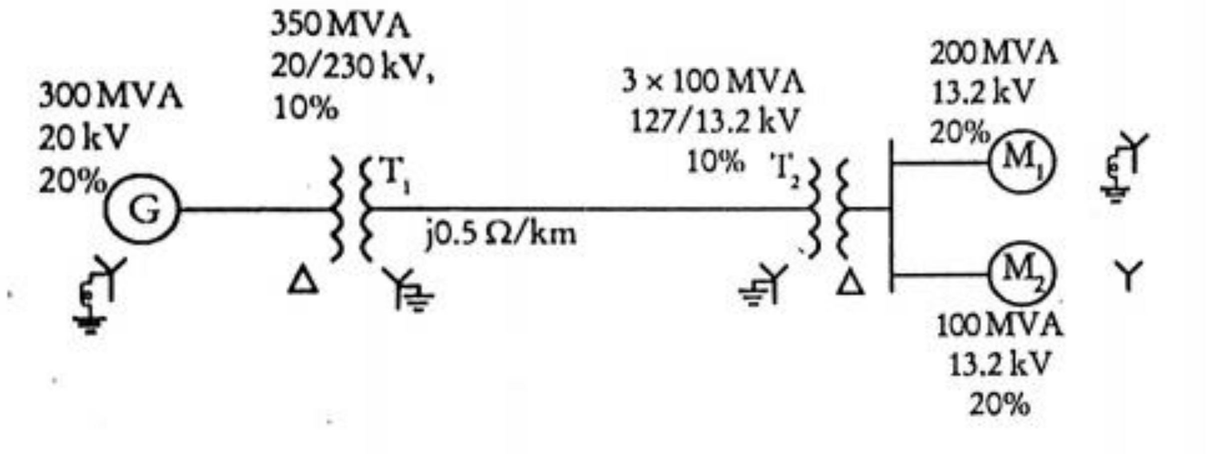


Figure: 1.13

Solution:

$MVA_{b,new}$=300MVA

$KV_{b,new}$=20KV

Reactance of Generator G

Since the generator rating and the base values are same, the generator p.u. reactance does not change

$\therefore X_{G,pu,new} = 20\% = 0.2p.u.$

Reactance of Transformer T_1

$$X_{T1,\,pu,new} = X_{T1\,pu,old} * \left(\frac{KV_{bT1,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bT1,old}}\right)$$

$$= 0.1 * \left(\frac{20}{20}\right)^2 * \left(\frac{300}{350}\right) = 0.0857$$

Reactance of Transmission line (TL)

Reactance of transmission line = 0.5Ω/Km

Total reactance X_{TL} = 0.5*64 = 32Ω

Base KV on HT side of T_1 = Base KV on LT side of T_1 * (HT voltage rating of T_1 / LT voltage rating of T_1)

= 20*(230/20)=230KV

$KV_{b,new}$=230KV

$$Z_b = \frac{(Kv_b, new)^2}{MVA_b} = \frac{230^2}{300} = 176.33\Omega$$

$$X_{TL,p.u} = \frac{32}{176.33} = 0.1815\,p.u$$

Reactance of Transformer T_2

Voltage ratio of line voltage of 3ϕ transformer bank =((√3*127)/13.2) = (220/13.2)KV

Base KV on LT side of T_2 = Base KV on HT side of T_2* (LT voltage rating of T_2/ HT voltage rating of T_2)

= 230 *(13.2/220) = 13.8KV

$KV_{b,new}$=13.8KV

$$X_{T2,\,pu,new} = X_{T2\,pu,old} * \left(\frac{KV_{bT2,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bT2,old}}\right)$$

$$= 0.1 * \left(\frac{13.2}{13.8}\right)^2 * \left(\frac{300}{3*100}\right) = 0.0915\,p.u$$

Reactance of Motor M_1

$$X_{M1,\,pu,new} = X_{M1\,pu,old} * \left(\frac{KV_{bM1,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bM1,old}}\right)$$

$$= 0.2 * \left(\frac{13.2}{13.8}\right)^2 * \left(\frac{300}{200}\right) = 0.0915\,p.u$$

Reactance of Motor M_2

$$X_{M2,pu,new} = X_{M2pu,old} * \left(\frac{KV_{bM2,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bM2,old}}\right)$$

$$= 0.2 * \left(\frac{13.2}{13.8}\right)^2 * \left(\frac{300}{100}\right) = 0.549\, p.u$$

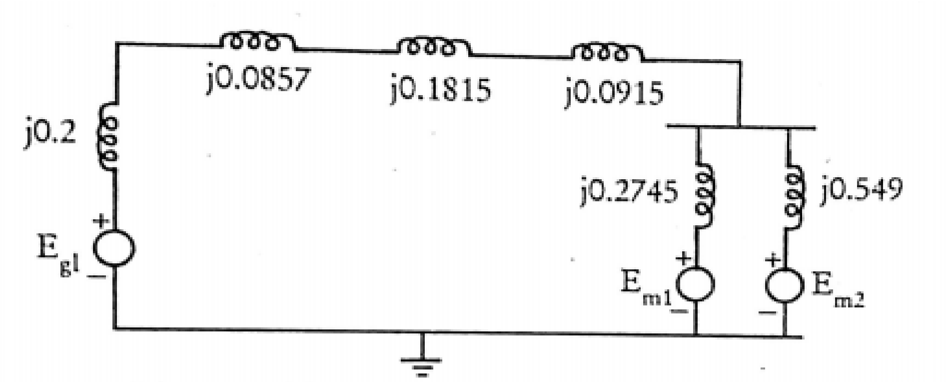


Figure :1.14

5.Draw the reactance diagram for the power system shown in fig. Neglect resistance and use a base of 100 MVA, 220KV in 50Ω line. The ratings of the generator, motor and transformer are given below.

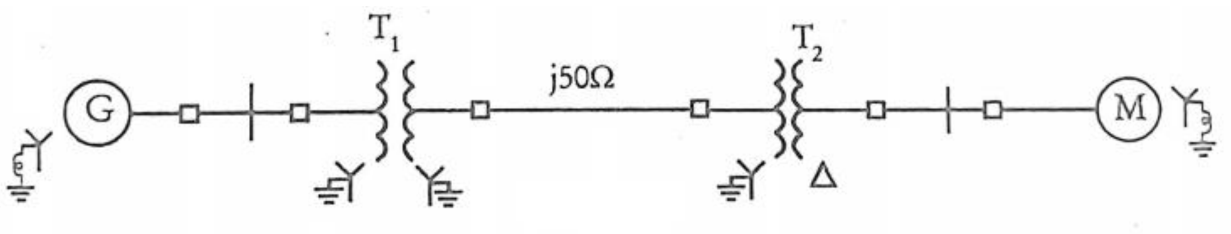


Figure:1.15

Generator: 40MVA,25KV,X''=20%

Synchronous motor:50MVA,11KV,X''=30%

Y-Y Transformer : 40MVA,33/220KV,X=15%

Y-Δ Transformer : 30MVA,11/220KV(Δ/Y), X=15%

Solution:

$MVA_{b,new}$=100MVA

$KV_{b,new}$=220KV

Reactance of Transmission line (TL)

$$Z_b = \frac{(Kv_b, new)^2}{MVA_b} = \frac{220^2}{100} = 484\Omega$$

$$X_{TL,p.u} = \frac{50}{484} = 0.1033 p.u$$

Reactance of Transformer T_1

Base KV on LT side of T_1 = Base KV on HT side of T_1* (LT voltage rating of T_1/ HT voltage rating of T_1)

= 220 *(33/220) = 33KV

$Kv_{b,new}$=33KV

$$X_{T2,pu,new} = X_{T2\,pu,old} * \left(\frac{KV_{bT2,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bT2,old}}\right)$$

$$= 0.15 * \left(\frac{11}{11}\right)^2 * \left(\frac{100}{30}\right) = 0.5 p.u$$

Reactance of Synchronous motor

$$X_{M,pu,new} = X_{M\,pu,old} * \left(\frac{KV_{bM,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bM,old}}\right)$$

$$= 0.3 * \left(\frac{11}{11}\right)^2 * \left(\frac{100}{50}\right) = 0.6 p.u$$

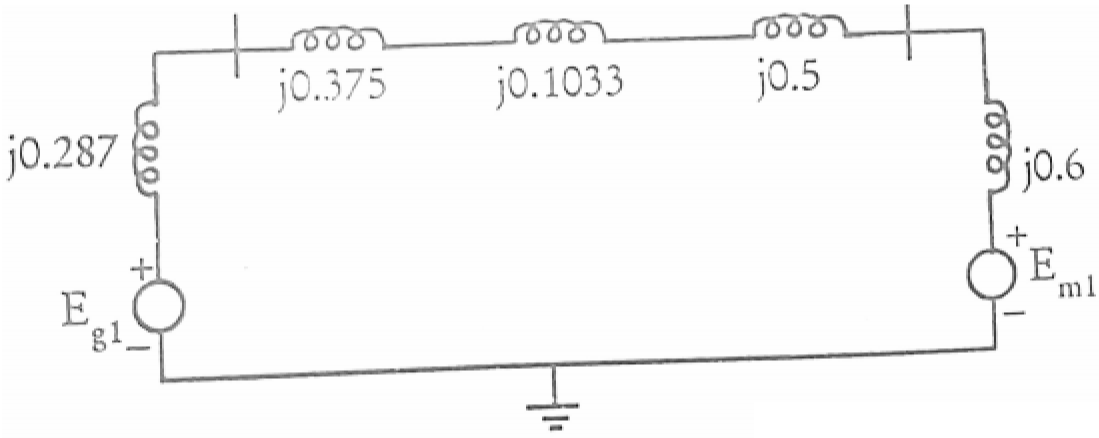


Figure: 1.16

6. A 15MVA, 8.5KV, 3- Phase generator has a substransient reactance of 20%. It is connected through a Δ - Y transformer to a high voltage transmission line having a total series reactance of 70Ω.The load end of the line has Y-Y step down transformer. Both transformer banks are composed of single Phase transformers connected for 3-Phase operation. Each of three transformers composing three phase bank is rated 6667KVA, 10/100KV, with a reactance of 10%. The load represented as impedance, is drawing 10MVA at 12.5KV and 0.8pf lagging. Draw the single line diagram of the power network. Choose a base of 10MVA,12.5KV in the load circuit and determine the reactance diagram. Determine also the voltage at the terminals of the generator.

Solution:

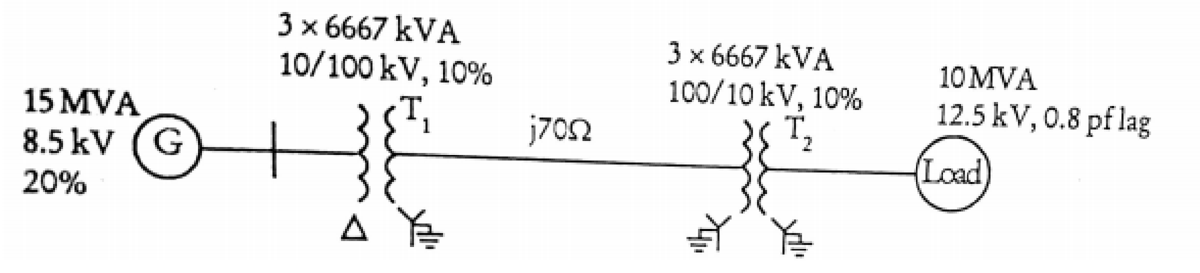


Figure :1.17

$MVA_{b,new}$=10MVA

$KV_{b,new}$=12.5KV

Reactance of Transformer T2

Voltage ratio of line voltage of transformer

T2=(100*√3KV/10*√3KV) = (173.2KV/17.32KV)

3 – Phase KVA rating of Transformer T2 = 3*6667=20,000KVA=20MVA

∴ KVb,old=17.32KV (LT side)

MVAb,old =20MVA

$$X_{T2,\,pu,new} = X_{T2\,pu,old} * \left(\frac{KV_{bT2,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bT2,old}}\right)$$

$$= 0.1 * \left(\frac{17.32}{12.5}\right)^2 * \left(\frac{10}{20}\right) = 0.096\,p.u$$

Reactance of Transmission line

Base KV on HT side of T_2 = Base KV on LT side of T_2* (HT voltage rating of T_2/ LT voltage rating of T_2)

= 12.5 *(173.2/17.32) = 125KV

$Kv_{b,new}$=125KV

$$Z_b = \frac{(Kv_b, new)^2}{MVA_b} = \frac{125^2}{10} = 1562.5\Omega$$

$$X_{TL,p.u} = \frac{70}{1562.5} = 0.0448\,p.u$$

Reactance of Transformer T1

Voltage ratio of line voltage of transformer

T1=(10KV/100*√3KV) = (10KV/173.2KV)

3 – Phase KVA rating of Transformer T2 =

3*6667=20,000KVA=20MVA

$\therefore$ KVb,old=173.2KV (HT side)

MVAb,old =20MVA

$$X_{T1,\,pu,new} = X_{T1\,pu,old} * \left(\frac{KV_{bT1,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bT1,old}}\right)$$

$$= 0.1 * \left(\frac{173.2}{125}\right)^2 * \left(\frac{10}{20}\right) = 0.096\,p.u$$

Reactance of Generator

Base KV on LT side of T_1 = Base KV on HT side of T_1* (LT voltage rating of T_1/ HT voltage rating of T_1)

= 125 *(10/173.2) = 7.217KV

$Kv_{b,new}$=7.217KV

$$X_{G,\,pu,new} = X_{G\,pu,old} * \left(\frac{KV_{bG,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{bG,old}}\right)$$

$$= 0.2 * \left(\frac{8.5}{7.217}\right)^2 * \left(\frac{10}{15}\right) = 0.185\,p.u$$

Load

This can be represented as constant current load

p.f of load = 0.8lag

$\therefore$ p.f angle = $-\cos^{-1}0.8 = -36.87°$

Complex load power = $10\angle -36.87°$ MVA

p.u value of load (Power) = Actual load MVA/ Base value of MVA

= $10\angle -36.87°$ /10 = $1\angle -36.87°$ p.u

p.u value of load voltage = Actual load voltage/Base voltage

= 12.5KV/12.5KV = 1.0 p.u

Let I = Load current in p.u

V=Load voltage in p.u

V*I=p.u value of load

$\therefore I = 1\angle -36.87^\circ / 1.0 = 1\angle -36.87^\circ$

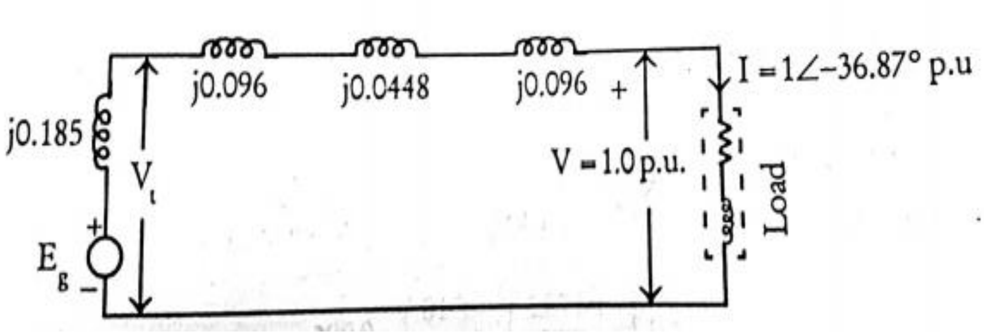


Figure:1.18

Terminal voltage of the Generator

$V_t = V + I(j0.096 + j0.096 + j0.0448)$

$= 1.0 + 1\angle -36.87^\circ * 0.2368\angle 90^\circ$

$= 1.0 + 0.2368 + j0.1894$

$= 1.1421 + j0.1894$

$= 1.1577\angle 9.4^\circ$ p.u

Actual value of generator terminal voltage

= p.u value of voltage * Base KV on LT side of Transformer T_1

$= 1.1577\angle 9.4^\circ * 7.217 = 8.355\angle 9.4^\circ$

BUS ADMITTANCE MATRIX

Bus

- **The meeting point of various components in a power system is called a bus**
- **The bus is a conductor made of copper or aluminum having negligible resistance**

- **The buses are considered as points of constant voltage in a power system**

Bus admittance matrix

- **The matrix consisting of the self and mutual admittance of the network of a power system is called Bus admittance matrix**
- **It is given by the admittance matrix Y in the node basis matrix equation of a power system. Denoted as Y_{bus}**
- **It is a symmetrical matrix**

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

- **The diagonal elements of bus admittance matrix are called self admittances of the buses**
- **Off – diagonal elements are called mutual admittances of the buses**

Formula for determining Y_{bus} after eliminating the last row and Column

$$Y_{jk,new} = Y_{jk,old} - \frac{Y_{jn} Y_{nk}}{Y_{nn}}$$

j=1,2,3…(n-1) ; K=1,2,3……(n-1) ; n= last row and column to be eliminated

Direct Inspection method

The Guidelines to form bus admittance matrix by Indirect Inspection method are:

1. **The diagonal element Y_{jj} is given by sum of all the admittances connected to node j**
2. **The off diagonal elements Y_{jk} is given by negative of the sum of all the admittances connected between node j and node k**

6. **For the network shown in Fig, form the bus admittance matrix. Determine the reduced admittance by eliminating node 4. The values are marked in p.u**

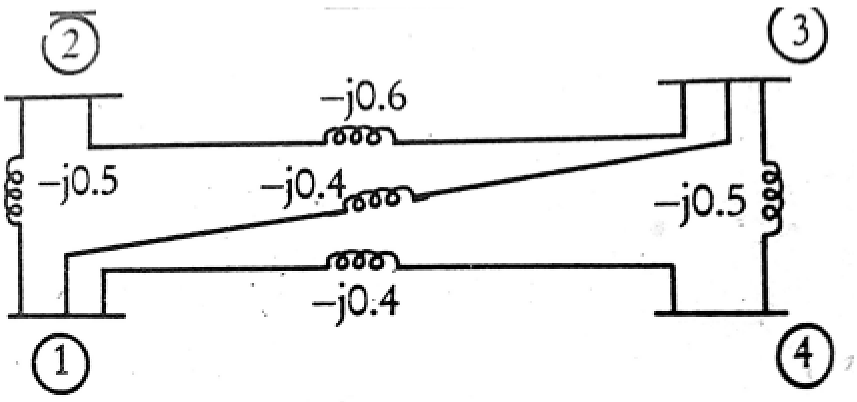


Figure: 1.19

Solution: Direct Inspection Method

The Y_{bus} matrix of the network is,

$$Y_{bus} = \begin{bmatrix} -(j0.5+j0.4+j0.4) & j0.5 & j0.4 & j0.4 \\ j0.5 & -(j0.5+j0.6) & j0.6 & 0 \\ j0.4 & j0.6 & -(j0.6+j0.5+j0.4) & j0.5 \\ j0.4 & 0 & j0.5 & -(j0.5+j0.4) \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

The elements of new bus admittance matrix after eliminating the 4th row and 4th column is given by

$$Y_{jk,new} = Y_{jk,old} - \frac{Y_{jn}Y_{nk}}{Y_{nn}}$$

n =4 ; j=1,2,3 ; K=1,2,3

The bus admittance matrix is symmetrical, ∴ $Y_{kj,\,new} = Y_{jK,new}$

$$Y_{11,new} = Y_{11,old} - \frac{Y_{14}Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4)(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12,new} = Y_{12,old} - \frac{Y_{14}Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4*0)}{-j0.9} = j0.5$$

$$Y_{13,new} = Y_{13,old} - \frac{Y_{14}Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4)(j0.5)}{-j0.9} = j0.622$$

$$Y_{21,new} = Y_{12,new} = j0.5$$

$$Y_{22,new} = Y_{22,old} - \frac{Y_{24}Y_{42}}{Y_{44}} = -j1.1 - \frac{(0)(0)}{-j0.9} = -j1.1$$

$$Y_{23,new} = Y_{23,old} - \frac{Y_{24}Y_{43}}{Y_{44}} = j0.6 - \frac{(0)(j0.5)}{-j0.9} = j0.6$$

$$Y_{31,new} = Y_{13,new} = j0.622$$

$$Y_{32,new} = Y_{23,new} = j0.6$$

$$Y_{33,new} = Y_{33,old} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5)(j0.5)}{-j0.9} = -j1.222$$

The reduced admittance matrix after eliminating 4th row is

$$Y_{bus} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.222 \end{bmatrix}$$

8. Determine the bus admittance matrix of the system whose reactance diagram is shown in fig. the currents and admittances are given in p.u. Determine the reduced bus admittance matrix after eliminating node-3

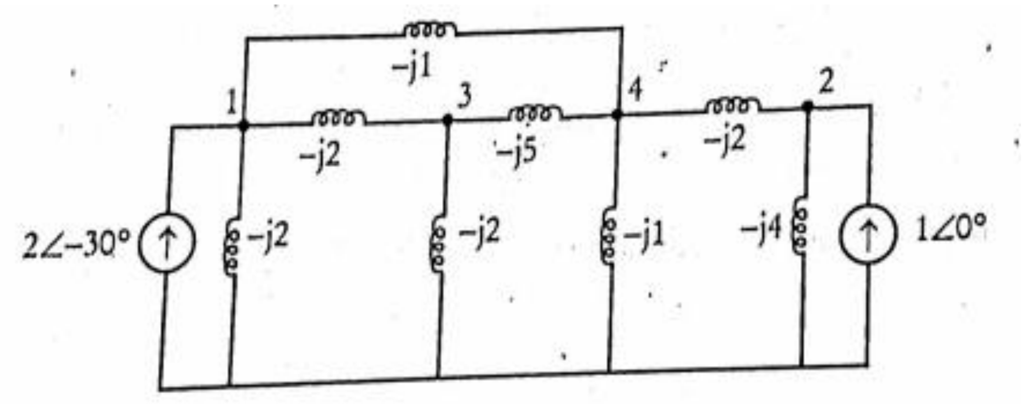


Figure:1.20

Solution:

$$Y_{bus}=\begin{bmatrix}-(j2+j2+j1) & 0 & j2 & j1\\ 0 & -(j2+j4) & 0 & j2\\ j2 & 0 & -(j2+j2+j5) & j5\\ j1 & j2 & j5 & -(j1+j5+j2+j1)\end{bmatrix}$$

$$Y_{bus}=\begin{bmatrix}-j5 & 0 & j2 & j1\\ 0 & -j6 & 0 & j2\\ j2 & 0 & -j9 & j5\\ j1 & j2 & j5 & -j9\end{bmatrix}$$

For eliminating node 3, the bus admittance matrix is re arranged by interchanging row 3 and then row 4 and then interchanging column 3 & column 4.

After interchanging row 3 & row 4 of Y_{bus} matrix,

$$Y_{bus}=\begin{bmatrix}-j5 & 0 & j2 & j1\\ 0 & -j6 & 0 & j2\\ j1 & j2 & j5 & -j9\\ j2 & 0 & -j9 & j5\end{bmatrix}$$

After interchanging column 3 & column 4 of Y_{bus} matrix,

$$Y_{bus} = \begin{bmatrix} -j5 & 0 & j1 & j2 \\ 0 & -j6 & j2 & 0 \\ j1 & j2 & -j9 & j5 \\ j2 & 0 & j5 & -j9 \end{bmatrix}$$

$$Y_{jk,new} = Y_{jk,old} - \frac{Y_{jn}Y_{nk}}{Y_{nn}}$$

$$Y_{11,new} = Y_{11,old} - \frac{Y_{14}Y_{41}}{Y_{44}} = -j5 - \frac{(j2)(j2)}{-j9} = -j4.556$$

$$Y_{12,new} = Y_{12,old} - \frac{Y_{14}Y_{42}}{Y_{44}} = 0 - \frac{(j2*0)}{-j9} = 0$$

$$Y_{13,new} = Y_{13,old} - \frac{Y_{14}Y_{43}}{Y_{44}} = j1 - \frac{(j2)(j5)}{-j9} = j2.111$$

$$Y_{21,new} = Y_{12,new} = 0$$

$$Y_{22,new} = Y_{22,old} - \frac{Y_{24}Y_{42}}{Y_{44}} = -j6 - \frac{(0)(0)}{-j9} = -j6$$

$$Y_{23,new} = Y_{23,old} - \frac{Y_{24}Y_{43}}{Y_{44}} = j2 - \frac{(0)(j5)}{-j9} = j2$$

$$Y_{31,new} = Y_{13,new} = j2.111$$

$$Y_{32,new} = Y_{23,new} = j2$$

$$Y_{33,new} = Y_{33,old} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j9 - \frac{(j5)(j5)}{-j9} = -j6.222$$

The reduced admittance matrix after eliminating bus 3 is

$$Y_{bus} = \begin{bmatrix} -j4.556 & 0 & j2.111 \\ 0 & -j6 & j2 \\ j2.111 & j2 & -j6.222 \end{bmatrix}$$

9. For the given 5-bus system form the admittance matrix by direct inspection method.

Line	Impedance, Z (ohms)	Half Line Charging Admittance, L (mho)
1-2	0.01+j0.05	-j0.02
1-4	0.07+j0.02	-j0.03
2-3	0.05+j0.11	-j0.025
2-4	0.04+j0.20	-j0.12
1-5	0.06+j0.14	-j0.01
3-5	0.02+j0.05	-j0.02
4-5	0.06+j0.14	-j0.025

Table:1.2

Solution:

$Y_{bus}(1,1) = 1/(0.01+j0.05) + 1/(0.07+j0.02) + 1/(0.06+j0.14) – j0.01 –$

$j0.02 – j0.03$

$= 1/(0.051\angle 1.37)+1/(0.073 \angle 0.28) +1/(0.152 \angle 1.17-$

$j0.01 – j0.02 – j0.03$

$= 19.61 \angle -1.37+13.69 \angle -0.28+6.58 \angle -1.17 – j0.01 –j0.02 –J0.03$

$Y_{bus}(1,1)=3.91-j19.22+13.16-j3.78+2.57-j6.06-j0.01-j0.02-j0.03$

$= 19.64-j29.11$

$Y_{bus}(2,2) = 1/(0.01+j0.05) + 1/(0.05+j0.11) + 1/(0.04+j0.2) – j0.02 – j0.025 – j0.012$

$= 8.23 + j31.6$

$Y_{bus}(3,3) = 1/(0.05+j0.11) + 1/(0.02+j0.05) – j0.02 – j0.025$

$= 10.31-j24.82$

$Y_{bus}(4,4) = 1/(0.04+j0.2) + 1/(0.07+j0.02) + 1/(0.06+j0.14) –$

j0.025 – j0.012 – j0.03

= 16.76 – j30.12

$Y_{bus}(5,5)$ = 1/(0.06+j0.14) + 1/(0.02+j0.05) + 1/(0.06+j0.14) –

j0.025 – j0.02 – j0.01

= 12.05 – j29.35

$Y_{bus}(1,2)$ = $Y_{bus}(2,1)$ = -1/(0.01+j0.05) = -3.85 + j19.2

$Y_{bus}(1,3)$ = $Y_{bus}(3,1)$ = 0

$Y_{bus}(1,4)$ = $Y_{bus}(4,1)$ = -1/(0.07+j0.02) = -13.21+ j3.77

$Y_{bus}(1,5)$ = $Y_{bus}(5,1)$ = -1/(0.06+j0.14) = -2.59 + j6.03

$Y_{bus}(2,3)$ = $Y_{bus}(3,2)$ = -1/(0.05+j0.11) = -3.42+j7.73

$Y_{bus}(2,4)$ = $Y_{bus}(4,2)$ = -1/(0.04+j0.2) = -0.96+j4.81

$Y_{bus}(2,5)$ = $Y_{bus}(5,2)$ = 0

$Y_{bus}(3,4)$ = $Y_{bus}(4,3)$ = 0

$Y_{bus}(3,5)$ = $Y_{bus}(5,3)$ = -1/(0.02+j0.05) = -6.9+j17.24

$Y_{bus}(4,5)$ = $Y_{bus}(5,4)$ = -1/(0.06+j0.14) = -2.59+j6.03

$$Y_{bus} = \begin{bmatrix} 19.64 - j29.11 & -3.85 + j19.2 & 0 & -13.21 + j3.77 & -2.59 + j6.03 \\ -3.85 + j19.2 & 8.23 + j31.6 & -3.42 + j7.73 & -0.96 + j4.81 & 0 \\ 0 & -3.42 + j7.73 & 10.31 - j24.82 & 0 & -6.9 + j17.24 \\ -13.21 + j3.77 & -0.96 + j4.81 & 0 & 16.76 - j30.12 & -2.59 + j6.03 \\ -2.59 + j6.03 & 0 & -6.9 + j17.24 & -2.59 + j6.03 & 12.05 - j29.35 \end{bmatrix}$$

Analytical method or Singular transformation Method

$Y_{bus} = [A]^T [y] [A]$

A – Incidence Matrix

y – primitive Ybus

10. For the given system form the admittance matrix by analytical method

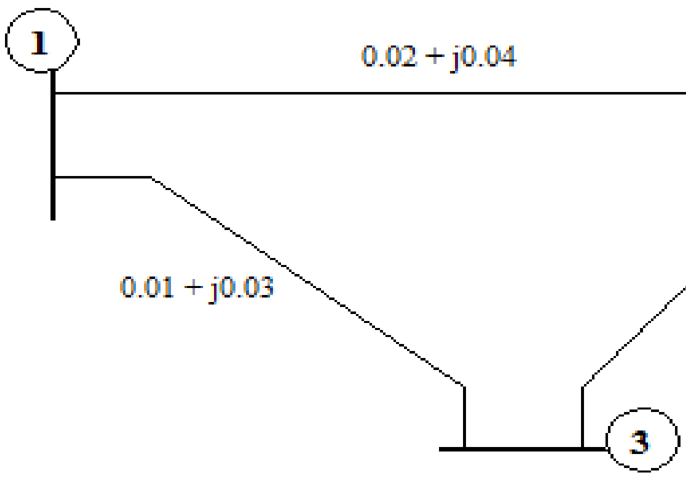


Figure: 1.21

Solution:

$$Z = \begin{bmatrix} 0.02 + j0.04 & 0 & 0 \\ 0 & 0.0125 + j0.025 & 0 \\ 0 & 0 & 0.01 + j0.03 \end{bmatrix}$$

Columns are nodes and rows are elements (lines)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$Y_{bus} = A^T yA$$

$$Y_{bus} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{(0.02 + j0.04)} & 0 & 0 \\ 0 & \frac{1}{(0.0125 + j0.025)} & 0 \\ 0 & 0 & \frac{1}{(0.01 + j0.03)} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

11. For the given system form the admittance matrix by analytical method

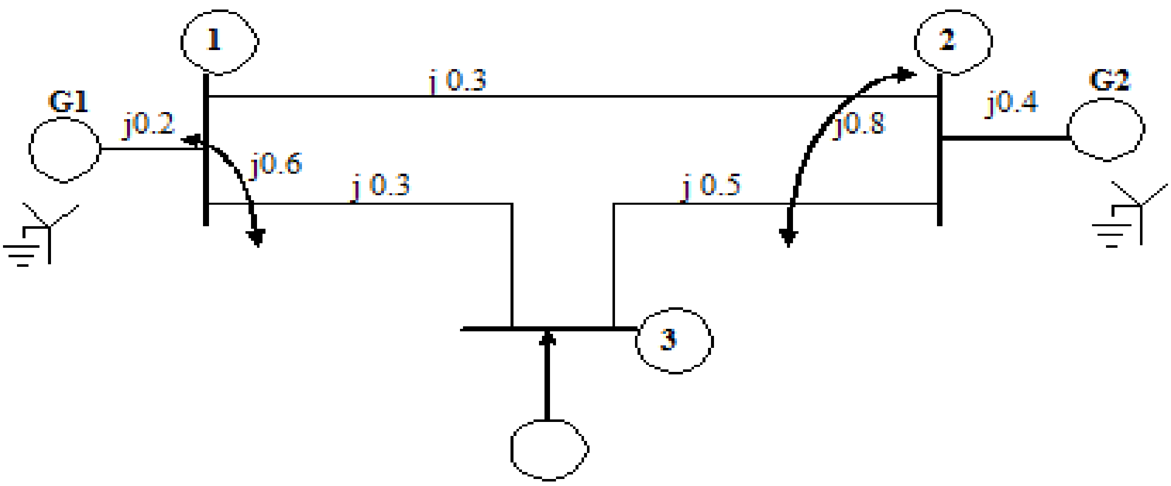


Figure:1.22

Solution:

Node 1 to Node 0 – 1st element

Node 1 to Node 2 - 2nd element

Node 1 to Node 3 – 3rd element

Node 2 to Node 3 – 4th element

Node 2 to Node 0 – 5th element

$$ElementalNodeIncidcenceMatrix(\hat{A}) = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$Incidcence\,Matrix(A) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} j0.2 & 0 & j0.6 & 0 & 0 \\ 0 & j0.3 & 0 & j0.8 & 0 \\ j0.6 & 0 & j0.3 & 0 & 0 \\ 0 & j0.8 & 0 & j0.5 & 0 \\ 0 & 0 & 0 & 0 & j0.4 \end{bmatrix}$$

$$Y_{bus} = A^T yA$$

$$Y_{bus} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{j0.2} & 0 & \frac{1}{j0.6} & 0 & 0 \\ 0 & \frac{1}{j0.3} & 0 & \frac{1}{j0.8} & 0 \\ \frac{1}{j0.6} & 0 & \frac{1}{j0.3} & 0 & 0 \\ 0 & \frac{1}{j0.8} & 0 & \frac{1}{j0.5} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{j0.4} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j15 & j2.083 & j6.25 \\ j2.083 & -j5.33 & j0.75 \\ j6.25 & j0.75 & -j5.33 \end{bmatrix}$$

Need of System analysis in planning and operation of power System

Load Flow Studies:

- **It is a steady state behavior of the power system under normal conditions & its dynamic behavior under small scale disturbances**
- **In Load flow studies, the main concentration is on transmission with generators & loads modeled by the complex powers. The transmission system may be a primary or sub transmission system**
- **The transmission system is to be designed in such a manner that power system operation is reliable and economical & no difficulties arise during its operation**
- **But these two objectives are conflicting, so more concentration is needed in load flow studies**
- **Now power system is highly complicated consisting of hundreds of buses & transmission lines**
- **So load flow involves extensive calculations**

Short Circuit Analysis:

- **It is the abnormal system behavior under conditions of fault during operation**
- **In a large interconnected power system, heavy currents flowing during short circuits must be interrupted through a circuit breaker.**
- **So maximum current that circuit breaker can withstand momentarily has to be determined**
- **For selection of circuit breakers, the initial current that flows on occurrence of a short circuit & the transient current that flows at the time of circuit interruption has to be calculated from short circuit studies**

Stability Studies:

- **The stability of an interconnected power system is its ability to return to its normal or stable operation after having been subjected to some form of disturbances**
- **Stability is considered as an essential part of power system planning for a long time**
- **During a fault, electrical power from nearby generators is reduced drastically, while power from remote generators is scarcely affected**
- **In some cases, the system will be stable even with a sustained fault, whereas other system will be stable only if the fault is cleared rapidly**
- **Whether the system is stable on occurrence of a fault depends not only on the system itself, but also on the type of the fault, location of the fault, rapidity on clearing the fault and method used in clearing the fault**
- **Thus for a reliable, economical operation of power system, the need of system analysis like load flow analysis, short circuit analysis, stability analysis is essential to have effective planning & operation of power system**

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UNIT – II – Power System Analysis – SEE1302

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POWER FLOW ANALYSIS

Problem definition – bus classification – derivation of power flow equation – solution by Gauss seidel and Newton Raphson methods by polar form – PV bus adjustments for both methods – computation of slack bus power, line flow and transmission lines

Power flow study or load flow study

- **The study of various methods of solution to power system networks is referred to as load flow study**
- **The solution provides the voltages at various buses, power flowing in various lines and line losses**

Information's obtained from a load flow study

- **Magnitude and phase of bus voltages, real and reactive power flowing in each line and the line losses**
- **Load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied**

Need for load flow study

- **It is essential to decide the best operation of existing system and for planning the future expansion of the system**
- **It is also essential for designing a new power system**

Work involved in a load flow study or How a load flow study is performed?

i. **Representation of the system by single line diagram**

ii. **Determining the impedance diagram using the informations in single line diagram**

iii. **Formulation of network equations**

iv. **Solution of network equations**

Quantities associated with each bus in a system

i. **Real Power (P)**

- ii. **Reactive Power (Q)**
- iii. **Magnitude of Voltage (|V|)**
- iv. **Phase angle of voltage (δ)**

Classification of buses

- i. **Load bus or PQ bus (P and Q are specified)**
- ii. **Generator bus or voltage controlled bus or PV bus (P and V Specified)**
- iii. **Slack bus or swing bus or reference bus (|V| and δ are specified)**

PQ bus

- **A bus is called PQ bus or load bus when real and reactive components of power are specified for the bus.**
- **In a load bus the voltage is allowed to vary within permissible limits**

PV bus or Voltage Controlled bus or Generator bus

- **A bus is called voltage controlled bus if the magnitude of voltage |V| and real power (P) are specified for it.**
- **In a voltage controlled bus the magnitude of the voltage is not allowed to change**

Slack bus

- **A bus is called swing bus (or Slack bus) when the magnitude and phase of bus voltage are specified for it**
- **The swing bus is the reference bus for load flow solution and it is required for accounting for line losses.**
- **Usually one of the generator bus is selected as the swing bus**

Need of Swing bus

- **The slack bus is needed to account for transmission line losses**
- **In a power system the total power generated will be equal to sum of power consumed by loads and losses**

- **In a power system only the generated power and load power are specified for buses**
- **The slack bus is assumed to generate the power required for losses**
- **Since the losses are unknown the real and reactive power are not specified for slack bus**
- **They are estimated through the solution of load flow equations**

Formulation of Load flow equations using Y_{bus} matrix

- **The load flow equations can be formed using either the mesh or node basis equations of power system**
- **From the point of computer time and memory, the nodal admittance formulation using the nodal voltages as the independent variables is the most economic**

$$Y_{bus} * V = I$$

Where,

Y_{bus} **- Bus admittance matrix of order (nxn)**

V - Bus (node) voltage matrix of order (nx1)

I - Sources current matrix of order (nx1)

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2p} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{p1} & Y_{p2} & \cdots & Y_{pp} & \cdots & Y_{pn} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{np} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_n \end{bmatrix}$$

I_p = Current injected to bus p

V_p=Voltage at bus p

$$I_p = Y_{p1}V_1 + Y_{p2}V_2 + \cdots + Y_{pp}V_p + \cdots Y_{pn}V_n$$

$$\therefore I_p = \sum_{q=1}^{p-1} Y_{pq}V_q + Y_{pp}V_p + \sum_{q=p+1}^{n} Y_{pq}V_q$$

S_p=Complex power of bus p

P_p=Real power of bus p

Q_p=Reactive power of bus p

$$S_p = P_p + jQ_p$$

$$S_p = V_p I_p^*$$

$$\therefore V_p I_p^* = P_p + jQ_p$$

The load flow problem can be handled more conveniently by use of I_p rather than I_p*

$$(V_p I_p^*)^* = (P_p + jQ_p)^*$$

$$V_p^* I_p = P_p - jQ_p$$

$$\therefore I_p = \frac{P_p - jQ_p}{V_p^*}$$

$$Y_{p1}V_1 + Y_{p2}V_2 + \cdots + Y_{pp}V_p + \cdots Y_{pn}V_n = \frac{P_p - jQ_p}{V_p^*}$$

- **Iterative methods are used to solve load flow problems**
- **The reason to use iterative methods is the load (or power) flow equations are nonlinear algebraic equations and explicit solution is not possible**

Iterative Methods

i. **Gauss seidel (G-S) method**

ii. **Newton Raphson (N-R) method**

Operating constraints imposed in the load flow studies

i. **Reactive power limits for generator buses**

ii. **Allowable change in magnitude of voltage for load buses**

Flat Voltage Start

- **In iterative methods of load flow solution, the initial voltages of all buses except slack bus are assumed as 1+j0 p.u**

Gauss Seidel Method

$$Y_{p1}V_1 + Y_{p2}V_2 + \cdots + Y_{pp}V_p + \cdots Y_{pn}V_n = \frac{P_p - jQ_p}{V_p^*}$$

$$\sum_{q=1}^{p-1} Y_{pq}V_q + Y_{pp}V_p + \sum_{q=p+1}^{n} Y_{pq}V_q = \frac{P_p - jQ_p}{V_p^*}$$

$$Y_{pp}V_p = \left[\frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq}V_q - \sum_{q=p+1}^{n} Y_{pq}V_q\right]$$

$$V_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_p^*} - \sum_{q=1}^{p-1} Y_{pq}V_q - \sum_{q=p+1}^{n} Y_{pq}V_q\right]$$

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right] \quad \text{----- (1)}$$

V_i^k – k^{th} iteration value of bus voltage V_i

V_i^{k+1} – $(k+1)^{th}$ iteration value of bus voltage V_i

$$\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^{p-1} Y_{pq}V_q + Y_{pp}V_p + \sum_{q=p+1}^{n} Y_{pq}V_q$$

$$\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^{p-1} Y_{pq}V_q + \sum_{q=p}^{n} Y_{pq}V_q$$

$$\therefore P_p - jQ_p = V_p^*\left[\sum_{q=1}^{p-1} Y_{pq}V_q + \sum_{q=p}^{n} Y_{pq}V_q\right]$$

$$\therefore P_p^{k+1} - jQ_p^{k+1} = (V_p^k)^*\left[\sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} + \sum_{q=p}^{n} Y_{pq}V_q^k\right]$$

