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**SCHOOL OF BUILDING AND ENVIRONMENT**

**DEPARTMENT OF CIVIL ENGINEERING**

# UNIT – I – BASIC CONCEPTS – SCIA1502

# UNIT – I – BASIC CONCEPTS

**Properties of Reinforced concrete materials - Stress - Strain diagrams of concrete and steel-Design philosophies (Principles of Elastic method, ultimate load and limit state method)-Design code and specification- Behavior of RC beams in bond development length and Anchorage-Loading standard as per IS 875-Behavior of RC structural systems under gravity and lateral load.**

## INTRODUCTION

Concrete is a mixture of cement, aggregates, water and admixtures in an adequate proportion. It is a rock like material. Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions. It has *enough strength in compression*, but has *little strength in tension*. Due to this, *concrete is weak in bending, shear and torsion*. Hence to overcome the issues *reinforcement in concrete* was obtained to withstand the properties required for a structure.

### Properties of Fresh Concrete

Concrete should be such that it can be transported, placed, compacted and finished without harmful segregation. The mix should maintain its uniformity and not bleed excessively; these two are collectively called as workability .Bleeding is movement and appearance of water at the surface of freshly-placed concrete, due to settlement of heavier particles. Consistency is a measure of its wetness and fluidity .Measured by the slump test and Workability dependent on water content, fineness of cement, and surface area of aggregates.

### Properties of Hardened Concrete:

The important properties of concrete are

a. Compressive strength

b. Tensile strength

c. Shear strength

d. Bond strength

e. Density

f. Impermeability

g. Durability

h. Ductility

Among these properties, compressive strength of concrete is the most valuable and can be easily tested in laboratory.

This is generally measured on concrete cubes or cylinders.

- ❖ Many of the properties of concrete can be inferred from compressive strength, using correlation that has been established.
- ❖ Quality of concrete depends on the compressive strength.
- ❖ Dependent on strength (compressive, tension and flexure), Modulus of elasticity, Durability, Creep and shrinkage
- ❖ Concrete is classified under different grades depending upon its characteristic strength.

***"Characteristic strength is defined as the strength of material below which not more than 5 percent of the test results are expected to fall".***

- ❖ Strength of concrete varies for the same concrete mix, which give different compressive strength in laboratory tests.
- ❖ Variability in strength evidently depends on degree of quality control.

- ❖ Variability in strength is measured in terms of either the "Standard Deviation" or the Coefficient of Variation (COV), which is the ratio of standard deviation to mean strength.
- ❖ Due to significant variability in strength, it is necessary to ensure that the designer has a reasonable assurance of a certain minimum strength of concrete.
- ❖ Characteristic strength provides minimum guaranteed strength.

**What is a stress-strain diagram?**

- ➢ It is a tool for understanding **material behavior under load.**
- ➢ A stress strain diagram **help engineers to select the right materials for specific loading conditions**. In other words, It is a graph that represents how a part behaves under an increasing load, and used by engineers when selecting materials for specific designs
- ➢ A stress-strain diagram generally contains three regions:
    - ❖ **Elastic region**: This portion is generally represented as a linear relationship between stress and strain. If the load is released the specimen will return to its original dimensions.
    - ❖ **Plastic region**: In this portion, the specimen begins to yield. The maximum strength of the specimen occurs in this zone. The specimen endures some permanent deformation that remains after the load is released.
    - ❖ **Rupture**: The point at which a specimen breaks into two parts.

**<u>Stress - Strain diagram of concrete</u>**

- ❖ Concrete is mostly used in compression that is why **its compressive stress strain curve is of major attention**.
- ❖ The stress and strain of concrete is **obtained by testing concrete cylinder specimen at age of 28 days**, using compressive test machine.
- ❖ The stress strain curve of concrete allows designers and engineers to **anticipate the behavior of concrete** used in building constructions.
- ❖ Stress strain curve of concrete is a graphical representation of concrete behavior under load.
- ❖ It is produced by plotting **concrete compressive strain at various interval of concrete compressive stress** (which is the loading).
- ❖ The S-S curve for hardened concrete is **almost linear**.

- ❖ The aggregate is more rigid than the cement paste and will therefore deform less( that is it will have lower strain) under same applied stress.
- ❖ The **S-S curve of concrete lies between the aggregate and the cement paste**.
- ❖ However, this relationship is **non-linear over the most of the range**, that is why micro cracks are formed.
- ❖ The cracks are formed
- ❖ at the interface between aggregate particles and cement paste as a result of the differential movement between the two phases
- ❖ Within the cement paste.
- ❖ These cracks are formed as result of **changes in temperature, moisture and application of load.**
- ❖ The experimental or actual stress strain curve for concrete is very difficult to use in design. Therefore, IS code 456:2000 has simplified as below.

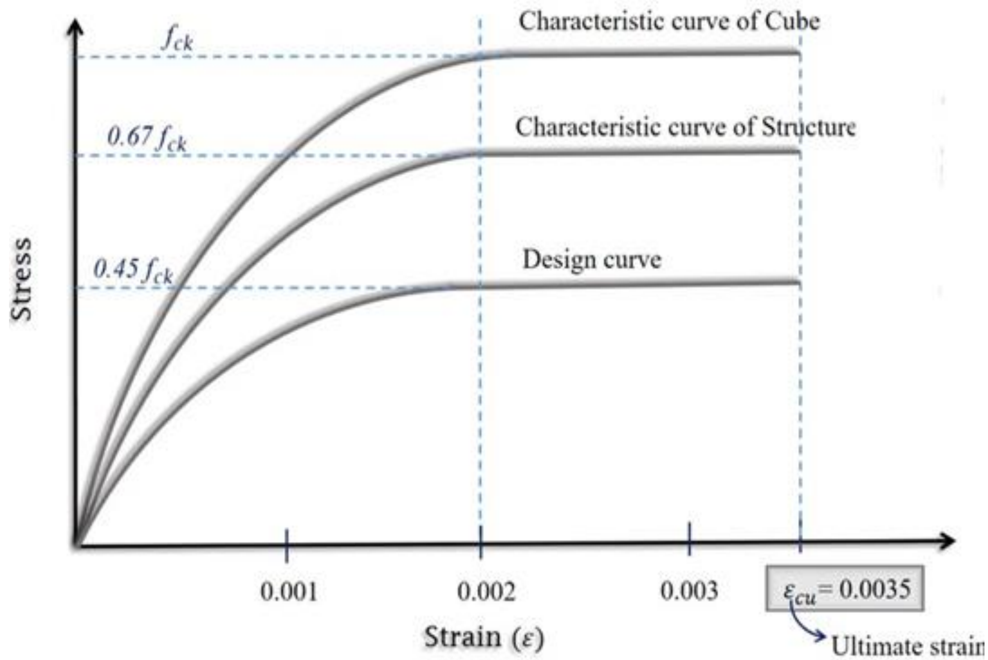



***Fig 1 Stress Strain Curve for Concrete***

- ❖ For design purpose, the compressive strength of concrete in the structure in taken as 0.67 times the characteristic strength.
- ❖ The 0.67 factor is introduced to account for the difference in the strength indicated by a cube test and the strength of concrete in actual structure.

- ❖ The partial safety factor equal to 1.5 is applied in addition to this 0.67 factor.
- ❖ The initial portion of the curve is parabolic. After a strain of 0.002 (0.2%), the stress becomes constant with increasing load, until a strain of 0.0035 is reached and here the concrete is assumed to have failed.
- ❖ **As per IS specifications, for M30 grade concrete.**
- ❖ **Up to 0.002 of strain the curve will be parabolic for a concrete cube**
- ❖ **And then the slope becomes zero up to ultimate strain of 0.0035**

**<u>As per IS specifications</u>**

a) The earlier given was for the characteristic strength of a standard 150x150x150 mm size of concrete cube.

b) But when the design is for a entire structure then the compressive strength of the concrete will get reduced due to the influence of the soil as given in the figure.

c) For design purpose Limit state method is adopted as in IS codes. So a partial safety factor of 1.5 is considered.

d) Therefore the design stress of concrete is taken as [(0.67*$f_{ck}$ )/ 1.5] = 0.45 $f_{ck}$

e) Hence for the design problems the design stress of concrete is taken as 0.45 $f_{ck}$ and the ultimate strain of concrete is taken as 0.0035.

f) Thus finally results in a design curve as shown in figure.

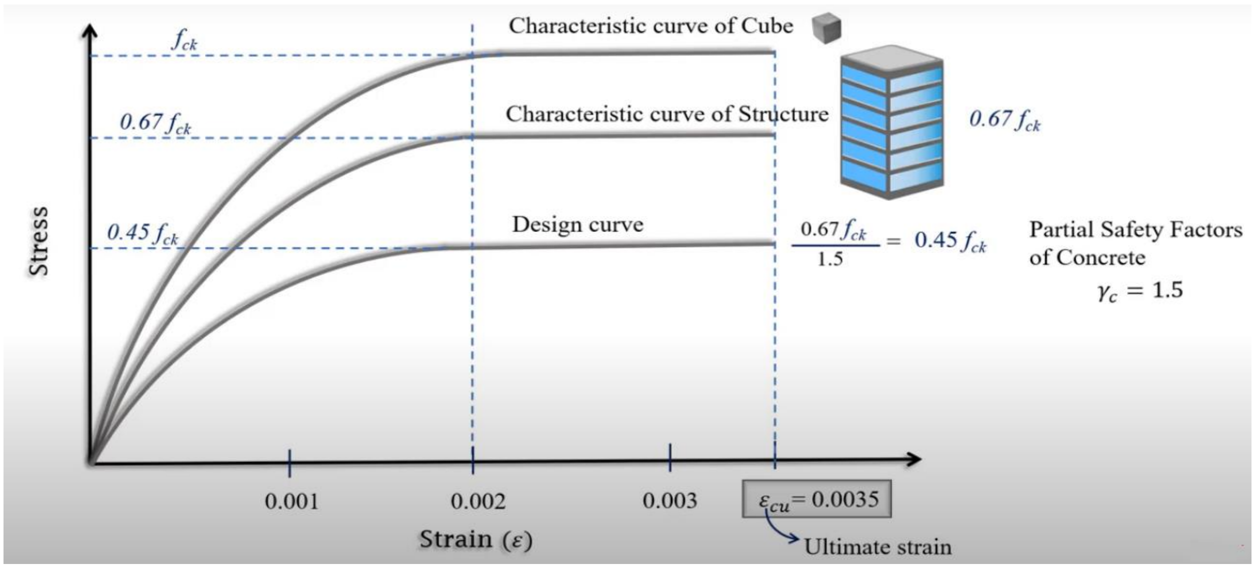



*Fig 2 Stress Strain design Curve for Concrete cube*

## Stress - Strain diagram of mild steel

- If the force is considerably large the material will experience elastic deformation but the ratio of stress and strain will not be proportional. (Point A to B). This is the elastic limit.
- Beyond that point the material will experience plastic deformation.
- The point where plastic deformations starts is the yield point which is shown in the figure as point B. 0-B is the upper yield point.
- Resulting graph will not be straight line anymore. C is the lower yield point.
- D is the maximum ultimate stress.
- E is the breaking stress. It is the area of the whole curve (point 0-E). Energy absorbed at unit volume up to breaking point.
- If tensile force is applied to a steel bar it will have some extension.
- If the force is small the ratio of the stress and strain will remain proportional

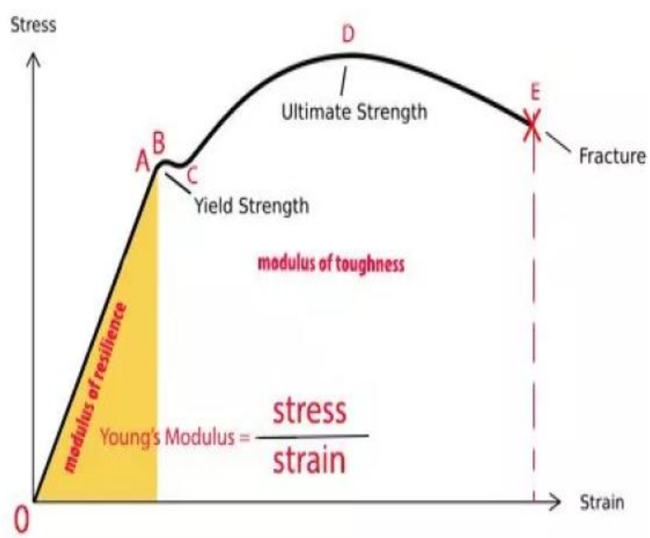



*Fig 3 Stress Strain Curve for mild steel*

## Stress - Strain diagram HYSD (High Yield Strength Deformed) bars

- There is no specified yield point for HYSD bars, the code gives the characteristic curve as given above.
- So considering the strain value of 0.002 the yield stress is noted.
- The design curve almost same as mild steel and the design stress is also 0.87 $f_y$.
- But the strain at this point is different as given above.

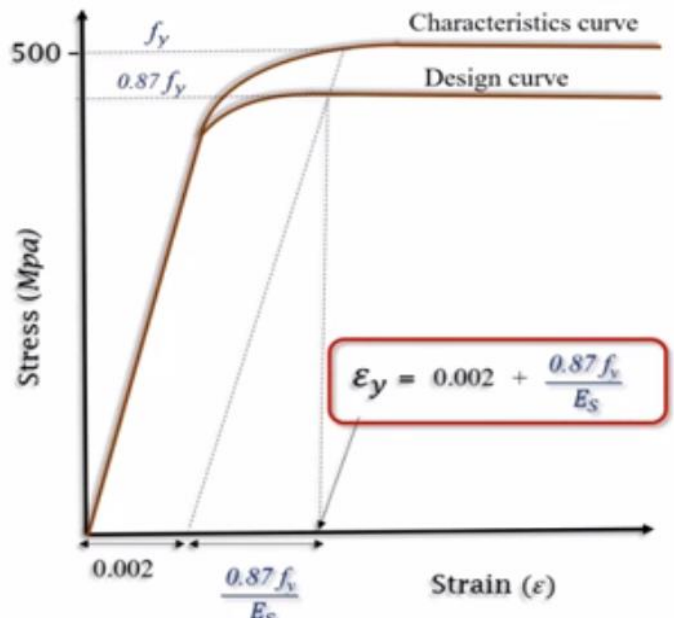



***Fig 4 Stress Strain Curve for HYSD bars***

**DESIGN PHILOSOPHIES**

- A design philosophy is a set of assumptions and procedures which are used to meet the conditions of serviceability, safety, economy and functionality of the structure. Some of the design philosophies that has been used by engineers are

    1. Working Stress Method (WSM)
    2. Ultimate Load Method (ULM)
    3. Limit State Method (LSM)

Working Stress Method

- The sections of the members of the structure are designed assuming straight line stress-strain relationships ensuring that at service loads the stresses in the steel and concrete do not exceed the allowable working stresses.
- The allowable stresses are taken as fixed proportions of the ultimate or yield strength of the materials.
- The B.Ms and forces that act on statically indeterminate structures are calculated assuming linear – elastic behaviour.
- Reinforced concrete sections behave in elastically at high loads. Hence elastic theory cannot give a reliable prediction of the ultimate strength of the

members because inelastic strains are not taken into account.

- ❖ For structures designed by the working stress method, the exact load factor is unknown and varies from structures to structure.

**Ultimate Load Method:**

- ❖ Sections of members of the structures are designed taking inelastic strains into account to reach ultimate (maximum) strength when an ultimate load, equal to the sum of each service load multiplied by its respective load factor, is applied to the structure.
- ❖ The beginning moments and forces that act as statically indeterminate structures at the ultimate load are calculated assuming non linear elastic behaviour of the structure up to the ultimate load. i.e., redistribution of same actions is taking place due to nonlinear relationship between actions and deformations.
- ❖ Ultimate strength design makes more efficient use of high strength reinforcement and smaller beam depths can be used without compression steel.
- ❖ Ultimate strength design allows the designer to assess the ductility of the structure in the post-elastic range.
- ❖ If the sections are designed based on ultimate strength design, there is a danger that although the load factor is adequate. The cracking and the deflections at the service loads may be excessive.
- ❖ Cracking may be excessive if the steel stresses are high or if the bars are badly distributed.
- ❖ Deflections may be critical if the shallow sections, which are possible in USD, are used and the stresses are high.
- ❖ To ensure a satisfactory design, the crack widths and deflections at service loads must be checked to make sure that this lies within reasonable limiting values, as per functional requirements of the structure. This is done by use of elastic theory.

**Limit state method of design**

The object of the design based on the limit state concept is to achieve an acceptable probability, that a structure will not become unsuitable in its lifetime for the use for which it is intended i.e. it will not reach a limit state

- ❖ A structure with appropriate degree of reliability should be able to withstand safely.

- ❖ All loads that are reliable to act on it throughout its life and it should also satisfy the sustainability requirements, such as limitations on deflection and cracking.
- ❖ It should also be able to maintain the required structural integrity, during and after accident, such as fires, explosion & local failure i.e. limit sate must be consider in design to ensure an adequate degree of safety and serviceability
- ❖ The most important of these limit states, which must be examine in design are as follows Limit state of collapse - Flexure, Compression, Shear and Torsion Limit state of serviceability
- ❖ This state corresponds to the maximum load carrying capacity.

**DESIGN CODE AND SPECIFICATION**

**Objectives**

- ➢ To provide adequate structural safety
- ➢ To specify simple design procedures, design table and formula
- ➢ To provide legal validity and to protect structural engineers
- ➢ To provide uniform set of guidelines

**Hand books are:**

- ➢ SP: 16 – 1980 - Design Aids for Reinforced Concrete to IS: 456 – 1978
- ➢ SP: 24 – 1983 - Explanatory handbook on Indian standard code of practice for plain and reinforced concrete
- ➢ SP: 34 – 1987 - Handbook on Concrete Reinforcement and Detailing

Code Book is

IS: 456 – 2000 - *Indian Standard* PLAIN AND REINFORCED CONCRETE CODE OF PRACTICE *(Fourth Revision)*

## Deflection Criteria

Deflection of structure or part thereof shall not adversely affect the appearance or efficiency of structure or finishes or partitions**.**

Deflection shall generally be limited to the following:

i. Final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from as-cast level of supports of floors, roofs and all other horizontal members should not normally exceed span/250.
ii. Deflection including effects of temperature, creep and shrinkage occurring after erection of partitions and application of finishes should not normally exceed (span/350) or 20mm whichever is less.

## Factors influencing limits on deflection in flexural members

- Aesthetic/psychological discomfort
- Crack width limitation
- Effect on attached structural and non structural elements
- Ponding in (roof) slabs

## DESIGN FOR BOND IN RC

- **Bond stress**: stress developed between surface of steel reinforcement and surrounding concrete by which slip occurs when concrete and steel act together in structural member by transferring stress between each other.
- **Slip**: relative displacement between reinforcement and surrounding concrete.
- Bond stresses are, in effect, longitudinal shearing stresses developed on surface between steel and surrounding concrete wherever stress in a bar changes.
- Importance of determining bond stress in tensile reinforcing bars has increased due to the following reasons
  - ✓ More frequent use of high grade steel with larger bar diameter and
  - ✓ Adopting refined ultimate strength design procedures

## DISTRIBUTION OF BOND STRESS

- Bond stress in a beam or wall is neither uniform nor gradually varying from point to point
- Very large bond stresses develop adjacent to tension crack

- ❖ Essentially ultimate bond stresses exist close by on the same bar, even reversed in direction in many cases.
- ❖ Thus there is a practical problem as to how to describe or measure or evaluate such a fluctuating stress condition.

**BOND MECHANISM**

Bond can be thought of as shearing stress or force between a bar and surrounding concrete Bond resistance is offered by

- ✓ Chemical Adhesion – gel like hydration products
- ✓ Friction–surface roughness and concrete shrinkage
- ✓ Mechanical Interaction - ribs in deformed bars Importance of Bond

Ultimate Limit State

- – Anchorage of Reinforcement
- – Control of Hinge rotations in flexural members
- – Maintenance of Composite action

Serviceability Limit State

- – Control of Flexural Cracking
- – Control of Member Deformations

**Factors affecting bond strength**

- ❖ Type of Reinforcement – Plain or Deformed
- ❖ Diameter of bars
- ❖ Grade of Concrete
- ❖ Cover to reinforcement
- ❖ Confinement of Concrete
- ❖ Direction of Casting
- ❖ Top bar effect
- ❖ Spacing of bars
- ❖ Relative rib area of bars
- ❖ Quality of reinforcement
- ❖ Coating to reinforcement

**Types of Bond**

- Flexural bond: arises in flexural members on account of shear or variation in bending moment-critical at points where shear is significant.
- Anchorage bond: arises over a length of anchorage provided for a bar or near the end of a reinforcing bar and resists pulling out of bar in tension.

