SATHYABAMA
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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - I

SYSTEM CONCEPTS – SEE1203

TYPES OF SYSTEMS

Control systems are basically classified as –

- Open-loop control system
- Closed-loop control system

In open-loop system the control action is independent of output. In closed-loop system control action is somehow dependent on output. Each system has at least two things in common, a controller and an actuator (final control element). The input to the controller is called reference input. This signal represents the desired system output.Open-loop control system is used for very simple applications where inputs are known ahed of time and there is no disturbance. Here the output is sensitive to the changes in ***disturbance*** inputs. Disturbance inputs are undesirable inputs that tend to deflect the plant outputs from their desired values. They must be calibrated and adjusted at regular intervals to ensure proper operation. Closed-loop systems are also called feedback control systems. Feedback is the property of the closed-loop systems which permits the output to be compared with the input of the system so that appropriate control action may be formed as a function of inputs and outputs. Feedback systems has the following features:

- reduced effect of nonlinearities and distortion
- Increased accuracy
- Increased bandwidth
- Less sensitivity to variation of system parameters
- Tendency towards oscillations
- Reduced effects of external disturbances

The general block diagram of a control system is shown below.

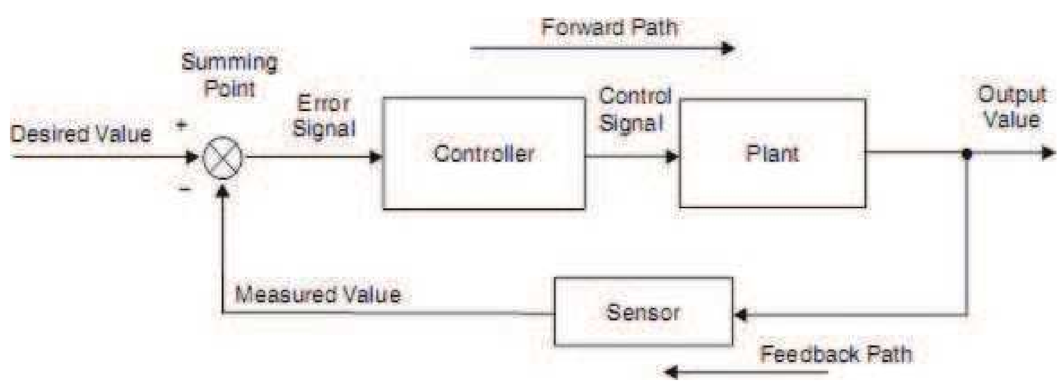


Figure: Closed-loop control system

Some Definitions

Reference input – It is the actual signal input to the control system.

Output (Controlled variable) – It is the actual response obtained from a control system. ***Actuating error signal*** – It is the difference between the reference input and feedback signal. ***Controller*** – It is a component required to generate control signal to drive the actuator.

Control signal – The signal obtained at the output of a controller is called control signal. ***Actuator*** – It is a power device that produces input to the plant according to the control signal, so that output signal approaches the reference input signal.
Plant – The combination of object to be controlled and the actuator is called the plant.
Feedback Element – It is the element that provides a mean for feeding back the output quantity in order to compare it with the reference input.
Servomechanism – It is a feedback control system in which the output is mechanical position, velocity, or acceleration.

EXAMPLE OF CONTROL SYSTEMS

Toilet tank filling system:

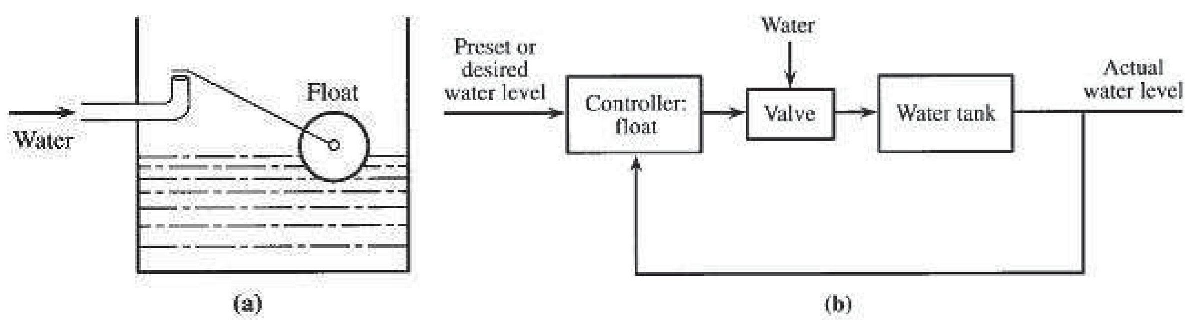


Figure: Toilet tank filling system

Position control system: [antenna]

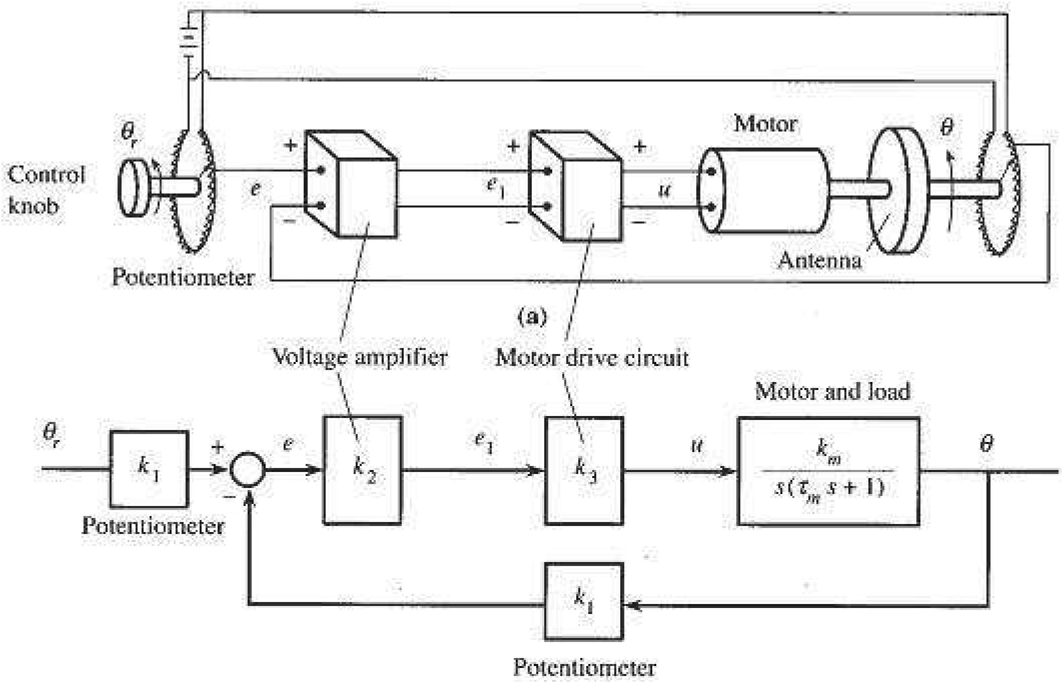


Figure: Position control system

Velocity control system: [audio/ video recorder]

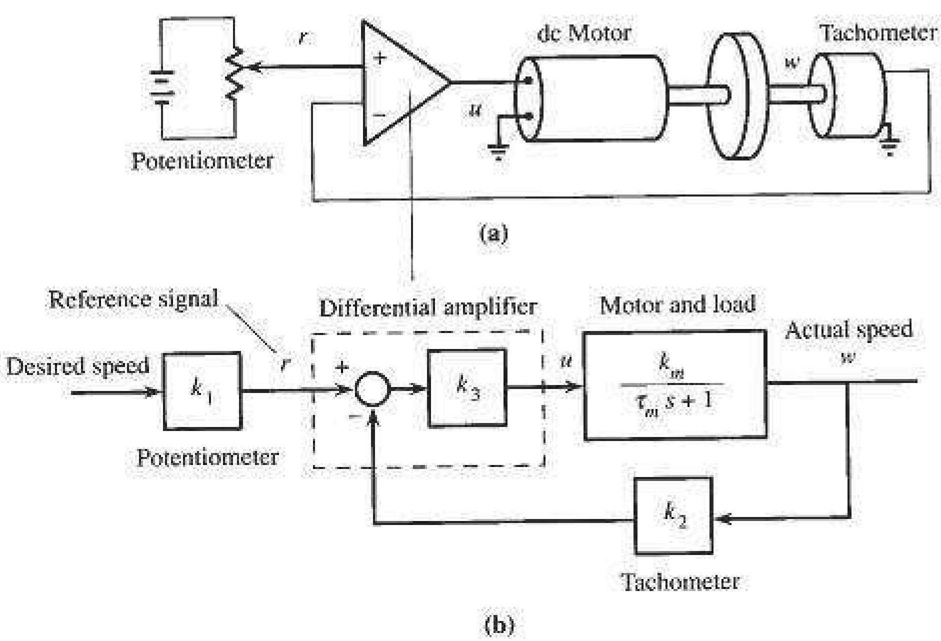


Figure: Velocity control system

Clothes Dryer:

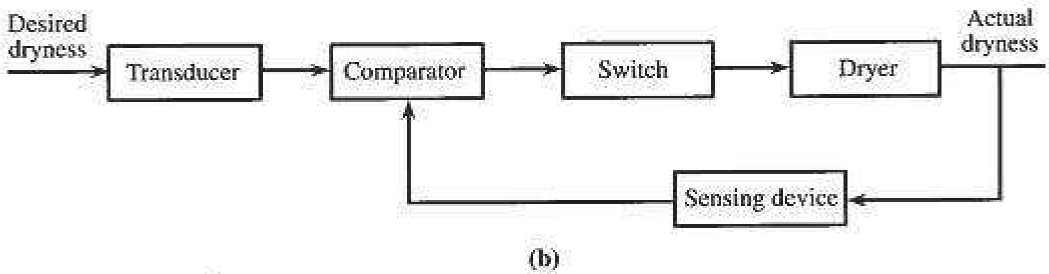


Figure: Automatic dryer

Temperature control system: [oven, refrigerator, house]

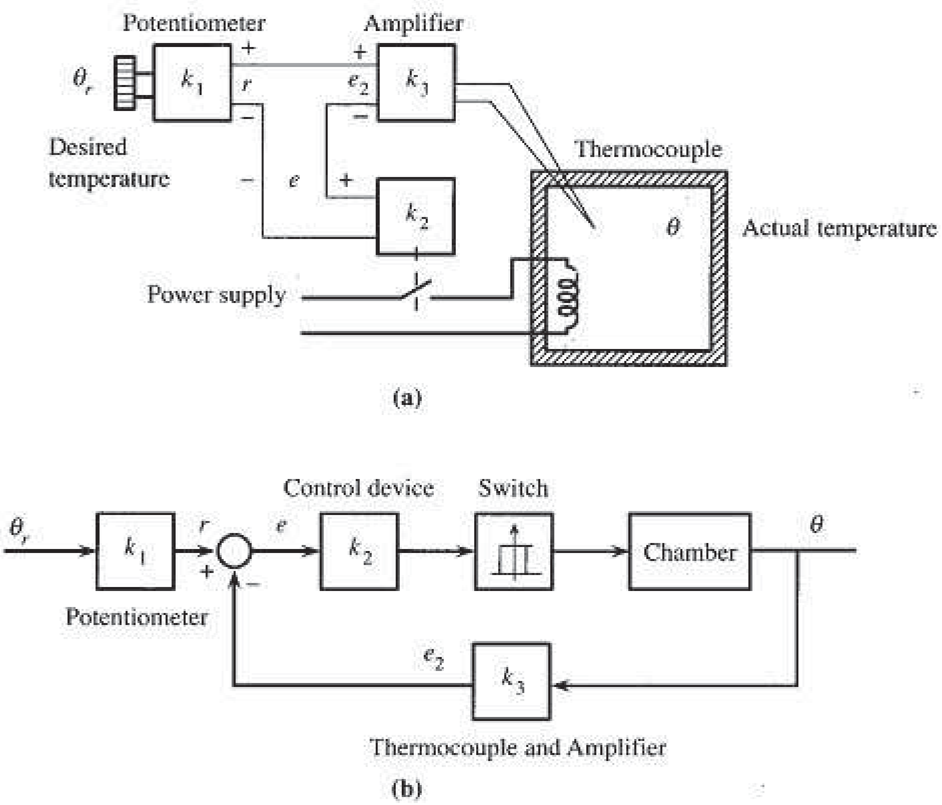
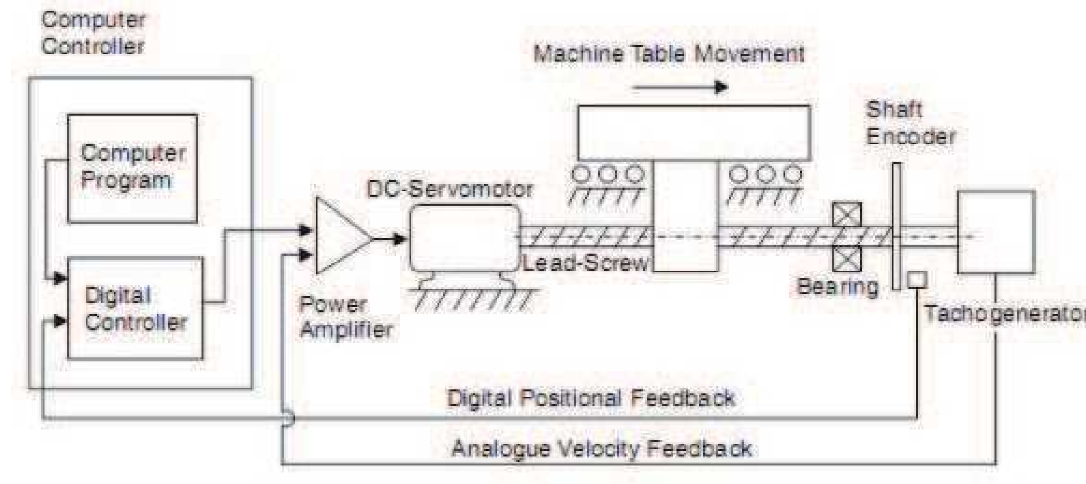
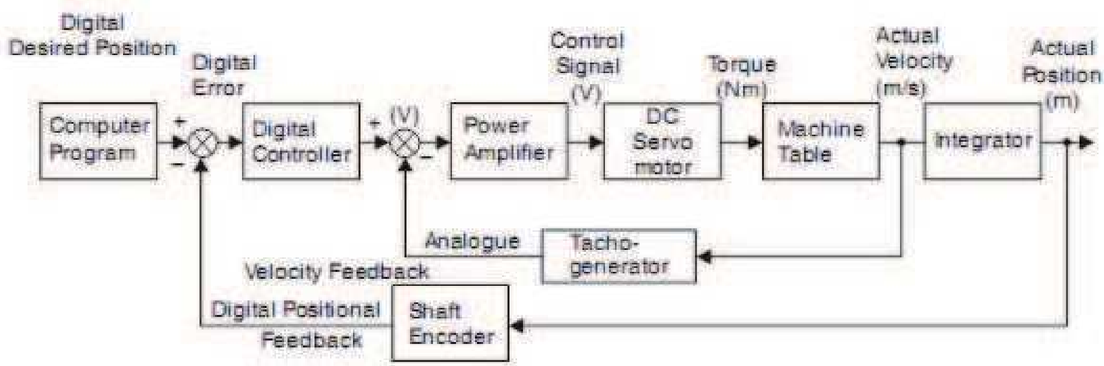


Figure: Temperature control system

Computer numerically controlled (CNC) machine tool:



(a)

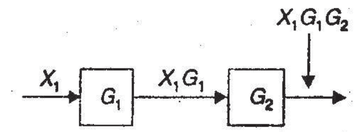
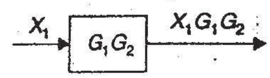
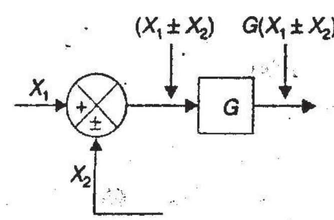
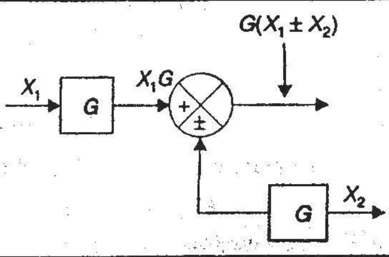
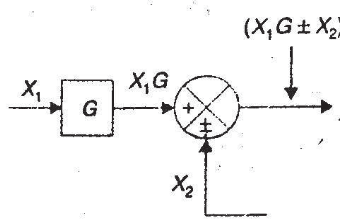
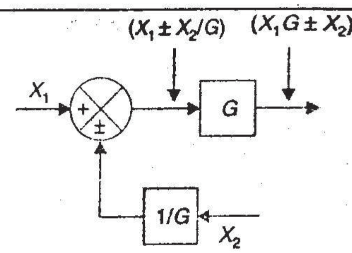
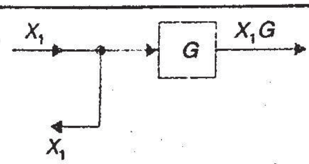
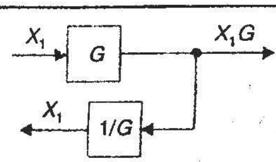
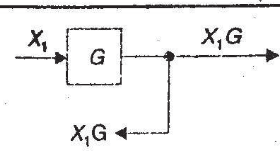
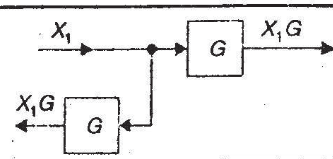
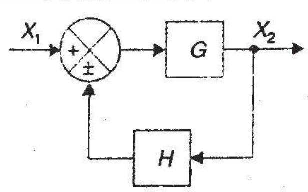
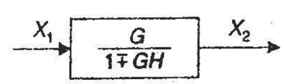


(b)

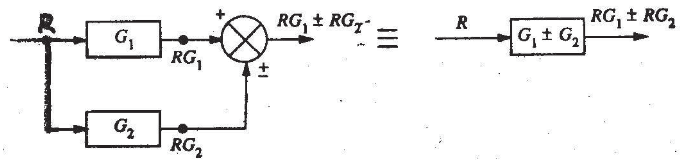
Figure: CNC machine tool control system

BLOCK DIAGRAM ALGEBRA

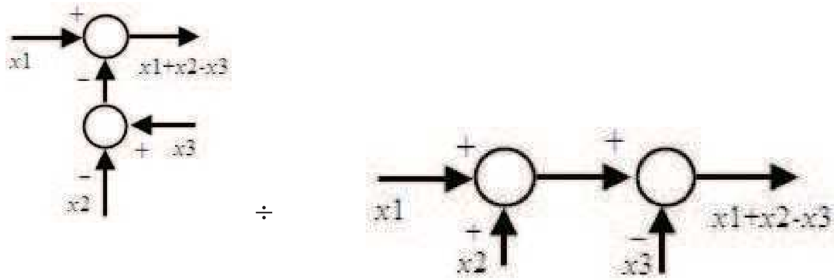
A complex system is represented by the interconnection of the blocks for individual elements. Evaluation of complex system requires simplification of block diagrams by block diagram rearrangement. Some of the important rules are given in figure below.

