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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - I
TRANSMISSION LINE THEORY – SECA1402

UNIT: I : TRANSMISSION LINE THEORY

Introduction:

Electrical signal containing information can be transmitted from one point in the system to another point in the system physically separated by a certain distance through different media like

1) Free space: The propagation is from a source (radio transmitter) to a load (radio receiver) where wave propagation is carried out through free space. The separation between the source and the load could be very large e.g. thousands of kms as in cellular communication or small (few kms) as in microwave communication within a city. This type of wave propagation is called unguided wave propagation.

2)Transmission Lines (connecting wires): In some applications the information has to be conveyed from one point to the other through connecting wires

E.g.: (i) Telephone lines from a telephone subscriber to the telephone exchange or vice versa

ii) Connection between a radio transmitter and its antenna or a connection between an antenna and a radio receiver

This type of propagation is called guided wave propagation because the electromagnetic wave is guided between the wires. These wires are called transmission lines. The waves are guided between boundaries of the transmission lines.

Thus a transmission line is a conductive method of guiding electrical energy from one place to another. It is employed not only to transmit energy, but also as circuit elements like inductors, capacitors, resonant circuits, filters, transformers and even insulators at very high frequencies. Also used as measuring devices and as an aid to obtain impedance matching.

The nature of transmission line and its performance depends upon

1. **The amount of power to be transmitted and**
2. **The frequencies involved**

Hence the transmission lines are categorized as

a) Power lines: used for transmission of large quantities of power over a fixed frequency

b) Communication lines: Used for transmission of small quantities of power over a band of frequencies

Types of Transmission Lines:

1. AN OPEN WIRE LINE (OR) PARALLEL WIRE TYPE:

These lines are the parallel conductors open to air hence called open wire lines. The conductors are separated by air as the dielectric are rigidly supported by cross arms put at a certain height from ground by means of galvanized iron poles or any other structure.

E.g. The telephone lines and the electrical power transmission lines

Electrical energy propagating through these lines set up electric fields between line conductors .These fields are at right angles to each other and to the direction of propagation which ia along the length of the line. This type of energy transmission is known as transverse electromagnetic mode of propagation(TEM).

Advantage: Less capacitance compared to underground cable

Disadvantage: 1.Initial cost is high due to the requirement of telephone posts and towers

2. Affected by atmospheric conditions like wind ,air, ice….maintenance is difficult

3. Possibility of shorting due to flying objects and birds

2. COAXIAL TYPE:

The radiation loss in an open two wire line is made negligible by employing a circular cylindrical conductor of smaller diameter placed coaxially inside another hollow conductor of larger diameter and of certain thickness. The resulting structure is called coaxial line. Here the inner tube is separated and insulated from the outer conductor by a dielectric medium which may be solid or gaseous.

The two conductors are at two different potentials. The fields re entirely confined to the space between the two conductors .No fields exist outside the outer conductor and similarly no external radiation can penetrate the outer tube and propagate inside.

Here the source is connected to the inner conductor of the coaxial line. At the other end load zL is connected to the inner conductor. The other end of source, load and the outer conductor of the coaxial line are all connected to the ground, hence the voltage between the inner conductor and the ground are different. Therefore coax is an unbalanced line.

The mode of wave propagation inside the coax is TEM mode. The electromagnetic wave is guided between the two tubes and the outer conductor acts as a shield to prevent leakage of signal from inside to outside and vice versa.

Coax are of three types

i) Flexible Coax: Use copper braided outer conductor, a thin center conductor and a low loss solid or foam polyethylene (PE) dielectric.

ii) Semi-rigid cables: Have solid dielectric and use thin outer conductor so that it could be bent for convenience while laying cables.

iii) Rigid cables: Have solid dielectric made of Teflon(PTFE).The space inside the tubes is essentially air with PTFE separating plugs at regular intervals

The coax can be used up to 3 GHz for transmitting large signal powers .Beyond this frequency the transmission of electromagnetic waves along the coax becomes difficult due to

a) **Losses that occur in the solid dielectric needed to support the conductors**
b) **Losses in conductors due to skin effect**

3. Waveguide: A transmission line consisting of a suitable shaped hollow conductor which may be filled with a dielectric material and is used to guide the EM waves of UHF propagated along its length, is called a waveguide

The walls of the waveguide are made of brass, copper or aluminum. During the propagation, the waves suffer multiple reflections at the walls of the guide and the resulting distribution associated with the wave causes the transmission mode.

Since the electric and magnetic fields are confined to the space within the guides, there is no loss of power through radiation. generally the medium inside the waveguide is air there is no loss of power due to dielectrics. However there is some power loss in the walls of the waveguide, but the loss is very small. The loss in a waveguide will be less than in coax.

Inside a waveguide, several modes of electromagnetic waves can propagate. The TEM mode does not exist, but either transverse electric (TE) or transverse magnetic(TM) modes can exist depending upon the mode of coupling the signal to the waveguide from the microwave source.

Advantage:

i) Higher power handling capability

ii) A simpler mechanical structure which reduces the fabrication cost

iii) Lower attenuation per unit length Disadvantage:

i) Larger cross sectional dimensions and a lower usable bandwidth than in a coax

Again the transmission line can be divided into two classes:

i) A balance transmission line is one where two signal wires are used to

propagate electromagnetic waves relative to some fixed potential, usually assumed to be ground. E.g.. A flat twin wire

ii) In an unbalanced line, one conductor forms the signal side ,while the other is the ground
E.g. coax since the shield wire is always connected with a ground point

LUMPED CONSTANTS

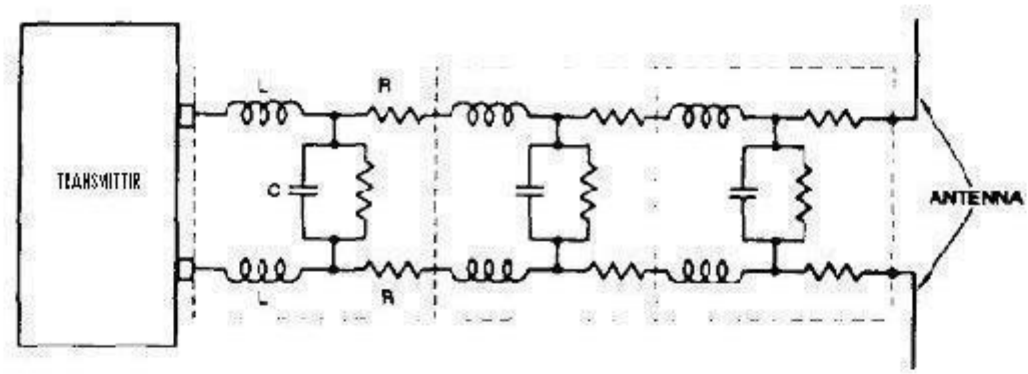

Fig:1: Transmission linea equivalent circuit

A transmission line has the properties of inductance, capacitance, and resistance just as the more conventional circuits have. Usually, however, the constants in conventional circuits are lumped into a single device or component. For example, a coil of wire has the property of inductance. When a certain amount of inductance is needed in a circuit, a coil of the proper dimensions is inserted.

The inductance of the circuit is lumped into the one component. Two metal plates separated by a small space, can be used to supply the required capacitance for a circuit. In such a case, most of the capacitance of the circuit is lumped into this one component. Similarly, a fixed resistor can be used to supply a certain value of circuit resistance as a lumped sum. Ideally, a transmission line would also have its constants

of inductance, capacitance, and resistance lumped together, as shown in figure 1 unfortunately, this is not the case. Transmission line constants are as described in the following paragraphs.

DISTRIBUTED CONSTANTS

Transmission line constants, called distributed constants, are spread along the entire length of the transmission line and cannot be distinguished separately. The amount of inductance, capacitance, and resistance depends on the length of the line, the size of the conducting wires, the spacing between the wires, and the dielectric (air or insulating medium) between the wires.

Resistance of a Transmission Line

The transmission line shown has electrical resistance along its length depending upon its cross sectional area. This resistance is usually expressed in ohms per unit length

Inductance of a Transmission Line

When current flows through a wire, magnetic lines of force are set up around the wire. As the current increases and decreases in amplitude, the field around the wire expands and collapses accordingly. The energy produced by the magnetic lines of force collapsing back into the wire tends to keep the current flowing in the same direction. This represents a certain amount of inductance, which is expressed in micro henrys per unit length.

Capacitance of a Transmission Line

Capacitance also exists between the transmission line wires. The two parallel wires act as plates of a capacitor and that the air between them acts as a dielectric. The capacitance between the wires is usually expressed in Pico farads per unit length. This electric field between the wires is similar to the field that exists between the two plates of a capacitor.

SECONDARY CONSTANTS OF A TRANSMISSION LINE:

i) Characteristic

If the line is infinitely long then the ratio will always produce a constant Impedance referred as Characteristic impedance (Z0).It is a complex quantity, it will vary with frequency

ii) Propagation

It is the measure of the signal received in terms of line attenuation per unit length and Phase shift per unit length.

TRANSMISSION LINE

A circuit with distributed parameter requires a method of analysis somewhat different from that employed in circuits of lumped constants. Since a voltage drop occurs across each series increment of a line, the voltage applied to each increment of shunt

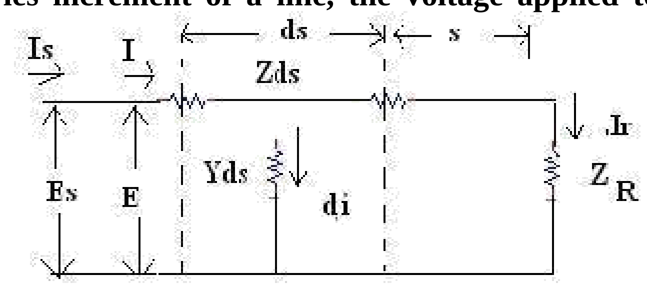

Fig:2: Transmission line with sending and receiving end

Hence the line current around the loop is not a constant, as is assumed in lumped constant circuits, but varies from point to point along the line. Differential circuit equations that describes that action will be written for the steady state, from which general circuit equation will be defined as follows.

R= series resistance, ohms per unit length of line (includes both wires) L= series inductance, henrys per unit length of line

C= capacitance between conductors, faradays per unit length of

G= shunt leakage conductance between conductors, mhos per unit length of

ωL = series reactance, ohms per unit length of line Z = R+jωL ωL = series susceptance, mhos per unit length of line Y =

S = distance to the point of observation, measured from the receiving end of the line I = Current in the line at any point

E=V= voltage between conductors at any point l = length of line

Variation in voltage is given by

$$-\frac{dV}{dx} = I(R + j\omega L)$$

Variation in current is given by

$$-\frac{dI}{dx} = V(G + j\omega C)$$

The solution of the above equation is given by

$$V(x) = ae^{-px} + be^{px}$$

$$I(x) = ce^{-px} + de^{px}$$

The above equation suggests that the line will contain two waves, one travelling in the positive 'x' direction (e^{-px}) .These are called incident waves which decay exponentially. The other term represents a wave propagating in negative 'x' direction. These are known as reflected wave.

An alternative solution in hyperbolic function is given by

$$e^{px} = coshpx + sinhpx$$

$$e^{-px} = coshpx - sinhpx$$

= (+)

The solution of the above equation is given by

V=a $[coshpx + sinhpx]$ + b$[coshpx - sinhpx]$

The above equation suggests that the line will contain two waves, one travelling in the positive 'x' direction ($^{-}$) .These are called incident waves which decay exponentially.

The other term represents a wave propagating in negative 'x' direction. These are known as reflected wave.

= (a + b) $cosh px$+(a-b) $sinh px$

v = A $cosh px$+B $sinh px$

Similarly

I=c [$cosh px + sinh px$]+d[$cosh px - sinh px$]

= (c +d) $cosh px$+(c-d) $sinh px$

= C $cosh px$+$D sinh px$

The four constants A, B, C and D can be simplified to two constants A & B by

$$-\frac{d}{dx}[\mathrm{A}\ cosh px + \mathrm{B}\ sinh px]=I(R+j\omega L)$$

$$=-\sqrt{\frac{G+j\omega C}{R+j\omega L}}[A sinh px + B cosh px]$$

$$I = -\frac{1}{Z_0}[A sinh px + B cosh px]$$

By applying boundary conditions at sending end V=V_s ,I=I_s and x=0

We get V_s=A and B= - $I_s Z_0$

Therefore V =Vs $cosh px$- $-$ Is$Z_0 sinh px$

$$I = I_s cosh px - \frac{V_s}{Z_0} sinh px$$

INFINITE LINE:

Long lines are called as infinite line. A signal fed into a line of infinite length could not reach the far end in a finite time. If a line of finite length is considered, then all power fed into it will be absorbed. As we move away from the input terminals towards load, the current and voltage become zero at an infinite distance. Hence the transmission line analysis begins with an infinite time in order to separate input conditions from output conditions.

An AC voltage is applied at a distance x from the sending end point of the infinite line. Current at any point at a distance x from the sending end is

At sending end ,x=0,I=Isi

Isi=c+d

At the receiving

end, x=∞ , I=0

0=c x= ∞

In this either c=0 or ∞=0, but ∞ cannot be equal to zero, so only possibility is c=0

And d= Isi

Therefore I= Isi ⁻

At the receiving end, x=∞, V=0

a=0, since e∞ ≠ 0 and b= Vsi

Therefore V= Vsi⁻
Equations are infinite line equation

OTHER UNITS OF A TRANSMISSION LINE:

WaveIength

The distance the wave travels along the line while the phase angle is changed through 2π radians is called wavelength.

$$\lambda = 2\pi / \beta$$

The change of 2π in phase angle represents one cycle in time and occurs in a

distance of one wavelength,

$$\lambda = v/f$$

VeIocity

$$V_P = f\,\lambda$$

=

This is the velocity of propagation along the line based on the observation of the change in the phase angle along the line.It is measured in miles/second if ß is in radians per meter.

Group Velocity:

If the transmission line is such that different frequencies travel with different velocities,then the line or the medium is said to be dispersive. In that cases, signals are propagated with a velocity known as group velocity. It is less than phase velocity. The inter-relation between group velocity and phase velocity is given by

=

corresponding phase constants

Distortions:

Wave-form Distortion:

In general α is a function of frequency. All the frequencies transmitted on a line will then not be attenuated equally. A complex applied voltage, such as voice voltage containing many frequencies, will not have all frequencies transmitted with equal attenuation, and the received for will be identical with the input waveform at the sending end. This variation is known as frequency distortion.

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Phase Distortion

It is apparent that ω and β do not both involve frequency in same manner and that the velocity of propagation will in general be some function of frequency.

All the frequencies applied to a transmission line will not have the same time of transmission , some frequencies delayed more than the others. For an applied voice voltage waves the received waves will not be identical with the input wave form at the receiving end, since some components will be delayed more than those of the other frequencies. This phenomenon is known as deIay or phase distortion.

Frequency distortion is reduced in the transmission of high quality radio

broadcast programs over wire line by use of equalizers at line terminals. These circuits are networks whose frequency and phase characteristics are adjusted to be inverse to those of the lines, resulting in an over all uniform frequency response over the desired frequency band.

Delay distortion is relatively minor importance to voice and music transmission because of the characteristics of ear. It can be very series in circuits intended for picture transmission, and applications of the co axial cable have been made to over come the difficulty.

In such cables the internal inductance is low at high frequencies because of skin effect, the resistance is small because of the large conductors, and capacitance is small because of the use of air dielectric with a minimum spacers.

DISTORTIONLESS LINE:

If a transmission line is free from frequency distortion and phase distortion then a line is said to be lo less or distortion less line.Inorder to avoid distortions in the

signal transmission, the attenuation constant α, the phase velocity Vp must be made independent of frequency and the phase shift constant β must be with multiples of angular frequency.

Conditions for a distortion less line:

The distortion less condition can be attained in two methods i)by making use of propagation constant calculations

ii) From the expressions of attenuation constant and phase shift constant

CONDITION FOR MINIMUM ATTENUATION:

To calculate the value of L,the other line parameter R,C,G & ω are considered as constant,only L may be varied

solving and reducing we get,

$R/L= G/C$

If only L is variable, the attenuation is minium when,

L=RC/G H/Km

In practice L is normally less than the desired vlue

Similarly,

C= LG/R F/Km

When R=0 and G=0, the attenuation constant is zero.

TELEPHONE CABLE

Telephone cable find wide application in the field of communication.Hence it is desirable to investigate their behavior as a special case of transmission line.

Telephone cable is formed by two wires ,insulated from each other by a layer of oil impregnated paper and then twisted in pairs.

structural view of a Telephone wire

Fig:3:Structure of telephone cable

A large number of such pairs are combined inside a protective lead or plastic sheath to form one underground cable.such transmission lines are called as telephone cable.

sectional view of a telephone cable

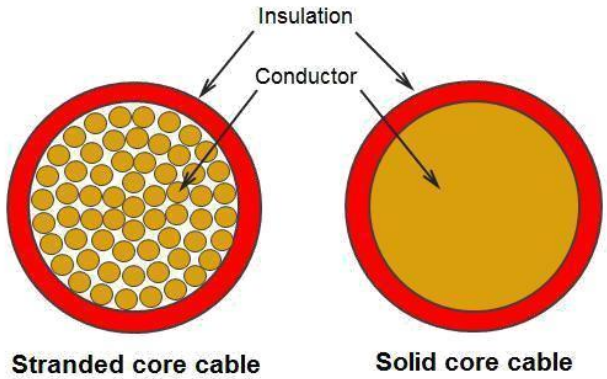

Fig:4: Sectional view of telephone cable

Small circle representing a cross sectional view of a twisted pair.

The telephone cables are primarily designed to transmit low voltages such a construction leads to abnormally low inductance(negligible inductance)&abnormally low conductance(negligible conductance) at audio frequencies(because of the proximity of the conductors and the presence of solid insulation.

Thus the reasonable assumptions, over the range of frequencies used in telephonic communication are

$\omega L << R$&$G << \omega c$

ω-Angular frequency.

The propagation constant of a loaded line is different from the already existing transmission line.The new propagation constant of a loaded line is identified with campbell'sformula.This is done by considering a

1.General transmission line analysis(or)

2.By adapting the notation of filter circuits.

CAMPBELL’S EQUATION

An analysis for the performance of a line loaded at uniform intervals can be obtained by considering a symmetrical section of line from the center of one loading coil to the center of the next, where the loading coil of the inductance is Zc.

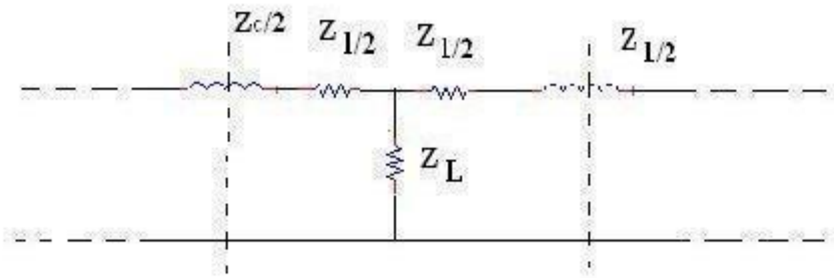

Fig:5: T-network equivalent circuit

The section line may be replaced with an equivalent T section having symmetrical series arms. Adopting the notation of filter circuits one of these series arms is called Z1/2 and is

Z1/2 = Z0/sinhpd [coshpd-1]

Where N is the number miles between loading coils and γ is the propagation constant per mile. Upon including half a loading coil, the equivalent series arm of the loaded section becomes

An equation relating that *γ* and the circuit element of a T section was already derived, which may be applied to the loaded T section as

So that the above equation reduces to

Coshp’d = Zcsinhpd/2Z0 +(coshpd)

This expression is known as CampbeII’s Equation and permits the determination of a value for *γ*' of a line section consisting partially of lumped land partially of distributed elements.Campbell’s equation makes possible the calculation of the effects of loading coils in

reducing attenuation and distortion on lines.

For a cable Z2 of the above figure is essentially capacitive and the cable capacitance plus lumped inductances appear similar to the circuit of the Iow pass fiIter

It is found that for frequencies below thw cutoff, given by

$$f_0=\frac{1}{\pi\sqrt{LC}}$$

The attenuation is reduced as expected, but above cutoff the attenuation rises as a result of filter action. This cutoff frequency forms a definite upper limit to successful transmission over cables.

It can be raised by reducing L but this expedient alloes the attenuation to rise. The cutoff frequency also be reduced by spacing the closer together, thus reducing C and more closely approximating the distributed constant line, but the cost increases rapidly.

In practice, a truIy distortion IessIine is not obtained by Ioading, because R and L are to some extent functions of frequency. Eddy current Iosses in the Ioading inductors aggravate this condition. However, a major improvement is obtained in the IoadedcabIe for a reasonabIe frequency range.

GENERAL EQUATION FOR A LINE WITH ANY TERMINATION:

A circuit with distributed parameter requires a method of analysis somewhat different from that employed in circuits of lumped constants. Since a voltage drop occurs across each series increment of a line, the voltage applied to each increment of shunt admittance is a variable and thus the shunted current is a variable along the line.

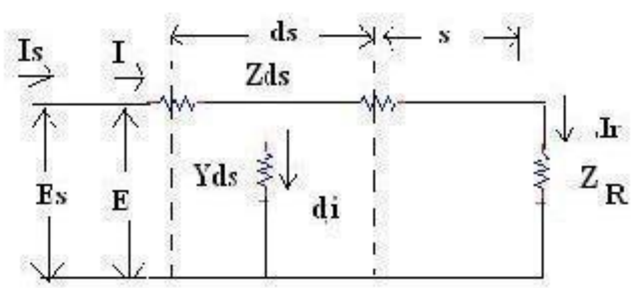

Fig:6: Transmission line- general equation

Hence the line current around the loop is not a constant, as is assumed in lumped constant circuits, but varies from point to point along the line. Differential circuit equations that describes that action will be written for the steady state, from which general circuit equation will be defined as follows.

R= series resistance, ohms per unit length of line(includes both wires) L= series inductance, henrys per unit length of line

C= capacitance between conductors, faradays per unit length of line G= shunt leakage conductance between conductors, mhos per

unit length Of line

ωL = series reactance, ohms per unit length of line Z = R+jωL

ωL = series susceptance, mhos per unit length of line Y = G+jωC

S = distance to the point of observation, measured from the receiving end of the line I = Current in the line at any point

E= voltage between conductors at any point l = length of line

The below figure illustrates a line that in the limit may be considered as made up of cascaded infinitesimal T sections, one of which is shown.

E = a $^{+}$ + b $^{-}$

ER = A+B

IR = C+D

By solving we get,
ES = (ZR+ZO)ER/ZR[$^{\surd}$ + (ZR-ZO/ZR+ZO) $^{-\surd}$]
IS = (ZR+ZO)IR/2ZO[$^{\surd}$ - (ZR-ZO/ZR+ZO) $^{-\surd}$]

REFLECTION:

When the transmission line is not correctly terminated, the travelling electromagnetic wve from generator at the sending end is reflected completely or partially at the termination

The phenomenon of setting up of a reflected wve at the load due to improper termination or due to impedance irregularity in a line called reflection.

REFLECTION COEFFICIENT:

The ratio of amplitudesw of reflected to incident voltage components at the receiving

end of a line is called the reflection coefficient by K.

K = ZR –Zo / ZR + Zo

INPUT IMPEDANCE:

The ratio of the voltage applied to the current flowing will give the input impedance of the transmission line. It is also known as characteristic impedance of the line for infinite line. Input impedance of open circuited and short circuited line:

Open and short circuited lines

As limited cases it is convenient to consider lines terminated in open circuit or short circuit, that is with *ZR* = ∞or *ZR* =0. The input impedance of a line of length l .
The open circuited condition is given by,
ZOC = Vs/Is = Z0 cothpl

Short circuited condition is given by,

ZSC = Z0 tanhpl
By multiplying the above two equations it can be seen that

$$Z0 = \sqrt{ZocZsc}$$

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DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - II
RADIO FREQUENCY TRANSMISSION LINES – SECA1402

UNIT II: RADIO FREQUENCY TRANSMISSION LINES

INT RODUCTION

When a line, either open wire or coaxial, is used at frequencies of a megacycle , it is found that certain approximations may be employed leading to simplified analysis of line

PARAMETERS OF OPENWIRE LINE AT RADIO FREQUENCY:

Approximations of a transmission line used at radio frequencies:

(i) **At radio frequencies the line has considerable skin effect, so that almost the entire current may be assumed to flow through the outer surface of the conductor. Thus the internal inductance of the wires may be considered to be zero.**
Li = 0

(ii) **At VHF, the inductive reactance is comparatively large with the series resistance R.**
$>>$ **R**

(iii) **The lines are well constructed, so that shunt conductance G may be considered to be zero at radio frequencies.**
G = 0

The analysis is made in either of two ways, depending on whether R is merely small with respect to ω L or R is small, the line is considered completely negligible compared with ω L.

If R is small, the line is considered one of small dissipation, and this concept is useful when lines are employed as circuit elements or where resonance properties are involved. If losses were neglected then infnite current or voltages would appear in calculations, and and physical reality would not be achieved.

In applications where losses may be neglegted, as in transmission of power at high efficiency, R may be considered as negligible, and the line as one of zereo dissipation. These methods will be studied separately.

From The above approximations, the line parameters of open wire line at radiofrequency:

1. **Loop inductance for open wire line: consider an open wire line having two circular conductors parallel to each other.**
   Let a-be radius of the conductor and d be the spacing between the two conductors
   In general,
   The self inductance of the parallel wire line system
   $L=\{\mu r+9.21 \log d/a\}10^{-7}$ henry/meter

Where μr is the relative permeability of the conducting material for non-magnetic material

But at high frequency due to skin effect, the internal inductance of the open wire line is Li=0 -7

Hence the self inductance at RF=L=9.21 10 log(d/a) henry/m

2. **Shunt capacitance:**

   The value of capacitance of a line is not affected by a skin effect or frequency, hence given as C=πεd/(ln(d/a))

3. **Loop resistance:**

   At radio frequency due to appreciable skin effect, the current flows over the surface of the conductor in a thin layer with a resultant reduction in effective cross section area or an increase in resistance of the conductor

   Rac=Rdc/2 (πfμ)

4. **Shunt conductance:** since the lines are well constructed that is filled with solid dielectric between the conductors there is no shunt conductance. G=0

Parameters of coaxial lines at RF:

At radio frequency due to skin effect, the current flows on the outer surface of the inner conductor and the inner surface of the outer conductor, which eliminates flux linkages due to internal conductor flux and the

1. Loop inductance: for the co-axial line

   L = /2 loge (b/a) henrys/m

   Where a- radius of a inner solid conductor

   b-inner surface radius of the outer hollow tubular conductor c-the outer surface radius

   μrd- relative permeability of the dielectric material μo- absolute permeability

2. Shunt capacitance: the capacitance of a co-axial line is not affected by frequency, except the relative permittivity of the dielectric , so that

   C = 2 d/ ln(b/a) farad/m

=   μ /

   Where, - is the di~~electric co~~nstant of the dielectric material - is the relative dielectric constant of the dielectric material

3. Loop resistance:

   Due to appreciable skin effect, thr resistance from the two thin walled tubes,

4. Shunt conductance: the shunt losses of air dielectric lines are zero, but many co axial lines employ solid dielectric materials and the conducting losses are measured in terms of the power factor of the material or iin terms of dissipation factor.

   For a good dielectric with small power factor angles the approxiamations is

G<< C
, which makes shuct conductance, G=0.

CONSTANTS FOR LINES OF ZERO DISSPATION:

- **(i) Internal inductance, due to skin effect Li = 0**
- **(ii) The inductive reactance is comparatively large with loop resistance, ≫**
- **(iii) Shunt conductance G = 0**

LOW DISSIPATION LINE: these lines are used where resonance properties areinvolved. Example transmission line as simple resistor, inductance, capacitor

If the loop resistance R is negligible in comparison with then the line is termed as

ZERO DISSIPATION LINES or LOSSLESS LINES. Example: transmission of power athigh efficiency is done through zero dissipation lines only. That is the transmission line which is used to transfer the power signal between the power amplifier and the antenna section at transmit end.

REPRESENTATION OF A RADIO FREQUENCY LINES:

- **(I) THE PRIMARY LINE CONSTANTS ARE SERIES LOOP INDUCTANCE 'L' and shunt capacitance 'C'.**
- **(II) Secondary line constants: for low dissipation lines**
  - **(a) Characteristic impedance:**
  - **(b) propagation constant**

If R is small and G is equal to zero at highfrequencies, then
= 0

VOLTAGES AND CURRENTS ON A DISSIPATION LESS LINE:

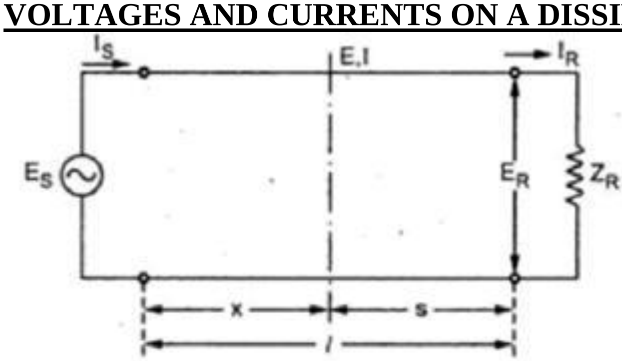


Fig:1: Transmission line of length l

The voltage at any distance s is measured from the receiving end of a line, terminated by the impedance Z_R, is given by E_s.
After grouping and simplifying, the voltage on a line is represented as,
$E_s = E_R \cos s + j I_R R0 \sin s$
Similarly,
The current on the line is given by, $I_s = I_R \cos s + j E_R/R0 \sin s$

STANDING WAVES

When the transmission line is not matched with its load i.e., load impedance is not equal to the characteristic impedance ($ZR = Z0$), the energy delivered to the load is reflected back to the source.

The combination of incident and reflected waves give rise to the standing waves.

STANDING-WAVE RATIO

The measurement of standing waves on a transmission line yields information about equipment operating conditions. Maximum power is absorbed by the load when ZL = Z0. If a line has no standing waves, the termination for that line is correct and maximum power transfer takes place.

$$VSWR = \frac{|V_{MAX}|}{|V_{MIN}|}$$

You have probably noticed that the variation of standing waves shows how near the rf line is to being terminated in Z0. A wide variation in voltage along the length means a termination far from Z0. A small variation means termination near Z0. Therefore, the ratio of the maximum to the minimum is a measure of the perfection of the termination of a line. This ratio is called the STANDING-WAVE RATIO (SWR) and is always expressed in whole numbers. For example, a ratio of 1:1 describes a line terminated in its characteristic impedance (Z0).

VoItage Standing-Wave Ratio

The ratio of maximum voltage to minimum voltage on a line is called the VOLTAGE ST ANDING-WAVE RATIO (VSWR). Therefore: The vertical lines in the formula indicate that the enclosed quantities are absolute and that the two values are taken without regard to polarity, Depending on the nature of the standing waves, the numerical value of VSWR ranges from a value of 1 (ZL = Z0, no standing waves) to an infinite value for theoretically complete reflection.

Since there is always a small loss on a line, the minimum voltage is never zero and the VSWR is always some finite value. However, if the VSWR is to be a useful quantity.the power losses along the line must be small in comparison to the transmitted power voltage. Since power is proportional to the square of the voltage, the ratio of the square of the maximum and minimum voltages is called the power standing- wave ratio. In a sense, the name is misleading because the power along a transmission line does not vary.