## WHAT IS DEVELOPMENT LENGTH?

Length of steel bar needed to be embedded into the column to establish the desired bond strength between concrete and steel

Holds Two Concrete Members together

Beam, Column, Footing etc

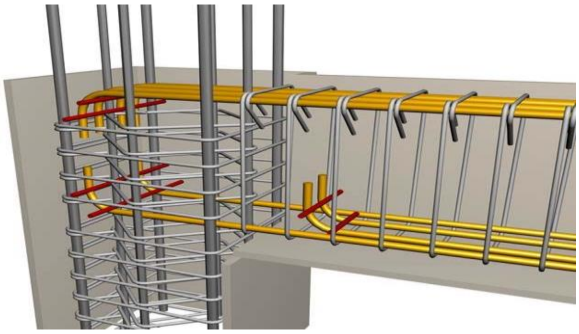
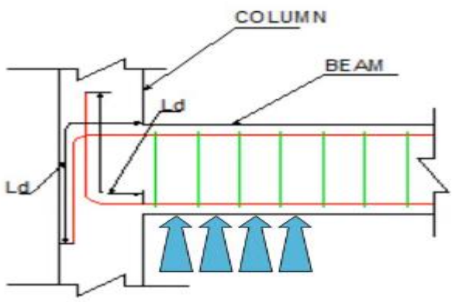
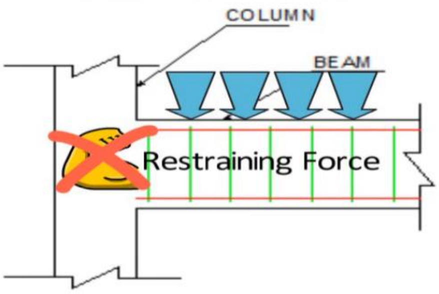


*Fig 5 Development length*




It acts as a Supporting Member for the Reinforced Beam in the Concrete Column

**What will happen if we dont provide Development Length?**




If we provide less development length

Bars will not break first

Reinforcement bars will split from Concrete

**Beam will come out of the Concrete Column**

**Table 1 Difference between three lengths**

| Development length | Anchorage length | Lap length |
|---|---|---|
| Development length must be provided for the required amount of reinforcement(as per moment) on either sides of every section in the member so that the reinforcement doesn't slip as it approaches the ultimate stress at that section | Anchorage is the length of reinforcement that needs to be embedded into the support for complete stress transfer.<br>**Anchorage bars** are different for bars in tension and compression.<br>**Bends** are taken ***as 4 times the diameter of the bar for 45 degree bend, 8 times the diameter of the bar for 90 degree bend, 16 times the diameter of the bar for 135 degree bend. U-hooks shall be taken 16 times the diameter of the bar***. | Lap is required when a reinforcement bar needs to be spliced at a section. This is again to ensure sufficient development length at every section. The bar that ends midway lacks the sufficient development length at the sections closer to its end and the bar that begins midway lacks the development length at the beginning sections. In order to satisfy both the requirements the bars are overlapped for a sufficient length on either sides of the intended splice point. |

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

# UNIT – II – BEAMS – SCIA1502

## UNIT – II – BEAMS

**Flexure of RCC beams of rectangular section - Balanced, Under Reinforced and Over Reinforced Sections - Analysis and Design of Singly and Doubly Reinforced beams - Design of flanged Sections (T and L beams) and Continuous Beams - Analysis and design with and without shear reinforcement as per IS: 456**

### BEAM

- A Beam is an inevitable horizontal or sloping structural element to resist the load of the structure.
- The main function of the beam is designed to resist the external or internal load such as wall, slab and floors of the building and distribute the load to the foundation through the column.
- The horizontal beam carries an only transverse (vertical) load and the sloping beams carry both transverse and axial load.

### LIMIT STATE METHOD OF DESIGN

- The object of the design based on the limit state concept is to achieve an acceptable probability, that a structure will not become unsuitable in it's lifetime use for which it is intended.
- A structure with appropriate degree of reliability should be able to withstand safety.
- It should also be able to maintain the required structural integrity, during and after accident, such as fires, explosion & local failure. i.e. limit sate must be consider in design to ensure an adequate degree of safety and serviceability

- The most important of these limit states, which must be examine in design are as follows Limit state of collapse
    - **Flexure**
    - **Compression**
    - **Shear**
    - **Torsion**
- This state corresponds to the maximum load carrying capacity.

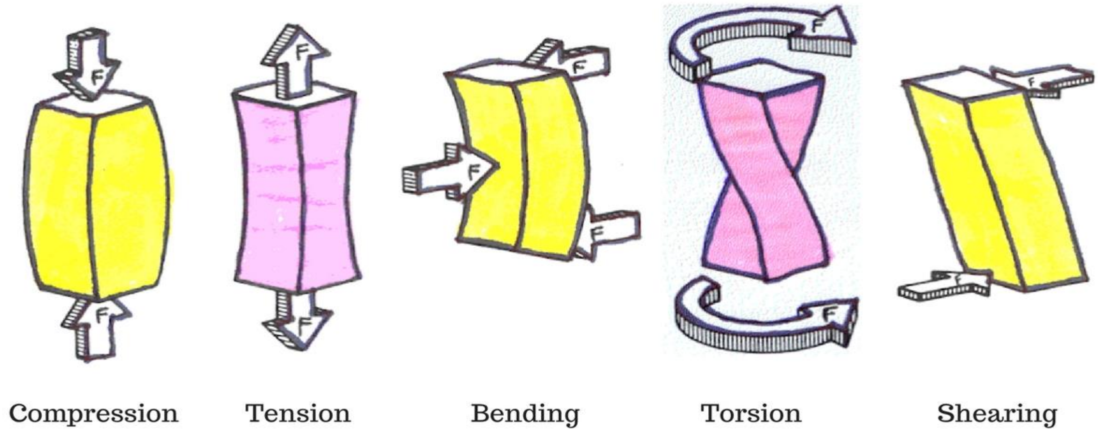



*Fig 1 Types of stresses*

## Limit State :

The acceptable limit for the safety and serviceability requirements before failure occur.

**Types of limit state**

1. Limit state of **collapse or failure**:
   - ❑ Flexural (Bending)
   - ❑ Compression
   - ❑ Shear
   - ❑ Torsion

   Based on *imaginary behaviour* of structure

2. Limit state of **serviceability:**
   - ❑ Deflection
   - ❑ Cracking
   - ❑ Vibration
   - ❑ Fire resistance
   - ❑ Etc.

   Based on *actual behaviour* of structure

**WHAT IS DEPTH OF NEUTRAL AXIS?**

- ➢ Neutral axis is the axis at which the stresses are zero and it is situated at the centre of gravity of the section, **which is neither compression nor tension.**
- ➢ The maximum depth of neutral axis is limited to ensure that tensile steel will reach its yield stress before concrete fails in compression, thus a brittle failure is avoided.

**WHAT IS EFFECTIVE DEPTH?**

- ➢ The effective depth of the beam is the **distance from the tension steel to the edge of the compression fiber**.

- Therefore, we can say that the effective depth of a beam section is a distance as measured from top fiber of beam to centroid of steel reinforcement.

**WHAT IS EFFECTIVE COVER?**

- Effective cover is taken as distance taken from bottom concrete fiber section from the center level of the reinforcement.

***Effective cover = overall depth – effective depth (OR) clear cover + (diameter of bar/2)***

**WHAT IS CLEAR COVER?**

- Clear cover is the distance measured from the exposed concrete surface (Without plaster and other finishes) to the nearest surface of the reinforcing bar.

**WHY EFFECTIVE COVER IS PROVIDED?**

- To protect the steel reinforcement bars (rebars) from environmental effects to prevent their corrosion;
- To provide thermal insulation, which protects the reinforcement bars from fire, and;
- To give reinforcing bars sufficient embedding to enable them to be stressed without slipping.

**FLEXURE OF RCC BEAMS OF RECTANGULAR SECTION**

- Flexural members are **slender members that deform primarily by bending moments** caused by concentrated couples or transverse forces.
- In modern construction, these **members may be** joists, **beams**, girders, lintels, and other specially named elements.
- But their **behavior** in every case **is essentially the same**.
- Unless otherwise specified in a problem, **flexural members will be referred to as beams** here.

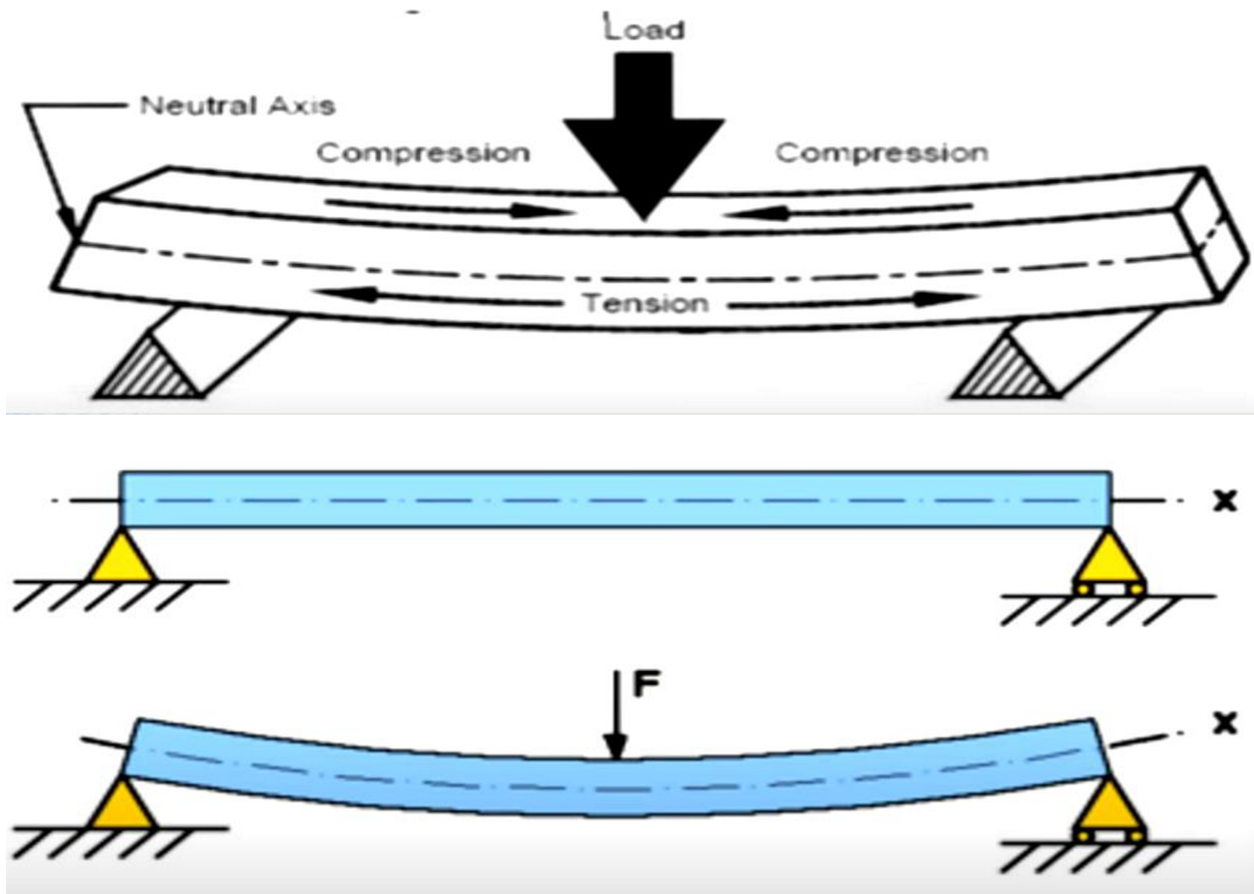



*Fig 2 stresses in a beam member*

**<u>Assumptions to determine Moment of resistance of Reinforced concrete beams</u>**

1. **Plane sections remain plane before and after bending**. This means that strains are proportional to distance from the neural axis.
2. Ultimate limit state of bending failure is assumed to have been reached when the strain in the concrete at the extreme bending compression fiber reaches 0.0035.
3. The stress distribution across compression face will correspond to the stress-strain diagram for concrete in compression.
4. The **tensile strength of concrete is neglected as the section is assumed to be cracked up to the neutral axis**.
5. **The stress in steel will correspond to the corresponding strain in the steel**
6. As given in assumption 2 above **that the reinforced concrete section in bending is assumed to fail when the compression strain in concrete reaches the failure strain** in bending compression equal to 0.0035.

## STRESS STRAIN PROFILE FOR BEAM

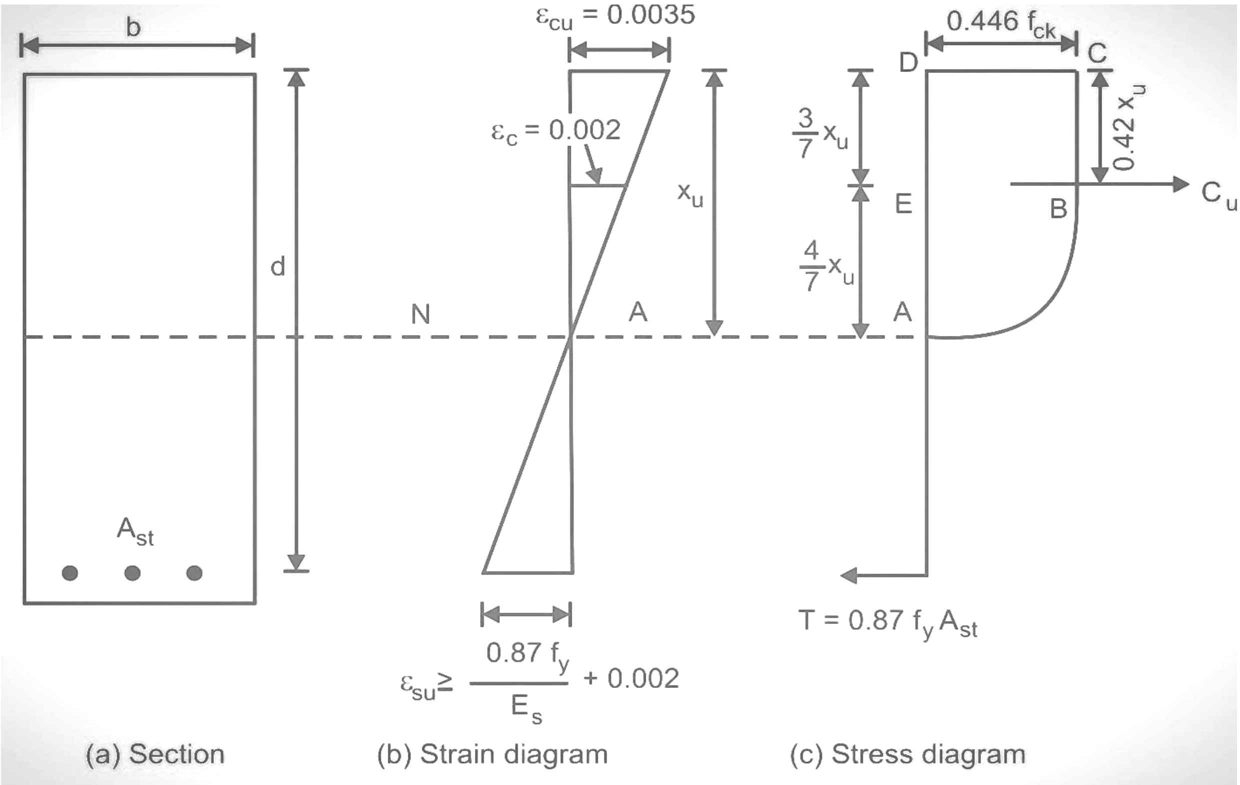



***Fig 3 stress strain profile for beam***

## TYPES OF REINFORCED CONCRETE BEAMS

- Singly reinforced beam
- Doubly reinforced beam
- Singly or doubly reinforced flanged beams
- Continuous beams

## SINGLY REINFORCED BEAM

- The beam that is longitudinally reinforced only in tension zone, it is known as singly reinforced beam.
- In such beams, the final bending moment and the stress because of bending are carried by the reinforcement, while this compression is carried by the concrete.
- But it is not possible to provide reinforcement only in the tension zone, because we need to tie the stirrups.
- Therefore, two rebars/ holding bars are used in the compression zone to tie the stirrups, and the rebars act as false members only to hold the stirrups

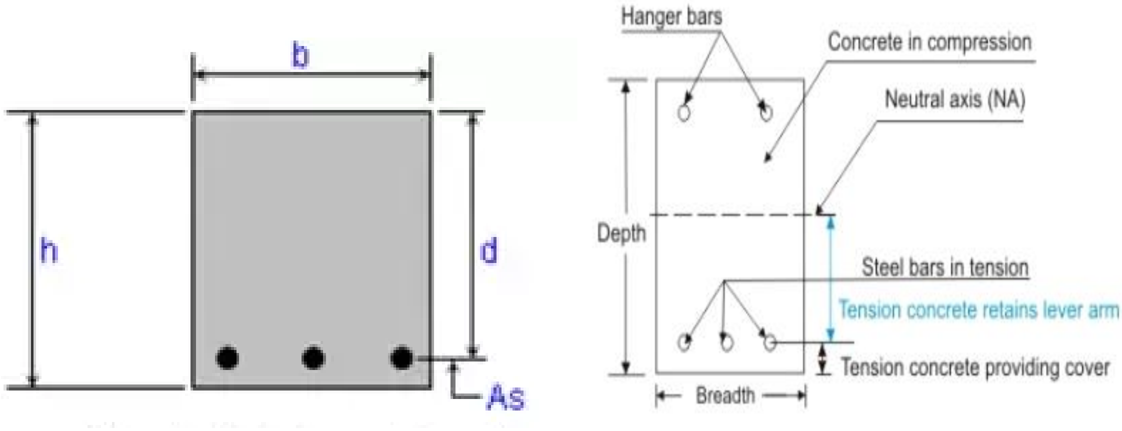



*Fig 4 Singly reinforced section*

## DOUBLY REINFORCED BEAM

- The beam that is reinforced with steel in the tension and compression zone is known as the doubly reinforced beam.
- The doubly reinforced beams have compression reinforcement in addition to the tension reinforcement, and this compression reinforcement can be on both sides of the beam (top or bottom face), depending on the type of beam, that is, simply supported or cantilever, respectively

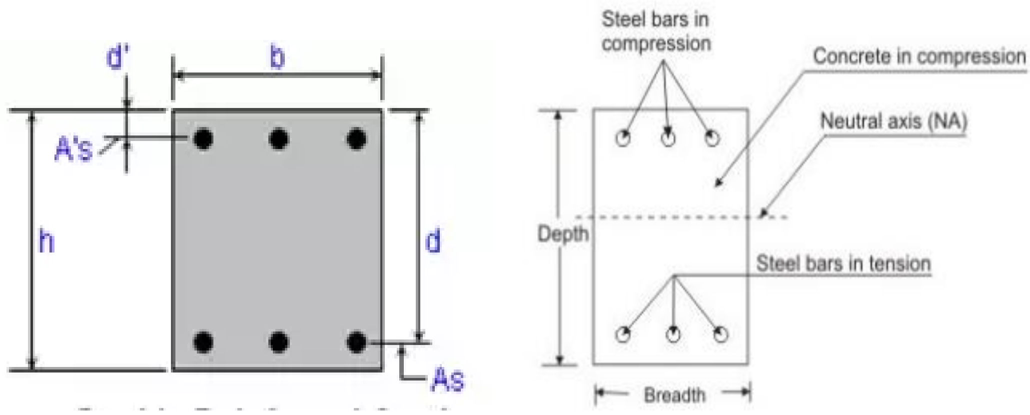



*Fig 5 Doubly reinforced section*

## WHY A DOUBLY REINFORCED BEAM?

- This type of beam will be considered necessary when, due to the consideration of headroom or architecture, the depth of the beam is restricted.
- And when the singly reinforced section is insufficient to resist the bending moment on the section additional tension and compression reinforcement are designed based on steel beam theory.
- The doubly reinforced beam (DRB) section is used where the span is more, where cross section will also be increased.

- Depth can be reduced and the $A_{st}$ can be increased.
- In DRB, the top and bottom reinforcement must be designed.

## SINGLY OR DOUBLY REINFORCED FLANGED BEAMS

- The flanged beam may be considered as a rectangular beam of width $b_f$ and effective depth d, if the neutral axis is in the flange as the concrete in tension is ignored. However, if the neutral axis is in the web, the compression is taken by the flange and a part of the web

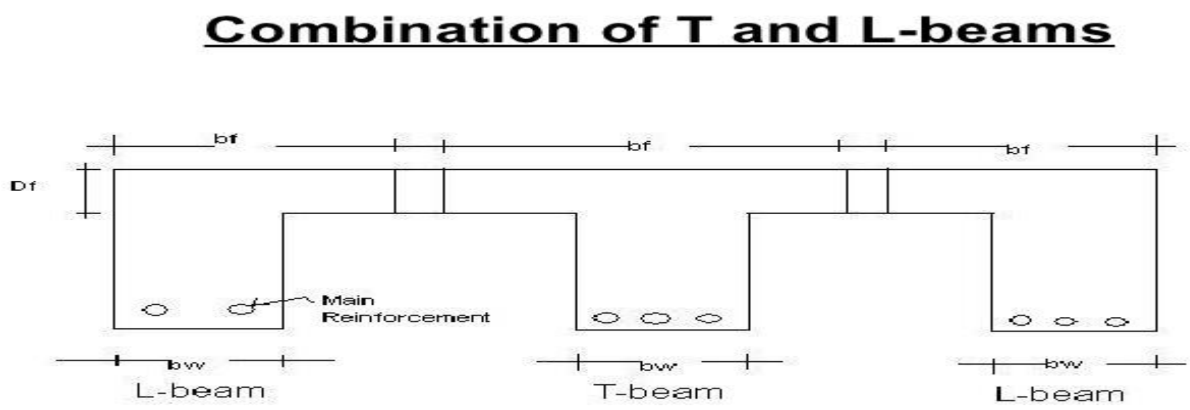



*Fig 6Flanged beams*

## CONTINUOUS BEAMS

- Beams are made continuous over the supports to increase structural integrity.
- A continuous beam provides an alternate load path in the case of failure at a section.
- In regions with high seismic risk, continuous beams and frames are preferred in buildings and bridges.
- A continuous beam is a statically indeterminate structure.