Rule	*Original diagram*	*Equivalent diagram*
1. Combining blocks in cascade		
2. Moving a summing point after a block		
3. Moving a summing point ahead of a block		
4. Moving a take off point after a block		
5. Moving a take off point ahead of a block		
6. Eliminating a feedback loop		

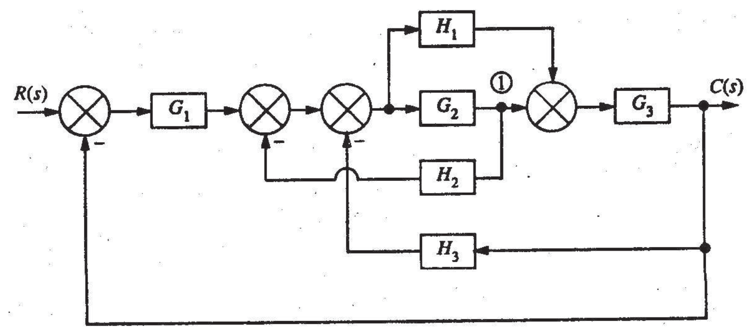
7. Combining Blocks in Parallel

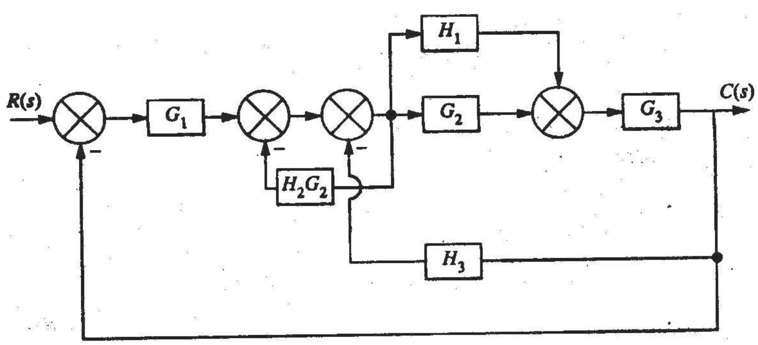


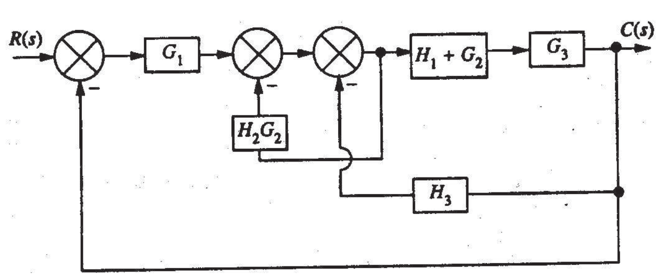
8. Moving summing point :

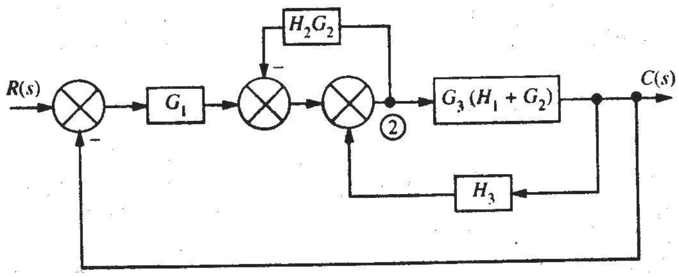


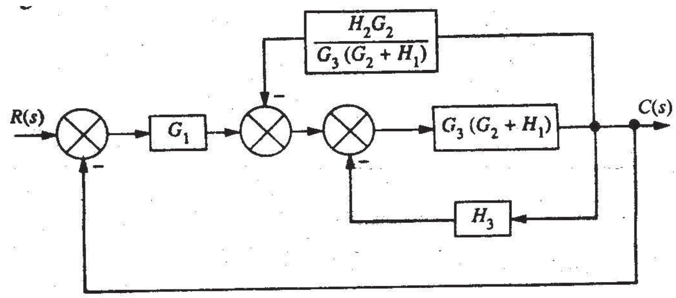
Example: Simplify the block diagram shown in Figure below.

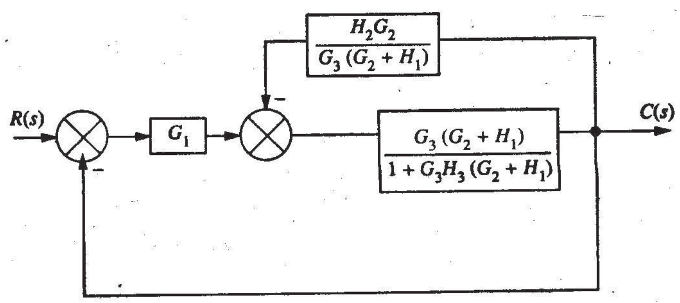


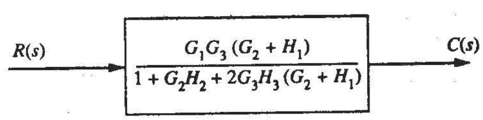




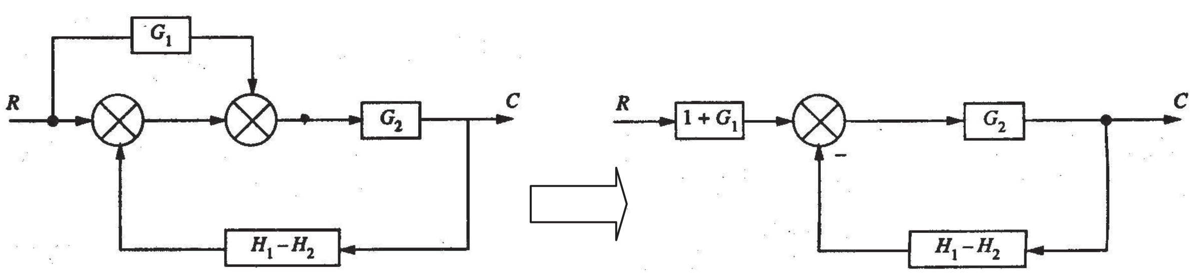


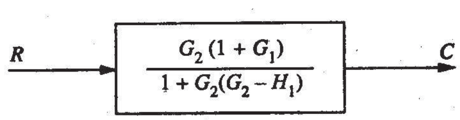






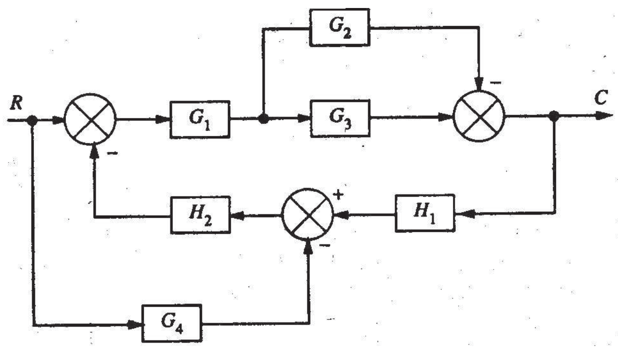
Example: Obtain the transfer function C/R of the block diagram shown in Figure below.



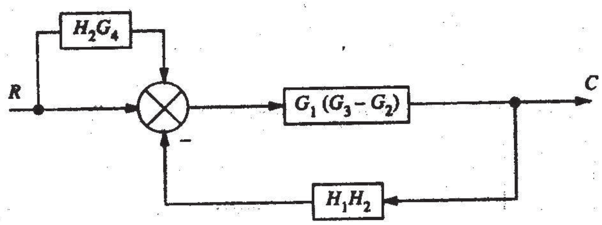


[Ans]

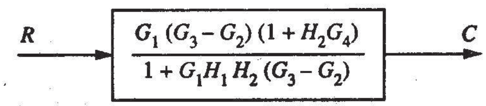
Example: Derive the transfer function of the system shown below.



(a)



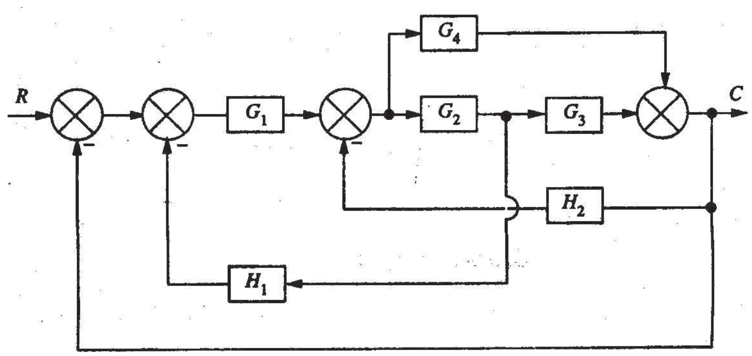
(b)

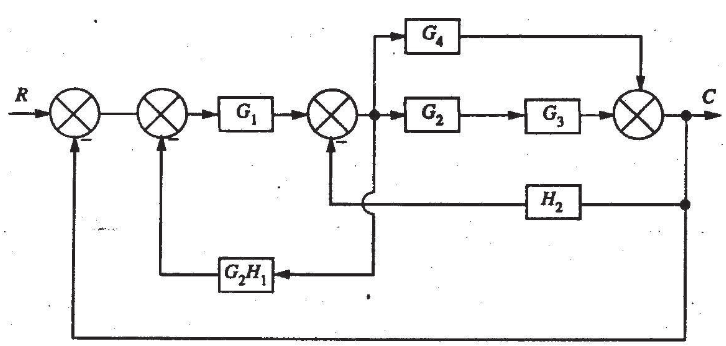


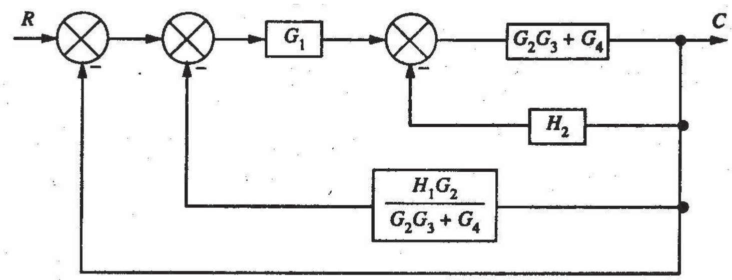
$$\frac{G_1 (G_3 - G_2)(1 + H_2 G_4)}{1 + G_1 H_1 H_2 (G_3 - G_2)}$$

[Answer]

Example: Derive the transfer function of the system shown below.

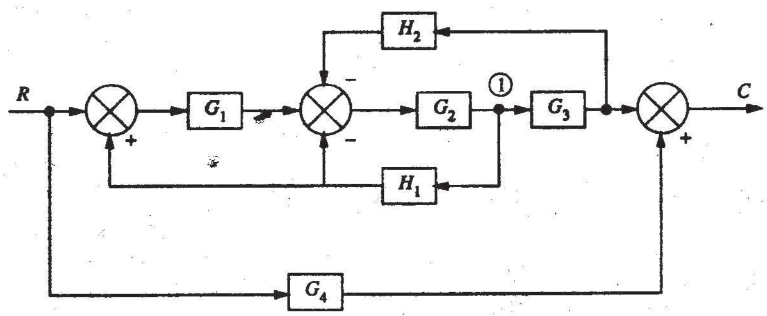


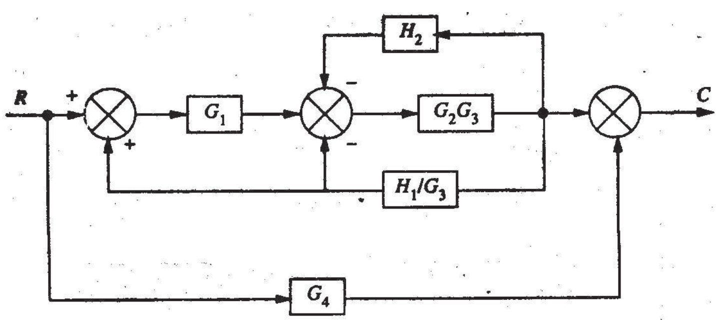




$$\frac{C}{R} = \frac{G_1 [G_2 G_3 + G_4)}{1 + H_2 [G_2 G_3 + G_4] + G_1 G_2 H_1 + G_1 [G_2 G_3 + G_4]}$$

Example: Find the transfer function of the following system.

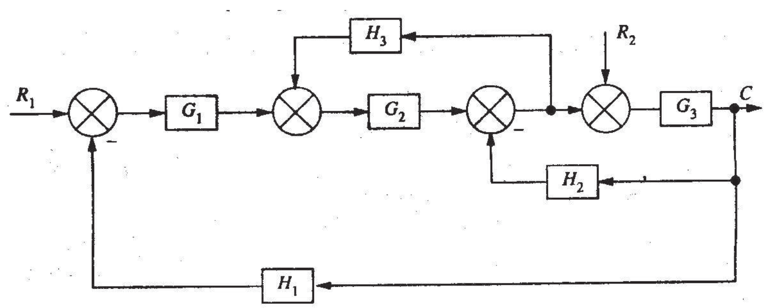




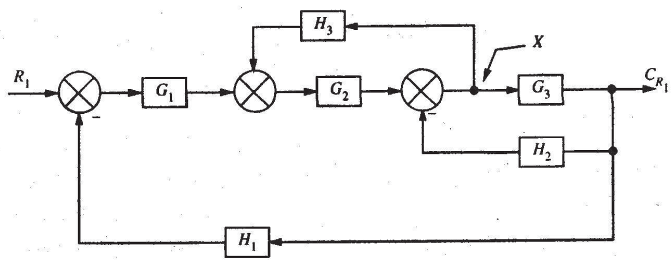
$$\frac{C}{R} = G_4 + \frac{G_1 G_2 G_3}{1 + H_2 G_2 G_3 + G_2 H_1 - H_1 G_1 G_2}$$

{Answer}

Example: Find the output of the system shown below.

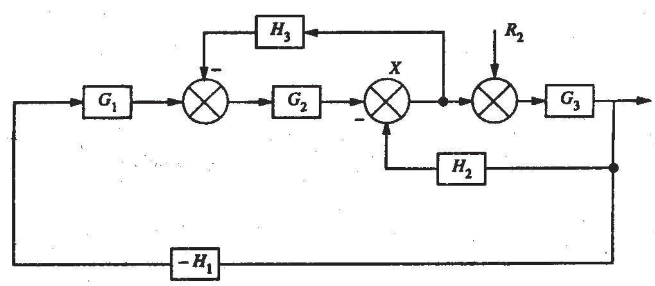


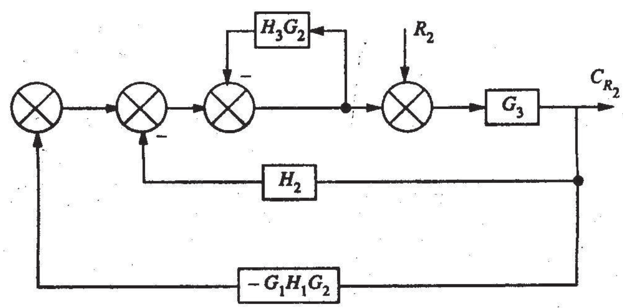
For Input R_1:



$$C_{R_1} = \left[\frac{G_1 G_2 G_3}{1 + G_3 H_2 + H_3 G_2 + G_1 G_2 G_3 H_1}\right] R_1 \qquad \text{.. (1)}$$

For input R_2:





$$C_{R_2} = \left[\frac{G_3 [1 + G_2 H_3]}{1 + G_2 H_3 + G_3 [G_1 G_2 H_1 + H_2]}\right] R_2 \qquad \text{.. (2)}$$

$$C = C_{R_1} + C_{R_2}$$

$$C = \left[\frac{G_1 G_2 G_3}{1 + G_3 H_2 + G_2 H_3 + G_1 G_2 G_3 H_1}\right] R_1 + \left[\frac{G_3 [1 + G_2 H_3]}{1 + G_2 H_3 + G_3 [G_1 G_2 H_1 + H_2]}\right] R_2$$

SIGNAL FLOW GRAPH

SFG is a diagram that represents a set of simultaneous linear algebraic equations which describe a system. Let us consider an equation, $Y = aX$. It may be represented graphically as,

where 'a' is called ***transmittance*** or transmission function.

Definitions in SFG

Node – A system variable, the value of which equals the sum of all incoming signals at the node.

Branch – A directed line segment joining two nodes.

Input/ Output node – node having only one outgoing/ incoming branch. *Path* – A traversal of connected branches in the direction of branch arrows. *Forward path* – A path from input to output node.

Loop – A closed path that originates and terminates on the same node.

Self-loop – A loop containing one branch.

Non-touching loops – Loops which do not have a common node.

… of a branch.

Construction of SFGs

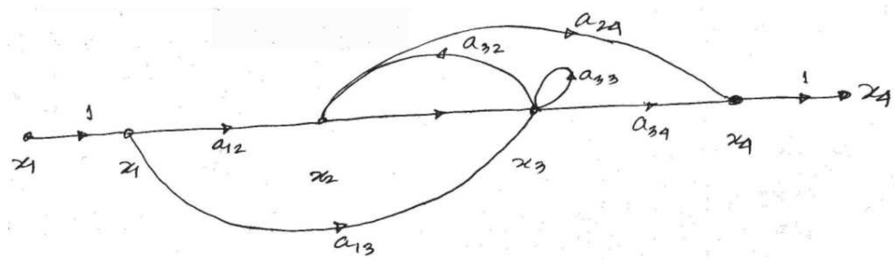
The SFG of a system can be constructed from the describing equations:

$$x_2 = a_{12} x_1 + a_{32} x_3$$

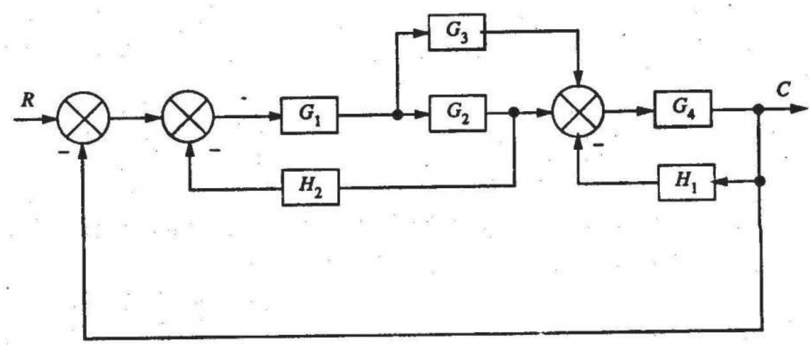
$$x_3 = a_{13} x_1 + a_{23} x_2$$

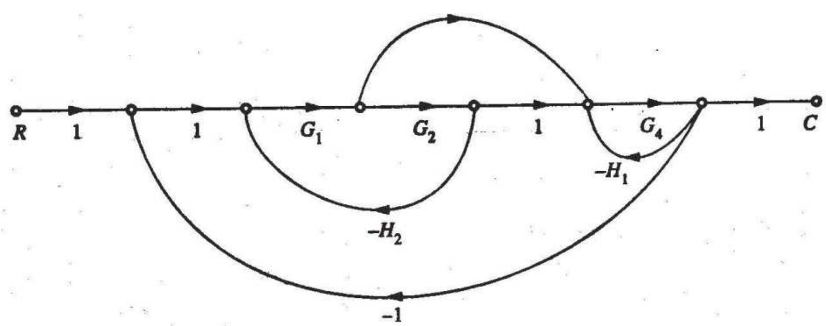
$$x_4 = a_{24} x_4 + a_{34} x_3$$

$+ a_{33} x_3$



SFG from Block Diagram





Each variable in the block diagram becomes a node, and each block becomes a branch.