Current Standing-Wave Ratio

The ratio of maximum to minimum current along a transmission line is called CURRENT ST ANDING- WAVE RATIO (ISWR). Therefore: This ratio is the same as that for voltages. It can be used where measurements are made with loops that sample the magnetic field along a line. It gives the same results as VSWR measurements.

STANDING WAVE RATIO

The ratio of the maximum to minimum magnitudes of voltage or current on a line having standing waves is called the standing wave ratio or voltage standing wave ratio (VSWR)

S = maximum magnitude/ minimum magnitude S | |+ | |/(| |-| |)

Maxima of voltage occurs at which the incident and reflected waves are in phase Minima of voltage occurs at which the incident and reflected waves are out of phase

Fig:2: co-axial able

This figure shows the relation between standing wave ratio S and reflection coefficient

Coaxial cable is used as a transmission line for radio frequency signals, in applications such as connecting radio transmitters and receivers with their antennas, computer network (Internet) connections, and distributing cable television signals. One advantage of coax over other types of transmission line is that in an ideal coaxial cable the electromagnetic field carrying the signal exists only in the space between the inner and outer conductors. This allows coaxial cable runs to be installed next to metal objects such as gutters without the power losses that occur in other transmission lines, and provides protection of the signal from external electromagnetic interference.

Coaxial cable differs from other shielded cable used for carrying lower frequency signals

such as audio signals, in that the dimensions of the cable are controlled to produce a repeatable and predictable conductor spacing needed to function efficiently as a radio frequency transmission line.

How it works

Coaxial cable cutaway

Like any electrical power cord, coaxial cable conducts AC electric current between locations. Like these other cables, it has two conductors, the central wire and the tubular shield. At any moment the current is traveling outward from the source in one of the conductors, and returning in the other. However, since it is alternating current, the current reverses direction many times a second. Coaxial cable differs from other cable because it is designed to carry radio frequency current. This has a frequency much higher than the 50 or 60 Hz used in mains (electric power) cables, reversing direction millions to billions of times per second. Like other types of radio transmission line, this requires special construction to prevent power losses:

If an ordinary wire is used to carry high frequency currents, the wire acts as an antenna, and the high frequency currents radiate off the wire as radio waves, causing power losses. To prevent this, in coaxial cable one of the conductors is formed into a tube and encloses the other conductor. This confines the radio waves from the central conductor to the space inside the tube. To prevent the outer conductor, or shield, from radiating, it is connected to electrical ground, keeping it at a constant potential.

The dimensions and spacing of the conductors must be uniform. Any abrupt change in the spacing of the two conductors along the cable tends to reflect radio frequency power back toward the source, causing a condition called standing waves. This acts as a bottleneck, reducing the amount of power reaching the destination end of the cable. To hold the shield at a uniform distance from the central conductor, the space between the two is filled with a semirigid plastic dielectric. Manufacturers specify a minimum bend
Radius to prevent kinks that would cause reflections. The connectors used with coax are designed to hold the correct spacing through the body of the connector.

Each type of coaxial cable has a characteristic impedance depending on its dimensions and materials used, which is the ratio of the voltage to the current in the cable. In order to prevent reflections at the destination end of the cable from causing standing waves, any equipment the cable is attached to must present an impedance equal to the characteristic impedance (called 'matching'). Thus the equipment "appears" electrically similar to a continuation of the cable, preventing reflections. Common values of characteristic impedance for coaxial cable are 50 and 75 ohms.

Description

Coaxial cable design choices affect physical size, frequency performance, attenuation, power handling capabilities, flexibility, strength and cost. The inner conductor might be solid or stranded; stranded is more flexible. To get better high-frequency performance, the inner conductor may be silver plated. Sometimes copper-plated iron wire is used as

an inner conductor.

The insulator surrounding the inner conductor may be solid plastic, a foam plastic, or may be air with spacers supporting the inner wire. The properties of dielectric control some electrical properties of the cable. A common choice is a solid polyethylene (PE) insulator, used in lower-loss cables. Solid Teflon (PTFE) is also used as an insulator. Some coaxial lines use air (or some other gas) and have spacers to keep the inner conductor from touching the shield.

Many conventional coaxial cables use braided copper wire forming the shield. This allows the cable to be flexible, but it also means there are gaps in the shield layer, and the inner dimension of the shield varies slightly because the braid cannot be flat. Sometimes the braid is silver plated. For better shield performance, some cables have a double-layer shield. The shield might be just two braids, but it is more common now to have a thin foil shield covered by a wire braid. Some cables may invest in more than two shield layers, such as "quad-shield" which uses four alternating layers of foil and braid. Other shield designs sacrifice flexibility for better performance; some shields are a solid metal tube. Those cables cannot take sharp bends, as the shield will kink, causing losses in the cable.

For high power radio-frequency transmission up to about 1 GHz coaxial cable with a solid copper outer conductor is available in sizes of 0.25 inch upwards. The outer conductor is rippled like a bellows to permit flexibility and the inner conductor is held in position by a plastic spiral to approximate an air dielectric.

Coaxial cables require an internal structure of an insulating (dielectric) material to maintain the spacing between the center conductor and shield. The dielectric losses increase in this order: Ideal dielectric (no loss), vacuum, air, Polytetrafluoroethylene (PTFE), polyethylene foam, and solid polyethylene. A low relative permittivity allows for higher frequency usage. An inhomogeneous dielectric needs to be compensated by a non-circular conductor to avoid current hot-spots.

Most cables have a solid dielectric; others have a foam dielectric which contains as much air as possible to reduce the losses. Foam coax will have about 15% less attenuation but can absorb moisture—especially at its many surfaces—in humid environments, increasing the loss. Stars or spokes are even better but more expensive. Still more expensive were the air spaced coaxials used for some inter-city communications in the middle 20th Century. The center conductor was suspended by

polyethylene discs every few centimeters. In a miniature coaxial cable such as an RG-62 type, the inner conductor is supported by a spiral strand of polyethylene, so that an air space exists between most of the conductor and the

inside of the jacket. The lower dielectric constant of air allows for a greater inner diameter at the same impedance and a greater outer diameter at the same cutoff frequency, lowering ohmic losses. Inner conductors are sometimes silver plated to smooth the surface and reduce losses due to skin effect. A rough surface prolongs the path for the current and concentrates the current at peaks and thus increases ohmic losses.

The insulating jacket can be made from many materials. A common choice is PVC, but some applications may require fire-resistant materials. Outdoor applications may require the jacket to resist ultraviolet light and oxidation. For internal chassis connections the insulating jacket may be omitted.

The ends of coaxial cables are usually made with RF connectors.

Open wire transmission lines have the property that the electromagnetic wave propagating down the line extends into the space surrounding the parallel wires. These lines have low loss, but also have undesirable characteristics. They cannot be bent, twisted or otherwise shaped without changing their characteristic impedance, causing reflection of the signal back toward the source. They also cannot be run along or attached to anything conductive, as the extended fields will induce currents in the nearby conductors causing unwanted radiation and detuning of the line. Coaxial lines solve this problem by confining the electromagnetic wave to the area inside the cable, between the center conductor and the shield. The transmission of energy in the line occurs totally through the dielectric inside the cable between the conductors. Coaxial lines can therefore be bent and moderately twisted without negative effects, and they can be strapped to conductive supports without inducing unwanted currents in them. In radio-frequency applications up to a few gigahertz, the wave propagates primarily in the transverse electric magnetic (TEM) mode, which means that the electric and magnetic fields are both perpendicular to the direction of propagation. However, above a certain cutoff frequency, transverse electric (TE) and/or transverse magnetic (TM) modes can also propagate, as they do in a waveguide. It is usually undesirable to transmit signals above the cutoff frequency, since it may cause multiple modes with different phase velocities to propagate, interfering with each other. The outer diameter is roughly inversely proportional to the cutoff frequency. A propagating surface-wave mode that does not involve or require the outer shield but only a single central conductor also exists in coax but this mode is effectively suppressed in coax of conventional geometry and common impedance. Electric field lines for this TM mode have a longitudinal component and require line lengths of a half-wavelength or longer.

REFLECTION COEFFICIENT:

Reflection coefficient is the ratio of the reflected wave amplitude and incident wave amplitude at the receiving end of the line. It is denoted as K, which is having magnitude and phase values.

Voltage reflection coefficient:
KVRC = (ZR-ZO)/(ZR+ZO)
Current reflection coefficient: -
KCRC = (ZR-ZO)/(ZR+ZO)

Reflection loss:

If the line is not terminated by its characteristic impedance, then reflection occurs which in turn increases the power loss, which is termed as reflection loss.

Reflection loss is defined as the ratio of power absorbed by the load to the incident power. It is denoted by the letter Fl.

Fl = reflection loss = 10 log10[power to load/incident power]

By derivation we get,

$$Fl = 10 \log_{10} (2* \quad /Zs+ZR)^2$$

Fr is representing a reflection factor which defines a level of mismatch between two impwdwnces.

Fr in terms of K is given by,

$$Fr = (2* \quad /Zs+ZR)$$

Relation between SWR and reflection coefficient

S = 1+K/1-K

Reflection coefficient in terms of SWR is given by

K=S-1/S+1

Input impedance in terms of reflection coefficient

$$Zin = R0[\ 1+k\ {-2} \quad / 1-K\ {-2}$$

Practical types

1. Coaxial cable

Coaxial lines confine the electromagnetic wave to the area inside the cable, between the center conductor and the shield. The transmission of energy in the line occurs totally through the dielectric inside the cable between the conductors. Coaxial lines can therefore be bent and twisted (subject to limits) without negative effects, and they can be strapped to conductive supports without inducing unwanted currents in them. In radio-frequency applications up to a few gigahertz, the wave propagates in the transverse electric and magnetic mode (TEM) only, which means that the electric and magnetic fields are both perpendicular to the direction of propagation (the electric field

is radial, and the magnetic field is circumferential). However, at frequencies for which the wavelength (in the dielectric) is significantly shorter than the circumference of the cable, transverse electric (TE) and transverse magnetic (TM) waveguide modes can also propagate.

When more than one mode can exist, bends and other irregularities in the cable geometry can cause power to be transferred from one mode to another.

The most common use for coaxial cables is for television and other signals with bandwidth of multiple megahertz. In the middle 20th century they carried long distance telephone connections.

2. Microstrip

A microstrip circuit uses a thin flat conductor which is parallel to a ground plane.

Microstrip can be made by having a strip of copper on one side of a printed circuit board (PCB) or ceramic substrate while the other side is a continuous ground plane. The width of the strip, the thickness of the insulating layer (PCB or ceramic) and the dielectric constant of the insulating layer determine the characteristic impedance. Microstrip is an open structure whereas coaxial cable is a closed structure.

Stripline

A stripline circuit uses a flat strip of metal which is sandwiched between two parallel ground planes. The insulating material of the substrate forms a dielectric. The width of the strip, the thickness of the substrate and the relative permittivity of the substrate determine the characteristic impedance of the strip which is a transmission line.

Stripline is a conductor sandwiched by dielectric between a pair of groundplanes, much like a coax cable would look after you ran it over with your small-manhood indicating SUV (let's not go there...) In practice, "classic" stripline is usually made by etching circuitry on a substrate that has a groundplane on the opposite face, then adhesively attaching a second substrate (which is metalized on only one surface) on top to achieve the second groundplane. Stripline is most often a "soft-board" technology, but using low-temperature co-fired ceramics (LTCC), ceramic stripline circuits are also possible.

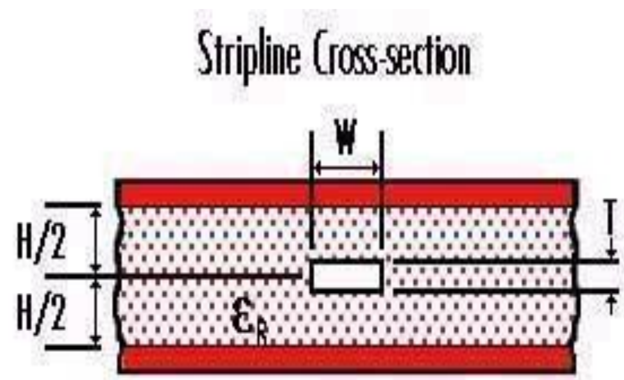


Fig:3:cross sectional view of stripline

All kinds of interesting circuits can be fabricated if a third layer of dielectric is added along with a second interior metal layer, for example, a stack-up of 31 mil Duroid, then 5 mil Duroid, then 31 mil Duroid (Duroid is a trademark of the Rogers Corporation). Transmission lines on either of the interior metal layers behave very nearly like "classic" stripline, the slight asymmetry is not a problem. Excellent "broadside" couplers can be made by running transmission lines parallel to each other on the two surfaces. We'll add more about this later!

Other variants of the stripline are offset strip line and suspended air stripline (SAS).

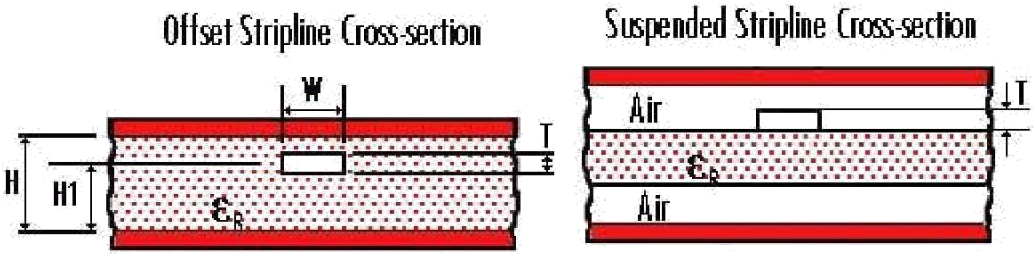


Fig:4: Off set and suspended stripline

For stripline and offset stripline, because all of the fields are constrained to the same dielectric, the effective dielectric constant is equal to the relative dielectric constant of the chosen dielectric material. For suspended stripline, you will have to calculate the effective dielectric constant, but if it is "mostly air", the effective dielectric constant will be close to 1.

Advantages and disadvantages of stripline

Stripline is a TEM (transverse electromagnetic) transmission line media, like coax. The filling factor for coax is unity, and "Keff" is equal to ER. This means that it is non-dispersive. Whatever circuits you can make on microstrip (which is quasi-TEM), you can make better using stripline, unless you run into fabrication or size constraints. Stripline filters and couplers always offer better bandwidth than their counterparts in microstrip, and the rolloff of stripline BPFs can be quite symmetric (unlike microstrip). Stripline has no lower cutoff frequency (like waveguide does).

But is stripline really non-dispersive at all frequencies? Read about the low frequencydispersion of TEM media, something to think about when you are designing between 10 MHz and 1 GHz...

Another advantage of stripline is that fantastic isolation between adjacent traces can be achieved (as opposed to microstrip). The best isolation results when a picket-fence of vias surrounds each transmission line, spaced at less than 1/4 wavelength. Stripline can be used to route RF signals across each other quite easily when offset stripline is used.

Disadvantages of stripline are two: first, it is much harder (and more expensive) to fabricate than microstrip, some old guys would even say it's a lost art. Lumped-element and active components either have to be buried between the groundplanes (generally a tricky proposition), or transitions to microstrip must be employed as needed to get the components onto the top of the board.

The second disadvantage of stripline is that because of the second groundplane, the strip widths are much narrower for a given impedance (such as 50 ohms) and board thickness than for microstrip. A common reaction to problems with microstrip circuits is to attempt to convert them to stripline. Chances are you'll end up with a board thickness that is four times that of your microstrip board to get equivalent transmission line loss.

That means you'll need forty mils thick stripline to replace ten mil thick microstrip! This is one of the reasons that softboard manufacturers offer so many thicknesses.

3. Balanced lines

A balanced line is a transmission line consisting of two conductors of the same type, and equal impedance to ground and other circuits. There are many formats of balanced lines, amongst the most common are twisted pair, star quad and twin-lead.

4. Twisted pair

Twisted pairs are commonly used for terrestrial telephone communications. In such cables, many pairs are grouped together in a single cable, from two to several thousand. The format is also used for data network distribution inside buildings, but in this case the cable used is more expensive with much tighter controlled parameters and either two or four pairs per cable.

5. Single-wire line

Unbalanced lines were formerly much used for telegraph transmission, but this form of communication has now fallen into disuse. Cables are similar to twisted pair in that many cores are bundled into the same cable but only one conductor is provided per circuit and there is no twisting. All the circuits on the same route use a common path for the return current (earth return). There is a power transmission version of single-wire earth return in use in many locations.

Waveguide

Waveguides are rectangular or circular metallic tubes inside which an electromagnetic wave is propagated and is confined by the tube. Waveguides are not capable of transmitting the transverse electromagnetic mode found in copper lines and must use some other mode. Consequently, they cannot be directly connected to cable and a mechanism for launching the waveguide mode must be provided at the interface.

6. Microwave transmission line:

Microwave transmission is thet ransmission of information or energy by

electromagnetic waves whose wavelengths are measured in small numbers of centimetre; these are called *microwaves*This part of the radio

spectrum ranges across frequencies of roughly 1.0 gigahertz (GHz) to 30 GHz. These correspond to wavelengths from 30 centimeters down to 0.1 cm.

Microwaves are widely used for point-to-point communications because their small wavelength allows conveniently-sized antennas to direct them in narrow beams, which can be pointed directly at the receiving antenna. This allows nearby microwave equipment to use the same frequencies without interfering with each other, as lower frequency radio waves do. Another advantage is that the high frequency of microwaves gives the microwave band a very large information-carrying capacity; the microwave band has a bandwidth 30 times that of all the rest of the radio spectrum below it. A disadvantage is that microwaves are limited to line of sight propagation; they cannot pass around hills or mountains as lower frequency radio waves can.

Microwave radio transmission is commonly used in point-to-point communicationsystems on the surface of the Earth, in satellite communications, and in deep spaceradio communications. Other parts of the microwave radio band are used for radars, radio navigation systems, sensor systems, and radio astronomy.

The next higher part of the radio electromagnetic spectrum, where the frequencies are above 30 GHz and below 100 GHz, are called "millimeter waves" because their

wavelengths are conveniently measured in millimeters, and their wavelengths range from 10 mm down to 3.0 mmRadio waves in this band are usually

stronglyattenuated by the Earthly atmosphere and particles contained in it, especially during wet weather. Also, in wide band of frequencies around 60 GHz, the radio waves are strongly attenuated by molecular oxygen in the atmosphere. The electronic technologies needed in the millimeter wave band are also much more difficult to utilize than those of the microwave band

Wireless transmission of information

- **One-way (e.g. television broadcasting) and two-way telecommunication using communications satellite**
- **Terrestrial microwave relay links in telecommunications networks including backbone or backhaul carriers in cellular networks linking BTS-BSC and BSC-MSC.**

7. Super conducting transmission line

The obvious advantage of superconducting transmission lines is they have no resistive losses in the bulk. If superconducting transmission lines had no other sources of power dissipation, the choice between types of transmission lines would be easy. We would simply calculate the cost of conventional power lines and subtract the cost of the power that is dissipated in transporting the electricity.

Then, we would compare it to the cost of making and cooling superconducting transmission lines.

Of course, real superconducting cables have other sources of loss which must also be factored in. There are a number of major sources of losses in superconducting transmission lines, many of them fundamentally different from those in conventional transmission lines. There are a number of relatively small losses due to need to cool the line. No cooling system is perfectly efficient, so there is some loss of liquid nitrogen needed to cool the line. Typical values for cooling efficiency are estimated to be on the order of 10% [3]. Furthermore, there are losses due to the imperfect efficiency of the liquid nitrogen pumping system itself, as well as hydraulic losses due to the flow friction in the circulating liquid nitrogen.

Similar to conventional transmission lines, superconducting transmission lines also have shield and dielectric losses, which can be calculated using the same physical models. Unlike conventional lines, superconducting transmission lines have conductor AC losses. There is no generally accepted physical model to describe these losses, so much of the data is empirical. There are also losses due to imperfect thermal insulation of the superconducting cable. The result is a thermal leak between the cold liquid nitrogen and the warm surroundings. The losses can be reduced but not eliminated by creating a vacuum between the superconducting cable and the thermal insulator. Finally, there are small losses due to joints and terminations of cables.

A High Temperature Superconducting (HTS) power cable is a wire-based device that carries large amounts of electrical current. There are two types of HTS cables.

Warm Dielectric Cable

The warm dielectric cable configuration features a conductor made from HTS wires wound around a flexible hollow core (figure 1). Liquid nitrogen flows through the core, cooling the HTS wire to the zero resistance state. The conductor is surrounded by conventional dielectric insulation. The efficiency of this design reduces losses.

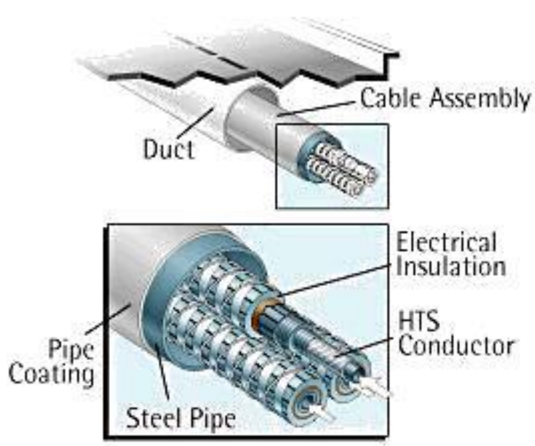


Fig:5:Construction of a warm dielectric HTS cable.

Cryogenic Dielectric Cable

The cryogenic dielectric is a coaxial configuration comprising an HTS conductor cooled by liquid nitrogen flowing through a flexible hollow core and an HTS return conductor, cooled by circulating liquid nitrogen. This represents an enhancement to the warm dielectric design, providing even greater ampacity, further reducing losses and entirely eliminating the need for dielectric fluids.

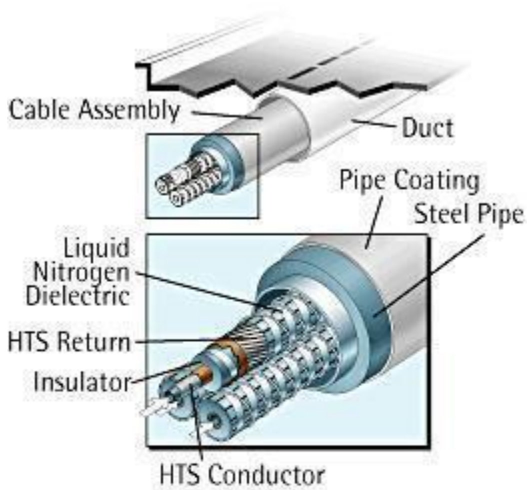


Fig:6:Construction of a cryogenic dielectric HTS cable.

Where and How are They Used?

HTS transmission cables would be used for power transmission and distribution in urban areas throughout the United States and the world.

What are the Benefits?

- **Can meet increasing power demands in urban areas via retrofit applications carrying two to five times more power than conventional cable**
- **Eliminates need to acquire new rights of way**
- **Replaces overhead transmission lines when environmental and other concerns prohibit their installation**
- **Enhanced overall system efficiency due to exceptionally low losses**
- **Increased utility system operating flexibility**
- **Reduced electricity costs**

With an estimated 80,000 miles of existing underground cable throughout the world, High Temperature Superconducting (HTS) cables will provide enormous benefits to a utility industry that is poised for growth and is faced with an ever rising demand for electricity faced with an ever rising demand for electricity and tightening constraints on

siting flexibility.

Conventional underground power transmission cables are utilised to transmit large amounts of power to congested urban areas. Conventional (copper -based) cables are capable of transmitting power (40 to 600 MVA) at high voltages (40 to 345 kV) through integrated underground duct systems. Existing duct systems limit the size of the conventional cables and the amount of power that can be transmitted through them.

When will HTS Power Transmission Cables be Available?

- **First HTS cable installation in a utility network is scheduled for the year 2000.**
- **The first HTS coaxial HTS cable demonstration is scheduled for that same year.**
- **The first commercial sales of HTS cable wires are expected shortly after 2001.**

Characteristics of Planar Transmission Lines

In general, planar transmission lines consist of strip metallic conductors, usually produced by some photographic process, on a non-conducting substrate. Typical substrate materials are slabs of dielectric, ferrite, or high resistivity semiconductors. In most cases, there are metal ground planes that can either be printed on the same substrate or be a part of the metal housing of MIC. This allows the characteristic impedance (*Z*0) of the line to be controlled by defining the dimensions in a single plane.

It is to be noted in Fig. (1.1) that the substrate materials with permittivity *r* are denoted by gray areas, and conductors and ground planes by bold lines. The region with free permittivity free space or air.

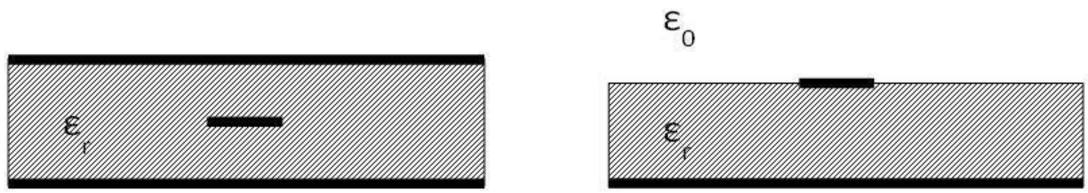
1. Stripline

The earliest form of planar transmission lines was stripline which is illustrated in Fig.

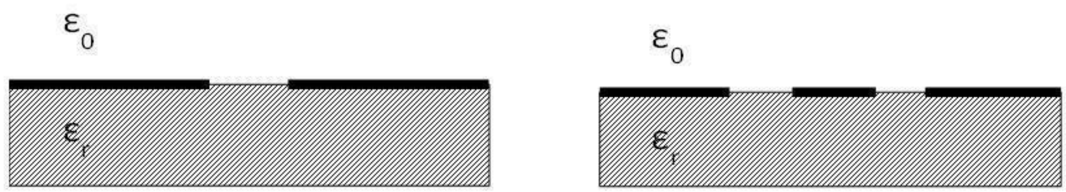
(1.1a). Striplines are essentially modifications of the two wire lines and coaxial lines. It consists of a strip conductor centered between two parallel ground planes with two equal slabs of a dielectric, ferrite, or semiconductor medium separating the center conductor from the ground planes. Usually, the medium is a solid material, but in some applications air is the actual dielectric used. The advantages of striplines are good electromagnetic shielding and low attenuation losses, which make them suitable for high-quality factor (*Q*) and low-interference applications. Transverse electric and

magnetic (TEM) waves propagate within the stripline. Such waves have electric and magnetic components in a plane transverse to the direction of propagation.

However, Striplines require strong symmetry and thereby present difficulties in the design of many circuit functions. Also, the tuning of circuits becomes difficult, because it requires destruction of the symmetry to access the center conductor. Any vertical asymmetry in the stripline structure could couple to waveguide-type modes bounded by the ground planes and the side walls. Also, with few exceptions of circuit configuration, the stripline structure is not convenient for incorporating chip elements and associated bias circuitry.



(a) Stripline ***(b) Microstrip Line***



(c) Slotline (d) Coplanar Waveguide (CPW)

(e) Coplanar Strip Line (CPS) ***(f) Finline***

Fig:7: ***Common transmission line structures suited to planar fabrication.***

2. Microstrip Line

The microstrip line is transmission line geometry with a single conductor trace on one side of a dielectric substrate and a single ground plane on the other side is shown in Fig.

(1.1b). Since it is an open structure, microstrip line has a major fabrication advantage over the stripline. It also features ease of interconnection and adjustments.

In the microstrip line, the electromagnetic fields exist partly in the air above the dielectric substrate and partly within the substrate itself. For most practical purposes, microstrip can be treated as a TEM transmission line with an effective relative permittivity (*eff*) that is a weighted average between air and the substrate material. But, the actual propagation of electromagnetic waves in microstrip is not purely TEM due to the combination of an open air space and a dielectric medium. Thus, it is usually assumed that the electromagnetic field in the microstrip line is quasi-TEM. It is largely TEM, but in reality microstrip lines, unlike striplines are dispersive, which means that the wave velocity varies with frequency rather than remaining a constant. This results in the varying of [redacted] $Z0$ with the frequency of the transmitted signal.

For microwave device applications, microstrip generally offers the smallest sizes and the easiest fabrication. MIC using microstrip can be designed for frequencies ranging from a few gigahertz, or even lower, upto at least many tens of gigahertz. However, it does not offer the highest electrical performance. Attenuation losses and power handling are compromised.

3. Slotline

The slotline consists of a narrow gap in the conductive coating on one side of the dielectric substrate, shown in Fig. (1.1c). The other side of the substrate is bare. Slotline has the following advantages:

1. **It is easy to fabricate because it requires only single-sided board etching.**
2. **Shunt mounting of elements is possible without holes through the substrate, since conductors are placed on only one side of the substrate.**

3. **It can be incorporated with microstrip lines for new types of circuits.**
4. **The substrate gives it rigidity.**
5. **The substrate concentrates the field density between the plates, suppressing higher-order modes or radiation.**

The disadvantage of the slotline is that its *Q*-factor is low (around 100), so it is relatively lossy. Another disadvantage arises from the fact that the field configuration deviates greatly from TEM. Thus, the dominant mode is similar to the dominant mode in rectangular waveguide; it is mainly a TE (transverse electric) field. This result in a highly dispersive behaviour, which means that slotline is not usually applicable for broadband applications. The presence of both longitudinal and transverse RF magnetic fields in slotline provides elliptic polarization that is useful for non-reciprocal ferrite circulators and isolators.

4. Coplanar Waveguide

The coplanar waveguide (CPW) structure consists of a center strip width with two parallel ground planes equidistant from it on either side, as shown in Fig. (1.1d). The center conductor and ground planes are located in one plane on the substrate surface. Coplanar Waveguides have the advantages of:

1. **Low dispersion;**
2. **No need for via holes, which introduce undesirable parasitic inductances and limit performance at high frequencies;**
3. **Ease of attaching both shunt and series circuit elements because of no need for via holes;**
4. **Simple realizations of short-circuited ends.**

The gap in the CPW is usually very small and supports electric fields primarily concentrated in the dielectric. With little fringing field in the air space, the CPW exhibits low dispersion. Like microstrip and stripline, CPW supports a quasi-TEM dominant mode. At higher frequencies, the field becomes less-TEM, and more TE in nature. The

magnetic field is elliptically polarized and the CPW becomes suitable for non-reciprocal ferrite devices, as with slotline.

In CPW, two fundamental modes are supported: the coplanar mode and the parasitic slotline mode. Air bridges between ground planes have to be applied to suppress the undesired slotline mode. However, these bridges increase insertion losses and make fabrication costly. Like slotline, the *Q*-factor of CPW is low (~150). Besides the parasitic mode and low-*Q* problems, CPW also have other disadvantages: heat sinking capabilities are poor, substrates are required to be relatively thick, and there are higher ohmic losses due to the concentration of its current near the metal edges.

5. Coplanar Strip Line

A coplanar strip line (CPS) consists of two conducting strips on the same substrate surface with one strip grounded and no other conducting layer, as shown in Fig. (1.1e)

.It is a complimentary structure of the CPW and is used as an area efficient variation of it. It also supports quasi-TEM mode and is less dispersive than slotline and microstrip line. The CPS has advantages over the parallel strip line because its two strips are on the same substrate surface for convenient connections. In MIC the wire bonds have always presented reliability and reproducibility problems. The CPS eliminates the difficulties involved in connecting the shunt elements between the hot and ground strips. As a result, reliability is increased, reproducibility is enhanced, and production cost is decreased.