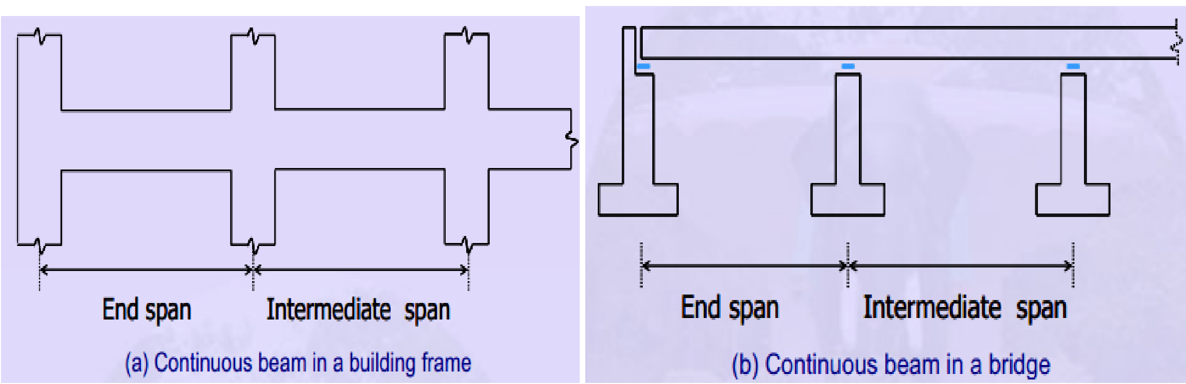



*Fig 7 Continuous beams*

**Balanced, Under-Reinforced and Over-Reinforced Beam Sections**

**Balanced Beam Section**

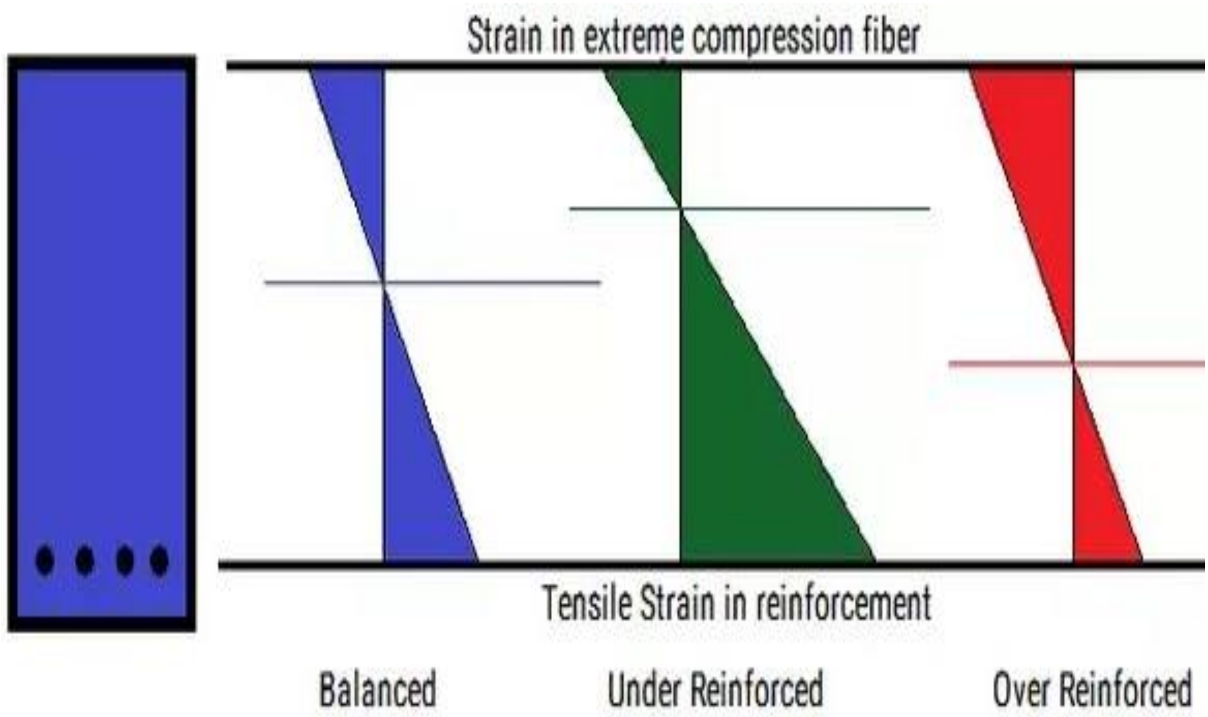
- Reinforced concrete beam sections in which the **tension steel also reaches yield strain simultaneously as the concrete reaches the failure strain in bending** are called balanced sections.
- Steel bars, inside fail first and then the concrete (i.e.,) Steel failure to concrete failure

**Under-Reinforced Beam Section**

- Reinforced concrete beam sections in which the **steel reaches yield strain at loads lower than the load at which the concrete reaches failure strain** are called under-reinforced sections.
- The concrete has maximum yield strain (i.e.,) steel reaches yield strain at lower load than concrete

**Over-Reinforced Beam Sections**

- Reinforced concrete beam sections, in which **the failure strain in concrete is reached earlier than the yield strain of steel is reached**, are called over-reinforced beam sections.
- The concrete fails first then the steel (i.e.,) concrete failure to steel failure



***Depth of Neutral axis***

Consider a rectangular beam section,

b- Width of section

d- Effective depth

$A_{st}$- Area of steel reinforcement = $\pi d^2/4$

$X_u$- depth of neutral axis

For equilibrium of forces at the limit state of collapse    Pg. No. 96

*Limiting values of ($X_{umax}$/d) for different grades of steel forming table Ref Pg. no. 70*

| Grades of steel | $X_{u,max}/d$ | Expression for Mu limit |
|---|---|---|
| Fe250 | 0.53 | $0.149 f_{ck}.b.d^2$ |
| Fe415 | 0.48 | $0.138 f_{ck}.b.d^2$ |
| Fe500 | 0.46 | $0.149 f_{ck}.b.d^2$ |

*<u>Steps to calculate Mu:</u>*

Find d

*Step 1;* to find Mu, find depth, d.

To find d, we have to equate compression force = tension force (i.e)

So equate C =T

$0.36\ f_{ck}\ b\ X_u = 0.87\ f_y\ A_{st}$

Find neutral axis co-efficient which is $X_u/d$

$X_u = (0.87\ f_y\ A_{st}) / (0.36\ f_{ck}\ b)$

Divide both sides by d = $\frac{Xu}{d} = \frac{0.87\ .fy\ .Ast}{(0.36.fck.b.d)}$

*Step 2;* From table in IS 456-2000 Pg No. 96

To find $\left(\frac{Xu}{d}\right)$max,    Fe250 = 0.53

Fe415 = 0.48

Fe500 = 0.46

From this we can get that whether beam is under reinforced or over reinforced or balanced.

i. $\left(\frac{Xu}{d}\right)_{max} < 0.48$
ii. $\left(\frac{Xu}{d}\right)_{max} = 0.48$
iii. $\left(\frac{Xu}{d}\right)_{max} > 0.48$

***For Fe415:***

If Xu/d is equal to 0.48, then it is balanced section

[ Mu = Mu,lim]

If Xu/d is less than limiting value 0.48, under reinforced section

[Mu<Mu,lim]

If Xu/d is greater than 0.48, than it is over reinforced section

[Mu >Mu,lim]

Analysis Problems

**Q1. A rectangular reinforced concrete beam of width 200 mm & it is reinforced with 2 steel bars of 20 mm diameter and effective depth of 400 mm. If M20 grade concrete and Fe 415 steel as used. Estimate the ultimate moment of resistant.**

*Given Data:*

b=200mm,

d=400mm,

$f_{ck}$ = 20 N/mm²

$A_{st}$ = = 2 * [(π*20²)/4] = 628.32 mm²

Ast = 628.32 mm²

<u>Solution</u>*:*

Depth of Neutral axis:

$$\frac{Xu}{d} = \frac{0.87\,.fy\,.Ast}{(0.36.fck.b.d)}$$

$$= \frac{0.87*415*628.32}{(0.36*20*200*400)}$$

=0.39

$$\frac{Xu}{d} = 0.394 \qquad \left(\frac{Xu}{d}\right)\text{max} = 0.48$$

$$\left(\frac{Xu}{d}\right) < \left(\frac{Xu}{d}\right)\text{max} = 0.394 < 0.48$$

Therefore section is under reinforced section

Moment of resistance

$$\text{Mu} = 0.87\,.\,\text{fy}.\,\text{Ast}\,.\,\text{d}\left[1 - \left(\frac{\text{Ast.fy}}{\text{b.d.fck}}\right)\right]$$

$$= 0.87 * 415 * 628.32 * 400\left[1 - \frac{628.32*415}{200*400*20}\right]$$

$= 75.95*10^6$ N.mm

= 75.95 KN.m

Q2. A reinforced concrete beam of rectangular section 200mm wide and 550mm deep. It is reinforced with the 4 bars of 25mm diameter at effective depth of 500mm. Using M20 grade concrete and Fe415 HYSD bars. Calculate the safe moment of resistance of section.

*Given Data:*

b =200mm, d = 500mm, fck = 20N/mm² $f_y$ = 415 N/mm²

Ast = n$[\frac{\pi d^2}{4}]$

Ast = 4[π*25²/4] = 1963.49 mm²

<u>*Solution:*</u>

Depth of neutral axis

$$\frac{Xu}{d} = \frac{0.87\ .fy\ .Ast}{(0.36.fck.b.d)}$$

$$= \frac{0.87*415*1963.49}{0.36*20*200*500}$$

$$= 0.98$$

$\left(\frac{Xu}{d}\right) > \left(\frac{Xu}{d}\right)_{max}$ (i.e) 0.98 > 0.48

(i.e) It is over reinforced concrete

Moment of Resistance, Mu:

$M_u = 0.138*f_{ck}*b*d^2$ (or) $M_u = 0.36(X_{umax}/d)\ [1-0.42(X_{umax}/d)]\ b*d^2 *f_{ck}$

$M_u = 0.138*20*200*500^2$ (or) $M_u = 0.36*0.48[1-(0.42*0.48)]\ 20*200*500^2$

$M_u = 138*10^6$ N mm (or) $M_u = 138*10^6$ N mm

$M_u$ =138 KN.m

Q3. A reinforced concrete beam of 300mm wide is reinforced with 1436mm² of Fe415 HYSD bars at an effective depth of 500mm, if M20 grade concrete is used estimate the moment of resistance of the section.

*Given Data:*

b =300mm, d = 500mm, $f_{ck}$ = 20N/mm² $f_y$ = 415 N/mm² $A_{st}$=1436mm²

<u>*Solution:*</u>

Depth of Neutral axis

$$\frac{Xu}{d} = \frac{0.87\ .fy\ .Ast}{(0.36.fck.b.d)}$$

$$=\left[\frac{(0.87*415*1436)}{(0.36*20*300*500}\right]$$

$$=0.48$$

$\left(\frac{Xu}{d}\right) = \left(\frac{Xu}{d}\right)_{max}$ => it is a balanced section

Moment of resistance, Mu:

$M_u = 0.138*f_{ck}*b*d^2$ (or) $M_u = 0.36(X_{umax}/d)[1-0.42(X_{umax}/d)]\ b*d^2*f_{ck}$

$M_u= 0.138*20*300*500^2$ (or) $M_u = 0.36*0.48[1-(0.42*0.48)]\ 20*300*500^2$

$M_u = 207*10\text{^}6$ N mm (or) $M_u = 206.9*10\text{^}6$ N mm

$M_u$=207 KN.m

Q4. Determine area of reinforced steel required for singly reinforced concrete section having a breadth of 300mm, effective depth of 600mm to resist a factor moment of 200KN.m. Adopt fck=20N/mm² and fy = 415 N/mm².

*Given Data:*

b =300mm, d = 600mm, fck = 20N/mm² fy = 415 N/mm², Mu (factor moment)= 200KN.m

<u>*Solution:*</u>

Limiting Moment of resistance, Mu,lim:

Mu,lim = 0.138*fck*b*d²

=0.138*20*300*600² =298KNm

Here Mu<Mu,lim = 200< 298KNm

i. If Mu<Mu,lim = under reinforced section
ii. If Mu=Mu,lim = Balanced reinforced section
iii. If Mu>Mu,lim = Over reinforced section

Therefore here it is under reinforced section.

[IS 456-2000 pg.no 96, For Mu]

$$* \text{Mu} = 0.87\,.\text{fy}.\text{Ast}\,.\,\text{d}\left[1 - \left(\frac{\text{Ast.fy}}{\text{b.d.fck}}\right)\right]$$

$$200*10\text{^}6 = 0.87*415*Ast*600[1-\frac{Ast*415}{300*600*20}]$$

200*10^6 = 216630 Ast – 24.91 Ast²

$$Ast = -b \pm (\sqrt{b^2}-4ac)/2a$$

$$= -216630 \pm (\frac{\sqrt{216630^2-4*24.9*200*10\text{^}6)}}{(2*24.9)}$$ Ast = 7649.4 mm² (+ve) (or) 1050.5 mm² (-ve)

**DOUBLY REINFORCED SECTION:**

**Problem:**

Q1. A doubly reinforced section/beam having a rectangular section 250mm wide and 540mm over all depth is reinforced with 2 bars of 12mm φ in the compression side and 4bars of

20mm φ in the tension side. The effective cover to bar 40mm using M20 grade concrete and Fe415 steel. Estimate the flexural strength of the section using IS 456-2000

*Given Data:*

b =250mm, D = 540mm, fck = 20N/mm² fy = 415 N/mm²

*Step 1:-*

Asc – area of steel in compression side

Asc = $[\frac{\pi d^2}{4}]$ 2 = 2*π*12² / 4 = 226.08 mm²

Ast – area of steel in tension side

Ast = $[\frac{\pi d^2}{4}]$ 2 = 2*π*20² / 4 = 1256 mm²

Effective cover, d' = 40mm

*Step 2:-* Effective depth d = D-d' = 540 – 40 = 500mm

Xumax/d = 0.48 (constant)

Xumax/d = 0.48*500 = 240mm

*Step 3:-* $fsc = 0.0035\left\{\frac{[Xumax - d']}{Xumax}\right\} * Es$

=0.0035{[240 – 40]/240 } *2*10^5

= 583.33 N/mm²

fsc value should not be greater than 0.87fy

fsc= 583.33 > 0.87 fy

fsc= 0.87*415 = 361.05 N/mm²

*Step 4:-* $Ast^2 = \frac{Asc.fsc}{0.87.fy} =$ 226.08 mm²

Ast = $Ast_1 + Ast_2$

$Ast_1$ = Ast - $Ast_2$ = 1256 – 226.08 = 1029.92

$Ast_1$ = 1030 mm²

*Step 5:-* Check for Xu,

$\frac{Xu}{d} = \frac{0.87\ .fy\ .Ast}{(0.36.fck.b.d)}$

Xu $=\left[\frac{(0.87*415*1030)}{(0.36*20*250)}\right]$

= 206.6 < 240mm

Here $Ast_1$, we have to substitute minimum area of steel section.

Xu< Xumax, Therefore section is under reinforced.

*Step:-6* for doubly reinforced section Mu will be

Mu= 0.87 .fy. $Ast_1$ (d-0.42.Xu) + fsc. Asc(d-d') [refer Pg No. 96 ]

= 191 KNm

Q2. A doubly reinforced concrete section has width of 300mm & it is reinforced with tension steel area of 2455mm² at an effective depth of 600mm. Compression steel area 982mm² is provided at an effective cover of 60mm using M20 grade concrete & Fe415 steel. Estimate the ultimate moment capacity of the section.

*Given Data:*

b =300mm, d = 600mm, fck = 20N/mm² fy = 415 N/mm²

Ast = 2455 mm² (tension), Asc = 982 mm²(compression), d' = 600mm

*Solution:*

*Step 1:-* Xumax/ d = 0.48

Xumax = 0.48 *600 = 288mm

*Step 2:-*

$$fsc = 0.0035\left\{\frac{[Xumax - d']}{Xumax}\right\} * Es$$

=0.0035{[288 – 60]/288 } *2*10^5

= 554.16 N/mm²

fsc should not be greater than 0.87fy

0.87 * fy = 361.05 N/mm²

fsc=554 is greater than 0.87 fy, so take 361 N/mm² as fsc

fsc = 361 N/mm²

*Step 3:-* $Ast_2 = \frac{Asc.fsc}{0.87.fy}$ = 982 mm²

$Ast = Ast_1 + Ast_2$

$Ast_1 = Ast - Ast_2$ = 2455 – 982 = 1473 mm²

*Step 4:-* Check Xu

$$\frac{Xu}{d} = \frac{0.87\,.fy\,.Ast}{(0.36.fck.b.d)}$$

$$Xu = \left[\frac{0.87*415*1473}{0.36*20*300}\right] = 246.21 \text{ mm}$$

Xu< Xumax = 246.21 < 288

It is under reinforced.

*Step 6:-*

Mu = 0.87 .fy. $Ast_1$ (d-0.42.Xu) + $F_{sc}$. Asc (d-d')

= 455 KN.m

**ULTIMATE SHEAR STRENGTH OF RC SECTION**

[Pg No: 48, 72, 73 for problems]

RC members are generally subjected to max shear forces normally near the support section of simply supported flexural members. In continuous beams the support section are subjected to shear couple with moments. The types of shear failure absorbed in RC members are:

i. Shear tension (or) Diagonal tension failure
ii. Flexure shear failure
iii. Shear compression failure
iv. Shear bond failure

Nominal shear stress

$\tau_v = V_u/bd$

where,

$V_u$ - ultimate shear force at section

$\tau_v$ - Nominal shear stress

b - Breadth

d - Effective depth

PROBLEMS

Q1. A RCB has a support section with a width of 250mm and effective depth of 500m. The support section is reinforced with 3 bars of 20mm on tension side 8mmφ. 2 legged stirrups are provided at a spacing of 200mm centre using M20 grade concrete of Fe415 steel bars. Calculate the shear strength of the support section.

Given Data:

b =250 mm, d = 500 mm, fck = 20 N/mm² fy = 415 N/mm²

Ast = $[\frac{\pi d^2}{4}]$ n, Ast $=3*\pi*20^2/4 = 942.48$mm²

$A_{SV} = 2*\pi*8^2/4 = 100.53$mm²,

$S_v = 200$mm

To find: $V_u$

*Step 1:-* Percentage of tension [refer Pg No. 73]

$Pt = \frac{100Ast}{bd} = \frac{100*942.48}{250*500}$ = 0.75 [refer table 19 IS 456-2000 & read out the design shear strength of concrete Ʈc corresponding to M20 grade concrete]

Ʈc = 0.56 N/mm²

*Step2:-* Shear resisted by concrete

$V_{uc}$ = Ʈc*b*d [refer Pg No. 72]

= 0.56*250*250 = 70*10^3 N or 70KN

*Step 3:-* Shear resisted by Vertical links/ stirrups

$$Vus = \frac{Asv\ (0.87\ fy)\ d}{Sv}$$ [Refer Pg No. 73]

= 100.53(0.87*415)500]/200

= 90.74 KN

*Step 4:-* Total shear resistance

$V_u = V_{us} + V_{us}$ = 70 + 90 = 160 KN

---

Common width of beams are 150, 200, 230, 250 & 300mm [Pg No. 37]

| Space range | Loading | Span(depth ratio L/d) |
|---|---|---|
| 3 to 4m | Light | 15 to 20 |
| 5 to 10m | Medium/heavy | 12 to 15 |
| 5 to 10m | Heavy | 10 to 12 |

LIMIT STATE DESIGN OF BEAMS

PROBLEMS

Q1. Design a singly reinforcement concrete beam with suitable following data.

Clear span = 4m, width of support = 300mm, service load = 5 KN/m, fck = 20 N/mm², fy=415 N/mm².

Given Data:

L= 4m, width =300mm = 0.3 m, fck = 20N/mm² fy = 415 N/mm²

Partial safety factor = 1.5

*Solution:*

*Step1:-* Cross-sectional dimension

Assuming the span depth ratio L/d = 20 [Pg No. 37]

L/d = 20 => d= L/20

Effective depth, d = 4000/20 = 200mm

Cover = 50mm, overall depth, D = 200+50 =250mm = 0.25m

b = 200mm = 0.2m

*Step2:-* Effective span (Adopt least value)

i. Clear span + effective depth = effective span => 4+0.2 = 4.2m

ii. Centre to centre of support = effective span => 4+ 0.3/2 +0.3/2 = 4.3m

Effective span, L = 4.2m (least value)

*Step 3:-* Load calculation (for singly reinforced beam design)

Self wt. of beam = b * D* unit wt. of concrete

= 0.2*0.25*25 = 1.25KN/m

Live load = 5 KN/m (given)

Total Load = 1.25 + 5 = 6.25 KN/m

Ultimate Load = Total load * partial safety factor

= 6.25 * 1.5 = 9.38 KN/m

Wu = 9.38 KN/m

*Step4:-* Shear force & bending moment

$Mu = \frac{Wu*l^2}{8}$ [Ultimate moment]

$= 9.38 * 4.2^2 /8 = 20.67$ KN.m

$Vu = \frac{Wu*l}{2}$ [Shear forces]

= 9.38 * 4.2 /2 = 19.7KN

*Step 5:-* Check for Mu, lim

Mu, lim = $0.138*fck*b*d^2$

$= 0.138*20*200*200^2$ = 22.08 KN.m

Mu < Mu, lim = (20.67< 22.08) => Under reinforced section

*Step 6:- Area* of steel

$Mu = 0.87 . fy. Ast . d \left[1 - \left(\frac{Ast.fy}{b.d.fck}\right)\right]$ [pg no – 92]

$20.67*10^6 = 0.87 * 415 * Ast * 200 \left[1 - \frac{Ast*415}{200*200*20}\right]$

Ast (req) = 349.65 $mm^2$.

i. Using 16mm φ

n = Ast (req)/ast => $349.65/\pi*16^2/4$ = 1.74 say 2 Nos

ii. Spacing = 1000 ast / Ast => 1000* 201.06/349.65 = 575 mm [Sv should not be less than 50mm & should not be greater than 300mm]

Max spacing 300mm. So using change of dia of rod 16mm 2 bars. To hanger bar 10mm dia

*Step 7:-* Check for shear stress

V = Vu/bd = (19.68 * 10^3) / (200*200)  [Pg No 72]

= 0.492 N/mm²

Ast (provided) = 2*$\pi$*16² / 4 = 402mm²

Percentage of tension reinforced = 100Ast (provided)/bd   = 1

[Refer table 19, take $\tau_c$ value]

$\tau_c$ = 0.62 N/mm²

$\tau_v$ = 0.492 N/mm²

$\tau_v < \tau_c$

$\frac{Asv}{b\ Sv} \geq \frac{0.4}{0.87\ fy}$  [Pg. No 48]

Provide the nominal shear reinforcement using 6mm 2 leg stirrups.

Asv = 2*$\pi$*6² /4 = 56.54 mm²

Sv = Asv * 0.87*fy / b*0.4   = 56.54 * 0.87*415 / 200* 0.4 = 255mm

Sv ⊳ 0.75d (i.e.) 255> 0.75*200 => 255>150

Adopt spacing of stirrups as 150mm centres.

*Step 8:-*   Check for deflection method

[pg no. 38 Refer fig 5.1, check Kt value]

Kt = 1.05

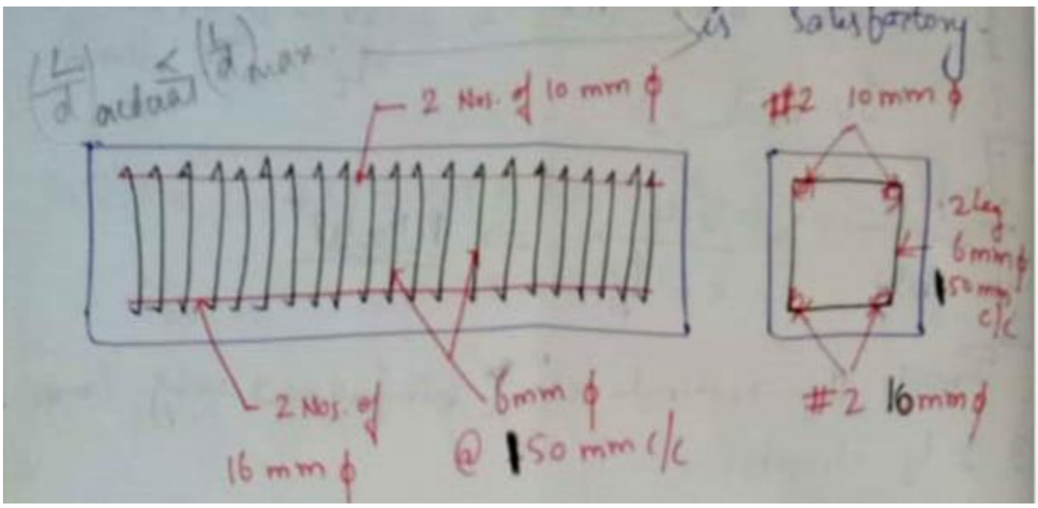
(L/d) max = (L/d) basic. $K_t$. $K_c$. $K_f$

= 20* 1.05*1*1

= 21

(L/d) actual = 4200/200 = 21

(L/d) max = (L/d) actual => therefore deflection control is satisfactory.