Reactive Power of bus p during (k+1)th iteration

$$Q_p^{k+1} = (-1) * \operatorname{Im}\left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\} \qquad \text{----- (2)}$$

Computation of Slack bus power and line flows

$$P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q^k$$

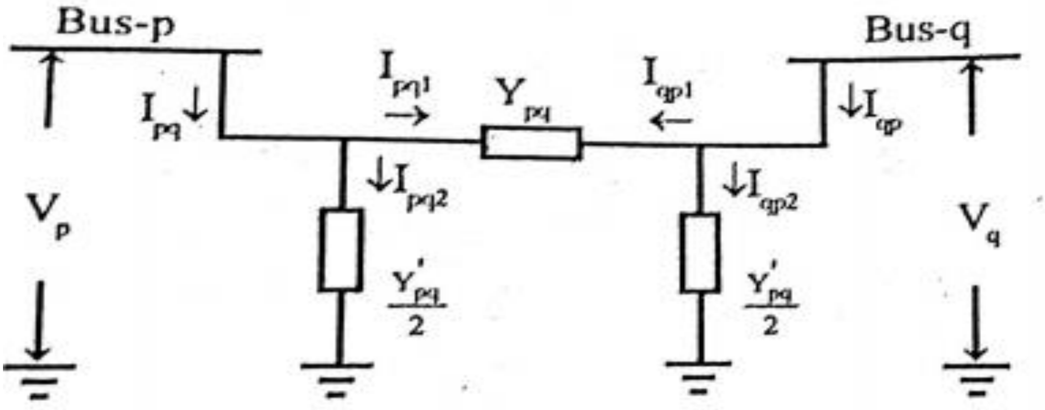


Figure: 2.1

Y_{pq} – series admittances

$Y'_{pq}/2$- Shunt admittances

$$I_{pq} = I_{pq1} + I_{pq2} = (V_p - V_q)Y_{pq} + V_p \frac{Y'_{pq}}{2}$$

$$I_{pq} = I_{pq1} + I_{pq2} = (V_p - V_q)Y_{pq} + V_p \frac{Y'_{pq}}{2}$$

Complex power Injected by bus p in line pq

$$S_{pq} = P_{pq} - jQ_{pq} = V_p^* I_{pq} = V_p^* \left[(V_p - V_q)Y_{pq} + V_p \frac{Y'_{pq}}{2} \right]$$

$$I_{qp} = I_{qp1} + I_{qp2} = (V_q - V_p)Y_{pq} + V_q \frac{Y_{pq}^{'}}{2}$$

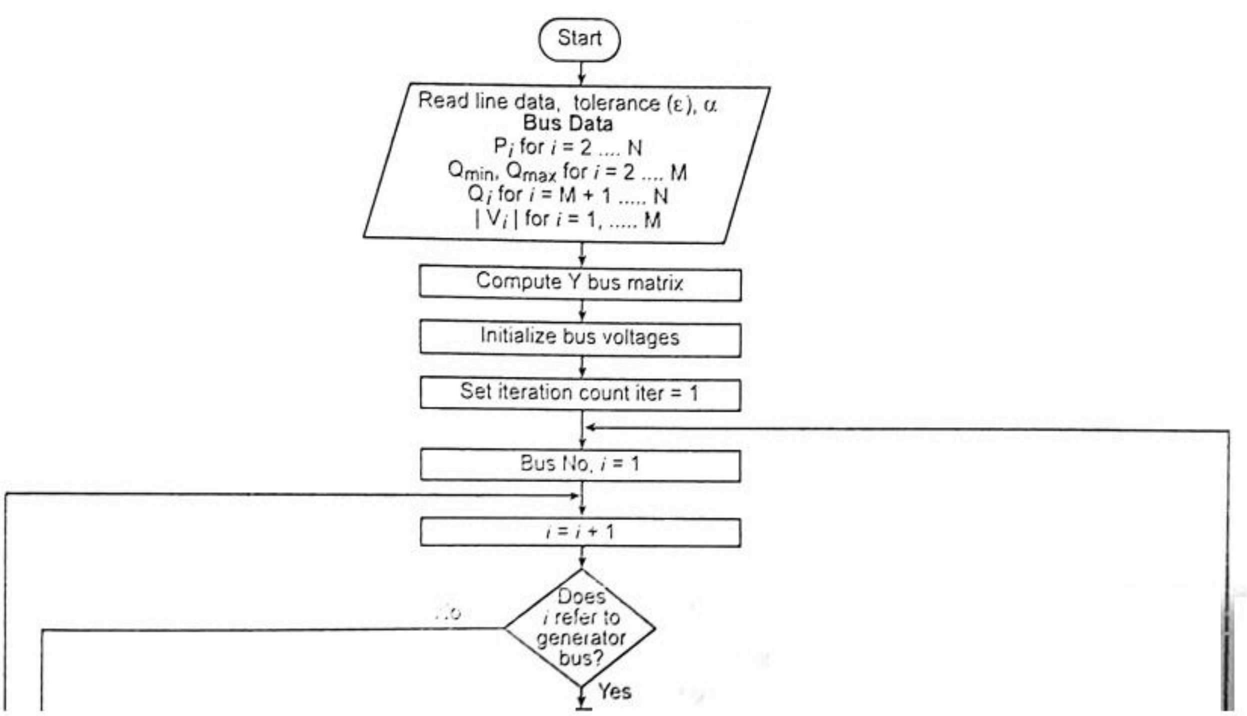
Complex power Injected by bus q in line pq

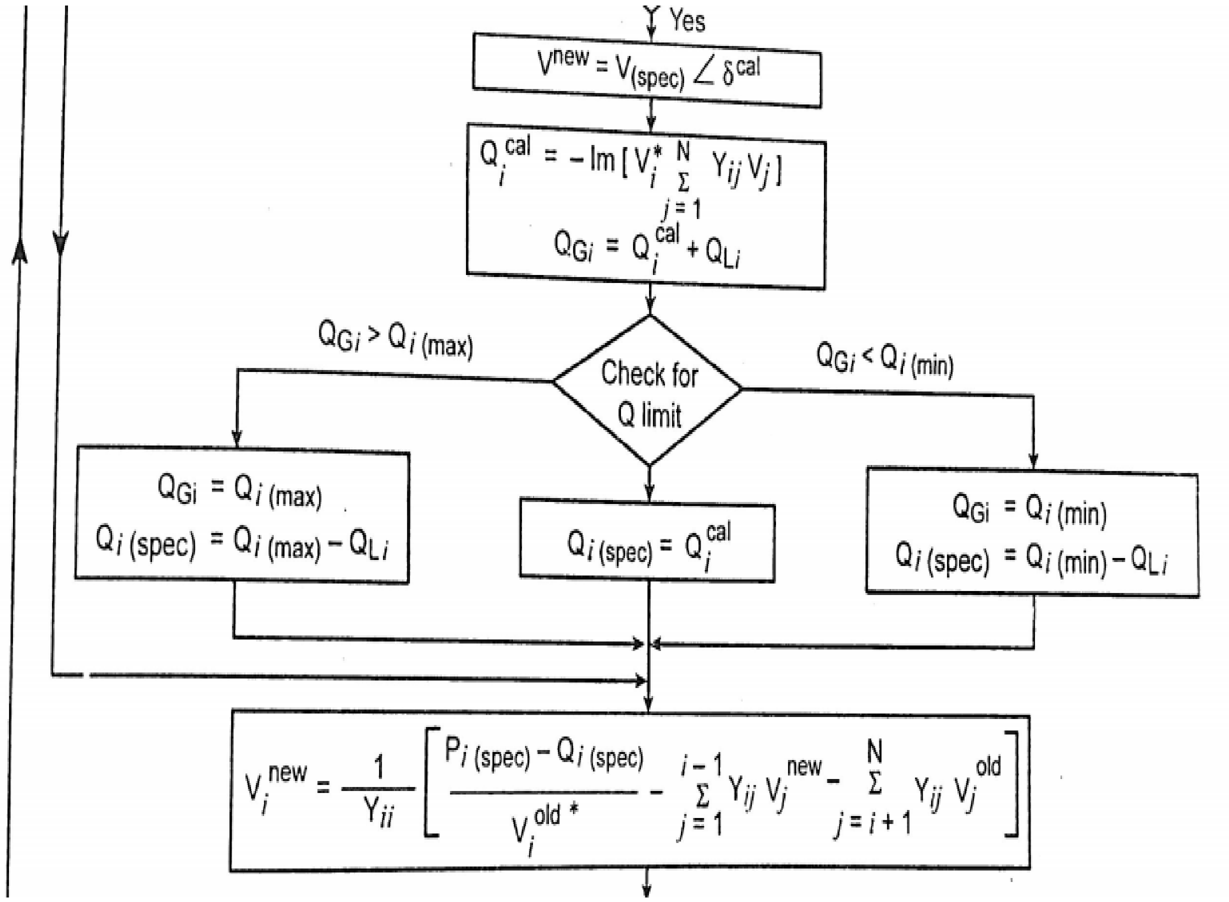
$$S_{qp} = P_{qp} - jQ_{qp} = V_q^* I_{qp} = V_q^* \left[(V_q - V_p)Y_{pq} + V_p \frac{Y_{pq}^{'}}{2} \right]$$

Power loss in the transmission line – pq

$$S_{pq,loss} = S_{pq} + S_{qp}$$

Flow chart of Load Flow Analysis using Gauss Siedel Method




Yes
$V^{new} = V_{(spec)} \angle \delta^{cal}$
$Q_i^{cal} = -\mathrm{Im}\left[V_i^* \sum_{j=1}^{N} Y_{ij} V_j\right]$
$Q_{Gi} = Q_i^{cal} + Q_{Li}$
$Q_{Gi} > Q_{i\,(max)}$
Check for Q limit
$Q_{Gi} < Q_{i\,(min)}$
$Q_{Gi} = Q_{i\,(max)}$
$Q_{i\,(spec)} = Q_{i\,(max)} - Q_{Li}$
$Q_{i\,(spec)} = Q_i^{cal}$
$Q_{Gi} = Q_{i\,(min)}$
$Q_{i\,(spec)} = Q_{i\,(min)} - Q_{Li}$
$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_{i\,(spec)} - Q_{i\,(spec)}}{V_i^{old\,*}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} V_j^{old}\right]$

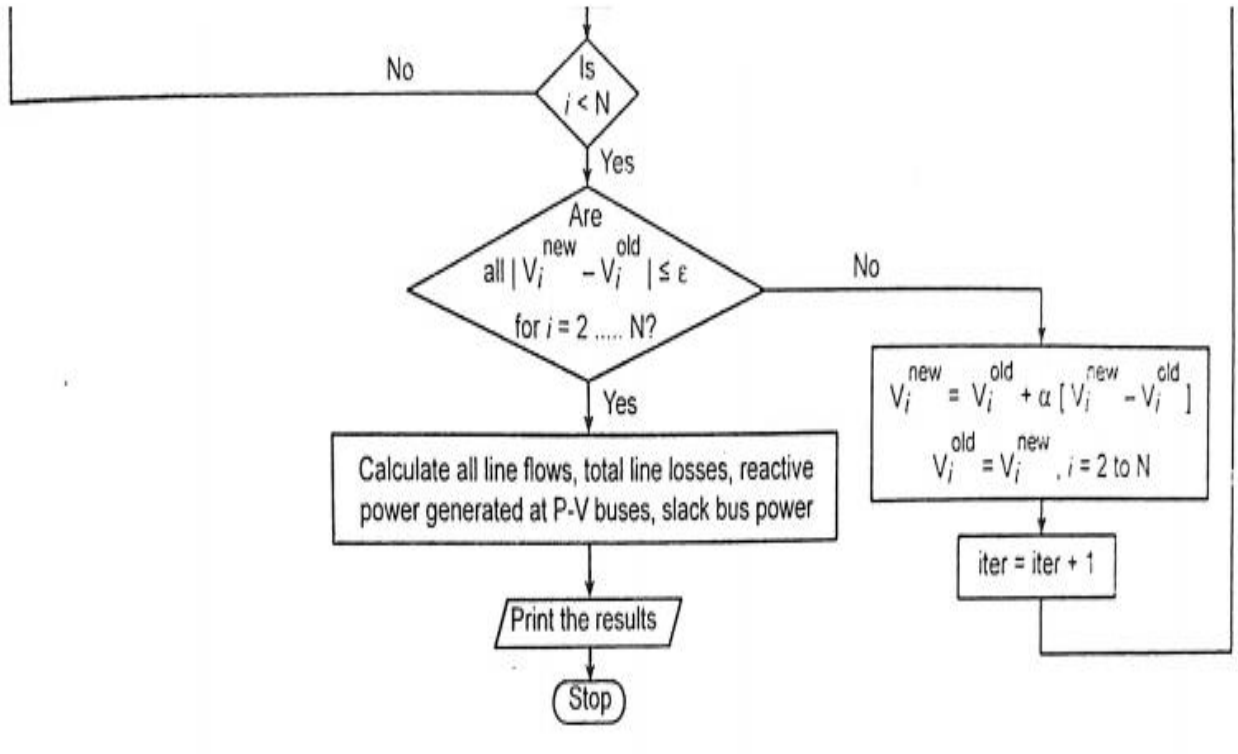


Figure:2.2

Algorithm for load flow solution by Gauss seidel method

Step 1: Form Y – bus matrix

Step 2: Assume $V_i = V_{i(spec)} \angle 0°$ at all generator buses

Step 3: Assume $V_i = 1 \angle 0° = 1 + j0$ at all load buses

Step 4: set iteration count=1 (k =1)

Step 5: Let bus number i=1

Step 6: If 'i' refers to generator bus go to step no.7, otherwise go to step 8

Step7(a): If 'i' refers to the slack bus go to step 9, otherwie go to step 7(b)

Step 7(b) : Compute Q_i using,

$$Q_i^{k+1} = (-1) * \operatorname{Im}\left\{ (V_i^k)^* \left[\sum_{j=1}^{i-1} Y_{ij} V_j^{k+1} + \sum_{j=i}^{n} Y_{ij} V_j^k \right] \right\}$$

$Q_{Gi} = Q_i^{cal} + Q_{Li}$

Check for Q limit violation

If $Q_{i(min)} < Q_{Gi} < Q_{i(max)}$, then $Q_{i(spec)} = Q_i^{cal}$

If $Q_{i(min)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

If $Q_{i(max)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

If Q_{limit} is violated , then treat this bus as P-q bus till convergence is obtained

Step 8: Compute V_i using the equation,

$$V_i^{k+1} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^k)^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{k+1} - \sum_{j=i+1}^{n} Y_{ij} V_j^k \right]$$

Step 9: If i is less than number of buses, increment i by 1 and go to step 6

Step 10: Compare two sucessive iteration values for Vi

If $V_i^{K+1} - V_i^k <$ tolerance, go to step 12

Step 11: Update the new voltage as

$V^{k+1} = V^k + \alpha(V^{K+1} - V^k)$

$V^k = V^{k+1}$

K=K+1; go to step 5

Step 12: Compute relevant quantities:

Slack bus power,

$$S_i = P_i - Q_i = V^* I = V_i^* \sum_{j=1}^{N} Y_{ij} V_j$$

Line flows,

$$S_{ij} = P_{ij} + jQ_{ij}$$

$$= V_i\left[V_i^* - V_j^*\right]Y_{ijseries}^* + |V_i|^2\, Y_{ii}^*$$

$$P_{Loss} = P_{ij} + P_{ji}$$

$$Q_{Loss} = Q_{ij} + Q_{ji}$$

Step 13: Stop the execution

1. **In the system shown in fig, generators are connected to all the four buses, while loads are at buses 2 and 3. The specifications of the buses and line impedances are given in the tables. Assume that all the buses other than slack bus are PQ type. By taking a flat voltage profile, determine the bus voltages at the end of first Gauss seidel iteration**

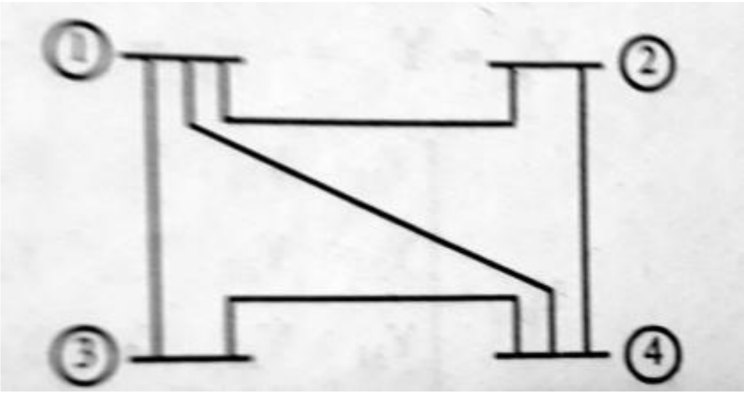


Figure:2.3

Bus code	P	Q	V
1	-	-	$1.05\angle 0°$
2	0.5	-0.2	-
3	-1.0	0.5	-
4	0.3	-0.1	-

Table:2.1

Line	R in p.u	X in p.u
1-2	0.05	0.15
1-3	0.10	0.30
1-4	0.20	0.40
2-4	0.10	0.30
3-4	0.05	0.15

Table: 2.2

$Z_{12}=0.05+j0.15$ p.u $\quad Z_{24}=0.10+j0.30$ p.u

$Z_{13}=0.10+j0.30$ p.u $\quad Z_{34}=0.05+j0.15$ p.u

$Z_{14}=0.20+j0.40$ p.u

$Y_{12}=1/Z_{12} = 1/(0.05+j0.15) = 2-j6 \quad Y_{13}=1/Z_{13}=1/(0.10+j0.30)=1-j3$

$Y_{14} =1/Z_{14}=1/(0.20+j0.40)=1-j2 \quad Y_{24}=1/Z_{24}=1/(0.10+j0.30)=1-j3$

$Y_{34}=1/Z_{34}=1/(0.05+j0.15)=2-j6$

$Y_{11}=Y_{12}+Y_{13}+Y_{14}=2-j6+1-j3+1-j2=4-j11$

$Y_{22}=Y_{12}+Y_{24}=2-j6+1-j3=3-j9$

$Y_{33}=Y_{13}+y_{34}=1-j3+2-j6=3-j9$

$Y_{44}=Y_{14}+Y_{24}+Y_{34}=1-j2+1-j3+2-j6=4-j11$

$Y_{12}=Y_{21}=-Y_{12}=-(2-j6)=-2+j6$

$Y_{13}=Y_{31}=-Y_{13}=-(1-j3)=-1+j3$

$Y_{14}=Y_{41}=-Y_{14}=-(1-j2)=-1+j2$

$Y_{23}=Y_{32}=0$

$Y_{24}=Y_{42}=-Y_{24}=-(1-j3)=-1+j3$

$Y_{34}=Y_{43}=-Y_{34}=-(2-j6)=-2+j6$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 4-j11 & -2+j6 & -1+j3 & -1+j2 \\ -2+j6 & 3-j9 & 0 & -1+j3 \\ -1+j3 & 0 & 3-j9 & -2+j6 \\ -1+j2 & -1+j3 & -2+j6 & 4-j11 \end{bmatrix}$$

$$V_1^0 = V_1^1 = \cdots\cdots = V_1^n = V_1 = 1.05 + j0\, p.u$$

$$V_2^0 = 1 + j0 \qquad V_3^0 = 1 + j0 \qquad V_4^0 = 1 + j0 \qquad K = 0$$

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2^1 = \frac{1}{3-j9}\left[\frac{0.5+j0.2}{1-j0} - (-2+j6)(1.05+j0) - 0*(1+j0) - (-1+j3)(1+j0)\right]$$

$$= \frac{1}{3-j9}[0.5 + j0.2 + 2.1 - j6.3 + 1 - j3]$$

$$= \frac{3.6 - j9.1}{3 - j9} = \frac{9.7862\angle -68.42^\circ}{9.4868\angle -71.57^\circ} = 1.0316\angle 3.15^\circ = 1.0300 + j0.0567\, p.u$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$V_3^1 = \frac{1}{3-j9}\left[\frac{-1-j0.5}{1-j0} - (-1+j3)(1.05+j0) - 0*(1.0300+j0.0567) - (-2+j6)(1+j0)\right]$$

$$= \frac{1}{3-j9}[-1 - j0.5 + 1.05 - j3.15 + 2 - j6]$$

$$= \frac{2.05 - j9.65}{3 - j9} = \frac{9.8653\angle -78.01^\circ}{9.4868\angle -71.57^\circ} = 1.0399\angle -6.44^\circ = 1.0333 - j0.01166\, p.u$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$V_4^1 = \frac{1}{4-j11}\left[\frac{0.3+j0.1}{1-j0} - (-1+j2)(1.05+j0) - (-1+j3)*(1.0300+j0.0567) - (-2+j6)(1.0333-j0.01166)\right]$$

$$= \frac{1}{4-j11}\left[0.3+j0.1+1.05-j2.1-(-1.2001+j3.0333)-(-1.367+j6.433)\right]$$

$$= \frac{3.9171-j11.4663}{4-j11} = \frac{12.1169\angle -71.14^\circ}{11.7047\angle -70.02^\circ} = 1.0352\angle -1.12^\circ = 1.0350 - j0.0202\,p.u$$

The bus voltages at the end of first Gauss seidel iteration are

$$V_1^1 = 1.05 + j0 = 1.05\angle 0^\circ\ p.u$$

$$V_2^1 = 1.0300 + j0.0567 = 1.0316\angle 3.15^\circ\ p.u$$

$$V_3^1 = 1.0333 - j0.01166 = 1.0399\angle -6.44^\circ\ p.u$$

$$V_4^1 = 1.0350 - j0.0202 = 1.0352\angle -1.12^\circ\ p.u$$

2. In Problem 1, let the bus 2 be a PV bus (Generator bus) with $|V_2|$ =1.07p.u. the reactive power constraint of the generator bus is $0.3 \le Q_2 \le 1.0$. With other data remaining same (Except Q_2), calculate the bus voltages at the end of first G-S iteration

Solution:

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$$Y_{bus}=\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}=\begin{bmatrix} 4-j11 & -2+j6 & -1+j3 & -1+j2 \\ -2+j6 & 3-j9 & 0 & -1+j3 \\ -1+j3 & 0 & 3-j9 & -2+j6 \\ -1+j2 & -1+j3 & -2+j6 & 4-j11 \end{bmatrix}$$

$$V_1^0=V_1^1=\cdots\cdots=V_1^n=V_1=1.05+j0\,p.u$$

$$V_3^0=1+j0 \quad V_4^0=1+j0 \quad V_2^0=1.07+j0 \quad K=0$$

$$Q_p^{k+1}=(-1)*\mathrm{Im}\left\{(V_p^k)^*\left[\sum_{q=1}^{p-1}Y_{pq}V_q^{k+1}+\sum_{q=p}^{n}Y_{pq}V_q^k\right]\right\}$$

$$Q_{2,cal}^1=(-1)\,\mathrm{Im}\left\{(V_2^0)^*\left[Y_{21}V_1^1+Y_{22}V_2^0+Y_{23}V_3^0+Y_{24}V_4^0\right]\right\}$$

$$Q_{2,cal}^1=(-1)\,\mathrm{Im}\{(1.07-j0)[(-2+j6)(1.05+j0)+(3-j9)(1.07+j0)+(0*(1-j0))+(-1+j3)(1+j0)]\}$$

$$=(-1)\,\mathrm{Im}\{1.07[-2.1+j6.3+3.21-j9.63-1+j3]\}$$

$$=(-1)\,\mathrm{Im}\{1.07[0.11-j0.33]\}=0.3531p.u$$

Q'_{2cal}=0.3531. The given Q limits are $0.3 \le Q_2 \le 1.0$. the calculated Q_2 is within the limits. So bus 2 is treated as PV bus.

Now Q_2=0.3531, p_2=0.5, $|V_2|$=1.07

$$V_{p,temp}^{k+1}=\frac{1}{Y_{pp}}\left[\frac{P_p-jQ_p}{(V_p^k)^*}-\sum_{q=1}^{p-1}Y_{pq}V_q^{k+1}-\sum_{q=p+1}^{n}Y_{pq}V_q^k\right]$$

$$V_{2,temp}^1=\frac{1}{Y_{22}}\left[\frac{P_2-jQ_2}{(V_2^0)^*}-Y_{21}V_1^1-Y_{23}V_3^0-Y_{24}V_4^0\right]$$

$$V_{2,temp}^1=\frac{1}{3-j9}\left[\frac{0.5-j0.3531}{1.07-j0}-(-2+j6)(1.05+j0)-(0*(1+j0))-(-1+j3)(1+j0)\right]$$

$$=\frac{1}{3-j9}[0.4673-j0.33+2.1-j6.3+1-j3]$$

$$= \frac{3.5673 - j9.63}{3 - j9} = \frac{10.2695\angle -69.67^\circ}{9.4868\angle -71.57^\circ} = 1.0825\angle 1.9^\circ$$

$$\therefore \delta_2^1 = \angle V_{2,temp}^1 = 1.9^\circ$$

$$\therefore V_2^1 = |V_2|_{spec} \angle \delta_2^1 = 1.07\angle 1.9^\circ = 1.0694 + j0.0355\ p.u$$

Bus 3 and bus 4 are load buses

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$V_3^1 = \frac{1}{3 - j9}\left[\frac{-1 - j0.5}{1 - j0} - (-1 + j3)(1.05 + j0) - (0*(1.0694 + j0.0355)) - (-2 + j6)(1 + j0)\right]$$

$$= \frac{1}{3 - j9}\left[-1 - j0.5 + 1.05 - j3.15 + 2 - j6\right] = \frac{2.05 - j9.65}{3 - j9} = \frac{9.8653\angle -78.01^\circ}{9.4868\angle -71.57^\circ}$$

$$= 1.0399\angle -6.44^\circ = 1.0333 - j0.1166\, p.u$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$V_4^1 = \frac{1}{4 - j11}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j2)(1.05 + j0) - (-1 + j3)*(1.0694 + j0.0355)) - (-2 + j6)(1.0333 - j0.01166)\right]$$

$$= \frac{1}{4 - j11}\left[0.3 + j0.1 + 1.05 - j2.1 - (-1.1759 + j3.1727) - (-1.367 + j6.433)\right]$$

$$= \frac{3.8929 - j11.6057}{4 - j11} = \frac{12.2412\angle -71.46^\circ}{11.7047\angle -70.02^\circ}$$

$$= 1.0458\angle -1.44^\circ = 1.0455 - j0.0263\, p.u$$

The bus voltages at the end of first Gauss seidel iteration are

$$V_1^1 = 1.05 + j0\,p.u = 1.05\angle 0^\circ$$

$$V_2^1 = 1.0694 + j0.0355 = 1.07\angle 1.9^\circ\ p.u$$

$$V_3^1 = 1.0333 - j0.1166 = 1.0399\angle -6.44^\circ\ p.u$$

$$V_4^1 = 1.0458\angle -1.44^\circ = 1.0455 - j0.0263\,p.u$$

3. In Problem 2, let the reactive power constraint of the generator bus is 0.4 ≤ Q_2 ≤ 1.0. With other data remaining same (Except Q_2), calculate the bus voltages at the end of first G-S iteration

Solution:

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 4-j11 & -2+j6 & -1+j3 & -1+j2 \\ -2+j6 & 3-j9 & 0 & -1+j3 \\ -1+j3 & 0 & 3-j9 & -2+j6 \\ -1+j2 & -1+j3 & -2+j6 & 4-j11 \end{bmatrix}$$

$$V_1^0 = V_1^1 = \cdots\cdots = V_1^n = V_1 = 1.05 + j0\,p.u$$

$$V_3^0 = 1 + j0 \qquad V_4^0 = 1 + j0 \qquad V_2^0 = 1.07 + j0 \qquad K = 0$$

$$Q_p^{k+1} = (-1) * \operatorname{Im}\left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

$$Q_{2,cal}^1 = (-1)\operatorname{Im}\left\{ (V_2^0)^* \left[Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0 + Y_{24}V_4^0 \right] \right\}$$

$$Q_{2,cal}^1 = (-1)\operatorname{Im}\{(1.07 - j0)[(-2 + j6)(1.05 + j0) + (3 - j9)(1.07 + j0) + (0*(1 - j0)) + (-1 + j3)(1 + j0)]\}$$

$$= (-1)\operatorname{Im}\{1.07[-2.1 + j6.3 + 3.21 - j9.63 - 1 + j3]\}$$

$$= (-1)\operatorname{Im}\{1.07[0.11 - j0.33]\} = 0.3531\,p.u$$

Q'_{2cal}=0.3531. The given Q limits are $0.4 \leq Q_2 \leq 1.0$. the calculated Q_2 is less than the specified lower limit. So bus 2 is treated as PQ bus.

Now Q_2=0.4, p_2=0.5, $|V_2|$=1.0

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2^1 = \frac{1}{3-j9}\left[\frac{0.5-j0.4}{1-j0} - (-2+j6)(1.05+j0) - (0*(1+j0)) - (-1+j3)(1+j0)\right]$$

$$= \frac{1}{3-j9}\left[0.5 - j0.4 + 2.1 - j6.3 + 1 - j3\right]$$

$$= \frac{3.6-j9.7}{3-j9} = \frac{10.3465\angle -69.64^\circ}{9.4868\angle -71.57^\circ} = 1.0906\angle 1.93^\circ = 1.0900 + j0.0367\,p.u.$$

Bus 3 and bus 4 are load buses

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$V_3^1 = \frac{1}{3-j9}\left[\frac{-1-j0.5}{1-j0} - (-1+j3)(1.05+j0) - (0*(1.0900+j0.0367)) - (-2+j6)(1+j0)\right]$$

$$= \frac{1}{3-j9}\left[-1 - j0.5 + 1.05 - j3.15 + 2 - j6\right] = \frac{2.05-j9.65}{3-j9} = \frac{9.8653\angle -78.01^\circ}{9.4868\angle -71.57^\circ}$$

$$= 1.0399\angle -6.44^\circ = 1.0333 - j0.1166\,p.u$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$V_4^1 = \frac{1}{4-j11}\left[\frac{0.3+j0.1}{1-j0} - (-1+j2)(1.05+j0) - (-1+j3)*(1.09+j0.0367)) - (-2+j6)(1.0333-j0.01166)\right]$$

$$= \frac{1}{4-j11}\left[0.3+j0.1+1.05-j2.1-(-1.2001+j3.2333)-(-1.367+j6.433)\right]$$

$$= \frac{3.9171-j11.6663}{4-j11} = \frac{12.3063\angle -71.44^\circ}{11.7047\angle -70.02^\circ}$$

The bus voltages at the end of first Gauss – Seidel iteration are,

V_1^1=1.05+j0=1.05∠0° p.u.

V_2^1=1.09+j0.0367=1.0906∠1.93° p.u.

V_3^1=1.0333-j0.116=1.0399∠-6.44° p.u.

V_4^1=1.0511-j0.0261=1.0514∠-1.42° p.u.

4. Figure shows a three bus power system.

Bus 1: Slack bus, V=1.05∠0° p.u.

Bus 2: PV bus, |V|=1.0 p.u., P_g=3p.u.

Bus 3: PQ bus, P_L=4p.u., Q_L=2p.u.

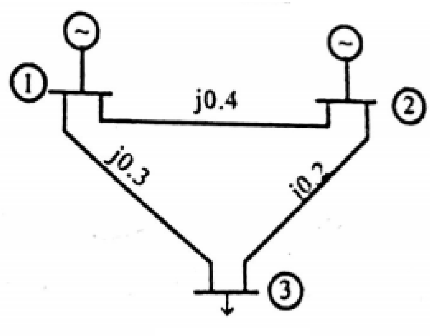

Figure:2.4

Carry out one iteration of load flow solution by Gauss seidel method. Neglect limits on reactive power generation

Solution:

The line impedances are

z_{12}=j0.4p.u.

z_{13}=j0.3p.u.

z_{23}=j0.2p.u.

The line admittances are

$y_{12}=1/z_{12}=1/j0.4=-j2.5$p.u.

$y_{13}=1/z_{13}=1/j0.3=-j3.333$p.u.

$y_{23}=1/z_{23}=1/j0.2=-j5$p.u.

$Y_{11}=y_{12}+y_{13}=-j2.5-j3.33=-j5.833$

$Y_{22}=y_{12}+y_{23}=-j2.5-j5=-j7.5$

$Y_{33}=y_{13}+y_{23}=-j3.333-j5=-j8.333$

$Y_{12}=Y_{21}=-y_{12}=-(-j2.5)=j2.5$

$Y_{13}=Y_{31}=-y_{13}=-(-j3.333)=j3.333$

$Y_{23}=Y_{32}=-y_{23}=-(-j5)=j5$

$$Y_{bus}=\begin{bmatrix}Y_{11} & Y_{12} & Y_{13}\\ Y_{21} & Y_{22} & Y_{23}\\ Y_{31} & Y_{32} & Y_{33}\end{bmatrix}=\begin{bmatrix}-j5.833 & j2.5 & j3.333\\ j2.5 & -j7.5 & j5\\ j3.333 & j5 & -j8.333\end{bmatrix}$$

The initial Values are:

$V_1^0=1.05\angle 0°=1.05+j0$ p.u.

$V_2^0=1.0\angle 0°=1.0+j0$ p.u.

$V_3^0=1.0\angle 0°=1.0+j0$ p.u.

Bus 1 is a slack bus,so its voltage will not change in any iteration

$V_1^1=v_1^0=1.05\angle 0°=1.05+j0$ p.u.

$$Q_p^{k+1} = (-1) * \mathrm{Im}\left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

$$Q_{2,cal}^1 = (-1)\,\mathrm{Im}\left\{ (V_2^0)^* \left[Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0 \right] \right\}$$

$$Q_{2,cal}^1 = (-1)\,\mathrm{Im}\left\{ (1 - j0)\left[j2.5(1.05 + j0) + (-j7.5)(1 + j0) + j5(1 + j0) \right) \right]\right\}$$

$$= (-1)\,\mathrm{Im}\{ j2.625 - j7.5 + j5 \} = -0.125\, p.u$$

Now, Q_2=-0.125, P_2=3,V_2^0=1+j0,$|V_2|_{spec}$=1.0

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k \right]$$

$$V_{2,temp}^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 \right]$$

$$V_{2,temp}^1 = \frac{1}{-j7.5} \left[\frac{3 + j0.125}{1 - j0} - (j2.5)(1.05 + j0) - (j5)(1 + j0) \right]$$

$$= \frac{1}{-j7.5} [3 + j0.125 - j2.625 - j5]$$

$$= \frac{1}{-j7.5} [3 - j7.5] = 1 + j0.4 = 1.077\angle 21.8^\circ$$

$$\therefore \delta_2^1 = \angle V_{2,temp}^1 = 21.8^\circ$$

$$\therefore V_2^1 = |V_2|_{spec} \angle \delta_2^1 = 1.0\angle 21.8^\circ = 0.92849 + j0.37137\, p.u$$

Bus 3 is a load bus. $\therefore$ P_3=-P_L=-4 and Q_3=-Q_L=-2

$$V_p^{k+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k \right]$$

$$V_3^1 = \frac{1}{-j8.333} \left[\frac{-4 + j2}{1 - j0} - (-j3.333)(1.05 + j0) - (j5)(0.92849 + j0.37137) \right]$$

$$=\frac{1}{-j8.333}\left[-4+j2-j3.49965+1.85685-j4.64245\right]=\frac{-2.14315-j6.1421}{-j8.3333}=\frac{6.50527\angle-109.24^\circ}{8.3333\angle-90^\circ}$$

$$=0.78064\angle-19.24^\circ=0.73704-j0.25724\,p.u$$

The bus voltages at the end of first Gauss seidel iteration are,

V_1^1=1.05+j0=1.05∠0° p.u.

V_2^1=0.92849+j0.37137=1.0∠21.8° p.u.

V_3^1=0.73704-j0.25724=0.78064∠-19.24° p.u.

5. The System data for a load flow solution are given in Tables below. Determine the voltages at the end of first iteration by Gauss seidel method. Take α = 1.6

Buscode	Admittance
1 - 2	2 − j 8
1 - 3	1 − j4
2 -3	0.666 − j2.664
2 - 4	1 − j4
3 - 4	2 − j8

Table:2.3

Buscode	P	Q	V	Remarks
1	-	-	1.06∠0°	Slack
2	0.5	0.2	-	PQ
3	0.4	0.3	-	PQ
4	0.3	0.1	-	PQ

Table 2.4

Solution:

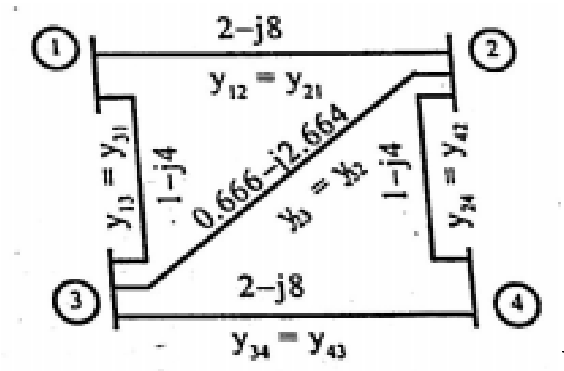

Figure 2.5

$Y_{11}=y_{12}+y_{13}=2-j8+1-j4=3-j12$

$Y_{22}=y_{12}+y_{23}+y_{24}=2-j8+0.666-j2.664+1-j4=3.666-j14.664$

$Y_{33}=y_{13}+y_{23}+y_{34}=1-j4+0.666-j2.664+2-j8=3.666-j14.664$

$Y_{44}=y_{24}+y_{34}=1-j4+2-j8=3-j12$

$Y_{12}=Y_{21}=-(2-j8)=-2+j8$

$Y_{13}=Y_{31}=-y_{13}=-(1-j4)=-1+j4$

$Y_{14}=Y_{41}=0$

$Y_{23}=Y_{32}=-y_{23}=-(0.666-j2.664)=-0.666+j2.664$

$Y_{24}=Y_{42}=-y_{24}=-(1-j4)=-1+j4$

$Y_{34}=Y_{43}=-y_{34}=-(2-j8)=-2+j8$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

$$V_1^0 = V_1^1 = \cdots\cdots = V_1^n = V_1 = 1.06 + j0\,p.u$$

$$V_2^0 = 1 + j0 \qquad V_3^0 = 1 + j0 \qquad V_4^0 = 1 + j0 \qquad K = 0$$

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^k\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2^1 = \frac{1}{3.666 - j14.664}\left[\frac{-0.5 + j0.2}{1 - j0} - (-2 + j8)(1.06 + j0) - (0.666 + j2.664)(1 + j0) - (-1 + j4)(1 + j0)\right]$$

$$= \frac{-0.5 + j0.2 + 2.12 - j8.48 + 0.666 - j2.664 + 1 - j4}{3.666 - j14.664}$$

$$= \frac{3.286 - j14.944}{3.666 - j14.664} = \frac{15.3010\angle -77.6^\circ}{15.1153\angle -75.96^\circ} = 1.0123\angle -1.64^\circ = 1.0119 - j0.0290\, p.u$$

$$V_{p,acc}^{k+1} = V_p^k + \alpha(V_p^{k+1} - V_p^k)$$

$$V_{2,acc}^1 = V_2^0 + \alpha(V_2^1 - V_2^0)$$

$$= 1 + 1.6(1.0119 - j0.0290 - 1)$$
$$= 1 + 1.6(0.0119 - j0.0290) = 1.0190 - j0.0464$$

$$V_2^1 = V_{2,acc}^1 = 1.0190 - j0.0464\, p.u. = 1.0201\angle -2.61^o\ p.u.$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$V_3^1 = \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0} - (-1 + j4)(1.06 + j0) - (0.666 + j2.664)(1.0190 + j0.0464) - (-2 + j8)(1 + j0)\right]$$

$$= \frac{-0.4 + j0.3 + 1.06 - j4.24 - (-0.5550 + j2.7455) + 2 - j8}{3.666 - j14.664}$$

$$= \frac{3.215 - j14.6855}{3.666 - j14.664} = \frac{15.0333\angle -77.65^\circ}{15.1153\angle -75.96^\circ} = 0.9946\angle -1.69^\circ = 0.9942 - j0.0293\, p.u$$

$$V_{3,acc}^1 = V_3^0 + \alpha(V_3^1 - V_3^0)$$

$$= 1 + 1.6(0.9942 - j0.0293 - 1)$$

$$= 1 + 1.6(-0.0058 - j0.0293) = 0.9907 - j0.0469$$

$$V_3^1 = V_{3,acc}^1 = 0.9907 - j0.0469\, p.u. = 0.9918\angle -2.71^o\ p.u.$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$V_4^1 = \frac{1}{3 - j12}\left[\begin{array}{l}\dfrac{-0.3 + j0.1}{1 - j0} - (0*1.06) - (-1 + j4)(1.0190 - j0.0464) \\ -(-2 + j8)(0.9907 - j0.0469)\end{array}\right]$$

$$= \frac{-0.3 + j0.1 - (-0.8334 + j4.1224) - (-1.6062 - j8.0194)}{3 - j12}$$

$$= \frac{2.1396 - j12.0418}{3 - j12} = \frac{12.2304\angle -79.92^\circ}{12.3693\angle -75.96^\circ} = 0.9888\angle -3.96^\circ = 0.9864 - j0.0683 p.u$$

$$V_{4,acc}^1 = V_4^0 + \alpha(V_4^1 - V_4^0)$$

$$= 1 + 1.6(0.9864 - j0.0683 - 1)$$

$$= 1 + 1.6(-0.0136 - j0.0683) = 0.9782 - j0.1033$$

$$V_4^1 = V_{4,acc}^1 = 0.9782 - j0.1093 p.u. = 0.9843\angle -6.38^o\ p.u.$$

The bus voltages at the end of first Gauss seidel iteration are,

V_1^1=1.06+j0=1.06∠0° p.u.