The main advantage of CPS is less sensitivity to the substrate thickness. Both the series and shunt components can be easily mounted and via is not needed. It doesn't

require any backside processing of the substrate and relatively large range of characteristic impedance can be obtained with it. However, the main drawback to CPS is due to the lack of shielding that causes stray coupling to other lines. This drawback could be improved by adding the coplanar ground planes on both sides of the CPS line .

6. Finlines

A finline consists of a totally shielding rectangular conducting box (like rectangular waveguide but avoiding waveguide modes) with a dielectric substrate fixed, usually centrally, across two of its faces. A metal circuit is deposited on one side of the substrate and a slot pattern in this metal forms the finline circuit. Its illustration is shown in Fig. (1.1f). These lines operate typically in the frequency range of 30 to 100 GHz. The main characteristics of the finline are broad bandwidth, moderate attenuation, low dispersion and compatibility with semiconductor elements. Losses in Finlines are

approximately on the order of 0.1 dB/ wavelength.

In finline, the substrate employed has low relative dielectric constant [redacted]r= 2.2)

substrates, and the resulting dominant mode is a combination of TE and TM modes, rather than a quasi-TEM. The resulting structure has a wider bandwidth and higher *Q* values than those of a microstrip line.

Since the characteristic impedance range of the finline is from about 10 to 400 Ω, it is greater than other printed transmission lines. Also, the finline structure is easy to mate with standard rectangular waveguide structures. Another advantage is that the guide wavelength in finline is longer than that in microstrip, thus permitting less stringent dimensional tolerances at high microwave frequencies. The finline produces circularly polarized fields. This is an advantage for non-reciprocal applications (isolators, circulators and phase shifters).

7.Applications of Planar Transmission Lines

Table-(1) compares the various transmission lines on the basis of their *Q*-factor, radiation, dispersion, impedance range, chip mounting and applications.

Table-(1): ***Characteristics and Applications of the Various Planar Transmission Lines***

Transmission Line	*Q*-Factor	Radiation	Dispersive	Impedance Range (Ω)	Chip Mounting	Applications
Stripline	400	Low	None	35-250	Poor	Blocking filters
Microstrip Line	250 (dielectric substrate) 100-150 (*Si, GaAs* substrate)	Low (for high) High (for low)	Low	20-120	Difficult for shunt; easy for series	Filters Hybrids High-*Q* resonators
Slotline	100	Medium	High	60-200	Easy for shunt; difficult for series	Antennas Phase shifters
CPW	150	Medium	Low	20-250	Easy for series and shunt	Filters Hybrids High-*Q* resonators
CPS	150	Medium	Low	20-250	Easy for series and shunt	Filters Resonators Mixer Modulator Feeding networks for printed antenna technology
Finline	500	None	Low	10-400	Fair	Bandpass filters Quadrature

						hybrids □ Transitions to waveguide □ Balanced mixer circuit

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23

SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - III

MATCHING, MEASUREMENTS AND INTERFERENCE – SECA1402

UNIT –III: MATCHING , MEASUREMENTS AND INTERFERENCE

Types of Transmission Line sections:

The Transmission line is characterised in terms of its length in wavelength.These are

(i) The one eighth wavelength line
(ii) The Half wave line
(iii) The quarter waveline

(i) The one-eighth wavelength line

The length of line is one-eigth wavelength long the input impedance of such a line with l =λ/8 is

Zin=Z0[Z+jz0tanβl/Z0+jZltanβl]
Here
L=λ/8;β=2Π/λ after substitution | in(λ/8)|=R0

(ii) The quarter wave line

The input impedance of a quarter line is Zin (λ/4),is Zin=Z0[Z+jz0tanβl/Z0+jZltanβl]
Here Z0=R0,β=2Π/λ,l=λ/4
After substituition
Zin=$R0^2$/ZL

(iii) The Half Wave line

Zin(λ/2)=R0[ZL+jR0tan(2Π/λ*λ/2) Zin(λ/2)=ZL

Impedance Matching

A transmission line is acting as a connecting links between a transmitter and an antenna or between an antenna and a receiver,which affects the efficienxcy of power transferAccording to maximum power transfer theorem, when the impedance of one is the complex conjugate of the other, the maximum power is absorbed by the load.Different types of impedance matching are

(i)stub matching

(ii)tapered section

(iii)quarter wave transformer matching

The Smith Chart diagram

Smith Chart is

a, (i)polar impedance

diagram (ii)It consists

of 2 sets of circles

(a) Constant resistance circles
(b) Constant reactance Circles

Application

1.This chart is applicable to the analysis of lossless line as well as lossy line. 2.It can be used as impedance and admittance diagram

3.Determination of input impedance 4.Conversion of impedance to admittance. 5.To measure a standing wave pattern directly

6.From this, the magnitudes of the reflection coefficient,reflected power,transmitted power and the load impedance can be calculated from it.

Characteristics of the smith chart are

1.The constant K and constant X loci forms,two families of orthogonal circles in the chart. 2.The upper half of thwe diagram represents +jX

3.The lower half of the diagram represents –jX

4.For admittance the constant r circles become constant g circle and the constant x circles become constant suscetance b circles.

5.The distance around the smith chart once is one-half wavelength(λ/2). 6.At a point of Zmin=1/ρ,there is a Vmin on the line

7.At a point of Zmax= ρ,there is a Vmax on the line

8.The horizontal radius to the right of the chart center corresponds to Vmax ,Imin,Zmax and ρ(SWR)

9. The horizontal radius to the left of the chart center corresponds to Vmin,Imax,Zmin and 1/ρ.

10.Since the Normalized admittance Y is a reciprocal of the normalized impedance,the cprresponding quantities in the admittance chart are 180 degree out of phase with those in the impedance chart.

11.The normalised impedance or admittance is repeated for every half wavelength of distance.

12.The distances are given in wavelength toward the generator and also toward the load.

Smith Chart for the Impedance Plot

It will be easier if we normalize the load impedance to the characteristic impedance of the transmission line attached to the load.

$$z = \frac{Z}{Z_o} = r + jx$$

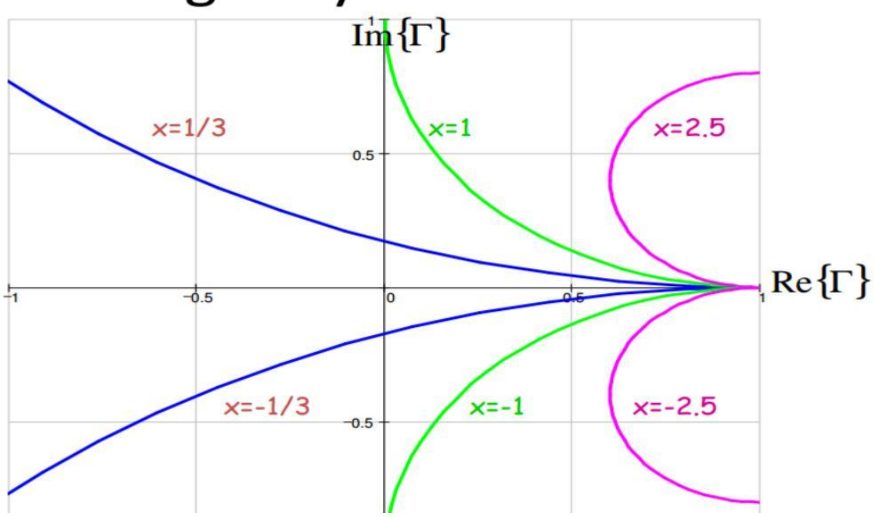
$$z = \frac{1+\Gamma}{1-\Gamma}$$

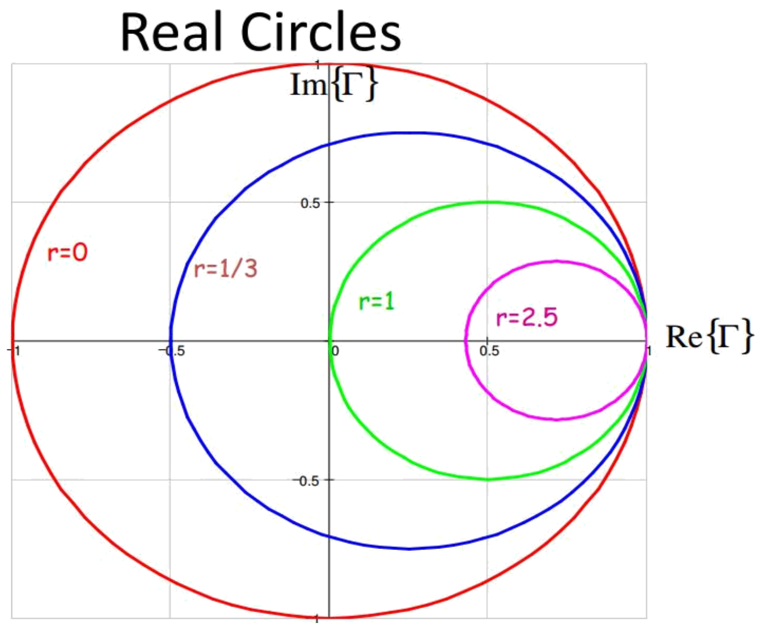
Since the impedance is a complex number, the reflection coefficient will be a complex number

$$\Gamma = u + jv$$

$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \qquad x = \frac{2v}{(1-u)^2+v^2}$$

Imaginary Circles

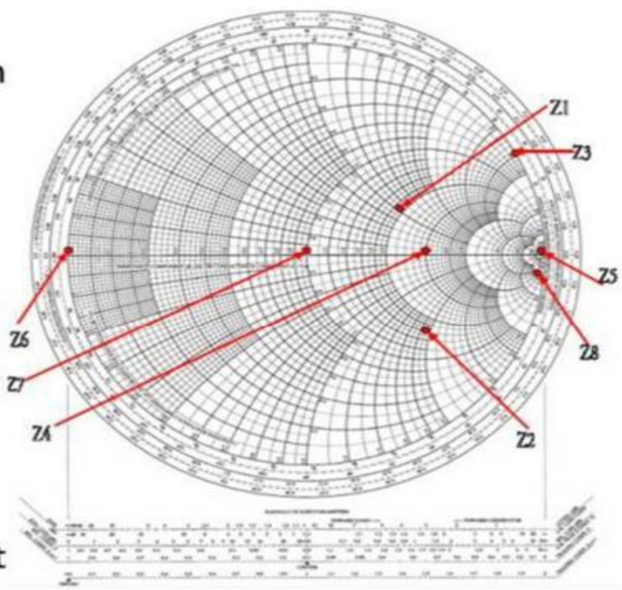




***how to plot impedance and obtain the normalized value**

•Impedance divided by lin impedance (50 Ohms)

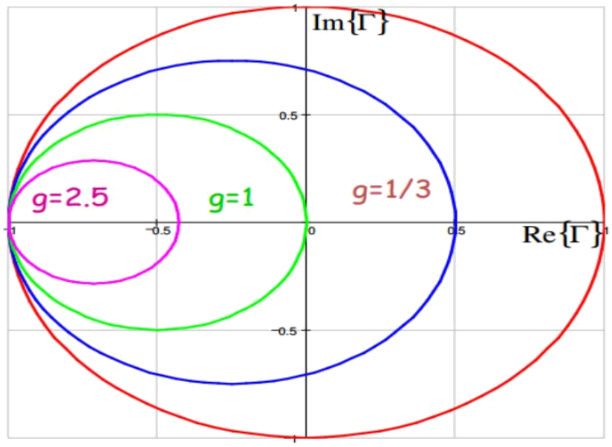
- Z1 = 100 + j50
- Z2 = 75 -j100
- Z3 = j200
- Z4 = 150
- Z5 = infinity (an open circuit)
- Z6 = 0 (a short circuit)
- Z7 = 50
- Z8 = 184 -j900



•Then, normalize and plot The points are plotted as follows:

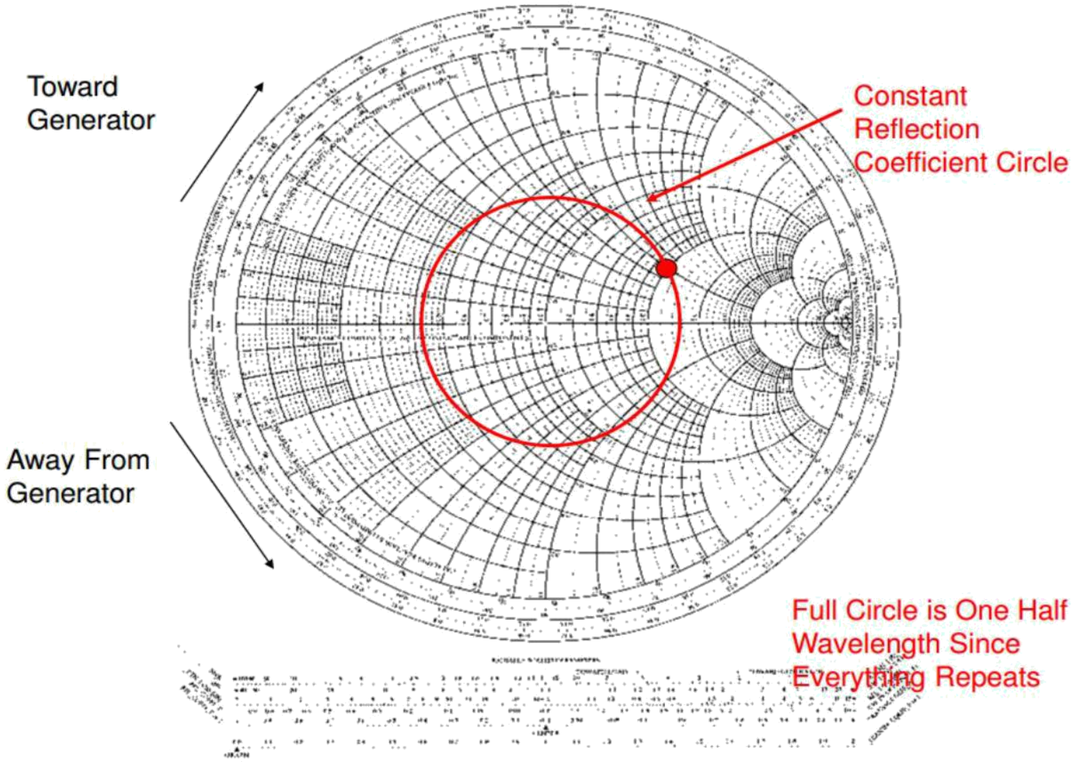
- z1 = 2 + j
- z2 = 1.5 -j2
- z3 = j4
- z4 = 3
- z5 = infinity
- z6 = 0
- z7 = 1
- z8 = 3.68 -j18S

Real admittance

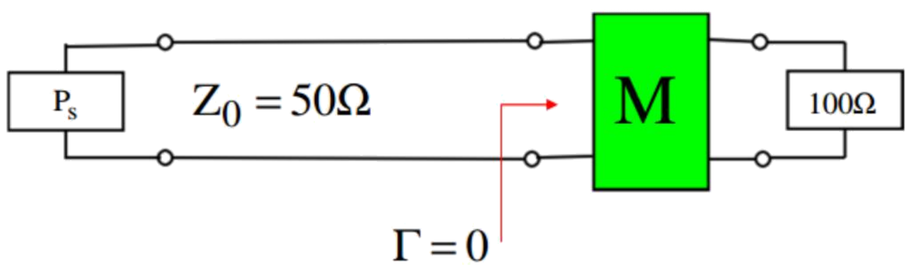


Matching

- For a matching network that contains elements connected in series and parallel, we will need two types of Smith charts
  - impedance Smith chart
  - admittance Smith Chart
- The admittance Smith chart is the impedance Smith chart rotated 180 degrees.
  - We could use one Smith chart and flip the reflection coefficient vector 180 degrees when switching between a series configuration to a parallel configuration.



Matching Example

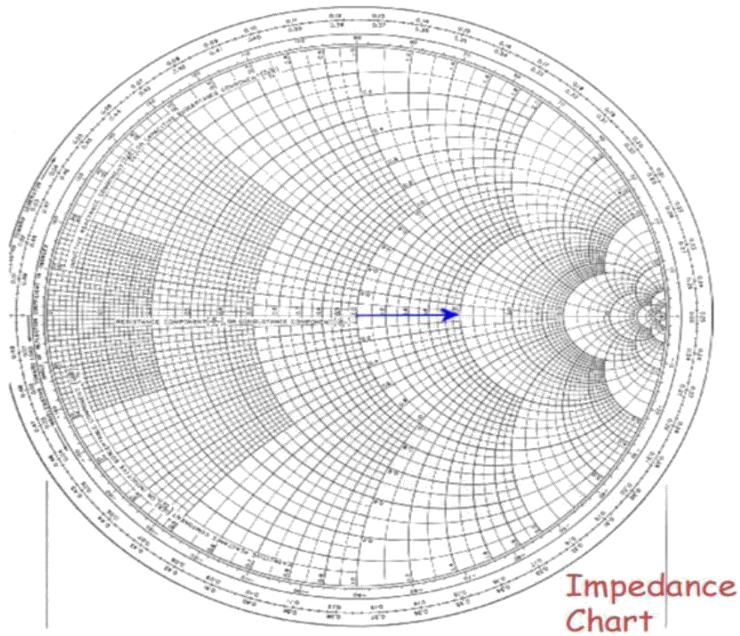


Match 100Ω load to a 50Ω system at 100MHz

A 100Ω resistor in parallel would do the trick but ½ of the power would be dissipated in the matching network. We want to use only lossless elements such as inductors and capacitors so we don't dissipate any power in the matching network

Matching Example

- We need to go from z=2+j0 to z=1+j0 on the Smith chart
- We won't get any closer by adding series impedance so we will need to add something in parallel.
- We need to flip over to the admittance chart



Matching Example

- We need to go from z=2+j0 to z=1+j0 on the Smith chart
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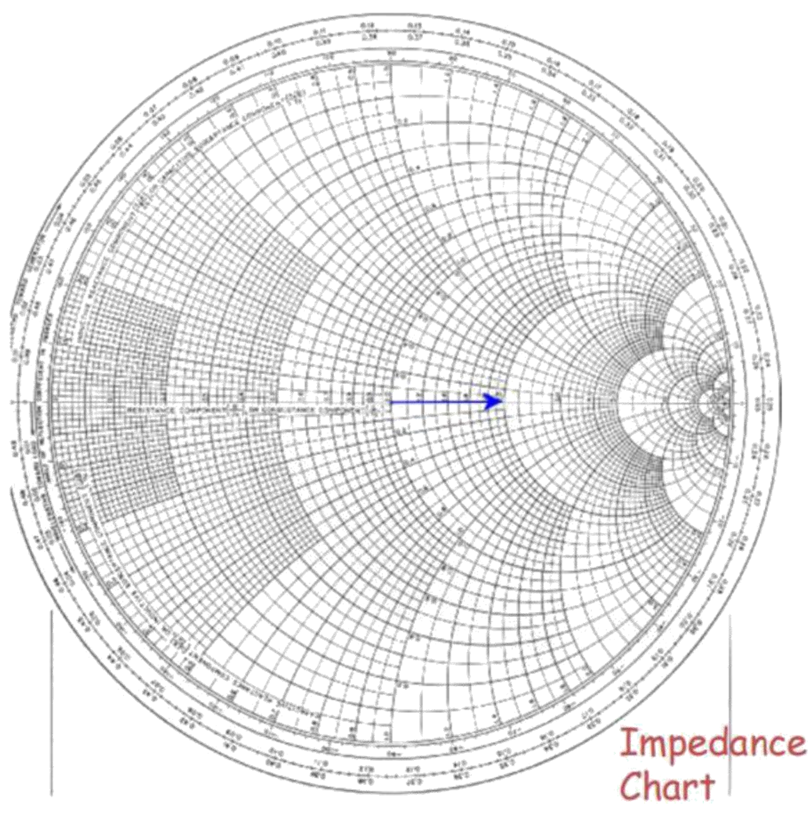
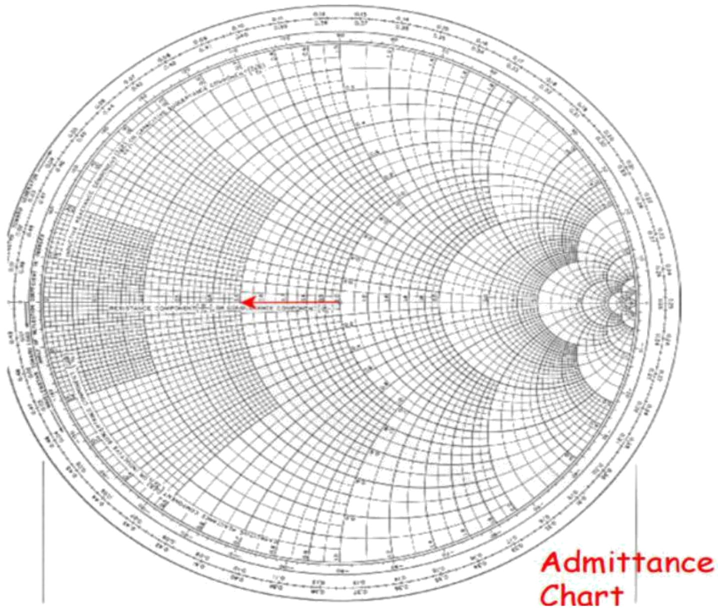


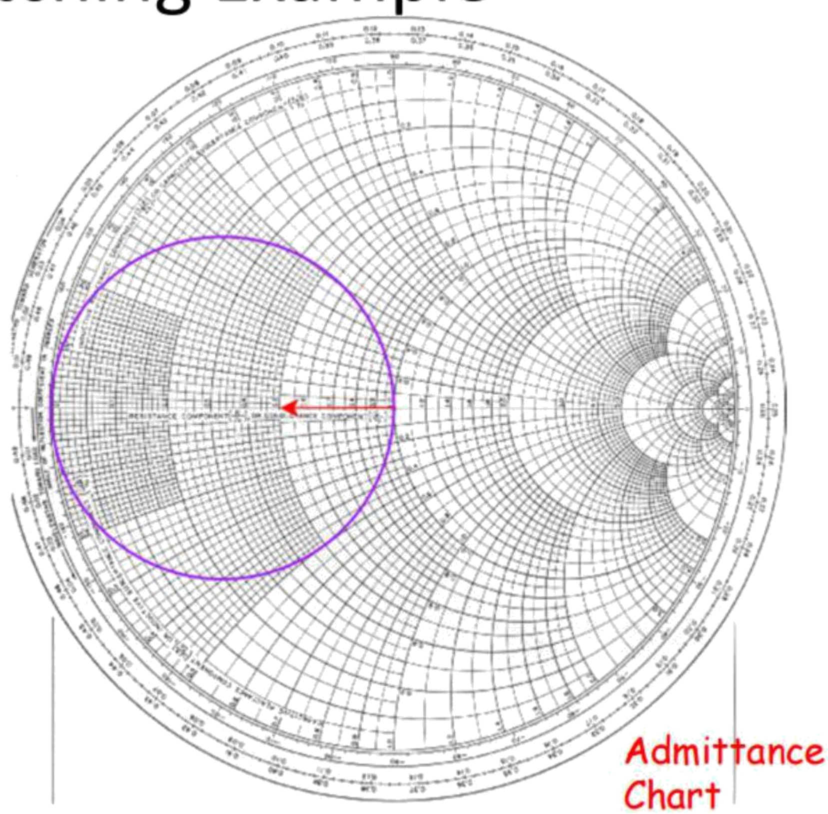
Matching Example

- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide.



Matching Example

- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance.



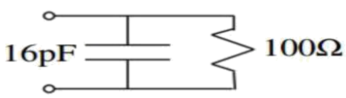
Matching Example

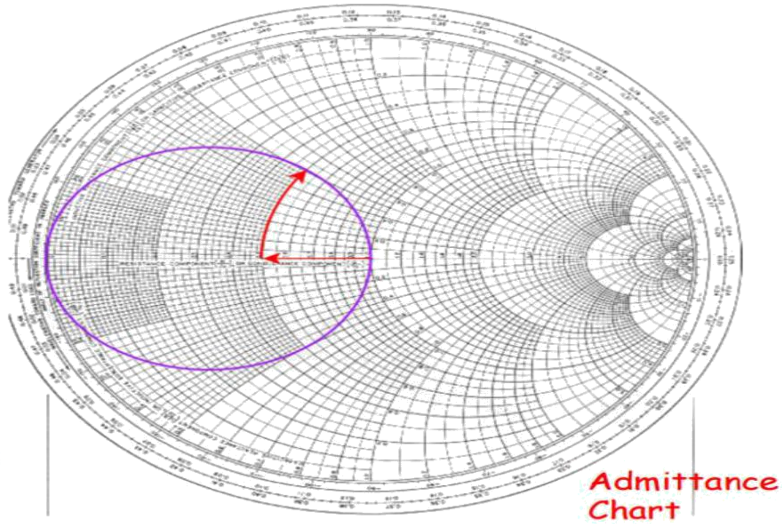
- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance jb = j0.5

$$jb = j0.5$$

$$\frac{j0.5}{50\Omega} = j2\pi(100\text{MHz})C$$

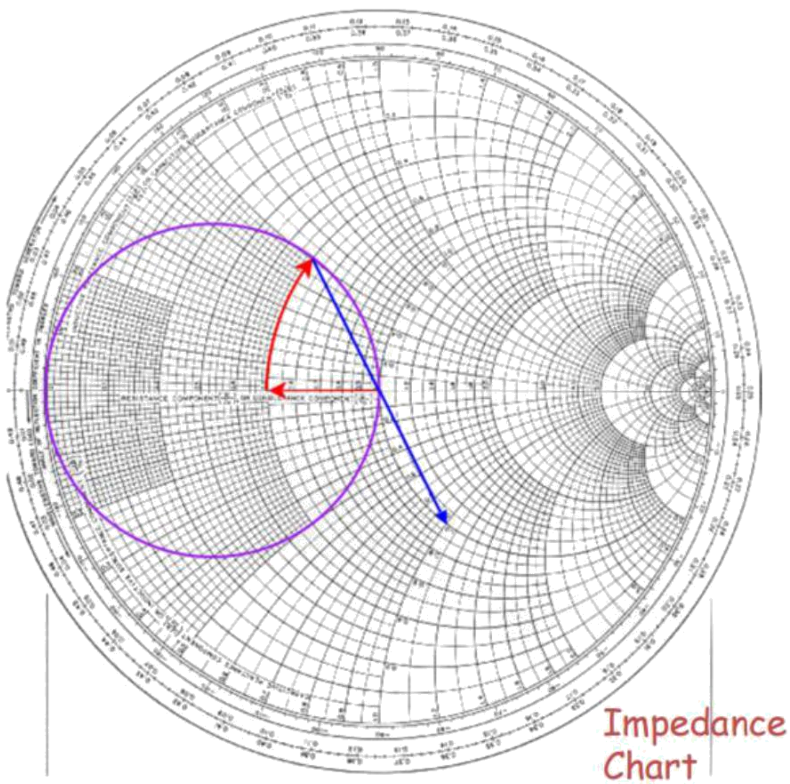
$$C = 16\text{pF}$$





Matching Example

- We will now add series impedance
- Flip to the impedance Smith Chart
- We land at on the r=1 circle at x=-1



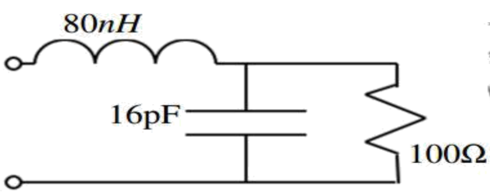
Matching Example

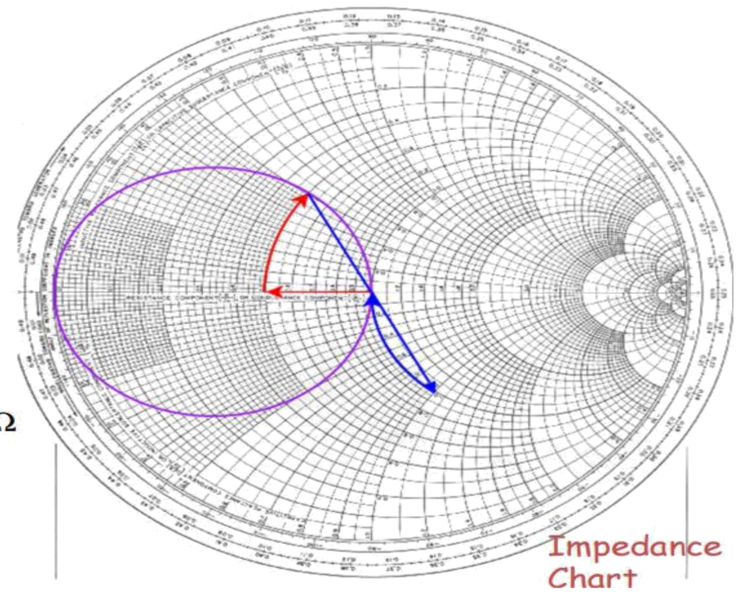
- Add positive imaginary admittance to get to z=1+j0

$$jx = j1.0$$

$$(j1.0)50\Omega = j2\pi(100\text{MHz})L$$

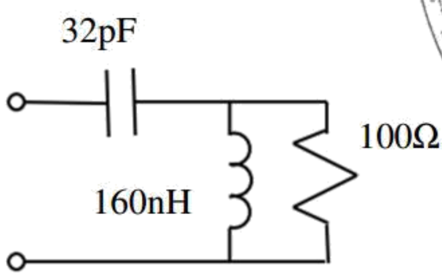
$$L = 80\text{nH}$$

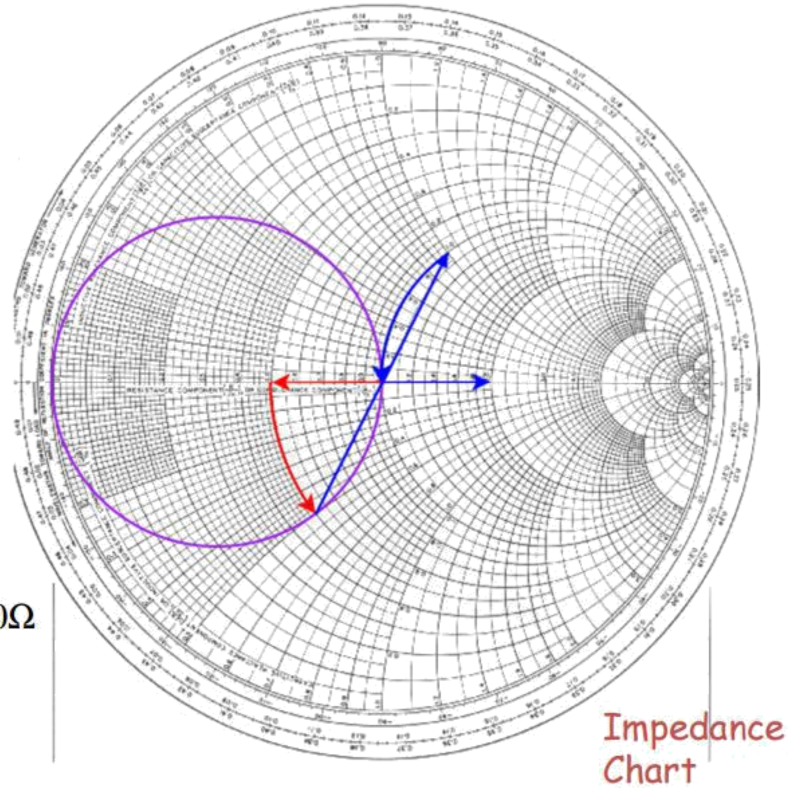




Matching Example

- This solution would have also worked





Smith Chart

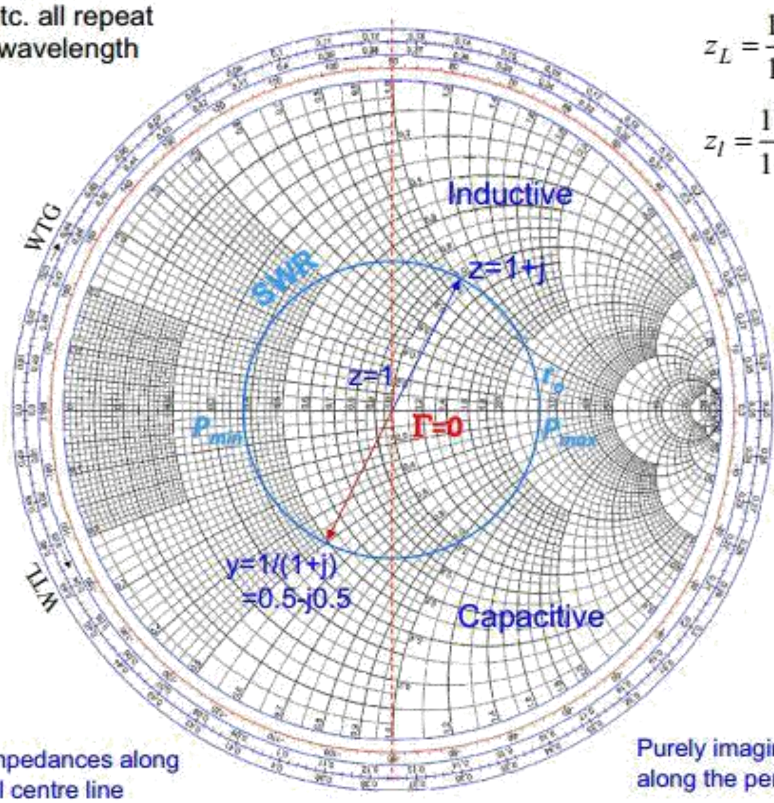
Impedances, voltages, currents, etc. all repeat every half wavelength

$$\Gamma = \Gamma_L = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} = r_L + jx_L$$

$$z_l = \frac{1+\Gamma_l}{1-\Gamma_l} \qquad \Gamma_l = \Gamma e^{-j2\beta l} = \Gamma e^{-j\frac{4\pi}{\lambda}l}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = r_o \, (\text{at} \, P \max)$$

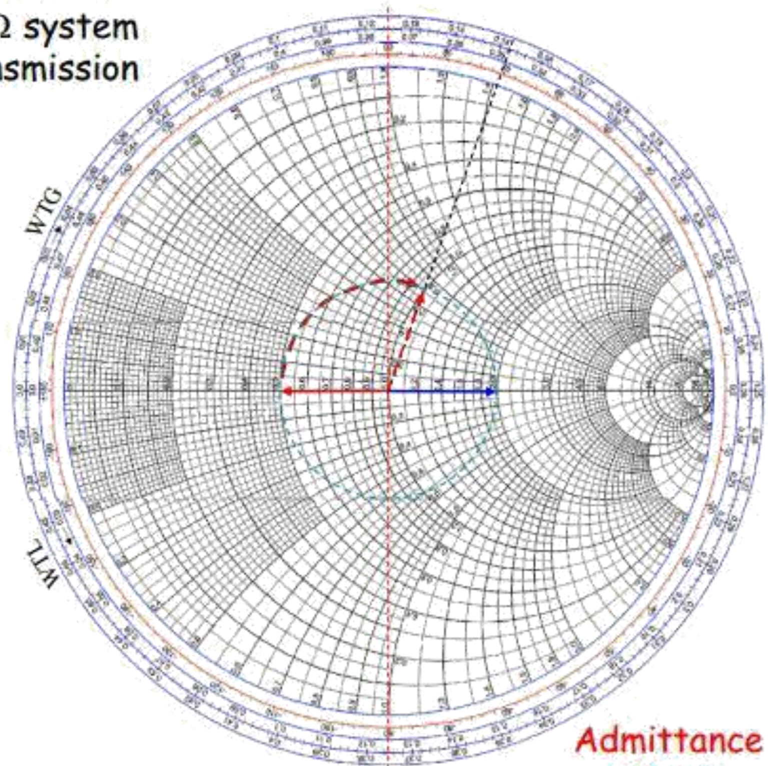


Purely real impedances along the horizontal centre line

Purely imaginary impedances along the periphery

Single Stub Tuner

Match 100Ω load to a 50Ω system at 100MHz using two transmission lines connected in parallel



Admittance Chart

- Flip to Admittance chart
- y=0.5+j0
- Adding length to Cable 1 rotates the reflection coefficient clockwise to g=1.