Mason's Gain Formulae

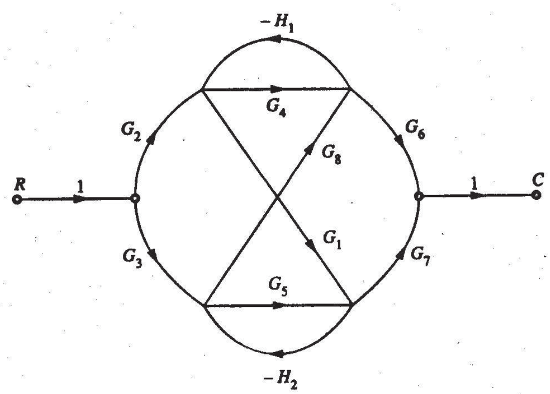
It is possible to write the overall transfer function of a system through inspection of SFG using Mason's gain formulae given by, $T = (\sum_i P_i O_i) / O$.

where T = overall gain of the system, P_i = path gain of *ith* forward path, O = determinant of SFG, O_i = value of O for that part of the graph not touching the *ith* forward path.

$O = 1 - \sum_j P_{j1} + \sum_j P_{j2} - \sum_j P_{j3} + \cdots$ = 1 – [sum of loop gain of all individual loops] + [sum of all gain-products of two non-touching loops] – [sum of all gain-products of three non-touching loops] + ……;

P_{jk} = *j*th product of *k* non-touching loops .

Example



1. There are 6 forward paths with path gains

$$P_1 = G_2 G_4 G_6$$
$$P_2 = G_3 G_5 G_7$$
$$P_3 = G_2 G_1 \cdot G_7$$
$$P_4 = G_3 G_8 G_6$$
$$P_5 = -G_2 G_1 \cdot H_2 G_8 \cdot G_6$$
$$P_6 = -G_3 G_8 H_1 G_1 G_7$$

2. There are three individual loops with loop gains

$$P_{11} = -H_1 G_4$$
$$P_{21} = -H_2 G_5$$
$$P_{31} = G_1 H_2 G_8 H_1$$

3. There is only one combination of two non-touching loops

$$P_{12} = H_1 H_2 G_4 G_5$$

4. There are no combinations of more than two non-touching loops.

5. Hence,

$$\Delta = 1 - [-H_1 G_4 - H_2 G_5 + G_1 H_2 G_8 H_1] + [H_1 H_2 G_4 G_5]$$
$$= 1 - G_1 H_2 G_8 H_1 + H_2 G_5 - G_1 H_2 G_8 H_1 + H_1 H_2 G_4 G_5$$

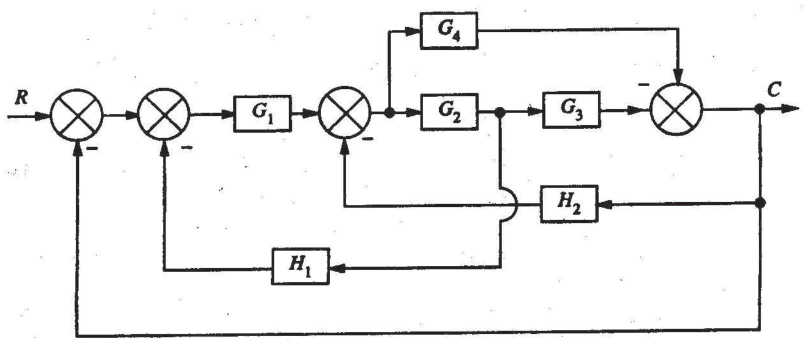
$\Delta_1 = 1 - (-H_2 G_5) = 1 + H_2 G_5$; $\Delta_2 = 1 - (-H_1 G_4) = 1 + H_1 G_4$;

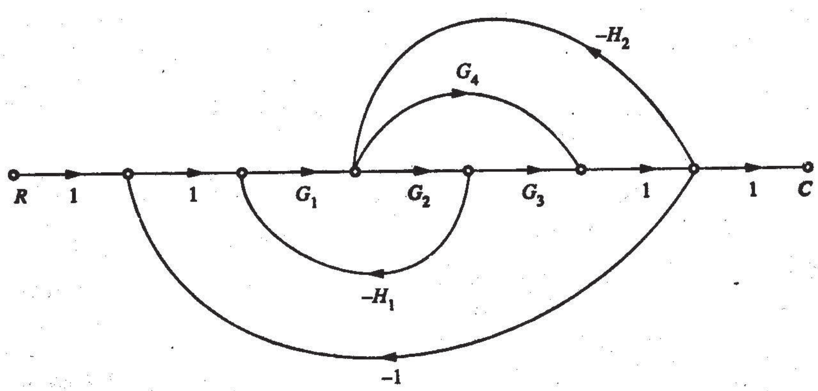
$O_3 = O_4 = O_5 = O_6 = 1$

Thus, $T = \dfrac{P_1 O_1 + P_2 O_2 + P_3 O_3 + P_4 O_4 + P_5 O_5 + P_6 O_6}{O}$, where P_1, O_1, O etc. are derived before.

Example

Draw the SFG and determine C/ R for the block diagram shown in Figure below.





$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2}$$ {Answer}

Example

For the system represented by the following equations, find the transfer function X(s)/U(s) by SFG technique.

$$x = x_1 + \alpha_3 u$$

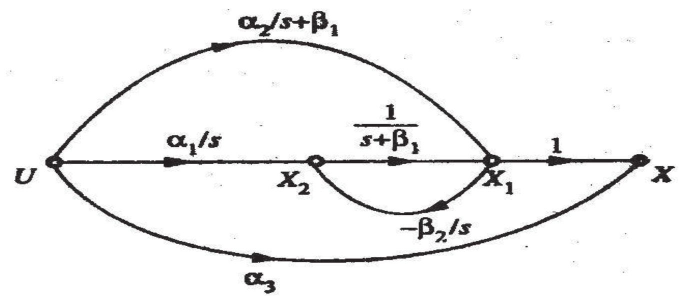
$$\dot{x}_1 = -\beta_1 x_1 + x_2 + \alpha_2 u$$
$$\dot{x}_2 = -\beta_2 x_1 + \alpha_1 u$$

We need to Laplace transform the given sets of equations in order to represent differentiated variables.

$$X = X_1 + \alpha_3 U$$

$$X_1 = \frac{1}{s+\beta_1} X_2 + \frac{\alpha_2}{s+\beta_1} u$$

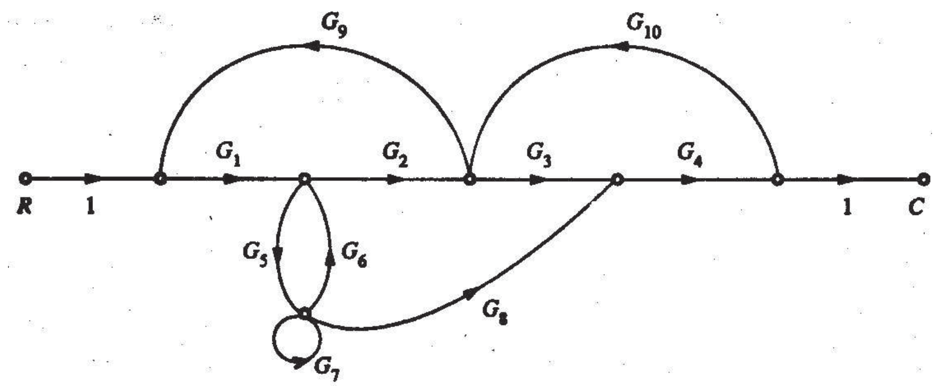
$$X_2 = -\frac{\beta_2}{s} X_1 + \frac{\alpha_1}{s} u$$



$$\frac{X(s)}{U(s)} = \frac{\alpha_1 + \alpha_2 s + \alpha_3 \bullet [s^2 + \beta_1 s + \beta_2]}{s^2 + \beta_1 s + \beta_2}$$ {Answer}

Example

Using Mason's gain formulae find C/R of the SFG shown in Figure below.



$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 (1 - G_7) + G_1 G_5 G_8 G_4}{1 - [G_1 G_2 G_9 + G_3 G_4 G_{10} + G_1 G_5 G_8 G_4 G_{10} G_9 + G_5 G_6 + G_7] + [G_1 G_2 G_9 G_7 + G_3 G_4 G_{10} G_5 G_6 + G_3 G_4 G_{10} G_7]}$$

TEXT / REFERENCE BOOKS :-

1. I.J.Nagarath and M.Gopal, "Control System Engineering", New Age International (P) Limited Publishers, 2nd Edition, 2009.
2. Kausuhio Ogata, "Modern Control Engineering", Prentice Hall of India PVT. Ltd, 5th Edition, 2010.
3. Richard Dorf, "Modern Control Systems", Pearson Education Ltd, 11th Edition, 2009.
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5. N.K.Sinha, "Control Systems", New Age International Private Limited Publishers, 3rd Edition, Reprint 2008.
6. A.Nagoorkani, "Control System", RBA Publications, 3rd Edition, reprint 2012.
7. U.A.Bakshi and S.C.Goyal, "Control System Engineering", Technical Publication, 2nd Revised Reprint 2007.

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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - II

TIME RESPONSE ANALYSIS OF CONTROL SYSTEM – SEE1203

Introduction :-

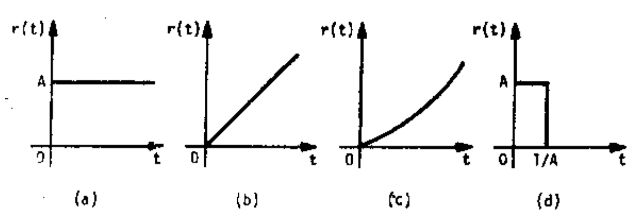
In time-domain analysis the response of a dynamic system to an input is expressed as a function of time. It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.

Usually, the input signals to control systems are not known fully ahead of time. In a radar tracking system, the position and the speed of the target to be tracked may vary in a random fashion. It is therefore difficult to express the actual input signals mathematically by simple equations. The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and constant acceleration. The dynamic behavior of a system is therefore judged and compared under application of standard test signals – an impulse, a step, a constant velocity, and constant acceleration. Another standard signal of great importance is a sinusoidal signal.

The time response of any system has two components: transient response and the steady-state response. Transient response is dependent upon the system poles only and not on the type of input. It is therefore sufficient to analyze the transient response using a step input. The steady-state response depends on system dynamics and the input quantity. It is then examined using different test signals by final value theorem.

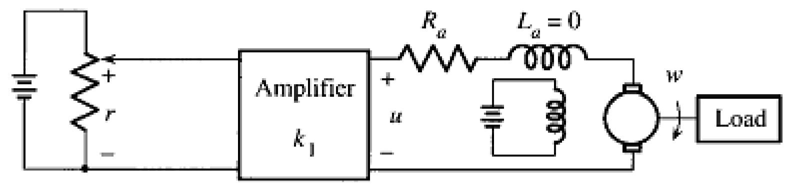
STANDARD TEST SIGNALS

a) Step signal:
b) Ramp signal:
c) Parabolic signal:
d) Impulse signal:



TIME-RESPONSE OF FIRST-ORDER SYSTEMS

Let us consider the armature-controlled dc motor driving a load, such as a video tape. The objective is to drive the tape at constant speed. Note that it is an open-loop system.



; If ,

;

is the steady-state final speed. If the desired speed is , choosing the motor will eventually reach the desired speed.

We are interested not only in final speed, but also in the speed of response. Here, is the time constant of motor which is responsible for the speed of response.

The time response is plotted in the Figure in next page. A plot of is shown, from where it is seen that, for the value of is less than 1% of its original value. Therefore, the speed of the motor will reach and stay within 1% of its final speed at 5 time constants.

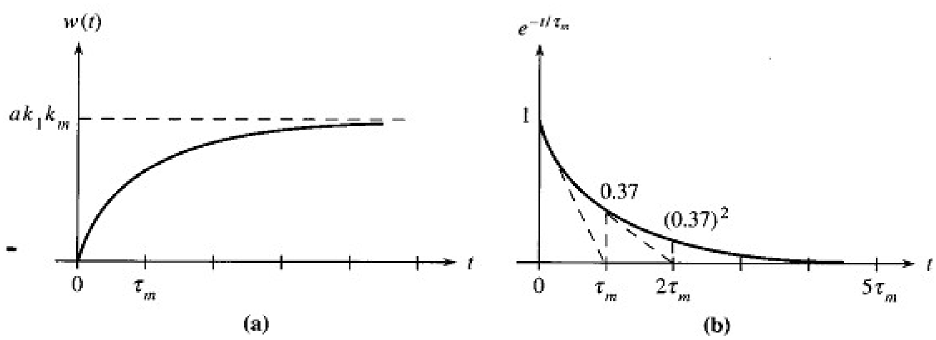
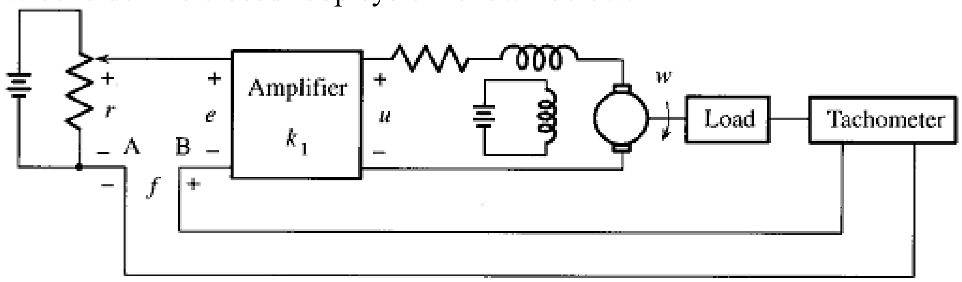
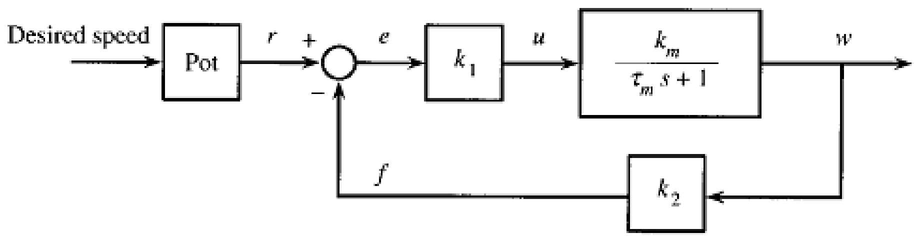


Figure: Time responses

Let us now consider the closed-loop system shown below.



(a)



(b)

Here,

where, and

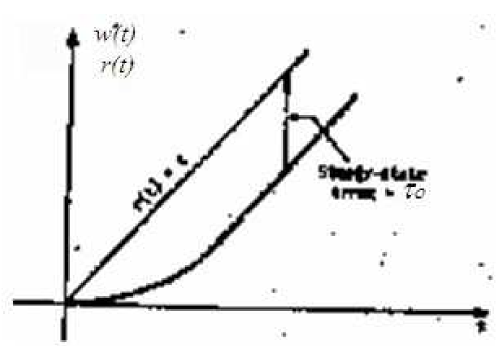
. If , the response would

be, .

If a is properly chosen, the tape can reach a desired speed. It will reach the desired speed in 5seconds. Here, . Thus, we can control the speed of response in feedback system.

Although the time-constant is reduced by a factor , in the feedback system, the motor gain constant is also reduced by the same factor. In order to compensate for this loss of gain, the applied reference voltage must be increased by the same factor.

Ramp response of first-order system



Let, for simplicity. Then, . Also, let, .

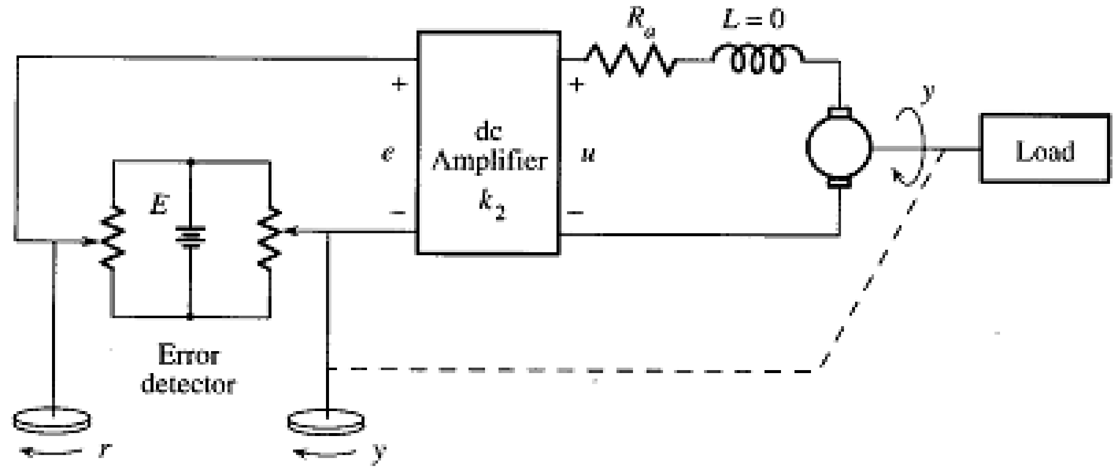
Then, ;

The error signal is,

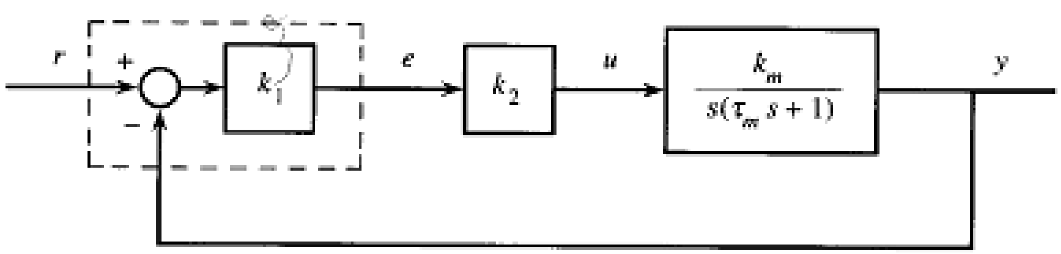
Or,

Thus, the first-order system will track the unit ramp input with a steady-state error , which is equal to the time-constant of the system.

TIME-RESPONSE OF SECOND-ORDER SYSTEMS



(a)



(b)

Consider the antenna position control system. Its transfer function from *r* to *y* is,

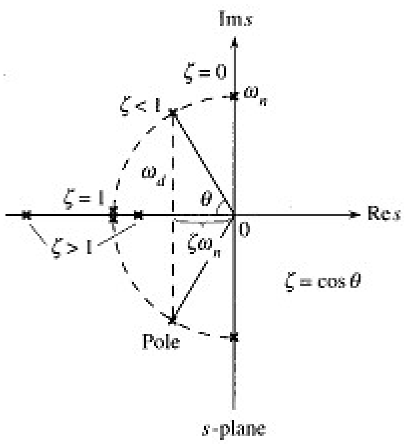
where, we define, and . The constant is called the *damping ratio* and is called the *natural frequency*. The system above is in fact a standard second order system.