V_2^1=1.019-j0.0464=1.0201∠-2.61° p.u.

V_3^1=0.9907-j0.0469=0.9918∠-2.7° p.u.

V_4^1=0.9782-j0.1093=0.9843∠-6.38° p.u.

When the generator bus is treated as load bus?

- **If the reactive power of a generator bus violates the specified limits then the generator bus is treated as load bus**

What will be the reactive power and bus voltage when the generator bus is treated as load bus?

- **When the generator bus is treated as load bus, the reactive power of the bus is equated to the limit it has violated, and the previous iteration value of bus voltage is used for calculating current iteration value**

Acceleration factor:

- **In Gauss Seidel method, the number of iterations can be reduced, if the correction voltage at each bus is multiplied by some constant**
- **It is used only for load bus**

Advantages of Gauss seidel Method:

1. **Calculations are simple and so the programming task is lesser**
2. **The memory requirements is less**
3. **Useful for small systems**

Disadvantages of Gauss Seidel Method:

1. **Requires large number of iterations to reach convergence**
2. **Not suitable for large systems**
3. **Convergence time increases with size of the system**

Newton Raphson Method

- **The set of nonlinear simultaneous (load flow) equations are approximated to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to first order approximation**

Jacobian Matrix

- **The matrix formed from the first derivatives of load flow equations is called Jacobian matrix and it is denoted by J**

How the elements of Jacobian matrix are computed?

- **The elements of Jacobian matrix will change in every iteration**

- **In each iteration, the elements of Jacobian matrix are obtained by partially differentiating the load flow equations with respect to a unknown variable and then evaluating the first derivatives using the solution of previous iteration**

Newton Raphson Method

- **The Gauss seidel algorithm is very simple but convergence become increasingly slow as the system size grows**
- **The Newton Raphson technique converges equally fast for large as well as small systems, usually in less than 4 to 5 iterations but more functional evaluations are required**
- **It has become very popular for large system studies**
- **The most widely used method for solving simultaneous non linear algebraic equations is the N-R method**
- **This method is a successive approximation procedure based on an initial estimate of the unknown and the use of Taylor series expansion**

The current entering bus i is given by

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

In polar form, $I_i = \sum_{j=1}^{n} |Y_{ij}||V_j| \angle \theta_{ij} + \delta_j$ **Where,** $Y_{ij}=|Y_{ij}|\angle\theta_{ij}$, $V_j=|V_j|\angle\delta_j$

Complex power at bus i,

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{j-1}^{N} Y_{ij} V_j$$

$$P_i - jQ_i = |V_i| \angle -\delta_i \sum_{j-1}^{N} |Y_{ij}||V_j| \angle(\theta_{ij} + \delta_j)$$

$$= \sum_{j-1}^{N} |V_i||Y_{ij}||V_j| \angle(\theta_{ij} + \delta_j - \delta_i)$$

Equating the real and imaginary parts,

$$P_i = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j|\cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j|\sin(\theta_{ij} + \delta_j - \delta_i)$$

Real power mismatch, $\Delta P_i^0 = P_i - P_i^0$

Reactive Power mismatch , $\Delta Q_i^0 = Q_i - Q_i^0$

In Matrix form,

$$\begin{bmatrix} \Delta P_2^0 \\ \vdots \\ \Delta P_N^0 \\ \cdots \\ \Delta Q_2^0 \\ \vdots \\ \Delta Q_N^0 \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P_2}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial P_2}{\partial \delta_N}\right)^0 & \vdots & \left(\frac{\partial P_2}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial P_2}{\partial |V_N|}\right)^0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \left(\frac{\partial P_N}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial P_N}{\partial \delta_N}\right)^0 & \vdots & \left(\frac{\partial P_N}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial P_N}{\partial |V_N|}\right)^0 \\ \cdots & \cdots\ \cdots & \cdots & \vdots & \cdots & \cdots\ \cdots\ \cdots & \cdots \\ \left(\frac{\partial Q_2}{\partial \delta_2}\right)^0 & & \left(\frac{\partial Q_2}{\partial \delta_N}\right)^0 & \vdots & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^0 & & \left(\frac{\partial Q_2}{\partial |V_N|}\right)^0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \left(\frac{\partial Q_N}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial Q_N}{\partial \delta_N}\right)^0 & \vdots & \left(\frac{\partial Q_N}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial Q_N}{\partial |V_N|}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta \delta_2^0 \\ \vdots \\ \Delta \delta_N^0 \\ \cdots \\ |\Delta V_2^0| \\ \vdots \\ |\Delta V_N^0| \end{bmatrix}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

The diagonal and off diagonal elements of J_1 are,

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i) \qquad j \neq i$$

The diagonal and off diagonal elements of J_2 are,

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{\substack{j=1 \\ \neq 1}}^{N} |V_j||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i) \qquad j \neq i$$

The diagonal and off diagonal elements of J_3 are,

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i) \qquad j \neq i$$

The diagonal and off diagonal elements of J_4 are,

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{\substack{j=1 \\ \neq 1}}^{N} |V_j||Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i) \qquad j \neq i$$

$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$

$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$

$$\begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

The new estimates of bus voltages are,

$$\delta_i^{new} = \delta_i^{old} + \Delta\delta_i^{old}$$

$$V_i^{new} = V_i^{old} + \Delta V_i^{old}$$

For PV buses or voltage Controlled Buses:

- **The Voltage magnitudes are specified for PV bus**
- **Let M be the number of generator buses.**
- **M equations involving ΔQ and ΔV and the corresponding columns of the Jacobian matrix are eliminated**
- **∴ There are (N-1) real power constraints and (N-1-M) reactive power constraints and the Jacobian matrix of order (2N-2-M) * (2N-2-M)**

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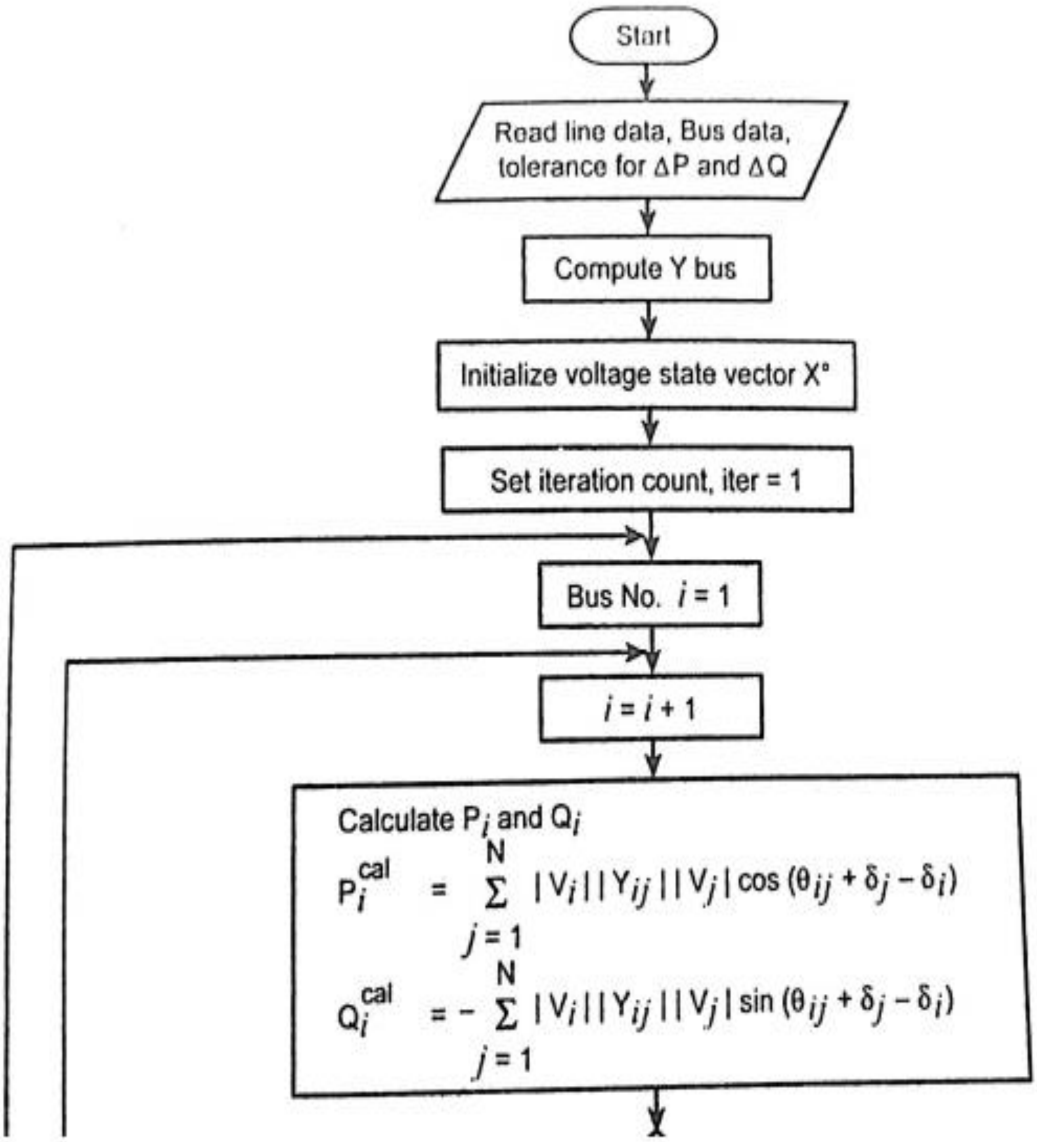
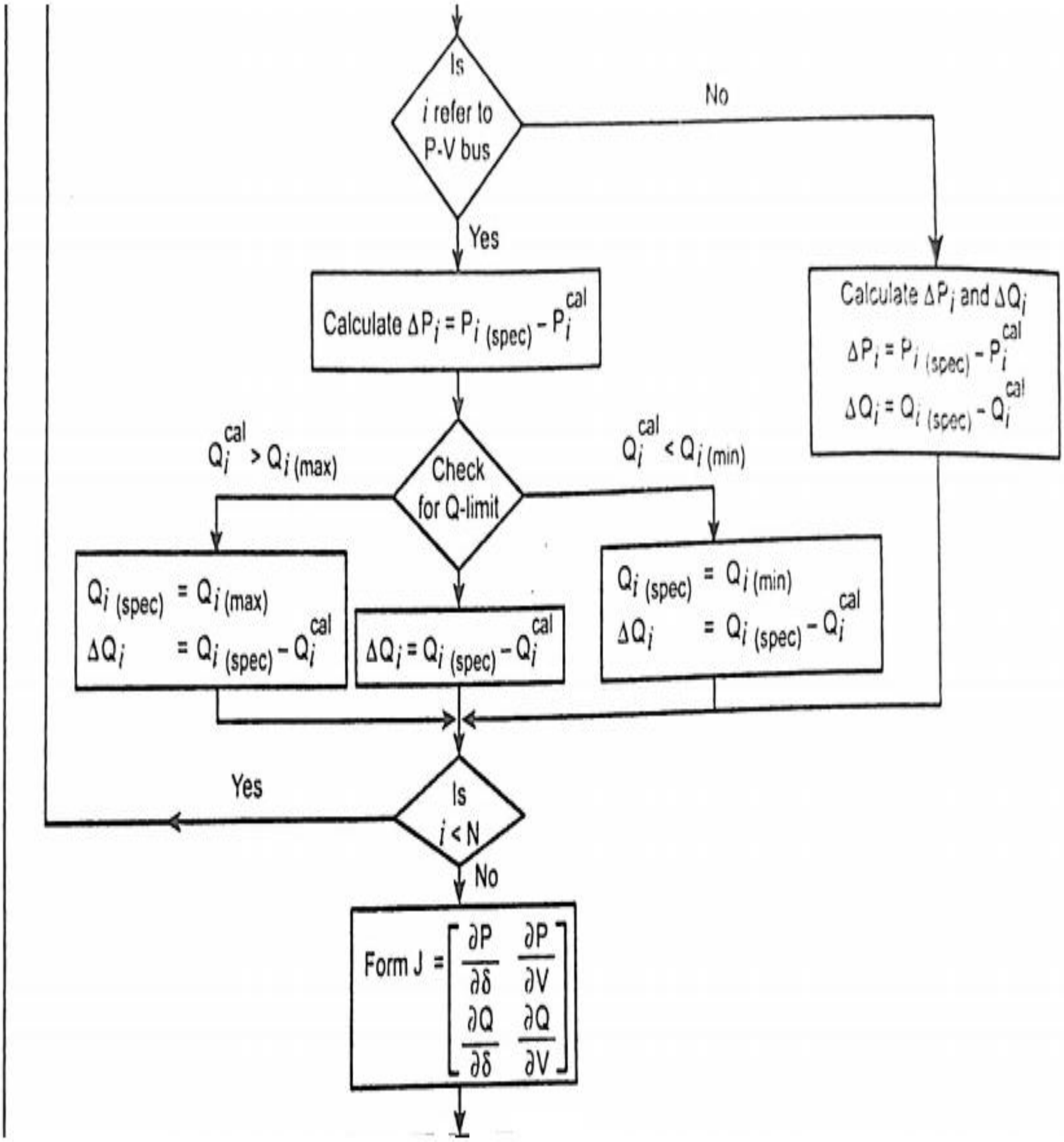
Flow chart of Load Flow Analysis using Newton Raphson Method




Is i refer to P-V bus
No
Yes
Calculate $\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
Calculate ΔP_i and ΔQ_i
$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_i^{cal} > Q_{i\,(max)}$
Check for Q-limit
$Q_i^{cal} < Q_{i\,(min)}$
$Q_{i\,(spec)} = Q_{i\,(max)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_{i\,(spec)} = Q_{i\,(min)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
Yes
Is $i < N$
No
Form $J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}$

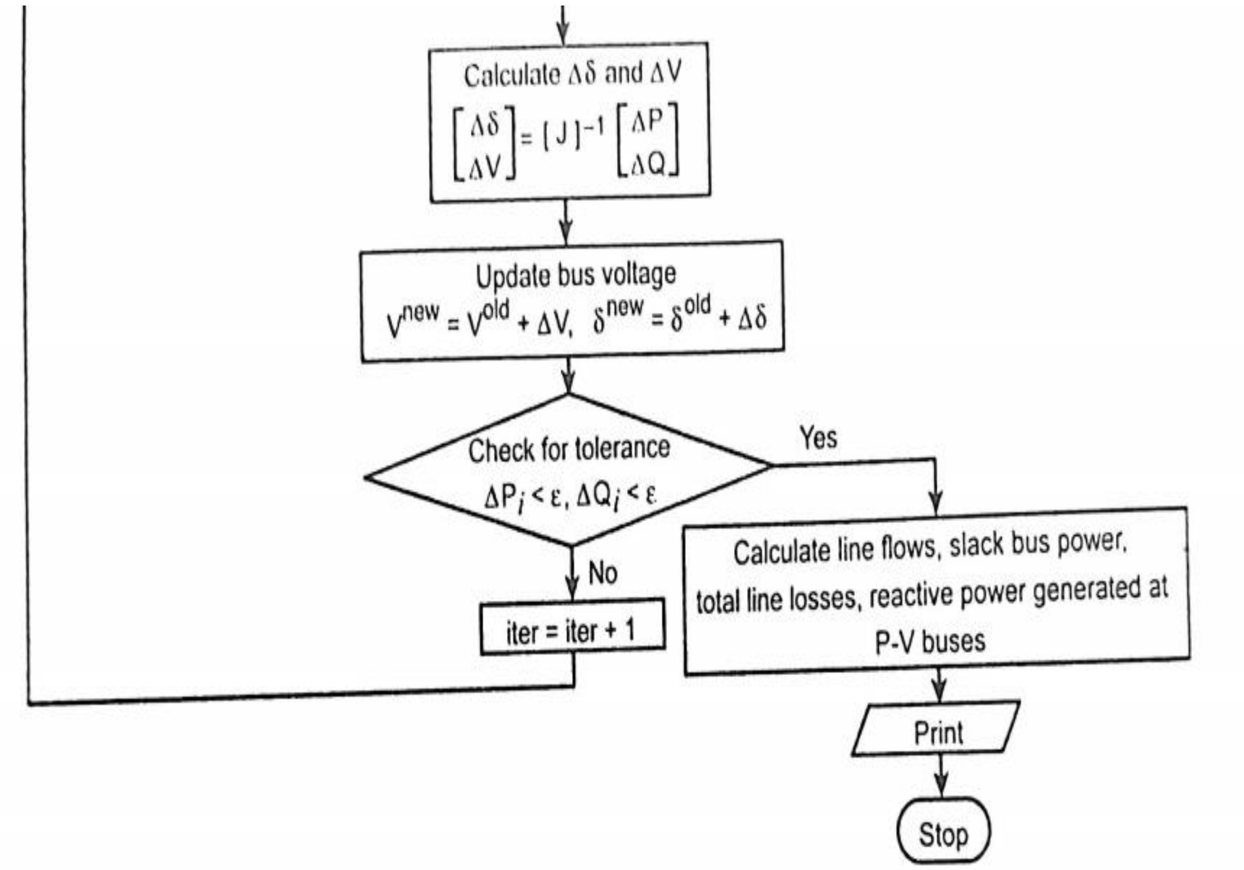


Figure: 2.6

Algorithm for Newton Raphson Method

Step 1: Formulate Y – bus matrix

Step 2: Assume flat start for starting voltage solution

δ_i^0 =0, for i=1, …….., N for all buses except slack bus

$|V_i^0|$=1.0, for i=M=1,M+2, ……..,N (for all PQ buses)

$|V_i|=|V_i|_{(spec)}$

Step 3: For load buses, calculate p_i^{cal} and Q_i^{cal}

Step 4: for PV buses, check for Q-limit violation

If $Q_{i(min)} < Q_i^{cal} < Q_{i(max)}$, the bus acts as P-V bus

If $Q_i^{cal} > Q_{i(max)}$, $Q_{i(spec)}=Q_{i(max)}$

If $Q_i^{cal} < Q_{i(min)}$, $Q_{i(spec)}=Q_{i(min)}$, the P-V bus will act as P-Q bus

Step 5: Compute mismatch vector using

$$\Delta P_i = P_{i(spec)} - P_i^{cal}$$

$$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$$

Step 6: Compute $\Delta P_{i(max)}=\max|\Delta P_i|$; i=1,2,……., N except slack

$$\Delta Q_{i(max)}=\max|\Delta Q_i|;\ i=M+1,\ldots\ldots, N$$

Step 7: Compute Jacobian matrix using

$$J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta} & \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$$

Step 8: Obtain state correction vector

$$\begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Step 9: Update state vector using

$$V^{new} = V^{old} + \Delta V$$

$$\delta^{new} = \delta^{old} + \Delta\delta$$

Step 10: This procedure is continued until

$|\Delta P_i| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$, otherwise go to step 3

6. The one line diagram of a simple 3 bus power system with generators at buses 1 and 3 is shown. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from the bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by Newton Raphson method.

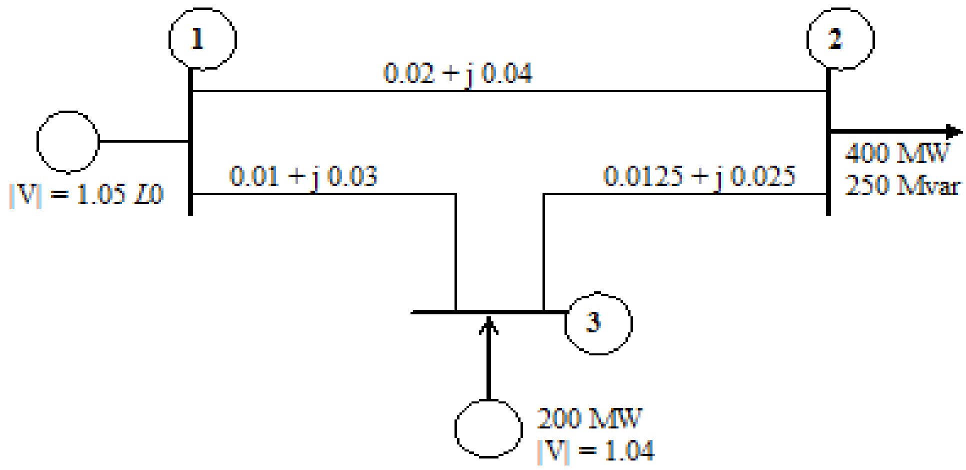


Figure: 2.7

Solution:

$$Y_{11} = \frac{1}{0.02 + j0.04} + \frac{1}{0.01 + j0.03} = 20 - j50 = 53.85\angle -68.3^\circ$$

$$Y_{12} = Y_{21} = \frac{1}{0.02 + j0.04} = 10 - j20 = 22.36\angle 116.56^\circ$$

$$Y_{13} = Y_{31} = \frac{1}{0.01 + j0.03} = 10 - j30 = 31.62\angle 108.43^\circ$$

$$Y_{22} = \frac{1}{0.02 + j0.04} + \frac{1}{0.0125 + j0.025} = 20 - j50 = 58.14\angle -63.6^\circ$$

$$Y_{23} = Y_{32} = \frac{1}{0.0125 + j0.025} = 16 - j32 = 35.70\angle 116.56^\circ$$

$$Y_{33} = \frac{1}{0.01 + j0.03} + \frac{1}{0.0125 + j0.025} = 26 - j62 = 17.23\angle -67.25^\circ$$

$$Y_{bus} = \begin{bmatrix} 53.85\angle -68.3^\circ & 22.36\angle 116.56^\circ & 31.62\angle 108.43^\circ \\ 22.36\angle 116.56^\circ & 58.14\angle -63.6^\circ & 35.70\angle 116.56^\circ \\ 31.62\angle 108.43^\circ & 35.70\angle 116.56^\circ & 17.23\angle -67.25^\circ \end{bmatrix}$$

Bus 1: Slack bus = V_1=1.05∠0° =$|V_1|$=1.05 ; δ_1=0°

Bus 2: load bus = P_2=400MW; Q_2=250MVA; $|V_2|$=1.0 ; δ_2=0°

Bus 3: Generator bus = P_3=200MVA; $|V_3|$=1.04; δ_3=0°

P_2^{sch}= -4.0p.u ; Q_2^{sch} =-2.5p.u; P_3^{sch}=2.0 p.u

$$P_i = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j|\cos(\theta_{ij}+\delta_j-\delta_i)$$

$$P_2 = |V_2||Y_{21}||V_1|\cos(\theta_{21}+\delta_1-\delta_2)+|V_2||Y_{22}||V_2|\cos(\theta_{22}+\delta_2-\delta_2)+|V_2||Y_{23}||V_3|\cos(\theta_{23}+\delta_3-\delta_2)$$

$$P_2 = |V_2||Y_{21}||V_1|\cos(\theta_{21}+\delta_1-\delta_2)+|V_2|^2|Y_{22}|\cos\theta_{22}+|V_2||Y_{23}||V_3|\cos(\theta_{23}+\delta_3-\delta_2)$$

$$P_2 = (1*22.36*1.05)\cos(116.56+0-0)+1^2*58.14\cos(-63.6)+(1*35.70*1.04)\cos(116.56+0-0) = -1.14$$

$$P_3 = |V_3||Y_{31}||V_1|\cos(\theta_{31}+\delta_1-\delta_3)+|V_3||Y_{32}||V_2|\cos(\theta_{32}+\delta_2-\delta_3)+|V_3|^2|Y_{33}|\cos\theta_{33}$$

$$P_3 = (1.04*31.62*1.05)\cos(108.43+0-0)+(1.04*35.70*1)\cos(116.56+0-0)+(1.04)^2*17.23\cos(-67.25) = 0.5616$$

$$Q_i = -\sum_{j-1}^{N} |V_i||Y_{ij}||V_j|\sin(\theta_{ij}+\delta_j-\delta_i)$$

$$Q_2 = -(|V_2||Y_{21}||V_1|\sin(\theta_{21}+\delta_1-\delta_2)+|V_2||Y_{22}||V_2|\sin(\theta_{22}+\delta_2-\delta_2)+|V_2||Y_{23}||V_3|\sin(\theta_{23}+\delta_3-\delta_2))$$

$$Q_2 = -[(1*22.36*1.05)\sin(116.56+0-0)+1^2*58.14\sin(-63.6)+(1*35.70*1.04)\sin(116.56+0-0)] = -2.28$$

$$\Delta P_2^0 = P_2^{sch} - P_2^0 = -4-(-1.14) = -2.86$$

$$\Delta P_3^0 = P_3^{sch} - P_3^0 = 2-(0.5616) = 1.4384$$

$$\Delta Q_2^0 = Q_2^{sch} - Q_2^0 = -2.5-(-2.28) = -0.22$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1\\ \neq i}}^{N} |V_i||Y_{ij}||V_j|\sin(\theta_{ij}+\delta_j-\delta_i)$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||Y_{21}||V_1|\sin(\theta_{21}+\delta_1-\delta_2)+|V_2||Y_{23}||V_3|\sin(\theta_{23}+\delta_3-\delta_2) = 54.28$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||Y_{ij}||V_j|\sin(\theta_{ij}+\delta_j-\delta_i)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||Y_{23}||V_3|\sin(\theta_{23}+\delta_3-\delta_2) = -33.28$$

$$llly\ \frac{\partial P_3}{\partial \delta_2} = -33.28$$

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{\substack{j=1\\ \neq 1}}^{N} |V_j||Y_{ij}|\cos(\theta_{ij}+\delta_j-\delta_i)$$

$$\frac{\partial P_2}{\partial |V_2|} = 2|V_2||Y_{22}|\cos\theta_{22}+|V_1||Y_{21}|\cos(\theta_{21}+\delta_1-\delta_2)+|V_3||Y_{23}|\cos(\theta_{23}+\delta_3-\delta_2) = 24.86$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(\theta_{ij}+\delta_j-\delta_i) \quad j \neq i$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32}+\delta_2-\delta_3) = -16.64$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1\\ \neq i}}^{N} |V_i||Y_{ij}||V_j|\cos(\theta_{ij}+\delta_j-\delta_i)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||Y_{21}||V_1|\cos(\theta_{21}+\delta_1-\delta_2)+|V_2||Y_{23}||V_3|\cos(\theta_{23}+\delta_3-\delta_2) = -27.14$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||Y_{ij}||V_j|\sin(\theta_{ij}+\delta_j-\delta_i) \qquad j \neq i$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||Y_{23}||V_3|\sin(\theta_{23}+\delta_3-\delta_2) = 16.64$$

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{\substack{j=1 \\ \neq 1}}^{N} |V_j||Y_{ij}|\sin(\theta_{ij}+\delta_j-\delta_i)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -2|V_2||Y_{22}|\sin\theta_{22} - |V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2) - |V_3||Y_{23}|\sin(\theta_{23}+\delta_3-\delta_2) = 49.72$$

$$\begin{bmatrix} -2.86 \\ 1.43 \\ -0.22 \end{bmatrix} = \begin{bmatrix} 54.28 & -33.28 & 24.86 \\ -33.28 & 66.04 & -16.64 \\ -27.14 & 16.64 & 49.72 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \\ \Delta|V_2|^0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \\ \Delta|V_2|^0 \end{bmatrix} = \begin{bmatrix} 54.28 & -33.28 & 24.86 \\ -33.28 & 66.04 & -16.64 \\ -27.14 & 16.64 & 49.72 \end{bmatrix}^{-1} \begin{bmatrix} -2.86 \\ 1.43 \\ -0.22 \end{bmatrix}$$

$$\Delta\delta_2^0 = -0.0453$$

$$\Delta\delta_3^0 = -0.0077$$

$$\Delta|V_2^0| = -0.0265$$

$$\Delta\delta_2^1 = 0 + (-0.0453) = -0.0453$$

$$\Delta\delta_3^1 = 0 + (-0.0077) = -0.0077$$

$$\Delta|V_2^1| = 1 + (-0.0265) = 0.9735$$

Advantages of Newton Raphson method

1. **The N- R method is faster, more reliable and the results are accurate**
2. **Requires less number of iterations for convergence**
3. **The Number of iterations are independent of size of the system (number of buses)**
4. **Suitable for large size system**

Disadvantages of Newton Raphson method

1. **The programming is more complex**
2. **The memory requirement is more**
3. **Computational time per iteration is higher due to large number of calculations per iteration**

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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – III – Power System Analysis – SEE1302

SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS

SYMMETRICAL SHORT CIRCUIT STUDIES

Need for Short circuit study – Bus impedance matrix formation – Symmetrical short circuit analysis using Zbus- Computations of short circuit capacity, post fault voltage and current

Fault

- **A fault in a circuit is any failure which interferes with the normal flow of current.**
- **The faults are associated with abnormal change in current, voltage and frequency of the power system**
- **The faults may cause damage to the equipments if it is allowed to persist for a long time**
- **Hence every part of a system has been protected by means of relays and circuit breakers to sense the faults and to isolate the faulty part from the healthy part in the event of fault**

Why faults occur in a power system?

- **Insulation failure of equipments**
- **Flashover of lines initiated by a lightning stroke**
- **Permanent damage to conductors and towers**
- **Accidental faulty operations**

Classification of faults

Method I

- **Shunt Fault: Due to short circuits in conductors**
- **Series Fault: Due to open conductors**

Method II

- **Symmetrical faults: The fault currents are equal in all the phases and can be analysed on per phase basis**

- **Unsymmetrical faults: The fault currents are unbalanced and so they are analysed using symmetrical components**

Various Types of shunt Faults

i. **Line to ground fault**
ii. **Line to line fault**
iii. **Double line to ground fault**
iv. **Three Phase fault**

Various Types of Series Faults

i. **One open conductor fault**
ii. **Two open conductor fault**

Symmetrical fault

i. **Three Phase Fault**

Unsymmetrical fault

i. **Line to ground fault**
ii. **Line to line fault**
iii. **Double line to ground fault**
iv. **One or two open conductor faults**

Methods of reducing short circuit current

- **By providing neutral reactance**
- **By introducing a large value of shunt reactance between buses**

Differences in representation of power system for load flow and short circuit studies

- **For load flow studies both the resistances and reactances are considered whereas for fault analysis the resistances are neglected**

- **For load flow studies the bus admittance matrix is useful whereas for short circuit studies bus impedance matrix is used**
- **The load flow study is performed to determine the exact voltages and currents whereas in short circuit studies the voltages can be safely assumed as 1 pu and the prefault current can be neglected**

Rank the various faults in the order of severity

i. **3 – Phase fault**
ii. **Double line to ground fault**
iii. **Line to line fault**
iv. **Single line to ground fault**
v. **Open conductor faults**

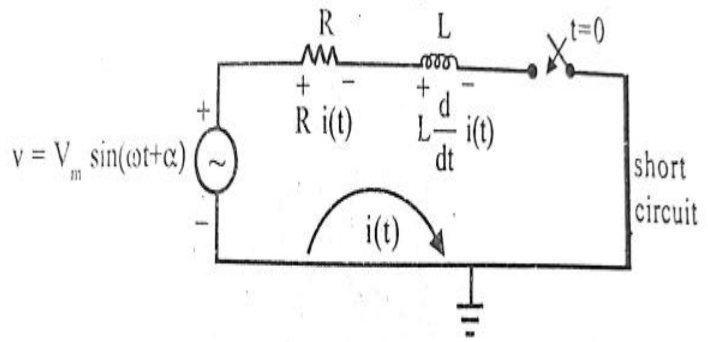
Relative frequency of occurrence of various types of faults

Type of Fault	Relative frequency of occurrence
i. 3 – Phase fault	5%
ii. Double line to ground fault	10%
iii. Line to line fault	15%
iv. Single line to ground fault	70%

Reason for transients during short circuits

- **The faults or short circuits are associated with sudden change in currents**
- **Most of the components of the power system have inductive property which opposes any sudden change in currents and so the faults (short circuits) are associated with transients**

Waveform of a short circuit current on a transmission line



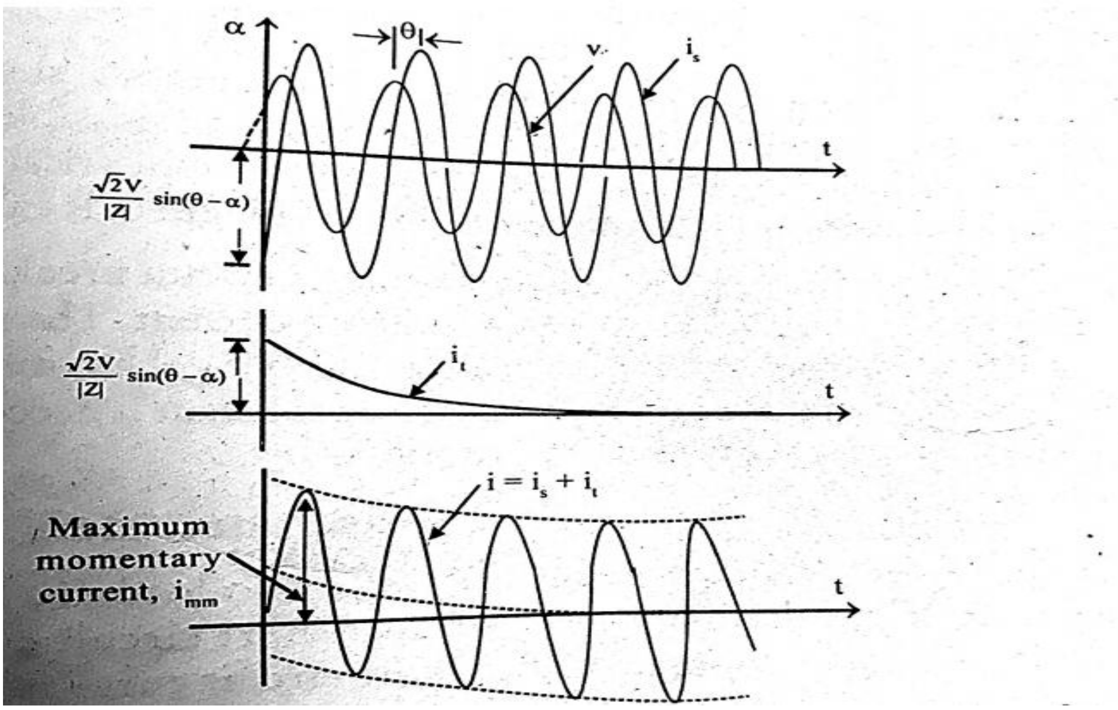


Figure: 3.1

Doubling Effect

- **If a symmetrical fault occurs when the voltage wave is going through zero then the maximum momentary short circuit current will be double the value of maximum symmetrical short circuit current.**
- **This effect is called doubling effect**

DC off –set Current

- **The unidirectional transient component of short circuit current is called DC off set current**

Oscillogram of short circuit current when an unloaded generator is subjected to symmetrical fault

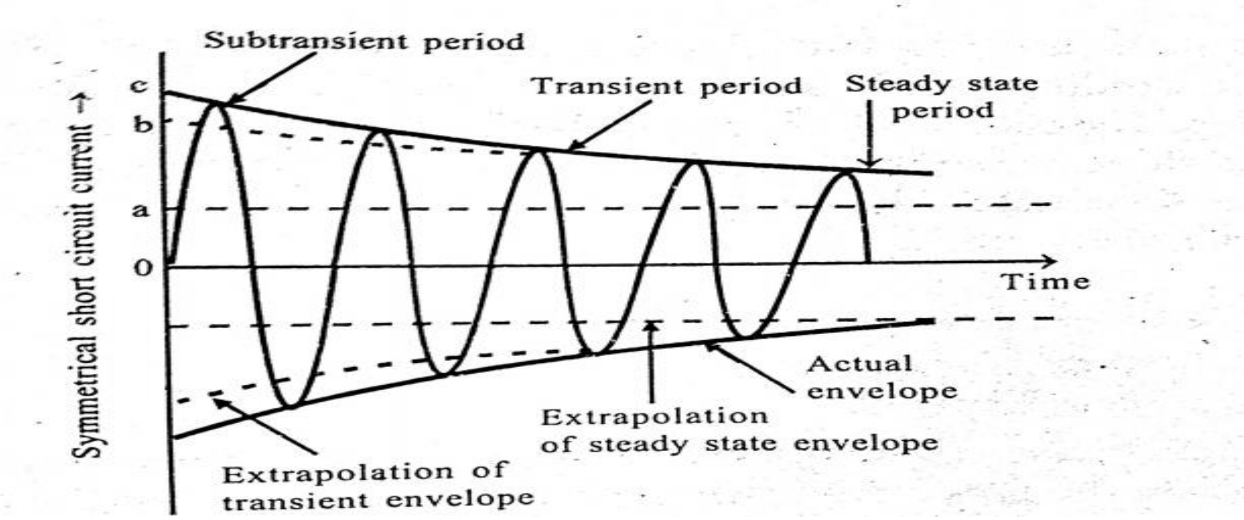


Figure 3.2

Subtransient Symmetrical rms current, I'' =oc/√2

Transient Symmetrical rms current, I'=ob/√2

Steady state symmetrical rms current, I =oa/√2

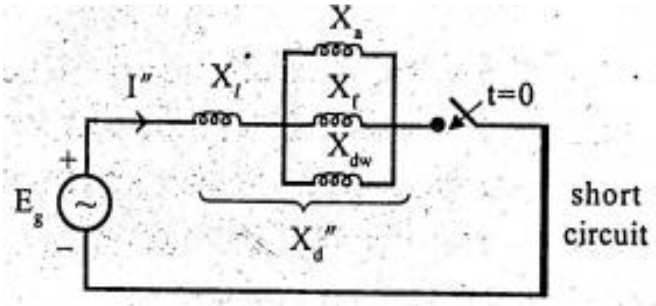
Subtransient reactance, X_d''=|E_g|/|I''|

Transient reactance, X_d'=|Eg|/|I'|

Synchronous reactance, $X_d=|E_g|/|I|$

Subtransient reactance:

- **It is the ratio of induced emf on no load and the subtransient symmetrical rms current, (i.e, it is the reactance of a synchronous machine under subtransient condition)**



$$X_d'' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f} + \frac{1}{X_{dW}}}$$

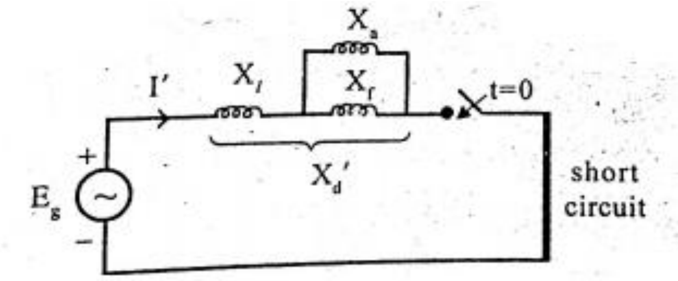
Figure: 3.3

Significance of subtransient reactance in short circuit studies:

- **It is used to estimate the initial value of fault current immediately on the occurrence of the fault**
- **The maximum momentary short circuit current rating of the circuit breaker used for protection or fault clearing should be less than this initial fault current**

Transient reactance:

- **It is the ratio of induced emf on no load and the transient symmetrical rms current. (i.e, it is the reactance of a synchronous machine under transient condition)**



$$X_d^{'} = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f}}$$

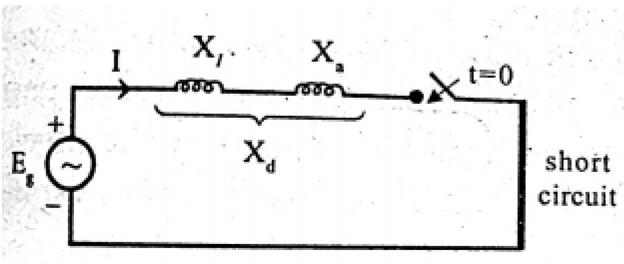
Figure: 3.4

Significance of transient reactance in short circuit studies:

- **It is used to estimate the transient state fault current**
- **Most of the circuit breakers open their contacts only during this period**
- **Therefore for a C.B used for fault clearing, its interruption short circuit current rating should be less than the transient fault current**

Synchronous reactance:

- **It is the ratio of induced emf and the steady state rms current (i.e, it is the reactance of a synchronous machine under steady state condition).**
- **It is the sum of leakage reactance and the reactance representing armature reaction**



$$X_d = X_l + \frac{1}{\frac{1}{X_a}} = X_l + X_a$$

Figure:3.5

Need for Short circuit studies or fault analysis

- **The short circuit studies are essential in order to design or develop the protective schemes for various parts of the system**
- **The selection of protective devices like current and voltage sensing devices, protective relays , circuit breakers mainly depends on various elements that may flow in fault conditions**

Fault Calculations

- **The fault condition of a power system can be divided into subtransient, transient and steady state periods**
- **The currents in the various parts of the system and in the fault are different in these periods**
- **The estimation of these currents for various types of faults at various locations in the system are commonly referred to as fault calculations**

Analysis of Symmetrical faults:

- **The symmetrical faults are analysed using per unit reactance diagram of the power system**
- **Once the reactance diagram is formed, then the fault is simulated by short circuit**
- **The currents and voltages at various parts of the system can be estimated by**

i. **Kirchoff's method**

ii. **Thevenin's theorem**

iii. **Bus impedance matrix**

1. A Synchronous generator and motor are rated for 30,000KVA, 13.2KV and both have sub - transient reactance of 20% . The line connecting them has a reactance of 10% on the base of machine ratings. The motor is drawing 20,000KW at 0.8 pf leading. The terminal voltage of the motor is 12.8KV. When a symmetrical three phase fault occurs at motor terminals, find the sub – transient current in generator, motor and at the fault point.