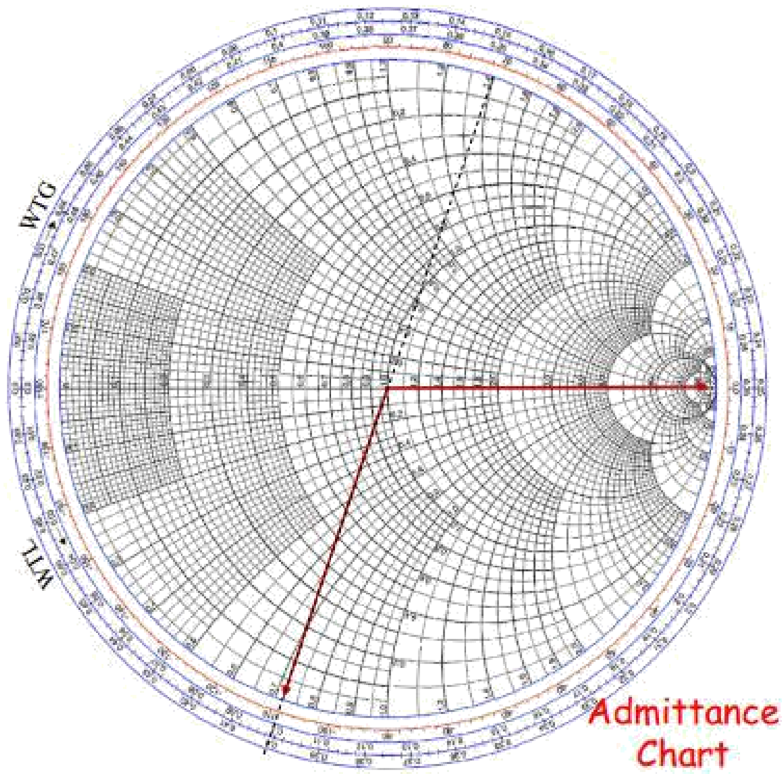
$$l_1 = 0.152\lambda$$

- y_{l1}=1+j0.72

Single Stub Tuner

- An short stub of zero length has an admittance=j∞
- By adding enough cable to the short stub, the admittance of the stub will reach to -0.72

$l_2 = (0.401\text{-}0.25)\lambda = 0.151\lambda$

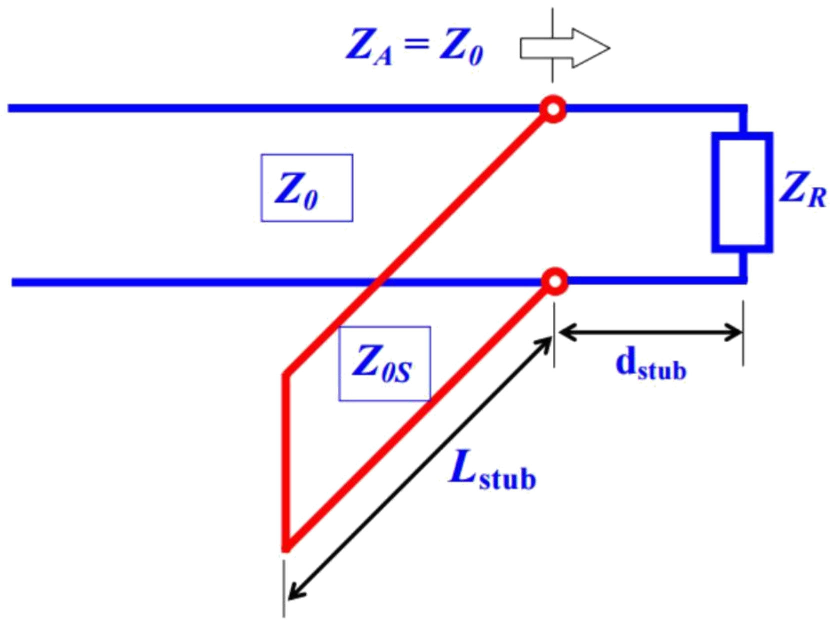


Summary

Impedance matching is necessary to: – reduce VSWR – obtain maximum power transfer • Lump reactive elements and a single stub can be used. • A quarter-wave line can also be used to transform resistance values, and act as an impedance inverter. •
These matching network types are narrow-band: they are designed to operate at a single frequency only.

❑ Single stub impedance matching

Impedance matching can be achieved by inserting another transmission line (stub) as shown in the diagram below



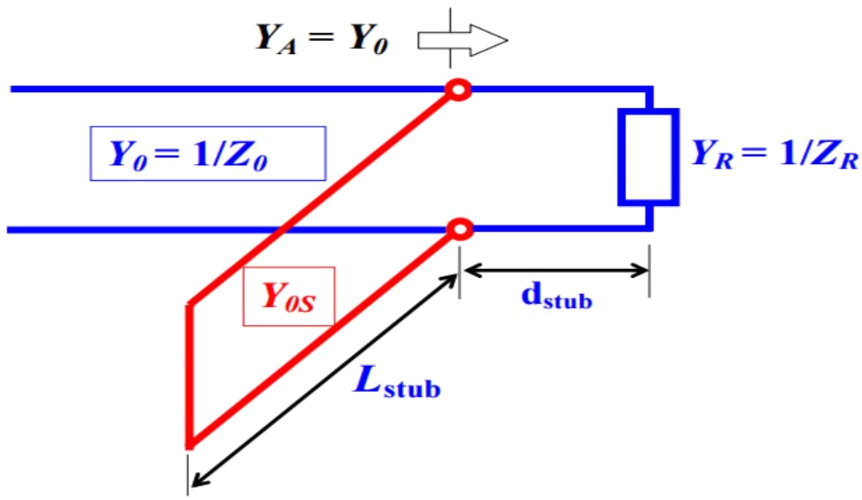
There are two design parameters for single stub matching:

- ❑ **The location of the stub with reference to the load d_{stub}**
- ❑ **The length of the stub line L_{stub}**

Any load impedance can be matched to the line by using single stub technique. The drawback of this approach is that if the load is changed, the location of insertion may have to be moved.

The transmission line realizing the stub is normally terminated by a short or by an open circuit. In many cases it is also convenient to select the same characteristic impedance used for the main line, although this is not necessary. The choice of open or shorted stub may depend in practice on a number of factors. A short circuited stub is less prone to leakage of electromagnetic radiation and is somewhat easier to realize. On the other hand, an open circuited stub may be more practical for certain types of transmission lines, for example microstrips where one would have to drill the insulating substrate to short circuit the two conductors of the line.

Since the circuit is based on insertion of a parallel stub, it is more convenient to work with admittances, rather than impedances.



In order to complete the design, we have to find an appropriate location for the stub. Note that the input admittance of a stub is always imaginary (inductance if negative, or capacitance if positive)

$$Y_{\text{stub}} = jB_{\text{stub}}$$

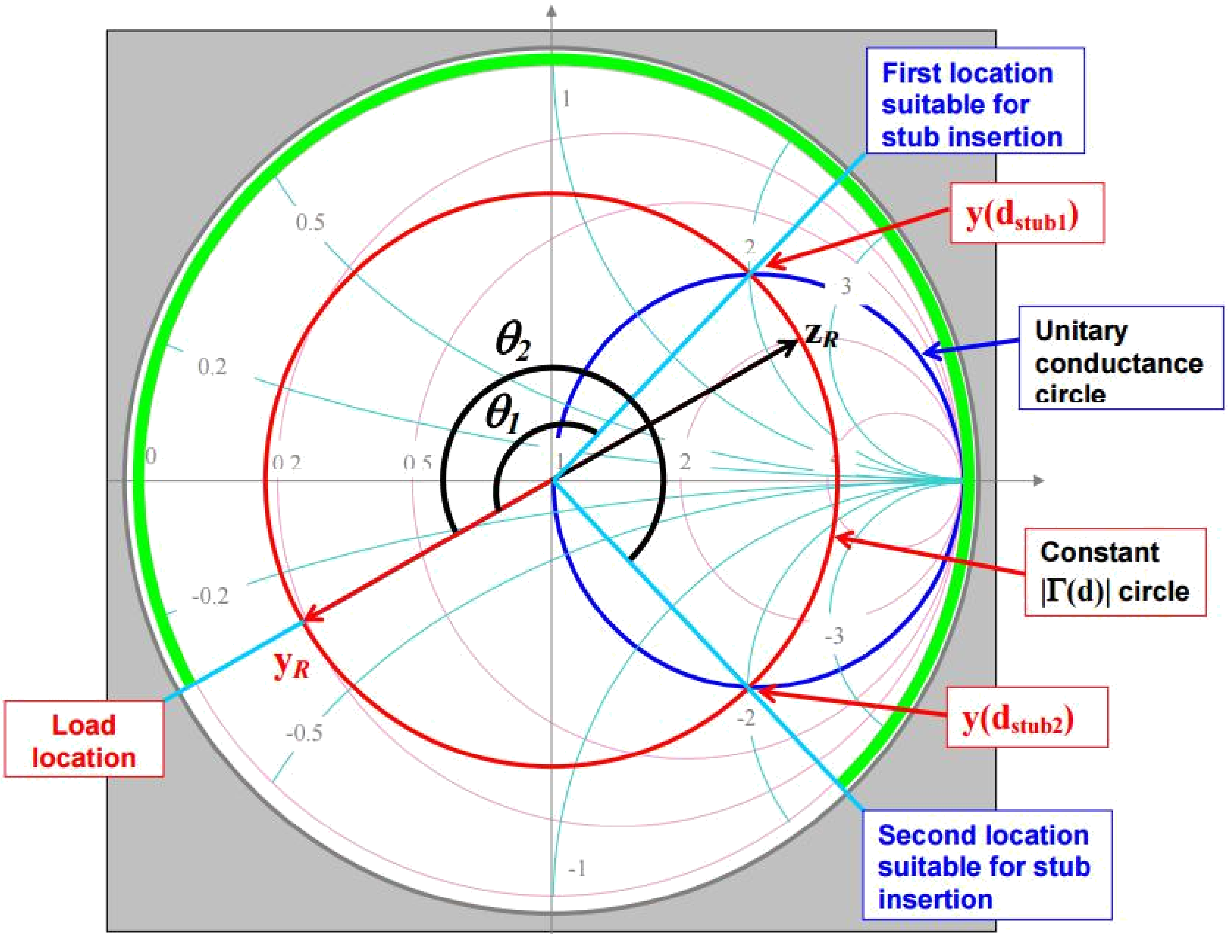
A stub should be placed at a location where the line admittance has real part equal to Y_0

$$Y(\mathrm{d}_{\text{stub}}) = Y_0 + jB(\mathrm{d}_{\text{stub}})$$

For matching, we need to have

$$B_{\text{stub}} = -B(\mathrm{d}_{\text{stub}})$$

Depending on the length of the transmission line, there may be a number of possible locations where a stub can be inserted for impedance matching. It is very convenient to analyze the possible solutions on a Smith chart.


First location suitable for stub insertion
y(d_stub1)
Unitary conductance circle
Constant |Γ(d)| circle
y(d_stub2)
Second location suitable for stub insertion
Load location
θ2
θ1
zR
yR
1
0.5
0.2
0
0.2
0.5
1
2
3
-0.2
-0.5
-1
-2
-3

The red arrow on the example indicates the load admittance. This provides on the "admittance chart" the physical reference for the load location on the transmission line. Notice that in this case the load admittance falls outside the unitary conductance circle. If one moves from load to generator on the line, the corresponding chart location moves from the reference point, in clockwise motion, according to an angle θ (indicated by the light green arc)

$$\theta = 2\beta d = \frac{4\pi}{\lambda} d$$

The value of the admittance rides on the red circle which corresponds to constant magnitude of the line reflection coefficient, $|\Gamma(d)| = |\Gamma_R|$, imposed by the load.

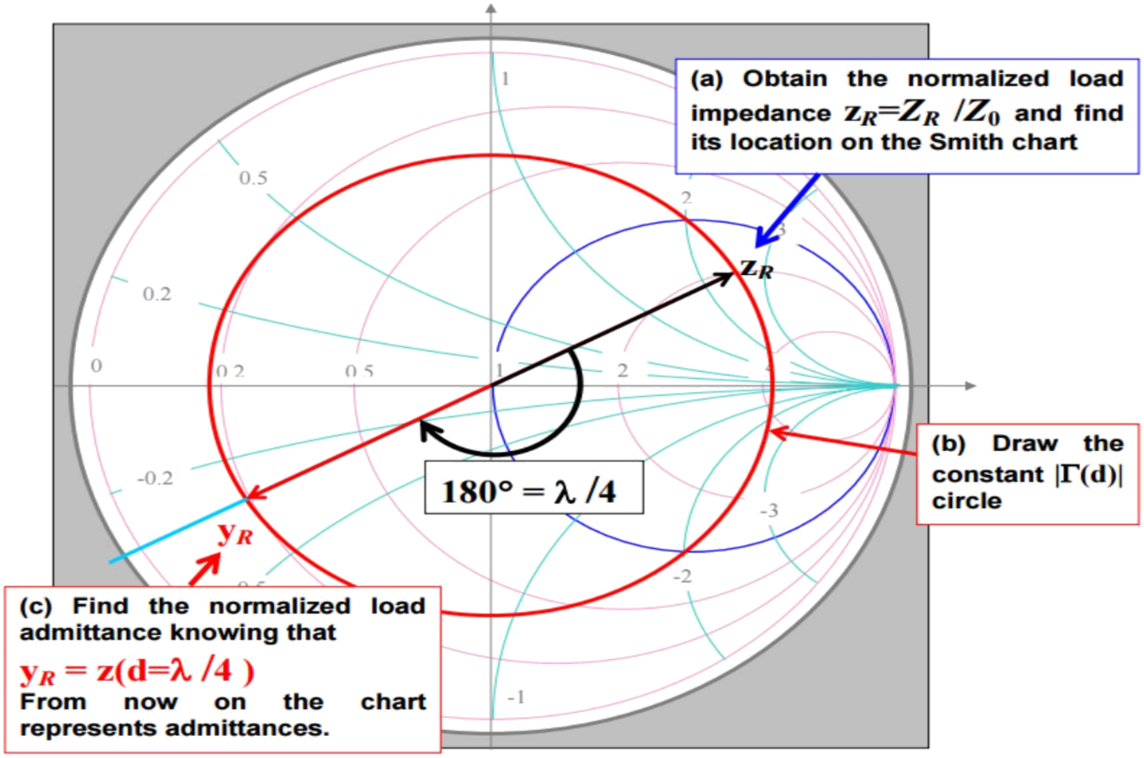
Every circle of constant $|\Gamma(d)|$ intersects the circle $\mathrm{Re}\{y\} = 1$ (unitary normalized conductance), in correspondence of two points. Within the first revolution, the two intersections provide the locations closest to the load for possible stub insertion.

Single stub matching problems can be solved on the Smith chart graphically, using a compass and a ruler. This is a step-by-step summary of the procedure:

(a) Find the normalized load impedance and determine the corresponding location on the chart.

(b) Draw the circle of constant magnitude of the reflection coefficient $|\Gamma|$ for the given load.

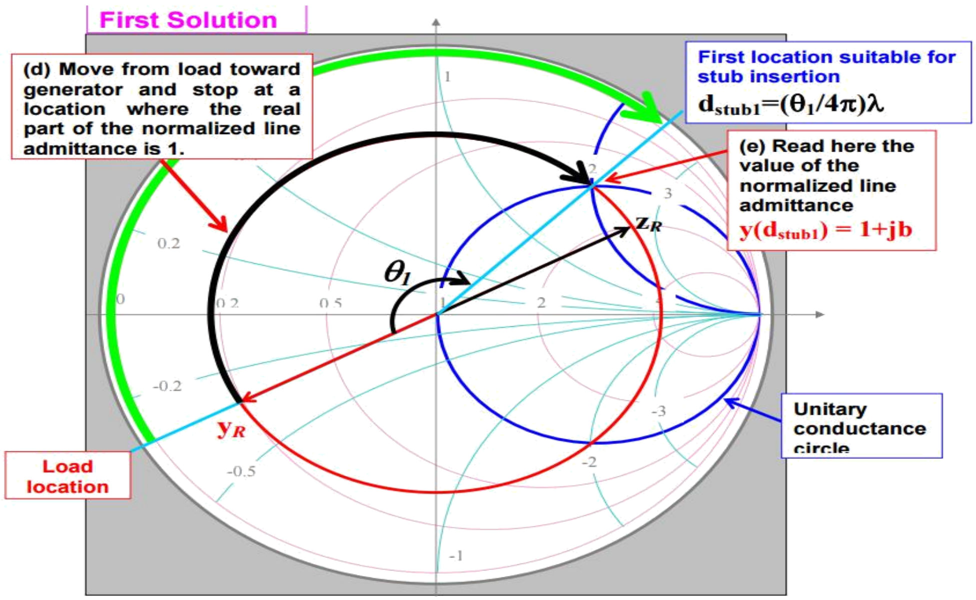
(c) Determine the normalized load admittance on the chart. This is obtained by rotating 180° on the constant $|\Gamma|$ circle, from the load impedance point. From now on, all values read on the chart are normalized admittances.



(d) Move from load admittance toward generator by riding on the constant $|\Gamma|$ circle, until the intersections with the unitary normalized conductance circle are found. These intersections correspond to possible locations for stub insertion. Commercial Smith charts provide graduations to determine the angles of rotation as well as the distances from the load in units of wavelength.

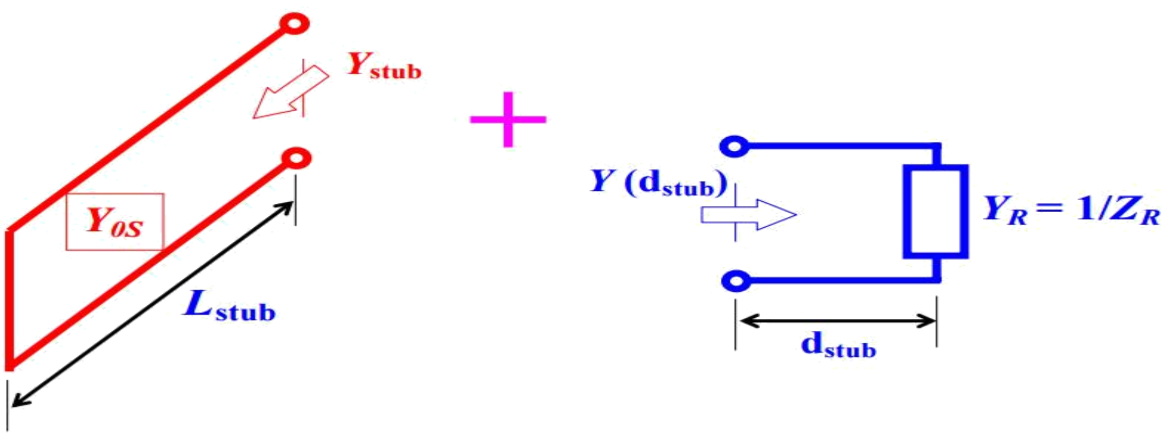
(e) Read the line normalized admittance in correspondence of the stub insertion locations determined in (d). These values will always be of the form

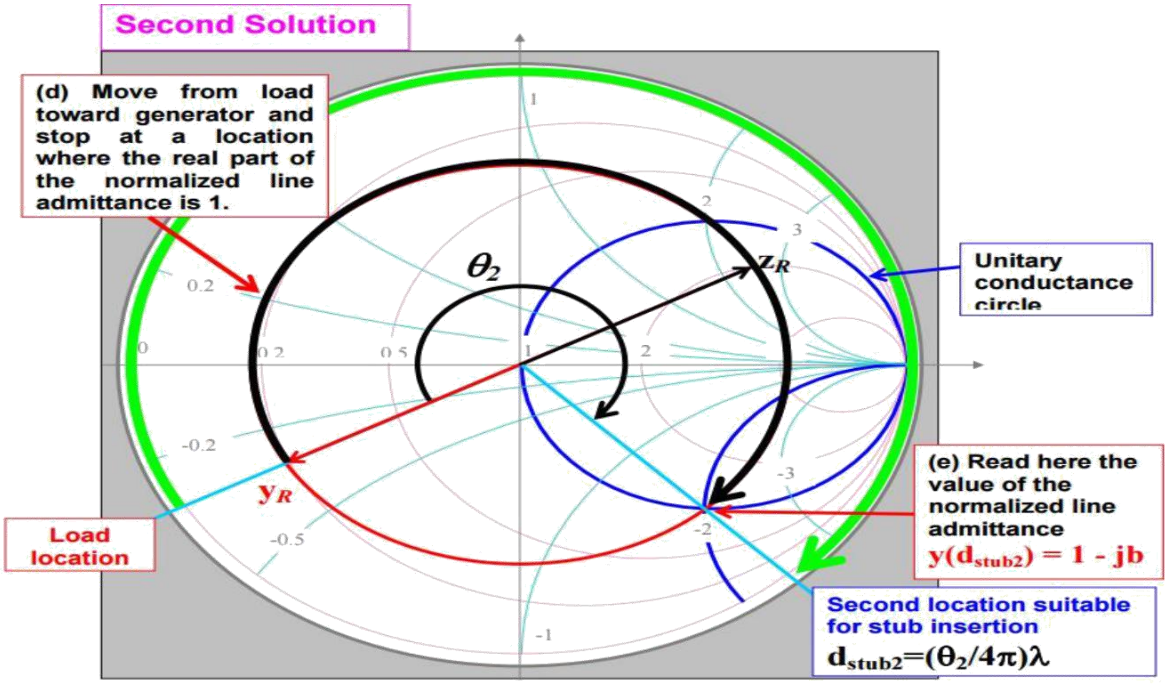
$$y(\mathrm{d_{stub}}) = 1 + jb \qquad \text{top half of chart}$$
$$y(\mathrm{d_{stub}}) = 1 - jb \qquad \text{bottom half of chart}$$



For proper impedance match:

$$Y_A = \underbrace{Y_{\text{stub}}}_{\text{Input admittance of the stub line}} + \underbrace{Y(\mathrm{d}_{\text{stub}})}_{\text{Line admittance at location } \mathrm{d}_{\text{stub}} \text{ before the stub is applied}} = Y_0 = \frac{1}{Z_0}$$





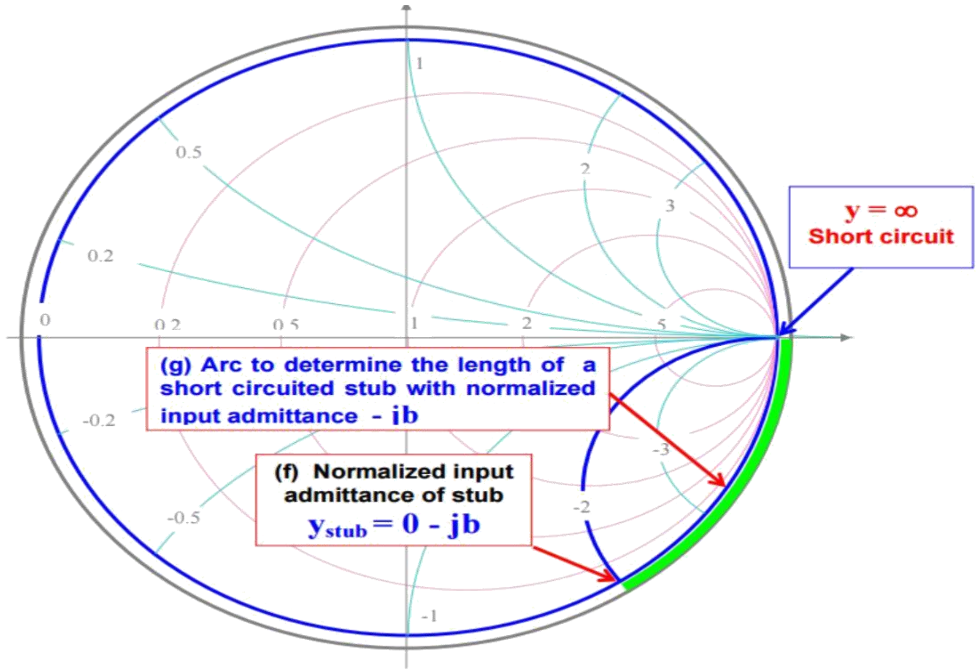
(f) **Select the input normalized admittance of the stubs, by taking the opposite of the corresponding imaginary part of the line admittance**

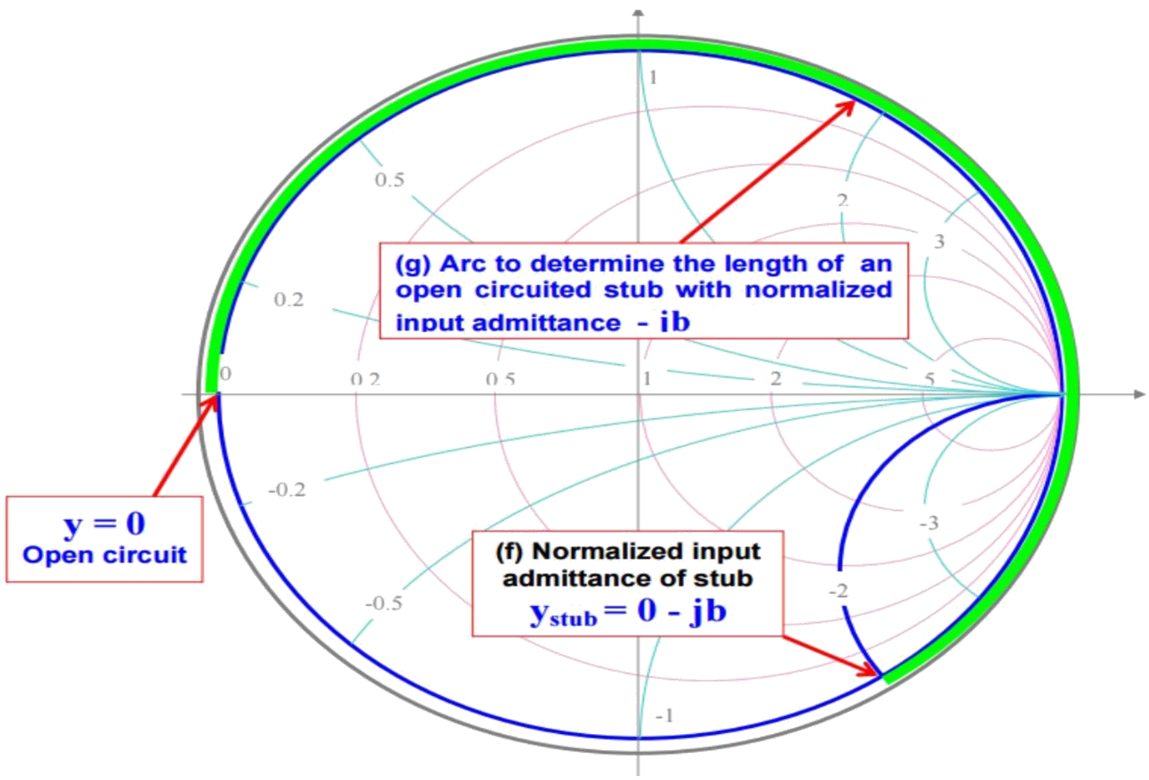
$$\text{line: } y(\mathrm{d_{stub}}) = 1 + jb \quad \rightarrow \quad \text{stub: } y_{\text{stub}} = -jb$$