*Step 9:-* Design using SP – 16 design tables

Mu /(d*d²) = 20.67*10^6 / 200*200 = 2.58

[ref table 2 of SP -16 and read out Pt = 1.005 (or) say 1

Ast = (Pt . b.d)/100 = 1.005*200*200 /100 = 350mm²

Hence Ast is the same as that computed using theoretical equations.

**DOUBLY REINFORCED SECTION**

Q1. Design a RCB of rectangular section using the following data. Effective span = 5m, width of beam = 250mm, overall depth = 500mm, working load = 40 KN/m, effective cover = 50mm. M20grade concrete & Fe 415 grade steel is used.

Given Data:

L =5m, b =250mm, D = 500mm, d' = 50mm, d = D-d' = 500-50 = 450mm

fck = 20N/mm² fy = 415 N/mm²

W = 40 KN/m

*Solution:*

*Step 1:-* Cross-section dimension

b =250mm, D = 500mm, d' = 50mm, d = 450mm, L=5000mm

*Step 2:-* Load Calculation

Working load, W = 40 KN/m

Ultimate Load = Total load * partial safety factor

= 40 * 1.5 = 60 KN/m

Wu = 60 KN/m

*Step 3:-* Shear force & bending moment

$Mu = \frac{Wu * l^2}{8}$ [Ultimate moment]

= 60 * 5² /8 = 187.5 KN.m

$Vu = \frac{Wu * l}{2}$ [Shear forces]

= 60 * 5 /2 = 150KN

*Step 5:-* Check for Mu, lim

Mu, lim = 0.138*fck*b*d²

= 0.138*20*250*450² = 139.73KNm say 140 KN.m

Mu >Mu, lim => Over reinforced section (i.e.) it is doubly reinforced section

Mu-Mulim = 187.5 – 140 [Pg No. 96]

= 47.5 KN.m

$$fsc = 0.0035\left\{\frac{[Xumax - d']}{Xumax}\right\} * Es$$

Xumax/d = 0.48 => Xumax = 0.48*d = 0.48*450

$$\text{fsc} = 0.0035\left\{\frac{[(0.48*450) - 50]}{0.48*50}\right\} * 2 * 10^5$$

= 538 N/mm²

<u>Check:</u> fsc > 0.87 fy

538 > 361 (so adopt fsc as 361 N/mm²)

Mu, Mulim = fsc. Asc (d-d') [Pg No – 96]

187*10^6 – 140*10^6 = 361* Asc(450-50)

Asc = 328mm²

i. Use 16mm φ

n = Ast/ast = 328/(π*16²/4) = 1.63 ≈ 2 nos

therefore provide 2 nos of 16mm φ

$$Ast_2 = \frac{Asc.fsc}{0.87.fy}$$ [Pg No- 96]

=328*361 / (0.87*415)

= 328 mm²

$$Ast_1 = \left[\frac{0.36.fck.b\ .Xumax}{0.87fy}\right]$$

{from Xumax/d = [0.87fy. Ast/(0.36. fck. b .d)] }

= 0.36*20*250*(0.48*450)/0.87*415

= 1077 mm²

Ast = $Ast_1$ + $Ast_2$

= 1077 + 328 = 1405 mm²

ii. Use 25mm φ

n = Ast/ast = 1405/(π*25²/4) = 3 nos

therefore provide 3 nos of 25mm φ

*Step 5:-* Check for shear stress

Ʈc = Vu/bd = (150 * 10^3) / (250*450) [Pg No 72]

= 1.33 N/mm²

Percentage of tension reinforced, pt = 100Ast/bd = 100*1405/250*450 = 1.25

Ʈc = 0.67 [refer table 19 Pg N0. 73]

Ʈv > Ʈc => therefore shear reinforcement shall be provided.

Vus = Vu – Ʈc bd [Pg No – 73]

Vus = 74625 N

Use 8mm φ 2 legged stirrups

Asv = $[\frac{\pi d^2}{4}]$ *2 = $[\frac{\pi * 8^2}{4}]$ *2 = 100.53 mm²

$Vus\ = \frac{Asv\ (0.87\ fy)\ d}{Sv}$

Therefore $Sv\ = \frac{Asv\ (0.87\ fy)\ d}{Vus}$

= 100.53(0.87*415)450/74625

Sv = 218.87 mm, say 200mm

Provide 2 legged 8mm φ stirrups @ 200mm c/c.

*Step 6:-* Check for deflection

(L/d)max = (L/d)basic. Kt. Kc. Kf [pg No. 37, clause 23.2.1]

= 20 * Kt. Kc. Kf

[Pg No 38, below fig 4]

$*\ fs\ =\ 0.58\ fy\ \left[\frac{Area\ of\ crosssection\ of\ steel\ required}{Area\ of\ crosssection\ of\ steel\ provided}\right]$

= 0.58. fy [1405/1472.62] Ast(provided) = 3*π*25² / 4 = 1472.62 mm²

= 230 N/mm².

Kt = 0.9 [ from graph]

% of tension = 1.25

fs= 230 N/mm²

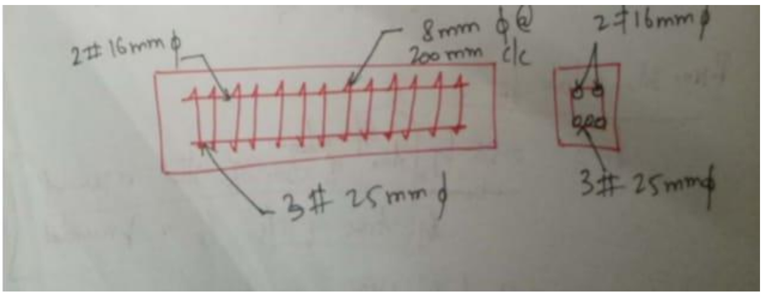
% of compression reinforcement = 100Ast/bd => 100*328 / 250*450 = 0.29

Kc = 1 [for doubly RCB]

- ⇨ (L/d)max = 20*0.9*1.09*1 = 19.62
- ⇨ (L/d) actual = 5000/450 = 11.1

(L/d) max > (L/d)actual

Therefore deflection is in control




**FLANGE BEAM** [T-Beam and L- Beam]

Design parameters of T- beam [refer Pg No 36, of IS 456-2000]

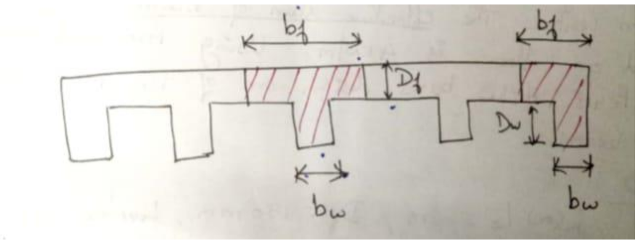
- ⇨ Effective width of flange (bf)

  bf ⊳ bw+1/2 of clear distance to the adjacent beam

- ⇨ For T-beam

  bf $= \frac{Lo}{6} + bw + 6D^{6}$

- ⇨ For L –beam
- ⇨ bf $= \frac{Lo}{12} + bw + 3D_{6}$




b- actual width of flange

L (or) Lo – effective span

bf – effective width of flange

bw – width of web(or) rib

Df – flange thickness

Effective depth – Span/depth(L/d) ratio is assumed in the range of 12 to 20 depending on the span range and degree of loading.

Width of web – the nominal range varies from 150 to 400mm. It is influenced by the width of the supporting column or the width of the support.

Flange thickness – It is generally same as the thickness of the slab between the ribs (or) web. The thickness of the slab varies from 100 to a max. of 250mm.

PROBLEM

Q1. A T-beam slab floor of an office comprises of its slab 150mm thick spanning b/w ribs spaced at 3m centres. The effective span of beam is 8m. Live load on the floor is 4 KN/m. Using M20 grade concrete & Fe415 HYSD bars design one of the intermediate T-beams.

Given Data:

L(or) Lo = 8m, b =3m, Df = 150mm, fck = 20N/mm² fy = 415 N/mm² Live load = 4KN/m

<u>*Solution:*</u>

*Step 1:-* Assume basic L/d as 20 for simply supported beam. For T beam, assume width of rib as 300mm (bw)

(i.e) bw = 300mm.

Flange width = 3m

The ratio of web to flange width (web/flange width) = 300/3000 = 0.1

Reduction factor, Kf = 0.8 [for 0.1, Kf = 0.8] [refer Pg. No – 39, fig 6]

Span/depth = 20* Kf = 20*0.8 = 16

L/d = 16 => d = L/16 = 8000/16= 500mm

Assuming, d' = 50mm, D = d+d' = 50+500 =550mm

D = 550mm, d=500mm, d' =50mm, bw = 300mm & Df = 150mm

*Step 2:-* Load calculation

Self wt of slab = b* Df * unit wt of concrete

= 3*0.15*25 = 11.25 KN/m

Floor finish = 1.8 KN/m [assumed] (i.e) 0.6*3

Self wt of rib = bw * Dw * unit wt of concrete

= 0.3 * 0.4*25 = 3 KN/m

Plaster finish = 0.3 to 0.5 KN/m [assume]

Total dead load = 11.25 + 3+ 1.8 [if needed add plaster finish too, here it didn't added]

= 16.5 KN/m

Live load = 4 KN/m

Total load = 16.5 + 4 = 20.5 KN/m

Ultimate load = Total load * partial safety factor

= 20.5 * 1.5 = 30.75 KN/m

*Step 3:-* Bending moment and shear force

$Mu = \frac{Wu*l^2}{8}$ [ultimate moment]

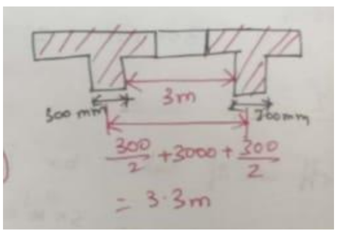
= 30.75 * 8² /8 = 246 KN.m

$Vu = \frac{Wu*l}{2}$ [shear forces]

= 30.75 * 8 /2 = 123KN

*Step 4:-* Effective width of flange [adopt least value]

i. bf = $\frac{Lo}{6} + bw + 6D^6$= $\frac{8000}{6} + 300 + 6 * 150$ = 2.53m



ii. bf = C/C of ribs

= 0.15 + 3+0.15 = 3.3m

bf = 2530 mm[least value]

*Step 5:-* Moment capacity of flange

Muf = 0.36. fck. bf . Df. (d-0.42 Df) [not in code book]

= 0.36*20*2530*150*(500 – (0.42*150))

= 1.19 * 10^9 = 1190 KNm

Mu < Muf (i.e) Xu < Df

It is under reinforced section

*Step 6:-* Reinforcement

Mu= 0.87 .fy. Ast .d [1 – (Ast .fy/ b. d. fck)]

This section is considered as rectangular section, so b = bf

246*10^6 = 0.87*415*Ast*500[1- (Ast*415/2530*500*20)

246*10^6 = 180.525* 10^3 * Ast – (74,92 * 10^6 * Ast$^2$)/25.3* 10^6

74.92*10^6 * Ast$^2$ - 4.56*10^12 Ast + 6.22*10^15 = 0

[use 991 caluculator & solve equation]

Ast = 1396mm$^2$ [required]

- ⇨ No of bars = Ast/ast

  Use 25mmφ steel rod

  n = 1396 / (π*25$^2$/4) = 3 Nos

  provide 3 bars of 25mm φ and 2 hanger bars of 12mm φ on compression zone.

*Step 7:-* Check for shear stress

% of tension, Pt = 100 Ast/bd = (100*1396 )/ (300*500) = 0.93

From table 19, Pg No.73, $\tau_c$ = 0.62 N/mm$^2$

$\tau_v$ = Vu /$b_w$. d = 123*10^3 /(300*500) = 0.82 N/mm$^2$

$\tau_v$ > $\tau_c$ => shear reinforcement shall be provided.

$V_{us}$ = $V_u$ – $\tau_c$ . b.d [pg No 73]

=123*10^3 –(0.62*300*500)

=30*10^3 N = 30KN

- ⇨ Use 8mm φ 2 legged stirrups

  Asv = $[\frac{\pi d^2}{4}]$ *2 = 2* $\frac{\pi * 8^2}{4}$ = 100.53 mm$^2$

  Vus = Asv (0.87 fy) d/Sv [pg no 73(a)]

  Sv = 100.53(0.87*415)500/(30*10^3)

Sv = 600mm

Provide 2 legged stirrups of 8mm φ at 600mm c/c

*Step 8:-* check for deflection

(L/d)max = (L/d)basic * Kf *Kc *Kt

= 20* Kt * Kf*Kc

fs = 0.58 fy [area of cross-section of steel required / area of cross-section of steel provided]

= 0.58* 415*[1396/1472.62]

= 228 N/mm².

Kt = 1.1 [pg No 38 chart]

Kf = 0.8 [refer step 1]

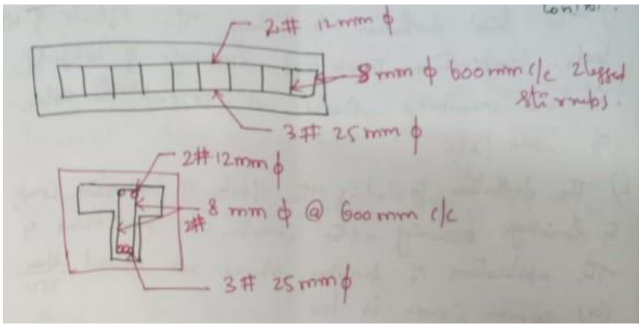
Kc = 1 [constant]

(L/d)max = 20*1.1*0.8*1 = 17.6

(L/d)actual = 8000/500 = 16

(L/d)actual < (L/d)max

The deflection is satisfactory & in control.




**CONTINUOUS BEAMS**

These are commonly used in multi storey building of secural base in perpendicular direction. The upper limit of span/depth ratio is 26, which can be modified by factors Kc, Kf and Kt.

IS 456-2000 code delection limits:

Excessive deflections of flexural members like beam & slabs may cause distress to users and result in cracking of partitions. Hence IS 456-2000 code clause 23.2 prescribes the limiting deflection as :

1. The final deflection including the effects of all loads, temperature, creep and shrinkage of horizontal structural members should not exceed the value of span/250.

2. The deflection including the effects of temperature, creep & shrinkage occurring after erection of partitions and the applications of finishes should not exceed span/350 (or) 20mm whichever is less.

In case of beams supporting heavy loads, the span/depth ratio of 10-12 is recommended from practical considerations.

Q1. Design a continuous reinforcement concrete beam of rectangular section to support a dead load 10KN/m and a service live load of 15 KN/m over 3 simply supported span of 8m each. Adopt M20 grade concrete and Fe415 HYSD bars.

Given Data:

L = 8m, fck = 20 N/mm², fy = 415 N/mm², dead load = 10KN/m

liveload = 15 KN/m

*Solution :*

*Step 1:-* Cross-section dimension :

Assuming L/d = 12 [refer previous para]

d = L/12 = 8000/12 = 666.67 say650mm

d' = 50mm [assume]

D = d + d' = 650 + 50 = 700mm

b = 300mm [assume]

*Step 2:-* Load calculation

Self wt of beam = b * D * unit wt of concrete

= 0.3 * 0.7* 25 = 5.25 KN/m

Dead load = 10 KN/m

Total dead load = 5.25 + 10 = 15.25 KN/m

Live load = 15 KN/m

(i.e) g = 15.25 KN/m , q = 15 KN/m

*Step 3:-* Bending moment and Shear force

Negative bending moment @ Interior support

[refer table 12, bending moment co-efficent – clause 22.5.1]

i. Mu(-ve) = $1.5[\frac{gL^2}{10} + \frac{qL^2}{9}]$ KN.m

Positive bending moment @ centre of span

ii. Mu(+ve) = $1.5[\frac{gL^2}{12} + \frac{qL^2}{10}]$ KN.m

Max. shear force @ support section

iii. $V_u$ = [1.5*0.6*(g+q)]*L KN

1. Mu(-ve) = $1.5[\frac{15.25*8^2}{10} + \frac{15*8^2}{9}]$ KN.m = 306 KN.m
2. Mu(+ve) = $1.5[\frac{15.25*8^2}{12} + \frac{15*8^2}{10}]$ KN.m = 266KN.m
3. $V_u$ = [1.5*0.6*(15.25+15)] KN = 272.25 KN

*Step 4:-* Check for Mu, lim:

Mu,lim = $0.138*20*300*650^2$

= 349.83 KN.

Mu = 306 KN.m & Mu,lim = 349.8 KN.m

Mu < Mu,lim => it is under reinforced section.

*Step 5:-* Area of Steel

Mu= 0.87 .fy. Ast .d [1 – (Ast .fy/ b. d. fck)]

For Mu(-ve)

306*10^6 = 0.87*415*Ast*650[1- (Ast*415/300*650*20)

Ast = 1566 $mm^2$

⇨ Assume 25mm φ

n = Ast/ast = $1566/(\pi * 25^2 /4)$ = 3.19 nos say 4.

Provide 4 Nos of 25 mm φ of rod.

Ast(prov) = $4*(\pi * 25^2 /4)$ = 1962.5 $mm^2$

For Mu(+ve)

266*10^6 = 0.87*415*Ast*650[1- (Ast*415/300*650*20)

Ast = 1317.3 $mm^2$ say 1318$mm^2$

⇨ Assume 25mm φ

n = Ast/ast = $1318/(\pi * 25^2 /4)$ = 2.68 nos say 2.[even no. is better]

Provide 2 Nos of 25 mm φ of rod.

*Step 6:-* Check for Shear stress

$\tau_v$ = Vu /$b_w$. d = 272*10^3 /(300*650) = 1.39 N/$mm^2$

% of tension, Pt = 100 Ast(prov)/bd = (100*1962.5 )/ (300*650) = 1.006

$\tau_c$ = 0.62 N/$mm^2$ [ from table 19]

$\tau_v$ > $\tau_c$ => shear reinforcement shall be provided.

$V_{us}$ = $V_u$ – $\tau_c$ . b.d [pg No 73]

=272.3*10^3 –(0.6*300*650)

=155300KN

⇨ Use 8mm φ 2 legged stirrups

$Asv = [\frac{\pi d^2}{4}] *2 \quad = 2* \frac{\pi * 8^2}{4} \quad = 100.53 \ mm^2$

Vus = Asv (0.87 fy) d/Sv [pg no 73(a)]

Sv = 100.53(0.87*415)650/(155300)

Sv = 150mm

Provide 2 legged stirrups of 8mm φ at 150mm c/c

*Step 8:-* check for deflection

(L/d)max = (L/d)basic * Kf *Kc *Kt

= 20* Kt * Kf*Kc

fs = 0.58* Ast(r)/Ast(b) = 192.06

Kt = 1.2 [pg No 38 chart]

Kf = 1

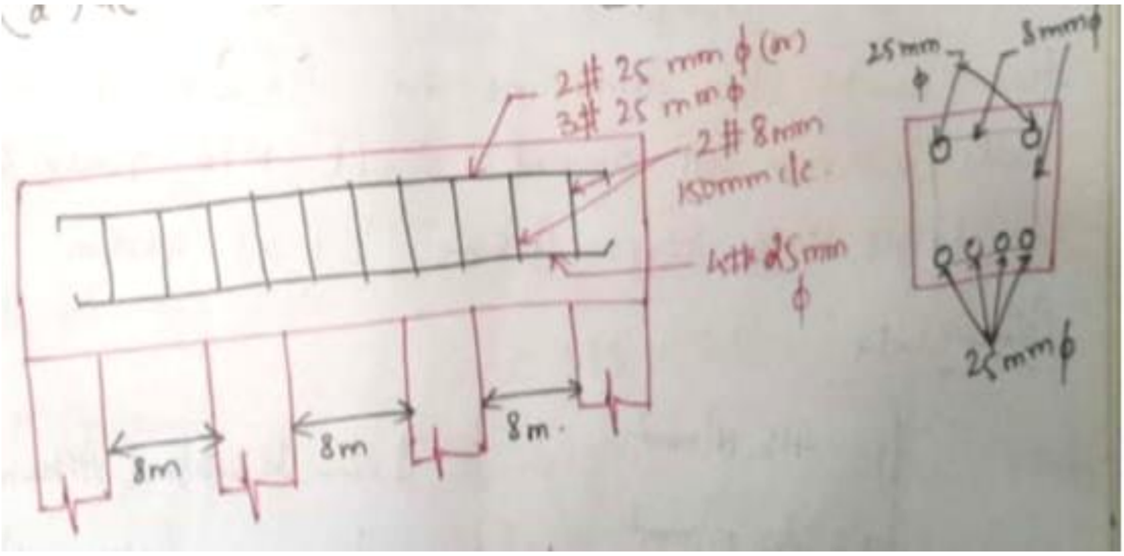
Kc = 1 [constant]

(L/d)max = 20*1.2*1*1 = 31.2 (or) 12*1.2*1*1 = 14.4

(L/d)actual = 8000/650 = 12.3

(L/d)actual < (L/d)max

The deflection is satisfactory & in control.



SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

# UNIT – III – SLABS – SCIA1502

# UNIT – III – SLABS

Analysis and Design of one way and Analysis - Design of two way slab for uniformly distributed load, various Boundary conditions and corner effects - Design of continuous slabs - Using code Coefficient.

## INRODUCTION

The reinforced concrete slabs are the most common type of structural element used to cover floors and roofs of buildings. Varying conditions and stipulations ask for the selection of appropriate and cost-effective concrete slab, keeping in view, the type of building, architectural layout, aesthetic features, and the span length.

The slab may be supported by walls or by reinforced concrete beams usually cast monolithically (hugely) with the slab or by structural steel beams or by columns, or by the ground.