Solution:

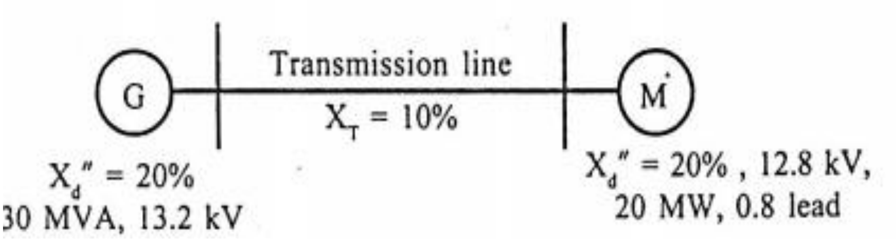


Figure: 3.6

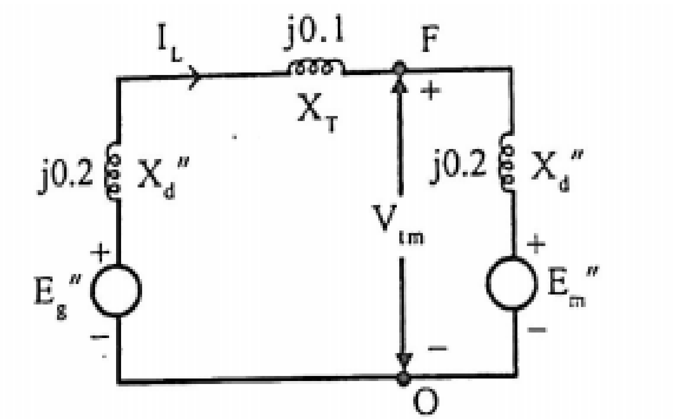


Figure: 3.7

Base Values:

MVA_b=30MVA; KV_b=13.2KV

Base current, $I_b = KVA_b/(\sqrt{3} * KV_b) =((30 * 1000) / (\sqrt{3} * 13.2)) = 1312.16$ A

Actual Value of prefault Voltage at fault point, V_{tm} = 12.8KV

p.u. value of prefault voltage at fault point, V_{tm} = Actual Value / Base Value

= 12.8 / 13.2 = 0.9697 p.u.

Actual Value of real power of the load, P_m = 20MW, 0.8 lead

p.u. value of real power of the load, P_m = Actual Value / Base Value

= 20/30 = 0.6667 p.u.

When voltage, current and power are expressed in p.u., then in 3 – phase circuits

$$P = VI \cos\Phi$$

Where cosΦ = power factor of the load

∴ **p.u. value of magnitude of load current,** $|I| = P_m / (V_{tm} \cos\Phi)$

$$= (0.6667/(0.9697 * 0.8))$$

$$= 0.8594 \text{ p.u.}$$

Take Terminal Voltage of motor V_{tm} as reference vector, so the load current will lead the terminal voltage of motor with an angle $\cos^{-1} 0.8$

$$\therefore V_{tm} = 0.9697 \angle 0°$$

$$I_L = 0.8594 \angle \cos^{-1} 0.8 = 0.8594 \angle 36.9° \text{ p.u.}$$

Method – 1: Using Kirchoff's Theorem

Prefault Condition

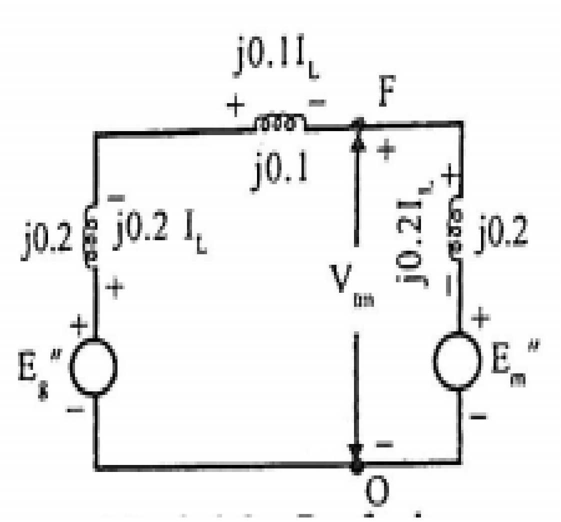


Figure:3.8

$$E_g" = j0.2\ I_L + j0.1\ I_L + V_{tm}$$

$$= j0.3\ I_L + V_{tm}$$

$= 0.3\angle 90° * 0.8594\angle 36.9° + 0.9697\angle 0°$

$= 0.2578\angle 126.9° + 0.9697\angle 0°$

$= -0.1548 + j0.2062 + 0.9697$

$= 0.8149 + j0.2062 = 0.8406\angle 14.2°$ p.u.

$E_m'' + j0.2I_L = V_{tm}$

$\therefore\ E_m'' = V_{tm} - j0.2I_L$

$= 0.9697\angle 0° - (0.2\angle 90° * 0.8594\angle 36.9°)$

$= 0.9697\angle 0° - (0.1719\angle 126.9°)$

$= 0.9697 - (-0.1032 + j0.1375)$

$= 1.0729 - j\,0.1375 = 1.0817\angle -7.3°$ p.u.

Fault Condition

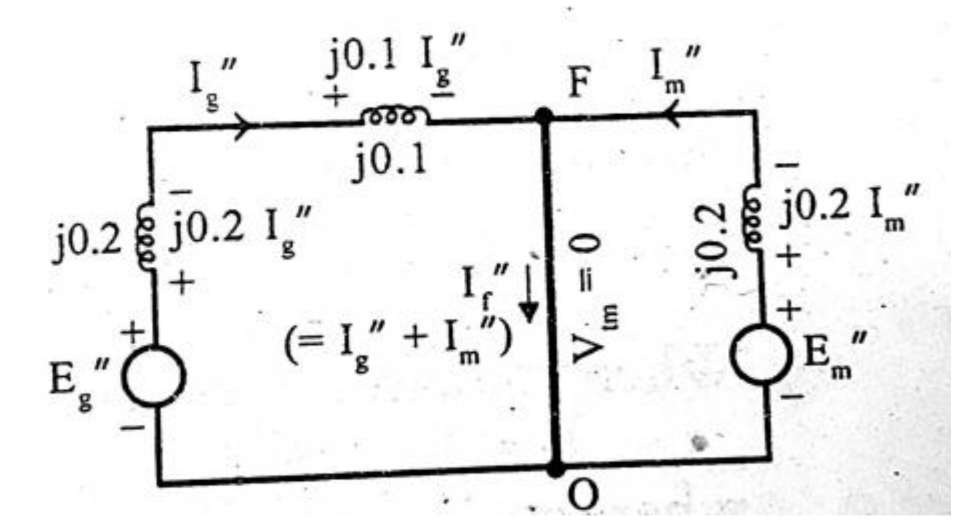


Figure: 3.9

$j0.2\ I_g'' + j0.1\ I_g'' = E_g''$

$j0.3I_g'' = E_g''$

$I_g'' = E_g'' / j0.3$

$= ((0.8406 \angle 14.2°) / (0.3 \angle 90°))$

$= 2.802 \angle 75.8°$ p.u.

$j0.2\ I_m'' = E_m''$

$I_m'' = E_m'' / j0.2$

$= ((1.0817 \angle -7.3°) / (0.2 \angle 90°))$

$= 5.4085 \angle -97.3°$ p.u.

$I_f'' = I_g'' + I_m''$

$= 2.802 \angle 75.8° + 5.4085 \angle -97.3°$

$= 0.687 - j2.716 - 0.687 - j5.365$

$= -j8.081 = 8.081 \angle -90°$ p.u.

Actual Value of fault current can be obtained by multiplying the p.u. values with base current

$I_g'' = 2.802 \angle 75.8° * 1312.16$

$= 3676.67 \angle 75.8°$ A $= 3.67667 \angle 75.8°$ KA

$I_m'' = 5.4085 \angle -97.3° * 1312.16$

$= 7096.8 \angle -97.3°$ A $= 7.0968 \angle -97.3°$ KA

$I_f'' = 8.081 \angle -90° * 1312.16$

$= 10603.56 \angle -90°$ A $= 10.60356 \angle -90°$ KA

Method – 2 : Using thevenin's theorem

To find Fault current

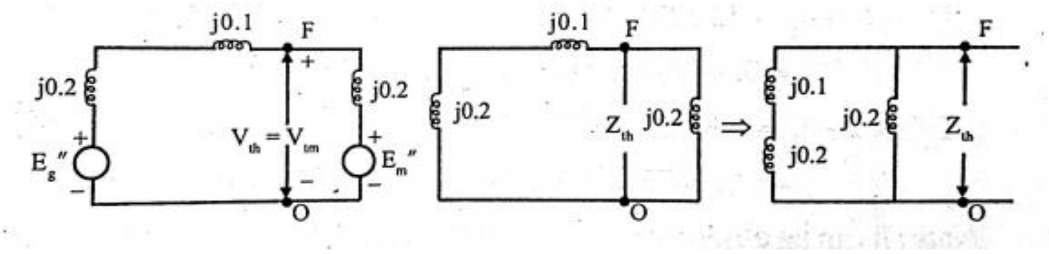


Figure: 3.10

Thevenin's equivalent impedance, Z_{th} = ((j0.1+j0.2)*j0.2) / (j0.1+j0.2)+j0.2

= j0.12

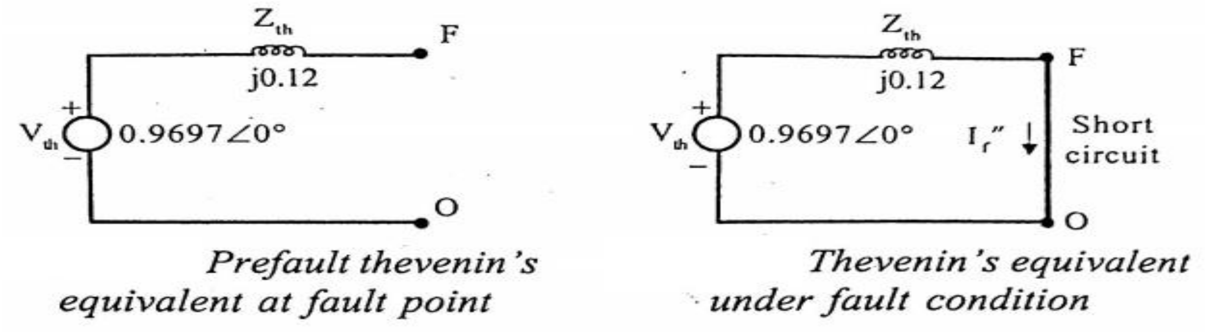


Figure: 3.11

Current in the fault = I_f'' = V_{th} / Z_{th} = 0.9697 $\angle 0°$ / 0.12 $\angle 90°$ = 8.081$\angle$-90° p.u

To find the change in current due to fault

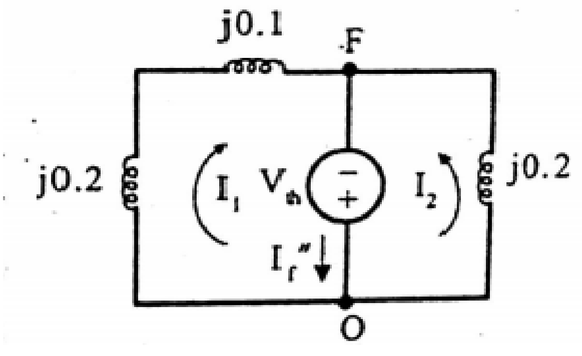


Figure: 3.12

$\mathbf{I_1 = V_{th} / j0.2+j0.1 = 0.9697 \angle 0° / 0.3 \angle 90°}$

$\mathbf{= 3.2323 \angle 90°}$

$\mathbf{I_2 = V_{th} / j0.2 = 0.9697 \angle 0° / 0.2 \angle 90°}$

$\mathbf{= 4.8485 \angle -90°}$

To find the sub - transient fault current in motor and generator

$\mathbf{I_g'' = I_1+I_L = 3.2323 \angle 90° + 0.8594 \angle 36.9° = -j3.2323 +0.6872+j0.516}$

$\mathbf{= 0.6872 -j2.7163 = 2.802 \angle -75.8°}$ **p.u.**

$\mathbf{I_m'' = I_2 - I_L = 4.8485 \angle -90° - 0.8594 \angle 36.9° =-j4.8485 - (0.6872+j0.516)}$

$\mathbf{= -0.6872 -j\,5.3645 = 5.4083 \angle -97.3°}$ **p.u.**

Note: The currents calculated by both the methods are same.

$\mathbf{I_g'' = 2.802 \angle 75.8° * 1312.16 = 3676.67 \angle 75.8°\ A = 3.67667 \angle 75.8°\ KA}$

$\mathbf{I_m'' = 5.4085 \angle -97.3° * 1312.16 = 7096.8 \angle -97.3°\ A = 7.0968 \angle -97.3°\ KA}$

$\mathbf{I_f'' = 8.081 \angle -90° * 1312.16 = 10603.56 \angle -90°\ A = 10.60356 \angle -90°\ KA}$

2. A 3 – Phase, 5MVA, 6.6KV alternator with a reactance of 8% is connected to a feeder of series impedance of 0.12+j0.48 ohms/phase per Km. The transformer is rated at 3 MVA, 6.6KV/33KV and has a reactance of 5%. Determine the fault current supplied by the generator operating under no load with a voltage of 6.9KV, when a 3 – Phase symmetrical fault occurs at a point 15Km along the feeder.

Solution:

Base Values

$\mathbf{MVA_b=5\ MVA}$

$\mathbf{KV_b=6.6KV}$

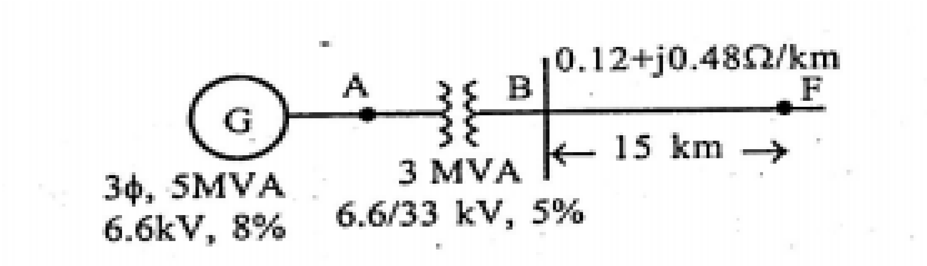


Figure: 3.13

To Find generator reactance

Since the rating of the generator is chosen as base value, the p.u. reactance of the generator will be same as the specified value.

∴ p.u. reactance of the generator, X_d = 8% = 0.08 p.u.

To find transformer reactance

$$X_{pu,new} = X_{pu,old} * \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 * \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

∴ p.u. reactance of transformer, X_T $= 0.05 * \left(\frac{6.6}{6.6}\right)^2 * \left(\frac{5}{3}\right) = 0.0833\,p.u.$

To find feeder reactance

The base impedance, $Z_b = (KV_b)^2 / MVA_b = 33^2 / 5 = 217.8\ \Omega$/ Phase

Actual impedance of the feeder for a length of 15Km

Z_{feed} = impedance / Km * length

= (0.12 + j0.48) * 15 = 1.8+j7.2 Ω/ Phase

∴ p.u. value of the impedance of the feeder,

$Z_{feed,\ p.u.}$ = Actual impedance / Base impedance

=(1.8+j7.2) / 217.8

= 0.0083 + j 0.0331 p.u.

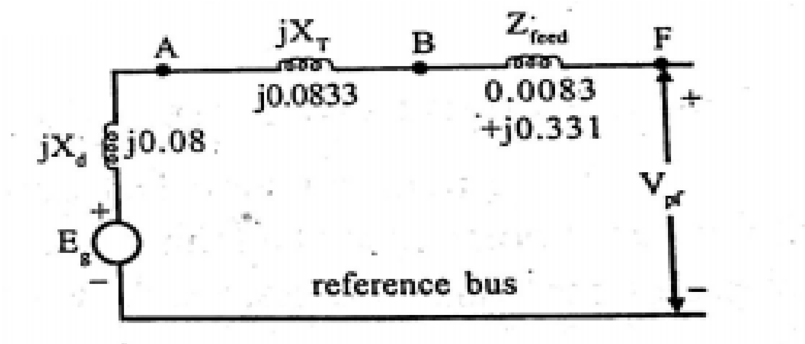


Figure:3.14

To find E_g & V_{pf}

- **Here the generator is not delivering any load current and so the induced emf of the generator will be same as operating voltage**

Actual Value of induced emf, E_g = 6.9 Kv

p.u. Value of induced emf = Actual Value / Base Value

= 6.9 / 6.6 = 1.0455 p.u.

- **The open circuit p.u. value of voltage is same at every point in a series path irrespective of their actual voltages**

$\therefore V_{pf} = 1.0455$ **p.u.**

To find fault current

$Z_{th} = j X_d + j X_T + Z_{feed}$

$= j0.08+j0.0833+0.0083+j0.0331$

$= 0.0083 + j0.1964$ p.u. $= 0.1966 \angle 87.6°$ p.u.

$= 0.1966 \angle 87.6°$ p.u.

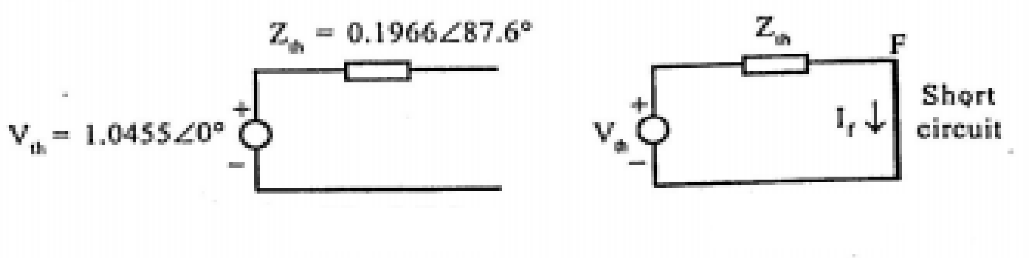


Figure:3.15

∴ **p.u. value of fault current, $I_f = V_{th} / Z_{th} = 1.0455 \angle 0° / 0.1966 \angle 87.6°$**

$= 5.3179 \angle -87.6°$ p.u.

Base current, $I_b = KVA_b / \sqrt{3}\ KV_b = (5 * 1000) / (\sqrt{3} *33) = 87.4773$ A

∴ **Actual Value of fault current, I_f = p.u. value of $I_f * I_b$**

$= (5.3179 \angle -87.6°) * 87.4773$

$= 4652 \angle -87.6°$ amps.

3. For the radial network shown in fig, a 3 – phase fault occurs at point F. Determine the fault current.

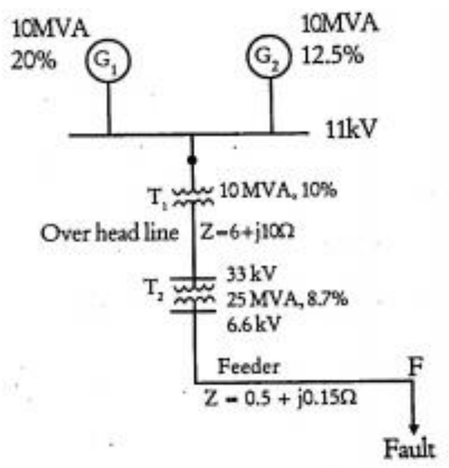


Figure: 3.16

Solution:

Base Values

Choose Generator 1 ratings as base value

MVA_b=10MVA

KV_b=11KV

To find the generator reactances

- **Since the generator ratings are chosen as base values, the p.u. reactance of the generators will remain same**

p.u. reactance of generator-1 , X_{d1} = 20% = 0.2 p.u.

p.u. reactance of generator-2 , X_{d2} = 12.5% = 0.125 p.u.

To find the reactance of T_1

- **The base values referred to LT side of transformer is same as chosen base and so its reactance is same as specified value**

p.u. reactance of transformer – T_1, XT_1 = 10% = 0.1 p.u.

To find the p.u. impedance of overhead line

Base KV on HT side of transformer – T_1 = 11 * (33 / 11)=33KV

Base impedance $Z_b = Kv_b^2 / MVA_b = 33^2 / 10 = 108.9\ \Omega$ / Phase

Actual Impedance of overhead line = 6+j10Ω

∴ p.u. impedance overhead line, Z_{TL} = Actual / Base = (6+j10) / 108.9

= 0.0551 +j0.0918 p.u.

To find the reactance of T_2

$X_{pu,new} = X_{puold} * (KV_{b,old} / KV_{b,new})^2 * (MVA_{b,new} / MVA_{b,old})$

p.u. reactance of transformer T_2, $XT_2 = 0.087 * (33/33)^2 * (10/25)$

= 0.0348 p.u.

To find the p.u. impedance of the feeder

Base KV on LT side of transformer, T_2 = 33 * (6.6 / 33) = 6.6KV

Base impedance, $Z_b = KV_b^2 / MVA_b$ = 6.62 /10 =4.356 Ω/Phase

Actual impedance of feeder =0.5+j0.15 Ω/ Phase

∴ **p.u. impedance of the feeder, Z_{fed} = Actual impedance / Base impedance**

= 0.5 +j0.15 / 4.356 =0.1148 +j0.0344 p.u.

To find thevenin's equivalent at fault point

- **The thevenin's voltage at the fault point is prefault voltage**

$V_{th} = 1\angle 0°$ p.u.

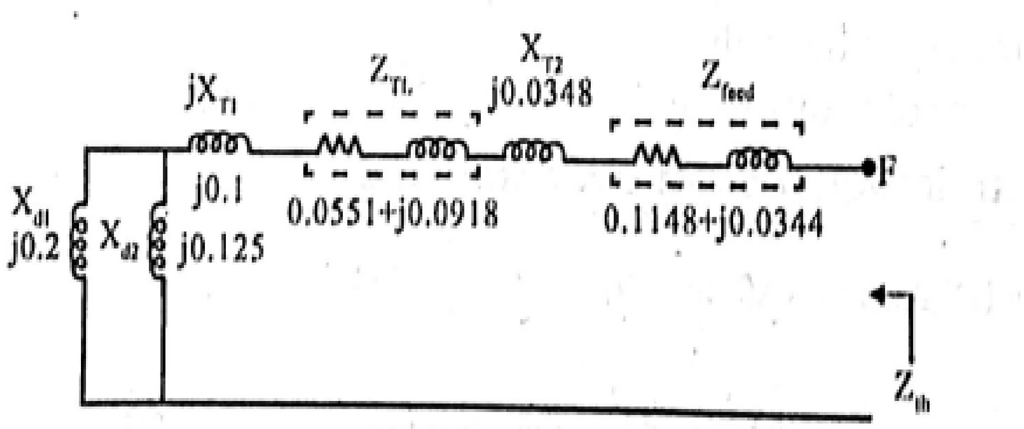


Figure: 3.17

Z_{th} = ((j0.2 * j0.125) / (j0.2 + j0.125))

+(j0.1+0.0551+j0.0918+j0.0348+0.1148+j0.0344)

= j0.0769 +0.1699+j0.261

= 0.1699+j0.3379

=0.3782∠63.3° p.u.

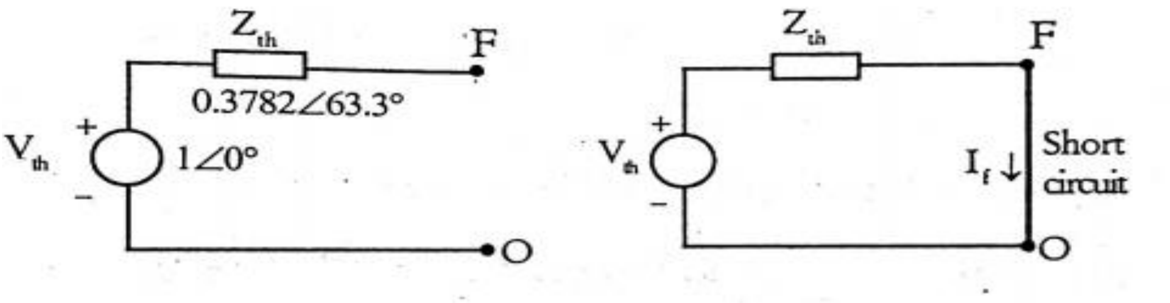


Figure:3.18

To find fault current

p.u. value of fault current, $I_f = V_{th} / Z_{th} = (1\angle 0°) / (0.3782 \angle -63.3°$ p.u.

The base current, $I_b = KVA_b / \sqrt{3} * KV_b$

$= (10 * 1000) / (\sqrt{3} * 6.6)$

$= 874.77A$

The actual value of fault current, I_f = p.u. valu of fault current * Base current

$= (2.6441 \angle -63.3°) * 874.77$

$= 2313 \angle -63.3°$ KA

Bus impedance Matrix in Fault calculations

- **The bus impedance matrix can be used to estimate the fault at any point of the system.**
- **Usually this method is useful for large system**

For a n bus system,

$Z_{bus}\ I = V$

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2k} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{k1} & Z_{k2} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_k \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \\ \vdots \\ V_n \end{bmatrix}$$

Where I_1, I_2,--------I_n are currents injected to buses 1,2,-------,n respectively

V_1, V_2,-------V_n are voltages at buses 1,2, ------,n respectively

Let a three phase fault occur in bus K

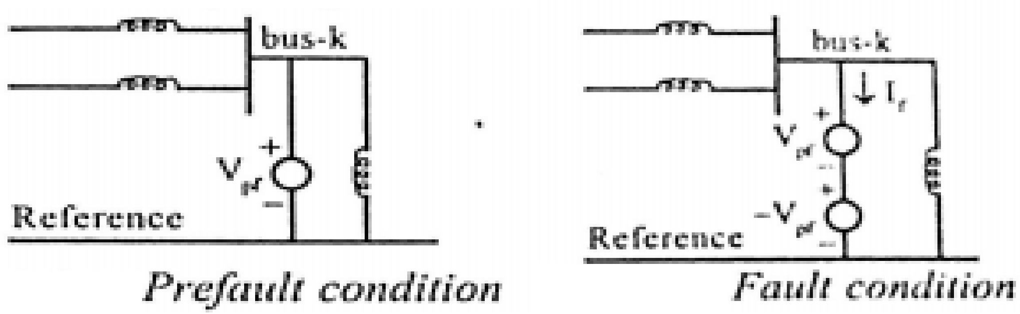


Figure:3.19

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ -V_{pf} \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2k} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{k1} & Z_{k2} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

$$\Delta V_1 = -I_f Z_{1k}$$
$$\Delta V_2 = -I_f Z_{2k}$$
$$\vdots$$
$$-V_{pf} = -I_f z_{kk}$$
$$\vdots$$
$$\Delta V_n = -I_f Z_{nk}$$

∴ The fault current in bus k, $I_f = V_{pf} / Z_{kk}$

- **In general the change in bus q voltage due to three phase fault in bus k is given by**

$$\Delta V_q = -I_f Z_{qk}$$

- **The voltage at a bus after a fault in bus K is given by sum of prefault bus voltage and change in bus voltage**
- **Since the system is unloaded system, the prefault voltage at all buses be**

$$V_{pf} = 1.0 \text{ p.u.}$$

$$V_1 = V_{pf} + (-I_f Z_{1k}) = 1\angle 0^\circ - I_f Z_{1k}$$
$$V_2 = V_{pf} + (-I_f Z_{2k}) = 1\angle 0^\circ - I_f Z_{2k}$$
$$\vdots$$
$$V_k = V_{pf} - V_{pf} = 0$$
$$\vdots$$
$$V_n = V_{pf} + (-I_f Z_{nk}) = 1\angle 0^\circ - I_f Z_{nk}$$

- **The fault current flowing through the lines can be estimated from the knowledge of line impedances**

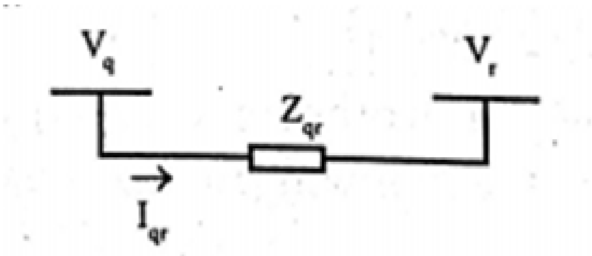


Figure: 3.20

$I_{qr} = (V_q - V_r) / Z_{qr} = (V_q - V_r) * Y_{qr}$

4. The bus impedance matrix of four bus system with values in p.u. is given by,

$$Z_{bus} = j\begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.12 \end{bmatrix}$$

In this system generators are connected to buses 1 and 2 and their subtransient reactances were included when finding Z_{bus}. If prefault current is neglected, find subtransient current in p.u. in the fault of a 3 phase on bus 4. Assume prefault voltage as 1 p.u. If the subtransient reactance of generator in bus 2 is 0.2 p.u. find the subtransient fault current supplied by generator.

Solution:

Let I_f'' be the sub transient current in the fault on bus 4

$I_f'' = V_{pf} / Z_{44}$

$V_{pf} = 1\angle 0°$ p.u.

$\therefore I_f'' = 1\angle 0° / j0.12 = -j8.333 = 8.333 \angle -90°$ p.u.

The voltage at bus 2, when there is a 3 phase fault in bus 4 is given by

$V_2 = V_{pf} + (-I_f'' Z_{24})$

$V_2 = 1\angle 0° + (8.333 \angle -90°) * j0.09 = 1 + 8.333 \angle -90° * 0.09 \angle 90°$

$= 1\text{-}0.74997 = 0.25003 = 0.25\angle 0°$ p.u.

The subtransient fault current delivered by generator at bus 2 ,

$$I_{g2}'' = (E_{g2}'' - V_2) / jX_{d2}''$$

$$= (1\angle 0° - 0.25\angle 0°) / j0.2$$

$$= (1-0.25) / 0.2\angle 90°$$

$$= 3.75\angle -90°$$

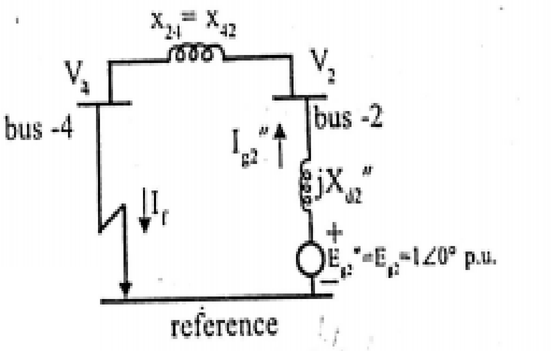


Figure: 3.21

5. **Find the fault current and post fault voltages for the given system shown below and fault being occurred at bus 2.**

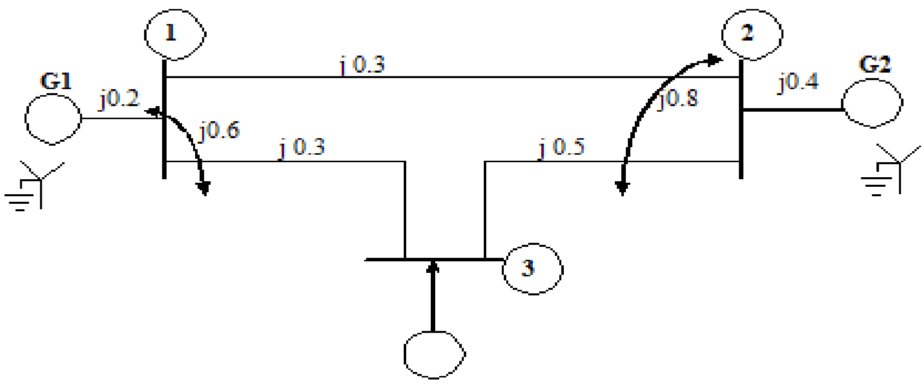


Figure:3.22

Solution:

Node 1 to Node 0 – 1st element

Node 1 to Node 2 - 2nd element

Node 1 to Node 3 – 3rd element

Node 2 to Node 3 – 4th element

Node 2 to Node 0 – 5th element

$$ElementalNodeIncidcenceMatrix(\hat{A}) = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$IncidcenceMatrix(A) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} j0.2 & 0 & j0.6 & 0 & 0 \\ 0 & j0.3 & 0 & j0.8 & 0 \\ j0.6 & 0 & j0.3 & 0 & 0 \\ 0 & j0.8 & 0 & j0.5 & 0 \\ 0 & 0 & 0 & 0 & j0.4 \end{bmatrix}$$

$$ElementalNodeIncidcenceMatrix(\hat{A}) = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$Incidcence\,Matrix(A) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} j0.2 & 0 & j0.6 & 0 & 0 \\ 0 & j0.3 & 0 & j0.8 & 0 \\ j0.6 & 0 & j0.3 & 0 & 0 \\ 0 & j0.8 & 0 & j0.5 & 0 \\ 0 & 0 & 0 & 0 & j0.4 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -\mathrm{j}15 & \mathrm{j}2.083 & \mathrm{j}6.25 \\ \mathrm{j}2.083 & -\mathrm{j}5.33 & \mathrm{j}0.75 \\ \mathrm{j}6.25 & \mathrm{j}0.75 & -\mathrm{j}5.33 \end{bmatrix} \qquad Z_{bus} = inv(Y_{bus})$$

$$Z_{bus} = \begin{bmatrix} \mathrm{j}0.167 & \mathrm{j}0.0945 & \mathrm{j}0.209 \\ \mathrm{j}0.0945 & \mathrm{j}0.245 & \mathrm{j}0.145 \\ \mathrm{j}0.209 & \mathrm{j}0.145 & \mathrm{j}0.452 \end{bmatrix}$$

Fault Bus at n^{th} bus = $V_{pre\ fault}$ / $Z_{bus\ (n,n)}$

Fault is 2^{nd} bus

$V_{pre\ fault}$ = 1.0 pu

:. I_{fault} = 1/ $Z_{(2,2)}$ = 1/ 0.245j = -4.0816j

Δ V(1) = I_{fault} * Z(1,2) = -4.0816j * 0.0945j = -0.386

Δ V(2) = I_{fault} * Z(2,2) = -4.0816j * 0.245j = -1

Δ V(3) = I_{fault} * Z(3,2) = -4.0816j * 0.145j = -0.592

$V(1) = V_{pre\ fault} - \Delta V(1) = 1.0 + (-0.386) = 0.614$

$V(2) = V_{pre\ fault} - \Delta V(2) = 1.0 + (-1.0) = 0.0$

$V(3) = V_{pre\ fault} - \Delta V(3) = 1.0 + (-0.592) = 0.408$

Bus Impedance Matrix

- **The matrix consisting of driving point impedances and transfer impedances of the network of a power system is called bus impedance matrix**
- **It is given by the inverse of bus a bus admittance matrix (Y_{bus}) and it is denoted as Z_{bus}**
- **The bus impedance matrix is symmetrical**
- **Diagonal Elements – Driving point impedances**
- **Off – Diagonal Elements – Transfer impedances**

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

Methods for Forming bus impedance matrix

Method 1: Form the bus admittance matrix (Y_{bus}) and then take its inverse to get bus impedance matrix (Z_{bus})

Method 2: Directly form bus impedance matrix (Z_{bus})from the reactance diagram. This method utilizes the techniques of modifications of existing bus impedance matrix due to addition of new bus (Building Block method)

Forming Z_{bus} using Building Block method

Case i: Adding an element from a new bus to a reference bus

Case ii: Adding an element from a Existing bus to a new bus

Case iii: Adding an element from a Existing bus to a reference bus

Case iv: Adding an element between two existing buses

6. Determine Z_{bus} for system whose reactance diagram is shown in fig where the impedance is given in p.u. preserve all the three nodes

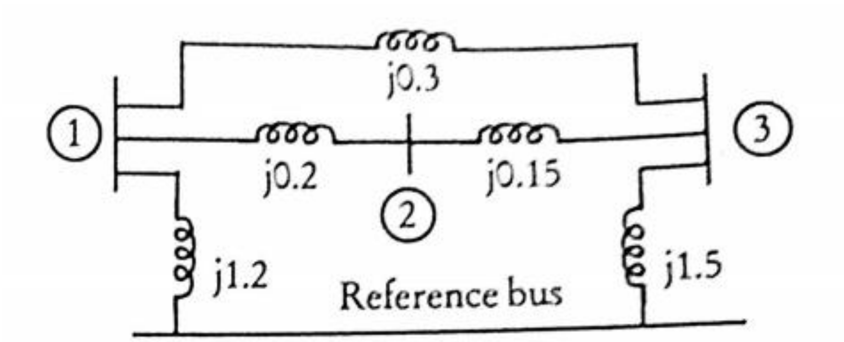
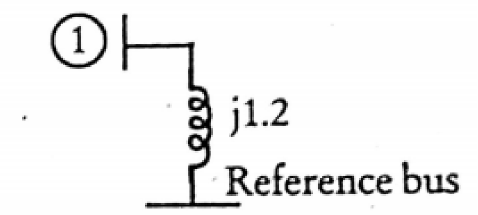
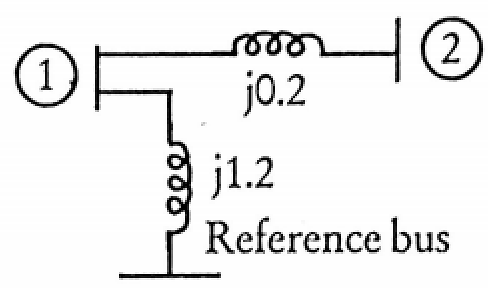


Figure:3.23

Solution:

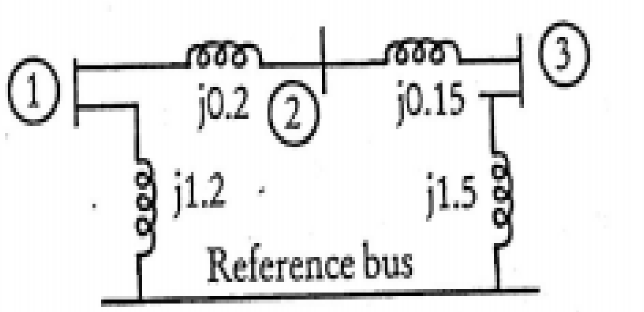


$$Z_{bus} = [j1.2]$$



$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.4 + j0.15 \end{bmatrix}$$

$$= \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$



$$Z_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j1.55 + j1.5 \end{bmatrix}$$

$$Z_{jk,new} = Z_{jk,old} - \frac{Z_{jn} Z_{nk}}{Z_{nn}}$$

$$Z_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

n =4 ; j=1,2,3 ; K=1,2,3

$$Z_{11,new} = Z_{11,old} - \frac{Z_{14} Z_{41}}{Z_{44}} = j1.2 - \frac{j1.2 * j1.2}{j3.05} = j0.728$$

$$Z_{12,new} = Z_{12,old} - \frac{Z_{14} Z_{42}}{Z_{44}} = j1.2 - \frac{j1.2 * j1.4}{j3.05} = j0.649$$

$$Z_{13,new} = Z_{13,old} - \frac{Z_{14} Z_{43}}{Z_{44}} = j1.2 - \frac{j1.2 * j1.55}{j3.05} = j0.590$$

$$Z_{21,new} = Z_{12,new} = j0.649$$

$$Z_{22,new} = Z_{22,old} - \frac{Z_{24} Z_{42}}{Z_{44}} = j1.4 - \frac{j1.4 * j1.4}{j3.05} = j0.757$$

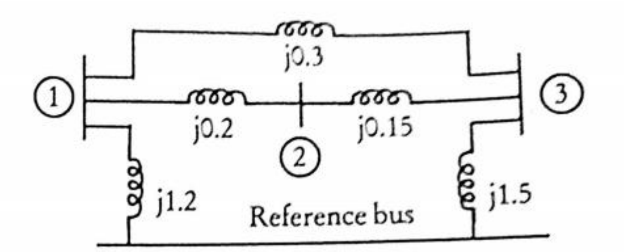
$$Z_{23,new} = Z_{23,old} - \frac{Z_{24} Z_{43}}{Z_{44}} = j1.4 - \frac{j1.4 * j1.55}{j3.05} = j0.689$$

$$Z_{31,new} = Z_{13,new} = j0.590$$

$$Z_{32,new} = Z_{23,new} = j0.689$$

$$Z_{33,new} = Z_{33,old} - \frac{Z_{34} Z_{43}}{Z_{44}} = j1.55 - \frac{j1.55 * j1.55}{j3.05} = j0.762$$

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 \\ j0.649 & j0.757 & j0.689 \\ j0.590 & j0.689 & j0.762 \end{bmatrix}$$



$Z_{44}=Z_{11}+Z_{33}-2*Z_{13}+Z_b$; Where Z_b = j0.3

$\therefore$ **Z_{44}=j0.728 + j0.762 -2(j0.59)+j0.3**

= j0.61

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.728 - j0.590 \\ j0.649 & j0.757 & j0.689 & j0.649 - j0.689 \\ j0.590 & j0.689 & j0.762 & j0.590 - j0.762 \\ j0.728 - j0.590 & j0.649 - j0.689 & j0.590 - j0.762 & j0.61 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.138 \\ j0.649 & j0.757 & j0.689 & -0.04 \\ j0.590 & j0.689 & j0.762 & -j0.172 \\ j0.138 & -0.04 & -j0.172 & j0.61 \end{bmatrix} \qquad Z_{jk,new} = Z_{jk,old} - \frac{Z_{jn}Z_{nk}}{Z_{nn}}$$

n =4 ; j=1,2,3 ; K=1,2,3

$$Z_{11,new} = Z_{11,old} - \frac{Z_{14}Z_{41}}{Z_{44}} = j0.728 - \frac{j1.38 * j0.138}{j0.61} = j0.697$$

$$Z_{12,new} = Z_{12,old} - \frac{Z_{14}Z_{42}}{Z_{44}} = j0.649 - \frac{j0.138 * (-j0.04)}{j0.61} = j0.658$$

$$Z_{13,new} = Z_{13,old} - \frac{Z_{14}Z_{43}}{Z_{44}} = j0.59 - \frac{j0.138 * (-j0.172)}{j0.61} = j0.629$$

$$Z_{21,new} = Z_{12,new} = j0.658$$

$$Z_{22,new} = Z_{22,old} - \frac{Z_{24}Z_{42}}{Z_{44}} = j0.757 - \frac{(-j0.04)*(-j0.04)}{j0.61} = j0.754$$

$$Z_{23,new} = Z_{23,old} - \frac{Z_{24}Z_{43}}{Z_{44}} = j0.689 - \frac{(-j0.04)*(-j0.172)}{j0.61} = j0.678$$

$$Z_{31,new} = Z_{13,new} = j0.629$$

7. Determine Z_{bus} for system whose reactance diagram is shown in fig where the impedance is given in p.u. preserve all the three nodes.