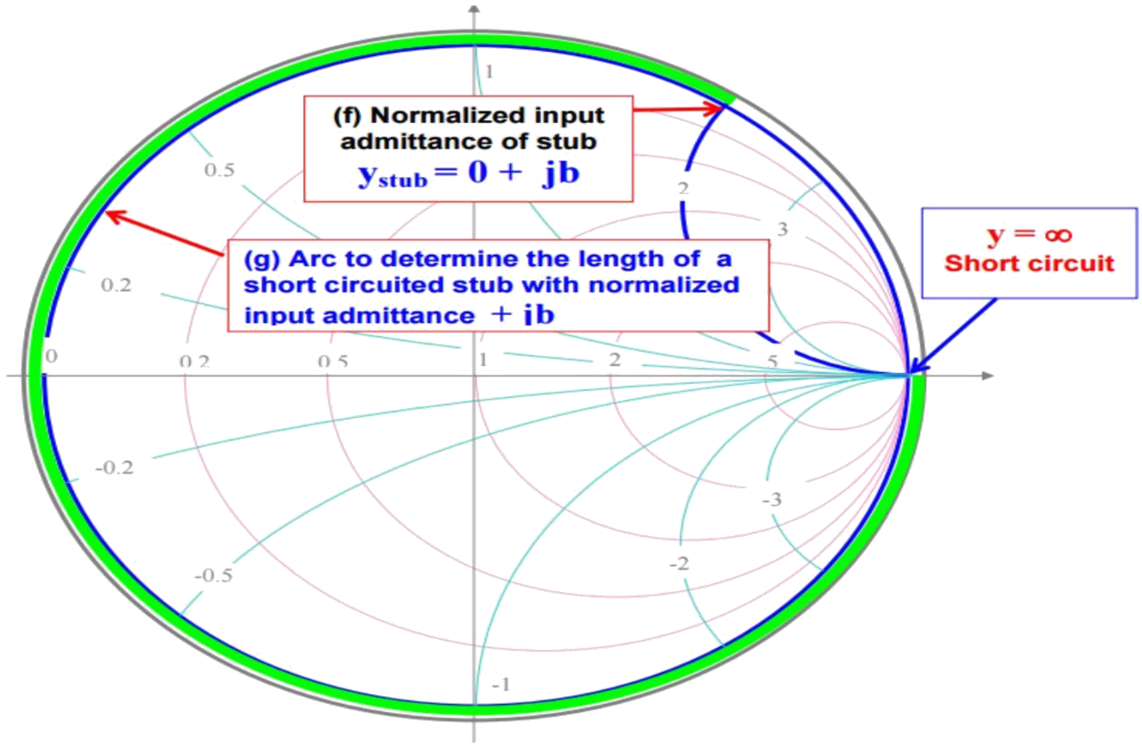
$$\text{line: } y(\mathrm{d_{stub}}) = 1 - jb \quad \rightarrow \quad \text{stub: } y_{\text{stub}} = +jb$$

(g) **Use the chart to determine the length of the stub. The imaginary normalized admittance values are found on the circle of zero conductance on the chart. On a commercial Smith chart one can use a printed scale to read the stub length in terms of wavelength. We assume here that the stub line has characteristic impedance Z_0 as the main line. If the stub has characteristic impedance $Z_{0S} \neq Z_0$ the values on the Smith chart must be renormalized as**

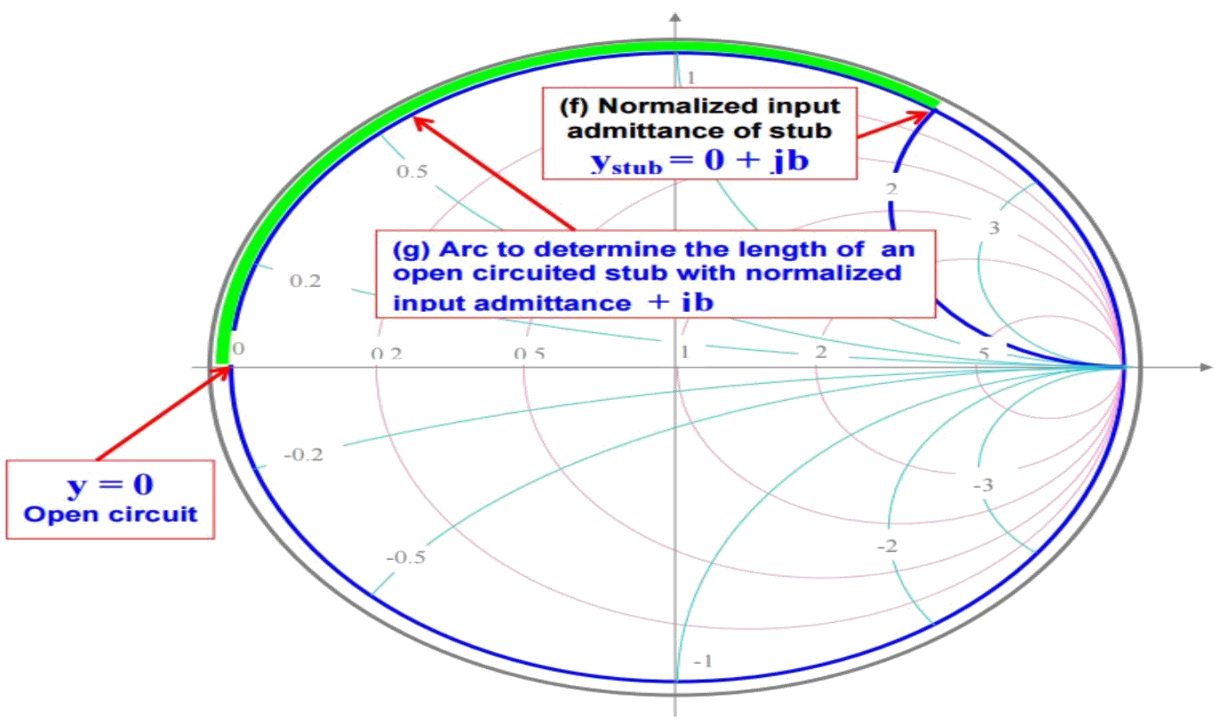
$$\pm jb' = \pm jb \frac{Y_0}{Y_{0s}} = \pm jb \frac{Z_{0s}}{Z_0}$$


1
0.5
2
3
0.2
0
0.2
0.5
1
2
5
y = ∞
Short circuit
(g) Arc to determine the length of a short circuited stub with normalized input admittance - jb
-0.2
-3
(f) Normalized input admittance of stub
ystub = 0 - jb
-0.5
-2
-1

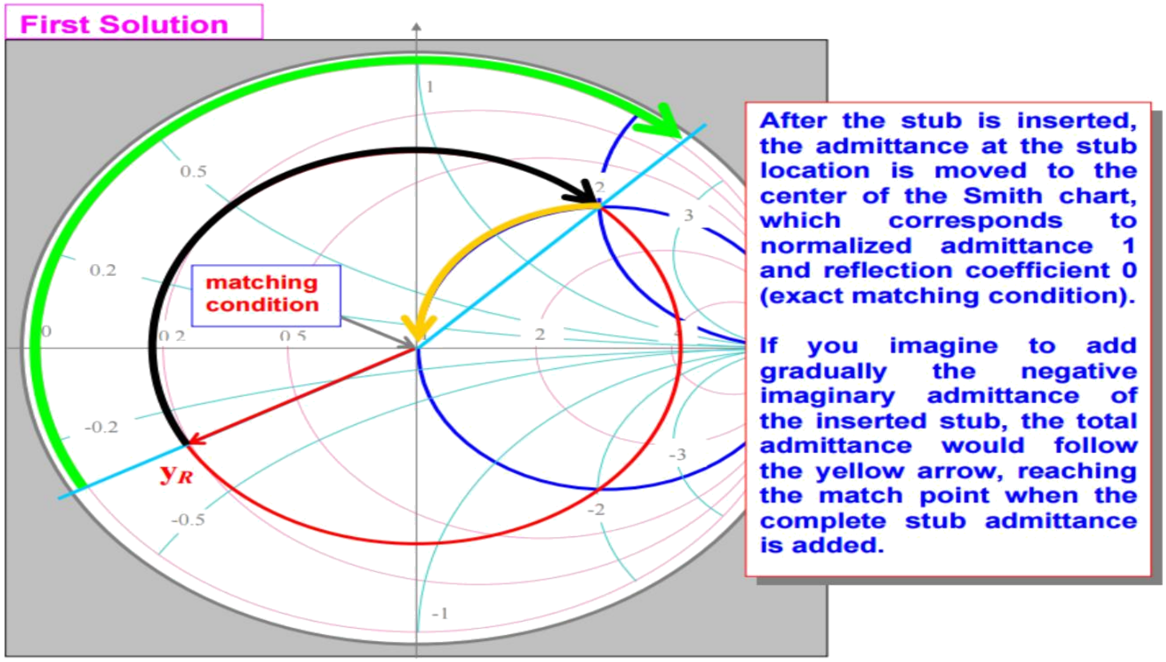

1
0.5
2
3
(g) Arc to determine the length of an open circuited stub with normalized input admittance - jb
0.2
0
0.2
0.5
1
2
5
-0.2
y = 0
Open circuit
-3
(f) Normalized input admittance of stub
ystub = 0 - jb
-0.5
-2
-1

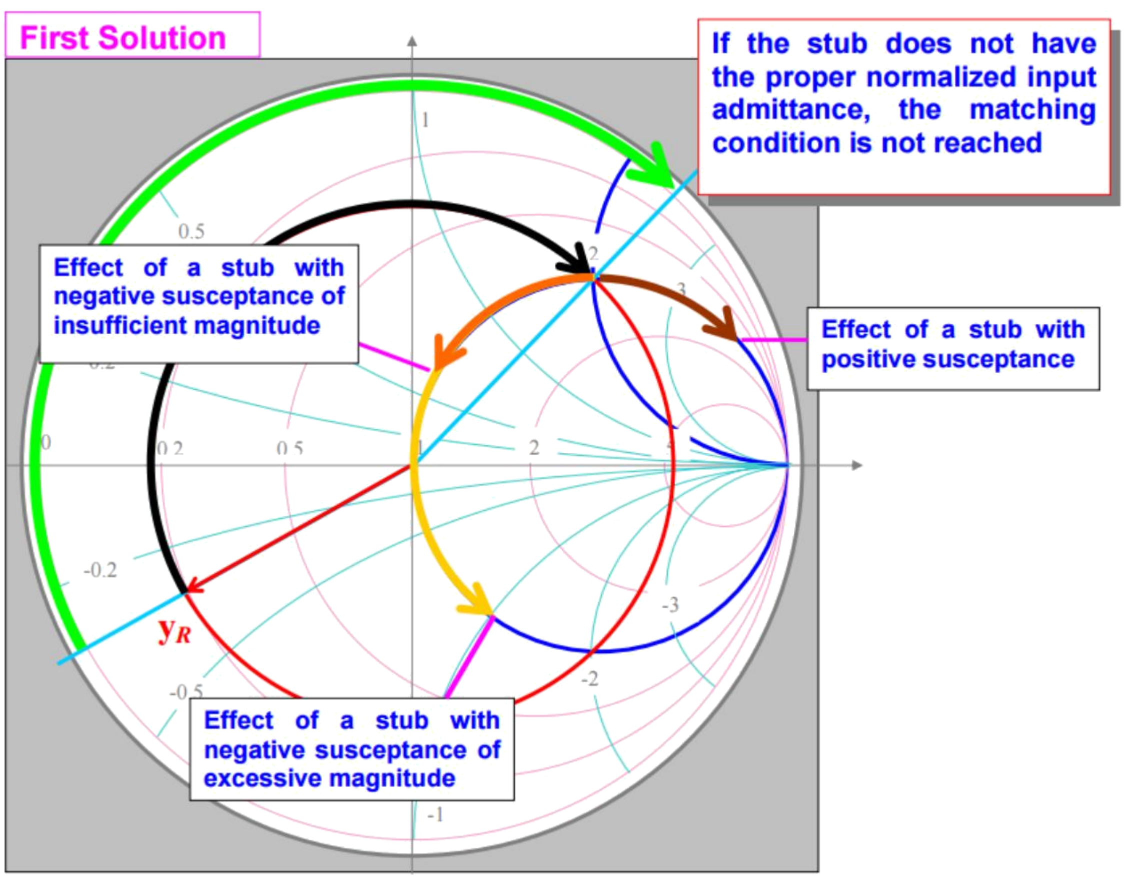


(f) Normalized input admittance of stub
$y_{stub} = 0 + jb$
(g) Arc to determine the length of a short circuited stub with normalized input admittance $+jb$
$y = \infty$
Short circuit



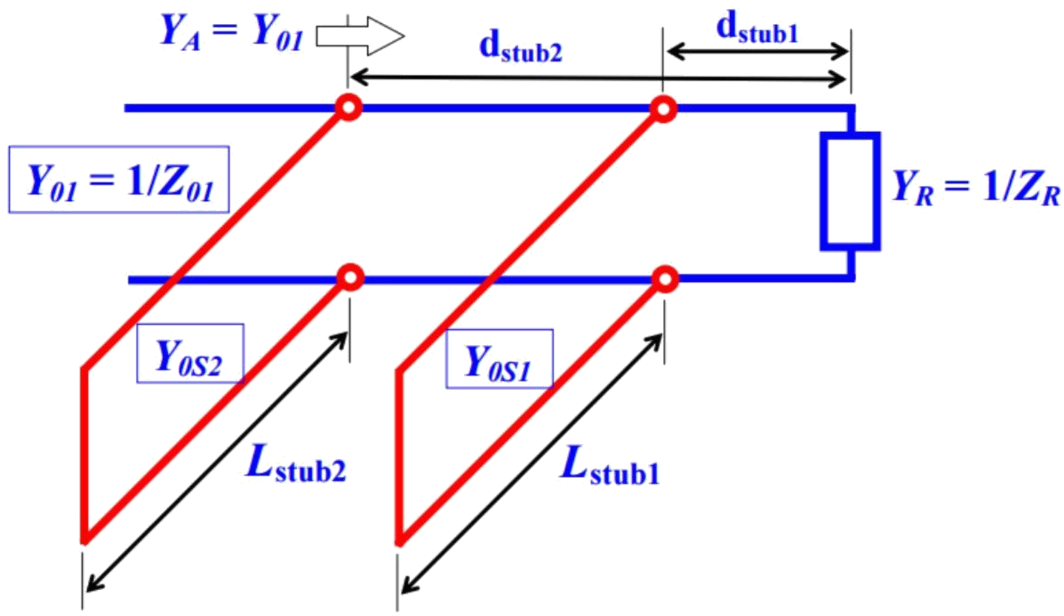
(f) Normalized input admittance of stub
$y_{stub} = 0 + jb$
(g) Arc to determine the length of an open circuited stub with normalized input admittance $+jb$
$y = 0$
Open circuit


First Solution
matching
condition
yR
After the stub is inserted, the admittance at the stub location is moved to the center of the Smith chart, which corresponds to normalized admittance 1 and reflection coefficient 0 (exact matching condition).
If you imagine to add gradually the negative imaginary admittance of the inserted stub, the total admittance would follow the yellow arrow, reaching the match point when the complete stub admittance is added.


First Solution
If the stub does not have the proper normalized input admittance, the matching condition is not reached
Effect of a stub with negative susceptance of insufficient magnitude
Effect of a stub with positive susceptance
yR
Effect of a stub with negative susceptance of excessive magnitude

❑ Double stub impedance matching

Impedance matching can be achieved by inserting two stubs at specified locations along transmission line as shown below



There are two design parameters for double stub matching:

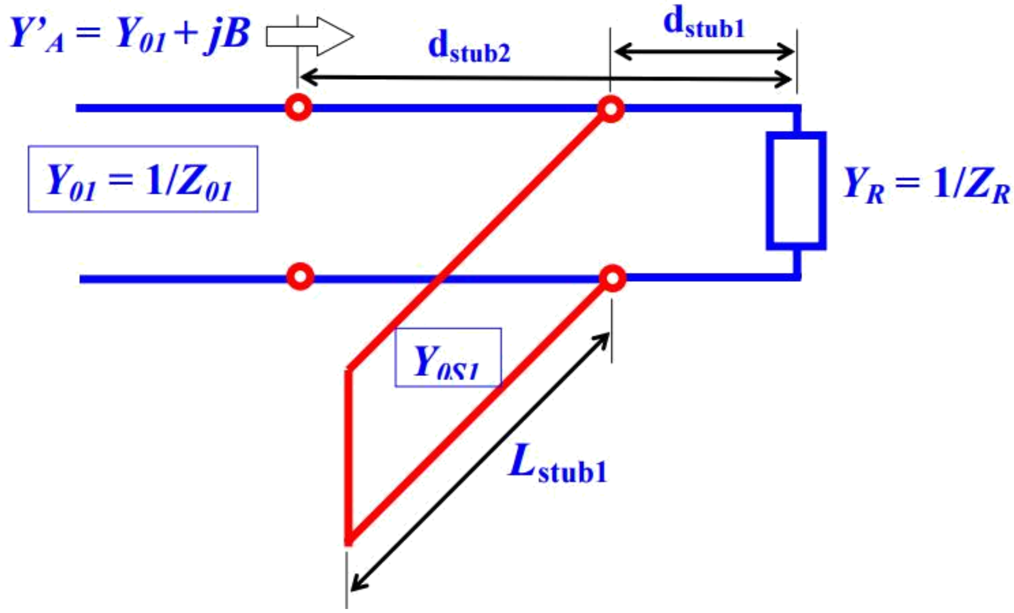
- ❑ **The length of the first stub line L_{stub1}**
- ❑ **The length of the second stub line L_{stub2}**

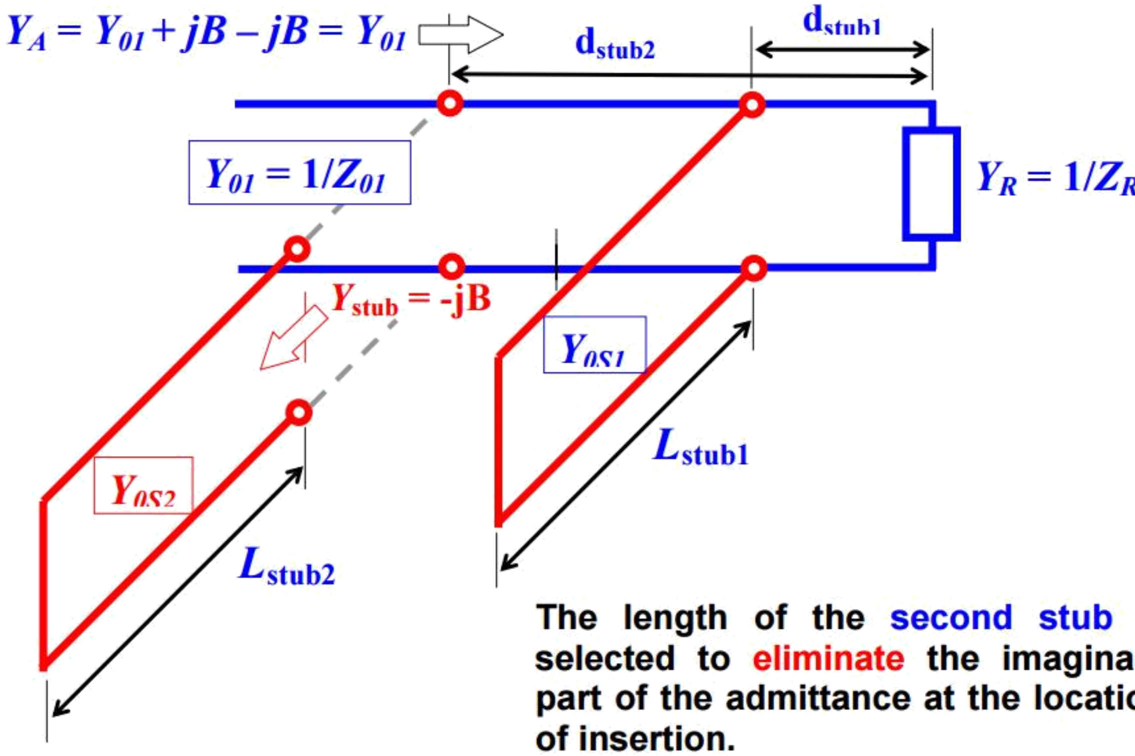
In the double stub configuration, the stubs are inserted at pre-determined locations. In this way, if the load impedance is changed, one simply has to replace the stubs with another set of different length.

The drawback of double stub tuning is that a certain range of load admittances cannot be matched once the stub locations are fixed.

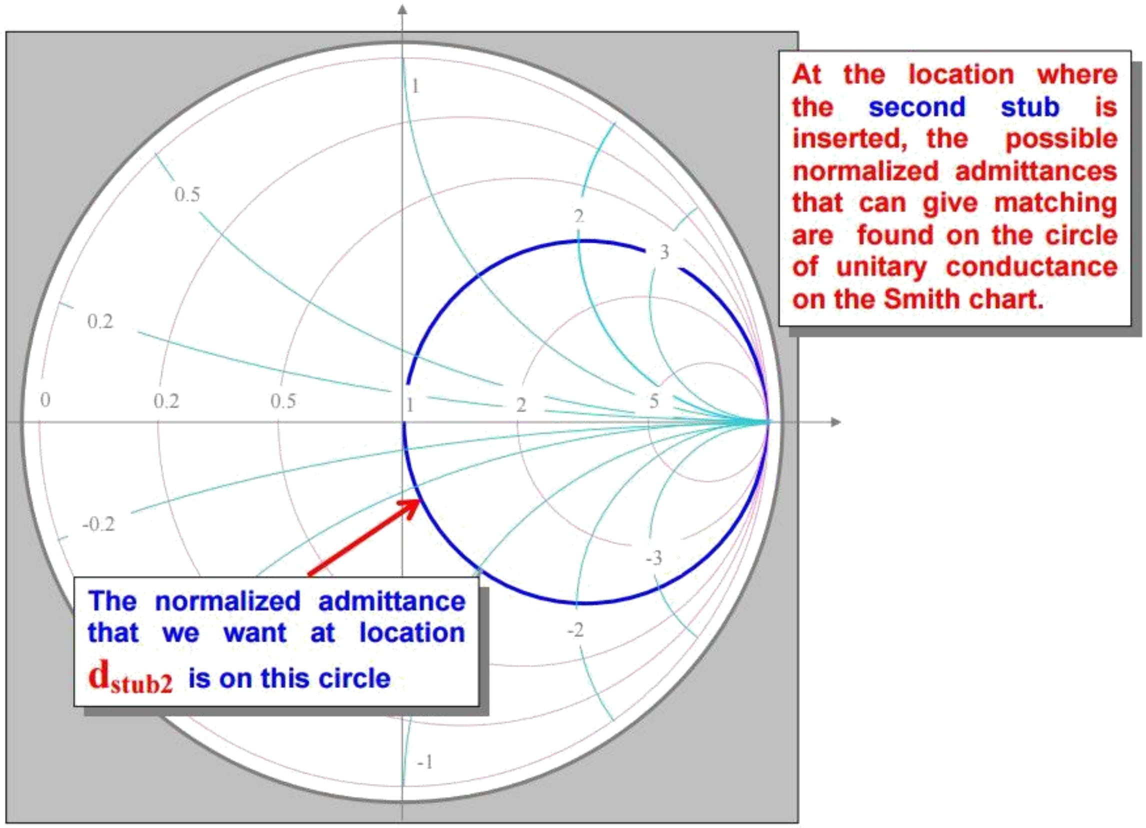
Three stubs are necessary to guarantee that match is always possible.

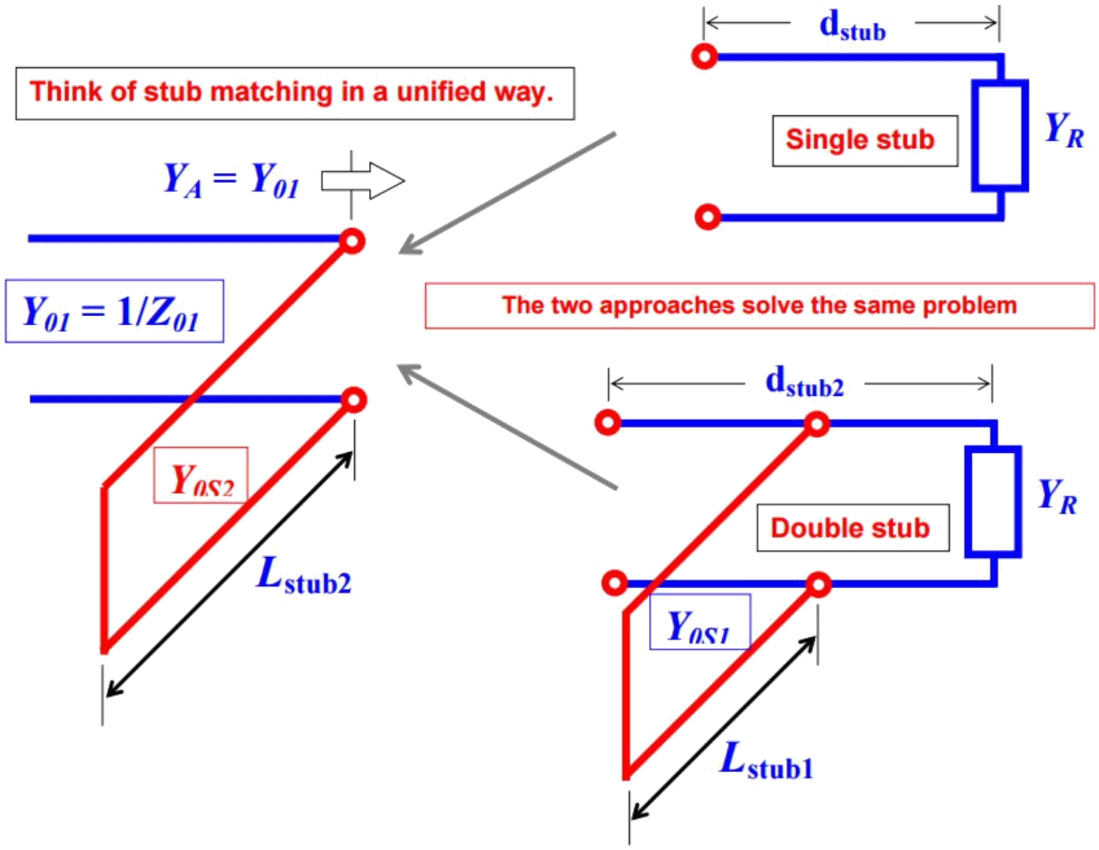
The length of the first stub is selected so that the admittance at the location of the second stub (before the second stub is inserted) has real part equal to the characteristic admittance of the line





The length of the second stub is selected to eliminate the imaginary part of the admittance at the location of insertion.


At the location where the second stub is inserted, the possible normalized admittances that can give matching are found on the circle of unitary conductance on the Smith chart.
The normalized admittance that we want at location d_{stub2} is on this circle
0
0.2
0.5
1
2
5
0.2
0.5
1
2
3
-0.2
-1
-2
-3



Given the load impedance, we need to follow these steps to complete the double stub design:

(a) Find the normalized load impedance and determine the corresponding location on the chart.

(b) Draw the circle of constant magnitude of the reflection coefficient $|\Gamma|$ for the given load.

(c) Determine the normalized load admittance on the chart. This is obtained by rotating -180° on the constant $|\Gamma|$ circle, from the load impedance point. From now on, all values read on the chart are normalized admittances.

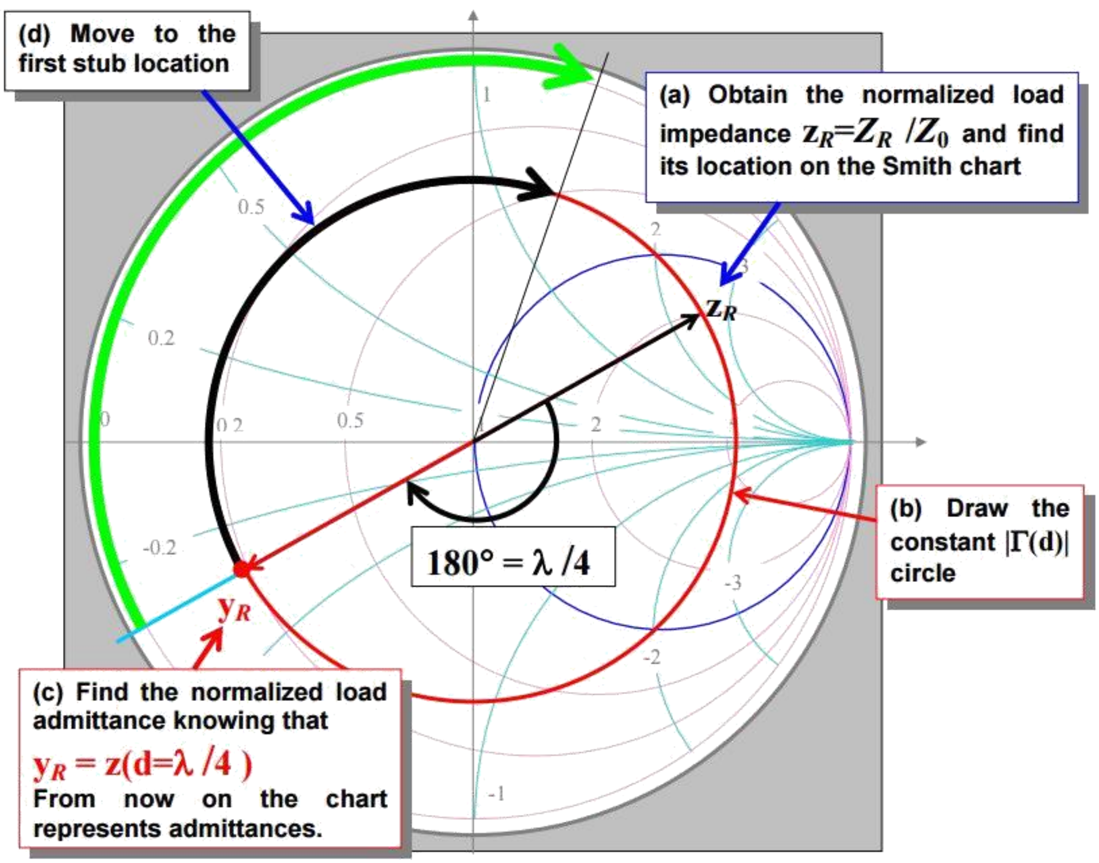
(d) Find the normalized admittance at location d_{stub1} by moving clockwise on the constant $|\Gamma|$ circle.