There are 16 different types of Slabs in Construction. Some of them are outdated and many of them are frequently used everywhere.

1. Flat Slab
2. Conventional Slab
   i. One Way Slab
   ii. Two Way Slab
3. Hollow core ribbed Slab or Hollow core slab
4. Hardy Slab
   i. Waffle Slab
   ii. Dome Slab
5. Pitch roof slab
6. Post tension slab
7. Cable suspension slab
8. Pre Tension Slab
9. Low roof slab
10. Projected slab
11. Grads Slab/ Slab on grade
12. Sunken Slab

*Miscellaneous Slabs*

i. Room Chajja or Loft
ii. Kitchen Slab
iii. Lintels
iv. Sun Shade slab

WHAT IS A SLAB?

- A slab is a structural element, made of concrete, that is used to create flat horizontal surfaces such as floors, roof decks and ceilings.

- A slab is generally several inches thick and supported by beams, columns, walls, or the ground.
- Concrete slabs can be prefabricated off-site and lowered into place or may be poured in-situ using formwork.
- If reinforcement is required, slabs can be pre-stressed or the concrete can be poured over rebar positioned within the formwork.

On the basis of reinforcement provided, beam support, and the ratio of the spans, slabs are generally classified into ***one-way slab*** **and** ***two-way slab.***

**ONE WAY SLAB**

- The one way slab is supported on two sides and the ratio of long to short span is greater than two.
- Generally all the Cantilever slabs are one Way slab.
- Chajjas and verandahs are a practical example of one way slab.

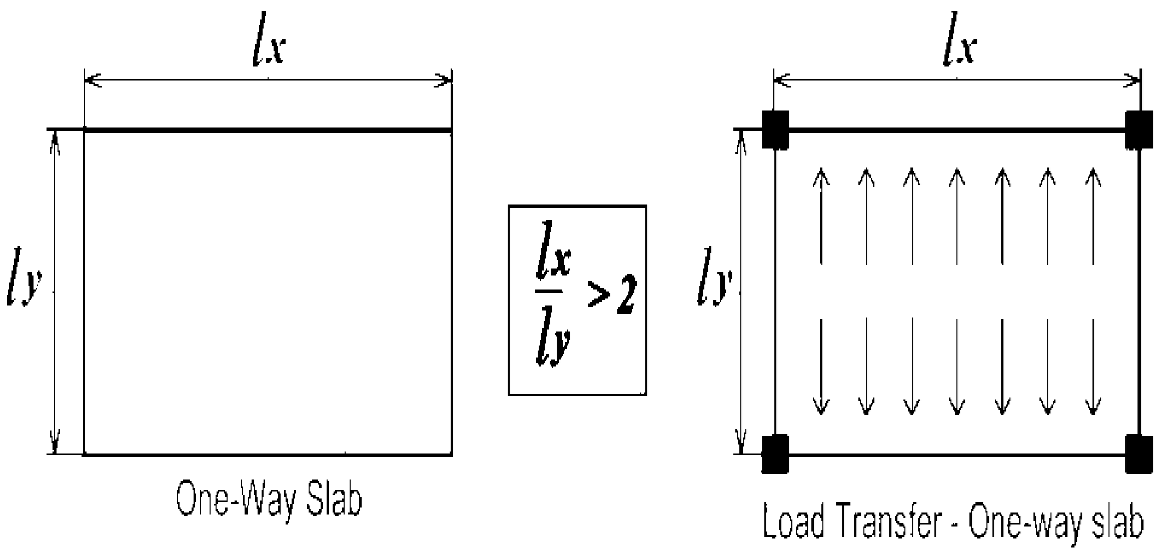



***Fig 1 One way slab***

**TWO WAY SLAB**

- The two way slab is supported on four sides and the ratio of long to short span is smaller than two.
- These types of slabs are used in constructing floors of a multi-storeyed building.

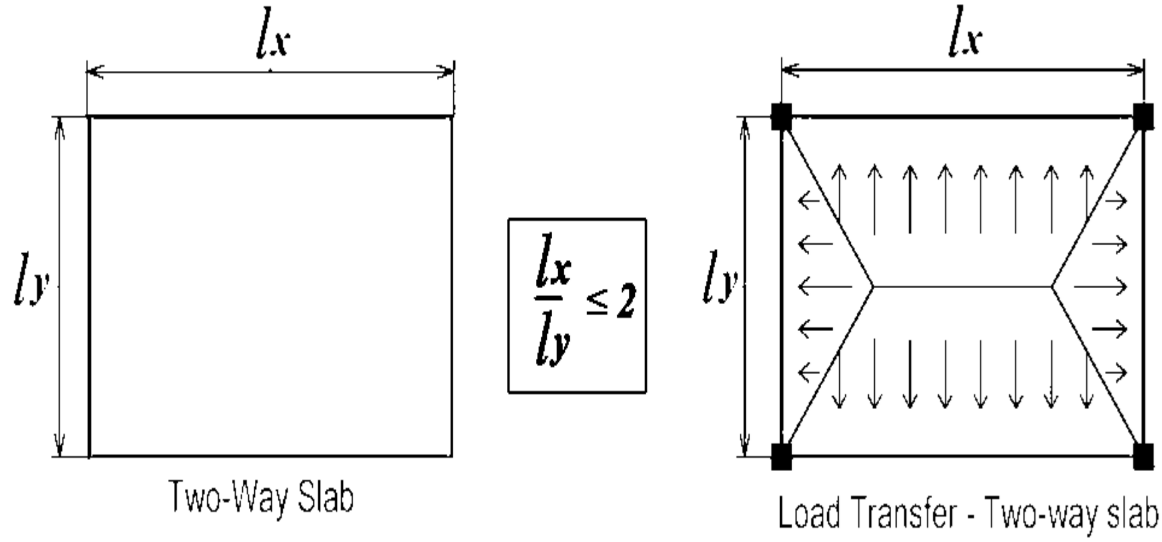
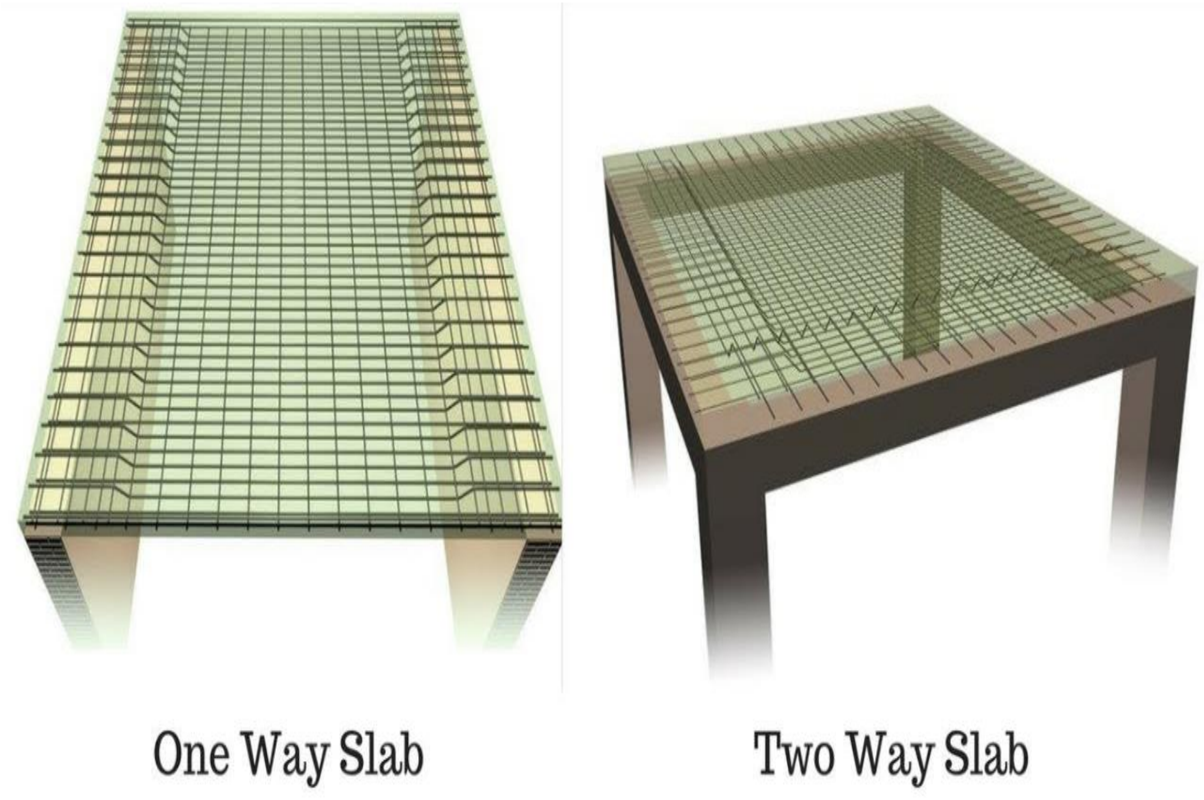



*Fig 2 Two way slab*



| DESCRIPTION | ONE WAY SLAB | TWO WAY SLAB |
|---|---|---|
| Ratio of longer span to shorter span | Greater than or equal to 2. | Less than 2. |
| Main reinforcement | Main reinforcement is required and provided in only one direction for one way slab. | Main reinforcement is required and provided in both directions for two way slab. |
| Bending | Bending occurs in one direction. | Bending occurs in both directions. |
| Support | One-way slab is supported by beams in simply two sides. | Two-Way Slab is supported by beams in all four sides. |
| Depth | Depth required is more. | Depth required is less. |
| Load distribution | Load distribution pattern in one-way slab is in one direction for shorter span. | Load distribution pattern in two-way slab is in four sides. |
| Deflection or Deformation | The deflected shape of the one-way slab is cylindrical. | Whereas the deflected shape of the two-way slab is a dish or saucer-like shape. |
| Example | Chajja and Varandha are practical examples of one-way slab | Whereas two-way slabs are used in constructive floors of the Multistorey building. |
| Economy | The one-way slab is economical up to a span of 3.6 meters. | Whereas the two-way slab is economical for the panel sizes up to 6m × 6m. |
| Quantity of steel | In one-way slab quantity of steel is less. | In two-way slab quantity of steel is more as compared to the one-way slab. |

**DESIGN PROBLEMS**

Problem : 1) Design a ss Rcc slab for an office floor having clear dimensions are 4×10 m of width, walls 230mm all around . Adopt M20 grade concrete and Fe415 HYSD bars . Assume L.L as 4KN/m .

<u>Given data :</u>

$f_y$=415 N/mm$^2$

$f_{ck}$ =20 N/mm$^2$

width of the wall =230 mm

Size of slab = 4 × 10 m

L.L = 4 KN /m ( Assumed)

B = 1m ( Assumed)

$$\frac{LY}{LX} = \frac{10}{4} = 2.5 > 2$$

Then it is one way slab.

<u>Step :1) Thickness of slab :</u>

$$\frac{l}{d} = 25 \Rightarrow d = \frac{4000}{25} = 160\text{mm}$$

Assuming ø as 10 mm and $d^1$= 20mm$^4$

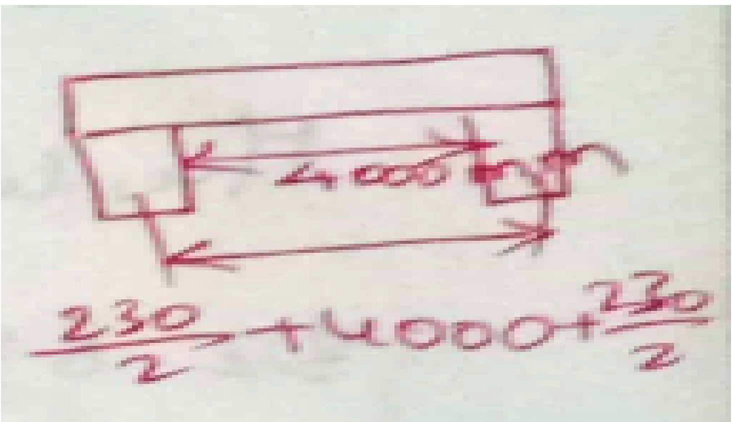
Effective cover = clear cover + ( dia. of rod )/2

= 20 + (10 /2)

= 25 mm

Overall depth, D = d +$d^1$= 160+25 =185mm.

<u>Step :2)Effective span :</u>




i) Effective span = clear span + effective depth

= 4 + 0.16

$= 4.16$

ii) Effective span = c/c of supports

$= \frac{0.23}{2} + 4 + \frac{0.23}{2}$

$= 4.23$ m

L = 4.16 ( least value is to be adopted)

<u>Step :3) Load calculation:</u>

Self weight of slab = b × D × unit wt of concrete

D . L = 1 × 0.185 × 25 = 4.626 KN/m

L . L = 4 KN/m , Floor finish =1.5 KN/m

Total load = 4.625 + 4 + 1.5 =10.125 KN/m

Ultimate load , $W_u$ = total load × p.s.f

= 10.125 × 1.5 = 15.1875 KN/m

<u>Step :4) shear force and bending moments :</u>

B.M $= \frac{WuL2}{8} = \frac{15.1875 \times 4.16 \times 4.16}{8}$

$\mu = 32.837$

S.F = $W_u$ L / 2 = (15.1875 ×4.16 ) / 2 = 31.57 KN

<u>Step :5) Check for $\mu_{u\,lim}$ :</u>

$\mu_{u\,lim} = 0.138 \times f_{ck} \times b \times d^2$

$= 0.138 \times 20 \times 1000 \times 160^2 = 70.65$ KNm

$\mu_u < \mu_{u\,lim}$, ⇒It is under reinforced section.

Step :6) Reinforcement( area of steel) :

$\mu_{u\ lim}$=0.87 $\times f_y \times$ Ast $\times$ d $\times(1 - \text{Ast by/bdfck})$

32.837 $\times 10^6$= 0.87 $\times$ 415 $\times$Ast $\times$ 160 $\times(1 -$ Ast$\times$ 415/1000$\times$160$\times$20)

Ast = 617.4 $mm^2$

Use 10mm ø rod as assumed in step 1)

$a_{st=}\pi(10)^2$ / 4=78.5$mm^2$

$S_v$=1000x $a_{st}$ /$A_{st}$

= 1000x78.5/617.4

= 127 ( say 130mm)

Step 7): Check for reinforcement:

$P_t$ = 100$A_{st}$/bd

= 100x617.4/1000x160

= 0.386

$A_{st}$=0.12% of c/s area

$=\frac{0.12}{100} \times b \ \times d$

=220$mm^2$

$\tau_c$= 0.4N/$mm^2$ ( from P.No: 73)

$\tau_v$ = $V_u$/bd = 31.6x$10^3$/1000x160

= 0.197N/$mm_2$

$\tau_c > \tau_v$( page No: 72)

Using 8 mm ø,

$$S_{v} = 100 \times \frac{ast}{Ast} = 100 \times \frac{\pi\frac{(8)2}{4}}{220} = 230mm$$

$\tau_c$x k = 0.4x1.24 = 0.496 ( Page No. 72 – 40.2.1.1)

$\tau_c(k) > \tau_v$( 40.3) (Min. shear shall be provided)

<u>Step 8) Check for deflection :</u>

$(L/d)_{max}$ = $(L/d)_{basic}$ x $(K_c)$x $(K_t)$x $(K_f)$

$K_c = K_f = 1$ & $K_t = 1$ (from figure)

$(L/d)_{max}$ = (4160/160)x 1x 1x 1 ( $f_s$= 207.176mm)

= 26

$f_s$ = 0.58 x $f_y$ ( $A_{st.\ req}$/ $A_{st\ prov}$) = 0.58 x 415x 617/$A_{st\ prov}$

$A_{st\ prov}$ = $A_{st\ req}$ + (100) or (50) for slab

= 717.4mm$^2$

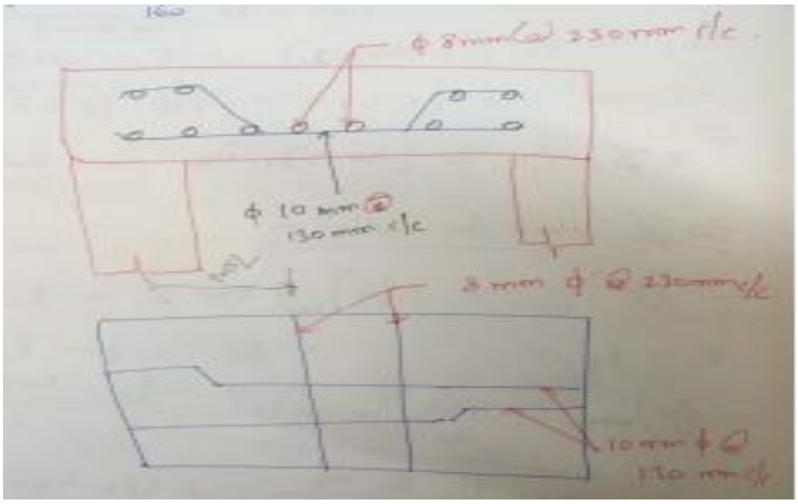
$(L/d)_{actual}$ = 400/160 = 25 { $((L/d)_{actual} < (L/d)_{max}$} [ Deflection is satisfactory and in control]

(Or)

$(L/d)_{max}$ = $(L/d)_{basic}$ x $(K_t)$x $(K_c)$x $(K_f)$

= 20x1.4x1x1 = 29

$(L/d)_{actual}$ = 4160/160 = 26<29 ( Therefore it is in control )




Problem 2) Design a two way slab for a room size 4×5 m with discontinuous and ss edges on all the sizes with corners prevented from lifting to support a live load of 4 KN/m.Adopt M20 grade concrete and Fe415 steel.

Given data :

$L_X$= 4m , $L_Y$ =5m,

$f_y$ =415 N/mm$^2$

$f_{ck}$ =20 N/mm$^2$

$$\frac{LY}{LX} = \frac{5}{4} = 1.25 < 2$$

It is one way slab.

Step :1) Thickness of slab :

$$\frac{l}{d} = 25 \Rightarrow d = \frac{4000}{25} = 160\text{mm}$$

( Assume 10mmø rod and clear cover as 25mm)

D= 160+25+(10/2)

=190 mm

effective span = clear span + effective depth

= 4 + 0.16 = 4.16

Step : 2) Load calculation:

Self weight of slab = b × D × unit wt of concrete

D . L = 1 × 0.19 × 25 = 4.75 KN/m

L . L = 4 KN/m , Floor finish =0.6 KN/m

Total load = 4.75 + 4 + 0.6 =9.35 KN/m

Ultimate load , $W_u$ = total load × p.s.f

= 9.35 × 1.5 = 14.02 KN/m

<u>Step :3) shear force and bending moments :</u>

$\frac{LY}{LX} = 1.25$( from Page No. 91 , table -26)

$\alpha$x = 0.075, $\alpha y = 0.056$

$\mu_x = \alpha x \times w_u (L_x)^2 = 0.075 \times 14.02 \times 4^2 = 16.82$ KNm

>-$L_x \Rightarrow$ -ve moment of continuous edge =0.025

>+$L_x \Rightarrow$ -ve moment at mid span =0.056

$\mu_y = \alpha y \times w_u (L_y)^2 = 0.056 \times 14.02 \times 5^2 = 19.64$ KNm

>-$L_y \Rightarrow$ -ve moment =0.047

>+$L_y \Rightarrow$ -ve moment =0.035

$V_{ux}$ =( $w_u L_x$)/2 =(14.02 × 04)/2 =28.04

<u>Step :4) Check for $\mu_{u\ lim}$ :</u>

$\mu_{u\ lim} = 0.138 \times f_{ck} \times b \times d^2$

$= 0.138 \times 20 \times 1000 \times 160^2 = 70.65$ KNm

$\mu_u < \mu_{u\ lim}$, ⇒It is under reinforced section.

<u>Step : 5) Reinforcement</u> :

$\mu_{u\ x} = 0.87 \times f_y \times Ast \times d \times (1 - Ast\ by/bdfck)$

$16.82 \times 10^6 = 0.87 \times 415 \times Ast \times 160 \times (1 - Ast \times 415/1000 \times 160 \times 20)$

Ast $= 303\ mm^2$

$\mu_{u\ y} = 0.87 \times f_y \times Ast \times d \times (1 - Ast\ by/bdfck)$

$19.64 \times 10^6 = 0.87 \times 415 \times Ast \times 160 \times (1 - Ast \times 415/1000 \times 160 \times 20)$

Ast $= 344\ mm^2$

Using 10mm ø of rod

$a_{st} = \pi(10)^2 / 4 = 78.54 mm^2$

$S_v = 1000x\ a_{st} / A_{st}$

$= 1000x(78.54/344)$ ⇒ adopt max $A_{st}$ from above

$= 259$ ( say 260mm)

Provide 10 mm ø rod for 260 mm c/c spacing.