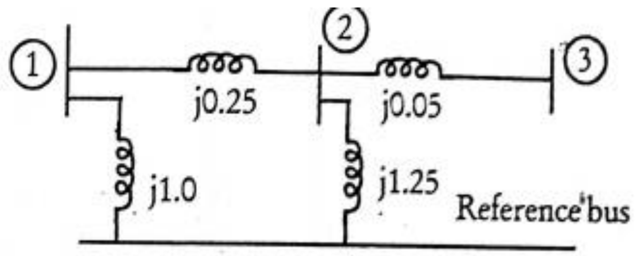
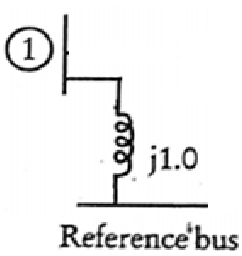
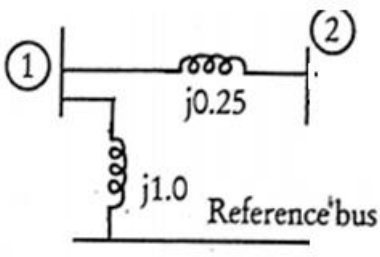


Figure:3.24

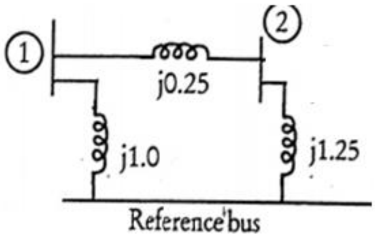
Solution:



$$Z_{bus} = [\ j\ 1.0\ ]$$



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0 + j0.25 \end{bmatrix} \quad Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1..25 \end{bmatrix}$$



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j1.25 + j1.25 \end{bmatrix}$$

$$Z_{jk,new} = Z_{jk,old} - \frac{Z_{jn} Z_{nk}}{Z_{nn}} \qquad Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

n =3 ; j=1,2 ; K=1,2

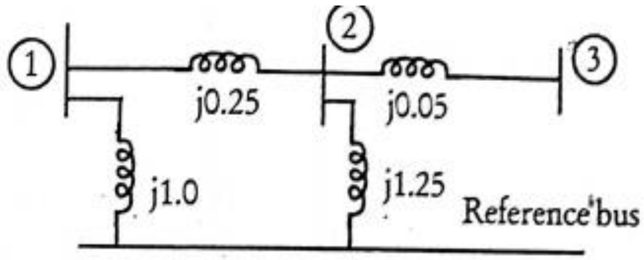
$$Z_{11,new} = Z_{11,old} - \frac{Z_{13} Z_{31}}{Z_{33}} = j1.0 - \frac{j1.0 * j1.0}{j2.5} = j0.6$$

$$Z_{12,new} = Z_{12,old} - \frac{Z_{13} Z_{32}}{Z_{33}} = j1.0 - \frac{j1.0 * j1.25}{j2.5} = j0.5$$

$$Z_{21,new} = Z_{12,new} = j0.5$$

$$Z_{22,new} = Z_{22,old} - \frac{Z_{23} Z_{32}}{Z_{33}} = j1.25 - \frac{j1.25 * j1.25}{j2.5} = j0.625$$

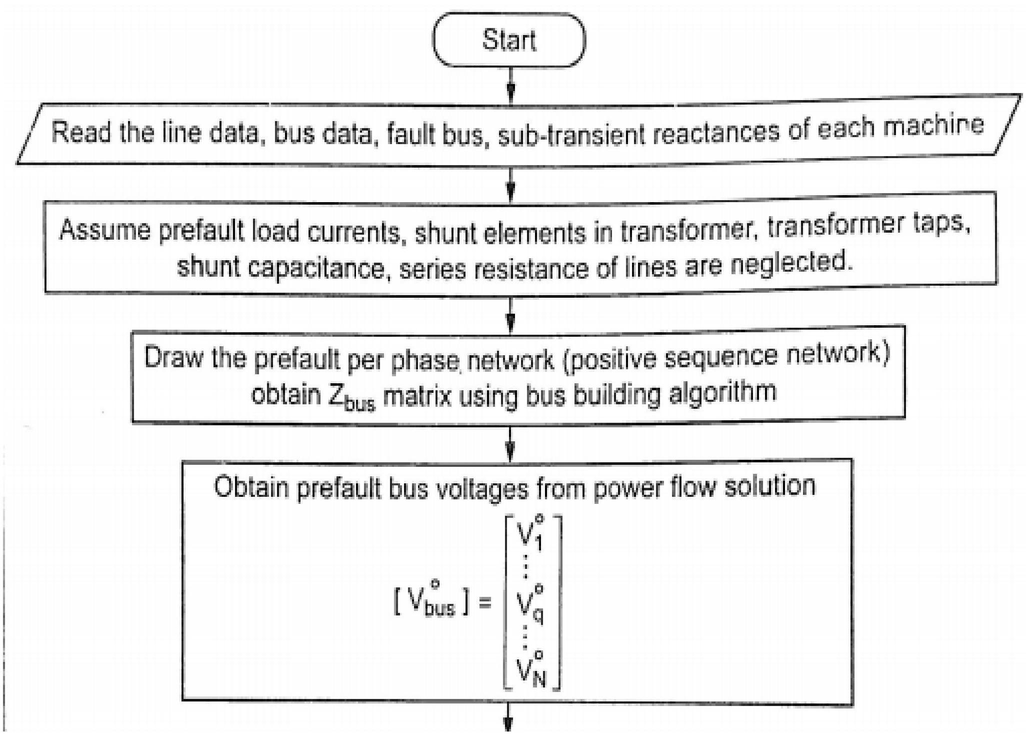
$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j1..625 \end{bmatrix}$$



$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.625 + j0.05 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

Flowchart of Symmetrical Fault Analysis using Z_{bus}



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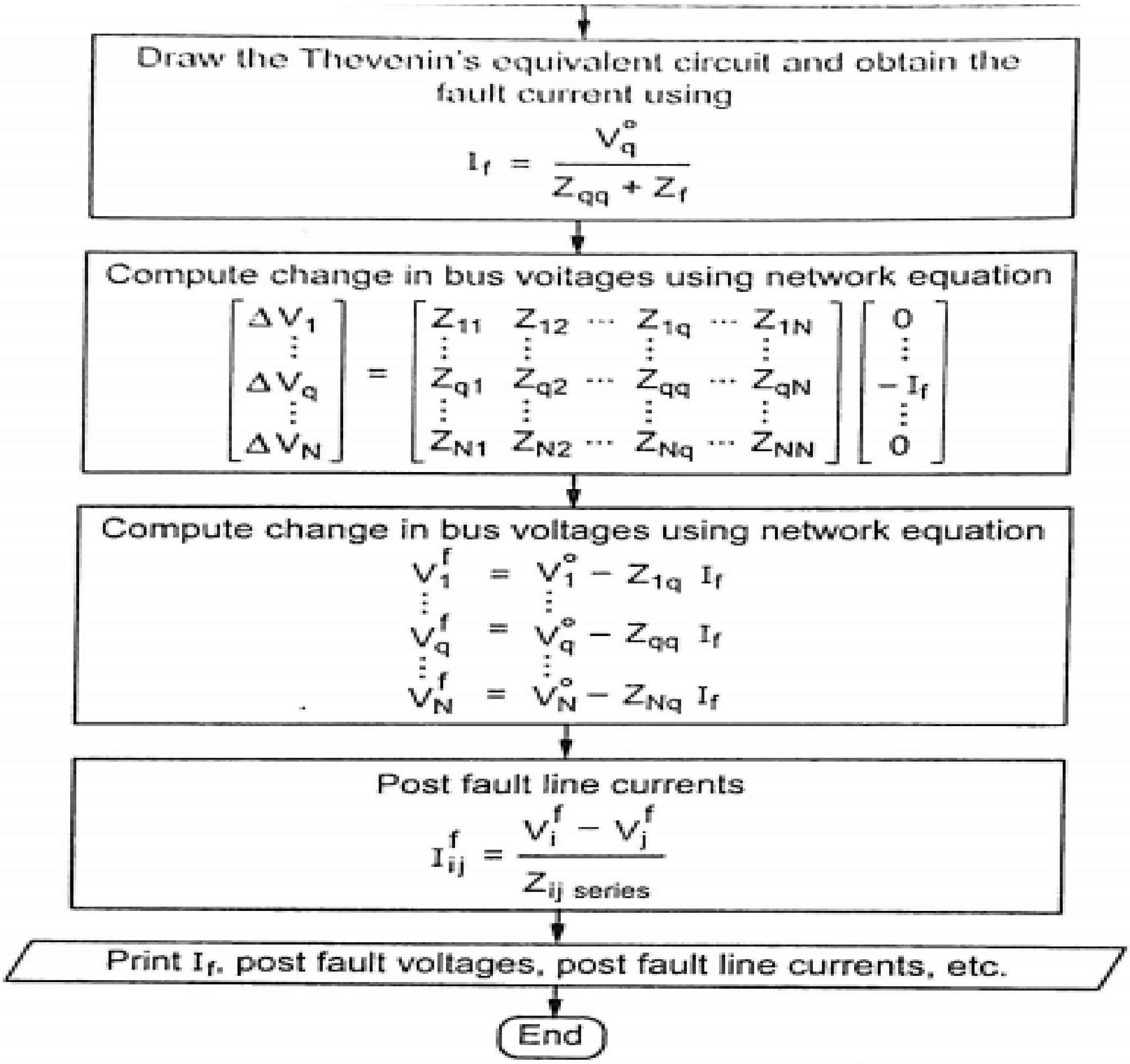

Figure: 3.25

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UNIT – IV – Power System Analysis – SEE1302

UNSYMMETRICAL SHORT CIRCUIT STUDIES

Symmetrical Component transformation – sequence impedance – sequence Networks – Unsymmetrical short circuit analysis for single line fault, line to line fault, double line to ground fault using Z_{bus} – Computations of short circuit capacity, post fault voltage and current

Symmetrical Components

- **An unbalanced system of N related vectors can be resolved into N systems of balanced vectors**
- **The N –sets of balanced vectors are called symmetrical components**
- **Each set consist of N vectors which are equal in length and having equal phase angles between adjacent vectors**

Symmetrical Components of three phase system:

1. **Positive sequence components**
2. **Negative sequence components**
3. **Zero sequence components**

Positive Sequence Components:

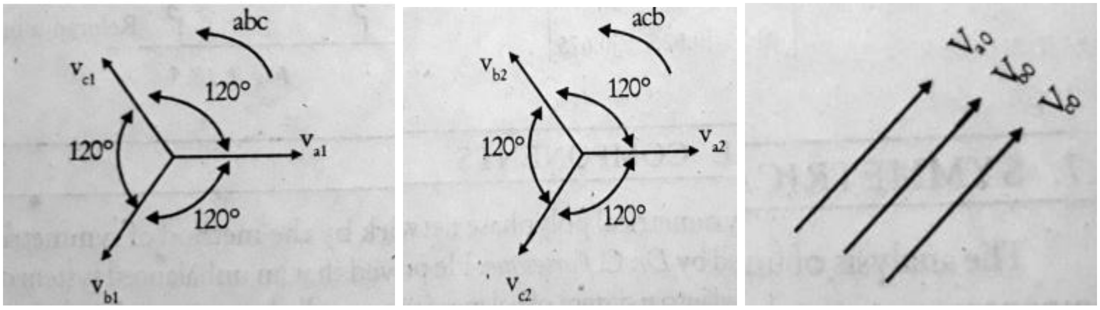
- **The positive sequence components of a 3 phase unbalanced vectors consists of three vectors of equal magnitude, displaced from each other by 120° in phase and having the same phase sequence as the original vectors**

Negative Sequence Components:

- **The negative sequence components of a 3 phase unbalanced vectors consists of three vectors of equal magnitude displaced from each other by 120° in phase and having the phase sequence opposite to that of the original vectors**

Zero Sequence Components:

- **The zero sequence components of a 3 phase unbalanced vectors consists of 3 phase vectors of equal magnitude and with zero phase displacement from each other**



Positive sequence components *Negative sequence components* *Zero sequence components*

Figure 4.1

Let V_a, V_b and V_c be the set of unbalanced voltage vectors with phase sequence abc. Each voltage vector can be resolved into positive, negative and zero sequence components.

Let V_{a1}, V_{b1} & V_{c1} = Positive sequence components of V_a, V_b & V_c respectively with phase sequence abc.

V_{a2}, V_{b2}, V_{c2} = Negative sequence components of V_a, V_b and V_c respectively with phase sequence acb.

V_{a0}, V_{b0} & V_{c0} = Zero sequence components of V_a, V_b & V_c respectively.

The operator **"a"** is defined as,

$$a = 1\angle 120° = 1\, e^{+j2\pi/3} = \cos 2\pi/3 + j\sin 2\pi/3 = -0.5 + j0.866$$

Since, $a = 1\angle 120° = -0.5 + j0.866$

$$a^2 = 1\angle 240° = -0.5 - j0.866$$

$$a^3 = 1\angle 360° = 1$$

$$1 + a + a^2 = 1 + (-0.5 + j0.866) + (-0.5 - j0.866) = 0$$

$$\therefore 1 + a + a^2 = 0$$

Computation of Unbalanced Vectors from their symmetrical components

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

Computation of balanced Vectors from their Unbalanced Vectors

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

1.

The voltages across a 3 phase unbalanced load are $V_a = 300 \angle 20°V$, $V_b = 360 \angle 90°V$ and $V_c = 500 \angle -140°V$. Determine the symmetrical components of voltages. Phase sequence is abc.

Solution:

The symmetrical components of V_a are given by the following matrix equations.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\therefore V_{a0} = \frac{1}{3}\left[V_a + V_b + V_c\right]$$

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2V_c\right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2 V_b + a V_c\right]$$

Given that $V_a = 300 \angle 20°V = 281.91 + j102.61\ V$

$V_b = 360 \angle 90°V = 0 + j360\ V$

$V_c = 500 \angle -140°V = -383.02 - j321.39\ V$

$$\therefore aV_b = 1\angle 120^\circ \times 360\angle 90^\circ = 360\angle 210^\circ = -311.77 - j180\ \text{V}$$

$$a^2V_b = 1\angle 240^\circ \times 360\angle 90^\circ = 360\angle 330^\circ = 311.77 - j180\ \text{V}$$

$$aV_c = 1\angle 120^\circ \times 500\angle -140^\circ = 500\angle -20^\circ = 469.85 - j171.01\ \text{V}$$

$$a^2V_c = 1\angle 240^\circ \times 500\angle -140^\circ = 500\angle 100^\circ = -86.82 + j492.40\ \text{V}$$

$$V_{a0} = \frac{1}{3}\left[V_a + V_b + V_c\right] = \frac{1}{3}(281.91 + j102.61 + 0 + j360 - 383.02 - j321.39)$$

$$= \frac{1}{3}(-101.11 + j141.22) = -33.70 + j47.07 = 57.89\angle 126^\circ\ \text{V}$$

$$V_{a1} = \frac{1}{3}\left[V_a + a\,V_b + a^2\,V_c\right] = \frac{1}{3}(281.91 + j102.61 - 311.77 - j180 - 86.82 + j492.40)$$

$$= \frac{1}{3}(-116.68 + j415.01) = -38.89 + j138.34 = 143.70\angle 106^\circ\ \text{V}$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2\,V_b + a\,V_c\right] = \frac{1}{3}(281.91 + j102.61 + 311.77 - j180 + 469.85 - j171.01)$$

$$= \frac{1}{3}(1063.53 - j248.40) = 354.51 - j82.80 = 364.05\angle -13^\circ\ \text{V}$$

We know that $V_{a0} = V_{b0} = V_{c0}$

$\therefore$ The zero sequence components are

$$V_{a0} = 57.89\angle 126^\circ\ \text{V}$$
$$V_{b0} = 57.89\angle 126^\circ\ \text{V}$$
$$V_{c0} = 57.89\angle 126^\circ\ \text{V}$$

We know that, $V_{b1} = a^2\,V_{a1}$; $V_{c1} = a\,V_{a1}$

$\therefore$ The positive sequence components are

$$V_{a1} = 143.70\angle 106^\circ\ \text{V}$$
$$V_{b1} = a^2\,V_{a1} = 1\angle 240^\circ \times 143.70\angle 106^\circ = 143.70\angle 346^\circ\ \text{V}$$
$$V_{c1} = a\,V_{a1} = 1\angle 120^\circ \times 143.70\angle 106^\circ = 143.70\angle 226^\circ\ \text{V}$$

We know that, $V_{b2} = a\,V_{a2}$; $V_{c2} = a^2\,V_{a2}$

$\therefore$ The negative sequence components are

$$V_{a2} = 364.05\angle -13^\circ\ \text{V}$$
$$V_{b2} = a\,V_{a2} = 1\angle 120^\circ \times 364.05\angle -13^\circ = 364.05\angle 107^\circ\ \text{V}$$
$$V_{c2} = a^2\,V_{a2} = 1\angle 240^\circ \times 364.05\angle -13^\circ = 364.05\angle 227^\circ\ \text{V}$$

2.

The symmetrical components of phase-a fault current in a 3-phase unbalanced system are $I_{a0} = 350\angle 90°$ A, $I_{a1} = 600\angle -90°$ A and $I_{a2} = 250\angle 90°$ A. Determine the phase currents I_a, I_b and I_c.

Solution:

The currents I_a, I_b and I_c are given by the following matrix equations.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\therefore I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

Given that $I_{a0} = 350\angle 90° = 0 + j350$

$I_{a1} = 600\angle -90° = 0 - j600$

$I_{a2} = 250\angle 90° = 0 + j250$

$$\therefore a I_{a1} = 1\angle 120° \times 600\angle -90° = 600\angle 30° = 519.62 + j300$$

$$a^2 I_{a1} = 1\angle 240° \times 600\angle -90° = 600\angle 150° = -519.62 + j300$$

$$a I_{a2} = 1\angle 120° \times 250\angle 90° = 250\angle 210° = -216.51 - j125$$

$$a^2 I_{a2} = 1\angle 240° \times 250\angle 90° = 250\angle 330° = 216.51 - j125$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = j350 - j600 + j250 = 0$$

$$I_b = I_{a0} + a^2 I_{a1} + aI_{a2} = j350 - 519.62 + j300 - 216.51 - j125$$
$$= -736.13 + j525 = 904.16\angle 145° \text{ A}$$

$$I_c = I_{a0} + a\,I_{a1} + a^2\,I_{a2} = j350 + 519.62 + j300 + 216.51 - j125$$
$$= 736.13 + j525 = 904.16\angle 35° \text{ A}$$

Sequence Impedance and sequence Networks:

- **The sequence impedances are the impedances offered by the devices or components for the like sequence component of the current**
- **The single phase equivalent circuit of a power system consisting of impedances to current of any one sequence only is called sequence network**

Positive , Negative and Zero sequence impedances:

- **The impedance of a circuit element for positive, negative and zero sequence component currents are called positive, negative and zero sequence impedance respectively**

Positive , Negative and Zero sequence reactance diagram:

- **The reactance diagram of a power system, when formed using positive, negative and zero sequence reactances are called positive, negative and zero sequence reactance diagram respectively**

Sequence Impedances and networks of generator:

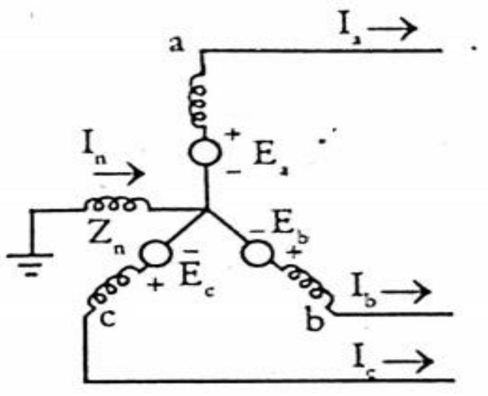


Figure: 4.2

Let E_a, E_b, E_c = Generated emf per phase in phase a, b and c respectively. (Positive sequence emf)

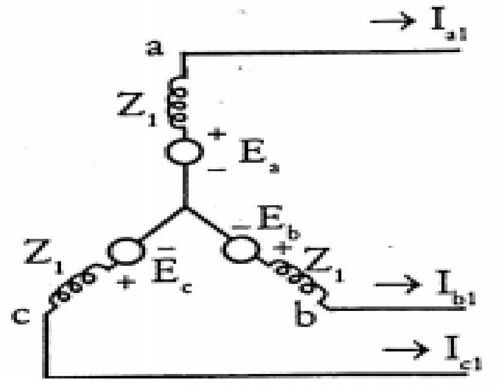
Z_1 = Positive sequence impedance per phase of generator.

Z_2 = Negative sequence impedance per phase of generator.

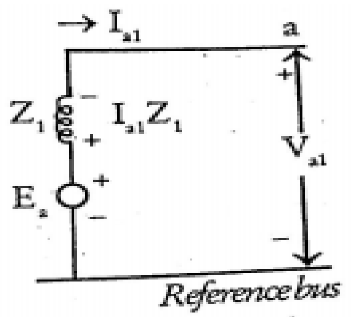
Z_{g0} = Zero sequence impedance per phase of generator.

Z_n = Neutral reactance.

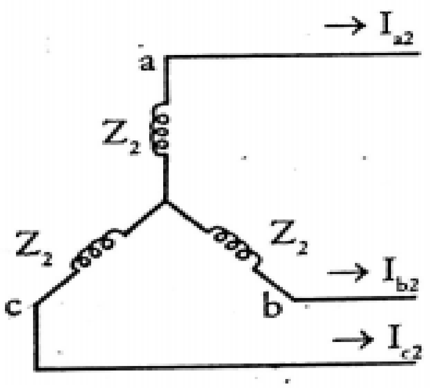
Z_0 = Total zero sequence impedance per phase of zero-sequence network of generator.



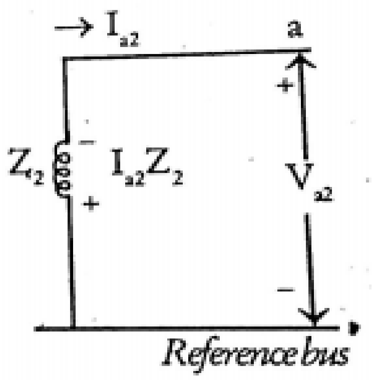
Positive-sequence current paths.



Positive-sequence network



Negative-sequence current paths.



Negative-sequence network

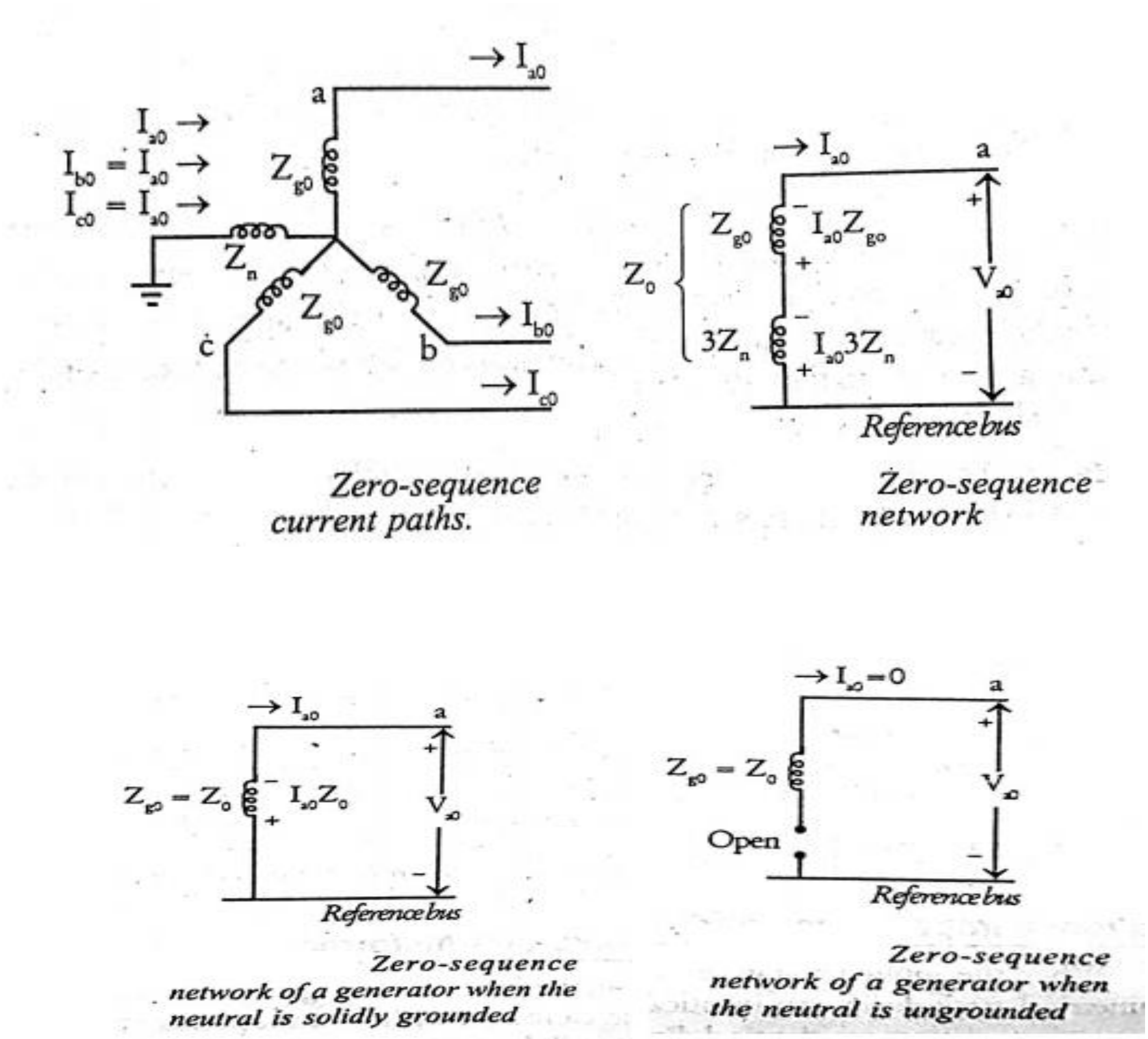


Figure:4.3

Sequence Impedances and networks of Transmission lines

Let, Z_1 = Positive sequence impedance of transmission line
Z_2 = Negative sequence impedance of transmission line
Z_0 = Zero sequence impedance of transmission line

The value of $Z_1 = Z_2$; Z_0 = 2 to 3.5 times the Z_1

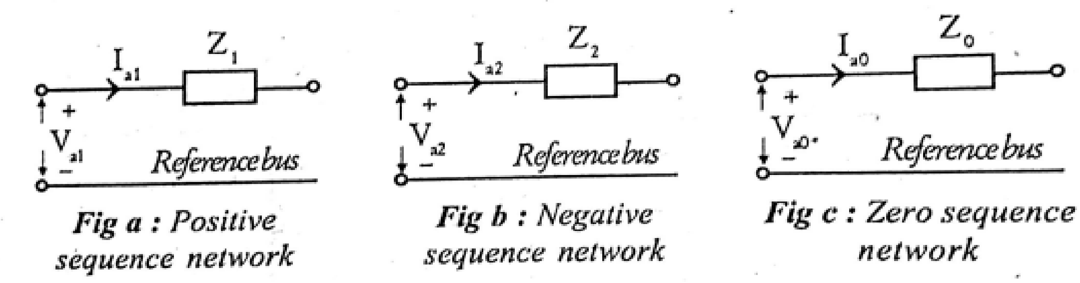


Figure: 4.4

Sequence Impedances and networks of Transformer:

Let, Z_1 = Positive sequence impedance of transformer

Z_2 = Negative sequence impedance of transformer

Z_0 = Zero sequence impedance of transformer

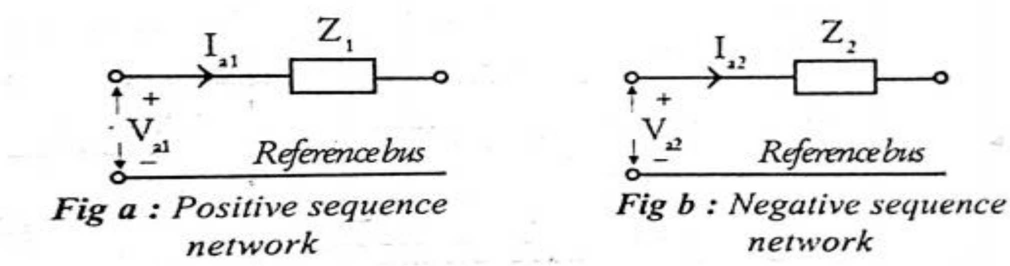
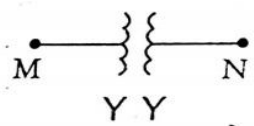
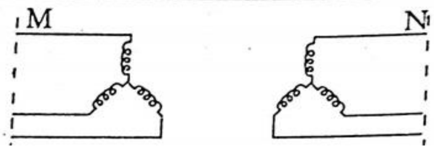
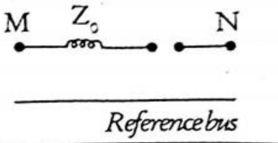


Figure: 4.5

The value of $Z_1 = Z_2 = Z_0$

Zero Sequence network of three phase transformer:

Configuration	Winding Connection Diagram	Zero sequence Network
 M N Y Y	 M N	 M Z_0 N Reference bus

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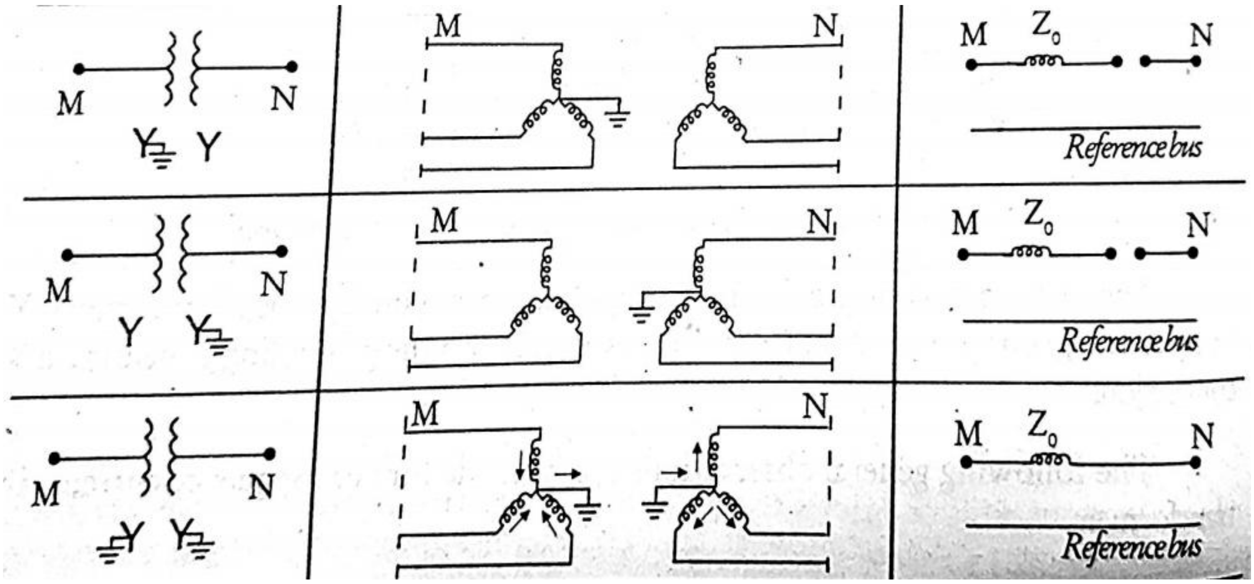
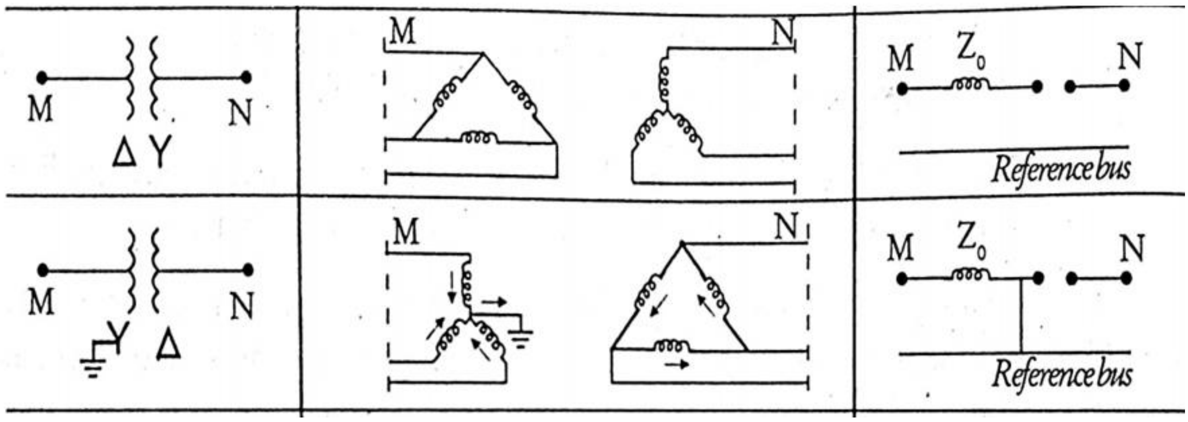
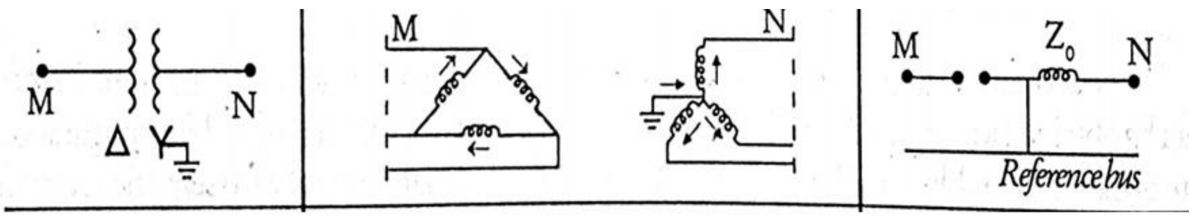






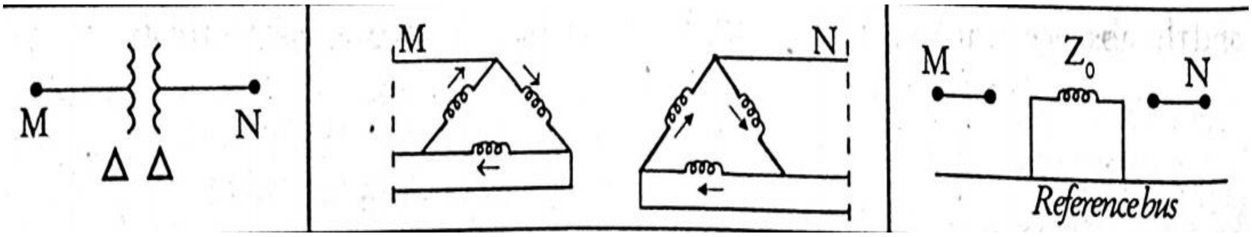


Figure: 4.6

Sequence Impedances and networks of Loads:

Let Z_{L1} = Positive sequence impedance of load
Z_{L2} = Negative sequence impedance of load
Z_{L0} = Zero sequence impedance of load.

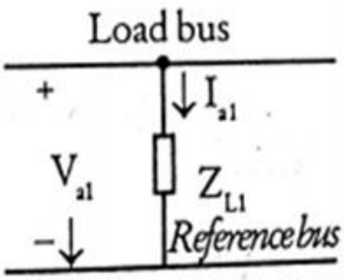


Fig a : Positive sequence network

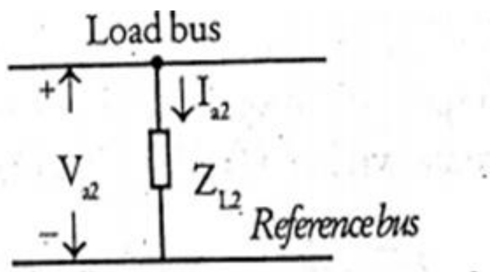
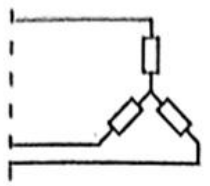
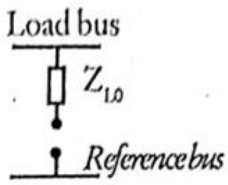
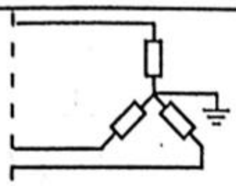
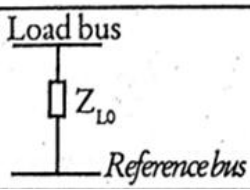


Fig b : Negative sequence network

Zero sequence networks of loads

Connection Diagram of load	Zero sequence network
	
	

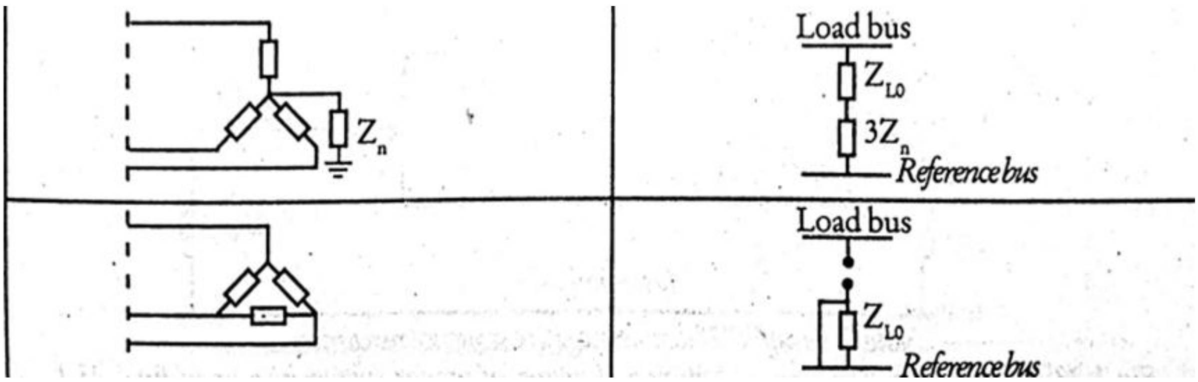


Figure: 4.7

3.

Determine the positive, negative and zero sequence networks for the system shown in fig 1.25.1. Assume zero sequence reactances for the generator and synchronous motors as 0.06 p.u. Current limiting reactors of 2.5 Ω are connected in the neutral of the generator and motor No.2. The zero sequence reactance of the transmission line is j300Ω.