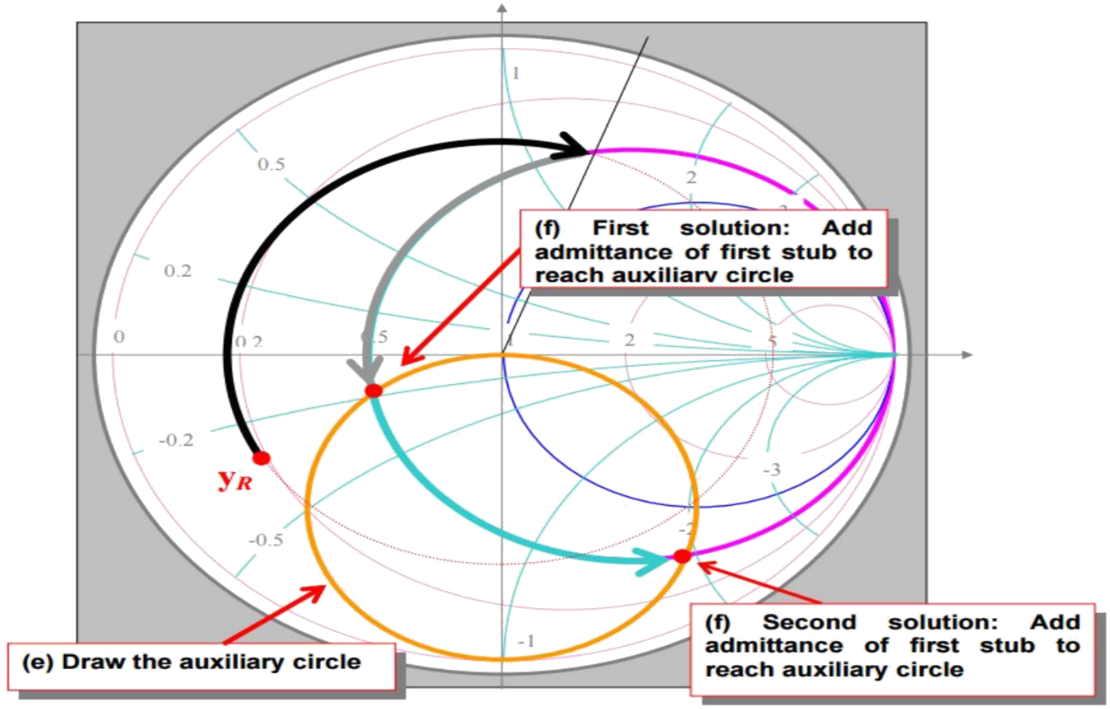
(e) Draw the auxiliary circle

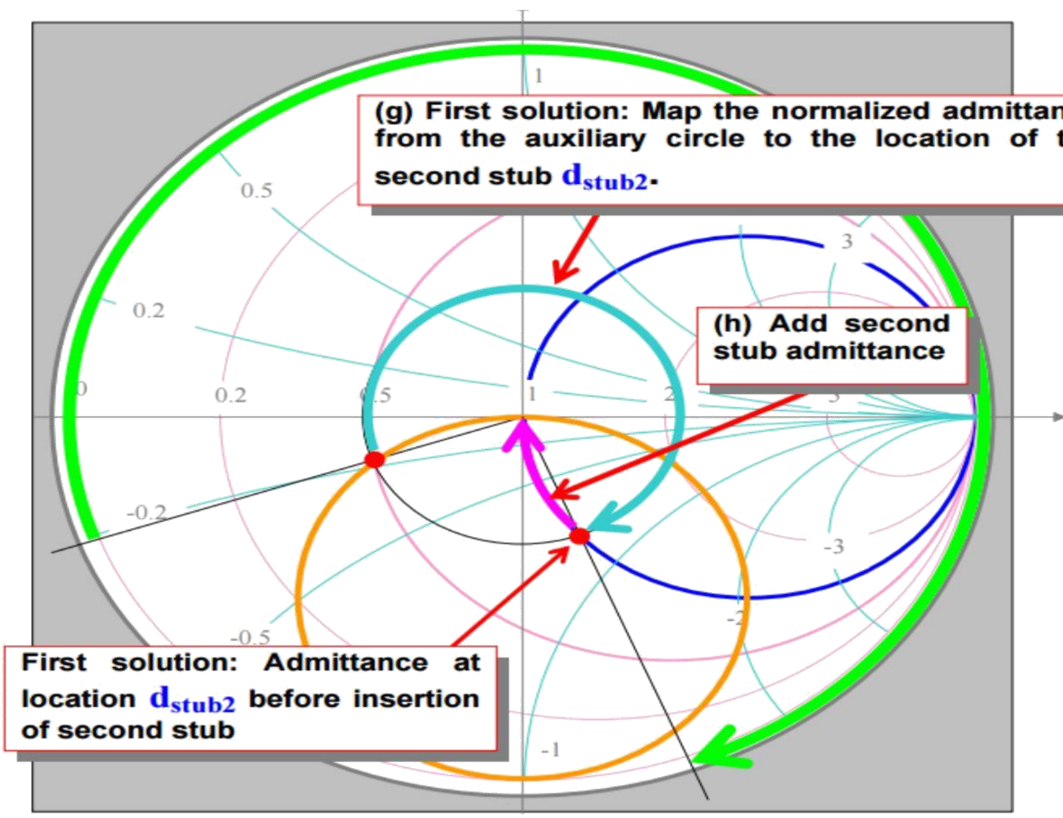
(f) Add the first stub admittance so that the normalized admittance point on the Smith chart reaches the auxiliary circle (two possible solutions). The admittance point will move on the corresponding conductance circle, since the stub does not alter the real part of the admittance

(g) Map the normalized admittance obtained on the auxiliary circle to the location of the second stub d_{stub2}. The point must be on the unitary conductance circle

(h) Add the second stub admittance so that the total parallel admittance equals the characteristic admittance of the line to achieve exact matching condition




(f) First solution: Add admittance of first stub to reach auxiliary circle
(f) Second solution: Add admittance of first stub to reach auxiliary circle
(e) Draw the auxiliary circle
y_R


(g) First solution: Map the normalized admittance from the auxiliary circle to the location of the second stub d_{stub2}.
(h) Add second stub admittance
First solution: Admittance at location d_{stub2} before insertion of second stub

1.

Smith Chart

Find

1) Reflection coefficient at load

$$z_R = 0.3 - j0.4 \Rightarrow \Gamma_R = 0.6e^{j227°}$$

2) SWR on the line

SWR=4.0

3) d_{min}

$$d_{min} = (0.5 - 0.435)\lambda = 0.065\lambda$$

4) Line impedance at 0.05λ to the left

$$50(0.26 - j0.09) = 13 - j4.5\Omega$$

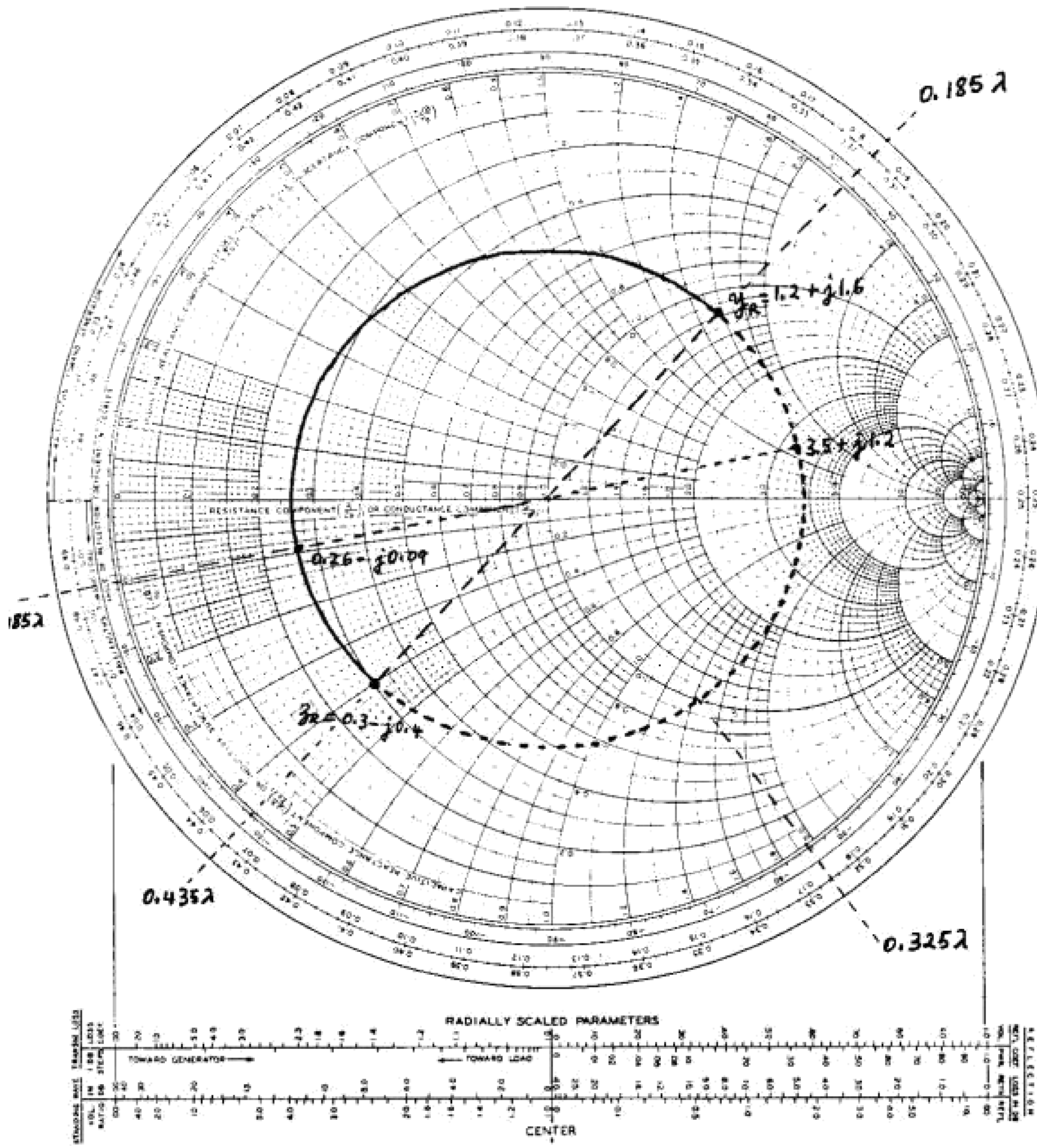
5) Line admittance at 0.05λ

$$(3.5 + j1.2)/50 = 0.068 + j0.025\ \mho$$

6) Location nearest to load where Real[y]=1

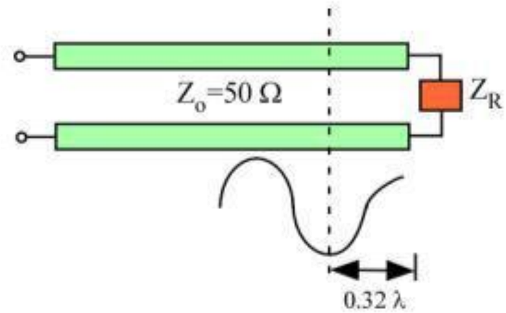
$$0.14\lambda = 0.325\lambda - j0.185\lambda = 0.14\lambda$$

IMPEDANCE OR ADMITTANCE COORDINATES



2.

VSW minimum occurs at 0.30λ from the termination of a lossless 50-Ω line. Find angle of reflection coefficient

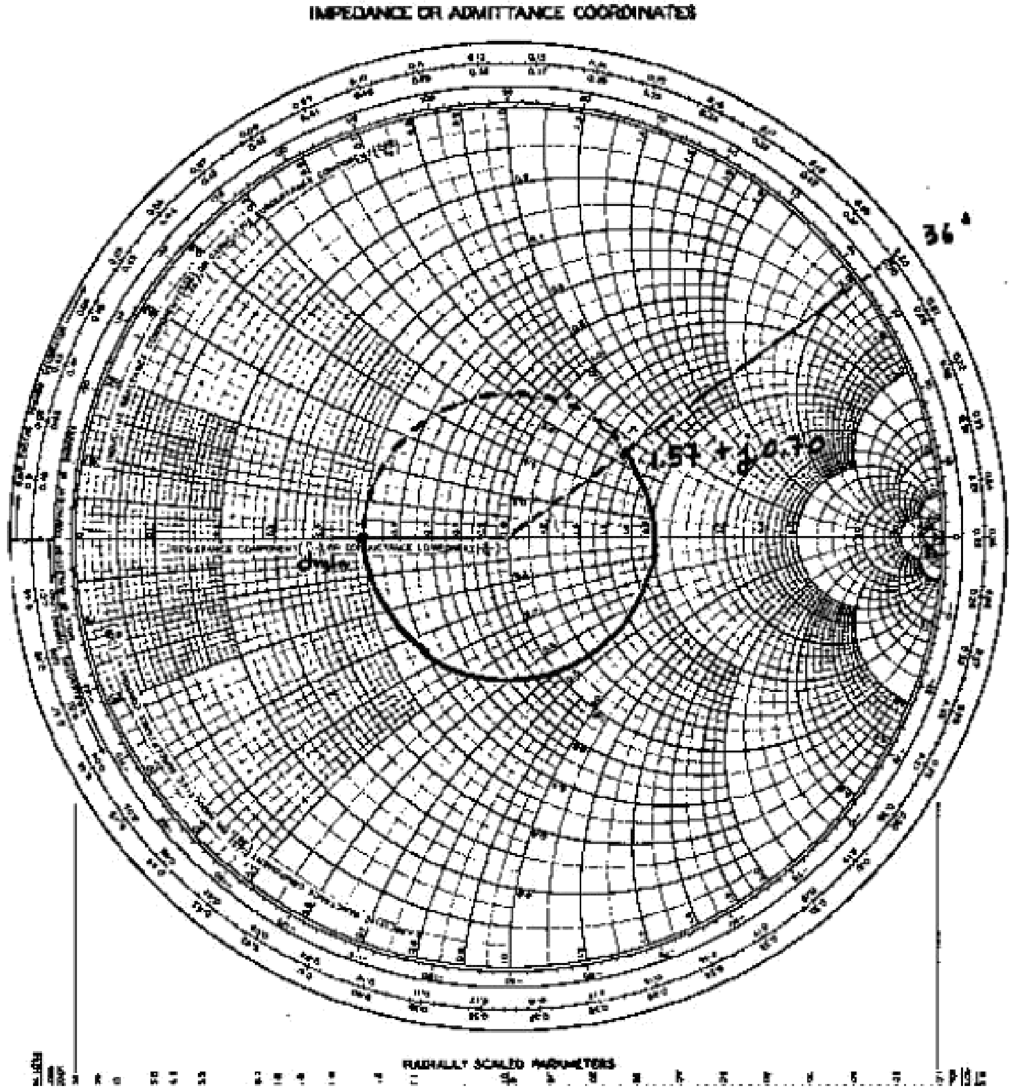


Answer: 36°

If the VSWR is 2.0 what is z_R?

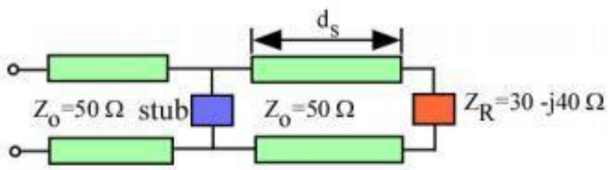
Answer: z_R =1.57+j0.7

Problem 2


IMPEDANCE OR ADMITTANCE COORDINATES
36°
1.57 + j0.70
RESISTANCE COMPONENT
RADIALLY SCALED PARAMETERS

3.

Single Stub



Find location and length of stub

$z_R = Z_R/50 = 0.6\text{-}j0.8$

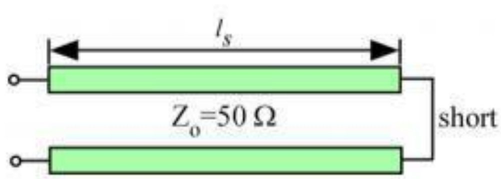
$y_R = 0.6+j0.8$

First Method

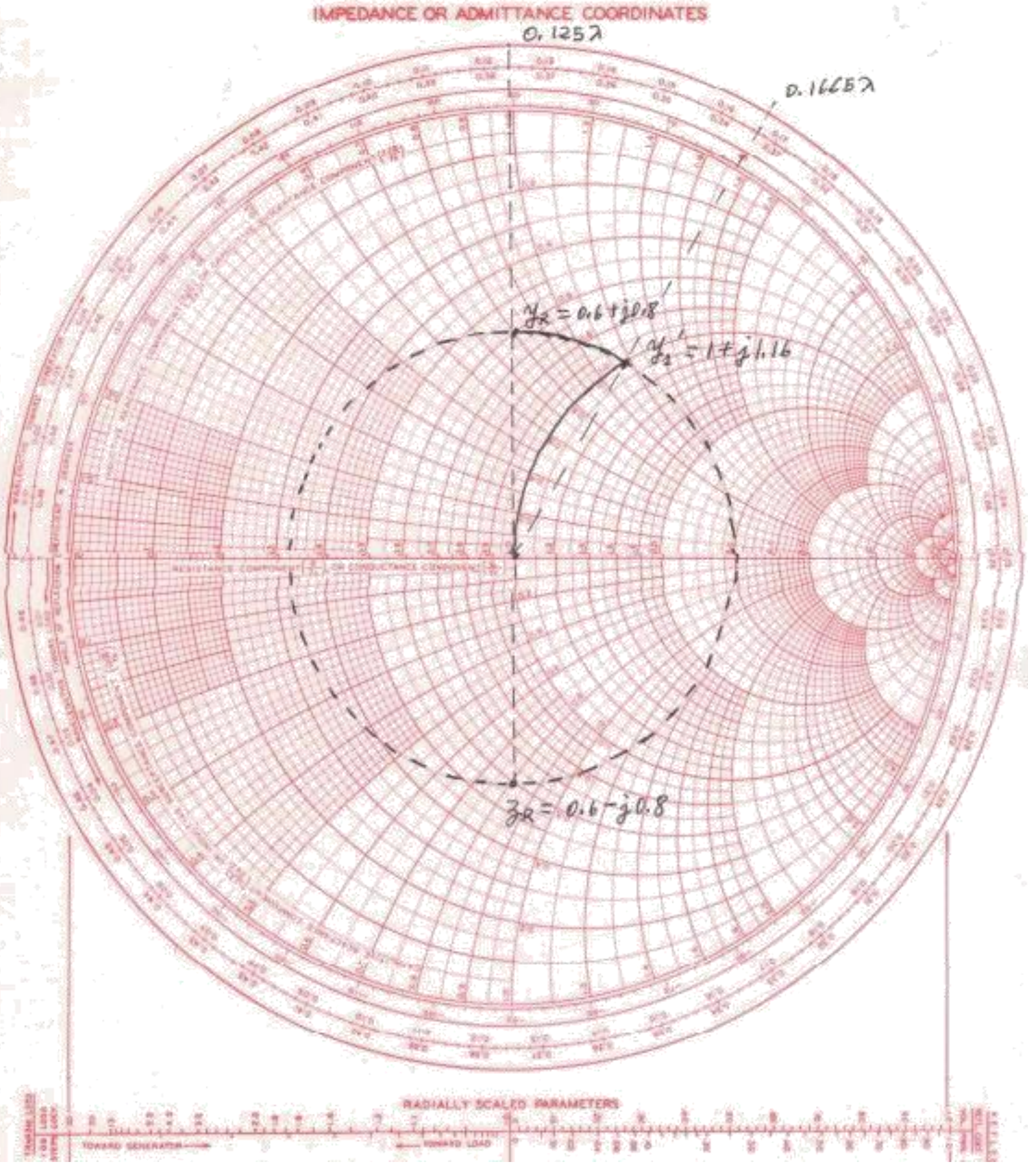
Rotate on constant VSWR circle from 0.125λ to 0.1665λ until intersection with unit conductance circle at $y_1' = 1 + j1.16$. Distance $d_s = (0.1665\lambda - 0.125\lambda) = 0.0415\lambda$ Move toward center of chart. Change in susceptance: j(0-0.16)=-j1.16

The length of the stub is such that

$$\frac{1}{tan\beta l_s} = -1.16 \text{ or } l_s = (0.363 - 0.25)\lambda = 0.113\lambda$$

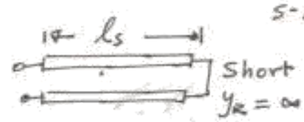


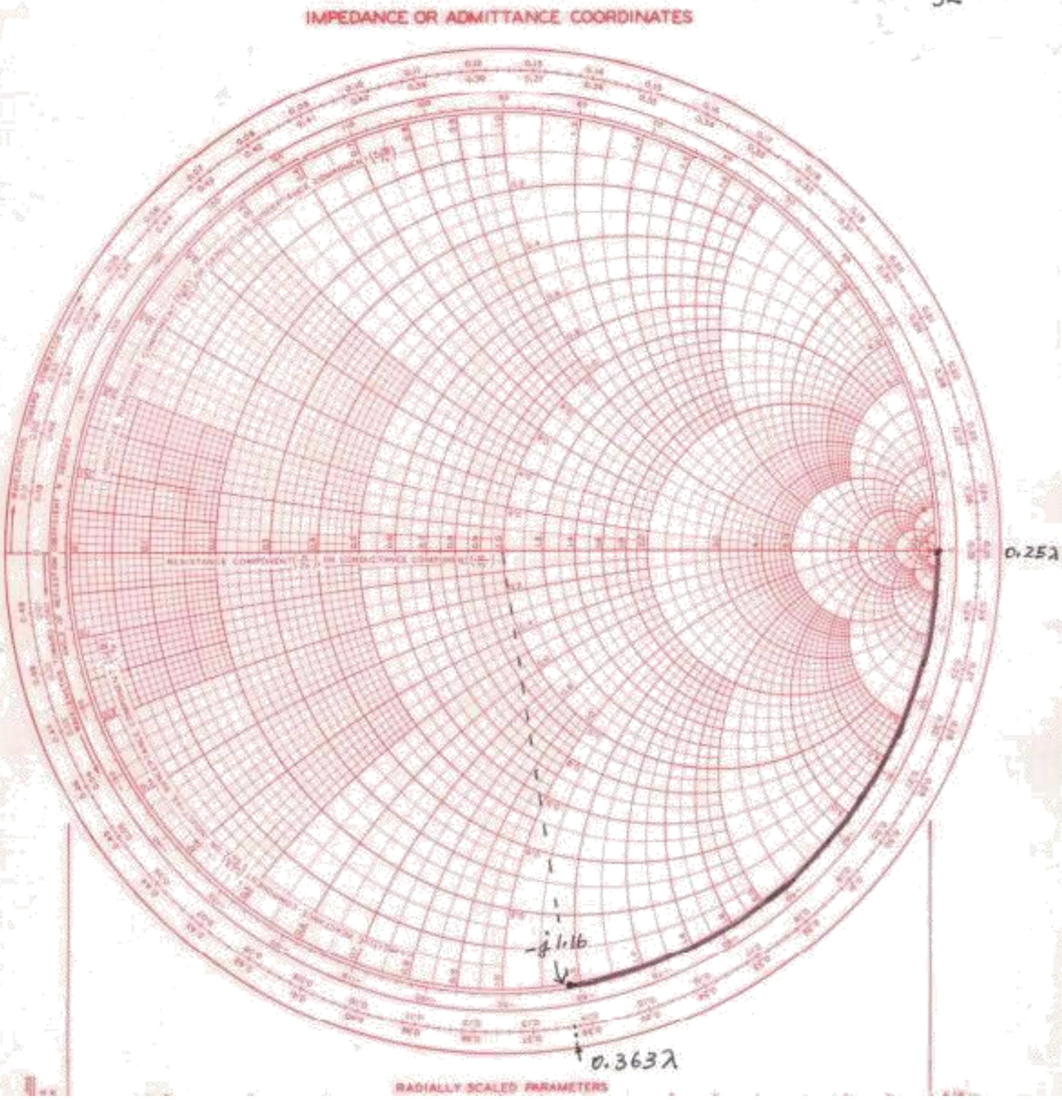
Single Stub
1st method



STUB

Determining length of stub





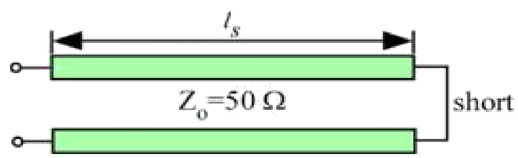
Second Method

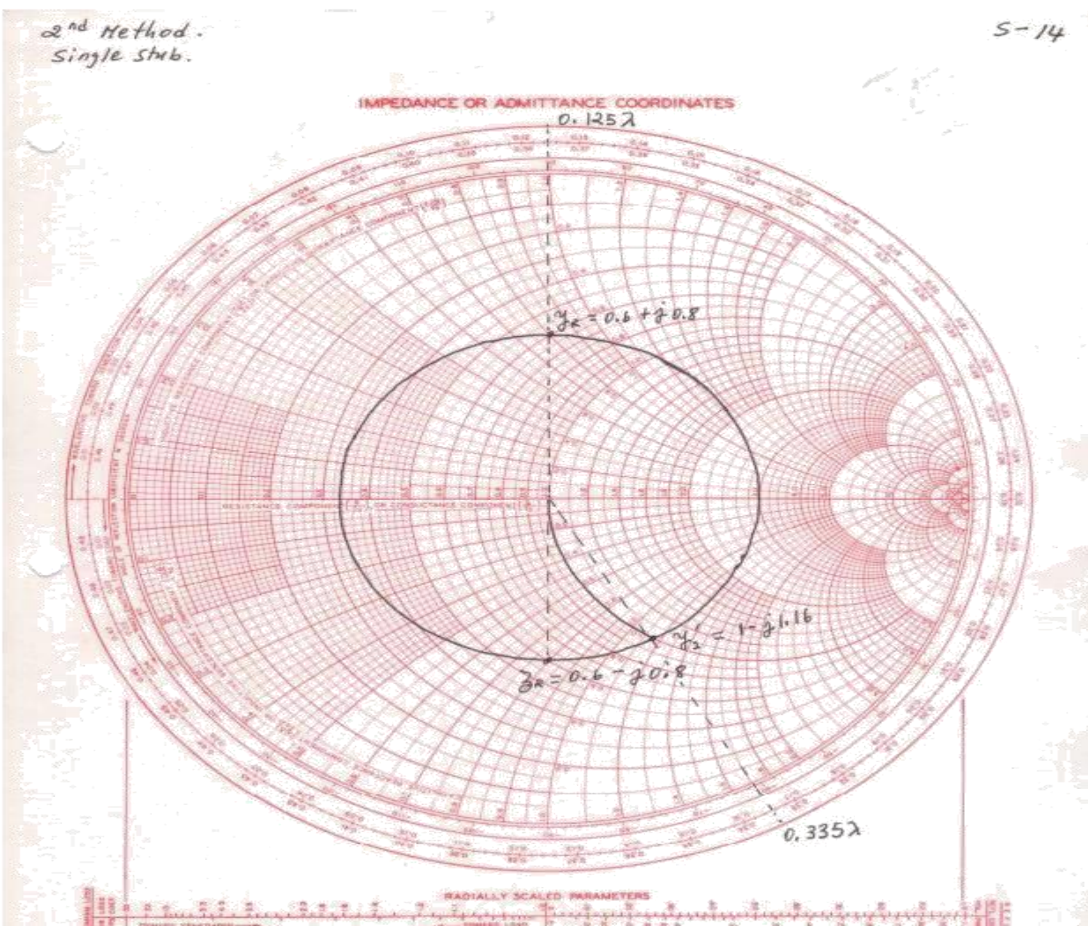
$y_R = 0.6 + j0.8$

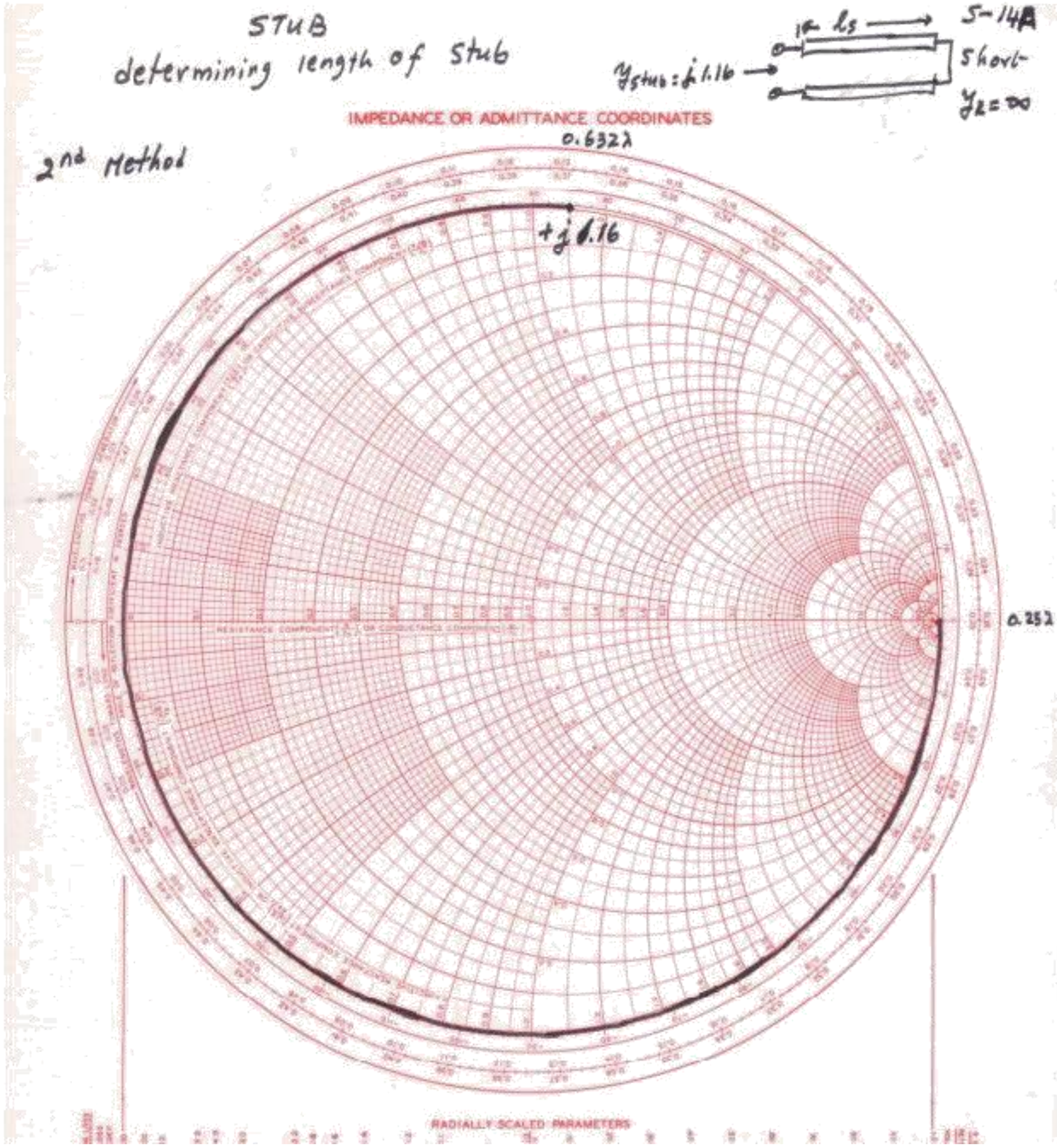
Rotate on constant VSWR circle from until intersection with unit conductance circle at 0.335λ at $y_1' = 1 - j1.16$. $d_s = (0.335\lambda - 0.125\lambda) = 0.210\lambda$ Move toward center of chart.