The transfer function has two poles and no zero. Its poles are,

.

Here, is called the *damping factor*, is called *damped or actual frequency*.

The location of poles for different are plotted in Figure below. For, the two poles are purely imaginary. If , the two poles are complex conjugate. All possible cases are described in a table shown below.



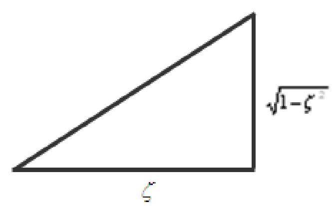
Damping Ratio	Poles	Remark
$\zeta = 0$	Pure imaginary	Undamped
$0 < \zeta < 1$	Complex conjugate	Underdamped
$\zeta = 1$	Repeated real poles	Critically damped
$\zeta > 1$	Two distinct real poles	Overdamped

Unit step response of second-order systems

Suppose,

; Or,

Performing inverse Laplace transform,

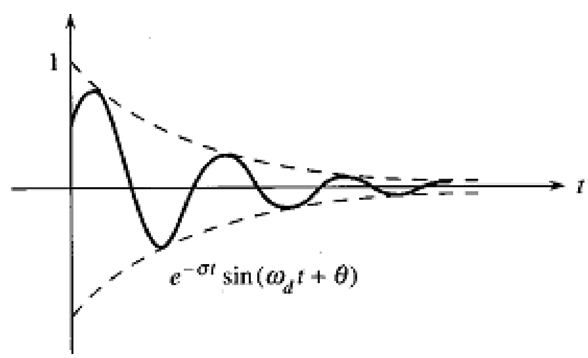


or,

or, ,where, and

or,

The plot of is shown in Figure.

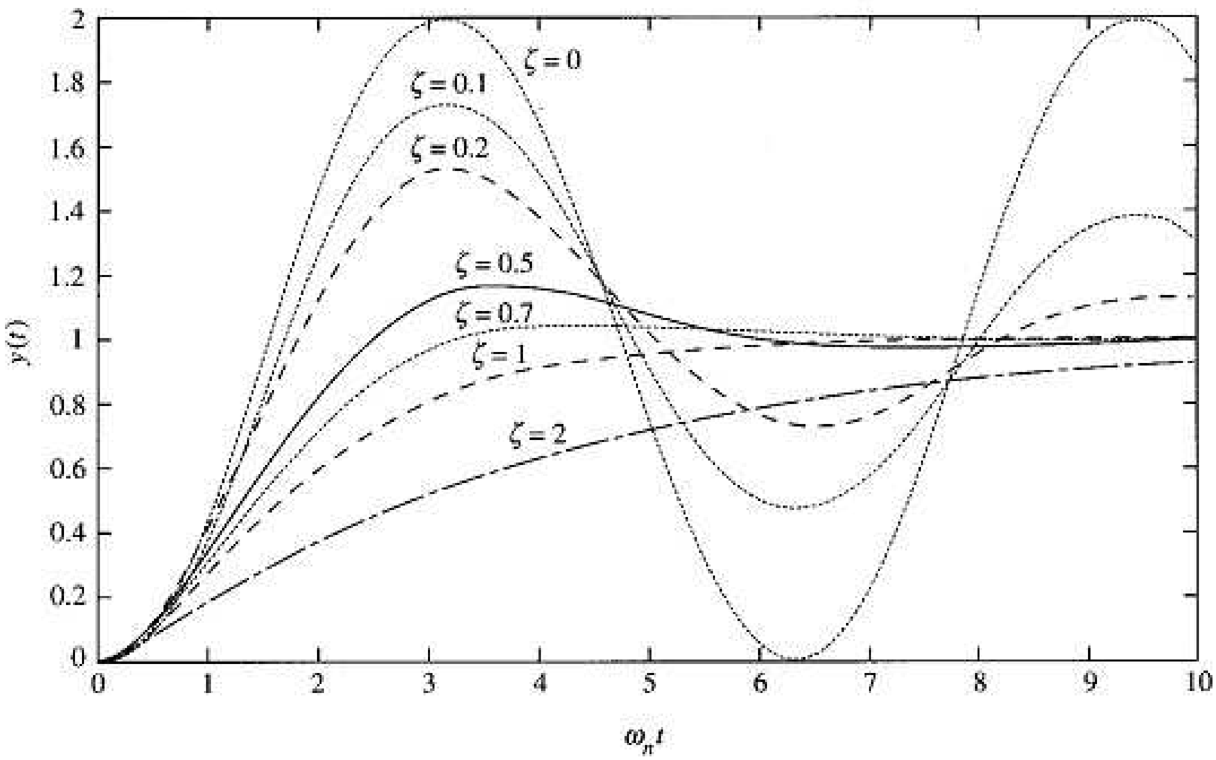


The steady-state response is,

Thus, the system has zero steady-state error. The pole of dictates the response,

.

The response for different is shown in Figure below.

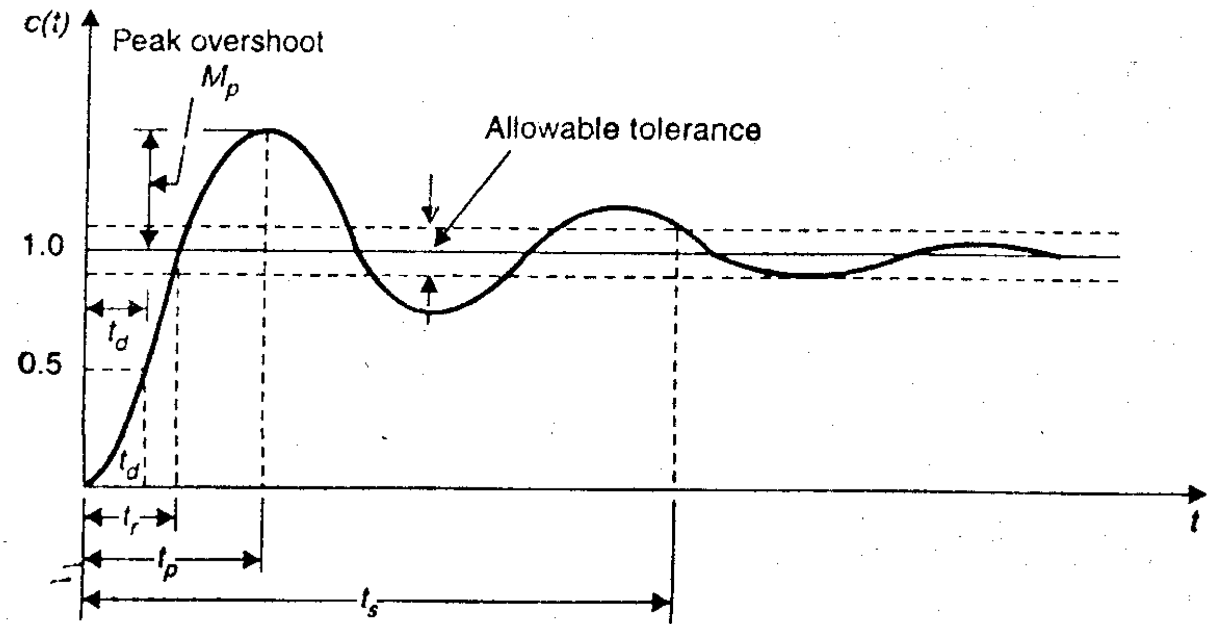


TIME DOMAIN SPECIALIZATION

Control systems are generally designed with damping less than one, i.e., oscillatory step response. Higher order control systems usually have a pair of complex conjugate poles with damping less than unity that dominate over the other poles. Therefore the time response of second- and higher-order control systems to a step input is generally of damped oscillatory nature as shown in Figure next (next page).

In specifying the transient-response characteristics of a control system to a unit step input, we usually specify the following:

1. Delay time,
2. Rise time,
3. Peak time,
4. Peak overshoot,
5. Settling time,
6. Steady-state error,



1. ***Delay time, :*** It is the time required for the response to reach 50% of the final value in first attempt.
2. ***Rise time, :*** It is the time required for the response to rise from 0 to 100% of the final value for the underdamped system.
3. ***Peak time, :*** It is the time required for the response to reach the peak of time response or the peak overshoot.
4. ***Settling time, :*** It is the time required for the response to reach and stay within a specified tolerance band (2% or 5%) of its final value.
5. ***Peak overshoot, :*** It is the normalized difference between the time response peak and the steady output and is defined as,

6. ***Steady-state error, :*** It indicates the error between the actual output and desired output as 't' tends to infinity.

.

Let us now obtain the expressions for the rise time, peak time, peak overshoot, and settling time for the second order system.

1. ***Rise time, :*** Put at ,,;.
2. ***Peak time, :*** Put and solve for ; .

,

Peak overshoot occurs at $k = 1$. .

3. ***Settling time, :*** For 2% tolerance band, , .

4. ***Steady-state error, :*** It is found previously that steady-state error for step input is zero.

Let us now consider ramp input, .

Then,

.

Therefore, the steady-state error due to ramp input is.

STEADY STATE ERRORS

The steady-state performance of a stable control system is generally judged by its steady-state error to step, ramp and parabolic inputs. For a unity feedback system,

,.

It is seen that steady-state error depends upon the input and the forward transfer function. The steady- state errors for different inputs are derived as follows:

ALGEBRIC CRITERIA

1. For unit-step input:

; is called position error constant.

2. For unit-ramp input:

; is called velocity error constant.

3. For unit-parabolic input:

; is called acceleration error const.

Types of Feedback Control System

The open-loop transfer function of a system can be written as,

If n = 0, the system is called type-0 system, if n = 1, the system is called type-1 system, if n = 2, the system is called type-2 system, etc. Steady-state errors for various inputs and system types are tabulated below.

Type of input	*Steady-state error*		
	Type-0 system	*Type-1 system*	*Type-2 system*
Unit-step	$1/(1+K_p)$	0	0
Unit-ramp	∞	$1/K_v$	0
Unit-parabolic	∞	∞	$1/K_a$
	$K_p = \lim_{s\to 0} G(s)$	$K_v = \lim_{s\to 0} sG(s)$	$K_a = \lim_{s\to 0} s^2G(s)$

ERROR CONSTANTS

The error constants for non-unity feedback systems may be obtained by replacing G(s) by G(s)H(s). Systems of type higher than 2 are not employed due to two reasons:

1. The system is difficult to stabilize.
2. The dynamic errors for such systems tend to be larger than those types-0, -1 and -2.

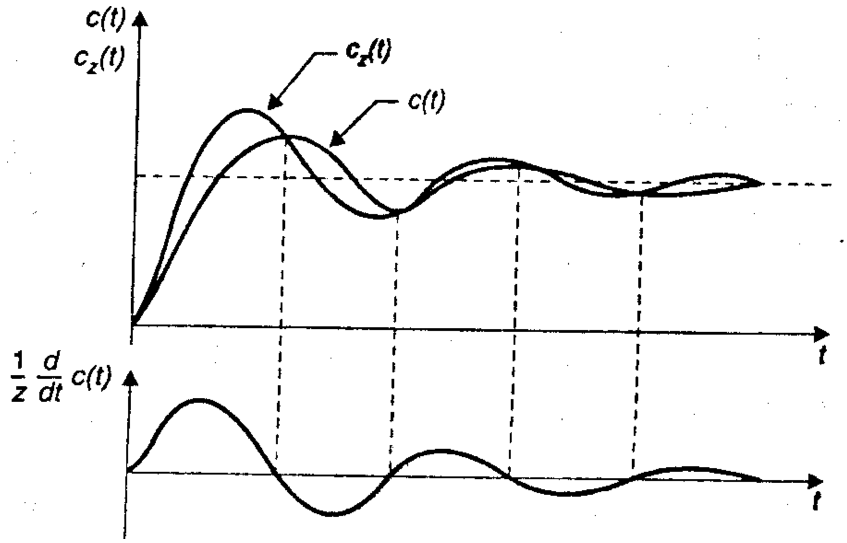
Effect of Adding a Zero to a System

Let a zero at s = -z be added to a second order system. Then we have,

.

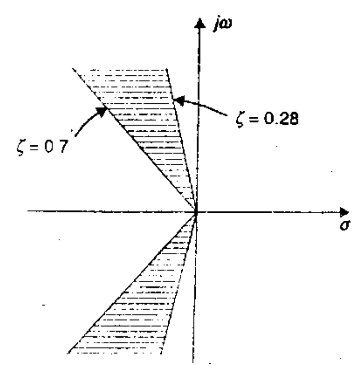
The multiplication term is adjusted to make the steady-state gain of the system unity. This gives $c_{ss} = 1$ when the input is unit step. Let $c_z(t)$ be the response of the system given by the above equation and $c(t)$ is the response without adding the pole. Manipulation of the above equation gives,

The effect of added derivative term is to produce a pronounced early peak to the system response which will be clear from the figure in the next page. Closer the zero to origin, the more pronounce the peaking phenomenon. Due to this fact, *the zeros on the real axis near the origin are generally avoided in design*. However, in a sluggish system the artful introduction of a zero at the proper position can improve the transient response. We can see from equation (03) that as z increases, i.e., the zero moves further into the left half of the s-plane, its effect becomes less pronounced.



Design Specifications of Second-order Systems

A control system is generally required to meet three time response specifications: steady-state accuracy, damping factor ζ (or peak overshoot, M_p) and settling time t_s. Steady-state accuracy requirement is met by suitable choice of *Kp, Kv, or Ka* depending on the type of the system. For most control systems ζ in the range of 0.7 – 0.28 (or peak overshoot of 5 – 40%) is considered acceptable. For this range of ζ, the closed-loop pole locations are restricted to the shaded region of the s-plane as shown in Figure.



For the antenna position control system,;;;. Here, is only the adjustable parameter. If we increase, will increase and thus settling time will decrease. At the same time, will decrease, this indicates the increase in peak overshoot. Thus by merely increasing gain, we cannot improve both transient and steady-state error specifications. We need to add additional components to the system. These are called compensators. It will allow improvement of both transient and steady-state specifications.

CONCEPTS OF STABILITY

BIBO stability: A system is said to be BIBO stable if for any bounded input, its output is also bounded. • Absolute stability: Stable /Unstable • Relative stability: Degree of stability (i.e. how far from instability) • A stable linear system described by a T.F. is such that all its poles have negative real parts

TEXT / REFERENCE BOOKS :-

1. I.J.Nagarath and M.Gopal, “Control System Engineering”, New Age International (P) Limited Publishers, 2nd Edition, 2009.
2. Kausuhio Ogata, “Modern Control Engineering”, Prentice Hall of India PVT. Ltd, 5th Edition, 2010.
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UNIT - III

THE CONCEPT OF STABILITY AND ROOT LOCUS TECHNIQUE– SEE1203

STABILITY

When a system is unstable, the output of the system may be infinite even though the input to the system was finite. This causes a number of practical problems. For instance, a robot arm controller that is unstable may cause the robot to move dangerously. Also, systems that are unstable often incur a certain amount of physical damage, which can become costly. Nonetheless, many systems are inherently unstable - a fighter jet, for instance, or a rocket at liftoff, are examples of naturally unstable systems. Although we can design controllers that stabilize the system, it is first important to understand what stability is, how it is determined, and why it matters.

The chapters in this section are heavily mathematical, and many require a background in linear differential equations. Readers without a strong mathematical background might want to review the necessary chapters in the Calculus and Ordinary Differential Equations books (or equivalent) before reading this material.

For most of this chapter we will be assuming that the system is linear, and can be represented either by a set of transfer functions or in state space. Linear systems have an associated characteristic polynomial, and this polynomial tells us a great deal about the stability of the system. Negativeness of any coefficient of a characteristic polynomial indicates that the system is either unstable or at most marginally stable. If any coefficient is zero/negative then we can say that the system is unstable. It is important to note, though, that even if all of the coefficients of the characteristic polynomial are positive the system may still be unstable. We will look into this in more detail below.

A system is defined to be **BIBO Stable** if every bounded input to the system results in a bounded output over the time interval $[t_0, \infty)$. This must hold for all initial times t_o. So long as we don't input infinity to our system, we won't get infinity output.

A system is defined to be **uniformly BIBO Stable** if there exists a positive constant *k* that is independent of t_0 such that for all t_0 the following conditions:

$$\|u(t)\| \leq 1$$

$$t \geq t_0$$

implies that

$$\|y(t)\| \leq k$$

There are a number of different types of stability, and keywords that are used with the topic of stability. Some of the important words that we are going to be discussing in this chapter, and the next few chapters are: **BIBO Stable**, **Marginally Stable**, **Conditionally Stable**, **Uniformly Stable**, **Asymptoticly Stable**, and **Unstable**. All of these words mean slightly different things.

Consider the system:

$$h(t) = \frac{2}{t}$$

We can apply our test, selecting an arbitrarily large finite constant M, and an arbitrary input x such that -M < x < M.

As M approaches infinity (but does not reach infinity), we can show that:

$$y_{-M} = \lim_{M\to\infty} \frac{2}{-M} = 0^-$$

And:

$$y_M = \lim_{M\to\infty} \frac{2}{M} = 0^+$$

So now, we can write out our inequality:

$$y_{-M} \leq y_x \leq y_M$$

$$0^- \leq x < 0^+$$

And this inequality should be satisfied for all possible values of x. However, we can see that when x is zero, we have the following:

$$y_x = \lim_{x\to 0} \frac{2}{x} = \infty$$

Which means that x is between -M and M, but the value y_x is not between y_{-M} and y_M. Therefore, this system is not stable.

Poles and StabilitY

When the poles of the closed-loop transfer function of a given system are located in the right-half of the S-plane (RHP), the system becomes unstable. When the poles of the system are located in the left-half plane (LHP) and the system is not improper, the system is shown to be stable. A number of tests deal with this particular facet of stability: The**Routh-Hurwitz Criteria**, the **Root-Locus**, and the **Nyquist Stability Criteria** all test whether there are poles of the transfer function in the RHP. We will learn about all these tests in the upcoming chapters.

If the system is a multivariable, or a MIMO system, then the system is stable if and only if *every pole of every transfer function* in the transfer function matrix has a negative real part and every transfer function in the transfer function matrix is not improper. For these systems, it is possible to use the Routh-Hurwitz, Root Locus, and Nyquist methods described later, but these methods must be performed once for each individual transfer function in the transfer function matrix.

Let us remember our generalized feedback-loop transfer function, with a gain element of K, a forward path Gp(s), and a feedback of Gb(s). We write the transfer function for this system as:

$$H_{cl}(s) = \frac{KGp(s)}{1 + H_{ol}(s)}$$

Where H_{cl} is the closed-loop transfer function, and H_{ol} is the open-loop transfAgain, we define the open-loop transfer function as the product of the forward path and the feedback elements, as such:

$H_{ol}(s) = KGp(s)Gb(s)$ <---Note this definition now contradicts the updated definition in the "Feedback" section.