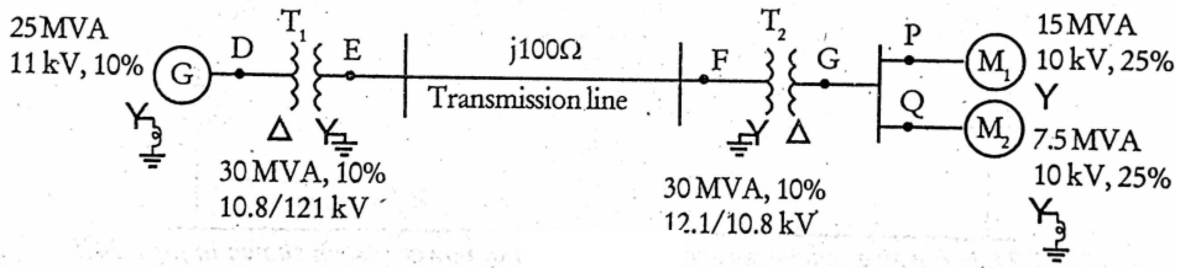


Figure: 4.8

Solution:

Let us choose the generator ratings as new base values for entire system.

Base megavoltampere, $MVA_{b,\,new} = 25\ MVA$

Base kilovolt, $kV_{b,\,new} = 11\ kV$

Sequence reactances of Generator G

Since the generator rating and the new base values are same, the generator p.u. reactances does not change. Also for generator the positive and negative sequence reactances are same.

$\therefore$ Positive sequence reactance of generator, $X_{G,1} = 10\,\% = 10/100 = 0.1$ p.u.

Negative sequence reactance of generator, $X_{G,2} = 0.1$ p.u.

Zero sequence reactance of generator, $X_{G,0} = 0.06$ p.u.

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{11^2}{25} = 4.84\Omega$$

$$\left.\begin{array}{l}\text{p.u. value of generator}\\ \text{neutral reactance}\end{array}\right\} X_{GN} = \frac{\text{Actual Neutral reactance}}{\text{Base impedance}} = \frac{2.5}{4.84} = 0.517 \text{ p.u.}$$

Sequence reactances of Transformer T_1

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Here, $X_{pu,old} = 10\% = 0.1$, $\quad kV_{b,old} = 10.8$ kV, $\quad MVA_{b,old} = 30$ MVA

$kV_{b,new} = 11$ kV, $\quad MVA_{b,new} = 25$ MVA

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of transformer } T_1\end{array}\right\} = 0.1 \times \left(\frac{10.8}{11}\right)^2 \times \left(\frac{25}{30}\right) = 0.08 \text{ p.u.}$$

In transformer the specified reactance is positive sequence reactance. Also we assume that the positive, negative and zero sequence reactances of the transformer are equal.

$\therefore$ Positive sequence reactance of transformer T_1, $X_{T1,1} = 0.08$ p.u.

Negative sequence reactance of transformer T_1, $X_{T1,2} = 0.08$ p.u.

Zero sequence reactance of transformer T_1, $X_{T1,0} = 0.08$ p.u.

Sequence reactances of Transmission line

$$\left.\begin{array}{l}\text{The base kV on HT side}\\ \text{of transformer } T_1\end{array}\right\} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$= 11 \times \frac{121}{10.8} = 123.24 \text{ kV}$$

Now, $kV_{b,new} = 123.24$ kV

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{(123.24)^2}{30} = 506.27\Omega$$

$$\left.\begin{array}{l}\text{p.u. reactance of}\\ \text{transmission line}\end{array}\right\} = \frac{\text{Actual reactance}}{\text{Base impedance}} = \frac{100}{506.27} = 0.198 \text{ p.u.}$$

The specified reactance in single line diagram is positive sequence reactance. Also the negative sequence reactance of a transmission line is same as that of positive sequence reactance.

$\therefore$ Positive sequence reactance of transmission line, $X_{TL,1} = 0.198$ p.u

Negative sequence reactance of transmission line, $X_{TL,2} = 0.198$ p.u

$$\left.\begin{array}{l}\text{p.u. value of zero sequence}\\ \text{reactance of transmission line}\end{array}\right\} X_{TL,0} = \frac{\text{Zero sequence reactance in }\Omega}{\text{Base impedance}} = \frac{300}{506.27} = 0.593 \text{ p.u.}$$

Sequence reactances of Transformer T_2

The ratings and winding connections of transformer T_1 and T_2 are identical and so the sequence reactances of T_1 and T_2 are same.

Positive sequence reactance of transformer T_2, $X_{T2,1} = 0.08$ p.u.

Negative sequence reactance of transformer T_2, $X_{T2,2} = 0.08$ p.u.

Zero sequence reactance of transformer T_2, $X_{T2,0} = 0.08$ p.u.

Sequence reactances of Synchronous motor M_1

$$\left.\begin{array}{l}\text{Base kV on LT side}\\ \text{of transformer } T_2\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 123.24 \times \frac{10.8}{121} = 11 \text{ kV}$$

Now, $kV_{b,new} = 11$ kV

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Here, $X_{pu,old} = 25\% = 0.25$; $kV_{b,old} = 10$ kV ; $MVA_{b,old} = 15$ MVA

$kV_{b,new} = 11$ kV ; $MVA_{b,new} = 25$ MVA

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_1\end{array}\right\} = 0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = 0.344 \text{ p.u.}$$

The reactance specified in single line diagram is positive sequence reactance. Also the negative sequence reactance of synchronous motor is same as that of positive sequence reactance.

$\therefore$ Positive sequence reactance of motor M_1, $X_{M1,1} = 0.344$ p.u.

Negative sequence reactance of motor M_1, $X_{M1,2} = 0.344$ p.u.

$$\left.\begin{array}{l}\text{The zero sequence reactance}\\ \text{of motor } M_1 \text{ on new bases}\end{array}\right\} X_{M1,0} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = 0.083 \text{ p.u.}$$

Sequence reactances of Synchronous motor M_2

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Here, $X_{pu,old} = 25\% = 0.25$; $kV_{b,old} = 10$ kV ; $MVA_{b,old} = 7.5$ MVA
$kV_{b,new} = 11$ kV ; $MVA_{b,new} = 25$ MVA

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_2\end{array}\right\} = 0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = 0.689 \text{ p.u.}$$

The reactance specified in single line diagram is positive sequence reactance. Also the negative sequence reactance of synchronous motor is same as that of positive sequence reactance.

$\therefore$ Positive sequence reactance of motor M_2, $X_{M2,1} = 0.689$ p.u.

Negative sequence reactance of motor M_2, $X_{M2,2} = 0.689$ p.u.

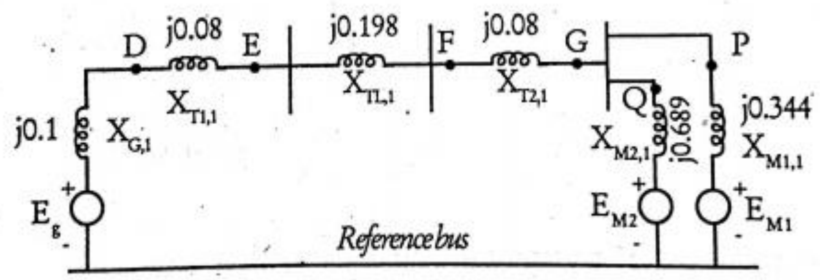
$$\left.\begin{array}{l}\text{The zero sequence reactance}\\ \text{of motor } M_1 \text{ on the new bases}\end{array}\right\} X_{M2,0} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = 0.165 \text{ p.u.}$$

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{11^2}{25} = 4.84\Omega$$

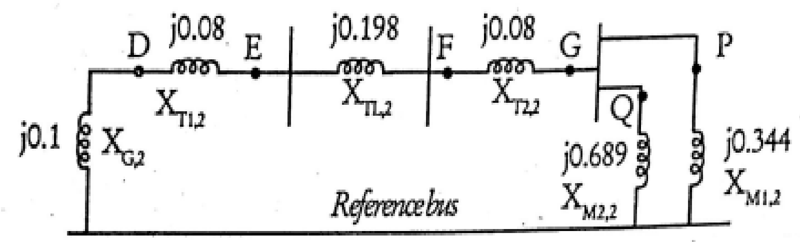
$$\left.\begin{array}{l}\text{p.u. value of motor}\\ \text{neutral reactance}\end{array}\right\} X_{MN} = \frac{\text{Actual neutral reactance}}{\text{Base impedance}} = \frac{2.5}{4.84} = 0.517 \text{ p.u.}$$

Positive sequence network



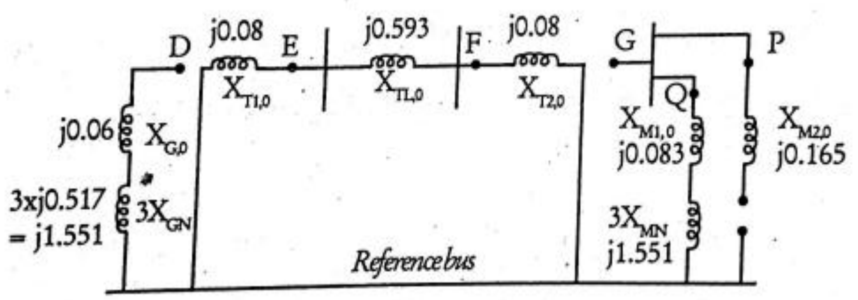
Positive sequence reactance diagram of the power system.

Negative sequence network



Negative sequence reactance diagram of the power system

Zero sequence network



Zero sequence reactance diagram of the power system.

Figure:4.9

Review of symmetrical components of unbalanced voltages and currents:

Computation of Unbalanced Vectors from their symmetrical components

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

Computation of balanced Vectors from their Unbalanced Vectors

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Review of Sequence networks of a generator:

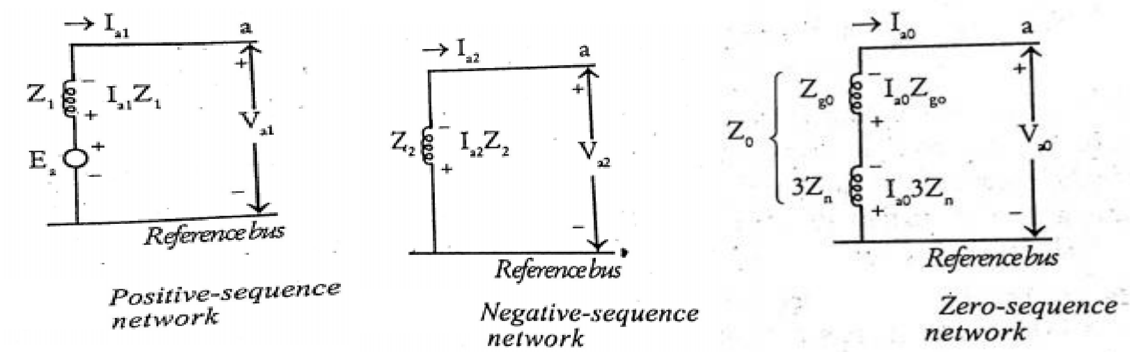
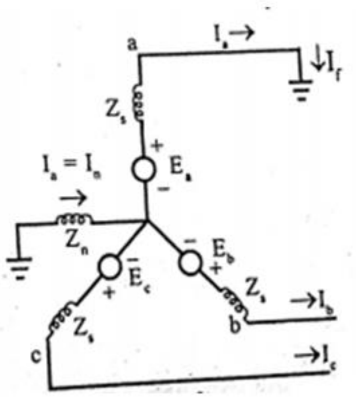


Figure: 4.10

$$\begin{aligned} V_{a0} &= -I_{a0}Z_0 \\ V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \end{aligned}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Single Line to Ground fault on an unloaded generator



Circuit diagram for single line-to-ground fault on phase-a of an unloaded generator

Figure: 4.11

The condition at the fault is expressed by the following equations

$$I_b = 0 \;;\quad I_c = 0 \;;\quad V_a = 0$$

The symmetrical components of the currents are given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_b = I_c = 0 \quad \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_{a0} = I_{a1} \text{ and } I_{a2} = I_{a1}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix}$$

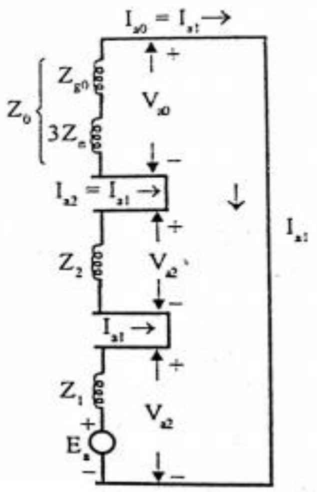
$$\begin{aligned} V_{a0} &= -Z_0 I_{a1} \\ V_{a1} &= E_a - Z_1 I_{a1} \\ V_{a2} &= -Z_2 I_{a1} \end{aligned} \qquad \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_{a0} + V_{a1} + V_{a2} = -I_{a1} Z_0 + E_a - I_{a1} Z_1 - I_{a1} Z_2$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

$$-I_{a1} Z_0 + E_a - I_{a1} Z_1 - I_{a1} Z_2 = 0$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$



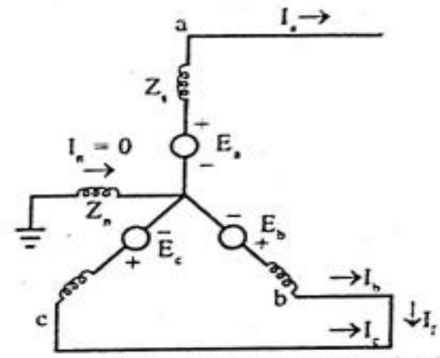
Connection of sequence network of an unloaded generator for line-to-ground fault on phase -a

$$I_f = I_a = 3\, I_{a1}$$

Figure: 4.12

If the neutral of the generator is not grounded, the zero-sequence network is open-circuited and Z_0 is infinite. Under this condition I_{a1} is zero and so I_{a2} & I_{a0} must be zero. Therefore no path exists for the flow of current in the fault unless the generator neutral is grounded.

Line to Line fault on an unloaded generator:



Circuit diagram for double line to ground fault between phase b & c in an unloaded generator

The conditions at the fault are expressed by the following equations

$$V_b = V_c \ ; \ I_a = 0 \ ;$$

$$I_b + I_c = 0 \Rightarrow I_b = -I_c$$

$$I_f = I_b = -I_c.$$

$$V_c = V_b \, ,$$

Figure: 4.13

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_{a1} = \frac{1}{3} (V_a + aV_b + a^2V_b)$$

$$V_{a1} = V_{a2} \, .$$

$$V_{a2} = \frac{1}{3} (V_a + a^2V_b + aV_b)$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_b = -I_c \text{ and } I_a = 0$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix}$$

$$I_{a0} = \frac{1}{3}\left[-I_c + I_c\right] = 0$$

$$I_{a1} = \frac{1}{3}\left[-a\, I_c + a^2 . I_c\right] \qquad I_{a2} = -I_{a1}$$

$$I_{a2} = \frac{1}{3}\left[-a^2\, I_c + a\, I_c\right] = -\frac{1}{3}\left[-a\, I_c + a^2\, I_c\right]$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

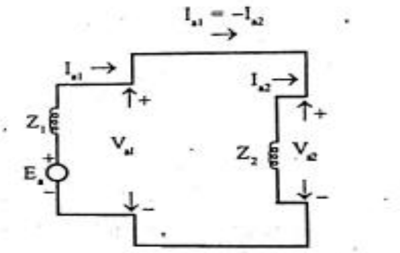
$; I_{a0} = 0 \text{ and } I_{a2} = -I_{a1}$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

$$V_{a0} = 0$$
$$V_{a1} = E_a - Z_1 I_{a1}$$
$$V_{a2} = Z_2 I_{a1} \qquad ; V_{a1} = V_{a2}.$$

$$E_a - Z_1 I_{a1} = Z_2 I_{a1}$$
$$\text{(or)} \quad I_{a1}(Z_1 + Z_2) = E_a$$
$$\therefore I_{a1} = \frac{E_a}{Z_1 + Z_2}$$

Since $V_{a1} = V_{a2}$, here the positive and negative sequence networks of the generator must be in parallel. Since $V_{a0} = 0$, the zero sequence network is shorted and so it need not be considered. (or Since Z_b does not enter into the equations, the zero-sequence network is not used). Since this type of fault does not involve ground, the neutral current $I_n = 0$. Hence the presence or absence of a grounded neutral at the generator does not affect the fault current.



Connection of the sequence networks of an unloaded generator for a line-to-line fault between phases b and c

Figure: 4.14

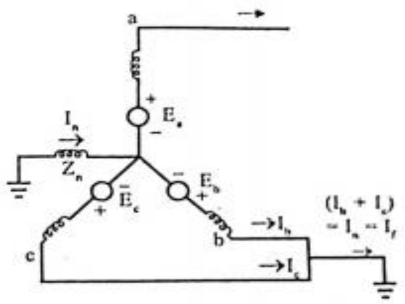
the fault current, $I_f = I_b = -I_c$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad I_b = I_{a0} + a^2 I_{a1} + a I_{a2} \quad I_{a0} = 0 \text{ and } I_{a2} = -I_{a1}$$

$$I_b = a^2 I_{a1} - a I_{a2} = I_{a1}(a^2 - a)$$

$$\therefore \text{Fault current, } I_f = I_b = I_{a1}(a^2 - a)$$

Double Line to Ground fault on an unloaded generator:



Circuit diagram for double line-to-ground fault between phase b & c in an unloaded generator

The conditions at the fault are expressed by the following equations

$$V_b = 0, \quad V_c = 0, \quad I_a = 0.$$

$$I_f = I_b + I_c. \quad V_b = V_c = 0,$$

Figure: 4.15

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = V_{a1} = V_{a2} = \frac{V_a}{3}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a0} = V_{a1} = V_{a2}$$

$$\begin{bmatrix} E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \\ E_a - I_{a1} Z_1 \end{bmatrix}$$

$$\text{Let, } \mathbf{Z} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \qquad \therefore \mathbf{Z}^{-1} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{E_a}{Z_1} \\ 0 \end{bmatrix} - \begin{bmatrix} \dfrac{E_a - I_{a1}Z_1}{Z_0} \\ \dfrac{E_a - I_{a1}Z_1}{Z_1} \\ \dfrac{E_a - I_{a1}Z_1}{Z_2} \end{bmatrix}$$

$$I_{a0} = -\left(\frac{E_a - I_{a1}Z_1}{Z_0}\right) = -\frac{E_a}{Z_0} + \frac{I_{a1}Z_1}{Z_0}$$

$$I_{a1} = \frac{E_a}{Z_1} - \left(\frac{E_a - I_{a1}Z_1}{Z_1}\right) = \frac{E_a}{Z_1} - \frac{E_a}{Z_1} + \frac{I_{a1}Z_1}{Z_1} = I_{a1}$$

$$I_{a2} = -\left(\frac{E_a - I_{a1}Z_1}{Z_2}\right) = -\frac{E_a}{Z_2} + \frac{I_{a1}Z_1}{Z_2}$$

$\because I_a = 0,$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$-\frac{E_a}{Z_0} + \frac{I_{a1}Z_1}{Z_0} + I_{a1} - \frac{E_a}{Z_2} + \frac{I_{a1}Z_1}{Z_2} = 0$$

$$I_{a1}\left(\frac{Z_1}{Z_0} + 1 + \frac{Z_1}{Z_2}\right) = \frac{E_a}{Z_0} + \frac{E_a}{Z_2}$$

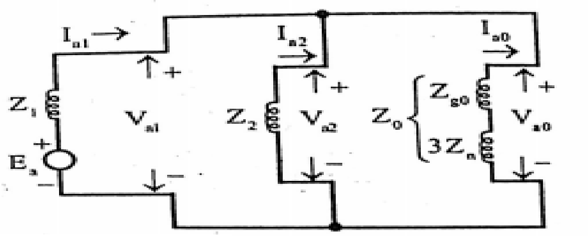
$$I_{a1}\left(1 + Z_1\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right)\right) = E_a\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right)$$

$$I_{a1}\left(1 + Z_1\left(\frac{Z_0 + Z_2}{Z_0Z_2}\right)\right) = E_a\left(\frac{Z_0 + Z_2}{Z_0Z_2}\right)$$

On multiplying throughout by $\frac{Z_0Z_2}{Z_0+Z_2}$ we get,

$$I_{a1}\left(\frac{Z_0Z_2}{Z_0+Z_2} + Z_1\right) = E_a$$

$$\therefore I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0Z_2}{Z_0+Z_2}}$$



Connection of the sequence networks of an unloaded generator for a double line-to-ground fault on phase b and c.

Under this fault conditon the sequence networks should be connected in parallel since the positive, negative, and zero-sequence voltages are equal during this fault. In the absence of a ground connection at the generator no current can flow into the ground at the fault. In this case Z_0 would be infinite and I_{a0} would be zero and so the fault will be similar to line-to-line fault.

Figure:4.16

$$I_f = I_b + I_c$$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} \quad ; \quad I_{a0} = -\frac{E_a}{Z_0} + \frac{I_{a1} Z_1}{Z_0} \quad ; \quad I_{a2} = -\frac{E_a}{Z_2} + \frac{I_{a1} Z_1}{Z_2}$$

$$I_{a2} = -I_{a1} \times \frac{Z_0}{Z_0 + Z_2} \ \& \ I_{a0} = -(I_{a1} - I_{a2})$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$
$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$$\therefore \text{Fault current, } I_f = I_b + I_c = I_{a0} + a^2 I_{a1} + a I_{a2} + I_{a0} + a I_{a1} + a^2 I_{a2}$$
$$= 2 I_{a0} + (a + a^2) I_{a1} + (a + a^2) I_{a2}$$
$$= 2 I_{a0} + (a + a^2)(I_{a1} + I_{a2})$$

Unsymmetrical Faults on Power Systems

Let Z_1 = Thevenin's impedance of positive sequence network.
Z_2 = Thevenin's impedance of negative sequence network.
Z_0 = Thevenin's impedance of zero sequence network.
V_{pf} = Prefault voltage at the fault point.

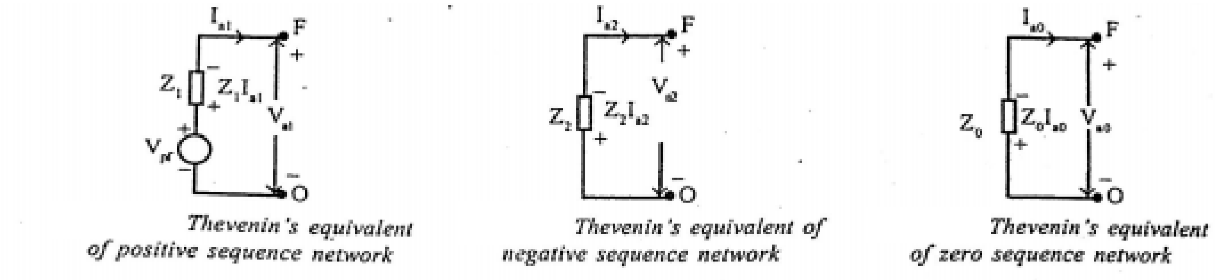
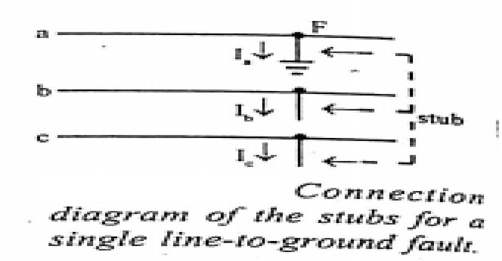


Figure:4.17

$$V_{a0} = -Z_0 I_{a0}$$
$$V_{a1} = V_{pf} - Z_1 I_{a1}$$
$$V_{a2} = -Z_2 I_{a2}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_{pf} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Single Line to Ground fault on Power System

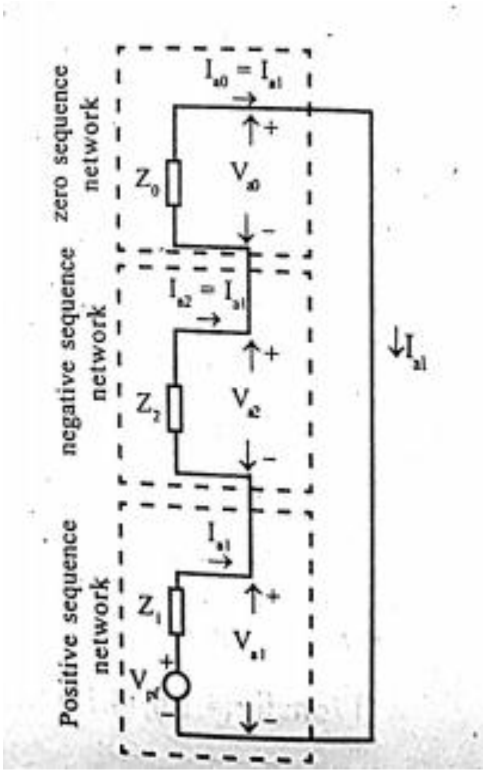


$$I_b = 0; \quad I_c = 0 \quad ; \quad V_a = 0$$

$$I_{a1} = I_{a2} = I_{a0} = \frac{I_a}{3}$$

$$I_{a1} = \frac{V_{pf}}{Z_1 + Z_2 + Z_0}$$

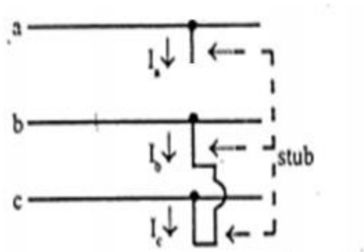
Figure: 4.18



Connection of sequence networks for a single line-to-ground fault in a power system

Figure: 4.19

Line to Line fault on Power System



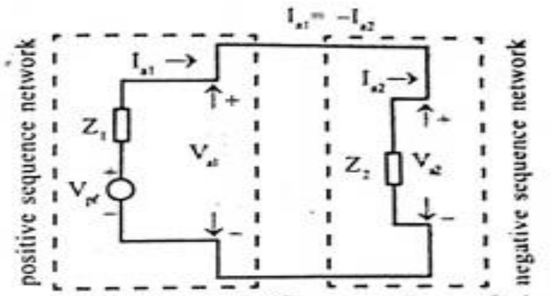
Connection diagram of the stubs for a line-to-line fault.

$$V_b = V_c \quad I_a = 0 \quad I_b = -I_c$$

$$V_{a1} = V_{a2}$$

$$I_{a1} = \frac{V_{pf}}{Z_1 + Z_2}$$

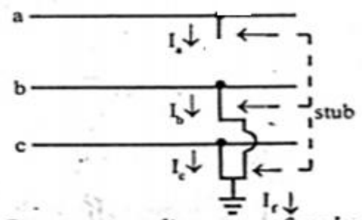
Figure: 4.20



Connection of the sequence networks for a line-to-line fault in a power system.

Figure: 4.21

Double Line to Ground fault on Power System



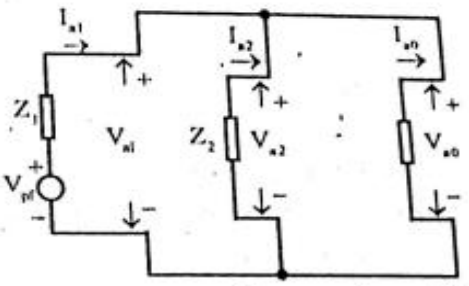
Connection diagram of stubs for a double line-to-ground fault in power system

Figure: 4.22

$$V_b = V_c = 0 \;;\; I_a = 0$$

$$V_{a1} = V_{a2} = V_{a0}$$

$$I_{a1} = \frac{V_{pf}}{Z_1 + (Z_2 Z_0 / (Z_2 + Z_0))}$$

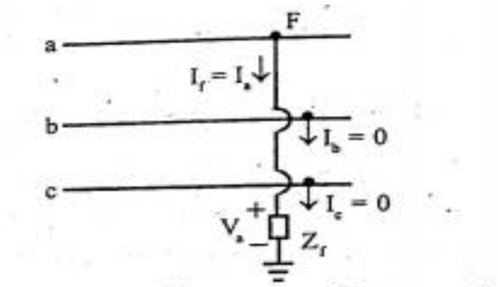


Connection of the sequence networks for a double line-to-ground fault in a power system.

Figure: 4.23

Unsymmetrical Faults on Power Systems through Impedance

Single Line to Ground fault on Power System through Impedance:



Connection diagram of stubs for a single line-to ground fault through an impedance

Figure: 4.24

At the fault point F, the following relations exist

$$I_b = 0; \quad I_c = 0\ ; V_a = Z_f I_a$$

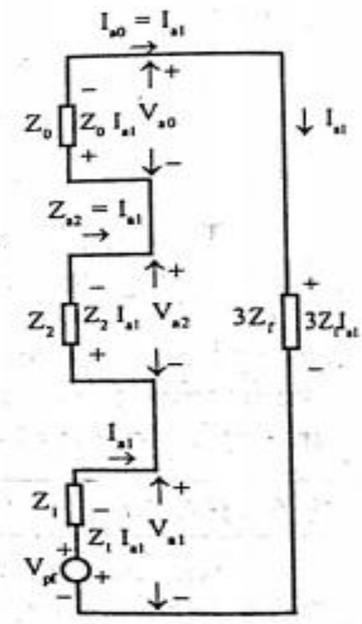
$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} I_a$$

$$I_a = 3\, I_{a1}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad V_a = V_{a1} + V_{a2} + V_{a0}$$

$$V_{a1} + V_{a2} + V_{a0} = Z_f I_a = 3Z_f\ I_{a1}$$



Connection of sequence networks for a single line to ground fault through an impedance Z_f

Figure: 4.25

$$I_{a1} = \frac{V_{pf}}{(Z_1 + Z_2 + Z_0) + 3Z_f}$$

$$\text{Fault current, } I_f = I_a = 3I_{a1} = \frac{3V_{pf}}{(Z_1 + Z_2 + Z_0) + 3Z_f}$$

$$V_{a1} = V_{pf} - Z_1 I_{a1}$$

$$V_{a2} = -Z_2 I_{a1}$$

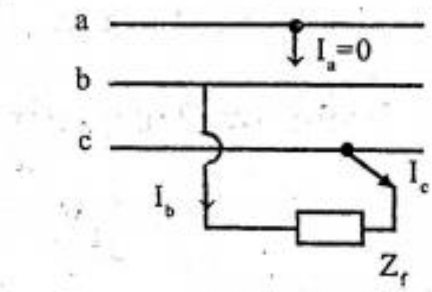
$$V_{a0} = -Z_0 I_{a1}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_{pf} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

From the symmetrical components of phase-a voltage, the phase voltages V_a, V_b and V_c are calculated using the following matrix equation.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

Line to Line fault on Power System through Impedance:



Connection diagram of stubs for a line-to-line fault through an impedance

$I_a = 0$ and $I_b = -I_c$

Also $V_b - V_c = I_b Z_f$; $\therefore V_c = V_b - I_b Z_f$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

Figure: 4.26

$$I_{a0} = \frac{1}{3}(I_b - I_b) = 0$$

$$I_{a1} = \frac{1}{3}(aI_b - a^2 I_b)$$

$$I_{a2} = \frac{1}{3}(a^2 I_b - aI_b) = -I_{a1}$$

$I_{a0} = 0$ and $I_{a2} = -I_{a1}$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

$$I_b = a^2 I_{a1} - aI_{a1} = I_{a1}(a^2 - a) = I_{a1}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3}\, I_{a1}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_c = V_b - I_b Z_f$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b - I_b Z_f \end{bmatrix}$$

From row-2 of matrix

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2 V_b - a^2 I_b Z_f\right]$$

$$\therefore 3V_{a1} = V_a + V_b(a + a^2) - a^2 Z_f I_b$$

From row-3 of matrix

$$V_{a2} = \frac{1}{3}\left[V_a + a^2 V_b + aV_b - aI_b Z_f\right]$$

$$\therefore 3V_{a2} = V_a + V_b(a^2 + a) - a\, Z_f I_b$$

$$3V_{a1} - 3V_{a2} = V_a + V_b(a + a^2) - a^2 Z_f I_b - V_a - V_b(a^2 + a) + aZ_f I_b$$
$$= (-a^2 + a) Z_f I_b$$
$$= \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) Z_f I_b = j\sqrt{3} Z_f I_b$$
$$\therefore V_{a1} - V_{a2} = \frac{1}{3} j\sqrt{3} Z_f I_b = j\frac{1}{\sqrt{3}} Z_f I_b$$

$$V_{a1} - V_{a2} = j\frac{1}{\sqrt{3}} Z_f\left(-j\sqrt{3} I_{a1}\right) = Z_f I_{a1}$$

$$\therefore V_{a1} - V_{a2} = Z_f I_{a1}$$

$$I_{a1} = \frac{V_{pf}}{Z_1 + Z_2 + Z_f}$$

The fault current, $I_f = I_b = -j\sqrt{3}\, I_{a1}$

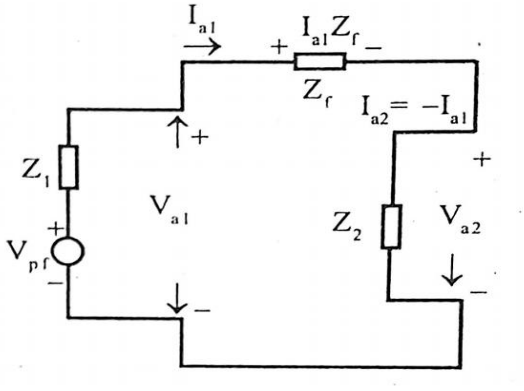


Figure: 4.27

Double line to ground fault on Power System through Impedance:

A double line to ground fault at point F in a power system, through a fault impedance Z_f can be represented by connecting three stubs as shown in Fig

The current and voltage conditions at the fault are

$I_a=0$

$V_b=V_c=Z_f(I_b+I_c)$

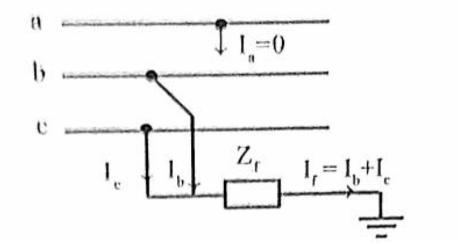


Figure: 4.28

The line currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$I_a=I_{a0}+I_{a1}+I_{a2}$

Since, $I_a=0$

$I_{a0}+I_{a1}+I_{a2}=0$ (or) $I_{a0}=-(I_{a1}+I_{a2})$

$I_b=I_{a0}+a^2I_{a1}+aI_{a2}$

$Ic=I_{a0}+aI_{a1}+a^2I_{a2}$

$I_b+I_c=I_{a0}+a^2I_{a1}+aI_{a2}+I_{a0}+aI_{a1}+a^2I_{a2}$

$=2I_{a0}+(a^2+a)I_{a1}+(a^2+a)I_{a2}$

W.K.T, $1+a+a^2=0,$ $\therefore$ $a+a^2=-1$

$I_b+I_c=2I_{a0}-I_{a1}-I_{a2}$

$= 2I_{a0}-(I_{a1}+I_{a2})$

$=2I_{a0}-(-I_{a0})$

$= 3I_{a0}$

The Symmetrical components of voltages after substituting $V_c=V_b$ **are given by**

$V_{a0}=(1/3)[V_a+V_b+V_b] =(1/3)[V_a+2V_b]$

$V_{a1}=V_{a2}= (1/3)[V_a+aV_b+a^2V_b]$

$=(1/3)[V_a+(a+a^2)V_b] = (1/3)[V_a-V_b]$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_{a0}-V_{a1}=(1/3)[V_a+2V_b] - (1/3)[V_a-V_b]$$

$$= (1/3)\,[V_a+2V_b-V_a+V_b] = V_b$$

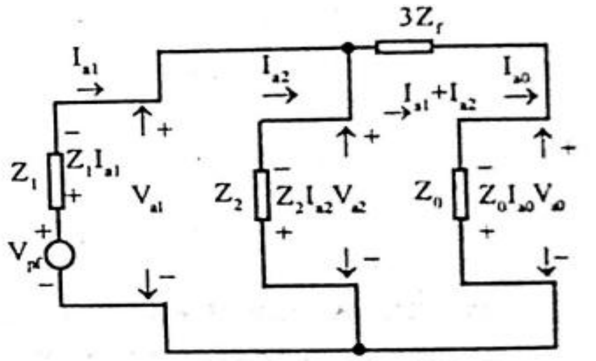
$$\therefore\ V_{a0}-V_{a1}=V_b$$

$$V_{a0}-V_{a1}=Z_f(I_b+I_c)$$

$$V_{a0}-V_{a1}=Z_f 3I_{a0}$$

$$\therefore\ V_{a0}=V_{a1}+3Z_fI_{a0} \qquad \text{(Since } V_{a1}=V_{a2})$$

Also, $V_{a0} = V_{a2}+3Z_fI_{ao}$



Connection of sequence networks for a double line-to-ground fault through an impedance

$$I_{a1} = \frac{V_{pf}}{Z_1 + \dfrac{Z_2(Z_0+3Z_f)}{Z_2+Z_0+3Z_f}}$$

Figure: 4.29

$$V_{a1}=V_{pf} - Z_fI_{a1}$$

$$V_{a2}=V_{a1}$$

$$I_{a2}=(V_{a2}/Z_2)$$

$$I_{a0}= -(I_{a1}+I_{a2})$$

Fault current, $I_f=I_b+I_c = 3I_{a0}$

4. Two 11 KV, 20MVA, three Phase star connected generators operate in parallel as shown in Figure. The positive, negative and zero sequence reactance's of each being respectively, j0.18,j0.15,j0.10 p.u. The star point of one of the generator is isolated and

that of the other is earthed through a 2.0Ω resistor. A single line to ground fault occurs at the terminals of one of the generators. Estimate (i) fault current, (ii) current in grounded resistor and (iii) voltage across grounding resistor

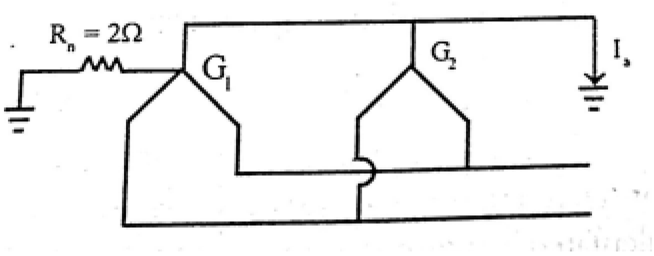


Figure: 4.30

Solution:

MVA_b=20MVA

KV_b=11KV

$Z_b = (KV_b)^2/MVA_b = (11)^2 / 20 = 6.05\ \Omega$

p.u. value of neutral resistance = Actual Value / Base Impedance

= 2/6.05 =0.3306 p.u

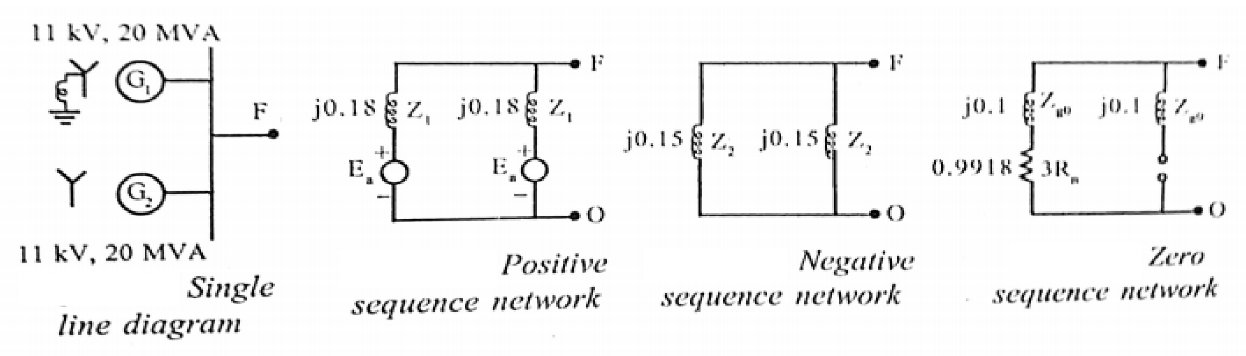


Figure: 4.31

The thevenin's equivalent of the sequence networks are shown as

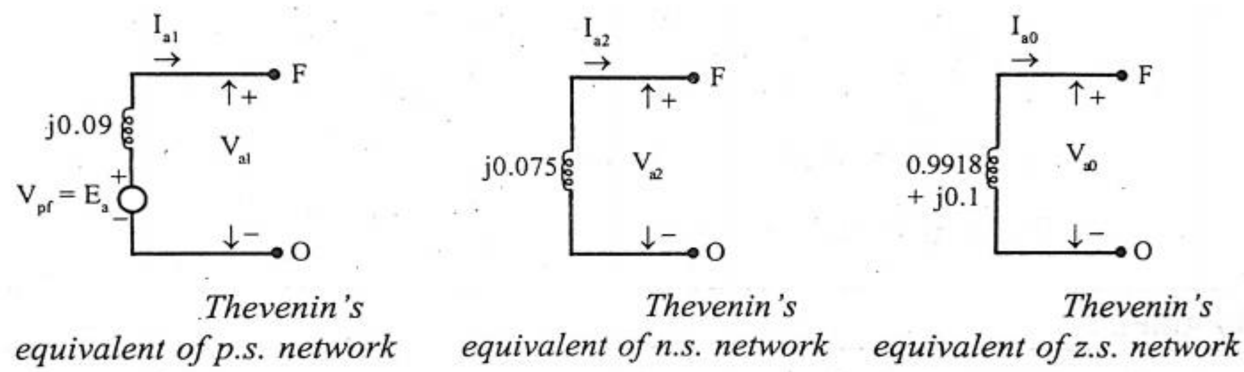


Figure: 4.32

For a single line to ground fault,

$I_{a1} = I_{a2} = I_{a0}$

$I_f = I_a = 3I_{a1}$

Hence the thevenin's equivalent of sequence networks are connected in series as shown in fig

The fault current is calculated by taking the prefault voltage V_{pf}=1 p.u.