<u>Step 6) Check for shear stress :</u>

% of tension , $P_t = 100A_{st}/bd$

$= (100x\ 344)/(1000x160)$

$= 0.193$

$Z_V = V_u/bd$

$= (28.06 \times 10^3)/1000 \times 160$

$=0.175$

$Z_c = 0.34$ ( from pg -73)

K= 1.26 ( from pg -72)

$K \times \tau_c = 0.43$

$K \times \tau_c > \tau_k$

Step 7) Check for deflection :

$(L/d)_{max} = (L/d)_{basic} \times (K_c) \times (K_t) \times (K_f)$

$K_c =$ p-37

$K_f =$ p-38

$K_t = \ldots.$

$f_s = 0.58 \times f_y ( A_{st.\ req} / A_{st\ prov})$

$A_{st\ prov} = A_{st\ req} + (100)$ or $(50)$ for slab

$= 344 + 50 = 395\ mm^2$

$f_s = 0.58 \times f_y ( A_{st.\ req} / A_{st\ prov}) = 0.58 \times 415 \times 344/395$

$= 210$ mm (pg 38)

$K_t = 1.78$ , $K_c = 1$, $K_f = 1$

$(L/d)_{basic} = 25$ ( $\Rightarrow 4000/160 = 25$)

$(L/d)_{max} = (L/d)_{basic} \times (K_t) \times (K_c) \times (K_f)$

$= 25 \times 1 \times 1 \times 1.78$

$= 44.5$

$(L/d)_{actual} = 4160/160 = 26$

$((L/d)_{actual} < (L/d)_{max}$ [ Deflection is satisfactory and in control]

**Step 9 CHECK FOR CRACKING**

i) Steel provided

It is more than mini (0.12%) = $\frac{0.12}{100} \times b \times D = 228\ mm^2$

ii) Spacing of main steel $< 3d$

$= 3 \times 160$

$= 480$ (260< 480 mm)

iii) Ø of reinforcement $< D/8$

10mm $< 190/8$

10 $< 23.75$

| |
|---|
| 0.12% ⇒ 0.12 × b ×D (Overall depth) |
| $S_v < 3d$ ⇒ Effective depth |
| Dia<D/8 ⇒ Overall depth |

**Step 10** TORSION REINFORCEMENT AT CORNERS

Area of reinforcement in each four layers $= 0.75 \times$ highest $A_{st}$

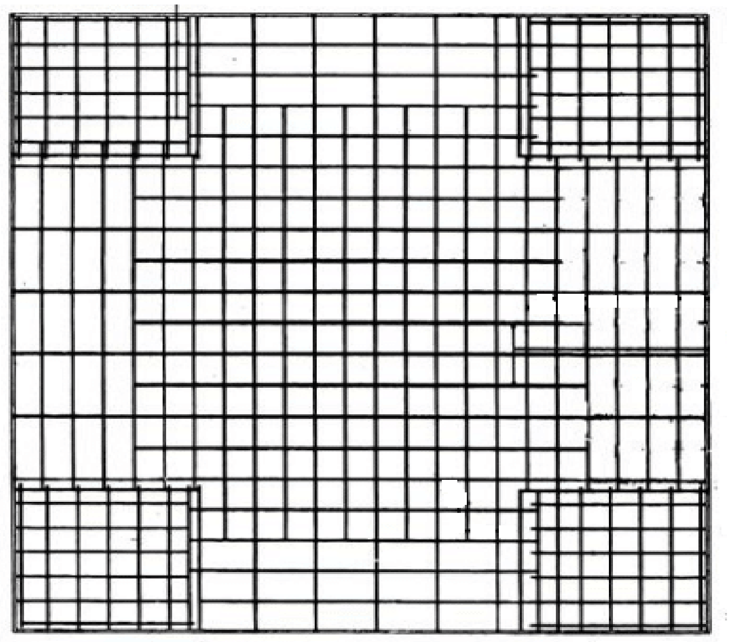
$= 0.75 \times 344$

**= 258 mm²**

Distance over torsion reinforcement is provided $= (1 \times \text{short span})/5$

$= 1 \times 4000/5$

**= 800**



**Provide 6mmø**

$S_v$ =1000 x [$a_{st}$ /$A_{st}$]

= 1000 x (($\pi$ x $6^2$)/4)/258

**$S_v$ =110mm**

Provide 6mmø @ 110mm c/c @ for all corners.

REINFORCEMENT IN EDGE STRIP

$A_{st}$ = 0.12 % of c/s area

=0.12% × b × d

= 228$mm^2$

Provide 10 mm ø

$S_v$ =1000x $a_{st}$ /$A_{st}$

= 1000 x (($\pi$ x $10^2$)/4)/228

=344mm (say 350 mm)

Problem 3) Design a two way slab for an office floor of size 4×6 m with discontinuous two adjacent edges. Adopt M20 grade concrete and Fe415 steel. Live load = 4 KN/m.

Given data :

Floor slab =4×6 m LX= 4m , LY =6m,

fy =415 N/$mm^2$

fck =20 N/$mm^2$

$LY/LX$= 6/4 = 1.5 < 2

It is one way slab.

Step 1) c/s Dimensions :

Assume l/d= 25 ⇒ $d$ =4000/25 = 160mm

D = d + $d^1$ = 160 + 25 = 185 mm

Assume $d^1$ = 25 mm

effective span = clear span + effective depth

= 4 + 0.16 = 4.16

Step : 2) Load calculation:

Self weight of slab = b × D × unit wt of concrete

D . L = 1 × 0.185 × 25 = 4.625 KN/m

L . L = 4 KN/m ,

Floor finish =1.5 KN/m

Total load = 4.75 + 4 + 1.5

=10.15 KN/m

Ultimate load , Wu = total load × p.s.f

= 10.5 × 1.5 = 15.19 KN/m

Step :3) shear force and bending moments :

Condition – Two adjacent edges

discontinuous Refer P.No: 91, Table -

26

>-Lx⇒ -ve moment of continuous edge =0.075

>+Lx⇒ -ve moment at mid span =0.056

>-Ly⇒ -ve moment =0.047

>+Ly⇒ -ve moment =0.035

-<u>ve moment:</u>

M ux = $\alpha$x × wu $(Lx)^2$ = 0.075 × 15.19 × $4.16^2$ = 19.72 KNm

Muy = $\alpha$y × wu $(Ly)^2$ = 0.047 × 15.19 × $4.16^2$ = 12.35 KNm

+<u>ve moment:</u>

M ux = $\alpha$x × wu $(Lx)^2$ = 0.056 × 15.19 × $4.16^2$ = 14.72 KNm

Muy = $\alpha$y × wu $(Ly)^2$ = 0.035 × 15.19 × $4.16^2$ = 9.2 KNm

<u>Step : 5) Reinforcement</u> :

M u =0.87 × fy × Ast × d ×(1 −[ Ast *fy/b*d*fck])

-M ux = 19.72 × $10^6$ N.mm ⇒ ASt =357.95$mm^2$

-Muy = 12.35 × $10^6$ N.mm ⇒ ASt =220.157$mm^2$

+M ux = 14.72 × $10^6$ N.mm ⇒ ASt =263.86$mm^2$

+Muy = 9.2 × $10^6$ N.mm ⇒ ASt =162.6$mm^2$

Step 6): Spacing

Assume 10mm ø rod

- Muy & Ast

Sv = 1000 ast/ Ast

= 219.446mm

+M ux & Ast

⇒ Sv = 297.683mm

- M uy & Ast

⇒ Sv =356.745mm

+M uy & Ast

⇒ Sv =482.760mm

| Location | | Ast | Spacing | |
|---|---|---|---|---|
| Short span | | | | |
| -ve | ⇒ | 357mm$^2$ | (219) 200mm | |
| +ve | ⇒ | 263mm$^2$ | (297) 280mm | |
| Long span | | | | |
| -ve | ⇒ | 220mm$^2$ | (357) 300mm | (Maximum) |
| +ve | ⇒ | 163mm$^2$ | (483) 300mm | (Maximum) |

Torsion reinforcement at corner :

Area of torsional steel in each of four layer

= 0.75 x Max. Ast = 0.75 x 357

$= 267.75 = 268mm^2$

Provide 4 layers of reinforcement at A

provide 8 mm ø

No. of bars = Ast/ast = $268/(\pi*8^2/4)$ = 5.3 say 6 bars

provide 4 layers of 8mm dia bars of 6 numbers at A.

At B 50 % of total torsional steel should be provided.

i.e; provide 3 numbers of 8mm ø steel rod at B.

At C& D torsional steel is not required.

CONTINUOUS SLAB

Problem 4) Design a continuous slab for an office floor . Slab is continuous over T- beams spaced at 4m intervals. Adopt M20 grade concrete and Fe415 steel. Assume Live load = 4 KN/m.

<u>Given data :</u>

L= 4m ,

fy =415 $N/mm^2$

fck =20 $N/mm^2$

L.L = 4 KN/m.

<u>Step 1) c/s Dimensions :</u>

Assume l/d = 26 ⇒ $d$ =4000/26 = 153.8mm , say 150mm

Assume d' = 25 mm

D = d + d' = 150 + 25 = 175 mm

d= 150 mm, D=175mm

Step : 2) Load calculation:

Self weight of slab = b × D × unit wt of concrete

D . L = 1 × 0.175 × 25 = 4.375 KN/m

Floor finish =1KN/m( assumed)

Total dead load (g) = 5.375 KN/m
L . L (q) = 4 KN/m

Step :3) shear force and bending moments :

$-Mu = 1.5 \{[gL^2/10]+[qL^2/9]\}$

$=1.5((5.375\times4^2)/10 + (4\times4^2)/9)$

= 23.56 KN.m

$+ Mu = 1.5 \{[gL^2/12]+[qL^2/10]\}$ = 20.35 KN.m

VL =( 1.5 × 0.6) (g+q) × L

=( 1.5 × 0.6)(5.375+ 4)×4

= 33.75KN

Step :4) Check for depth:

Mu +, Mu- Which one is greater

$M u lim =0.138 \times fck \times b \times d^2$

$+ve^- \Rightarrow$ $20.35\times10^6 = 0.138 \times 20 \times 1000 \times d^2$

d= 867<depth provided (150)

$-ve \Rightarrow 23.567 = 0.138 \times 20 \times 1000 \times d^2$

d= 92.407 <depth provided (150) ⇒satisfied

Step : 5) Reinforcement :

M u =0.87 × fy × Ast × d ×(1 − [Ast *fy/b*d*fck)

$23.56\times 10^{6} = 0.87 \times 415 \times Ast \times 150 \times (1 - Ast \times 415/1000\times150\times20)$

Ast =465.07 $mm^2$

Spacing , Sv=1000x ast /Ast

$= (1000\times(\pi \times 10^{\wedge}2\ ))/465.07$

= 168mm

Provide 10mm ø with 160 mm c/c spacing.

Step 6) Check for shear stress :

% of tension , $P_t = 100A_{st}/bd$

= (100x 344)/(1000x160)

= 0.193

$Z_V = V_u/bd$

$= (28.06 \times 10^3)/1000x160$

=0.175

$Z_c = 0.34$ ( from pg -73)

K= 1.26 ( from pg -72)

$K\times \tau_c = 0.43$

$K\times \tau_c > \tau_k$

Step 7) Check for deflection :

$(L/d)_{max} = (L/d)_{basic}$ x $(K_c)$x $(K_t)$x $(K_f)$

$K_c =$ p-37

$K_f =$ p-38

$K_t =$ ….

$f_s = 0.58$ x $f_y$ ( $A_{st.\ req}$/ $A_{st\ prov}$)

$A_{st\ prov} = A_{st\ req}$ + (100) or (50) for slab

$= 344 + 50 = 395\ mm^2$

$f_s = 0.58$ x $f_y$ ( $A_{st.\ req}$/ $A_{st\ prov}$) = 0.58 x 415x 344/395

= 210 mm (pg 38)

$K_t$= 1.78 , $K_c$ = 1, $K_f$= 1

$(L/d)_{basic}$ = 25 ( ⇒4000/160 =25)

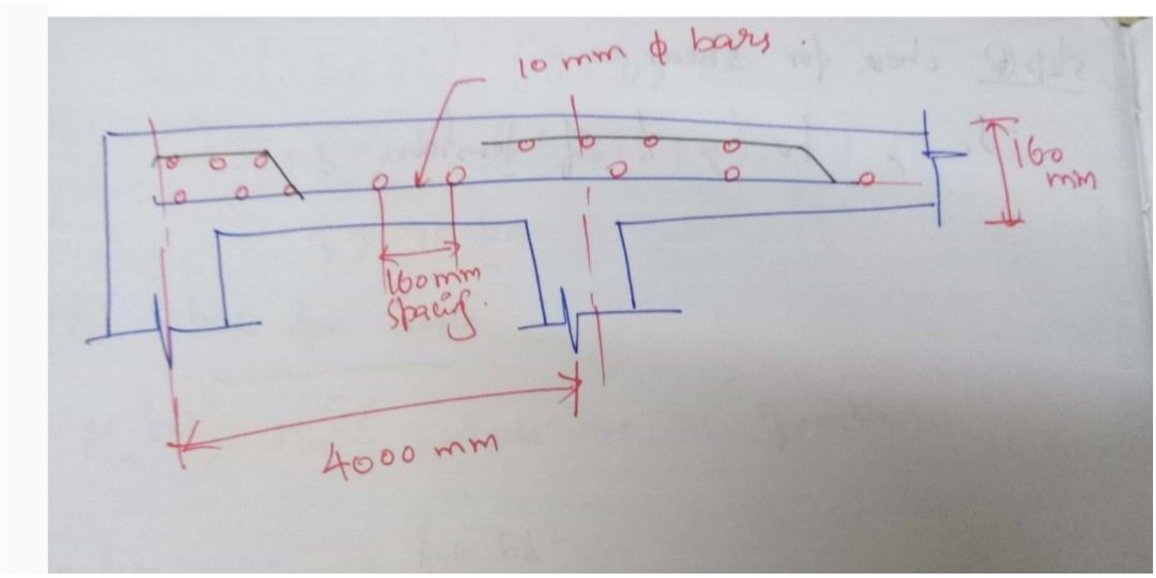
$(L/d)_{max} = (L/d)_{basic}$ x $(K_t)$x $(K_c)$x $(K_f)$

= 25 x 1 x 1 x 1.78

=44.5

$(L/d)_{actual}$ = 4160/160 = 26

($(L/d)_{actual} < (L/d)_{max}$ [ Deflection is satisfactory and in control]


10 mm ϕ bars
160 mm
160mm
spacing.
4000 mm

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

# UNIT – IV – COLUMNS – SCIA1502

# UNIT – VI – COLUMNS

Behaviour of Columns - Types of columns - Design of Axially loaded and Eccentrically loaded columns - Using IS: 456 & SP -16

## INTRODUCTION

A vertical member whose effective length is greater than 3 times its least lateral dimension carrying compressive loads is called as a column. Columns transfer the loads from the beams or slabs to the footings or foundations. The inclined member carrying compressive loads as In the case of frames and trusses is called as struts. The pedestal is a vertical compression member whose effective length is less than 3 times Its least lateral dimension.

WHAT IS A COLUMN?

- The columns in a structure carry the loads from the beams and slabs down to the foundations, and therefore they are primarily compression members, although they may also have to resist bending forces due to the continuity of the structure.
- IS 456:2000 defines the column as a compression member.
- It is also said that the column may fail in any of the following as given below:
    - Pure compression failure
    - Combined compression and bending failure
    - Failure by elastic in stability

## CLASSIFICATION OF COLUMN

- Based on type of reinforcement
    - Tied column
    - Spiral column
    - Composite column
- Based on type of loading
    - Axially loaded column
    - Columns with uni-axial eccentric loading
    - Columns with bi-axial eccentric loading
- Based on slenderness ratio
    - Long column
    - Short column
- Based on shape

- ❖ Square/Rectangular
- ❖ Circular
- ❖ T- Type
- ❖ L- Type
- ❖ V- Type
- ❖ Hexagon
- ❖ Arch type
- ❖ Y – Type
- ❖ Y type with Arch

BASED ON TYPE OF REINFORCEMENT

- ➢ Tied Column:

When the main longitudinal bars of the column are confined within closely spaced lateral ties, it is called as tied column.

- ➢ Spiral Column:

When the main longitudinal bars of the column are enclosed with in closely spaced and continuously wound spiral reinforcement, it is called as a spiral column.

- ➢ Composite Column:

When the longitudinal reinforcement is in the form of structural steel section or pipe with or without longitudinal bars, it is called as a composite column.

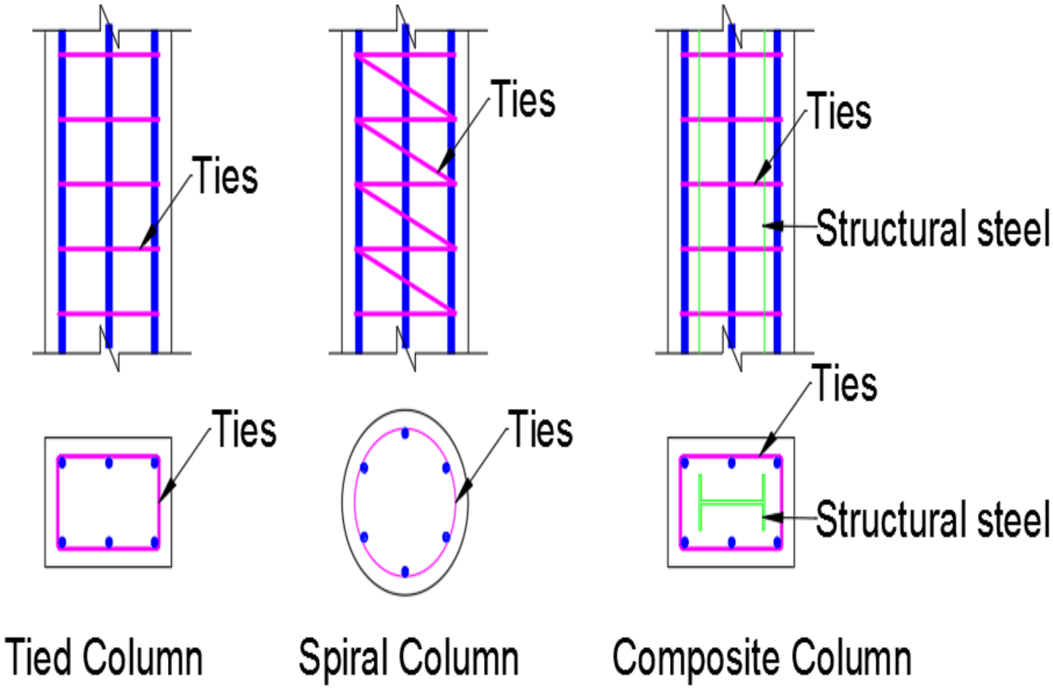



***Fig 1 Based on type of reinforcement***

**BASED ON TYPE OF LOADING**

AXIALLY LOADED COLUMN

- When the line of action of the compressive force coincides with the center of gravity of the cross-section of the column, it is called axially loaded column.
- The total load from top is acted on the centroid of the column.

ECCENTRICALLY LOADED COLUMN (UNI-AXIAL OR BIAXIAL)

- When the line of action of compressive force doesn't coincide with the center of gravity of the cross-section of the column, it is called as the eccentrically loaded column.

Columns with uni-axial eccentric loading

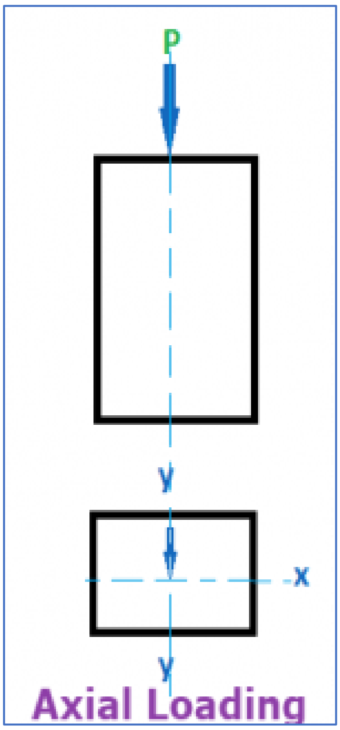
If load is acted eccentrically either on X or Y axis (anyone) then it is called as eccentrically loaded column (uni-axial)

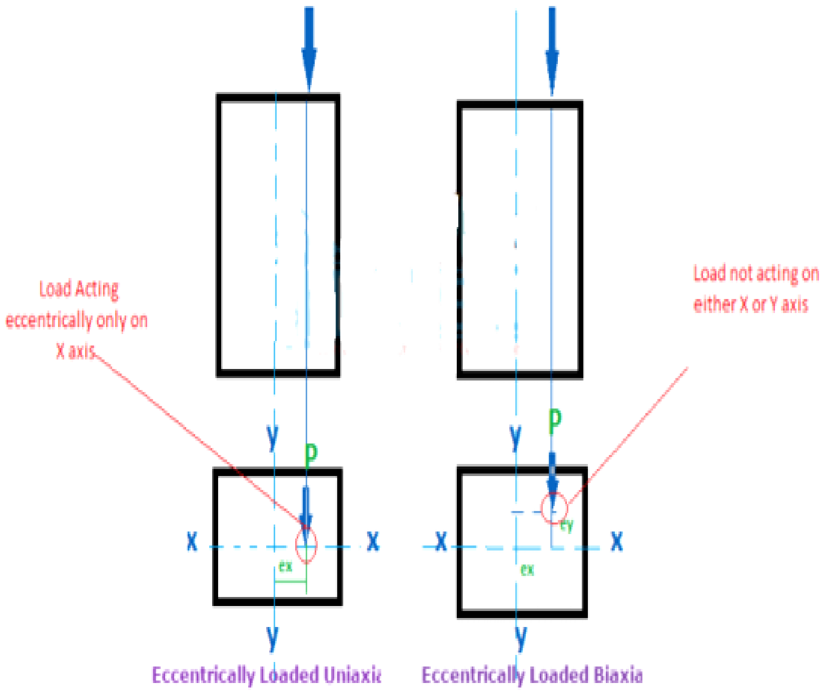
Columns with beam connected at one side are an example of Uni-axial eccentrically loaded column.

Columns with bi-axial eccentric loading

If load is not acting eccentrically on both the axis, which mean load won't act on either X or Y axis is called eccentrically loaded column (Biaxial)

Corner columns with beam connected at two side is an example of Biaxial eccentrically loaded column.



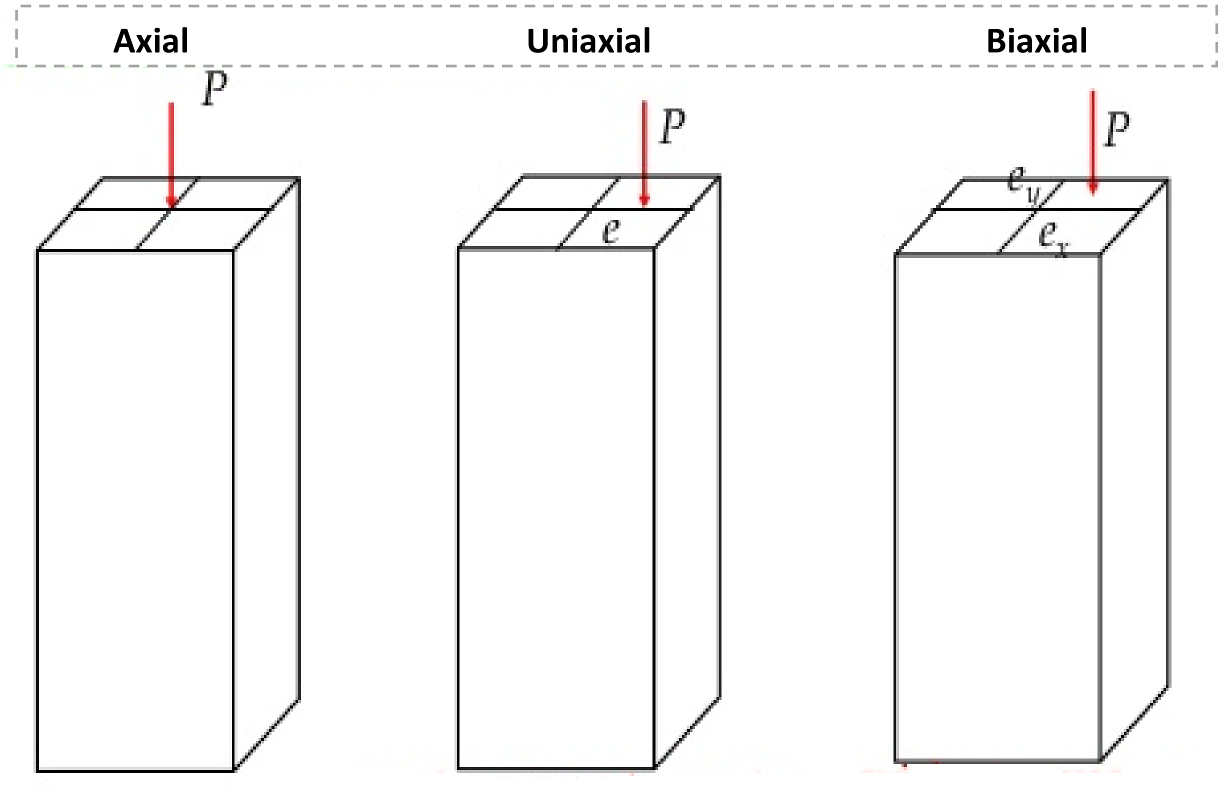



***Fig 2 Based on type of loading***

**BASED ON SLENDERNESS RATIO**

Depending upon the Slenderness ratio the columns are classified as

- short
- Intermediate
- long.