Now, we can define F(s) to be the **characteristic equation**. F(s) is simply the denominator of the closed-loop transfer function, and can be defined as such:

[Characteristic Equation]

$$F(s) = 1 + H_{ol} = D(s)$$

We can say conclusively that the roots of the characteristic equation are the poles of the transfer function. Now, we know a few simple facts:

1. The locations of the poles of the closed-loop transfer function determine if the system is stable or not
2. The zeros of the characteristic equation are the poles of the closed-loop transfer function.
3. The characteristic equation is always a simpler equation than the closed-loop transfer function.

These functions combined show us that we can focus our attention on the characteristic equation, and find the roots of that equation.

State-Space and Stability

As we have discussed earlier, the system is stable if the eigenvalues of the system matrix A have negative real parts. However, there are other stability issues that we can analyze, such as whether a system is *uniformly stable*, *asymptotically stable*, or otherwise. We will discuss all these topics in a later chapter.

Marginal Stability

When the poles of the system in the complex S-Domain exist on the complex frequency axis (the vertical axis), or when the eigenvalues of the system matrix are imaginary (no real part), the system exhibits oscillatory characteristics, and is said to be marginally stable. A marginally stable system may become unstable under certain circumstances, and may be perfectly stable under other circumstances.

ROUTH STABILITY CRITERION:

The Routh approximation method which has been suggested for the reduction of stable discrete time linear systems to guarantee stable models, uses the bilinear transformation. A stability theorem in the *z*-plane is presented which is shown to be an equivalent of the Routh criterion. An efficent method that avoids the bilinear transformation is presented by which the Routhdisrete models are derived directly in the *z*-plane.

In control system theory, the **Routh–Hurwitz stability criterion** is a mathematical test that is a necessary and sufficient condition for the stability of a linear time invariant (LTI)control system. The Routh test is an efficient recursive algorithm that English mathematician Edward John Routh proposed in 1876 to determine whether all the roots of thecharacteristic polynomial of a linear system have negative real parts.[1] German mathematician Adolf Hurwitz independently proposed in 1895 to arrange the coefficients of the polynomial into a square matrix, called the Hurwitz matrix, and showed that the polynomial is stable if and only if the sequence of determinants of its principal submatrices are all positive.[2] The two procedures are equivalent, with the Routh test providing a more efficient way to compute the Hurwitz determinants than computing them directly. A polynomial satisfying the Routh–Hurwitz criterion is called a Hurwitz polynomial.

The importance of the criterion is that the roots ***p*** of the characteristic equation of a linear system with negative real parts represent solutions e^{pt} of the system that are stable (bounded). Thus the criterion provides a way to determine if the equations of motion of a linear system have only stable solutions, without solving the system directly. For discrete systems, the corresponding stability test can be handled by the Schur–Cohn criterion, the Jury test and the Bistritz test. With the advent of computers, the criterion has become less widely used, as an alternative is to solve the polynomial numerically, obtaining approximations to the roots directly.

The Routh test can be derived through the use of the Euclidean algorithm and Sturm's theorem in evaluating Cauchy indices. Hurwitz derived his conditions differently.

Using Euclid's algorithm

The criterion is related to Routh–Hurwitz theorem. Indeed, from the statement of that theorem, we have $p - q = w(+\infty) - w(-\infty)$ where:

- *p* is the number of roots of the polynomial *f*(*z*) with negative Real Part;
- *q* is the number of roots of the polynomial *f*(*z*) with positive Real Part (let us remind ourselves that *f* is supposed to have no roots lying on the imaginary line);
- *w*(*x*) is the number of variations of the generalized Sturm chain obtained from $P_0(y)$ and $P_1(y)$ (by successive Euclidean divisions) where $f(iy) = P_0(y) + iP_1(y)$ for a real *y*.

By the fundamental theorem of algebra, each polynomial of degree *n* must have *n* roots in the complex plane (i.e., for an *f* with no roots on the imaginary line, $p + q = n$). Thus, we have the condition that f is a (Hurwitz) stable polynomial if and only if $p - q = n$ (the proof is given below). Using the Routh–Hurwitz theorem, we can replace the condition on$p$ and q by a condition on the generalized Sturm chain, which will give in turn a condition on the coefficients of f.

Using matrices

Let $f(z)$ be a complex polynomial. The process is as follows:

1. Compute the polynomials $P_0(y)$ and $P_1(y)$ such that $f(iy) = P_0(y) + iP_1(y)$ where y is a real number.
2. Compute the Sylvester matrix associated to $P_0(y)$ and $P_1(y)$.
3. Rearrange each row in such a way that an odd row and the following one have the same number of leading zeros.
4. Compute each principal minor of that matrix.
5. If at least one of the minors is negative (or zero), then the polynomial f is not stable.

Example

- Let $f(z) = az^2 + bz + c$ (for the sake of simplicity we take real coefficients) where $c \neq 0$ (to avoid a root in zero so that we can use the Routh–Hurwitz theorem). First, we have to calculate the real polynomials $P_0(y)$ and $P_1(y)$:

$$f(iy) = -ay^2 + iby + c = P_0(y) + iP_1(y) = -ay^2 + c + i(by).$$

Next, we divide those polynomials to obtain the generalized Sturm chain:

- $P_0(y) = ((-a/b)y)P_1(y) + c,$ yields $P_2(y) = -c,$
- $P_1(y) = ((-b/c)y)P_2(y),$ yields $P_3(y) = 0$ and the Euclidean division stops.

Notice that we had to suppose b different from zero in the first division. The generalized Sturm chain is in this case $(P_0(y), P_1(y), P_2(y)) = (c - ay^2, by, -c)$. Putting $y = +\infty$, the sign of $c - ay^2$ is the opposite sign of a and the sign of by is the sign of b. When we put $y = -\infty$, the sign of the first element of the chain is again the opposite sign of a and the sign of by is the opposite sign of b. Finally, $-c$ has always the opposite sign of c.

Suppose now that f is Hurwitz-stable. This means that $w(+\infty) - w(-\infty) = 2$ (the degree of f). By the properties of the function w, this is the same as $w(+\infty) = 2$ and $w(-\infty) = 0$. Thus, a, b and c must have the same sign. We have thus found the necessary condition of stability for polynomials of degree 2.

Routh–Hurwitz criterion for second, third, and fourth-order polynomials[edit]

In the following, we assume the coefficient of the highest order (e.g. a_2 in a second order polynomial) to be positive. If necessary, this can always be achieved by multiplication of the polynomial with -1.

- For a second-order polynomial, $P(s) = a_2s^2 + a_1s + a_0 = 0$, all the roots are in the left half plane (and the system with characteristic equation $P(s)$ is stable) if all the coefficients satisfy $a_n > 0$.
- For a third-order polynomial $P(s) = a_3s^3 + a_2s^2 + a_1s + a_0 = 0$, all the coefficients must satisfy $a_n > 0$, and $a_2a_1 > a_3a_0$
- For a fourth-order polynomial $P(s) = a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 = 0$, all the coefficients must satisfy $a_n > 0$, and $a_3a_2 > a_4a_1$ and $a_3a_2a_1 > a_4a_1^2 + a_3^2a_0$
- In general Routh stability criterion proclaims that all First column elements of Routh array is to be of same sign.

This criterion is also known as modified Hurwitz Criterion of stability of the system. We will study this criterion in two parts. Part one will cover necessary condition for stability of the system and part two will cover the sufficient condition for the stability of the system. Let us again consider the characteristic equation of the system as

1) Part one (necessary condition for stability of the system): In this we have two conditions which are written below: (a) All the coefficients of the characteristic equation should be positive and real. (b) All the coefficients of the characteristic equation should be non zero.

2) Part two (sufficient condition for stability of the system): Let us first construct routh array. In order to construct the routh array follow these steps: (a) The first row will consist of all the even terms of the characteristic equation. Arrange them from first (even term) to last (even term). The first row is written below: a0 a2 a4 a6............ (b) The second row will consist of all the odd terms of the characteristic equation. Arrange them from first (odd term) to last (odd term). The first row is written below: a1 a3 a5 a7. (c) The elements of third row can be calculated as:

(1) First element : Multiply a0 with the diagonally opposite element of next column (i.e. a3) then subtract this from the product of a1 and a2 (where a2 is diagonally opposite element of next column) and then finally divide the result so obtain with a1. Mathematically we write as first element

(2) Second element : Multiply a0 with the diagonally opposite element of next to next column (i.e. a5) then subtract this from the product of a1 and a4 (where a4 is diagonally opposite

element of next to next column) and then finally divide the result so obtain with a1. Mathematically we write as second elemenT

Similarly, we can calculate all the elements of the third row. (d) The elements of fourth row can be calculated by using the following procedure: (1) First element : Multiply b1 with the diagonally opposite element of next column (i.e. a3) then subtract this from the product of a1 and b2 (where b2 is diagonally opposite element of next column) and then finally divide the result so obtain with b1. Mathematically we write as first element

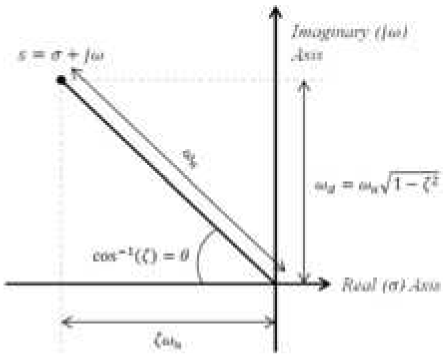
(2) Second element :Multiply b1 with the diagonally opposite element of next to next column (i.e. a5) then subtract this from the product of a1 and b3 (where b3 is diagonally opposite element of next to next column) and then finally divide the result so obtain with a1. Mathematically we write as second element

Similarly, we can calculate all the elements of the fourth row. Similarly, we can calculate all the elements of all the rows. Stability criteria if all the elements of the first column are positive then the system will be stable. However if anyone of them is negative the system will be unstable. Now there are some special cases related to Routh Stability Criteria which are discussed below: (1) Case one: If the first term in any row of the array is zero while the rest of the row has at least one non zero term. In this case we will assume a very small value (ε) which is tending to zero in place of zero. By replacing zero with (ε) we will calculate all the elements of the Routh array. After calculating all the elements we will apply the limit at each element containing (ε). On solving the limit at every element if we will get positive limiting value then we will say the given system is stable otherwise in all the other condition we will say the given system is not stable. (2) Case second : When all the elements of any row of the Routh array are zero. In this case we can say the system has the symptoms of marginal stability. Let us first understand the physical meaning of having all the elements zero of any row. The physical meaning is that there are symmetrically located roots of the characteristic equation in the s plane. Now in order to find out the stability in this case we will first find out auxiliary equation. Auxiliary equation can be formed by using the elements of the row just above the row of zeros in the Routh array. After finding the auxiliary equation we will differentiate the auxiliary equation to obtain elements of the zero row. If there is no sign change in the new routh array formed by using auxiliary equation, then in this we say the given system is limited stable. While in all the other cases we will say the given system is unstable

ROOT LOCUS:

In control theory and stability theory, **root locus analysis** is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain within a feedback system. This is a technique used as a stability criterion in the field of control systems developed by Walter R. Evans which can determinestability of the system. The root locus plots the poles of the closed loop transfer function in the complex S plane as a function of a gain parameter (see pole–zero plot).

Uses

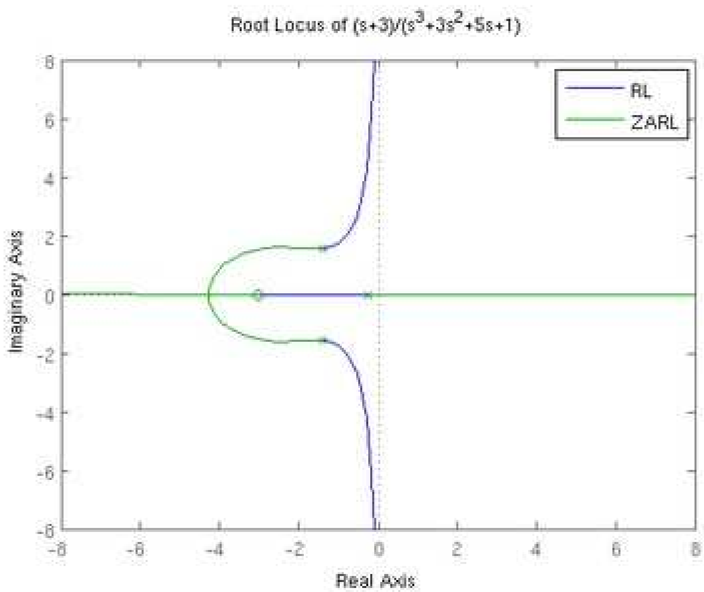


Effect of pole location on a second order system's natural frequency and damping ratio.

In addition to determining the stability of the system, the root locus can be used to design the damping ratio (ζ) and natural frequency (ω_n) of a feedback system. Lines of constant damping ratio can be drawn radially from the origin and lines of constant natural frequency can be drawn as arcs whose center points coincide with the origin. By selecting a point along the root locus that coincides with a desired damping ratio and natural frequency, a gain K can be calculated and implemented in the controller. More elaborate techniques of controller design using the root locus are available in most control textbooks: for instance, lag, lead, PI, PD and PID controllers can be designed approximately with this technique.

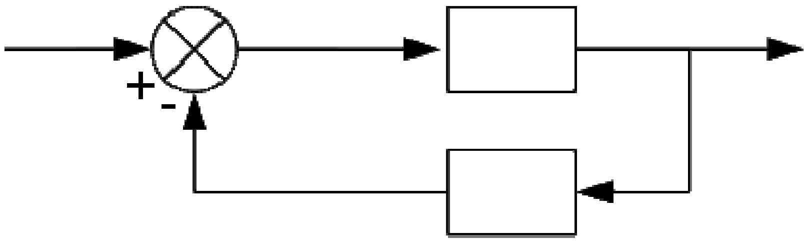
The definition of the damping ratio and natural frequency presumes that the overall feedback system is well approximated by a second order system; i.e. the system has a dominant pair of poles. This is often not the case, so it is good practice to simulate the final design to check if the project goals are satisfied.

Example



RL = root locus; ZARL = zero angle root locus

Suppose there is a feedback system whose input is the signal $X(s)$ and output is $Y(s)$. The feedback system forward path gain is $G(s)$; the feedback path gain is $H(s)$.



For this system, the overall transfer function is given by

$$T(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Thus the closed-loop poles (roots of the characteristic equation) of the transfer function are the solutions to the equation $1 + G(s)H(s) = 0$. The principal feature of this equation is that roots may be found wherever $G(s)H(s) = -1$.

In systems without pure delay, the product $G(s)H(s) = -1$ is a rational polynomial function and may be expressed as[2]

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_{m+n})}$$

where the $-z_i$ are the m zeros, the $-p_i$ are the $m + n$ poles, and K is a scalar gain. Typically, a root locus diagram will indicate the transfer function's pole locations for varying values of K. A root locus plot will be all those points in the s-plane where $G(s)H(s) = -1$ for any value of K.

The factoring of K and the use of simple monomials means the evaluation of the rational polynomial can be done with vector techniques that add or subtract angles and multiply or divide magnitudes. The vector formulation arises from the fact that each monomial term in the factored $G(s)H(s)$, $(s-a)$ for example, represents the vector from a to s. The polynomial can be evaluated by considering the magnitudes and angles of each of these vectors. According to vector mathematics, the angle of the result is the sum of all the angles in the numerator add minus the sum of all the angles in the denominator. Similarly, the magnitude of the result is the product of all the magnitudes in the numerator divided by the product of all the magnitudes in the denominator. It turns out that the calculation of the magnitude is not needed because K varies; one of its values may result in a root. So to test whether a point in the s-plane is on the root locus, only the angles to all the open loop poles and zeros need be considered. A graphical method that uses a special protractor called a "Spirule" was once used to determine angles and draw the root loci.

From the function $T(s)$, it can be seen that the value of K does not affect the location of the zeros. The root locus only gives the location of closed loop poles as the gain K is varied. The zeros of a system do not move.

Using a few basic rules, the root locus method can plot the overall shape of the path (locus) traversed by the roots as the value of K varies. The plot of the root locus then gives an idea of the stability and dynamics of this feedback system for different values of K.

Sketching root locus[edit]

- Mark open-loop poles and zeros
- Mark real axis portion to the left of an odd number of poles and zeros
- Find asymptotes

Let P be the number of poles and Z be the number of zeros:

$$P - Z = \text{number of asymptotes}$$

The asymptotes intersect the real axis at α (which is called the centroid) and depart at angle ϕ given by:

$$\phi_l = \frac{180^\circ + (l-1)360^\circ}{P - Z}, l = 1, 2, \ldots, P - Z$$

$$\alpha = \frac{\sum_P - \sum_Z}{P - Z}$$

where $\sum_P$ is the sum of all the locations of the poles, and $\sum_Z$ is the sum of all the locations of the explicit zeros.

- Phase condition on test point to find angle of departure
- Compute breakaway/break-in points

The breakaway points are located at the roots of the following equation:

$$\frac{dG(s)H(s)}{ds} = 0 \text{ or } \frac{d\overline{GH}(z)}{dz} = 0$$

Once you solve for z, the real roots give you the breakaway/reentry points. Complex roots correspond to a lack of breakaway/reentry.

z-plane versus s-plane

The root locus method can also be used for the analysis of sampled data systems by computing the root locus in the z-plane, the discrete counterpart of the s-plane. The equation $z = e^{sT}$ maps continuous s-plane poles (not zeros) into the z-domain, where T is the sampling period. The stable, left half s-plane maps into the interior of the unit circle of the z-plane, with the s-plane origin equating to $|z| = 1$ (because $e^0 = 1$). A diagonal line of constant damping in the s-plane maps around a spiral from (1,0) in the z plane as it curves in toward the origin. Note also that the Nyquist aliasing criteria is expressed graphically in the z-plane by the x-axis, where $\omega nT = \pi$.

The line of constant damping just described spirals in indefinitely but in sampled data systems, frequency content is aliased down to lower frequencies by integral multiples of the Nyquist frequency. That is, the sampled response appears as a lower frequency and better damped as well since the root in the *z*-plane maps equally well to the first loop of a different, better damped spiral curve of constant damping. Many other interesting and relevant mapping properties can be described, not least that *z*-plane controllers, having the property that they may be directly implemented from the *z*-plane transfer function (zero/pole ratio of polynomials), can be imagined graphically on a *z*-plane plot of the open loop transfer function, and immediately analyzed utilizing root locus.

Since root locus is a graphical angle technique, root locus rules work the same in the *z* and *s* planes.

The idea of a root locus can be applied to many systems where a single parameter *K* is varied. For example, it is useful to sweep any system parameter for which the exact value is uncertain in order to determine its behavior.

CONSTRUCTION OF ROOT LOCI:

To facilitate the application of the root-locus method for systems of higher order than 2nd, rules can be established. These rules are based upon the interpretation of the angle condition and the analysis of the characteristic equation. The rules presented aid in obtaining the root locus by expediting the manual plotting of the locus. But for automatic plotting using a computer these rules provide checkpoints to ensure that the solution is correct.