Change in susceptance: is +j1.16. Therefore, the length of the stub is such that

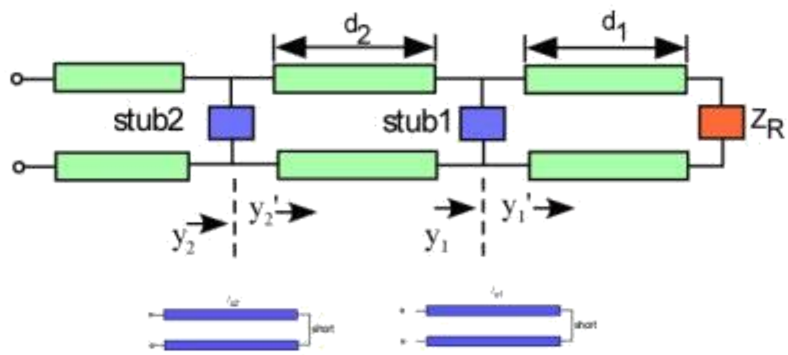
$$\frac{1}{tan\beta l} = +1.16 \text{ or } l_s = (0.632 - 0.25)\lambda = 0.382\lambda$$






STUB
determining length of Stub
$y_{stub} = j1.16$
l_s
5-14A
Short
$y_L = \infty$
IMPEDANCE OR ADMITTANCE COORDINATES
0.632λ
2nd Method
$+j1.16$
0.25λ
RADIALLY SCALED PARAMETERS

Analysis of double stub



1) Get y_R from Z_R

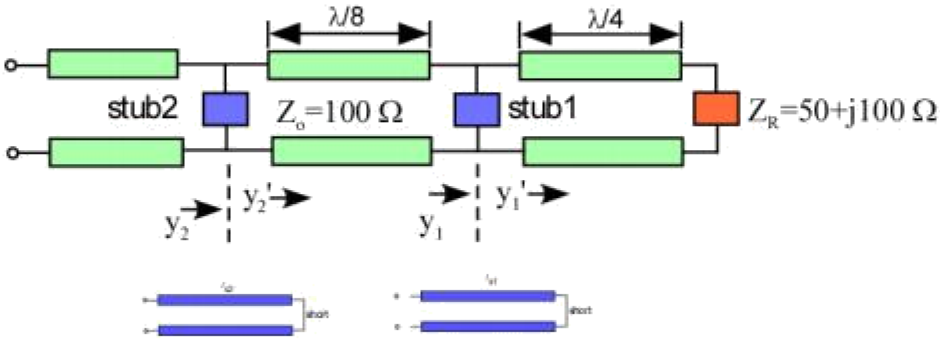
2) Rotate on constant VSWR circle by d_1 to get y_1'

3) Move on constant g circle by susceptance of stub1 to get to y_1.

4) Rotate on constant VSWR circle by d_2 until intersection with g =1 circle.

5) Move toward center of chart.1

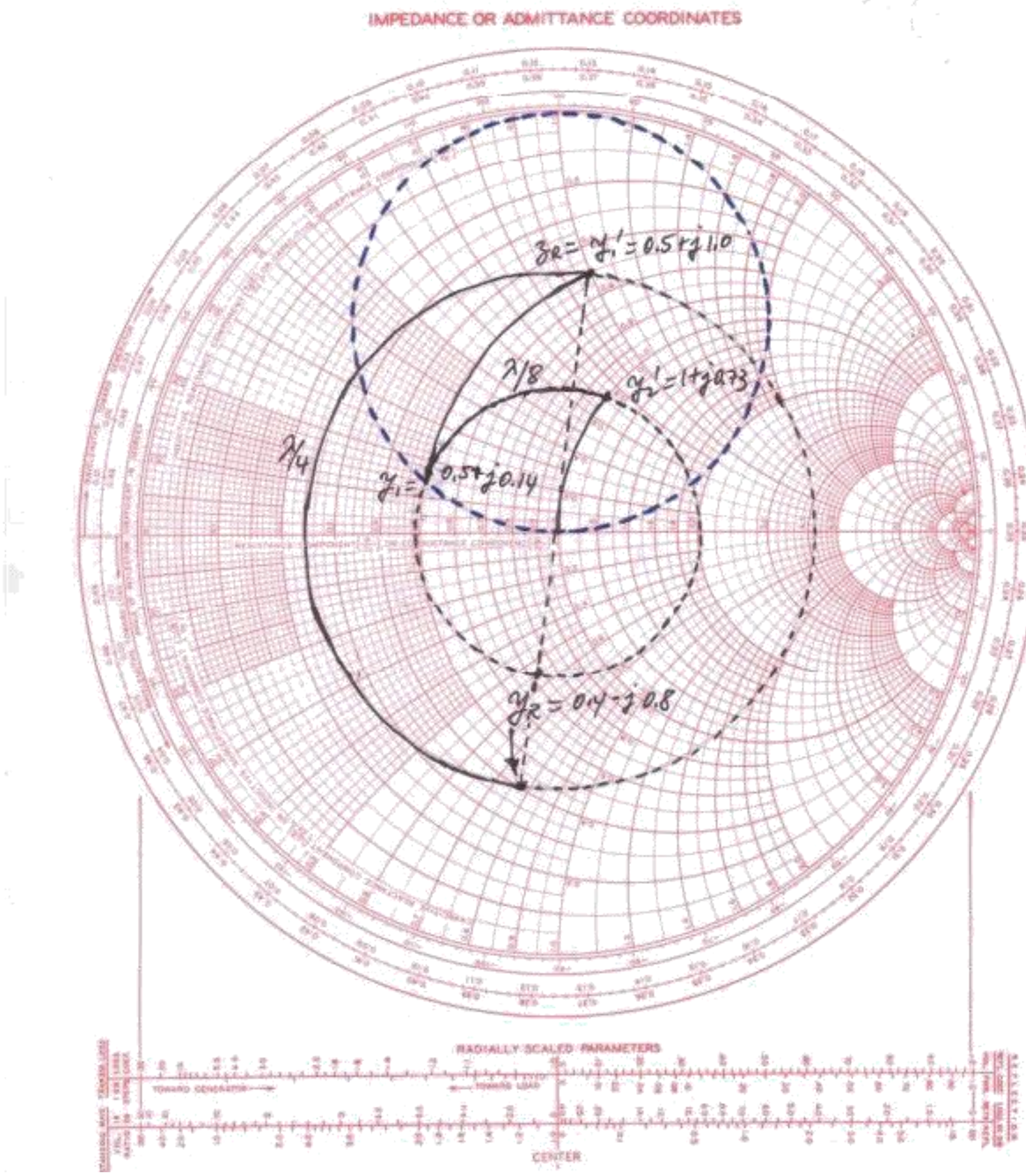
Double Stub – Example



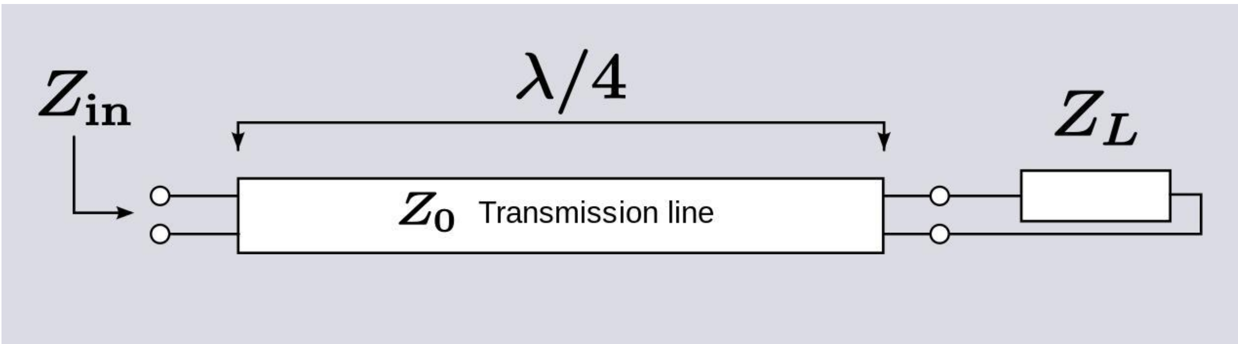
Find l_{s1} and l_{s2}

$$z_R = (50 + j100)/100 = 0.5 + j1.0$$

1) Normalize admittance, $y_R = 0.4 - j0.8$
2) Rotate by λ/4 toward source $y_1' = 0.5 + j1.0$
3) Draw auxiliary circle (g=1 circle rotated by λ/8)
4) Auxiliary circle intersects $g = 0.5$ circle at $y_1 =$ 0.5+j0.14.➔ $change = j(0.14 - 1.0) = -j0.86$,
5) Add stub $(0.388 - 0.25)\lambda = 0.138\lambda$
6) Rotation by λ/8 must end on $g = 1$ circle, $y_2' = 1 + j0.73$
7) Add stub2 such that $\dfrac{1}{j\,tan\,\beta l_{s2}} = -j0.73$
8) $l_{s2} = (0.4 - 0.25)\lambda = 0.15\lambda$


IMPEDANCE OR ADMITTANCE COORDINATES
$z_R = y_1' = 0.5 + j1.0$
$\lambda/8$
$y_2' = 1 + j0.73$
$\lambda/4$
$y_1 = 0.5 + j0.14$
$y_R = 0.4 - j0.8$
RADIALLY SCALED PARAMETERS
CENTER

Quarter-wave impedance transformer



A quarter-wave impedance transformer, often written as λ/4 impedance transformer, is a component used in electrical engineering consisting of a length of transmission or waveguide exactly one-quarter of a wavelength (λ) long and terminated in some known impedance. The device presents at its input the dual of the impedance with which it is terminated.

It is a similar concept to a stub; but whereas a stub is terminated in a short (or open) circuit and the length is chosen so as to produce the required impedance, the λ/4 transformer is the other way around; it is a pre-determined length and the termination is designed to produce the required impedance.

The relationship between the characteristic impedance, Z_0, input impedance, Z_{in} and load impedance, Z_L is: $\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$**]**

Theory of operation

A transmission line that is terminated in some impedance, Z_L, that is different from the characteristic impedance, Z_0, will result in a wave being reflected from the termination back to the source. At the input to the line the reflected voltage adds to the incident voltage and the reflected current subtracts (because the wave is travelling in the opposite direction) from the incident current. The result is that the input impedance of the line (ratio of voltage to current) differs from the characteristic impedance and for a line of length *l* is given by;[7]

$$Z_{\text{in}} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

where *γ* is the line propagation constant.

A very short transmission line, such as those being considered here, in many situations will have no appreciable loss along the length of the line and the propagation constant can be considered to be purely imaginaryphase constant, *iβ* and the impedance expression reduces to,[7]

$$Z_{\text{in}} = Z_0 \frac{Z_L + iZ_0 \tan(\beta l)}{Z_0 + iZ_L \tan(\beta l)}$$

Since *β* is the same as the angular wavenumber,

$$\beta = \frac{2\pi}{\lambda} \ ,$$

for a quarter-wavelength line,

$$l = \frac{\lambda}{4} \ , \quad \beta l = \frac{\pi}{2} \ ,$$

and the impedance becomes,taking the limit as the tangent function argument approaches $\frac{\pi}{2}$

$$Z_{\text{in}} = \lim_{\beta l \to \pi/2} Z_0 \frac{Z_L + iZ_0 \tan(\beta l)}{Z_0 + iZ_L \tan(\beta l)} = Z_0 \frac{iZ_0}{iZ_L} = \frac{{Z_0}^2}{Z_L}$$

which is the same as the condition for dual impedances;

$$\frac{Z_{\text{in}}}{Z_0} = \frac{Z_0}{Z_L}$$

Phenomenon of Corona

Electrical Transmission overhead line provides one of the most important property, which is a specific line voltage reveres occur. Transmission efficiency and therefore provides a degree of power in the Loss decreased. Moreover, because the line to get his prey life span can be much reduced. And bodies created by the induced current harmonics line charging current increases and the nearest line of the unwanted noise is telecommunication . Also If the line is dirty or rough weather - rainy when the bodies of thousands of adverse effects. In line due to the need to design the appropriate safety system, which provides line protection from the harmful effects can be.

Effects of Corona

i. Conductor all side Purple - Glow (violet glow) is observed.
ii. This hissing sound (hissing noise) generated by.
iii. Provides a certain amount of power is wasted. Which can be measured using meters. The transmission efficiency decreases.
iv. Provides for the weight of the gas line to the wire with the chemical reaction. The loss is received.
v. Max is rough and dirty conductors glow.

Factors affecting corona

i. Atmosphere.
ii. Conductor size.
iii. Spacing between conductors.
iv. Line Voltage..

The phenomenon of corona is accompanied by a hissing sound, production of ozone, power loss and radio interference. The higher the voltage is raised, the larger and higher the luminous envelope becomes, and greater are the sound, the power loss and the radio noise. If the applied voltage is increased to breakdown value, a flash-over will occur between the conductors due to the breakdown of air insulation.

The phenomenon of violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as corona. If the conductors are polished and smooth, the corona glow will be uniform throughout the length of the conductors, otherwise the rough points will appear brighter. With d.c. voltage, there is difference in the appearance of the two wires. The positive wire has uniform glow about it, while the negative conductor has spotty glow.

Explanation of corona formation

Some ionization is always present in air due to cosmic rays, ultraviolet radiations and radioactivity. Therefore, under normal conditions, the air around the conductors contains some ionized particles (i.e., free electrons and +ve ions) and neutral molecules. When p.d. is applied between the conductors, potential gradient is set up in the air which will have maximum value at the conductor surfaces. Under the influence of potential gradient, the existing free electrons acquire greater velocities. The greater the applied voltage, the greater the potential gradient and more is the velocity of free electrons.

When the potential gradient at the conductor surface reaches about 30 kV per cm (max. value), the velocity acquired by the free electrons is sufficient to strike a neutral molecule with enough force to dislodge one or more electrons from it. This produces another ion and one or more free electrons, which is turn, are accelerated until they collide with other neutral molecules, thus producing other ions. Thus, the process of ionisation is cummulative.The result of this ionization is that either corona is formed or spark takes place between the conductors.

Factors Affecting corona effect

The phenomenon of corona is affected by the physical state of the atmosphere as well as by the conditions of the line. The following are the factors upon which corona depends :

i) Atmosphere: As corona is formed due to ionization of air surrounding the conductors, therefore, it is affected by the physical state of atmosphere. In the stormy weather, the number of ions is more than normal and as such corona occurs at much less voltage as compared with fair weather.

(ii) Conductor size: The corona effect depends upon the shape and conditions of the conductors. The rough and irregular surface will give rise to more corona because unevenness of the surface decreases the value of breakdown voltage. Thus a stranded conductor has irregular surface and hence gives rise to more corona that a solid conductor.

(iii) Spacing between conductors: If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect. It is because larger distance between conductors reduces the electro-static stresses at the conductor surface, thus avoiding corona formation.

(iv) Line voltage :The line voltage greatly affects corona. If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed. However, if the line voltage has such a value that electrostatic stresses developed at

the conductor surface make the air around the conductor conducting, then corona is formed.

Advantages of Corona effect

(i) Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.

(ii) Corona reduces the effects of transients produced by surges.

Disadvantages of Corona effect

(i) Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.

(ii) Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.

(iii) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighboring communication lines.

Methods of Reducing Corona Effect

It has been seen that intense corona effects are observed at a working voltage of 33 kV or above. Therefore, careful design should be made to avoid corona on the sub-stations or bus-bars rated for 33kV and higher voltages otherwise highly ionized air may

cause flash-over in the insulators or between the phases, causing considerable damage to the equipment. The corona effects can be reduced by the following methods .

(i) By increasing conductor size: By increasing conductor size, the voltage at which corona occurs is raised and hence corona effects are considerably reduced.This is one of the reasons that ACSR conductors which have a larger cross-sectional area are used in transmission lines.

(ii) By increasing conductor spacing: By increasing the spacing between conductors, the voltage at which corona occurs is raised and hence corona effects can be eliminated. However, spacing cannot be increased too much otherwise the cost of supporting structure (e.g., bigger cross arms and supports) may increase to a considerable extent.

Important terms:

The phenomenon of corona plays an important role in the design of an overhead transmission line.Therefore, it is profitable to consider the following terms much used in the analysis of corona effects:

i) Critical disruptive voltage : It is the minimum phase-neutral voltage at which corona occurs.Consider two conductors of radii r cm and spaced d cm apart. If V is the phase-neutral potential, then potential gradient at the conductor surface is given by:

$$g = \frac{V}{r \log_e \frac{d}{r}} \text{ volts / cm}$$

In order that corona is formed, the value of g must be made equal to the breakdown strength of air.The breakdown strength of air at 76 cm pressure and temperature of 25ºC is 30 kV/cm (max) or 21·2 kV/cm (r.m.s.) and is denoted by go.IfVc is the phase-neutral potential required under these conditions, then,

$$g_o = \frac{V_c}{r \log_e \frac{d}{r}}$$

where g_o = breakdown strength of air at 76 cm of mercury and 25ºC = 30 kV/cm (*max*) or 21·2 kV/cm (*r.m.s.*)

∴ Critical disruptive voltage,

$$V_c = g_o\, r \log_e \frac{d}{r}$$

The above expression for disruptive voltage is under standard conditions i.e., at 76 cm of Hg and 25ºC.However, if these conditions vary, the air density also changes, thus altering the value of go.The value of go is directly proportional to air density. Thus the breakdown strength of air at a barometric pressure of b cm of mercury and temperature of tºC becomes δ go where

$$\delta = \text{air density factor} = \frac{3 \cdot 92b}{273 + t}$$

Under standard conditions, the value of δ = 1.

$$\therefore \quad \text{Critical disruptive voltage}, \; V_c = g_o\, \delta\, r \log_e \frac{d}{r}$$

Correction must also be made for the surface condition of the conductor.This is accounted for by multiplying the above expression by irregularity factor mo

$$\therefore \text{ Critical disruptive voltage, } \quad V_c = m_o g_o \delta r \log_e \frac{d}{r} \text{ kV/phase}$$

$$\begin{aligned} \text{where} \quad m_o &= 1 \text{ for polished conductors} \\ &= 0{\cdot}98 \text{ to } 0{\cdot}92 \text{ for dirty conductors} \\ &= 0{\cdot}87 \text{ to } 0{\cdot}8 \text{ for stranded conductors} \end{aligned}$$

(ii) Visual critical voltage: It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors. It has been seen that in case of parallel conductors, the corona glow does not begin at the disruptive voltage Vc but at a higher voltage Vv, called visual critical voltage. The phase-neutral effective value of visual critical voltage is given by the following empirical formula :

$$V_v = m_v g_o \delta r \left(1 + \frac{0{\cdot}3}{\sqrt{\delta r}}\right) \log_e \frac{d}{r} \text{ kV/phase}$$

where mv is another irregularity factor having a value of 1·0 for polished conductors and 0·72 to 0·82 for rough conductors.

iii) Power loss due to corona : Formation of corona is always accompanied by energy loss which is dissipated in the form of light, heat, sound and chemical action. When disruptive voltage is exceeded, the power loss due to corona is given by :

$$P = 242{\cdot}2\left(\frac{f+25}{\delta}\right)\sqrt{\frac{r}{d}}\,(V - V_c)^2 \times 10^{-5} \text{ kW / km / phase}$$

$$\begin{aligned} f &= \text{supply frequency in Hz} \\ V &= \text{phase-neutral voltage } (r.m.s.) \\ V_c &= \text{disruptive voltage } (r.m.s.) \text{ per phase} \end{aligned}$$

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DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - IV
ELECTROMAGNETIC WAVES – SECA1402

UNIT IV: ELECTROMAGNETIC WAVES

Maxwell's Equation

The waves guided or directed by the guided structures are called guided wave. In general wave equations are derived from Maxwell's equation. To obtain the solution of this problem it is essential to apply certain restrictions or boundary conditions to the Maxwell's equation.

Maxwell's Equations are a set of four vector-differential equations that govern all of electromagnetic (except at the quantum level, in which case we as antenna people don't care so much). They were first presented in a complete form by James Clerk Maxwell back in the 1800s. He didn't come up with them all on his own, but did add the displacement current term to Ampere's law which made them complete. The four equations (written only in terms of E and H, the electric field and the magnetic field), are given below

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon} \qquad \text{(Gauss' Law)}$$

$$\nabla \cdot \mathbf{H} = 0 \qquad \text{(Gauss' Law for Magnetism)}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \text{(Faraday's Law)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \text{(Ampere's Law)}$$

In Gauss' law, p_v is the volume electric charge density, J is the electric current density (in Amps/meter-squared),u is the permittivity and is the permeability.

The good news about this is that all of electromagnetic is summed up in these 4 equations. The bad news is that no matter how good at math you are, these can only be solved with an analytical solution in extremely simple cases. Antennas don't present a very simple case, so these equations aren't used a whole lot in antenna theory

The last two equations (Faraday's law and Ampere's law) are responsible for electromagnetic radiation. The curl operator represents the spatial variation of the fields, which are coupled to the time variation. When the E-field travels, it is altered in space, which gives rise to a time-varying magnetic field. A time-varying magnetic field then varies as a function of location (space), which gives rise to a time varying electric field. These equations wrap around each other in a sense, and give rise to a wave equation. These equations predict electromagnetic radiation as we understand it.

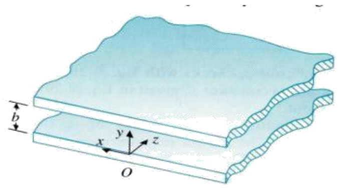


Fig1 parallel-plate waveguide

Consider a parallel-plate waveguide of two perfectly conducting plates separated by a distance b and filled with a dielectric medium having constitutive parameter as shown in Fig. 1. The plates are assumed be infinite in extent in the X direction.

a) Obtain the time-harmonic field expressions for TM modes in the guide.

b) Determine the cutoff frequency.

a) For TM modes, - Eq. becomes

$$\frac{d^2E_z^o(y)}{dy^2} + h^2E_z^o(y) = 0.$$ **----------------------------------1**

The general solution for Eq. (1) : - Boundary conditions (The tangential component of the electric field must vanish on the surface of the perfectly conducting plates.) :

(i) At y=0 Ez=0

(ii) (ii) At y=b Ez=0 - The value of the eigenvalue h :

$$h = \frac{n\pi}{b}, \qquad n = 1, 2, 3, \ldots .$$

Types of propagation:

TE waves

TM waves

TEM waves

Transverse electric (TE) wave has the magnetic field component in the direction of propagation, but no component of the electric field in the same direction. Hence the TE waves also known as M –waves or H-waves.

Transverse magnetic(TM) wave has the electric field in the direction of propagation, but no component of the magnetic field in the same direction. Hence the TM waves are also called E-waves.

Transverse electromagnetic (TEM) wave: No field in the direction of propagation Attenuation of parallel plane guides: When the electromagnetic wave propagates through the wave guide, the amplitude of the fields or the signal strength of the wave decreases as the distance from the source increases. This is because when the wave strikes the walls of the guide, the loss in the power takes place the attenuation factor is denoted by α.

A=Power lost per unit length/2*power transmitted.

Attenuation due to finite wall conductivity is inversely proportional to the square root of wall conductivity, but depends on the mode and the frequency in a complicated way.

Attenuation due to wall losses in rectangular copper waveguide : Figure 9-7. TE10 mode has the lowest attenuation in a rectangular waveguide. The attenuation constant increases rapidly toward infinity as the operating frequency approaches the cutoff frequency.

Causes for attenuation in waveguides: lossy dielectric and imperfectly conducting walls.

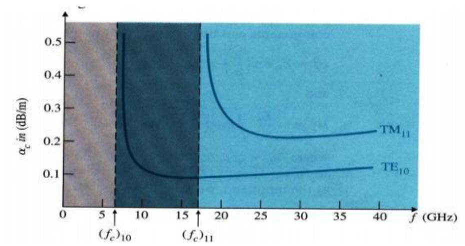


Fig 2: attenuation due to wall losses

Cut-off frequency:

The frequency at which wave motion ceases is called cut-off frequency

$$\boxed{f_c = \frac{n}{2b\sqrt{\mu\epsilon}} \qquad \text{(Hz)},}$$

Propagation Constant:

$$\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}.$$

Wave impedance:

It is the ratio of the component of the electric field to that of magnetic field. Wave impedance for TEM wave

$$Z_{\text{TEM}} = \frac{E_x^0}{H_y^0} = \frac{j\omega\mu}{\gamma_{\text{TEM}}} = \frac{\gamma_{\text{TEM}}}{j\omega\epsilon},$$

Wave impedance for TM and TE wave

$$\boxed{Z_{\text{TM}} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{\gamma}{j\omega\epsilon} \qquad (\Omega).}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} \qquad (\Omega),$$

Phase velocity:

It is the velocity at which energy propagates along a wave guide

$$u_p = \frac{\omega}{\beta} \qquad (\text{m/s}).$$

• The phase velocity and the wave impedance for TEM waves are independent of the frequency of the waves.

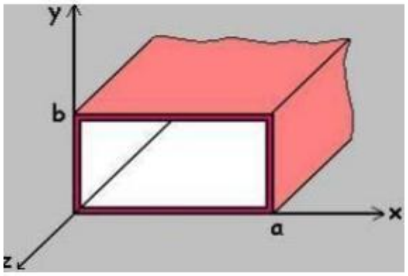
• TEM waves cannot exist in a single-conductor hollow (or dielectric-filled) waveguide of any shape.

Rectangular waveguides are the one of the earliest type of the transmission lines. They are used in many applications. A lot of components such as isolators, detectors, attenuators, couplers and slotted lines are available for various standard waveguide bands between 1 GHz to above 220 GHz.

A rectangular waveguide supports TM and TE modes but not TEM waves because we cannot define a unique voltage since there is only one conductor in a rectangular waveguide. The shape of a rectangular waveguide is as shown below. A material with permittivity e and permeability m fills the inside of the conductor.

A rectangular waveguide cannot propagate below some certain frequency. This frequency is called the cut-off frequency

Here, we will discuss TM mode rectangular waveguides and TE mode rectangular waveguides separately. Let's start with the TM mode



TM Modes

Consider the shape of the rectangular waveguide above with dimensions a and b (assume a>b) and the parameters e and m. For TM waves Hz = 0 and Ez should be solved from equation for TM mode;

Ñ ^{2} xy Ez ^{0} + h^2 Ez ^{0} = 0

Since E$_z$(x,y,z) = Ez ^{0} (x,y)e^{-gz}, we get the following equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+h^2\right)E_z^0(x,y)=0$$

If we use the method of separation of variables, that is Ez ^{0} (x,y)=X(x).Y(y) we get,

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2}=\frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2}+h^2$$

Since the right side contains x terms only and the left side contains y terms only, they are both equal to a constant. Calling that constant as kx ^{2} , we get;

$$\frac{d^2X(x)}{dx^2}+k_x^2X(x)=0$$

$$\frac{d^2Y(y)}{dy^2}+k_y^2Y(y)=0$$

Where ky ^{2} =h^2 -kx ^{2}

Now, we should solve for X and Y from the preceding equations. Also we have the boundary conditions of;

Ez ^{0} (0,y)=0

Ez ^{0} (a,y)=0

Ez ^{0} (x,0)=0

Ez ^{0} (x,b)=0

From all these, we conclude that

X(x) is in the form of sin k$_x$x, where k$_x$=mp/a, m=1,2,3,…

Y(y) is in the form of sin k$_y$y, where k$_y$=np/b, n=1,2,3,…

So the solution for Ez ^{0} (x,y) is

$$E_z^0(x,y)=E_0\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\ \text{(V/m)}$$

From ky 2 =h2 -kx 2 , we have;

$$\boxed{h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For TM waves, we have

$$H_x^0 = \frac{j w \varepsilon}{h^2} \frac{\partial E_z^0}{\partial y}$$

$$H_y^0 = -\frac{j w \varepsilon}{h^2} \frac{\partial E_z^0}{\partial x}$$

$$E_x^0 = -\frac{\gamma}{h^2} \frac{\partial E_z^0}{\partial x}$$

$$E_y^0 = -\frac{\gamma}{h^2} \frac{\partial E_z^0}{\partial y}$$

From these equations, we get

$$E_x^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$\boxed{E_y^0(x,y) = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

$$\boxed{H_x^0(x,y) = \frac{jw\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

$$\boxed{H_y^0(x,y) = -\frac{jw\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)}$$

where

$$\boxed{\gamma = j\beta = j\sqrt{w^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

Here, m and n represent possible modes and it is designated as the TMmn mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

When we observe the above equations we see that for TM modes in rectangular waveguides, neither m nor n can be zero. This is because of the fact that the field expressions are identically zero if either m or n is zero. Therefore, the lowest mode for rectangular waveguide TM mode is TM11 .

Here, the cut-off wave number is

$$\boxed{k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

And therefore

$$\boxed{\beta=\sqrt{k^2-k_c^2}}$$

The cut-off frequency is at the point where g vanishes. Therefore

$$f_c=\frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\,(Hz)$$

Since l=u/f, we have the cut-off wavelength

$$\boxed{\lambda_c=\frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}\,(m)}$$

At a given operating frequency f, only those frequencies, which have f_c<f will propagate. The modes with f<f_c will lead to an imaginary b which means that the field components will decay exponentially and will not propagate. Such modes are called cut-off or evanescent modes.

The mode with the lowest cut-off frequency is called the dominant mode. Since TM modes for rectangular waveguides start from TM_{11} mode, the dominant frequency is

$$(f_c)_{11}=\frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}\,(Hz)$$

The wave impedance is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for E_x and H_y (see the equations above);

$$Z_{TM}=\frac{E_x}{H_y}=\frac{\gamma}{jw\varepsilon}=\frac{j\beta}{jw\varepsilon}\Rightarrow Z_{TM}=\frac{\beta\eta}{k}$$

The guide wavelength is defined as the distance between two equal phase planes along the waveguide and it is equal to

$$\boxed{\lambda_g=\frac{2\pi}{\beta}}>\frac{2\pi}{k}=\lambda$$

which is thus greater than l, the wavelength of a plane wave in the filling medium. The phase velocity is

$$u_p=\frac{w}{\beta}>\frac{w}{k}=\frac{1}{\sqrt{\mu\varepsilon}}$$

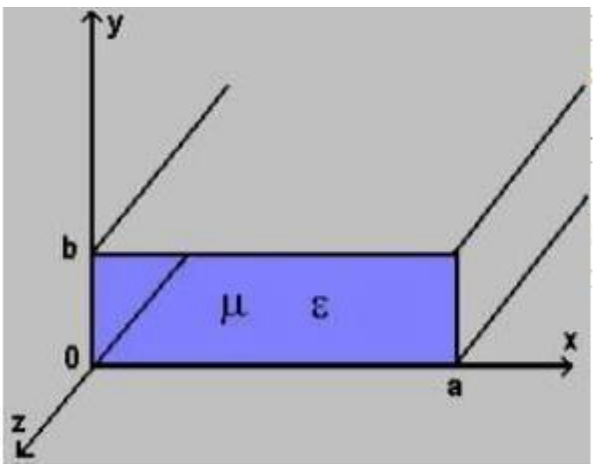
which is greater than the speed of light (plane wave) in the filling material.

Attenuation for propagating modes results when there are losses in the dielectric and in the imperfectly conducting guide walls. The attenuation constant due to the losses in the dielectric can be found as follows:

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

TE Modes

Consider again the rectangular waveguide below with dimensions a and b (assume a>b) and the parameters e and m.