From Fig,

$I_{a1} = 1\angle 0° \,/\, (j0.09+j0.075+0.9918+j0.1)$

$= 1 / (0.9918+j0.265)$

$= 1 / (1.0266 \angle 15°)$

$=0.9714 \angle -15°$ p.u.

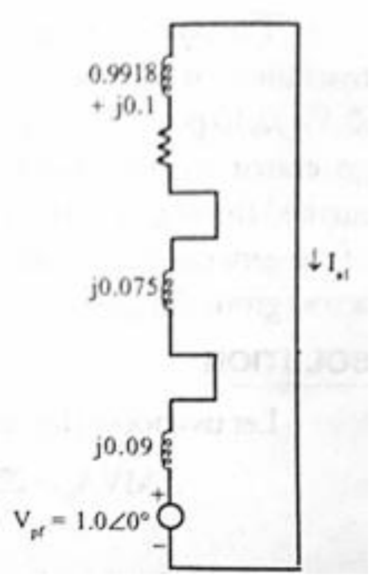


Figure: 4.33

i. **To find fault current**

Fault current, $I_f = I_a = 3I_{a1}$

$= 3 * 0.9741\angle -15°$

$= 2.9223 \angle -15°$ **p.u.**

Base current, $I_b = KVA_b / (\sqrt{3}*KV_b) = (20 * 1000) / (\sqrt{3}*11) = 1049.7$ A

∴ **Actual value of fault current = p.u. value of fault current * Base current**

$= 2.9223 \angle -15°$ **p.u. or** $3.0675 \angle -15°$ **KA**

ii. **To find the current through neutral resistor**

The current through the neutral resistor is same as that of fault current

∴ **Current through neutral resistor =** $2.9223 \angle -15°$ **p.u. or** $3.0675 \angle -15°$ **KA**

iii. **To find the voltage across grounding resistor**

From the thevenin's equivalent of zero sequence network, we get

The Voltage across grounding resistor = $3R_n I_{a0} = 3R_n I_{a1}$

$= 3*0.3306*0.9741\angle -15° = 0.9661 \angle -15°$ **p.u.**

Actual Value of voltage across grounding resistor

= p.u value of voltage * ($KV_b/\sqrt{3}$) **(Since KV_b is line value)**

= 0.9661 $\angle$-15° * (11/ $\sqrt{3}$) = 6.1356 $\angle$-15° KV

5. A salient pole generator without dampers is rated 20MVA, 13.8KV and has a direct axis sub - transient reactance of 0.25 per unit. The negative and zero sequence reactance are 0.35 and 0.10 per unit respectively. The neutral of the generator is solidly grounded. Determine the sub - transient current in the generator and the line to line voltages for sub - transient conditions when a line to line fault occurs at the generator operating unloaded at rated voltage. Neglect resistance.

Solution:

Base Values

MVA_b=20MVA

KV_b=13.8KV

Base current, $I_b = KVA_b / (\sqrt{3} * KV_b)$

= (20* 1000) / ($\sqrt{3}$ * 13.8)

= 836.7 A = 837 A

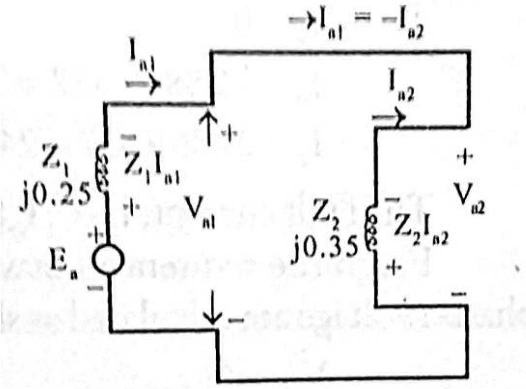


Figure: 4.34

$I_a = 0$; $I_b = -I_c$; $I_{a0} = 0$; $I_{a2} = -I_{a1}$; $V_b = V_c$; $V_{bc} = 0$

From Fig,

$I_{a1} = E_a / (Z_1 + Z_2)$

E_a is phase value value of induced emf.

$E_a = 1\angle 0°$ p.u

$\therefore I_{a1} = 1 / (j0.25+j0.35) = 1 / j0.6 = -j1.667 = 1.667 \angle -90°$ p.u.

$I_{a2} = -I_{a1} = j1.667$ p.u.

$I_{a0}=0$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$a=1\angle 120° = -0.5+j0.866$

$a^2 = 1\angle 240° = -0.5-j0.866$

$I_a=I_{a0}+I_{a1}+I_{a2}=0-j1.667+j1.667=0$

$I_b=I_{a0}+a^2I_{a1}+aI_{a2} = -j1.667(-0.5-j0.866) + j1.667\,(-0.5+j0.866)$

$=j0.833-1.443-j0.833-1.443 = -2.886$ p.u.

$I_c = I_{a0}+aI_{a1}+a^2I_{a2}=-aI_{a2}-a^2I_{a1}=-(a^2I_{a1}+aI_{a2}) = -I_b = 2.886$ p.u

Actual values of line currents are obtained by multiplying the p.u. values with base currents

$I_a=0$

$I_b=-2.886 * 837 =2416 \angle 180°$ A

$I_c=2.886 * 837 = 2416 \angle 0°$ A

The fault current, $I_f = |I_b| = 2416$ A = 2.416 KA

From the sequence networks of the generator, the symmetrical components of Phase a Voltage are calculated as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$V_{a0}=0$; $V_{a1}=E_a-I_{a1}Z_1$; $V_{a2}=V_{a1}$

$\therefore V_{a1} = V_{a2} = 1- (-j1.667)(j0.25) = 1-0.417 =0.583$ p.u.

$V_a = V_{ao}+V_{a1}+V_{a2} = 0.583+0.583 = 1.166\angle 0°$ p.u.

$V_b = V_{a0}+a^2V_{a1}+aV_{a2}$

$= 0.583\ (-0.5-j0.866)+0.583(-0.5+j0.866) = -0.583$ p.u.

$V_c=V_b = -0.583$ p.u.

Line voltages are

$V_{ab}= V_a - V_b = 1.166+0.583 = 1.749\angle 0°$ p.u.

$V_{bc} = V_b - V_c = -0.583 +0.583 =0$ p.u.

$V_{ca} = V_c - V_a = -0.583 -1.166 = 1.749 \angle 180°$ p.u

E_a=1p.u

Line Value of base voltage = 13.8KV

Phase Value of base voltage = (13.8 / √3) = 7.9KV

$V_{ab}=1.749 \angle 0° * 7.97 = 13.94 \angle 0°$ KV

$V_{bc} = 0$ KV

$V_{ca} = 1.749 \angle 180° * 7.97 = 13.94 \angle 180°$ KV

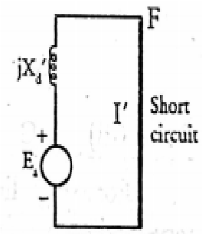
6. A generator of negligible resistance having 1 p.u. voltage behind transient reactance is subjected to different types of faults

Type of Fault	Resulting fault current in pu
3 – Phase	3.33
L – L	2.33
L – G	3.01

Calculate the per unit value of 3 sequence reactances.

Solution:

Case (i): 3 Phase fault



$E_g' = 1$ p.u. , $I' = 3.33$ p.u. **Figure: 4.35**

When load current is neglected, $E_g'' = E_g' = E_g = E_a$

$\therefore$ $X_d' = |E_g| / |I'| = 1 / 3.33 = 0.3$ p.u.

$\therefore$ W.K.T, the reactance during symmetrical fault is + sequence reactance

$\therefore$ + sequence reactance of generator, $X_1 = X_d' = 0.3$ p.u.

Case (ii): L – L Fault,

From Fig

$$I_{a1} = E_a / (jX_1 + jX_2)$$

$$\therefore |I_{a1}| = E_a / (X_1 + X_2)$$

Assume the line to line fault is between Phase b and Phase c. Hence the fault current is I_b.

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

For a line to line fault, $I_{a0} = 0$ and $I_{a2} = -I_{a1}$

$$\therefore I_b = 0 + a^2 I_{a1} - a I_{a1} = I_{a1}(a^2 - a)$$

$$= I_{a1}(-0.5 - j0.866 - (-0.5 + j0.866)) = I_{a1}(-j1.7321) = -j1.732 I_{a1}$$

Fault current, $I_f = |I_b| = 1.732\ |I_{a1}|$

$$\therefore |I_{a1}| = (I_f / 1.732)$$

$$I_f = 2.23 \text{ p.u.}$$

$$\therefore |I_{a1}| = (2.23 / 1.732) = 1.2875 \text{ p.u.}$$

$$1.2875 = (E_a / (X_1 + X_2))$$

$$\therefore X_1 + X_2 = Ea/1.2875$$

$X_2 = (E_a / 1.2875) - X_1 = (1/1.2875) - 0.3 = 0.48$ p.u.

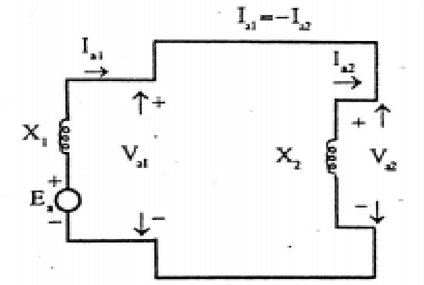


Figure: 4.36

Case (iii): L-G Fault

For a single line to ground fault, on a generator the sequence networks are connected in series as shown in Fig

For a single line to ground fault, the fault current is $|I_a|$ and it is equal to $3|I_{a1}|$

I_f=3.01 p.u.

$\therefore\ 3|I_{a1}| = I_f = 3.01$ (or) $|I_{a1}| = (3.01/3) = 1.0033$ p.u.

From Fig

$I_{a1} = (E_a / (jX_1+jX_2+jX_0))$

$\therefore\ |I_{a1}| = (E_a/(X_1+X_2+X_0))$

$X_1+X_2+X_0 = (E_a/|I_{a1}|)$

$X_0 = (E_a/|I_{a1}|) - X_1 - X_2$

$= (1/1.0033) - 0.3 - 0.48$

$= 0.22$ p.u.

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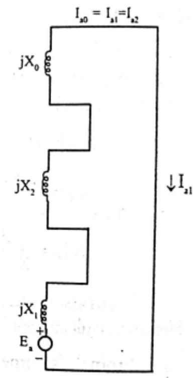
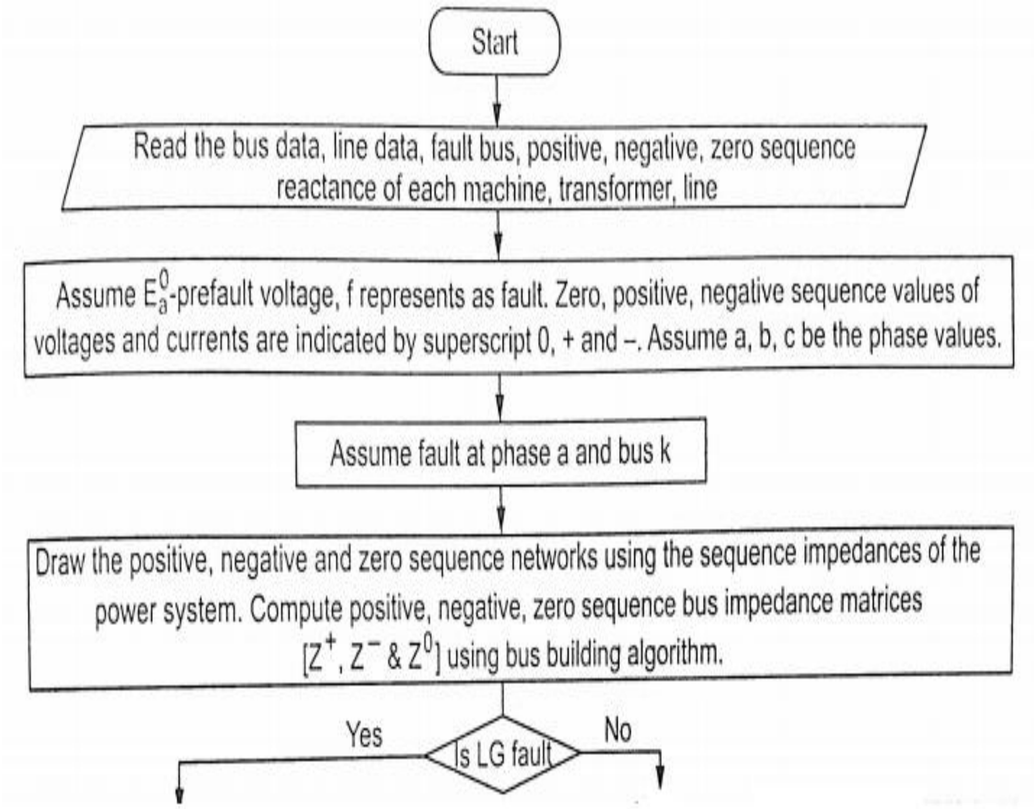


Figure: 4.37

Flowchart of UnSymmetrical Fault Analysis using Z_{bus}



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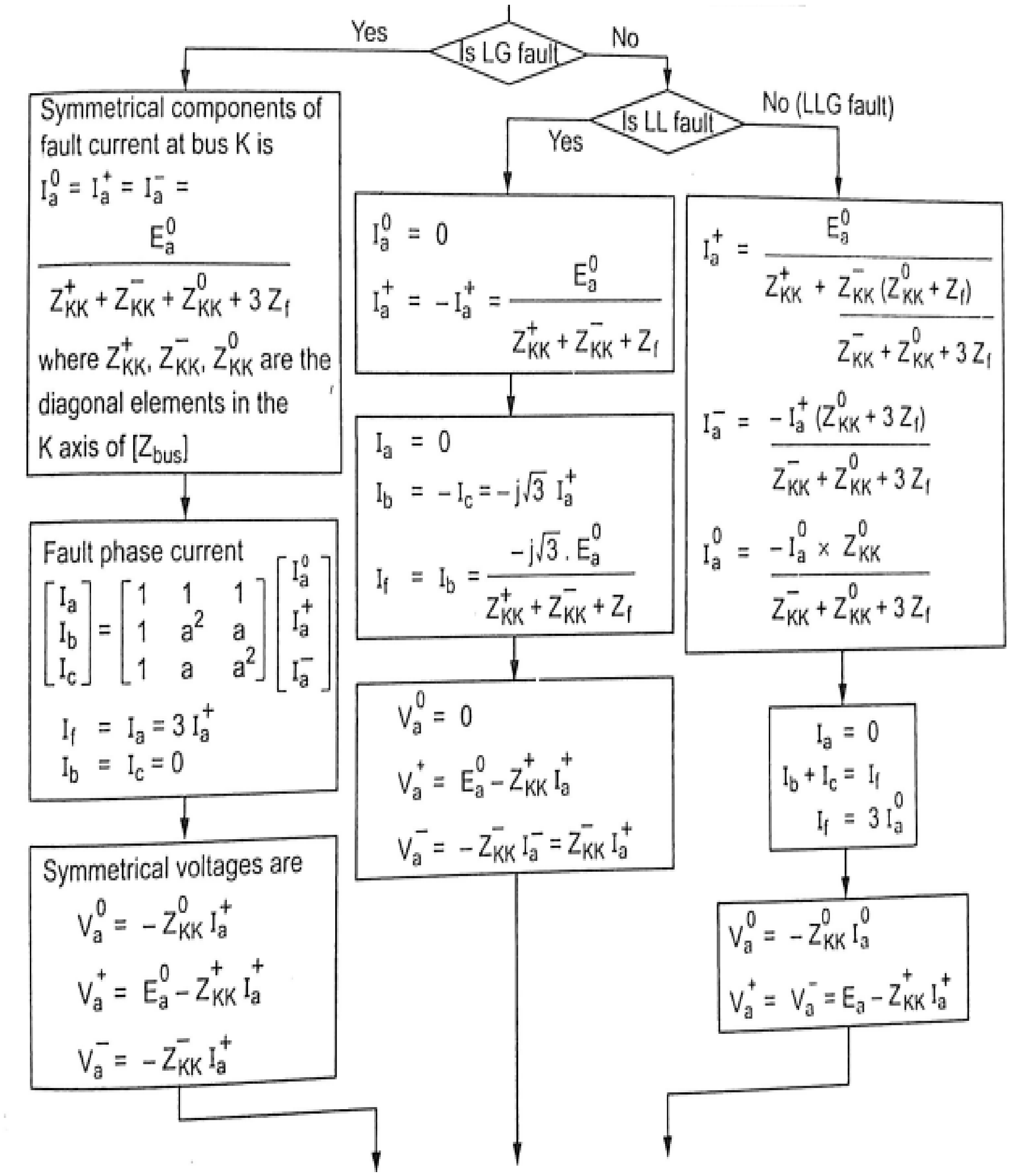

$$E_a = \text{P.F.}$$

Post fault zero sequence bus voltages

$$V_1^{f+} = V_{p.f} - Z_{1K}^{+} I_K^{f+}$$

$$\vdots$$

$$V_K^{f+} = V_{p.f} - Z_{KK}^{+} I_K^{f+}$$

$$\vdots$$

$$V_K^{f+} = V_{p.f} - Z_{nK}^{+} I_K^{f+}$$

Post fault negative sequence bus voltages

$$V_1^{f-} = -Z_{1K}^{-} I_K^{f-}$$

$$\vdots$$

$$V_K^{f-} = -Z_{KK}^{-} I_K^{f-}$$

$$\vdots$$

$$V_n^{f-} = -Z_{nK}^{-} I_K^{f-}$$

Post fault zero sequence bus voltages

$$V_1^{f0} = V_{p.f} - Z_{1K}^{0} I_K^{f0}$$

$$\vdots$$

$$V_K^{f0} = V_{p.f} - Z_{KK}^{0} I_K^{f0}$$

$$\vdots$$

$$V_K^{f0} = V_{p.f} - Z_{nK}^{0} I_K^{f0}$$

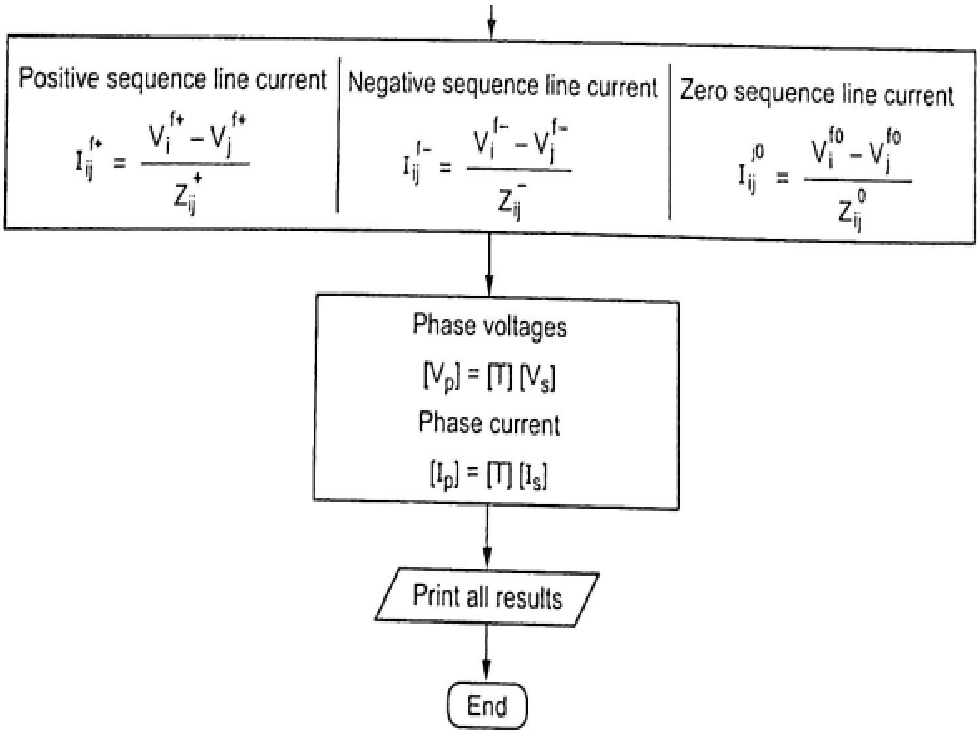

Figure: 4.38

7. For the given network shown below a solid single line to ground fault is occurred at bus 3. Perform the fault analysis and determine (a) Fault current (b) Bus voltages after fault and (c) Line currents after fault. Assume pre fault voltage 1 p.u./phase. Neglect the shunt admittance of the line. All values are given on 100 MVA base.

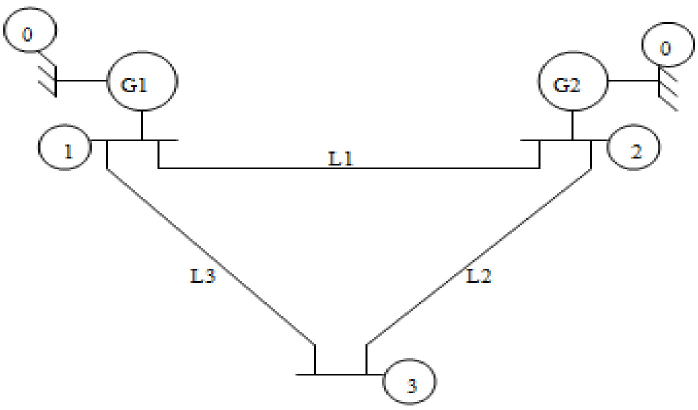

Figure: 4.39

Elements	Bus Code	Line Impedance		
		Zero Sequence	Positive Sequence	Negative Sequence
L1	1-2	j0.12	j0.2	j0.2
L2	2-3	j0.12	j0.2	j0.2
L3	1-3	j0.12	j0.2	j0.2
G1	1-0	j0.15	j0.3	j0.3
G2	2-0	j0.15	j0.3	j0.3

Table: 4.1

Solution:

$$[Y]^{(0)} = j \begin{pmatrix} 1/0.15 + 1/0.12 + 1/0.12 & -1/0.12 & -1/0.12 \\ -1/0.12 & 1/0.15 + 1/0.12 + 1/0.12 & -1/0.12 \\ -1/0.12 & -1/0.12 & 1/0.12 + 1/0.12 \end{pmatrix}$$

$$[Y]^{(0)} = j \begin{pmatrix} 23.33 & -8.33 & -8.33 \\ -8.33 & 23.33 & -8.33 \\ -8.33 & -8.33 & 16.66 \end{pmatrix}$$

$$[Z]^{(0)} = \text{inverse of } [Y]^{(0)}$$

$$[Z]^{(0)} = j \begin{pmatrix} 0.09078 & 0.05921 & 0.075 \\ 0.05921 & 0.09078 & 0.075 \\ 0.075 & 0.075 & 0.135 \end{pmatrix}$$

Similarly

$$[Z]^{(1)} = [Z]^{(2)} = j \begin{pmatrix} 0.17727 & 0.12272 & 0.15 \\ 0.12272 & 0.17727 & 0.15 \\ 0.15 & 0.15 & 0.25 \end{pmatrix}$$

Faulted bus number =3; pre fault phase voltage, E=1.0; for solid fault $Z_f = 0$

$$I_3^{(0)} = I_3^{(1)} = I_3^{(2)} = (\sqrt{3}) * E / (Z_{33}^{(0)} + Z_{33}^{(1)} + Z_{33}^{(2)} + 3Z_f)$$

$$= (\sqrt{3}) / (j0.135 + j0.25 + j0.25)$$

$= -j2.7276$ **p.u. Fault current at bus 3,** $I_f = 3 * I_3^{(1)} = -j8.1828$ **p.u.**

Bus voltages after fault is

Zero Sequence Voltages

$$V_1^{(0)} = - Z_{13}^{(0)} I_3^{(0)}$$

$$= - j0.075 * (-j2.7276) = -j0.2045 \text{ pu}$$

$$V_2^{(0)} = - Z_{23}^{(0)} I_3^{(0)} = -j0.2045 \text{ pu}$$

$$V_3^{(0)} = - Z_{33}^{(0)} I_3^{(0)} = -j0.3682 \text{ pu}$$

Positive Sequence Voltages

$$V_1^{(1)} = (\sqrt{3})E - Z_{13}^{(1)} I_3^{(1)}$$

$$= (\sqrt{3}) - j0.15*(-j2.7276)$$

$$= 1.3229 \text{ pu}$$

$$V_2^{(1)} = (\sqrt{3})E - Z_{23}^{(1)} I_3^{(1)} = 1.3229 \text{ pu}$$

$$V_3^{(1)} = (\sqrt{3})E - Z_{33}^{(1)} I_3^{(1)} = 1.0501 \text{ pu}$$

Negative Sequence Voltages

$$V_1^{(2)} = - Z_{13}^{(2)} I_3^{(2)}$$

$$= - j0.15 * (-j2.7276) = -0.4091 \text{ pu}$$

$$V_2^{(2)} = - Z_{23}^{(2)} I_3^{(2)} = -0.4091 \text{ pu}$$

$$V_3^{(2)} = - Z_{33}^{(2)} I_3^{(2)} = -0.6819 \text{ pu}$$

The line currents after fault are as follows,

Current in L1 element which connects buses 1 and 2

$$I_{12}^{(0)} = (V_1^{(0)} - V_2^{(0)}) / Z_{line\,12}^{(0)} = 0$$

Current in L2 element which connects buses 2 and 3

$$I_{23}^{(0)} = (V_2^{(0)} - V_3^{(0)}) / Z_{line\,23}^{(0)}$$

$$= (-0.2046 - (-0.3682)) / j0.12$$

$$= -j1.3638$$

Similarly

$I_{12}^{(0)} = 0$	$I_{12}^{(1)} = 0$	$I_{12}^{(2)} = 0$
$I_{23}^{(0)} = -j1.3638$	$I_{23}^{(1)} = -j1.3638$	$I_{23}^{(2)} = j1.3638$
$I_{13}^{(0)} = -j1.3638$	$I_{13}^{(1)} = -j1.3638$	$I_{13}^{(2)} = -j1.3638$
$I_{10}^{(0)} = j1.3638$	$I_{10}^{(1)} = -j4.4096$	$I_{10}^{(2)} = j1.3638$
$I_{20}^{(0)} = j1.3638$	$I_{20}^{(1)} = -j4.4096$	$I_{20}^{(2)} = j1.3638$

Above all currents are in pu.

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UNIT – V – Power System Analysis – SEE1302

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STABILITY AND SECURITY ANALYSIS

Distinction between steady state and transient state – Concepts of Stability and Security- Swing Equation – Solution to swing equation – step by step method – Power angle equation – Equal area criterion – critical clearing angle and time . Stability Analysis of single machine connected to infinite bus by modified Euler's method – Multi machine stability analysis using Runge kutta method

Stability

The Stability of a system is defined as the ability of power system to return to stable (Synchronous) operation when it is subjected to a disturbance

Steady State Stability

The steady state stability is defined as the ability of a power system to remain stable (i.e, without loosing synchronism) for small disturbance

Transient Stability

The transient stability is defined as the ability of a power system to remain stable (i.e, without loosing synchronism) for large disturbances.

Steady State Stability limit

- **The steady state stability limit is the maximum power that can be transmitted by a machine (or transmitting system) to a receiving system without loss of synchronism**
- **In Steady state the power transferred by synchronous machine (or power system) is always less than the steady state stability limit**

Transient Stability limit

- **The transient stability limit is the maximum power that can be transmitted by a machine (or transmitting system) to a fault or a receiving system during a transient state without loss of synchronism**
- **The transient stability limit is always less than the steady state stability limit**

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Classification of Stability Studies

Depending upon the nature of disturbance, stability studies can be classified as :

1. Steady state stability

It is concerned with the determination of upper limit of loading without loss of synchronism

2. Dynamic Stability

It is concerned with the study of nature of oscillations and its decay for small disturbances

3. Transient Stability

It is concerned with the study of dynamics of a system for large disturbances

Dynamics of synchronous Machine Rotor

E_{KE} - Kinectic energy of the rotor in MJ (Mega Joules)

J - Moment of inertia of the rotor in Kg-m^2

ω_{sm} - Synchronous angular speed of the rotor in mech.rad/sec

ω_s - Synchronous angular speed of rotor in elect.rad/sec

P - Number of poles in Synchronous machine

M - Moment of inertia of rotor in MJ-s/elec.rad or MJ-s/mech .rad

S - Power rating of Machine in MVA

H - Inertia constant in MJ/MVA or MW-s/MVA

F - Frequency in cycles/sec or HZ

δ_m - Angular displacement of rotor with respect to synchronously rotating reference frame in mech.rad

δ - Angular displacement of rotor with respect to synchronously rotating reference frame in elect.rad

θ_m - Angular displacement of rotor with respect to a stationary axis in mech.rad

θ - Angular displacement of rotor with respect to stationary axis in elec.rad

t - Time in seconds

T_m - Mechanical torque at the shaft of rotor (supplied by prime mover) in N-m

T_e - Net electromagnetic torque in N-m

T_a - Net accelerating torque in N-m

P_m - Mechanical Power input in p.u.

P_e - Electrical power output in p.u.

The Kinetic energy (in MJ) of the rotor of a synchronous machine is given by

$$E_{KE} = \frac{1}{2} J\omega_{sm}^2 * 10^{-6}$$

The mechanical and electrical angular speeds are related to the number of poles in synchronous machine as shown as

$$\omega_s = \frac{P}{2}\omega_{sm} \quad \textbf{or} \quad \omega_{sm} = \frac{2}{P}\omega_s$$

$$E_{KE} = \frac{1}{2} J\left(\frac{2}{P}\right)^2 \omega_s^2 * 10^{-6}$$

Let, $E_{KE} = \frac{1}{2} M\omega_s$

Where, $M = J\left(\frac{2}{P}\right)^2 \omega_s * 10^{-6}$

Here M is the moment of inertia in MJ-s/elec.rad. This is used popularly in stability studies

Another useful constant which is popularly used in stability studies is the inertia constant H. It is defined as:

H = Stored Kinetic energy in MJ at synchronous speed / Machine rating in MVA

$$H = \frac{E_{KE}}{S}$$

$$H = \frac{\frac{1}{2} J\omega_{sm}^2}{S} \qquad H = \frac{\frac{1}{2} M\omega_s}{S} \qquad M = \frac{2HS}{\omega_s}$$

We Know that , $\omega_s = 2\pi f$

$$M = \frac{2HS}{2\Pi f} = \frac{HS}{\Pi f}$$ **(in MJ-s/elec.rad)**

Sometimes it is required in MJ-s/elect.degree

$$M = \frac{HS}{180 f}$$ **(in MJ-s/elec.deg)**

The value of M can be expressed in per unit by selecting a base of MVA

Let, S_b=Base MVA

$$M_{p.u.} = \frac{SH / \Pi f}{S_b} \quad \textbf{Or} \quad M_{p.u.} = \frac{SH / 180 f}{S_b}$$

If the machine rating S is chosen as base Value, then S= Sb p.u. value of M with Machine rating as base MVA

$$M_{p.u.} = \frac{H}{\Pi f} \quad \text{or} \quad M_{p.u.} = \frac{H}{180 f}$$

1. **A 2 pole 50Hz, 11KV turbo alternator has a ratio of 100MW, power factor 0.85 lagging. The rotor has a moment of inertia of 10,000 Kgm^2. Calculate H and M.**

Solution:

Synchronous speed in rps, n_s = 2f/p =(2*50)/2 =50 rps

Synchronous speed in rad/sec, $\omega_s = 2\pi n_s = 2\pi * 50$ = 314.16 elect.rad/sec

Inertia constant M = $J(2/P)^2\omega_s * 10^{-6}$ in MJ-s/elect.rad

$\therefore M = 10000 *(2/2)^2 *314.16 * 10^{-6} = 3.146$ MJ-s /elec.rad

MVA rating of machine, S = P/pf = 100/0.85 =117.675 MVA

Base Values

KV_b=11KV

MVA_b=117.65MVA

Inertia constant, M in p.u., M_{pu} = (M in MJ-s/elect.rad)/MVA_b

= 3.1416 / 117.65 =0.0267 p.u.

Inertia constant, $H=\pi f M_{pu}=\pi*50*0.0267 = 4.194$ MW –s / MVA

2. Two power station A and B are located close together. Station A has four identical generator sets each rated 100MVA and having an inertia constant of 9MJ/MVA whereas the station B has 3 sets each rated 200MVA, 4MJ/MVA. Calculate the inertia constant of a single equivalent machine on a base of 100MVA .

Solution:

- **Assume that the machines are swinging coherently. For two machines swinging coherently the equivalent inertia constant, H_{eq} is given by**

$H_{eq} = (H_{1,mach}*S_{1,mach}) / S_{sys} + (H_{2,mach}*S_{2,mach}) / Ssys$

Where, Ssys – MVA rating of system

$S_{1,mach}$ & $H_{1,mach}$ – MVA rating and inertia constant of machine 1

$S_{2,mach}$ & $H_{2,mach}$ – MVA rating and inertia constant of machine 2

Station A

The station A has four machines of identical rating

$\therefore$ Equivalent inertia constant of station A = $H_A = 4((H_{mach}*S_{mach}) / S_{sys})$

= 4((9 * 100)/100)

=36 MJ/MVA

Station B

The station B has three machines of identical rating

$\therefore$ Equivalent inertia constant of station A = H_B = 3((H_{mach}*S_{mach}) / S_{sys})

= 3((4 * 200)/100)

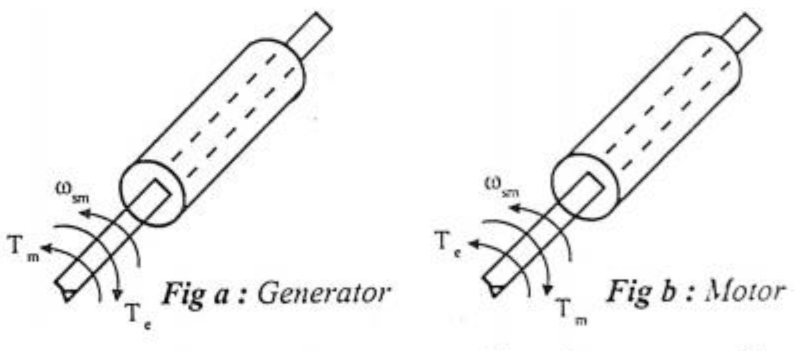
= 24 MJ/MVA

H_{eq} of System

Equivalent inertia constant of the system = H_{eq}=H_A+H_B = 36+24 = 60MJ/MVA

Swing Equation

- **The rotor of a synchronous machine is subjected to two torques, T_e and T_m which are acting in opposite directions as shown in Fig**



Torque acting on rotor of synchronous machine

Figure: 5.1

Where T_e – Net electrical or electromechanical torque in N-m

T_m – Mechanical or shaft torque supplied by the prime mover in N-m

- **Under steady state operating condition the T_e and T_m are equal and the machine runs at constant speed, which is called synchronous speed.**
- **If there is a difference between the two torques then the rotor will have an accelerating or decelerating torque, denoted as T_a**

T_a=T_m-T_e

- **Here T_m & T_e are positive for generators and T_m & T_e are negative for motors**
- **Let θ_m – Angular displacement for rotor with respect to stationary reference axis**
- **δ_m – Angular displacement of rotor with respect with synchronously rotating reference axis**
- **By Newton's second law of motion,**

$$T_a \alpha \frac{d^2\theta_m}{dt^2} \qquad \textbf{Or} \qquad T_a = J\frac{d^2\theta_m}{dt^2}$$

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e$$

The angular displacements θ_m & δ_m are related to synchronous speed by the following equation,

$$\theta_m = \omega_{sm} t + \delta_m$$

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

- **From the equation, the rotor angular velocity $d\theta_m/dt$ is constant and equal to**

ω_{sm} (Synchronous speed) only $d\delta_m/dt$ is zero.