Slenderness ratio is the ratio of effective length (L) to least lateral dimension (D) of the column.

According to IS 456:2000 code clause 25.1.2, when both the slenderness ratio's L / D are less than 12 it is termed as short column. If the ratio is equal to or more than 12 then it is termed as Long column.

Short Column

- If the ratio effective length of the column to the least lateral dimension is less than 12, the column is called as the short column.
- A short column fails by crushing (pure compression failure).

Long Column

- If the ratio effective length of the column to the least lateral dimension exceeds 12, it is called as long column.

- A long column fails by bending or buckling.

*Note:*

In case of Non-rectangular & non circular sections where the slenderness ratio is better defined in terms of radius of gyration rather than the lateral dimensions these definition of Short or long column is not applicable.

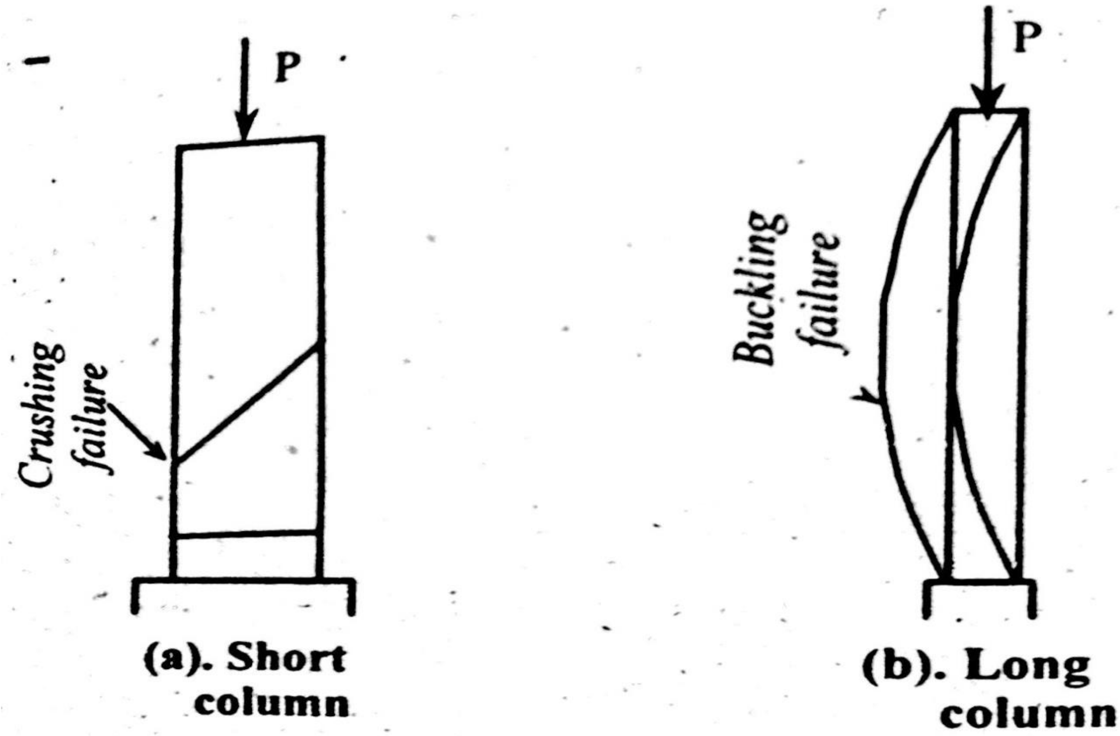



***Fig 3 Based on slenderness ratio***

**COLUMN FAILURES**

- The column may fail in any of the following as given below:
    - Pure compression failure
    - Combined compression and bending failure
    - Failure by elastic in stability

**Column Failure due to Pure Compression**

- When reinforced concrete columns are axially loaded, the reinforcement steel and concrete experiences stresses.
- When the loads are high compared to cross-sectional area of the column, the steel and concrete reach the yield stress and column fails without undergoing any lateral deformation.
- The concrete column is crushed and collapse of the column is due to the material failure.

- To overcome this, the concrete column should have sufficient cross-sectional area, so that the stress is under the specified limit.
- This type of failure is generally seen in case of pedestals whose height to least lateral dimension is less than 3 and does not experience bending due to axial loads.

**Column Failure due to Combined Compression and Failure**

- Short columns are commonly subjected to axial loads, lateral loads and moments. Short columns under the action of lateral loads and moments undergo lateral deflection and bending.
- Long columns undergo lateral deflection and bending even when they are only axially loaded.
- Under such circumstances when the stresses in steel and concrete reach their yield stress, material failure happens and RCC column fails.
- This type of failure is called combined compression and bending failure.

**Column Failure due to Elastic Instability**

- Long columns are very slender, i.e. its effective length to least lateral dimension is more than 12.
- Under such condition, the load carrying capacity of reinforced concrete columns reduces drastically for given cross-sectional area and percentage of reinforcement steel.
- When such type of concrete columns is subjected to even small loads, they tend to become unstable and buckle to any side.
- So, the reinforcement steel and concrete in such cases reach their yield stress even for small loads and fail due to lateral elastic buckling.
- This type of failure is unacceptable in practical concrete constructions. Code prevents usage of such long columns for slenderness ratio greater than 30 (for unbraced columns) for the use in concrete structures.

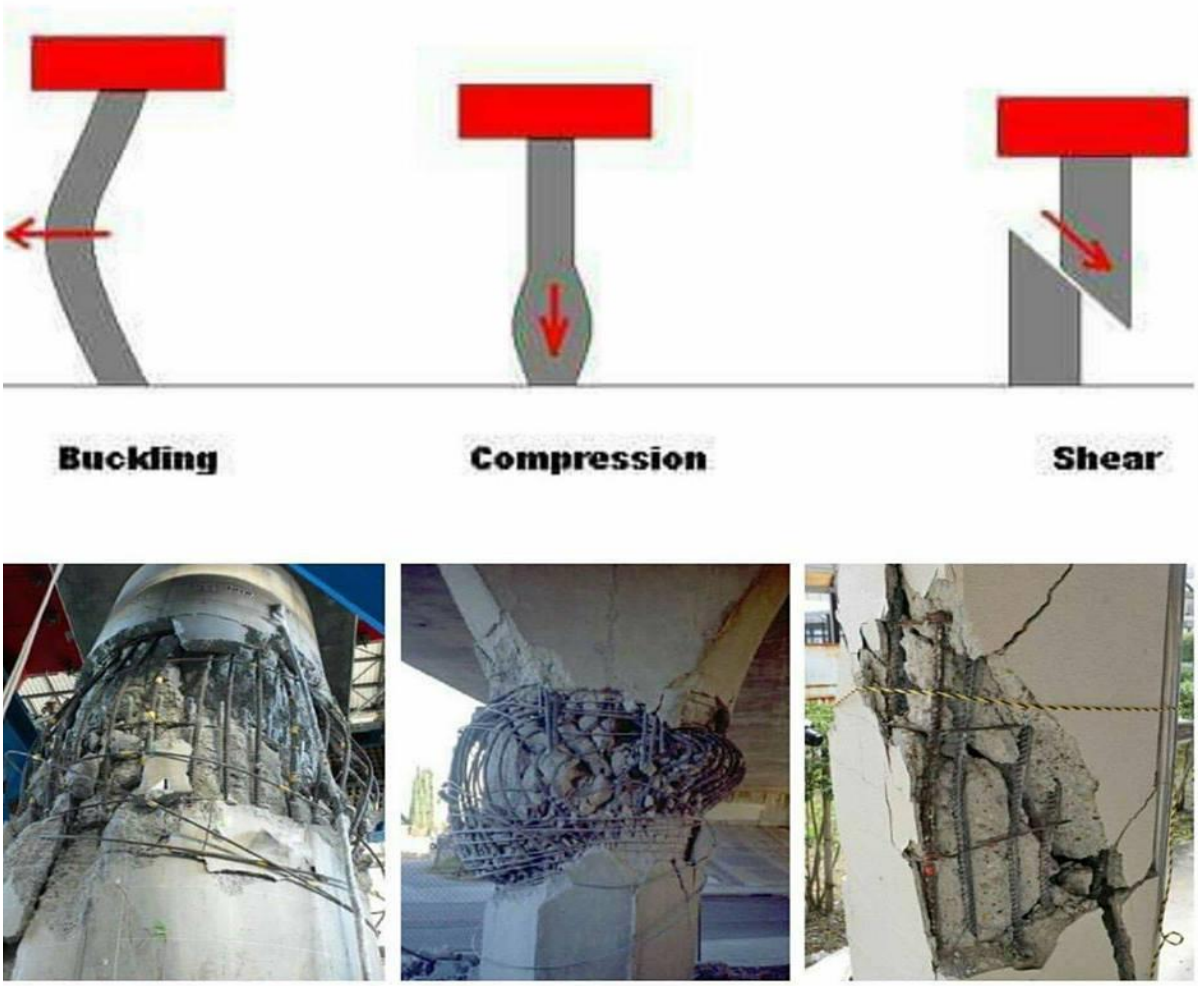



***Fig 4 Column Failure***

**PROBLEMS**

Design reinforcement in the column of size 400 x 600 mm subjected to an axial working load of 2000 KN. The column has an unsupported length of 3m and it is fixed against sideway in both directions. Adopt M20 and Fe 415.

STEP 1 : Given data

fck = 20 N/mm$^2$

fy =415 N/mm$^2$

Working load = 2000 KN

Unsupported Length = 3m

Column size(b x d) = 400 x 600mm

**STEP 2 : SLENDERNESS RATIO**

L/D =3000/400 (D is the Least Lateral Dimension)

=7.5 < 12

**STEP 3 : MIN. ECCENTRICITY**

$e_{min}$ = 3000/500 + 400/30

= 19.3 < 20mm (Pg. No. 42 clause 25.4)

$e_{max}$ = 3000/500 + 600/30

= 26 < 20mm

Check:

0.05 x D = 30 > $e_{ymin}$

0.05 x b = 20 > $e_{x\ min}$ (Pg. No. 71)

**STEP 4 : ULTIMATE LOAD, $P_u$**

$P_u$ = Working Load x PSF

= 1.5 x 2000

**$P_u$ = 3000 kN**

**STEP 5 : LONGITUDINAL REINFORCEMENT**

**$P_u$ = $0.4\ f_{ck}\ Ag + (0.67\ f_y - 0.4\ f_{ck})\ As_c$**

$A_g$ = 400 x 600 (Column area given)

= 24000 $mm^2$

**$3000 \times 10^3$ = $0.4 \times 20 \times 24000 + (0.6 \times 415 - 0.4 \times 20)\ A_{sc}$**

$A_{sc}$ = 3999.25 $mm^2$ say 4000 $mm^2$

Use 25 mm dia rod

n = $A_{sc} / a_{sc}$

= $4000 / (\pi \times 25^2 / 4)$

= 8.15 say 10 rods

In order to provide for column 10 nos. Can be split as 6 and 4 nos with 25mm and 20 mm respectively

$6 \times (\pi \times 25^2 / 4) + 4 \times (\pi \times 20^2/4) = 4201\ mm^2$

**STEP 6 : LATERAL TIES**

Tie Diameter < dia of bar / 4

= 25 / 4

= 6.25 < 8 mm

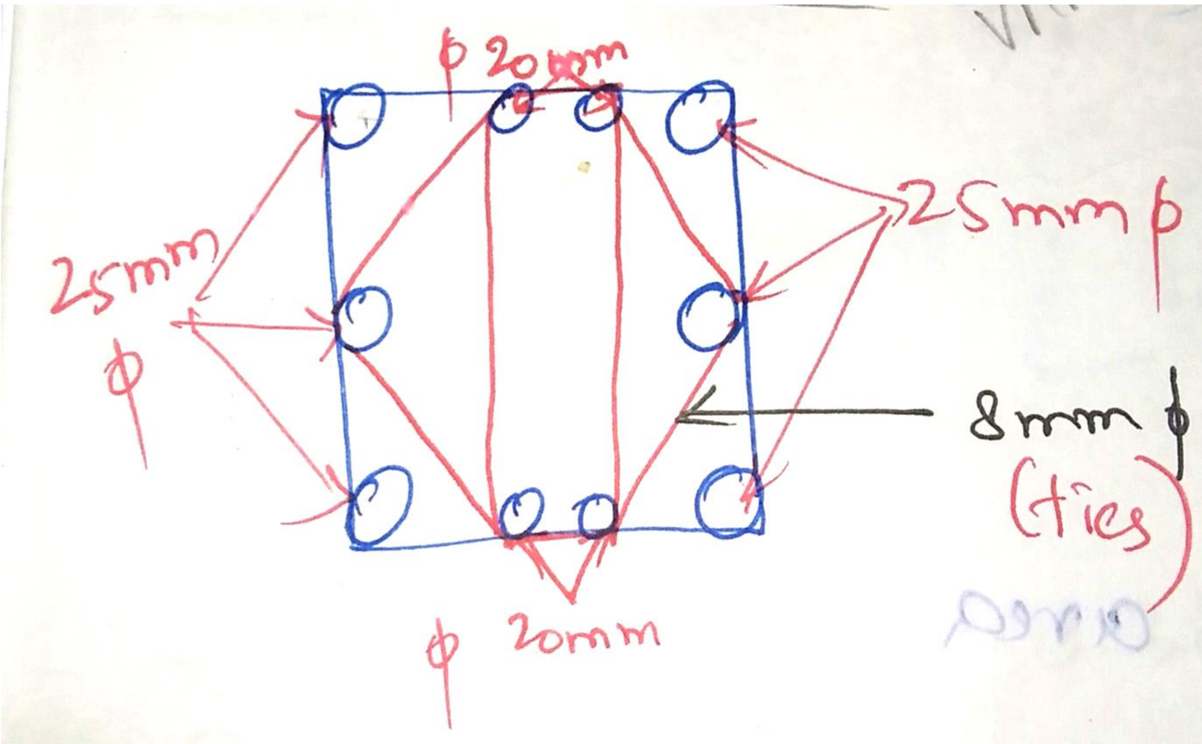
Tie spacing < 300 mm

16 x dia = 16 x 25

= 400 mm

**But take 300 mm as spacing which is always nominal**

**Provide 8 mm dia bars with spacing of 300 mm c/c**




**Design the reinforcement in the circular column of diameter 30mm with helical reinforcement to support a factored load of 1500 KN , the columns has an unsupported length of 3m & it is braced against side way. Adopt M20 & Fe 415 steel.**

STEP 1 : GIVEN DATA

Diameter = 300 mm

Unsupported Length , L = 3m

= 3000 mm

$P_u$ = 1500 KN

$f_{ck}$ = 20 N/mm$^2$

$f_y$ = 415 N/mm$^2$

STEP 2 : SLENDERNESS RATIO

$e_{min}$ = [ L/500 + D/300]

= [3000/500 +300/30]

= 16 < 20mm

Also 0.05 x D = 15 < 20 mm

STEP 3 : LONGITUDINAL REINFORCEMENT

$P_u$ = $0.4\ f_{ck}\ Ag + (0.67\ f_y - 0.4\ f_{ck})\ A_{sc}$

Ag = $\pi \times 300^2/4$

= $70650\ mm^2$

$A_{sc}$ = $3460\ mm^2$

$A_{scmin}$ = 0.8 % of C/Sn area

= $0.8/100 \times \pi \times 300^2/4$

= $560\ mm^2$

**Use 28 mm diameter**

n = $A_{sc}/\ a_{sc}$

= $3460 / (\pi \times 28^2 /4)$

= 5.6 say 6 Nos.

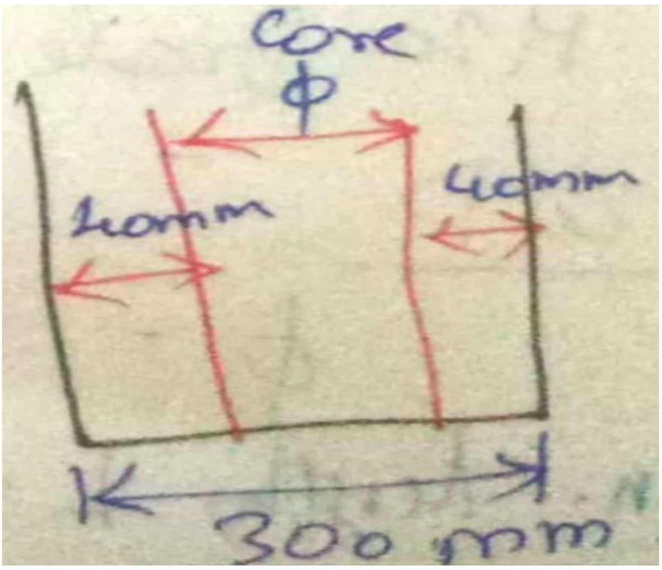
**Provide 6 Nos. of bars of 28 mm diameter**

STEP 4 : HELICAL REINFORCEMENT

Core diameter = [ 300 – (2x40)]

= 220 mm

**Assuming clear cover of 40 mm**




Area of core, $A_c$ = $(\pi \times 220^2 /4) - (6 \times \pi \times 28^2 /4)$

$A_c$ = $34319\ mm^2$

Volume of core = $34319 \times 10^3\ mm^3$

Gross area , $A_g$ = $(\pi \times 300^2 /4)$

= $70650\ mm^2$

**Use 8 mm diameter helical spirals at a pitch 'P' mm**

The volume of helical /m length is given by

$V_{ns}$ = $\pi$ ( 300 –(2x40)-8) x ($\pi$x$8^2$/4) x 1000/p

= 33.3 x $10^6$/p $mm^3$

According to clause 39.4.1 (IS: 456 )

**$V_{ns}$/ $V_c$ < 0.36 { ($A_g$ / $A_c$ ) – 1} $f_{ck}$/ $f_y$ (Pg. No 71 )**

33.3 x $10^6$/[p (34319 x $10^3$)] = 0.36 [(70650/34319) – 1] (20/415)

P = 52.7 mm < 75 mm

Codal restriction on pitch according to **clause 26.5.3.2**

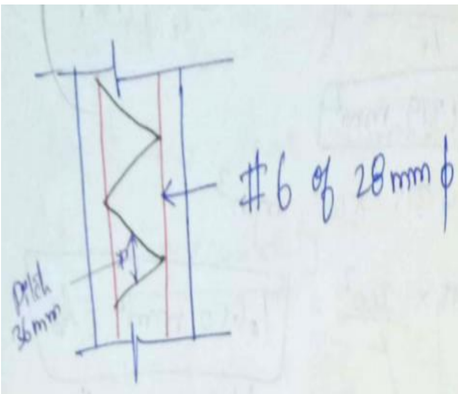
P < 75 mm (or) Core diameter /6 = 220/6 = 36.66 mm

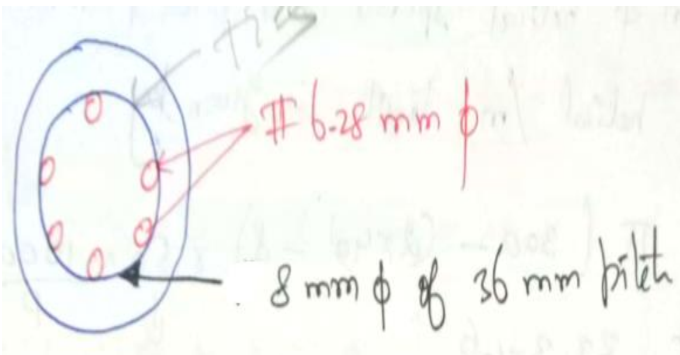
(or) p > 25

(or) 3 x diameter of helical rod

3x8 = 24

Therefore **provide 8 mm diameter of spiral at pitch of 36.6 mm**




**UNIAXIAL LOADING PROBLMES**

**Design the longitudinal & lateral reinforcement in rectangular reinforced concrete column of size 300 x 400 mm subjected to design ultimate load of 1200 KN and an ultimate moment of 200 KN-m with respect to major axis. Adopt M20 grade and Fe 415 steel.**

Given Data :

$P_u$ = 1200 KN

$M_u$ = 200 KNm

fck = 20 N/$mm^2$

fy = 415 N/$mm^2$

Column dim = 300 x 400 mm

STEP 1 : NON DIMENSIONAL PARAMETER

| | |
|---|---|
| **A** | $P_u/f_{ck}$ b D = 1200 x $10^3$/20 x 300 x400 = 0.5 |
| **B** | $M_u/f_{ck}$ b $D^2$ = 200 x $10^6$/20 x 300 x$400^2$ = 0.208 |