Though the angle and magnitude conditions can also be applied to systems having dead time, in the following we restrict to the case of the open-loop rational transfer functions according to Eq.

or

$$G_0(s) = k_0 \frac{b_0 + b_1 a + \ldots + b_{m-1} s^{m-1} + s^m}{a_0 + a_1 s + \ldots + a_{n-1} s^{n-1} + s^n} = k_0 \frac{N_0(s)}{D_0(s)} \ .$$

As this transfer function can be written in terms of poles and zeros s_{P_ν} and s_{Z_μ} ($\nu = 1, 2, \ldots n$; $\mu = 1, 2, \ldots, m$) $G_0(s)$ can be represented by their magnitudes and angles

$$G_0(s) = k_0 \frac{|s - s_{Z_1}| e^{j\varphi_{Z_1}} |s - s_{Z_2}| e^{j\varphi_{Z_2}} \ldots |s - s_{Z_m}| e^{j\varphi_{Z_m}}}{|s - s_{P_1}| e^{j\varphi_{P_1}} |s - s_{P_2}| e^{j\varphi_{P_2}} \ldots |s - s_{P_n}| e^{j\varphi_{P_n}}}$$

or

$$G_0(s) = k_0 \frac{\prod_{\mu=1}^{m} |s - s_{Z_\mu}|}{\prod_{\nu=1}^{n} |s - s_{P_\nu}|} e^{j\left(\sum_{\mu=1}^{m} \varphi_{Z_\mu} - \sum_{\nu=1}^{n} \varphi_{P_\nu}\right)} .$$

From Eq. (6.8) the *magnitude condition*

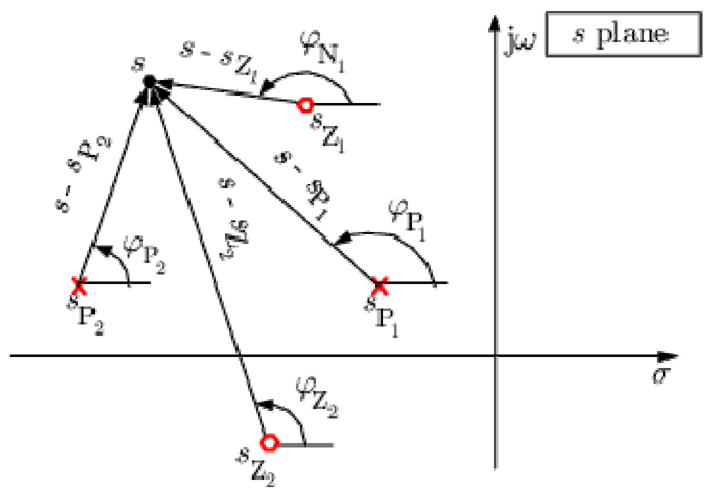
$$\frac{\prod_{\mu=1}^{m} |s - s_{Z_\mu}|}{\prod_{\nu=1}^{n} |s - s_{P_\nu}|} = \frac{1}{k_0}$$

and from Eq. the *angle condition*

$$\varphi(s) = \sum_{\mu=1}^{m} \varphi_{Z_\mu} - \sum_{\nu=1}^{n} \varphi_{P_\nu} = \pm 180^\circ (2\,k + 1)$$

for $k = 0, 1, 2, \ldots$

follows. Here φ_{Z_μ} and φ_{P_ν} denote the angles of the complex values $(s - s_{Z_\mu})$ and $(s - s_{P_\nu})$, respectively. All angles are considered positive, measured in the counterclockwise sense. If for each point the sum of these angles in the$s$plane is calculated, just those particular points that fulfil the condition in Eq. are points on the root locus. This principle of constructing a root-locus curve - as shown in Figure is mostly used for automatic root-locus plotting.



Pole-zero diagram for construction of the root locus

In the following the most important *rules for the construction of root loci* for $k_0 > 0$ are listed:

Symmetry

As all roots are either real or complex conjugate pairs so that the root locus is symmetrical to the real axis.

Number of branches

The number of branches of the root locus is equal to the number of poles n of the open-loop transfer function.

Locus start and end points

The locus starting points ($k_0 = 0$) are at the open-loop poles and the locus ending points ($k_0 = \infty$) are at the open-loop zeros. $(n - m)$ branches end at infinity. The number of starting branches from a pole and ending branches at a zero is equal to the multiplicity of the poles and zeros, respectively. A point at infinity is considered as an equivalent zero of multiplicity equal to $n - m$.

Real axis locus

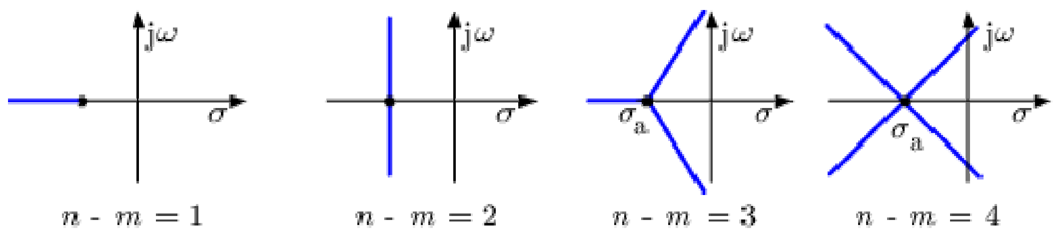
If the total number of poles and zeros to the right of a point on the real axis is odd, this point lies on the locus.

Asymptotes

There are $n - m$ asymptotes of the root locus with a slope of

$$\alpha_k = \arg s = \frac{\pm 180°(2k+1)}{n-m} .$$

For $(n-m) = 1, 2, 3$ and 4 one obtains the asymptote configurations as shown in Figure 6.4.



Asymptote configurations of the root locus

Real axis intercept of the asymptotes

The real axis crossing $(\sigma_a, j0)$ of the asymptotes is at

$$\sigma_a = \frac{1}{n-m}\left\{\sum_{\nu=1}^{n} \mathrm{Re}\, s_{P_\nu} - \sum_{\mu=1}^{m} \mathrm{Re}\, s_{Z_\mu}\right\} .$$

Breakaway and break-in points on the real axis

At least one breakaway or break-in point $(\sigma_B, j0)$ exists if a branch of the root locus is on the real axis between two poles or zeros, respectively. Conditions to find such real points are based on the fact that they represent multiple real roots. In addition to the characteristic equation for multiple roots the condition

$$\frac{d}{ds}[1 + G_o(s)] = \frac{d}{ds} G_o(s) = 0 .$$

must be fulfilled, which is equivalent to

$$\sum_{\nu=1}^{n} \frac{1}{s - s_{P_\nu}} = \sum_{\mu=1}^{m} \frac{1}{s - s_{Z_\mu}}$$

for $s = \sigma_B$. If there are no poles or zeros, the corresponding sum is zero.

Complex pole/zero angle of departure/entry

The angle of departure of pairs of poles with multiplicity r_{P_ϱ} is

$$\varphi_{P_\varrho,D} = \frac{1}{r_{P_\varrho}} \left\{ -\sum_{\substack{\nu=1 \\ \nu\neq\varrho}}^{n} \varphi_{P_\nu} + \sum_{\mu=1}^{m} \varphi_{Z_\mu} \pm 180^\circ(2k+1) \right\}$$

and the angle of entry of the pairs of zeros with multiplicity r_{Z_ϱ}

$$\varphi_{Z_\varrho,E} = \frac{1}{r_{Z_\varrho}} \left\{ -\sum_{\substack{\mu=1 \\ \mu\neq\varrho}}^{m} \varphi_{Z_\mu} + \sum_{\nu=1}^{n} \varphi_{P_\nu} \pm 180^\circ(2k+1) \right\} .$$

Rule 9 Root-locus calibration

The labels of the values of k_0 can be determined by using

$$k_0 = \frac{\prod_{\nu=1}^{n} |s - s_{P_\nu}|}{\prod_{\mu=1}^{m} |s - s_{Z_\mu}|} .$$

For $m = 0$ the denominator is equal to one.

Asymptotic stability

The closed loop system is asymptotically stable for all values of k_0 for which the locus lies in the left-half s plane. From the imaginary-axis crossing points the critical values $k_{0_{crit}}$ can be determined.

The rules shown above are for positive values of k_0. According to the angle condition of for negative values of k_0 some rules have to be modified. In the following these rules are numbered as above but labelled by a *.

Locus start and end points

The locus starting points ($k_0 = 0$) are at the open-loop poles and the locus ending points ($k_0 = -\infty$) are at the open-loop zeros. $(n-m)$ branches end at infinity. The number of starting branches from a pole and ending branches at a zero is equal to the multiplicity of the poles and zeros, respectively. A point at infinity is considered as an equivalent zero of multiplicity equal to $n-m$.

Real axis locus

If the total number of poles and zeros to the right of a point on the real axis is even including zero, this point lies on the locus.

Asymptotes

There are $n-m$ asymptotes of the root locus with a slope of

$$\alpha_k = \arg s = \frac{\pm 360^\circ k}{n-m} .$$

Complex pole/zero angle of departure/entry

The angle of departure of pairs of poles with multiplicity r_{P_ϱ} is

$$\varphi_{P_\varrho, D} = \frac{1}{r_{P_\varrho}} \left\{ -\sum_{\substack{\nu=1 \\ \nu \neq \varrho}}^{n} \varphi_{P_\nu} + \sum_{\mu=1}^{m} \varphi_{Z_\mu} \pm 360^\circ k \right\}$$

and the angle of entry of the pairs of zeros with multiplicity r_{Z_ϱ}

$$\varphi_{Z_\varrho,E} = \frac{1}{r_{Z_\varrho}} \left\{ -\sum_{\substack{\mu=1 \\ \mu\neq\varrho}}^{m} \varphi_{Z_\mu} + \sum_{\nu=1}^{n} \varphi_{P_\nu} \pm 360° k \right\} .$$

The root-locus method can also be applied for other cases than varying k_0. This is possible as long as $G_0(s)$ can be rewritten such that the angle condition according to Eq. and the rules given above can be applied. This will be demonstrated in the following two examples.

Given the closed-loop characteristic equation

$$a_0 + a_1 s + \ldots + a_{n-1} s^{n-1} + s^n = 0 ,$$

the root locus for varying the parameter a_1 is required. The characteristic equation is therefore rewritten as

$$1 + a_1 \frac{s}{a_0 + a_2 s^2 + \ldots + s^n} = 0 .$$

This form then correspondents to the standard form

$$1 + G_0(s) = 1 + a_1 \frac{N_0(s)}{D_0(s)} = 0$$

to which the rules can be applied. ■

Given the closed-loop characteristic equation

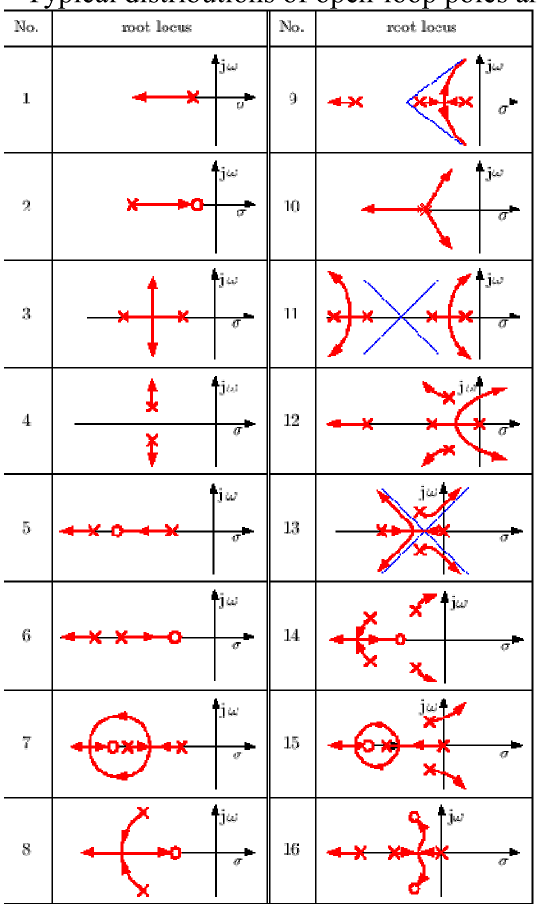
$$s^3 + (3 + \alpha)\, s^2 + 2s + 4 = 0 ,$$

it is required to find the effect of the parameter on the position of the closed-loop poles. The equation is rewritten into the desired form

$$1+\alpha\frac{s^2}{s^3+3s^2+2s+4}=0\ .$$

Using the rules 1 to 10 one can easily predict the geometrical form of the root locus based on the distribution of the open-loop poles and zeros. Table 6.2 shows some typical distributions of open-loop poles and zeros and their root loci.

Typical distributions of open-loop poles and zeros and the root loci



For the qualitative assessment of the root locus one can use a physical analogy. If all open-loop poles are substituted by a negative electrical charge and all zeros by a commensurate positive

one and if a massless negative charged particle is put onto a point of the root locus, a movement is observed. The path that the particle takes because of the interplay between the repulsion of the poles and the attraction of the zeros lies just on the root locus.

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UNIT - IV

FREQUENCY RESPONSE ANALYSIS – SEE1203

What is Frequency Response?

The response of a system can be partitioned into both the transient response and the steady state response. We can find the transient response by using Fourier integrals. The steady state response of a system for an input sinusoidal signal is known as the frequency response. In this chapter, we will focus only on the steady state response.

If a sinusoidal signal is applied as an input to a Linear Time-Invariant (LTI) system, then it produces the steady state output, which is also a sinusoidal signal. The input and output sinusoidal signals have the same frequency, but different amplitudes and phase angles.

Let the input signal be −

$$r(t)=A\sin(\omega 0t)r(t)=A\sin(\omega 0t)$$

The open loop transfer function will be −

$$G(s)=G(j\omega)G(s)=G(j\omega)$$

We can represent $G(j\omega)G(j\omega)$ in terms of magnitude and phase as shown below.

$$G(j\omega)=|G(j\omega)|\angle G(j\omega)G(j\omega)=|G(j\omega)|\angle G(j\omega)$$

Substitute, $\omega=\omega 0\omega=\omega 0$ in the above equation.

$$G(j\omega 0)=|G(j\omega 0)|\angle G(j\omega 0)G(j\omega 0)=|G(j\omega 0)|\angle G(j\omega 0)$$

The output signal is

$$c(t)=A|G(j\omega 0)|\sin(\omega 0t+\angle G(j\omega 0))c(t)=A|G(j\omega 0)|\sin(\omega 0t+\angle G(j\omega 0))$$

The amplitude of the output sinusoidal signal is obtained by multiplying the amplitude of the input sinusoidal signal and the magnitude of $G(j\omega)G(j\omega)$ at $\omega=\omega 0\omega=\omega 0$.

The phase of the output sinusoidal signal is obtained by adding the phase of the input sinusoidal signal and the phase of $G(j\omega)G(j\omega)$ at $\omega=\omega 0\omega=\omega 0$.

Where,

A is the amplitude of the input sinusoidal signal.

ω_0 is angular frequency of the input sinusoidal signal.

We can write, angular frequency $\omega 0\omega 0$ as shown below.

$$\omega 0=2\pi f0\omega 0=2\pi f0$$

Here, $f0f0$ is the frequency of the input sinusoidal signal. Similarly, you can follow the same procedure for closed loop control system.

Frequency Domain Specifications

The frequency domain specifications are resonant peak, resonant frequency and bandwidth.

Consider the transfer function of the second order closed loop control system as,

$$T(s)=C(s)R(s)=\omega 2ns2+2\delta\omega ns+\omega 2nT(s)=C(s)R(s)=\omega n2s2+2\delta\omega ns+\omega n2$$

Substitute, $s=j\omega$ in the above equation.

$$T(j\omega)=\frac{\omega_n^2}{(j\omega)^2+2\delta\omega_n(j\omega)+\omega_n^2}$$

$$\Rightarrow T(j\omega)=\frac{\omega_n^2}{-\omega^2+2j\delta\omega\omega_n+\omega_n^2}=\frac{\omega_n^2}{\omega_n^2\left(1-\frac{\omega^2}{\omega_n^2}+\frac{2j\delta\omega}{\omega_n}\right)}$$

$$\Rightarrow T(j\omega)=\frac{1}{\left(1-\frac{\omega^2}{\omega_n^2}\right)+j\left(\frac{2\delta\omega}{\omega_n}\right)}$$

Let, $\frac{\omega}{\omega_n}=u$ Substitute this value in the above equation.

$$T(j\omega)=\frac{1}{(1-u^2)+j(2\delta u)}$$

Magnitude of $T(j\omega)$ is -

$$M=|T(j\omega)|=\frac{1}{\sqrt{(1-u^2)^2+(2\delta u)^2}}$$

Phase of $T(j\omega)$ is -

$$\angle T(j\omega)=-\tan^{-1}\left(\frac{2\delta u}{1-u^2}\right)$$

Resonant Frequency

It is the frequency at which the magnitude of the frequency response has peak value for the first time. It is denoted by ω_r. At $\omega=\omega_r$, the first derivate of the magnitude of $T(j\omega)$ is zero.

Differentiate M with respect to u.

$$\frac{dM}{du}=-\frac{1}{2}\left[(1-u^2)^2+(2\delta u)^2\right]^{\frac{-3}{2}}\left[2(1-u^2)(-2u)+2(2\delta u)(2\delta)\right]$$

$$\Rightarrow \frac{dM}{du}=-\frac{1}{2}\left[(1-u^2)^2+(2\delta u)^2\right]^{\frac{-3}{2}}\left[4u(u^2-1+2\delta^2)\right]$$

Substitute, $u=u_r$ and $\frac{dM}{du}==0$ in the above equation.

$$0=-\frac{1}{2}\left[(1-u_r^2)^2+(2\delta u_r)^2\right]^{\frac{-3}{2}}\left[4u_r(u_r^2-1+2\delta^2)\right]$$

$$\Rightarrow 4u_r(u_r^2-1+2\delta^2)=0$$

$$\Rightarrow u_r^2-1+2\delta^2=0$$

$$\Rightarrow u_r^2=1-2\delta^2$$

$$\Rightarrow u_r=\sqrt{1-2\delta^2}$$

Substitute, $u_r=\frac{\omega_r}{\omega_n}$ in the above equation.

$$\frac{\omega_r}{\omega_n}=\sqrt{1-2\delta^2}$$

$$\Rightarrow \omega_r=\omega_n\sqrt{1-2\delta^2}$$

Resonant Peak

It is the peak (maximum) value of the magnitude of $T(j\omega)$. It is denoted by M_r.

At $u=u_r$, the Magnitude of $T(j\omega)$ is -

$$M_r=\frac{1}{\sqrt{(1-u_r^2)^2+(2\delta u_r)^2}}$$

Substitute, $u_r=\sqrt{1-2\delta^2}$ and $1-u_r^2=2\delta^2$ in the above equation.

$$M_r = \frac{1}{\sqrt{(2\delta^2)^2 + \left(2\delta\sqrt{1-2\delta^2}\right)^2}}$$

$$\Rightarrow M_r = \frac{1}{2\delta\sqrt{1-\delta^2}}$$

Resonant peak in frequency response corresponds to the peak overshoot in the time domain transient response for certain values of damping ratio δ. So, the resonant peak and peak overshoot are correlated to each other.

Bandwidth

It is the range of frequencies over which, the magnitude of $T(j\omega)$ drops to 70.7% from its zero frequency value.

At $\omega = 0$, the value of u will be zero.