- **Hence $d\delta_m/dt$ represents the deviation of the rotor speed from synchronism**

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \qquad J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

Let, $P_{m,act}$ - shaft power input to the machine neglecting losses (in MW)

$P_{e,act}$ - Electrical power developed in rotor (in MW)

$$P = \frac{2\Pi NT}{60} = \omega T$$

$$P_{m,act} = \omega_{sm} T_m \qquad \textbf{or} \qquad T_m = \frac{P_{m,act}}{\omega_{sm}}$$

$$P_{e,act} = \omega_{sm} T_e \qquad T_e = \frac{P_{e,act}}{\omega_{sm}}$$

$$J \frac{d^2\delta_m}{dt^2} = \frac{P_{m,act}}{\omega_{sm}} - \frac{P_{e,act}}{\omega_{sm}}$$

$$J\omega_{sm} \frac{d^2\delta_m}{dt^2} = P_{m,act} - P_{e,act}$$

Let, H – Inertia constant in MJ/MVA

S - power rating of machine in MVA

$$H = \frac{\frac{1}{2} J\omega_{sm}^2}{S} \qquad J\omega_{sm} = \frac{2HS}{\omega_{sm}}$$

$$\frac{2HS}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = P_{m,act} - P_{e,act}$$

W.K.T, ω_{sm}=(2/p)ω_s & δ_m=(2/p)δ

P – Number of poles in Synchronous machine

$$\frac{2HS}{2\omega_s / P} \frac{d^2(2\delta / p)}{dt^2} = P_{m,act} - P_{e,act}$$

$$\frac{2HS}{\omega_s} \frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act}$$

On substituting for ω_s=2πf

$$\frac{2HS}{2\Pi f} \frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act} \qquad \frac{HS}{\Pi f} \frac{d^2\delta}{dt^2} = P_{m,act} - P_{e,act}$$

p.u. value of mechanical power, P_m=$P_{m,act}$/S

p.u. value of electrical power, $P_e = P_{e,act}/S$

$$\frac{HS}{\Pi f}\frac{d^2\delta}{dt^2} = P_m S - P_e S \qquad \frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

- **This equation is called swing equation.**
- **It is the fundamental equation which governs the dynamics of the synchronous machine rotor**
- **This swing equation is a second order differential equation**

Power Angle Equation

- **The equation relating the electrical power generated (P_e) to the angular displacement of the rotor (δ) is called Power angle Equation**
- **The Power angle equation can be derived using the transient model of the generator, because for stability studies, the transient model of the generator is used**
- **The transient model of the generator is shown in fig**

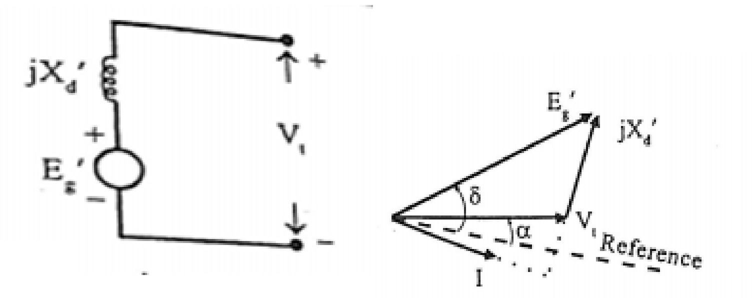


Figure: 5.2

- **Consider a single generator supplying power through a transmission system to a load or to a large system at other end.**
- **Such system can be represented by a 2 – bus network as shown in fig as a rectangular box representing the linear passive components (reactance's) of the system including the transient reactance of the generator**

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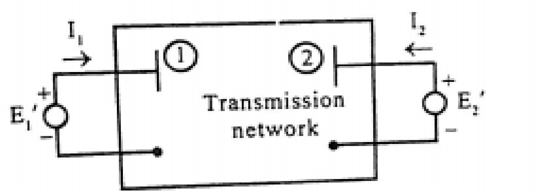


Figure: 5.3

Here E_1' – Transient internal voltage of the generator at bus 1

E_2' – Voltage at the receiving end. (This may be the voltage at infinite bus, or transient internal voltage of synchronous motor at bus 2)

The node basis matrix equation of 2 bus system of fig can be written as

$$I = Y_{bus} V$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1^{'} \\ E_2^{'} \end{bmatrix}$$

Where, I_1 & I_2 are the currents injected by the sources E_1' & E_2' respectively to the system

$$I_1 = Y_{11}E_1^{'} + Y_{12}E_2^{'}$$

$$S_1 = P_1 + jQ_1 = E_1^{'} I_1^{*}$$

$$P_1 + jQ_1 = E_1^{'}(Y_{11}E_1^{'} + Y_{12}E_2^{'})^{*}$$

$$= E_1^{'}((Y_{11}{}^{*}(E_1^{'})^{*} + Y_{12}{}^{*}(E_2^{'})^{*})$$

$$= (E_1^{'})(E_1^{'})^{*} Y_{11}{}^{*} + Y_{12}{}^{*}(E_1^{'})(E_2^{'})^{*}$$

$$= | E_1^{'} |^2 \; Y_{11}{}^{*} + Y_{12}{}^{*}(E_1^{'})(E_2^{'})^{*}$$

Let $E_1' = |E_1'|\angle\delta_1$ $\quad$ $Y_{11} = G_{11} + jB_{11} = |Y_{11}|\angle\theta_{11}$

$E_2' = |E_2'|\angle\delta_2$ $\quad$ $Y_{12} = G_{12} + jB_{12} = |Y_{12}|\angle\theta_{12}$

$$P_1 + jQ_1 = |E_1^{'}|^2 (G_{11} + jB_{11})^* + (Y_{12}\angle\theta_{12})^* |E_1^{'}| \angle\delta_1 (|E_2^{'}| \angle\delta_2)^*$$

$$= |E_1^{'}|^2 (G_{11} - jB_{11}) + |Y_{12}| \angle -\theta_{12}) |E_1^{'}| \angle\delta_1 |E_2^{'}| \angle -\delta_2$$

$$P_1 + jQ_1 = |E_1^{'}|^2 (G_{11} - jB_{11}) + |E_1^{'}||E_2^{'}||Y_{12}| \angle(\delta_1 - \delta_2 - \theta_{12})$$

$$= |E_1^{'}|^2 (G_{11} - jB_{11}) + |E_1^{'}||E_2^{'}||Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12}) + j|E_1^{'}||E_2^{'}||Y_{12}| \sin(\delta_1 - \delta_2 - \theta_{12})$$

$$P_1 = |E_1^{'}|^2 G_{11} + |E_1^{'}||E_2^{'}||Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

$$Q_1 = -|E_1^{'}|^2 B_{11} - |E_1^{'}||E_2^{'}||Y_{12}| \sin(\delta_1 - \delta_2 - \theta_{12})$$

Let $\delta = \delta_1 - \delta_2$

$\gamma = \theta_{12} - \pi/2$ **($\theta_{12} = \gamma + \pi/2$)**

$P_c = |E_1'|^2 G_{11}$

$P_{max} = |E_1'||E_2'||Y_{12}|$

$P_1 = P_e$

$$P_e = P_c + P_{max} \cos(\delta - \gamma - \Pi/2)$$

$$P_e = P_c + P_{max} \sin(\delta - \gamma)$$

- **This equation is called Power angle equation**

Pe - Electrical power generated by the Generator

Pc - power loss in the system

Pmax – maximum real power that can be delivered by the generator to an infinite bus

- **Assume bus 2 is an infinite bus .$\delta_2 = 0$. $\therefore \delta = \delta_1$**
- **The power angle equation can be further simplified by considering the network as purely reactive network (Neglect resistance)**

$G_{11}=0$, $\theta_{12}=\pi/2$. so $\gamma=0$

On Substituting,

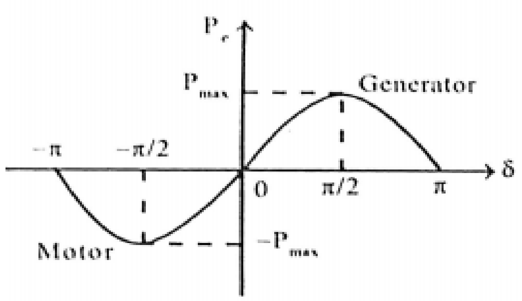
$P_e = P_{max} \sin\delta$. This equation is called simplified power angle equation

Where,

$$P_{max} = \frac{|E_1^{'}||E_2^{'}|}{X_{12}}$$

and X_{12} – Transfer reactance between bus 1 & 2

- **The graph or plot of P_e as a function of δ is called power angle curve.**



$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta$$

- **This equation is the swing equation in which the electrical power is expressed as a function of δ**

Steady State Stability

- **In steady state every synchronous machine has a limit for power transfer to a receiving system**
- **The steady state limit of a machine or transmitting system is defined as the maximum power that can be transmitted to the receiving system without loss of synchronism**

- **Consider a single synchronous machine delivering power to a large system through a transmitting system**
- **The power angle equation of such system developed can be used for the analysis of steady state stability if the transfer emfs are replaced by steady state emfs**

Let |E| = Magnitude of steady state internal emf of synchronous machine

|V| = Magnitude of voltage of receiving system

X = Transfer reactance between the synchronous machine and receiving system

Real power injected by machine to system, $P_e=P_{max}\sin\delta$

$$P_{max} = \frac{|E||V|}{X}$$

- **Let the system be operating with steady state power transfer with a torque angle δ_0**
- **In this operating condition, let the electrical power output be P_{e0}**
- **Now $P_m = P_{e0}$ under ideal conditions**
- **With the power input (Pm) remaining same, assume electrical power output increases by a small amount ΔP**

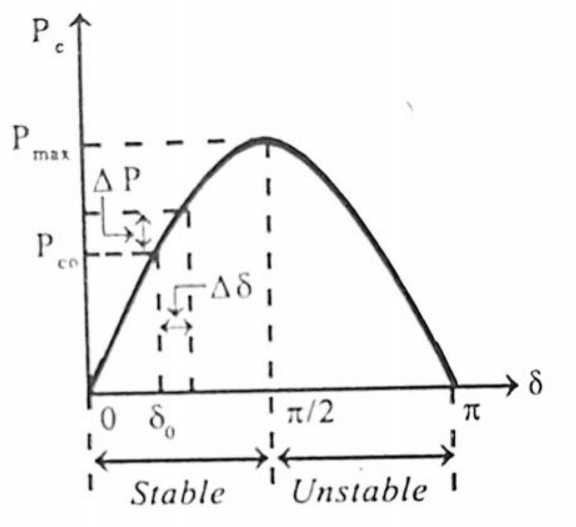


Figure: 5.4

- **Now the torque angle change by a small amount $\Delta\delta$**
- **So the new value of torque angle is $(\delta_0+\Delta\delta)$**
- **The electrical power output for this new torque angle of $(\delta_0+\Delta\delta)$ is**

$$P_{e0}+\Delta P=P_{max}\sin(\delta_0+\Delta\delta)$$

$$=P_{max}[\sin\delta_0\cos\Delta\delta+\cos\delta_0\sin\Delta\delta]$$

Since $\Delta\delta$ is a small incremental displacement from δ_0

$$\sin\Delta\delta \cong \Delta\delta \quad \text{and} \quad \cos\Delta\delta \cong 1$$

$$P_{e0}+\Delta P=P_{max}\sin\delta_0+(P_{max}\cos\delta_0)\Delta\delta$$

When $\delta=\delta 0$

$$P_e= P_{e0}=P_m\sin\delta_0$$

$$\therefore\ \Delta P=(P_{max}\cos\delta_0)\Delta\delta$$

The nonlinear swing equation can be linearized about the operating point for steady state analysis

$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2}=P_m-P_e$$

$$M\frac{d^2\delta}{dt^2}=P_m-P_e$$

For a torque of $\delta=(\delta_0+\Delta\delta)$, $P_e=(p_{e0}+\Delta P)$

$$M\frac{d^2(\delta_0+\Delta\delta)}{dt^2}=P_m-(P_{e0}+\Delta P)$$

Since δ_0 is constant and $P_m=P_{e0}$

$$M\frac{d^2\Delta\delta}{dt^2}=-\Delta P$$

$$M\frac{d^2\Delta\delta}{dt^2}=-(P_{max}\cos\delta_0)\Delta\delta$$

$$M\frac{d^2\Delta\delta}{dt^2}+(P_{\max}\cos\delta_0)\Delta\delta=0$$

Let, $\frac{d^2}{dt^2}=x^2$ **and** $P_{\max}\cos\delta_0=C$

$$Mx^2\Delta\delta+C\Delta\delta=0$$

$$(Mx^2+C)\Delta\delta=0$$

Since $\Delta\delta\neq 0$ $(Mx^2+C)=0$

This is the characteristic equation of the system for small changes. The stability is determined by the roots of the characteristic equation

$\therefore x^2=\frac{-C}{M}$ **or** $\therefore x=\pm\sqrt{\frac{-C}{M}}$

Case 1: When c is positive, (i.e, $P_{max}\cos\delta_0>0$)

- **In this case the roots are purely imaginary and conjugate**
- **Hence the system behaviour is oscillatory about δ_0**
- **In this analysis the resistances in the system have been neglected**
- **If resistances are included, then the roots will be complex conjugate and the response will be damped oscillatory**
- **$\therefore$ in a practical sysyem the system is stable for small increment in power provided, $P_{max}\cos\delta_0>0$**

Case 2: When c is negative, (i.e, $P_{max}\cos\delta_0<0$)

- **In this case the roots are real and equal in magnitude**
- **One of the root is positive and the other one is negative**
- **Due to positive root the torque angle increases without bound when there is a small increment in power and the machine will loose synchronism**

- **Hence the machine becomes unstable for small changes in power provided $P_{max}\cos\delta_0<0$**

Steady State limit

- **The term $P_{max}\cos\delta_0$ decides the steady state stability of the system and so it is called Synchronizing coefficient or stiffness of the synchronous machine**
- **From the power angle curve for generator action, the range of δ_0 as 0 to π**
- **The Fig is drawn using the equation $P_e = P_{max}\sin\delta$**

When $0 \le \delta_0 \le \pi/2$; $P_{max}\cos\delta_0$ and P_e are positive

When $\delta_0 = \pi/2$; $P_{max}\cos\delta_0 = 0$ and $P_e = P_{max}$

When $\pi/2 < \delta_0 \le \pi$; $P_{max}\cos\delta_0$ is negative and P_e is positive

- **From the above discussion, the synchronizing coefficient ($P_{max}\cos\delta_0$) and real power injected to the system (P_e) are positive when δ is in the range of 0 to $\pi/2$**
- **$\therefore$ the maximum power that can be transmitted without loss of stability occurs for $\delta = \delta_0 = \pi/2 = 90°$**
- **The maximum power transmitted is $P_{max} = \frac{|E||V|}{X}$**
- **Fig shows the stable and unstable steady state operating regions of a generator**
- **This concept is also applicable for power transfer from one system to another system if the transmitting system is represented by single equivalent generator**
- **In stable operating region of the system the damping should be sufficient to reduce the oscillations developed due to small changes in loads**
- **If oscillations exists for a long time then it may pose a problem to system security**
- **Practically the system has to be operated below the steady state stability limit**
- **The limit can be improved by reducing the reactance X or by increasing the voltages at sending end and /or at the receiving end**
- **The reactance can be reduced by introducing series capacitors in the transmission line**

- **Alternatively the line reactance can be reduced by using two parallel transmission lines**

Transient Stability

- **It is concerned with the study of system behaviour for large disturbances**
- **The Short circuits and switching heavy loads can be treated as transients**
- **The dynamics of the system under transient state are governed by the nonlinear swing equation developed**
- **Since the changes in δ is very large in the transient state, the swing equation cannot be linearized for a general solution**
- **So the solution has to be obtained by using any one of numerical techniques like point by point method , Runga kutta method and modified Euler's method**
- **The transient stability of a single machine connected to infinite bus bar can be easily determined by a simple criterion called equal area criterion**
- **The computational task involved in transient stability studies can be understood considering the transient state of a practical system**

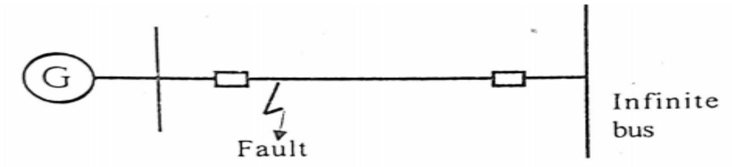


Figure: 5.5

- **Consider a single machine system feeding energy through a transmission line to an infinite bus**
- **Let the Circuit breakers (C.B) be auto closure type**
- **In this C.B will open its contact upon sensing a fault and after a small time it will close its contacts, if the fault still exist then again it will open its contact to permanently disconnect the faulty part**
- **This feature is useful in clearing transient faults**
- **Transient fault exists for a small time and it gets cleared when the circuit is opened**

- **Then the circuit can be closed for normal operation**
- **Most of the auto reclosure C.B will open and close the contacts twice before permanently disconnecting the circuit**
- **In majority of faults, first reclosure will be successful**
- **Hence system stability is improved by using autoreclosure C.B**

Steps involved in Transient stability study

1. **Calculate the transient internal emf and torque angle δ_0 using the prefault load currents**
2. **Determine an equation for power during the fault condition $P_e(\delta)$. If the fault is 3 – phase fault then power transferred to infinite bus is zero and the entire power goes to fault**
3. **Calculate $\delta(t)$ for various time instant by solving the swing equation using a numerical technique. The initial value of δ for the solution is δ_0**
4. **Assume the fault is cleared when the C.B open its contact for the first time. Now $Pe(\delta)=0$. Continue calculating $\delta(t)$ by taking previous step value as initial condition**
5. **Assume C.B close its contact and power feeding to infinite bus is resumed. For this situation find $Pe(\delta)$ and continue to calculate $\delta(t)$**
6. **Examine the variations of $\delta(t)$. If $\delta(t)$ goes through a maximum value and starts to reduce then the system is a stable system. On the other hand if $\delta(t)$ remains increasing for a specified length of time then the system is considered unstable**

Equal Area Criterion

1. **The system is stable if $d\delta/dt = 0$ at some time instant**
2. **The system is unstable if $d\delta/dt > 0$ for a sufficiently long time (typically 1 second or more)**

For a single machine infinite bus bar system, the stability criterion stated above can be converted to a simple condition as shown below

Consider the swing equation of a generator connected to infinite bus

$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

Let there be a change in P_e due to a large disturbance, with P_m remaining constant

P_m-P_e=P_a Where P_a is the accelerating power

$$M = \frac{H}{\Pi f}$$

$$M\frac{d^2\delta}{dt^2} = P_a$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

On Multiplying the equation by 2 dδ/dt,

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\frac{P_a}{M}$$

$$2\frac{d\delta}{dt}\frac{d}{dt}\frac{d\delta}{dt} = \frac{2}{M}P_a\frac{d\delta}{dt}$$

$$2\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}P_a\frac{d\delta}{dt}$$

$$2d\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}P_a d\delta$$

On integrating the equation

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta}P_a d\delta$$

Where δ_0 is the initial value of torque angle or rotor angle

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta}P_a d\delta}$$

For a stable system dδ/dt=0, at a particular time instance. so for a stable system

$$\sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta = 0}$$

The equation is zero if the integral of P_a is zero.

For $\frac{d\delta}{dt} = 0$ **The term** $\int_{\delta_0}^{\delta} P_a d\delta = 0$

- **The physical meaning of integration is the estimation of the area under the curve**
- **Hence the integral of p_a equal to zero area**
- **The condition of stability can be stated as:**

i. **The system is stable if the area under P_a - δ curve reduces to zero at some value of δ**

ii. **This is possible only if the positive (accelerating) area under P_a-δ curve is equal to the negative (decelerating) area under P_a-δ for a finite change in δ**

iii. **Hence this stability criterion is called equal area criterion of stability**

- **The equal area criterion of stability can be applied to any type of disturbances that may occur in a single machine infinite bus bar system**

Transient Stability analysis for a sudden change in mechanical input

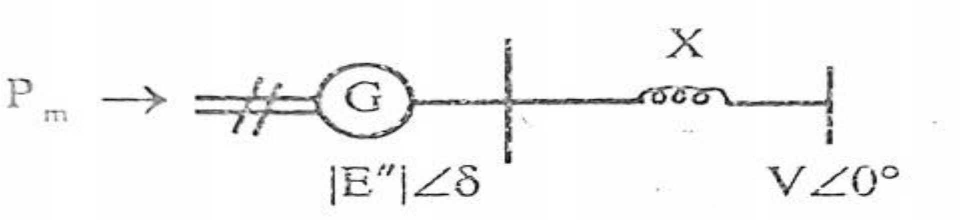


Figure: 5.6

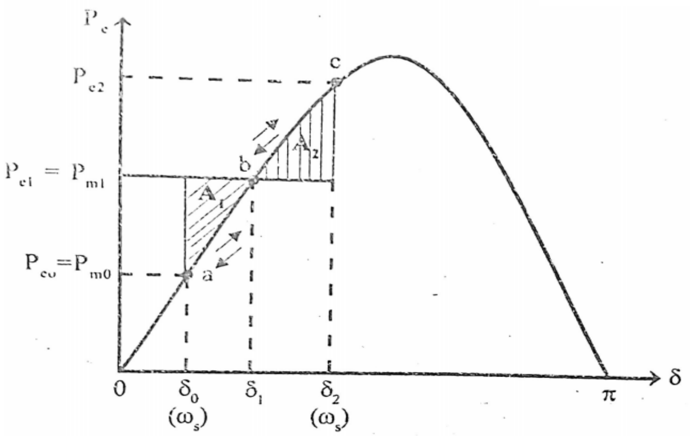


Figure: 5.7

- **Consider a single generator feeding energy to infinite bus as shown in fig**
- **The electrical power transmitted by the generator is given by**

$$P_e = \frac{|E'||V|}{X}\sin\delta = P_{max}\sin\delta$$

Where $P_{max} = \frac{|E'||V|}{X}$

- **Let the generator be operating in steady state with a torque angle δ_0.**
- **At this condition the mechanical power input is P_{mo} and the electrical output is P_{e0}**
- **Under ideal conditions $P_{m0}=P_{e0}$**
- **$P_{m0}=P_{e0}=P_{max}\sin\delta_0$**
- **In the power angle curve shown in fig, the steady state operating point is point a**

Let the mechanical input to the generator rotor be suddenly increased to P_{m1} by some adjustment in prime mover

- **Since the mechanical power is more than electrical power, the generator will have an accelerating power Pa given by**

$P_a=P_{m1}-P_e$

Where $P_e = P_{max} \sin\delta$

- **Due to accelerating power the rotor speed increases and so the rotor angle also increases**
- **This results in increased electrical power generation**
- **$\therefore$ the operating point will move upwards along the power angle curve.**
- **At Point b again the mechanical power Pm1 equals the electrical power P_{e1}, where P_{e1} is the electrical power output corresponding to torque angle δ_1**
- **Now the rotor angle cannot stay at this point because the inertia of the rotor will make the rotor to oscillate with respect to point b**
- **Hence the torque angle will continue to increase till point c, when the operating point moves from b to c, the electrical power is more than mechanical power**

  $\therefore$ the power P_a given by equation is negative and it is called decelerating power
- **In this region (i.e., from point b to c) the rotor angle δ increases but the rotor speed decreases due to decelerating power**
- **The point c is decided by the damping of the system**
- **At point c the speed of rotor will be equal to synchronous speed**
- **At point a the speed is synchronous speed (ω_s)**
- **From point a to b the speed increases and then from point b to c the speed decreases**
- **Once again at point c the speed is equal to synchronous speed (ω_s)**
- **Thus the rotor oscillates between point a and point c before settling to point b**
- **In Fig , the area A_1 is the accelerating area and area A_2 is the deceleration area**
- **The equal area criterion says that, the system is stable if**

$$\int_{\delta_0}^{\delta} P_a d\delta = 0$$

- **To satisfy the equation, the acceleration area A_1 should be equal to deceleration area A_2**

- **When the oscillation die out the system will settle to a new state**
- **In this new steady state, $P_{m1}=P_{e1}$**

  $\therefore$ **$P_{m1} = P_{e1}=P_{max}\sin\delta_1$**

- **The areas A_1 & A_2 can be evaluated as**

$$A_1 = \int_{\delta_0}^{\delta} (P_{m1} - P_e)d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m1})d\delta$$

- **Where $P_e= P_{max}\sin\delta$**
- **From the above discussion it is clear, that there is a upper limit for increase in mechanical power input P_m**
- **As the mechanical power is increased, a limiting condition is finally reached at a point where A_1 equals the area above $P_{m1,max}$ line as shown in fig**
- **The corresponding δ can be $\delta_{1,max}$**
- **Under this condition $\delta 2$ takes a maximum value of $\delta 2,max$**

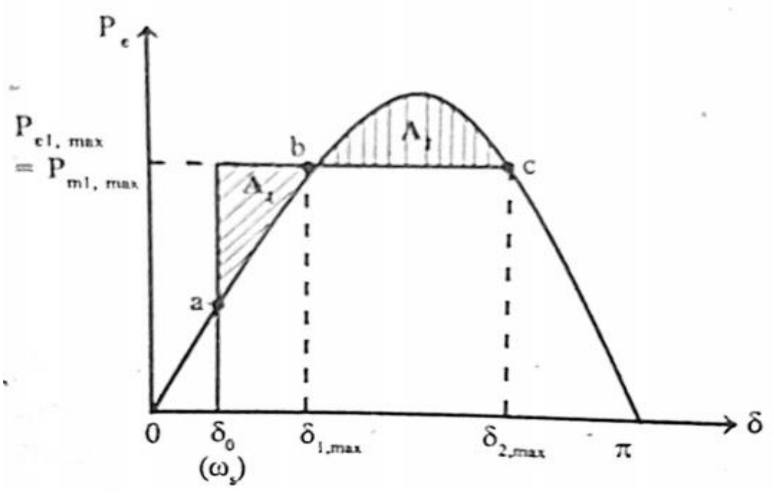


Figure: 5.8

Here $\delta_{2,max} = \pi - \delta_{1,max}$

$P_{e1,max} = P_{m1,max} = P_{max} \sin \delta_{1,max}$

$$\therefore \sin\delta_{1,\max} = \frac{P_{m1,\max}}{P_{\max}} \qquad \delta_{1,\max} = \sin^{-1}\left(\frac{P_{m1,\max}}{P_{\max}}\right) \qquad \delta_{2,\max} = \Pi - \sin^{-1}\left(\frac{P_{m1,\max}}{P_{\max}}\right)$$

- **From Fig it is understood, any further increase in $P_{m1,max}$ will make the area A_2 less than the area A_1**
- **This means that the acceleration power is more than the deceleration power**
- **Hence the system will have an excess kinetic energy which causes δ to increase beyond point c**
- **If the δ increases beyond c the deceleration power changes to acceleration power and so the system will become unstable**
- **The system will remain stable even though the rotor may oscillate beyond δ =90°, as long as the equal area criterion is met.**
- **Hence the condition of δ =90° for stability is applicable only for steady state stability and not for transient stability**

Clearing time and clearing angle

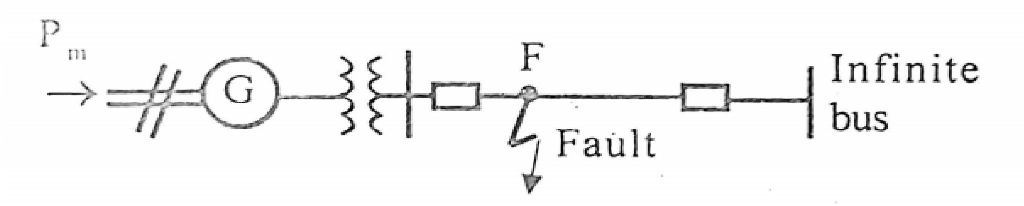


Figure: 5.9

- **Consider a single machine system shown in Fig**
- **Let the mechanical input be Pm and the machine is operating in steady state with torque angle δ0**
- **In the power angle curve, the operating point is Point a**

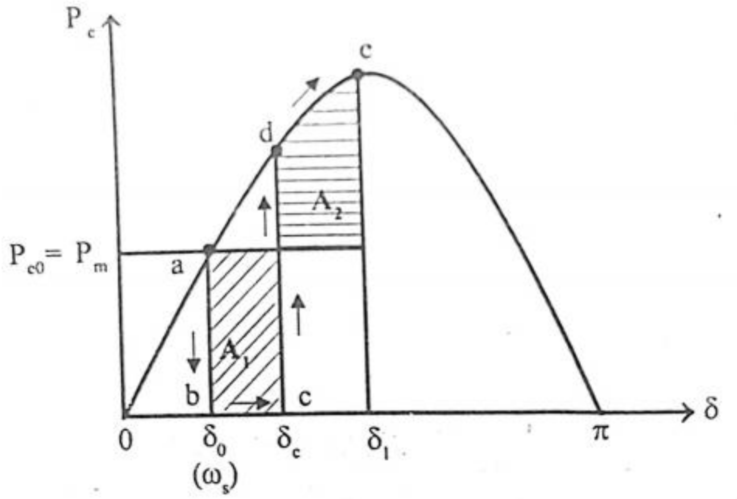


Figure: 5.10

- **Let a three fault phase fault occur at point F in the system**
- **Now Pe=0 and the operating point drops to b**
- **It means the power transferred to infinite bus is zero and the entire power generated is flowing through the fault**
- **Now the operating point moves along bc**
- **Let the fault be transient in nature and so the fault be cleared by opening of the C.B at point c where δ = δc and the correspondind time be tc**
- **Here tc is called clearing time and δc is called clearing angle**
- **It is assumed that the C.B closes its contact immediately after opening**
- **Hence normal operation is restored**
- **Now the operating point shifts to point d**
- **Now the rotor decelerates and the operating point moves along dc**
- **For this transient state, if an angle δ1 can be found such that A2=A1, then the system is found to be stable**
- **The stable system may finally settles down to the steady operating point a in an oscillatory manner due to damping in the system**

- **In the above discussion it is assumed that the fault is cleared at δ_c, but if the fault clearing is delayed then the angle δ_1 continue to increase to an upper limit δ_{max}**
- **This corresponds to a point where equal areas for A_1 and A_2 can be found for a given P_m as shown in fig**

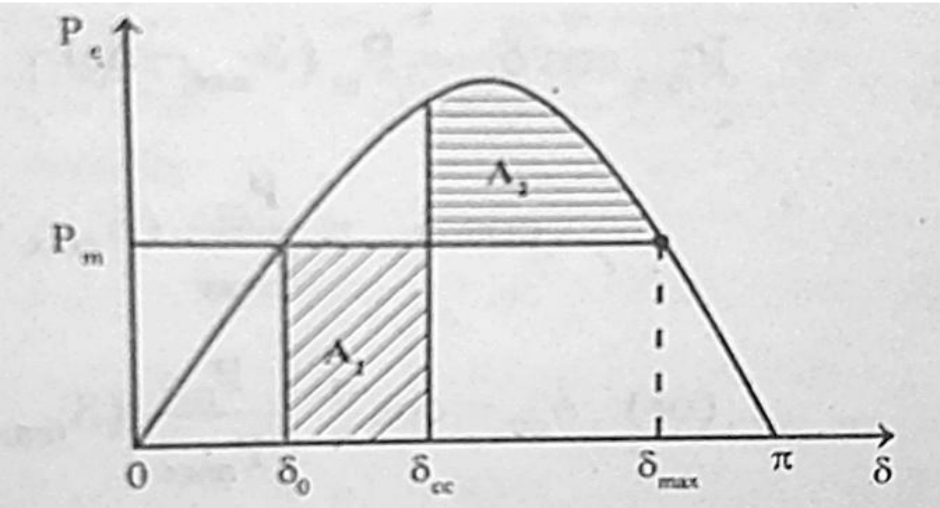


Figure: 5.11

- **For this situation the fault would have been cleared at an angle δ_{cc}**
- **This angle δ_{cc} is called critical clearing angle**
- **The time corresponding to this angle is called critical clearing time, t_{cc}**
- **If the fault is not cleared within critical time, then δ_1 would increase to a value greater than δ_{max}**
- **In this situation the area A_2 will be less than the area A_1 and so the system would be unstable**
- **For a 3 – Phase fault in simple systems, the equations for δ_{cc} and t_{cc} can be obtained as**

$$\delta_{max} = \pi - \delta_0$$

Under steady state for a given δ_0, $P_m = P_e$ and it is constant

$$\therefore\ P_m = P_e = P_{max} \sin \delta_0$$

The acceleration Power, $P_a=P_m-P_e$

When a three phase fault occurs, $P_e=0$

$\therefore P_a=P_m=\text{constant}$

The acceleration area A_1 can be evaluated by integrating P_a from $\delta=\delta_0$ to $\delta=\delta_{cc}$

$$\therefore A_1=\int_{\delta_0}^{\delta_{cc}} P_m d\delta = P_m[\delta]_{\delta_0}^{\delta_{cc}} = P_m(\delta_{cc}-\delta_0)$$

When the Power feeding is resumed after the fault, Pe = Pmax sinδ

Now, Pa=Pe-Pm = Pmax sinδ - Pm

The deacceleration area A2 can be evaluated by integrating Pa from from δ=δcc to δ=δmax

$$\therefore A_2=\int_{\delta_{cc}}^{\delta_{\max}} (P_{\max}\sin\delta - P_m)d\delta = \left[-P_{\max}\cos\delta - P_m\delta\right]_{\delta_{cc}}^{\delta\max}$$

$$=\left[-P_{\max}\cos\delta_{\max} - P_m\delta_{\max} + P_{\max}\cos\delta_{cc} + P_m\delta_{cc}\right]$$

$$=P_{\max}(\cos\delta_{cc}-\cos\delta_{\max}) - P_m(\delta_{\max}-\delta_{cc})$$

For a stable system A1=A2. Hence the equations of A1 & A2 can be equated to solve δcc

$$\therefore P_{\max}\cos\delta_{cc} = P_m\delta_{cc} - P_m\delta_0 + P_{\max}\cos\delta_{\max} + P_m\delta_{\max} - P_m\delta_{cc}$$

$$P_{\max}\cos\delta_{cc} = P_m(\delta_{\max}-\delta_0) + P_{\max}\cos\delta_{\max}$$

$$\therefore \cos\delta_{cc} = \frac{P_m}{P_{\max}}(\delta_{\max}-\delta_0) + \cos\delta_{\max}$$

$$\delta_{cc} = \cos^{-1}\left[\frac{P_m}{P_{\max}}(\delta_{\max}-\delta_0) + \cos\delta_{\max}\right]$$

Consider the swing equation of single machine system

$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

During a three phase fault, Pe=0

$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = P_m \qquad \frac{d^2\delta}{dt^2} = \frac{\Pi f P_m}{H}$$

On integrating the equation twice

$$\delta = \frac{\Pi f}{2H} P_m t^2 + \delta_0$$

Where δ_0 is the integral constant

When $\delta = \delta_{cc}$, $t = t_{cc}$; At t_{cc}

$$\delta_{cc} = \frac{\Pi f}{2H} P_m t_{cc}^2 + \delta_0$$

$$t_{cc} = \sqrt{\frac{2H(\delta_{cc} - \delta_0)}{\Pi f P_m}}$$

This equation is used to estimate the value of critical clearing time t_{cc}

Solution of swing equation by Point by Point method

Consider the swing equation of a power system

$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$M = \frac{H}{\Pi f}$$

$$P_e = P_{\max} \sin\delta$$

$$P_a = P_m - P_e = P_m - P_{\max}\sin\delta$$

$$\therefore M\frac{d^2\delta}{dt^2} = P_a$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

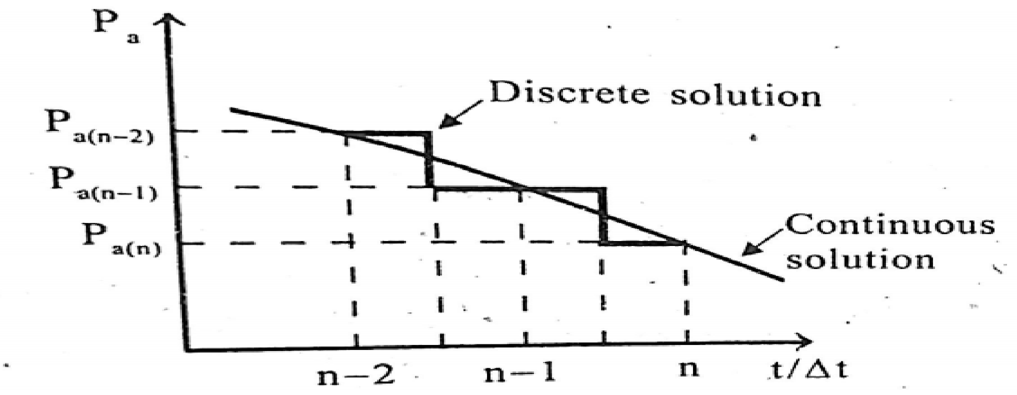
- **The equation is a nonlinear equation**
- **During transient state the δ is a function of time, t and so it can be denoted as $\delta(t)$**
- **In point by point method, the solution of $\delta(t)$ is obtained by dividing the time into small equal values of Δt**

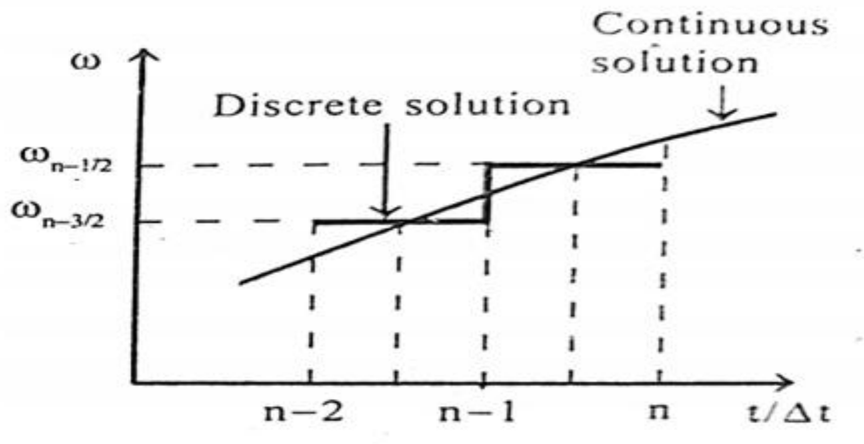
Assumptions:

1. **The accelerating power p_a computed at the beginning of an interval is assumed constant from the middle of the preceding interval to the middle of the interval being considered**
2. **The angular velocity is assumed constant throughout any interval. This constant value is the value corresponding to the midpoint of concerned interval**

- **The solution starts from the initial condition values, that corresponds to a stable operating point**

Let δ_0 be the angle corresponding to initial operating point





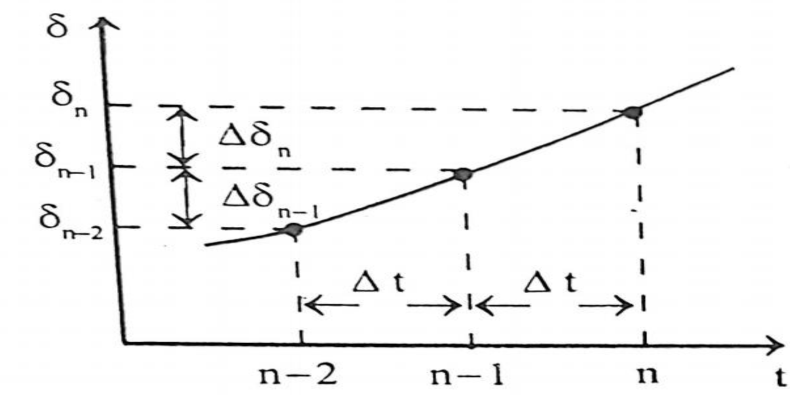


Figure: 5.12

δ_{n-1} – The value of δ at the end of (n-1) th interval

$\omega_{n-1/2}$ - The value of ω at the end of (n-1) th interval

$P_{a(n-1)}$ – Value of Pa at the end of (n-1)th interval

$P_{a(n-1)} = P_m - P_{max} \sin\delta_{n-1}$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \qquad \frac{d\omega}{dt} = \frac{P_a}{M} \qquad \frac{\Delta\omega}{\Delta t} = \frac{P_a}{M} \qquad \therefore \Delta\omega = \frac{\Delta t}{M} P_a$$

Let $\omega_{n-3/2}$ – the value of ω at the end of nth interval

For calculating nth interval value of ω, $\Delta\omega = \omega_{n-1/2} - \omega_{n-3/2}$, $P_a = P_{a(n-1)}$

$$\therefore \omega_{n-1/2} - \omega_{n-3/2} = \frac{\Delta t}{M} P_{a(n-1)}$$

$$\therefore \omega_{n-3/2} = \omega_{n-1/2} - \frac{\Delta t}{M} P_{a(n-1)}$$

For small changes in δ,

$$\omega = \frac{\Delta\delta}{\Delta t}$$

$$\Delta\delta = \Delta t\omega$$

For a change in δ in (n-1)th interval,

$$\Delta\delta_{n-1} = \Delta t\omega_{n-3/2}$$

For a change in δ in nth interval,

$$\Delta\delta_n = \Delta t\omega_{n-1/2}$$

δn – The Value of δ at the end of nth interval,

$$\delta_n = \delta_{n-1} + \Delta\delta_n$$

- **The above process of computation is repeated to obtain $P_{a(n)}$, $\Delta\delta_{(n+1)}$ and $\delta_{(n+1)}$**
- **The solution of δ(t) is thus obtained in discrete form over the desired length of time**
- **The normal desired length of time is 0.5 sec**
- **The continuous form of solution is obtained by drawing a smooth curve through discrete values**

Modified Euler's Method

- **This method is used to solve the swing equation**
- **In this the swing equation are transformed into the state variable form**

$$\frac{d\delta}{dt} = \Delta\omega$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right) = \frac{\Pi f}{H} p_a$$

$$\frac{d}{dt}(\Delta\omega) = \frac{\Pi f}{H} p_a$$

Computational algorithm using Modified Euler's Method for power system problems

1. **Obtain a load flow solution for the pre transient condition**
2. **Calculate the generator internal voltage behind transient reactance. The state vectors have finite values whereas all ω=0 under pre transient condition**
3. **Assume the occurrence of fault and initialize time i=0. calculate the reduced admittance matrix for this condition. Set count j=0.**

4. Determine the state derivatives and calculate the first state estimate

$$\delta_{i+1}^{*} = \delta_i + \left[\frac{d\delta}{dt} / \Delta\omega t\right](\Delta t)$$

$$\Delta\omega_{i+1}^{P} = \Delta\omega_i + \left[\frac{d\Delta\omega}{dt} / \delta_i\right](\Delta t)$$

5. Second estimate of the variable δ can be obtained if derivatives at $t_1=t_0+\Delta t$ so that the generated power can be calculated

6. Determine the average value of the state derivative and obtain the second estimate of the state variables and the second estimate of the internal voltage angle and machine angular speeds

$$\delta_{i+1}^{c} = \delta_i + \left[\frac{\frac{d\delta}{dt} / \Delta\omega_i + \frac{d\delta}{dt} / \Delta_{\omega i+1}^{P}}{2}\right](\Delta t)$$

$$\Delta\omega_{i+1}^{c} = \Delta\omega_i + \left[\frac{\frac{d\Delta\omega t}{dt}/\delta_i + \frac{d\Delta\omega t}{dt}/\delta_{i+1}}{2}\right](\Delta t)$$

7. Compute final internal voltage of the generation at the end of $[t_0+\Delta t]$ and print the results

8. Check if $t < t_{cc}$. If yes advance time by Δt and go to step 4

9. Check if j=0, if yes the nodal admittance matrix is changed corresponding to the post fault condition and a new reduced admittance matrix is obtained. Set j=j+1

10. Set i=i+1 and $t_1=t_0+\Delta t$ and $t_2=t_1+\Delta t$

11. Check if $t < t_{max}$. If yes go to step 8

12. Terminate the process of computation

The relation between δ and t for various generators are obtained and stability of the system can be estimated for a particular type of fault and particular clearing time

Runge Kutta Method

This method is the most powerful method for solving swing equation on digital computers

Algorithm

1. **Obtain a load flow solution for the pre transient condition**
2. **Calculate the generator internal voltages behind transient reactances**
3. **Assume the occurrences of a fault and calculate the reduce admittance matrix for the condition and initialize the time count k=0, initialize j=0**
4. **Determine the following conditions**

$K_1^k = f_1(\delta^k, \omega^k)\Delta t$

$I_1^k = f_2(\delta^k, \omega^k)\Delta t$

$K_2^k = f1[\delta^k + 1/2K_1^k \omega^k + 1/2I_1^k]\Delta t$

$I_2^k = f_2[\delta^k + 1/2K_1^k \omega^k + 1/2I_1^k]\Delta t$

$K_3^k=f_1[\delta^k+1/2K_2^k\omega^k+1/2I_2^k]\Delta t$

$I_3^k=f_2[\delta^k+1/2K_2^k\omega^k+1/2I_2^k]\Delta t$

$K_4^k=f_1[\delta^k+1/2K_3^k\omega^k+1/2I_3^k]\Delta t$

$I_4^k=f_2[\delta^k+1/2K_3^k\omega^k+1/2I_3^k]\Delta t$

5. Then compute the change in state vector

$\Delta\delta^k = 1/6\ [\ k_1^k+2K_2^k+2K_3^k+K_4^k]$

$\Delta\omega^k=1/6[I_1^k+2I_2^k+2I_3^k+I_4^k]$

6. Evaluate the internal voltage behind transient reactance

7. Check if t > tcc, if yes k=k+1 and go to step 4

8. Check if j=0, yes modify the network data and obtain a new reduced admittance matrix corresponding to post fault condition. Set j=j+1

9. Set k=k+1

10. Check if k < K_{max}, yes go to step 4

11. Then terminate the process