For TE waves Ez = 0 and Hz should be solved from equation for TE mode;

$Ñ^{2}{}_{xy}\, H_z + h^2 H_z = 0$

Since $H_z(x,y,z) = Hz^{0}\,(x,y)e^{-gz}$, we get the following equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)H_z^0(x,y) = 0$$

If we use the method of separation of variables, that is $Hz^{0}\,(x,y)=X(x).Y(y)$ we get,

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + h^2$$

Since the right side contains x terms only and the left side contains y terms only, they are both equal to a constant. Calling that constant as kx^{2} , we get;

$$\frac{d^2X(x)}{dx^2}+k_x^2X(x)=0$$

$$\frac{d^2Y(y)}{dy^2}+k_y^2Y(y)=0$$

where $k_y{}^2 = h^2 - k_x{}^2$

Here, we must solve for X and Y from the preceding equations. Also we have the following boundary conditions:

$$\frac{\partial H_z^0}{\partial x}=0(E_y=0) \text{ at } x=0$$

$$\frac{\partial H_z^0}{\partial x}=0(E_y=0) \text{ at } x=a$$

$$\frac{\partial H_z^0}{\partial y}=0(E_x=0) \text{ at } y=0$$

$$\frac{\partial H_z^0}{\partial y}=0(E_x=0) \text{ at } y=b$$

From all these, we get

$$H_z^0(x,y)=H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \text{ (A/m)}$$

From $k_y{}^2=h^2-k_x{}^2$, we have;

$$\boxed{h^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

For TE waves, we have

$$H_x^0=-\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x}$$

$$H_y^0=-\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_x^0=-\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_y^0=-\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial x}$$

From these equations, we obtain

$$E_x^0(x,y)=\frac{jw\mu}{h^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_y^0(x,y) = -\frac{jw\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_x^0(x,y) = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_y^0(x,y) = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

Where

$$\gamma = j\beta = j\sqrt{w^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

As explained before, m and n represent possible modes and it is shown as the TEmn mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

Here, the cut-off wave number is

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

and therefore,

$$\beta = \sqrt{k^2 - k_c^2}$$

The cut-off frequency is at the point where g vanishes. Therefore

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\ (Hz)$$

Since l=u/f, we have the cut-off wavelength

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}\ (m)$$

At a given operating frequency f, only those frequencies, which have f>fc will propagate. The modes with f<fc will not propagate

The mode with the lowest cut-off frequency is called the dominant mode. Since TE10 mode is the minimum possible mode that gives nonzero field expressions for rectangular waveguides, it is the dominant mode of a rectangular waveguide with a>b and so the dominant frequency is

$$(f_c)_{10} = \frac{1}{2a\sqrt{\mu\varepsilon}}\,(Hz)$$

The wave impedance is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for Ex and Hy (see the equations above);

$$Z_{TE} = \frac{E_x}{H_y} = \frac{jw\mu}{\gamma} = \frac{jw\mu}{j\beta} \Rightarrow Z_{TE} = \frac{k\eta}{\beta}$$

The guide wavelength is defined as the distance between two equal phase planes along the waveguide and it is equal to

$$\boxed{\lambda_g = \frac{2\pi}{\beta}} > \frac{2\pi}{k} = \lambda$$

which is thus greater than l, the wavelength of a plane wave in the filling medium. The phase velocity is

The attenuation constant due to the losses in the dielectric is obtained as follows:

$$\gamma = j\beta = j\sqrt{k^2 - k_c^2} = jk\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = jw\sqrt{\mu}\sqrt{\varepsilon + \frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

After some manipulation, we get

$$\alpha_d = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{k^2 \tan\delta}{2\beta}$$

Example:

Consier a length of air-filled copper X-band waveguide, with dimensions a=2.286cm, b=1.016cm. Find the cut-off frequencies of the first four propagating modes.

Solution: From the formula for the cut-off frequency

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \xrightarrow{air-filled} \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\,(Hz)$$

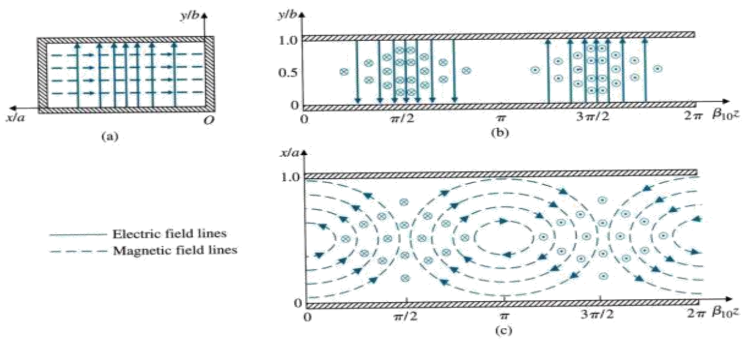


FIGURE 9-5 Field lines for TE_{10} mode in rectangular waveguide.

TEM waves:

The lowest possible values of m without making all the field components zero is 1.in other words,the lowest oreder mode with TE wave possible is TE_{10} mode

For the transverse magnetic TM waves the field expressions with m=0 are

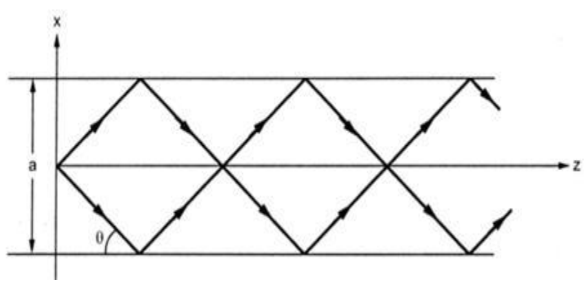
$$H_y = C_4 e^{-j\bar{\beta}z}$$

$$E_x = \frac{\bar{\beta}}{\omega\varepsilon} C_4 e^{-j\bar{\beta}z}$$

$$E_z = 0$$

So under this special case,the component of E in the direction of propagation E ia also zero indicationg that the electromagnetic field is entirely transverse.

Velocities of propagation:



Consider an electromagnetic wave travelling in positive z direction, propagates through a waveguide of width a as shown above

The angle θis the angle made by the wave with the walls of the guide. This angle made by the wave,with the walls and the direction of the wave ,depends on the frequency and the distance between two walls ie a.for each of the component waves,the component of electric field E will be in Y direction,while that of travel at the walls and nonzero at points between the walls.

The use of the terms *velocity of propagation* and *wave propagation speed* to mean a ratio of speeds is confined to the computer networking and cable industries. In a general science and engineering context, these terms would be understood to mean a true speed or velocity in units of distance per time while *velocity factor* is used for the ratio.

$$\mathrm{VF} = \frac{1}{c\sqrt{LC}}$$

Characteristics of TE and TM waves

The transverse electric and magnetic waves for parallel conducting planes exhibit some interesting properties as compared to those for uniform plane wavws propagating thought free space.

From the summary of the different field components for TE and Tm waves It is seen that for each of the components of E or H there is a sinusoidal standing wave distribution across the guide in the X-direction.This meanse that each of these components varies in magnitude.but not in phase,in the x-direction.

Following quantities for the TE and TM mode

1.propagation constant

2.cut-off frequency

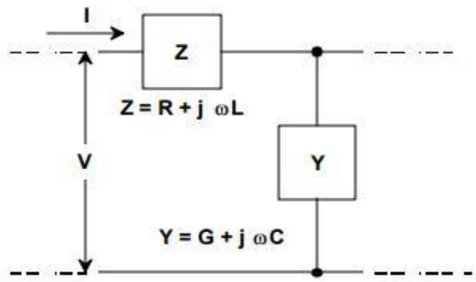
3.guide wavelength

4.velocities of wave propagation

5.wave impedance

Waves Impedances :

Consider a transmission line consisting of iterated incremental elements as shown here:



Z and Y are the impedance and admittance per unit length z

Z = R + jwL and Y = G + jwC,

where R is the series resistance per unit length z,

W/m L is the series inductance per unit length z,

H/m G is the shunt conductance per unit length z,

S/m C is the shunt capacitance per unit length z, F/m

The equations for V and I are

dV/ dz = ZI and dI/ dz = YV, simultaneous solution of which yields

d^2V/dz^2 = ZYV and $d^2I\ dz^2$ = ZYI; z here represents distance along the transmission line.

The solution of these equations is in the form of waves in the +z and -z direction, which for sinusoidal excitation take the form

$$\mathbf{V(z) = V_+e^{\omega t-j\gamma z} + V_-e^{\omega t+j\gamma z)}} \text{ and } \mathbf{I(z) = I_+e^{(\omega t-j\gamma z)} + I_+e^{(\omega t+j\gamma z)}}$$

The propagation constant g k is given by

$$\gamma = \alpha + \mathbf{j}\beta = \sqrt{\mathbf{ZY}}. \text{ For } \omega L >> R \text{ and } \omega C >> G \text{ (low or zero loss case),}$$

$$\beta = \omega\sqrt{\mathbf{LC}}$$

The voltage and current functions represent waves in each direction such that successive peaks and troughs move at a velocity

$$\mathbf{v} = \frac{\omega}{\beta} = \mathbf{f}\lambda, \text{ so } \beta = \frac{\mathbf{2p}}{\lambda}$$

To distinguish it from the free-space wavelength nomenclature l or lo, the wavelength on a waveguide or coaxial transmission line is often referred to as the guide wavelength lg.

For a single wave solution in one direction, the ratio V(z)/I(z) is the same everywhere on the line, and is defined as the characteristic impedance Zo, which for a lossless line is a real number

$$\mathbf{Z_0} = \frac{\mathbf{V_+}}{\mathbf{I_+}} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} = \sqrt{\frac{\mathbf{L}}{\mathbf{C}}},$$

where L and C are the inductance and capacitance per unit length

Thus we can rewrite the current equation as

$$I(z) = I_{+}e^{(\omega t-j\beta z)} + I_{+}e^{(\omega t+j\beta z)} = \frac{V_{+}}{Z_0}\, e^{j(\omega t-\beta z)} - \frac{V_{-}}{Z_0}\, e^{j(\omega t+\beta z)}$$

where the minus sign reflects the fact that the magnetic field, and hence the current, of the negative-going propagation is reversed compared to that of a positive-going wave. If both waves exist, the instantaneous voltage or current as function of location is the sum of voltages or currents of both waves. The characteristic impedance Zo is the ratio of voltage to current of either wave independently, but not necessarily their sum.

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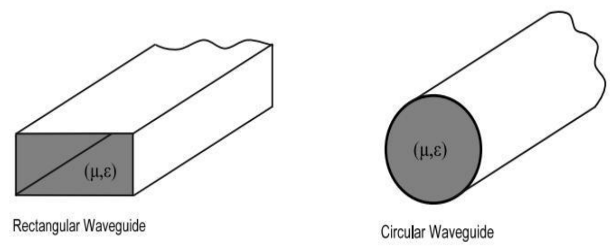
SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRONICS AND COMMMUNICATION ENGINEERING

UNIT - V
GUIDED WAVES AND WAVEGUIDE THEORY – SECA1402

Unit V: GUIDED WAVES AND WAVEGUIDE THEORY

Waveguides, like transmission lines, are structures used to guide electromagnetic waves from point to point. However, the fundamental characteristics of waveguide and transmission line waves (modes) are quite different. The differences in these modes result from the basic differences in geometry for a transmission line and a waveguide. Waveguides can be generally classified as either metal waveguides or dielectric waveguides. Metal waveguides normally take the form of an enclosed conducting metal pipe. The waves propagating inside the metal waveguide may be characterized by reflections from the conducting walls. The dielectric waveguide consists of dielectrics only and employs reflections from dielectric interfaces to propagate the electromagnetic wave along the waveguide.



Rectangular Waveguides

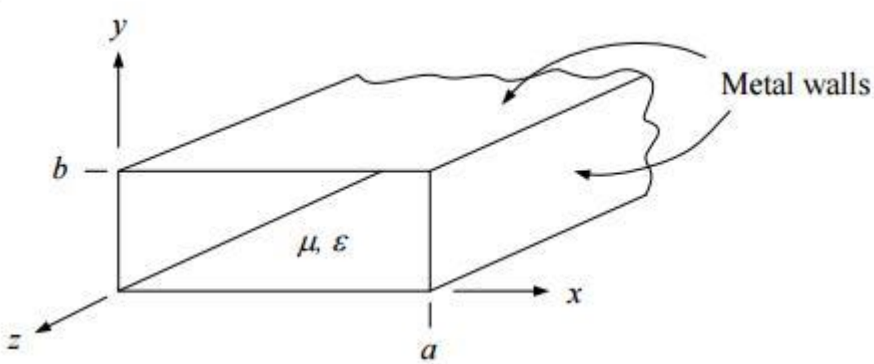


Figure 1: Rectangular Waveguide

The wave is propagating in z-direction as

Consider a rectangular waveguide with $0 < x < a$, $0 < y < b$ and $a > b$.

There are two types of waves in a hollow waveguide with only one conductor.

- **Transverse electric waves (TE-waves). E = (Ex, Ey, 0) and H = (Hx, Hy, Hz).**

• **Transverse magnetic waves (TM-waves). E = (Ex, Ey, Ez) and H = (Hx, Hy, 0).**

They need to satisfy the Maxwell's equations and the boundary conditions. The boundary conditions are that the tangential components of the electric field and the normal derivative of the tangential components of the magnetic field are zero at the boundaries

TE-waves

We now try to find the electromagnetic fields for TE-waves, when Ez is zero. The electromagnetic fields are obtained from Hz. The equation to be solved is

$$\nabla^2 H_z + k^2 H_z = 0$$
$$\frac{\partial H_z}{\partial x}(0,y,z) = \frac{\partial H_z}{\partial x}(a,y,z) = \frac{\partial H_z}{\partial y}(x,a,z) = \frac{\partial H_z}{\partial y}(x,b,z) = 0$$

where k = ω/c is the wave number. There are infinitely many solutions to this equation

$$H_{zmn}(x,y,z) = h_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\mathrm{j}k_z z}$$

The m, n values can take the values m = 0, 1, 2 . . . and n = 0, 1, 2 . . ., but (m, n) ≠ (0, 0). The Corresponding transverse electric and magnetic fields are obtained from Maxwell's equations. The spatial dependence of these components are

$$E_x \sim \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\mathrm{j}k_z z}$$
$$E_y \sim \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\mathrm{j}k_z z}$$
$$H_x \sim \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\mathrm{j}k_z z}$$
$$H_y \sim \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\mathrm{j}k_z z}$$

Each of these components satisfies the Helmholtz equation and the boundary conditions. The electromagnetic field corresponding to (m, n) is called a TE_{mn} mode. Thus there are infinitely many TE_{mn} modes. The k_z is the z−component of the wave vector. For a given frequency it is given by

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

This means that for m and n values such that

$$k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 > 0$$

$$f > \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

then k_z is real and the TE_{mn} mode is propagating.

For m and n values such that

$$k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 < 0$$

$$f < \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

then k_z is imaginary and the TE_{mn} mode is a non-propagating mode.

Cut-off frequency:

For a TE_{mn} mode the cut-off frequency is the frequency for which $k_z = 0$. This means that the mode is in between its propagating and non-propagating stages. The cut off frequency for the TE_{mn} mode is

$$f_{cmn} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}.$$

The fundamental mode TE_{10} (or) Dominant Mode

The fundamental mode or Dominant mode of a waveguide is the mode that has the lowest cut-off frequency. For a rectangular waveguide it is the TE_{10} mode that is the fundamental mode. It has

$$f_{c10} = \frac{c}{2a}.$$

The electric field of the fundamental mode is

$$\boldsymbol{E} = E_0 \sin\left(\frac{\pi x}{a}\right) e^{-\mathrm{j}k_z z}\boldsymbol{e}_y$$

It is almost always the fundamental mode that is used in the waveguide. It is then crucial to make sure that the frequency is low enough such that only the fundamental mode can propagate. Otherwise there will be more than one mode in the waveguide and since the modes travel with different speeds, as will be seen below, one cannot control the phase of the wave.

TM-Waves

The electromagnetic fields are obtained from

$$\nabla^2 E_z + k^2 E_z = 0$$
$$E_z(0,y,z) = E_z(a,y,z) = E_z(x,0,z) = E_z(x,b,z) = 0$$

where k = ω/c is the wave number. There are infinitely many solutions top this equation

$$E_{zmn}(x,y,z) = e_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\mathrm{j}k_z z}$$

The m, n values can take the values m = 1, 2 . . . and n = 1, 2 . . .. The corresponding transverse electric and magnetic fields are obtained from Maxwell's equations. The spatial dependence of these components is the same as for the TE-waves

$$E_x \sim \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\mathrm{j}k_z z}$$
$$E_y \sim \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\mathrm{j}k_z z}$$
$$H_x \sim \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\mathrm{j}k_z z}$$
$$H_y \sim \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\mathrm{j}k_z z}$$

The electromagnetic field corresponding to (m, n) is called a TE_{mn} mode. Thus there are infinitely many TE_{mn} modes.
For a given frequency k_z for the TE-modes is the same as for the TM-modes.

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Cut-off frequency

For a TMmn mode the cut-off frequencies are the same as for the TE_{mn} modes, i.e.,

$$f_{cmn} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Dominant Mode

The mode with the lowest cut-off frequency is called the *dominant mode*. Since TM modes for rectangular waveguides start from TM_{11} mode, the dominant frequency is

$$(f_c)_{11} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}\ (Hz)$$

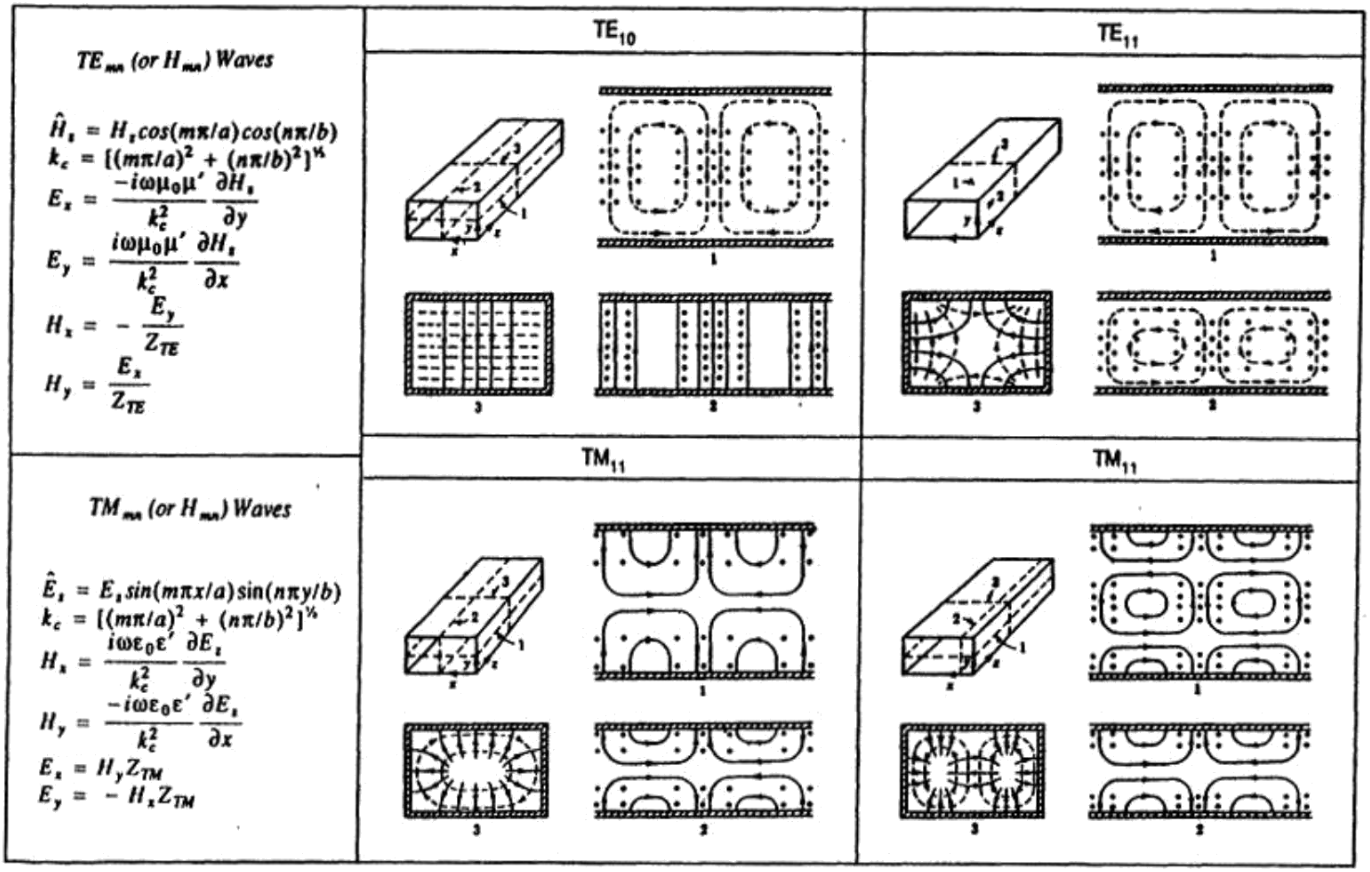


Figure 2: Field Configurations and Key expressions of calculation for modes in Rectangular Waveguides

Impossibility of TEM waves in Waveguides

TEM modes can only exist in two-conductor waveguides such as two-wire transmission lines, co-axial lines, parallel-plate waveguides, etc, but not in single-conductor waveguides such as rectangular waveguides and circular waveguides. This is because either longitudinal field components or longitudinal currents are required to support the transverse magnetic field components Hx and Hy which form close loops in the transverse plane. There are no longitudinal currents (not longitudinal surface currents) inside hollow waveguides and hence hollow waveguides cannot support TEM modes. But they can support TE and TM modes.

<u>Wave Impedances</u>

For any transverse electromagnetic wave , the wave impedance (in ohms) is defined as being approximately equal to the ratio of the electric and magnetic fields, and converges as a function of frequency to the intrinsic impedance of the dielectric:

$$\eta = \frac{|E|}{|H|}$$

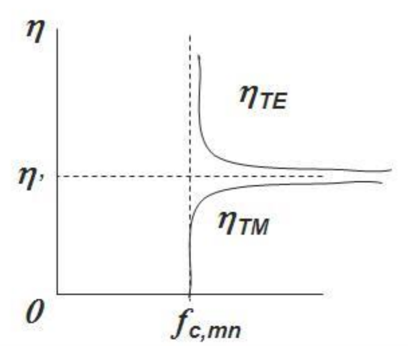
For TM mode

$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \eta' \sqrt{1-\left[\frac{f_c}{f}\right]^2}$$

For TE mode

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$$

Wave impedance varies with frequency and mode as follows



<u>**Circular Waveguides**</u>

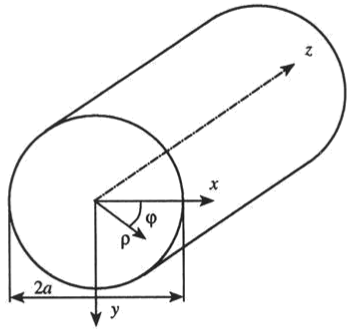


Figure 3: Circular Waveguide

The circular waveguide is occasionally used as an alternative to the rectangular waveguide. Like other wave guides constructed from a single, enclosed conductor, the circular waveguide supports transverse electric (TE) and transverse magnetic (TM) modes. These modes have a cutoff frequency, below which electromagnetic energy is severely attenuated. Wave Propagates in z-direction. There are two sets of modes, TE and TM modes, which can propagate in a cylindrical waveguide.

TE mode

For TE waves in a cylindrical waveguide, E_z=0 and $H_z \neq 0$, all other field components can be expressed in terms of H_z. The Maxwell's equations can be expanded in the cylindrical coordinate as

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} \pm j\beta_z E_\phi = -j\omega\mu H_\rho \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} \pm j\beta_z H_\phi = j\omega\varepsilon E_\rho$$

$$\mp j\beta_z E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \qquad \mp j\beta_z H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\varepsilon E_\phi$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \overline{E}_\phi\right) - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \phi} = -j\omega\mu H_z \qquad \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho H_\phi\right) - \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} = j\omega\varepsilon E_z$$

The ρ and φ components can be expressed in terms of E_z and H_z as follows

$$E_\rho = \frac{1}{\beta_z^2 - \beta^2}\left[\pm j\beta_z \frac{\partial E_z}{\partial \rho} + \frac{j\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$
$$E_\phi = -\frac{1}{\beta_z^2 - \beta^2}\left[\mp j\frac{\beta_z}{\rho}\frac{\partial E_z}{\partial \phi} + j\omega\mu\frac{\partial H_z}{\partial \rho}\right]$$
$$H_\rho = -\frac{1}{\beta_z^2 - \beta^2}\left[\frac{j\omega\varepsilon}{\rho}\frac{\partial E_z}{\partial \phi} \mp j\beta_z\frac{\partial H_z}{\partial \rho}\right]$$
$$H_\phi = \frac{1}{\beta_z^2 - \beta^2}\left[j\omega\varepsilon\frac{\partial E_z}{\partial \rho} \pm j\frac{\beta_z}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

TM Mode

The derivation for TM mode is the same except that we are solving for Ez. We can therefore write

$$E_z(\rho, \phi, z) = [A\sin(\nu\phi) + B\cos(\nu\phi)]\, J_\nu(k_c\rho)e^{-j\beta z}$$

The boundary condition in this case is as follows

$$E_z(a, \phi, z) = 0 \text{ or } J_\nu(k_c a) = 0.$$

This leads to

$$k_c a = p_{\nu n} \qquad \rightarrow \qquad k_c = \frac{p_{\nu n}}{a}$$

TE_{nl} (or H_{nl}) Waves

$$\hat{H}_z = H_z J_n(k_c r)$$
$$k_c = \frac{p'_{nl}}{a}$$
where p'_{nl} is the lth root of $J'_n(x)$
$$E_r = \frac{i\omega\mu_0\mu'}{rk_c^2}\frac{\partial H_z}{\partial \phi}$$
$$E_\phi = \frac{i\omega\mu_{0\mu'}}{k_c^2}\frac{\partial H_z}{\partial r}$$
$$H_r = -\frac{E_\phi}{Z_{TE}}$$
$$H_\phi = \frac{E_r}{Z_{TE}}$$

TM_{nl} (or E_{nl}) Waves

$$\hat{E}_z = J_n(k_c)\cos(n\phi)$$
$$k_c = \frac{p_{nl}}{a}$$
where p_{nl} is the lth root of J_n (x)
$$H_r = \frac{i\omega\varepsilon_0\varepsilon'}{rk_c^2}\frac{\partial E_z}{\partial \phi}$$
$$H_\phi = \frac{-i\omega\varepsilon_0\varepsilon'}{k_c^2}\frac{\partial E_z}{\partial r}$$
$$E_r = H_\phi Z_{TM}$$
$$E_\phi = -H_r Z_{TM}$$

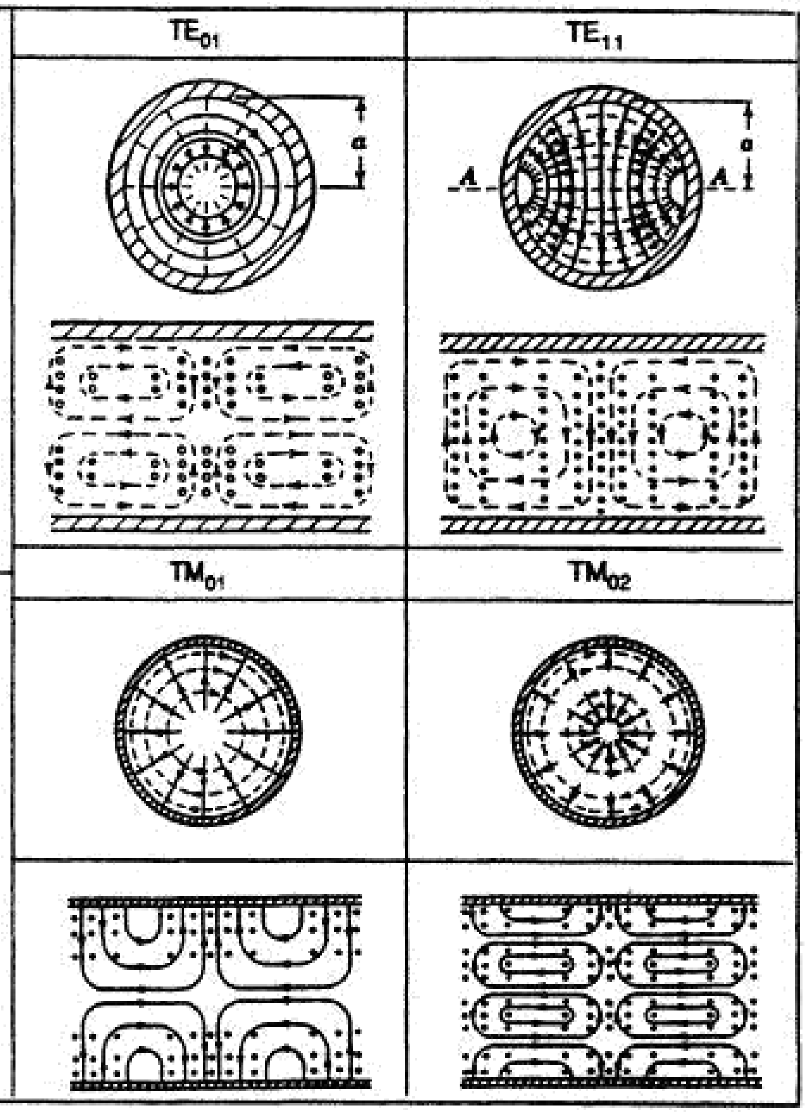


Figure 4: Field distributions and key expressions of calculation for modes in Circular Waveguides

Intrinsic Wave Impedance (η)

Intrinsic Wave Impedance in Circular Waveguide for TE mode is given as follows

$$\eta_{TE} = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1-\frac{f_c^2}{f^2}}} = \frac{\eta_0}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$\boxed{\eta_{TE} = \frac{\eta_0}{\sqrt{1-\frac{f_c^2}{f^2}}}}$$

Where η_o is the intrinsic impedance of a uniform plane wave in a lossless dielectric medium

$$\eta_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

Intrinsic Wave Impedance in Circular Waveguide for TM mode is given as follows

$$\eta_{TM} = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r} = \frac{\beta}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\frac{f_c^2}{f^2}} = \eta_0\sqrt{1-\frac{f_c^2}{f^2}}$$

$$\boxed{\eta_{TM} = \eta_0\sqrt{1-\frac{f_c^2}{f^2}}}$$

Power Flow in Waveguides

Power Flow in a Rectangular Waveguide (TE10) The time-average Poynting vector for the TE10 mode in a rectangular waveguide is given by

$$\langle \mathbf{P} \rangle = \frac{1}{2}\mathrm{Re}[\mathbf{E}\times\mathbf{H}^*] = \hat{\mathbf{z}}\frac{|E_o|^2}{2}\frac{\beta_z}{\omega\mu}\sin^2\frac{\pi x}{a}$$

$$\langle \mathrm{Power} \rangle = \int_0^a\int_0^b \frac{|E_o|^2}{2}\frac{\beta_z}{\omega\mu}\sin^2\frac{\pi x}{a}\,dxdy$$

$$\langle \mathrm{Power} \rangle = \frac{|E_o|^2}{4}\frac{\beta_z ab}{\omega\mu} = \frac{|E_o|^2 ab}{4\eta_{gTE_{10}}}$$

Therefore the time-average power flow in a waveguide is proportional to its cross-section area.

Attenuation Factor and Q of waveguides

Attenuation Factor α is given by

$$\alpha = \tfrac{1}{2} (P_M/P_T)$$

Where P_M is the Total Power Loss into the Conductor and P_T is the Power Transmitted

Q of the waveguide is defined as follows

$$Q = 2\pi \text{ (energy stored per cycle/energy lost per cycle)}$$

TEXT / REFERENCE BOOKS

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