Substitute, $u = 0$ in M.

$$M = \frac{1}{\sqrt{(1-0^2)^2 + (2\delta(0))^2}} = 1$$

Therefore, the magnitude of $T(j\omega)$ is one at $\omega = 0$.

At 3-dB frequency, the magnitude of $T(j\omega)$ will be 70.7% of magnitude of $T(j\omega)$ at $\omega = 0$.

i.e., at $\omega = \omega_B, M = 0.707(1) = \frac{1}{\sqrt{2}}$

$$\Rightarrow M = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1-u_b^2)^2 + (2\delta u_b)^2}}$$

$$\Rightarrow 2 = (1-u_b^2)^2 + (2\delta)^2 u_b^2$$

Let, $u_b^2 = x$

$$\Rightarrow 2 = (1-x)^2 + (2\delta)^2 x$$

$$\Rightarrow x^2 + (4\delta^2 - 2)x - 1 = 0$$

$$\Rightarrow x = \frac{-(4\delta^2 - 2) \pm \sqrt{(4\delta^2 - 2)^2 + 4}}{2}$$

Consider only the positive value of x.

$$x = 1 - 2\delta^2 + \sqrt{(2\delta^2 - 1)^2 + 1}$$

$$\Rightarrow x = 1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}$$

Substitute, $x = u_b^2 = \frac{\omega_b^2}{\omega_n^2}$

$$\frac{\omega_b^2}{\omega_n^2} = 1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}$$

$$\Rightarrow \omega_b = \omega_n \sqrt{1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}}$$

Bandwidth ω_b in the frequency response is inversely proportional to the rise time t_r in the time domain transient response.

Bode attenuation diagrams

If the absolute value $A(\omega)$ and the phase $\varphi(\omega)$ of the $G(j\omega) = A(\omega)\, e^{j\varphi(\omega)}$

frequency response are separately plotted over the ω

frequency , one obtains the

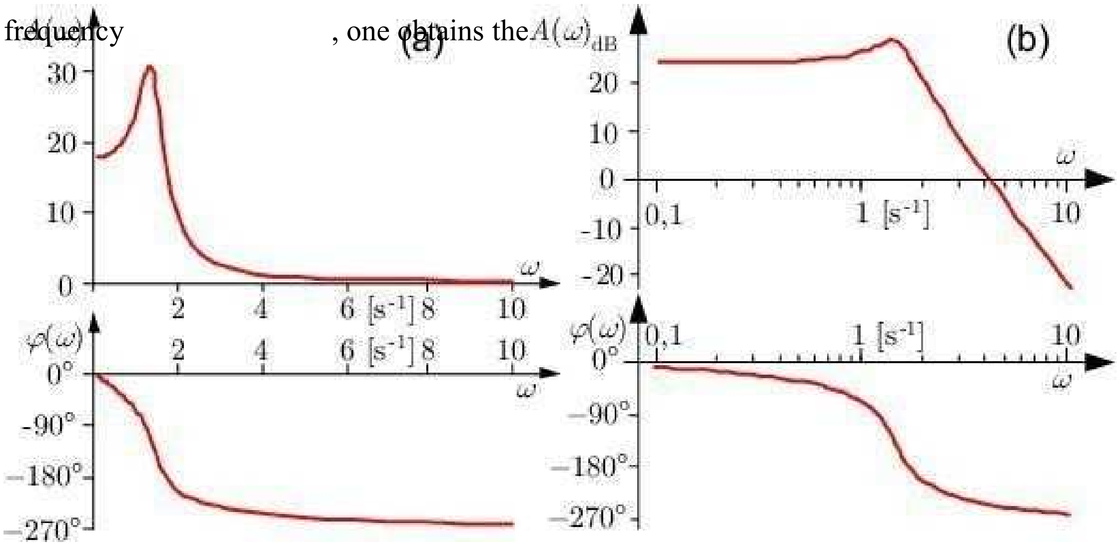


Figure 6.1: Plot of a frequency response: (a) linear, (b) logarithmic presentation (ω on a logarithmic sca

amplitude response and the *phase response*. Both together are the *frequency response characteristics*. $A(\omega)$ and ω are normally drawn with a logarithm and $\varphi(\omega)$ with a linear scale.

This representation is called a *Bode diagram* or *Bode plot*. Usually $A(\omega)$ will be specified in decibels [dB] By definition this is

$$A(\omega)_{dB} = 20 \log_{10} A(\omega) \quad [dB]$$

The logarithmic representation of the amplitude response $A(\omega)_{dB}$ has consequently a linear scale in this diagram and is called the *magnitude*.

Stability analysis using Bode plots:

The magnitude and phase relationship between sinusoidal input and steady state output of a system is known as frequency

- The polar plot of a sinusoidal transfer function G (jw) is plot of the magnitude of G (jw) versus the phase angle of G (jw) on polar coordinates as _co' varied from zero to infinity.

- The phase margin is that amount, of additional phase lag at the gain crossover frequency required to bring the system to the verge of
- The gain margin is the reciprocal of the magnitude l G(jw) l at the frequency at which the phase angle as
- The inverse polar plot at G (jw) is a graph of 1/G (jw) as a function of
- Bode plot is a graphical representation of the transfer function for determining the stability of control
- Bode plot is a combination of two plot - magnitude plot and phase
- The transfer function having no poles and zeros in the right -half s-plane are called minimum phase transfer
- System with minimum phase transfer function are called minimum phase
- systems. The transfer function having poles and zeros in the right half s-plane are called non- minimum phase transfer functions systems with non-minimum phase transfer function. are called non-minimum phase
- In bode plot the relative stability of the system is determined from the gain margin and phase margin. .
- If gain cross frequency is less than phase cross over frequency then gain margin and phase margin both are positive and system is
- If gain cross over frequency is greater than the phase crossover frequency than both gain margin and'phase margin are
- It gain cross over frequency is equal to me phase cross over trequency me gain marg and phase margin are zero and system is marginally
- The maximum value of magnitude is known as resonant
- The magnitude of resonant peak gives the information about the relative stability of the
- The frequency at which magnitude has maximum value is known as resonant frequency.
- Bandwidth is defined a the range of frequencies in which the magnitude of closed loop does not drop —3 db.

Example Problems:

Q1. Sketch the Bode Plot for the transfer function given by,

G(s) H(s) = 2 (s+0.25)/s² (s+1) (s+0.5)
and from Plot find (a) Phase and Gain cross rer frequencies (b) Gain Margin and Phase Margin. Is this System Stable?

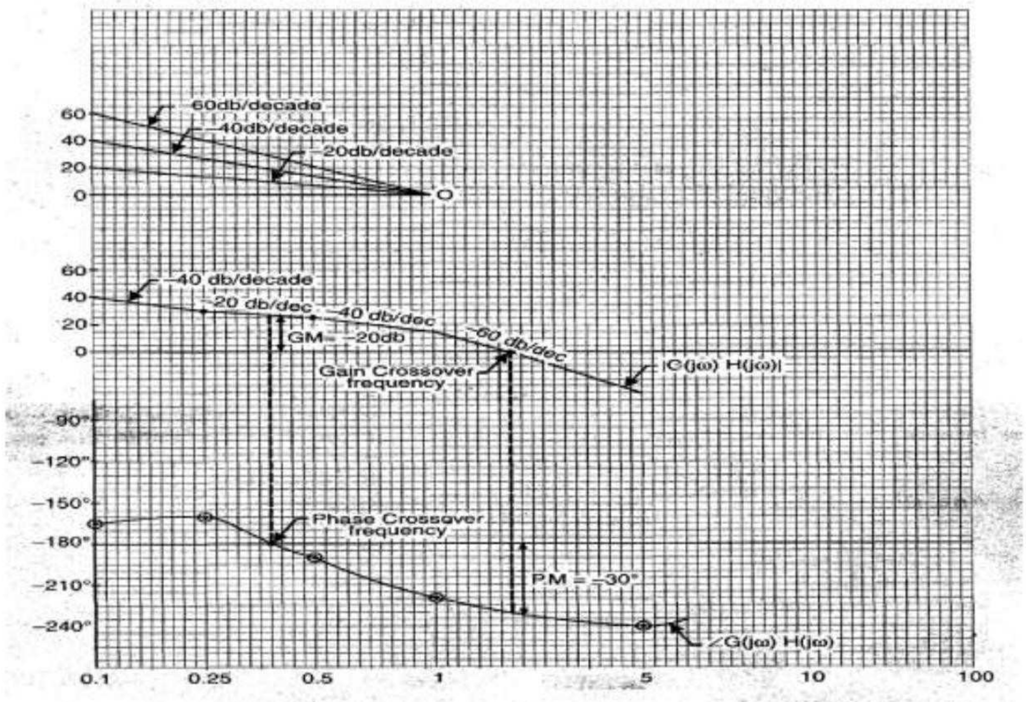
Ans. Given $G(s)\,H(s) = \dfrac{2(s+0.25)}{s^2(s+1)(s+0.5)}$

$$= \frac{\frac{2\times 0.25}{0.5}\left[\frac{s}{0.25}+1\right]}{s^2(s+1)\left[\frac{s}{0.5}+1\right]}$$

$$= \frac{1(4s+1)}{s^2(s+1)(2s+1)}$$

Put $s = j\omega$

This is type 2 system, hence initial slope of bode plot = –40 dB/decade and the plot intersects 0 dB axis at $\omega = \sqrt{K} = \sqrt{1} = 1$ rad/sec. The comer frequencies are :



$\omega = \frac{1}{4} = 0.25$ rad/sec

$\omega = \frac{1}{2} = 0.5$ rad/sec

$\omega = 1$ rad/sec.

Frequency range is considered from $\omega = 0.1$ rad/sec to $\omega = 10$ rad/sec.

The plot is as shown.

As initial slope of plot is –40 dB/dec and comer frequency is 0.25 rad/sec. The plot after $\omega = 0.25$ has slope = –20 dB/dec.

After $\omega = 0.5$, slope is –40 dB/decade

After $\omega = 1$, slope is –60 dB/dec.

Phase Angle :

$$\angle G(j\omega)\, H(j\omega) = \tan^{-1}(4\omega) - 180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega.$$

The phase angle for frequency range considered are calculated as :

ω	0.1	0.25	0.5	1	5
∠G (jω) H (jω)	–175.2	–175.2	–188	–212.4	–225

The gain crosses 0db axis at co = 1.24 rad/sec, the gain crossover frequency is co = rad/sec.

The phase crosses —180° line at co = 0.4 rad/sec, therefore phase

crossover frequency is co = 0.4 rad/sec.
At phase cross over the gain is 20 dB, therefore gain margin is —20 dB.
At gain crossover the phase angle is 2150, the phase margin is 180° + (—215°) = —35°. As both gain and phase margins are negative, the system is unstable.

Q3. Sketch the bode plot for the transfer function given by

$$G(s) = \frac{23.7\,(1+j\omega)\,(1+0.2j\omega)}{(j\omega)\,(1+3j\omega)\,(1+0.5j\omega)\,(1+0.1j\omega)}$$

and from plot find gain margin and

phase margin. Ans.

On 0)-axis mark the point at 23.7 rad/sec. since in denominator (jw) term is having power one, from 23.7 draw a line of slope —20 db/decade to meet y-axis. This will be the starting point.

Step 1.

From the starting point to I corner frequency (0.33) the slope of the line is —20 db/decade.

From I corner frequency (0.33) to second corner frequency (1) the slope of the line will be — 20 ÷ (—20) = —40 db/decade.

From II corner frequency to IV corner frequency (2) the slope of the line be —40 + (÷20) = —20 db/decade.

From III corner frequency to IV corner frequency, the slope of line will be —20 + (—20) = —40 db/decade.

From IV corner frequency (5) to V corner frequency the slope will be —40 ÷ (+20) = —20 db/decade.

After V corner frequency, the slope will be (—20) ÷ (—20) = —40 db/decade.

Step 2.

Draw the phase plot.

Step 3.

$$G(j\omega)H(j\omega) = \frac{k}{j\omega(j0.1\omega + 1)(j0.05\omega + 1)}.$$

Ans. Advantages of Bode Plot :
Please refer to Q. No. 1 (i) of May 2009.

As $G(j\omega)H(j\omega) = \frac{k}{j\omega(j0.1\omega + 1)(j0.5\omega + 1)}$

Comer frequencies are

$$\omega_1 = \frac{1}{0.1} = 10 \text{ rad/sec}$$

$$\omega_2 = \frac{1}{0.05} = 20 \text{ rad/sec}$$

Draw magnitude plot without K.
For phase plot

ω	Arg $j\omega$ ϕ_1	Arg $(1 + j0.1\omega)$ ϕ_2	Arg $(1 + j0.05\omega)$ ϕ_3	Resultant $\phi_1 + \phi_2 + \phi_3$
4	−90°	−21.8°	−11.3°	−123.1°
6	−90°	−30.96°	−16.69°	−137.65°
8	−90°	−38.56°	−21.8°	−150.36°
10	−90°	−45°	−26.56°	−161.56°
12	−90°	−50.19°	−30.96°	−171.46°
14	−90°	−54.46°	−35°	−179.48°
18	−90°	−60.9°	−42°	−192.9°
20	−90°	−63.43°	−45°	−198.43°

ω	$-\tan^{-1} j\omega$	$-\tan^{-1} 3\omega$	$-\tan^{-1} 0.5\omega$	$-\tan^{-1} 0.1\omega$	$\tan^{-1} \omega$	$\tan^{-1} 2\omega$	Resultant
0.1	−90°	−16.7°	−2.86°	−0.57°	+5.71°	1.14°	−103°
0.2	−90°	−31°	−5.71°	−1.14°	+11.3°	2.3°	−114.25°
0.5	−90°	−56.3°	−14.03°	−2.86°	+26.56°	5.71°	−130.92°
0.8	−90°	−67.4°	−21.8°	−4.57°	+38.65°	9.09°	−136.03°
1.0	−90°	−71.56°	−26.56°	−5.71°	+45°	11.3°	−137.5°
2.0	−90°	−80.54°	−45°	−11.3°	+63.43°	21.8°	−141.61°
5.0	−90°	−86.18°	−68.19°	−26.56°	+78.7°	45°	−147.23°
8.0	−90°	−87.61°	−76°	−38.65°	+82.87°	58°	−151.39°
10.0	−90°	−88°	−78.7°	−45°	+84.3°	63.4°	−154.0°
20.0	−90°	−89°	−84.3°	−63.43°	+87.13°	76°	−163.6°

What is Polar Plot in Control System

A polar plot is a plot of a function that is expressed in polar coordinates, with radius as a function of angle.

Polar plots are drawn between magnitude and phase.

If we have a sinusoidal transfer function G(jw), which is a complex function.

Therefore we can write-

G(jw)= Re[G(jw)] + jIm[G(jw)]

G(jw)= lG(jw)l angle(G(jw))

Therefore we can represent the transfer function G(jw), as a phasor of magnitude M and phase angle phi. This phase angle is measured positively in the counter clockwise direction.

The magnitude and the phase angle changes as the input frequency (w), is varied from zero to infinity.

So the locus obtained in the complex plane by the tip of the phasor G(jw) is called the polar plot.

Polar plots are also known as a Nyquist Plots.

Now we will understand the effect on shape of the polar plot on adding poles or zeros to the transfer function.

Polar Plots of Transfer Functions (Adding Poles and Zeros)

There are following three rules that are followed to trace Polar Plots in control systems, on adding poles and zeros to the transfer function-

1- Addition of a non-zero pole to a transfer function, results in further rotation of the polar plot through an angle of -90 degrees as w tends to infinity.

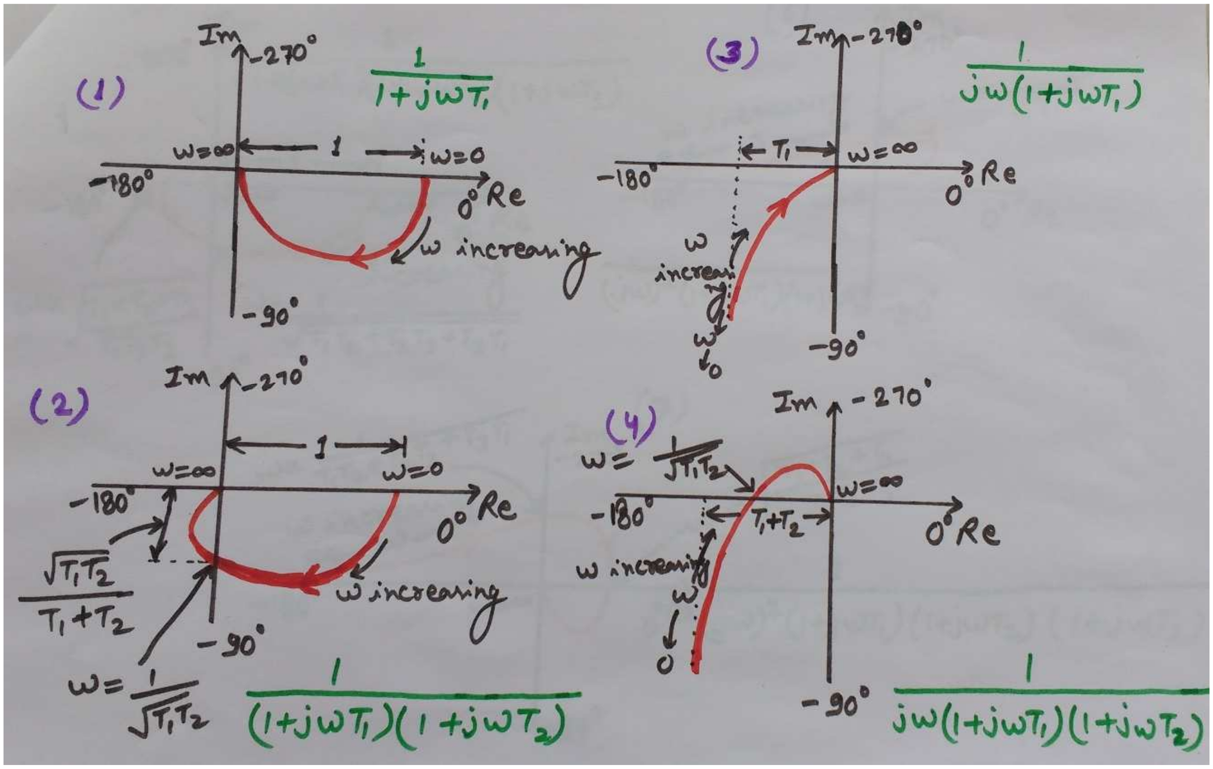
2- Adding a pole at the origin to a transfer function, rotates the polar plot at 0 and infinite frequencies by a further angle of -90 degrees.

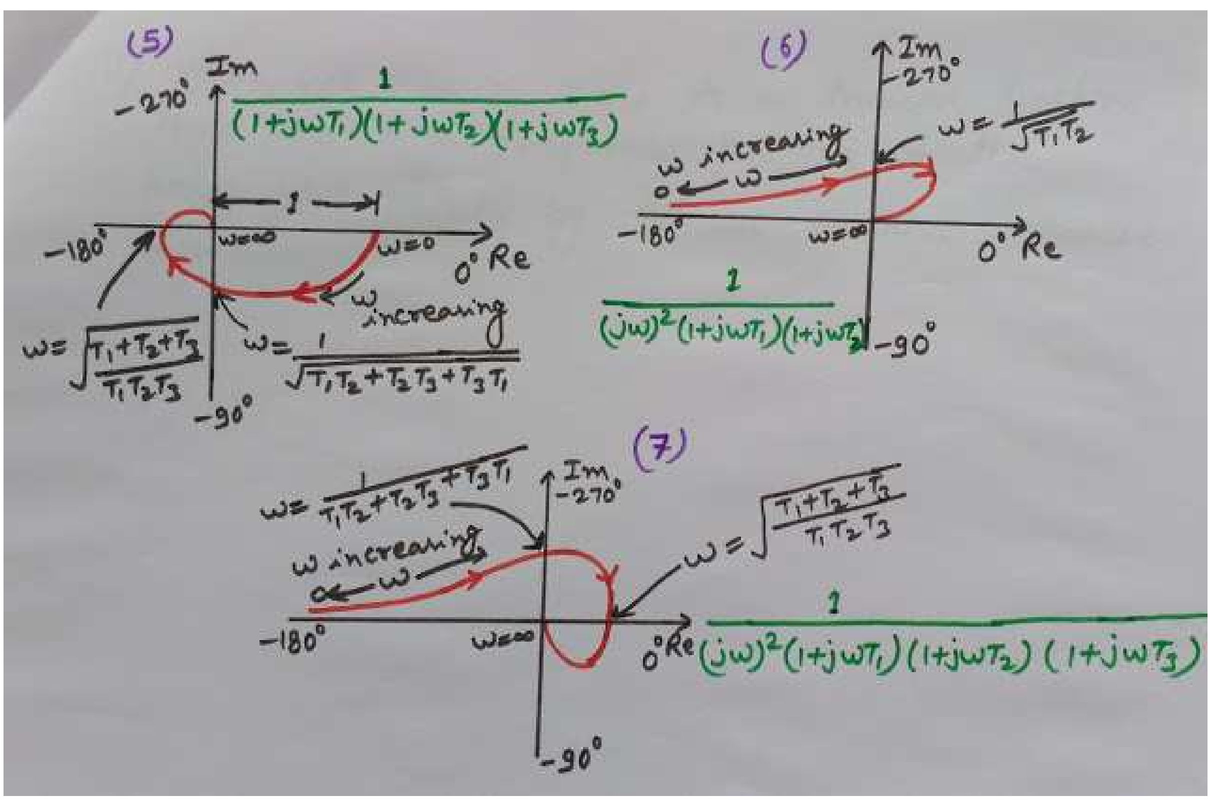
3- When we add a Zero to a transfer function, then the high frequency portion of the polar plot rotates by 90 degrees in counterclockwise direction.

We will use these rules to trace polar plots of different transfer functions with the help of a standard polar plot.

Here we will add non zero poles or poles at origin to the transfer function and will see how it affects the shape of the polar plots.

Now look at the images shown below-



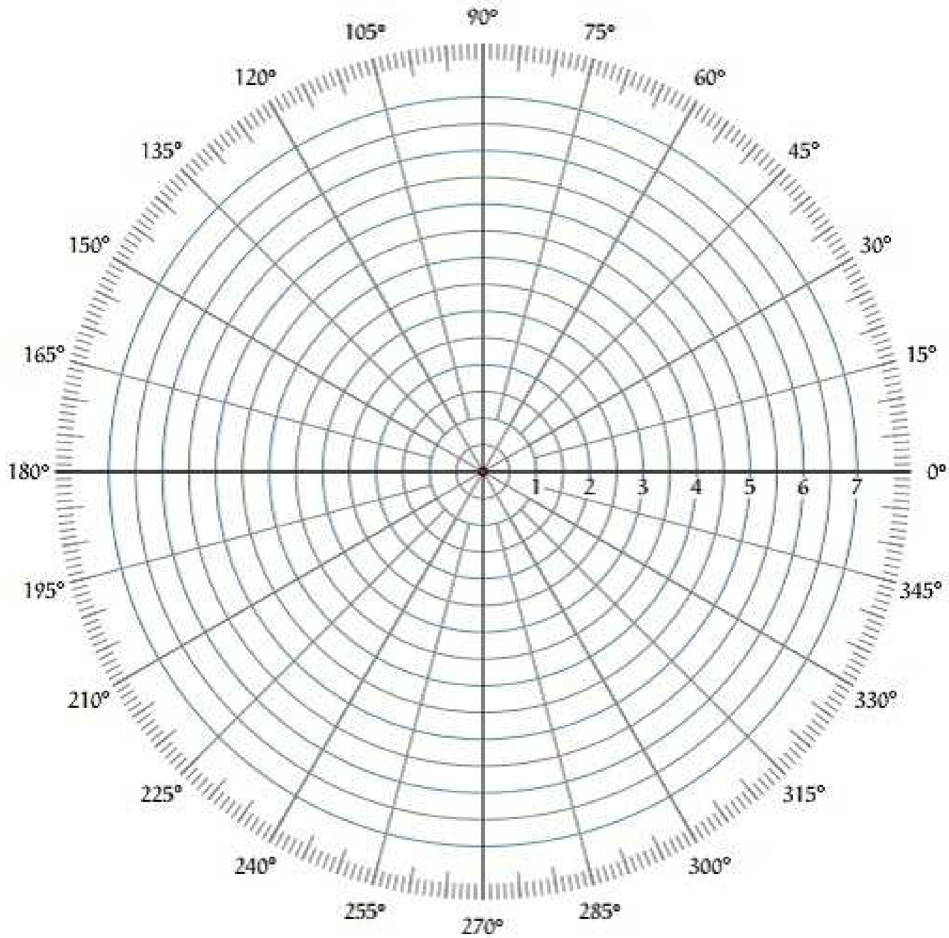


Polar plot is a plot which can be drawn between magnitude and phase. Here, the magnitudes are represented by normal values only.

The polar form of G(jω)H(jω)G(jω)H(jω) is

G(jω)H(jω)=|G(jω)H(jω)|∠G(jω)H(jω)G(jω)H(jω)=|G(jω)H(jω)|∠G(jω)H(jω)

The Polar plot is a plot, which can be drawn between the magnitude and the phase angle of G(jω)H(jω)G(jω)H(jω) by varying ωω from zero to ∞. The polar graph sheet is shown in the following figure.



This graph sheet consists of concentric circles and radial lines. The concentric circles and the radial lines represent the magnitudes and phase angles respectively. These angles are represented by positive values in anti-clock wise direction. Similarly, we can represent angles with negative values in clockwise direction. For example, the angle 2700 in anti-clock wise direction is equal to the angle −900 in clockwise direction.

Rules for Drawing Polar Plots

Follow these rules for plotting the polar plots.

- Substitute, s=jωs=jω in the open loop transfer function.
- Write the expressions for magnitude and the phase of G(jω)H(jω)G(jω)H(jω).
- Find the starting magnitude and the phase of G(jω)H(jω)G(jω)H(jω) by substituting ω=0ω=0. So, the polar plot starts with this magnitude and the phase angle.

- Find the ending magnitude and the phase of $G(j\omega)H(j\omega)G(j\omega)H(j\omega)$ by substituting $\omega=\infty\omega=\infty$. So, the polar plot ends with this magnitude and the phase angle.
- Check whether the polar plot intersects the real axis, by making the imaginary term of $G(j\omega)H(j\omega)G(j\omega)H(j\omega)$ equal to zero and find the value(s) of $\omega\omega$.
- Check whether the polar plot intersects the imaginary axis, by making real term of $G(j\omega)H(j\omega)G(j\omega)H(j\omega)$ equal to zero and find the value(s) of $\omega\omega$.
- For drawing polar plot more clearly, find the magnitude and phase of $G(j\omega)H(j\omega)G(j\omega)H(j\omega)$ by considering the other value(s) of $\omega\omega$.

Example

Consider the open loop transfer function of a closed loop control system.

$$G(s)H(s)=5s(s+1)(s+2)G(s)H(s)=5s(s+1)(s+2)$$

Let us draw the polar plot for this control system using the above rules.

Step 1 − Substitute, $s=j\omega s=j\omega$ in the open loop transfer function.

$$G(j\omega)H(j\omega)=5j\omega(j\omega+1)(j\omega+2)G(j\omega)H(j\omega)=5j\omega(j\omega+1)(j\omega+2)$$

The magnitude of the open loop transfer function is

$$M=5\omega(\omega2+1———\surd)(\omega2+4———\surd)M=5\omega(\omega2+1)(\omega2+4)$$

The phase angle of the open loop transfer function is

$$\phi=-900-\tan-1\omega-\tan-1\omega2\phi=-900-\tan-1\ \ \omega-\tan-1\ \ \omega2$$

Step 2 − The following table shows the magnitude and the phase angle of the open loop transfer function at $\omega=0\omega=0$ rad/sec and $\omega=\infty\omega=\infty$ rad/sec.

Frequency (rad/sec)	Magnitude	Phase angle(degrees)
0	∞	-90 or 270
∞	0	-270 or 90

So, the polar plot starts at (∞,−900) and ends at (0,−2700). The first and the second terms within the brackets indicate the magnitude and phase angle respectively.

Step 3 − Based on the starting and the ending polar co-ordinates, this polar plot will intersect the negative real axis. The phase angle corresponding to the negative real axis is −1800 or 1800. So, by equating the phase angle of the open loop transfer function to either −1800 or 1800, we will get the $\omega\omega$ value as 2–√2.
By substituting $\omega=2–\surd\omega=2$ in the magnitude of the open loop transfer function, we will get $M=0.83M=0.83$. Therefore, the polar plot intersects the negative real axis when $\omega=2–\surd\omega=2$ and the polar coordinate is (0.83,−1800).

So, we can draw the polar plot with the above information on the polar graph sheet.

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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - V

COMPENSATION AND CONTROLLERS – SEE1203

Introduction:

Automatic control systems have played a vital role in the advancement of science and engineering. In addition to its extreme importance in sophisticated systems in Space vehicles, missile- guidance, aircraft navigating systems, etc., automatic control system as become an important and integral part of manufacturing and industrial processes. Control of process parameters like pressure, temperature, flow, viscosity, speed, humidity, etc., in process engineering and tooling, handling and assembling mechanical parts in manufacturing industries among others in engineering field where automatic control systems are inevitable part of the system.

A control system is designed and constructed to perform specific functional task. The concept of control system design starts by defining the output variable(Speed, Pressure, Temperature Etc.,) and then determining the required specification (Stability, Accuracy, and speed of response). In the design process the designs must first select the control Media and then the control elements to meet the designed ends.

In actual practice several alternative can be analyzed and a final judgment can be made an overall performances and economy.

Systems have been categorized as manual and automatic systems. Based on the type of control needed most systems are categorized as - Manual & Automatic. In applications where systems are to be operated with limited or no supervision, then systems are made automatic and where system needs supervision the system is designed as manual. In the present-day context most of the systems are designed as automatic systems for which one of the important considerations was economics. However, the necessity for the system to be made as an automatic system is to make sure that the system performs with no scope for error which otherwise is prone to a lot of errors especially in the operations. Other classification of a system is based on the input and output relationships. Accordingly, in an Open Loop Control System the output is independent of the input and in a closed loop control system the output is dependant on the input. The term input refers to reference variable and the output is referred to as Controlled variable. Most of the systems are designed as closed loop systems where a feedback path with an element with a transfer function would help in bridging the relationship between the input and the output. A system can be

represented by the block diagram and from a s imple to a complicated system, reduction techniques can be used to obtain the overall transfer function of the system. Overall system Transfer function can also be obtained by another technique using signal flow analysis where the transfer function of the system is obtained from Mason‟ s gain formula. Once the system is designed, the response of the system may be obtained based on the type of input. This is studied in two categories of response namely response of the systemic time domain and frequency domain. The system thus conceived and designed needs to be analyzed based on the same domains. At this stage the systems are studied from the point of view of its operational features like Stability, Accuracy and Speed of Response. Development of various systems have been continuous and the history of the same go back to the old WATT‟ S
Speed Governor, which was considered as an effective means of speed regulation. Other control system examples are robot arm, Missile Launching and Guidance System, Automatic Aircr aft Landing System, Satellite based digital tracking systems,etc to name a few. In the design of the control systems, three important requirements are considered namely STABILITY, **ACCURACY and SPEED OF RESPONSE.**

Stable Systems are those where response to input must reach and maintain some useful value with in a reasonable period of time. The designed systems should both be Unstable Systems as unstable control systems produce persistent or even violent oscillations of the output and output will be driven to some extreme limiting value.

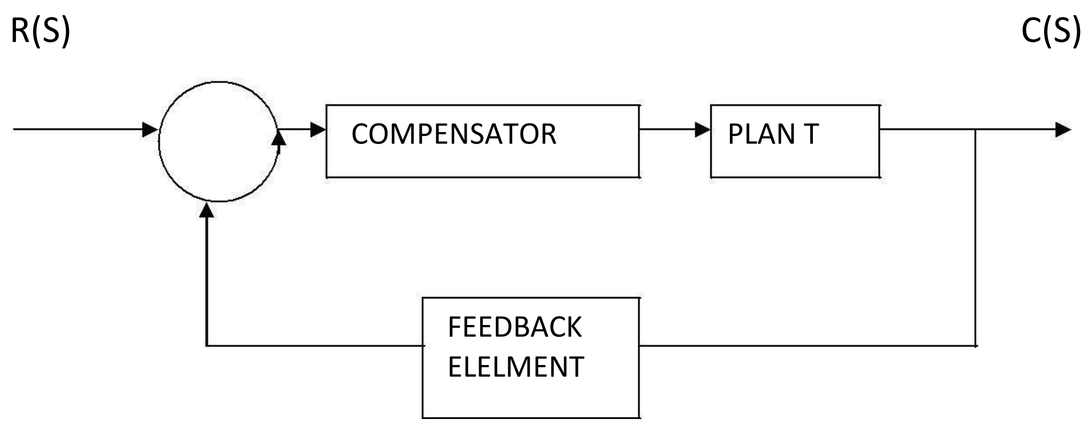
Systems are also designed to meet certain levels of **Accuracy. This is a** relative term with limits based upon a particular application. A time measurement system may be from a simple watch to a complicated system used in the sports arena. But the levels of accuracy are different in both cases. One used in sports arena must have very high levels of sophistication and must be reliable showing no signs of variations. However, this feature of the system is purely based on the system requirement. For a conceived, designed and developed system, the higher the levels of Accuracy expected, higher is the Cost.

The third important requirement comes by way of SPEED OF RESPONSE. System must complete its response to some input within an acceptable period of time. System has no value if the time required to respond fully to some input is far greater than the time interval between inputs

System Compensation

Compensation is the minor adjustment of a system in order to satisfy the given specifications. Specification refers to the objective of a system to perform and obtain the expected output after the system is provided with a proper input. Some of the needs o f the system compensation are as specified.

Basic Characteristics Of Lead, Lag And Lag-Lead Compensation:

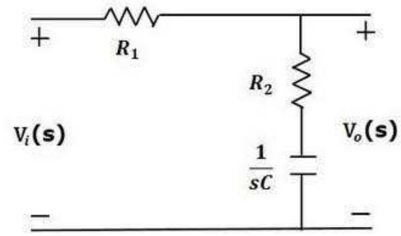


Lead compensation essentially yields an appropriate improvement in transient resonse and a small imrovement in steady state accuracy. Lag compensation on the other hand, yields an appreciable improvement in steady state accuracy ast the expense of increasing theh transient responsetime. Lag- lead compensation combines the characterisitcs of both lead compensation and lag compensation. The use of a lag- lead compensator raises the order of the system by two

(unless cancellation occurs between the zeroes of the lag- lead network and the poles of the uncompensated open- loop transfer function), which means that the system becomes more complex and it is more difficult to control the transient response behavior. The particular situation determines the type of the compensation to be used.

Lag Compensator

The Lag Compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied. The lag compensator circuit in the ‘s’ domain is shown in the following figure.



Here, the capacitor is in series with the resistor $R2$ and the output is measured across this combination.

The transfer function of this lag compensator is –

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha}\left(\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}\right)$$

Where,

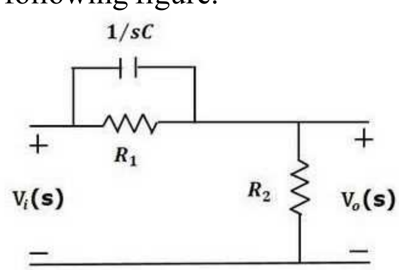
$$\tau = R_2 C$$

$$\alpha = \frac{R_1 + R_2}{R_2}$$

From the above equation, α is always greater than one. We know that, the phase of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer function. So, in order to produce the phase lag at the output of this compensator, the phase angle of the transfer function should be negative. This will happen when $\alpha > 1$.

Lead Compensator

The lead compensator is an electrical network which produces a sinusoidal output having phase lead when a sinusoidal input is applied. The lead compensator circuit in the 's' domain is shown in the following figure.



Here, the capacitor is parallel to the resistor $R1$and the output is measured across resistor

$R_2. The transfer function of this lead compensator is –

$$\frac{V_o(s)}{V_i(s)} = \beta\left(\frac{s\tau+1}{\beta s\tau+1}\right)$$

Where,

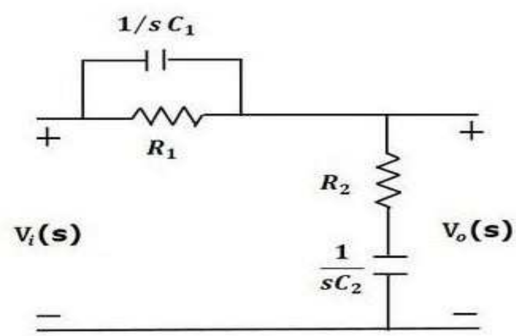
$$\tau = R_1C$$

$$\beta = \frac{R_2}{R_1+R_2}$$

We know that, the phase of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer function. nSo, in order to produce the phase lead at the output of this compensator, the phase angle of the transfer function should be positive. This will happen when $0<\beta<1$. Therefore, zero will be nearer to origin in pole-zero configuration of the lead compensator.

Lag-Lead Compensator

Lag-Lead compensator is an electrical network which produces phase lag at one frequency region and phase lead at other frequency region. It is a combination of both the lag and the lead compensators. The lag-lead compensator circuit in the 's' domain is shown in the following figure.



This circuit looks like both the compensators are cascaded. So, the transfer function of this circuit will be the product of transfer functions of the lead and the lag compensators.

$$\frac{V_o(s)}{V_i(s)} = \beta\left(\frac{s\tau_1+1}{\beta s\tau_1+1}\right)\frac{1}{\alpha}\left(\frac{s+\frac{1}{\tau_2}}{s+\frac{1}{\alpha\tau_2}}\right)$$

We know $\alpha\beta = 1$.

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \left(\frac{s+\frac{1}{\tau_1}}{s+\frac{1}{\beta\tau_1}}\right)\left(\frac{s+\frac{1}{\tau_2}}{s+\frac{1}{\alpha\tau_2}}\right)$$

Where,

$$\tau_1 = R_1C_1$$

$$\tau_2 = R_2C_2$$

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