STEP 2 : LONGITUDINAL REINFORCEMENT

Adopting an effective cover of 50 mm

d' = 50 mm

d'/D = 50/400 = 0.125 = 0.15

**Refer chart 33 sp 16 Page No. 118**

**<u>From chart 33</u>** $P/f_{ck}$ = 0.2 (Based on A and B )

P = 0.2 x $f_{ck}$ = 0.2 x 20 = 4

Asc = P*b*D / 100 = 4800 $mm^2$

Use 28mm dia bars and find out no. of bars

n = Ast /ast

= 4800/($\pi$ x $28^2$/4) = 7.7 = 8 nos

**Provide 8 nos of 28 mm dia bars**

STEP 3 : LATERAL TIES

Tie dia = diameter of rod / 4

= 28/4

= 7mm < 8 mm

Tie spacing > 300 mm

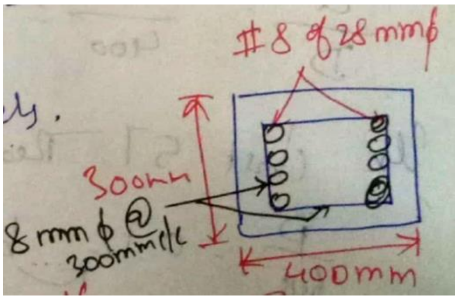
= 16 x d

= 16 x 28

= 448 > 300 mm

Adopt 300mm as spacing

Provide 8 mm dia bars with spacing 300 mm c/c



**Design a short circular column of dia 400 mm to support a factored axial load of 900 KN together with a factored moment of 100 KN-m. Adopt M20 Grade and Fe 415 Steel**

Given Data

$P_u$ = 900 KN

= $900 \times 10^3$ N

$M_u$ = 100 KNm

= $100 \times 10^6$ N-mm

$f_{ck}$ = 20 N/mm$^2$

$f_y$ = 415 N/mm$^2$

Dia, D = 400 mm

STEP 1 : NON DIMENSIONAL PARAMETER

| | |
|---|---|
| **A** | $P_u/f_{ck} D^2$ = $900 \times 10^3/20 \times 400^2$ = 0.28 |
| **B** | $M_u/f_{ck} D^3$ = $100 \times 10^6/20 \times 400^3$ = 0.78 |

STEP 2 : LONGITUDINAL REINFORCEMENT

Adopting an effective cover of 50 mm

d' = 50 mm

d'/D = 50/400 = 0.125 = 0.15

**Use chart 57,**

P/fck = 0.12 (Based on A and B )

P = 0.12 x fck = 0.12 x 20 = 2.4

Asc = $(P\pi D^2 / 4) \times (1/100)$ (from same chart)

= $2.4 \times \pi \times 400^2/400$

= 3014.4 mm$^2$

**Use 25 mm dia bars**

n = Ast /ast

= $3014.4/(\pi \times 25^2/4)$

= 6.14 say 6 nos

**Provide 6 nos of 25 mm dia bars**

STEP 3 : LATERAL TIES

Tie dia = diameter of rod / 4

= 25/4

= 6.25 mm < 8 mm

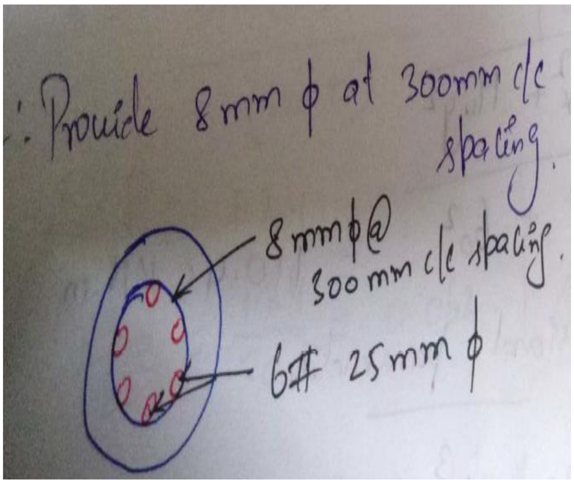
Tie spacing > 300 mm

= 16 x d

= 16 x 25

= 400 > 300 mm

**Adopt 300mm as spacing. Provide 8 mm dia bars with spacing 300 mm c/c**




**BIAXIAL LOADING PROBLMES**

**A short column located at the corners of multi storey building is subjected to an axial factored load of 2000 KN together with factored moments of 75 KNm and 60 KNm acting in perpendicular planes. The size of the column is fixed as 450 x 450 mm. Adopting M20 grade concrete and Fe415 HYSD bars. Design the reinforcement in the column section.**

STEP 1: Given Data :

$P_u$ = 2000 KN

$M_{ux}$ = 75 KNm

$M_{uy}$ = 60 KNm

$f_{ck}$ = 20 N/mm$^2$

$f_y$ = 415 N/mm$^2$

b = 450 mm

D = 450 mm

STEP 2: EQUIVALENT MOMENT

Mu = $\sqrt{M_{ux}^2 + M_{uy}^2}$

= $\sqrt{[75^2 + 65^2]}$

= 110.45 Kn.m

STEP 3 : NON DIMENSIONAL PARAMETER

**A** $P_u/f_{ck}$ b D = 2000 x $10^3$/20 x 450 x450 = **0.49**

**B** $M_u/f_{ck}$ b $D^2$ = 110 x $10^6$/20 x 450 x$450^2$ = **0.06**

STEP 4 : LONGITUDINAL REINFORCEMENT

Adopting an effective cover of 50 mm

d' = 50 mm

d'/D = 50/450 = 0.1

Refer chart 44 of SP:16 and read out $P/f_{ck}$ value [for four sides]

$P/f_{ck}$ = 0.06

P = 0.06 x $f_{ck}$ = 0.06 x 20 = 1.2

$A_{sc}$ = P*b*D / 100 = 2430 $mm^2$

Use 20mm$^{\phi}$ bars and find out no. of bars

n = $A_{sc}$ /$a_{sc}$

= 2430/(π x $20^2$/4) = 8 nos

**Provide 8 nos of 20 mm dia bars**

$A_{sc}$ (provided) = 8*(π*$20^2$ / 4)

= 2512 $mm^2$

Find P value by using $A_{sc}$ (provided)

P = [$A_{sc}$ (provided) * 100] / (b*D)

[The formula arrived from $A_{sc}$ = (P*b*D) /100]

= 1.24

P/ $f_{ck}$ = 1.24 / 20

= 0.062

Refer Chart 44 and read out $M_{ux}$ / $f_{ck}bD^2$

Corresponding to the value of P/ $f_{ck}$ *b*D and P/ $f_{ck}$ value read out $M_{ux}$ / $f_{ck}bD^2$

That is P/ $f_{ck}$ *b*D =0.49 and P/ $f_{ck}$ =0.062

From chart 44, $M_{ux}$ / $f_{ck}bD^2$ = 0.06

$M_{ux}$= 0.06 * 20 * 450 * $450^2$

= 109.3*$10^6$

= 110 Kn.m

Therefore, we take $M_{ux1}$=$M_{uy1}$=110 Kn.m

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**SCHOOL OF BUILDING AND ENVIRONMENT**

**DEPARTMENT OF CIVIL ENGINEERING**

# UNIT – V – FOOTINGS – SCIA1502

# UNIT – V – FOOTINGS

**Types of footing - SBC of soil - Design loads - Design of axially and eccentric loaded rectangular footing - Combined rectangular footing.**

## INTRODUCTION

Footings are structural elements that transmit column or wall loads to the underlying soil below the structure. Footings are designed to transmit these loads to the soil without exceeding its safe bearing capacity, to prevent excessive settlement of the structure to a tolerable limit, to minimize differential settlement, and to prevent sliding and overturning. The settlement depends upon the intensity of the load, type of soil, and foundation level. Where possibility of differential settlement occurs, the different footings should be designed in such away to settle independently of each other.

Foundation design involves a soil study to establish the most appropriate type of foundation and a structural design to determine footing dimensions and required amount of reinforcement. Because compressive strength of the soil is generally much weaker than that of the concrete, the contact area between the soil and the footing is much larger than that of the columns and walls.

## DESIGN OF FOOTINGS

### ***General***

Most of the structures built by us are made of reinforced concrete. Here, the part of the structure above ground level is called as the superstructure, where the part of the structure below the ground level is called as the substructure. Footings are located below the ground level and are also referred as foundations. Foundation is that part of the structure which is in direct contact with soil. The R.C. structures consist of various structural components which act together to resist the applied loads and transfer them safely to soil. In general the loads applied on slabs in buildings are transferred to soil through beams, columns and footings. Footings are that part of the structure which is generally located below ground Level. They are also referred as foundations. Footings transfer the vertical loads, Horizontal loads, Moments, and other forces to the soil.

The important purposes of foundation are as follows;

1. To transfer forces from superstructure to firm soil below.

2. To distribute stresses evenly on foundation soil such that foundation soil neither fails nor experiences excessive settlement.

3. To develop an anchor for stability against overturning.

4. To provide an even surface for smooth construction of superstructure.

Due to the loads and soil pressure, footings develop Bending moments and Shear forces. Calculations are made as per the guidelines suggested in IS 456 2000 to resist the internal forces.

**TYPES OF FOUNDATIONS**

Based on the position with respect to ground level, Footings are classified into two types;

1. Shallow Foundations

2. Deep Foundations

Shallow Foundations are provided when adequate SBC is available at relatively short depth below ground level. Here, the ratio of $D_f$ / B < 1, where $D_f$ is the depth of footing and B is the width of footing. Deep Foundations are provided when adequate SBC is available at large depth below ground level. Here the ratio of $D_f$ / B >= 1.

***<u>Types of Shallow Foundations</u>***

The different types of shallow foundations are as follows:

- ✓ Isolated Footing
- ✓ Combined footing
- ✓ Strap Footing
- ✓ Strip Footing
- ✓ Mat/Raft Foundation
- ✓ Wall footing

Some of the popular types of shallow foundations are briefly discussed below.

<u>Isolated Column Footing</u>

These are independent footings which are provided for each column. This type of footing is chosen when

- ✓ SBC is generally high
- ✓ Columns are far apart
- ✓ Loads on footings are less

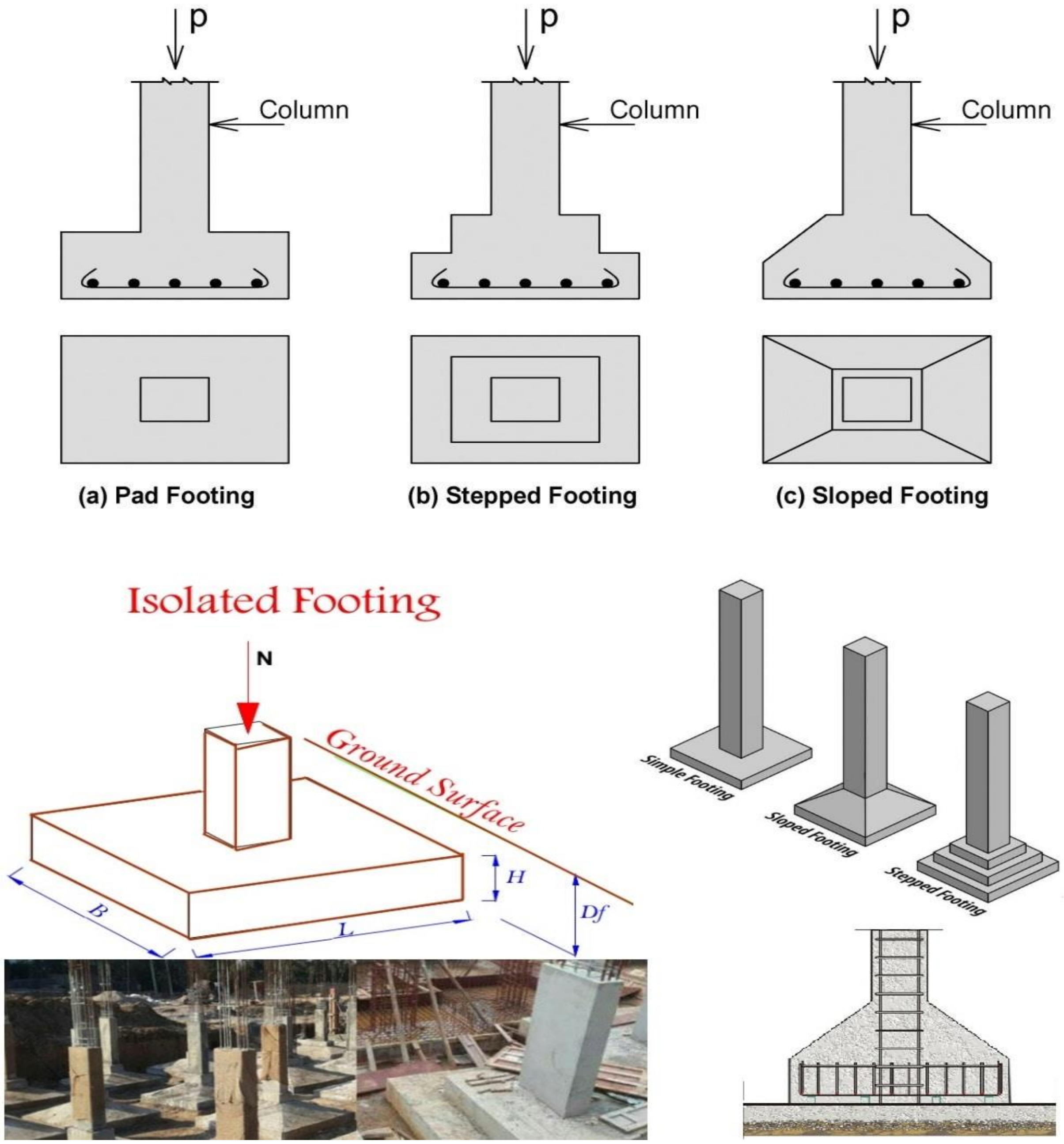


***Fig 1 Isolated Column Footing***

The isolated footings can have different shapes in plan. Generally it depends on the shape of column cross section Some of the popular shapes of footings are;

a) Square

b) Rectangular

c) Circular

The isolated footings essentially consist of bottom slab. These bottom Slabs can be flat, stepped or sloping in nature. The bottom of the slab is reinforced with steel mesh to resist the two internal forces namely bending moment and shear force.

<u>Combined Column Footing</u>

These are common footings which support the loads from 2 or more columns. Combined footings are provided when

- ✓ SBC is generally less
- ✓ Columns are closely spaced
- ✓ Footings are heavily loaded

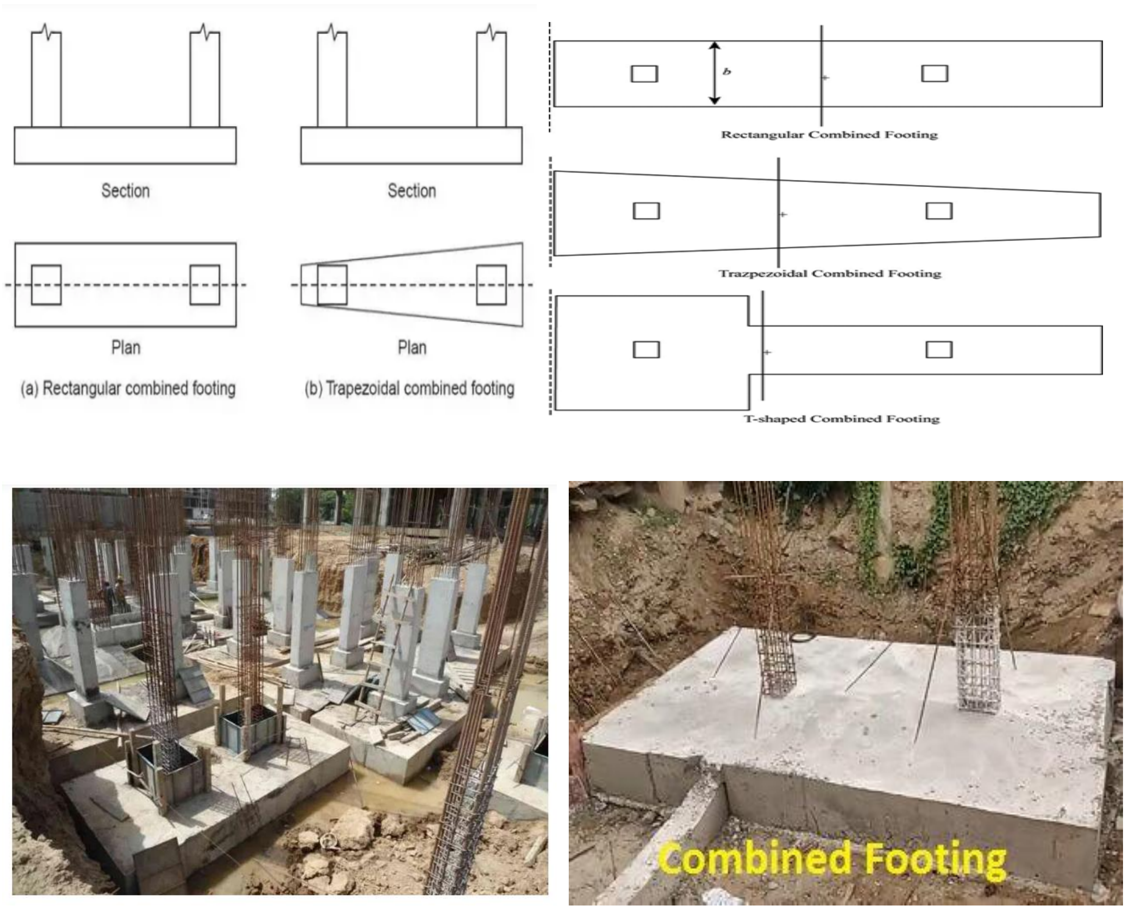


***Fig 2 Combined Column Footing***

In the above situations, the area required to provide isolated footings for the columns generally overlap. Hence, it is advantageous to provide single combined footing. In some cases the columns are located on or close to property line. In such cases footings cannot be extended on one side. Here, the footings of exterior and interior columns are connected by the combined footing.

Combined footings essentially consist of a common slab for the columns it is supporting. These slabs are generally rectangular in plan. Sometimes they can also be trapezoidal in plan (refer Fig. 2). Combined footings can also have a connecting beam and a slab arrangement, which is similar to an inverted T – beam slab.

<u>Strap Footing</u>

An alternate way of providing combined footing located close to property line is the strap footing. In strap footing, independent slabs below columns are provided which are then connected by a strap beam. The strap beam does not remain in contact with the soil and does not transfer any pressure to the soil. Generally it is used to combine the footing of the outer column to the adjacent one so that the footing does not extend in the adjoining property.

<u>Strip Footing</u>

Strip footing is a continuous footing provided under columns or walls

<u>Mat Foundation</u>

Mat foundation covers the whole plan area of structure. The detailing is similar to two way reinforced solid floor slabs or flat slabs. It is a combined footing that covers the entire area beneath a structure and supports all the walls and columns. It is normally provided when

- ✓ Soil pressure is low
- ✓ Loads are very heavy

- ✓ Spread footings cover > 50% area

***<u>Types of Deep Foundations</u>***

Deep foundations are provided when adequate SBC is available at large depth below GL. There are different types of deep foundations. Some of the common types of deep foundations are listed below.

- ✓ Pile Foundation
- ✓ Pier Foundation
- ✓ Well Foundation

**BEARING CAPACITY OF SOIL**

The safe bearing capacity of soil is the safe extra load soil can withstand without experiencing shear failure. The Safe Bearing Capacity (SBC) is considered unique at a particular site. But it also depends on the following factors:

- ✓ Size of footing
- ✓ Shape of footing
- ✓ Inclination of footing
- ✓ Inclination of ground
- ✓ Type of load
- ✓ Depth of footing etc.

SBC alone is not sufficient for design. The allowable bearing capacity is taken as the smaller of the following two criteria

- ✓ Limit states of shear failure criteria (SBC)

- ✓ Limit states of settlement criteria

Based on ultimate capacity, i.e., shear failure criteria, the SBC is calculated as

SBC = Total load / Area of footing

Usually the Allowable Bearing Pressure (ABP) varies in the range of 100 $kN/m^2$ to 400 $kN/m^2$. The area of the footing should be so arrived that the pressure distribution below the footing should be less than the allowable bearing pressure of the soil. Even for symmetrical Loading, the pressure distribution below the footing may not be uniform. It depends on the Rigidity of footing, Soil type and Conditions of soil. In case of Cohesive Soil and Cohesion less Soil the pressure distribution varies in a nonlinear way. However, while designing the footings a linear variation of pressure distribution from one edge of the footing to the other edge is assumed. Once the pressure distribution is known, the bending moment and shear force can be determined and the footing can be designed to safely resist these forces.

**DEPTH OF FOOTING**

As per Rankine's theory

Minimum depth of foundation = $P/w\ (1-\sin\Phi\ /1+\sin\Phi)^2$ where

P = gross bearing capacity

w = density of soil

Φ = angle of response of soil

***<u>Design of Isolated Column Footing</u>***

The objective of design is to determine

- Area of footing
- Thickness of footing
- Reinforcement details of footing (satisfying moment and shear considerations)
- Check for bearing stresses and development length

This is carried out considering the loads of footing, SBC of soil, Grade of concrete and Grade of steel. The method of design is similar to the design of beams and slabs. Since footings are buried, deflection control is not important. However, crack widths should be less than 0.3 mm.

The steps followed in the design of footings are generally iterative. The important steps in the design of footings are;

- Find the area of footing (due to service loads)
- Assume a suitable thickness of footing
- Identify critical sections for flexure and shear
- Find the bending moment and shear forces at these critical sections (due to factored loads)
- Check the adequacy of the assumed thickness
- Find the reinforcement details
- Check for development length
- Check for bearing stresses

Limit state of collapse is adopted in the design pf isolated column footings. The various design steps considered are;

- Design for flexure
- Design for shear (one way shear and two way shear)
- Design for bearing

- Design for development length

The materials used in RC footings are concrete and steel. The minimum grade of concrete to be used for footings is M20, which can be increased when the footings are placed in aggressive environment, or to resist higher stresses.

Cover: The minimum thickness of cover to main reinforcement shall not be less than 50 mm for surfaces in contact with earth face and not less than 40 mm for external exposed face. However, where the concrete is in direct contact with the soil the cover should be 75 mm. In case of raft foundation the cover for reinforcement shall not be less than 75 mm.

Minimum reinforcement and bar diameter: The minimum reinforcement according to slab and beam elements as appropriate should be followed, unless otherwise specified. The diameter of main reinforcing bars shall not be less 10 mm. The grade of steel used is either Fe 415 or Fe 500.

**ONE-WAY SHEAR**

One-way shear in footing is considered similar to that of slabs. Considering the footing as a wide beam, the critical section is taken along a vertical plane extending the full width of the footing, located at a distance equal to the effective depth of footing (i.e., considering a dispersion angle of 45°) from the face of the column, pedestal, or wall.

**TWO-WAY SHEAR**

The behaviour of footing in two-way (punching) shear is identical to that of two-way flat slabs supported on columns.

The critical section for the two-way shear is taken at a distance d/2 from the periphery of the column

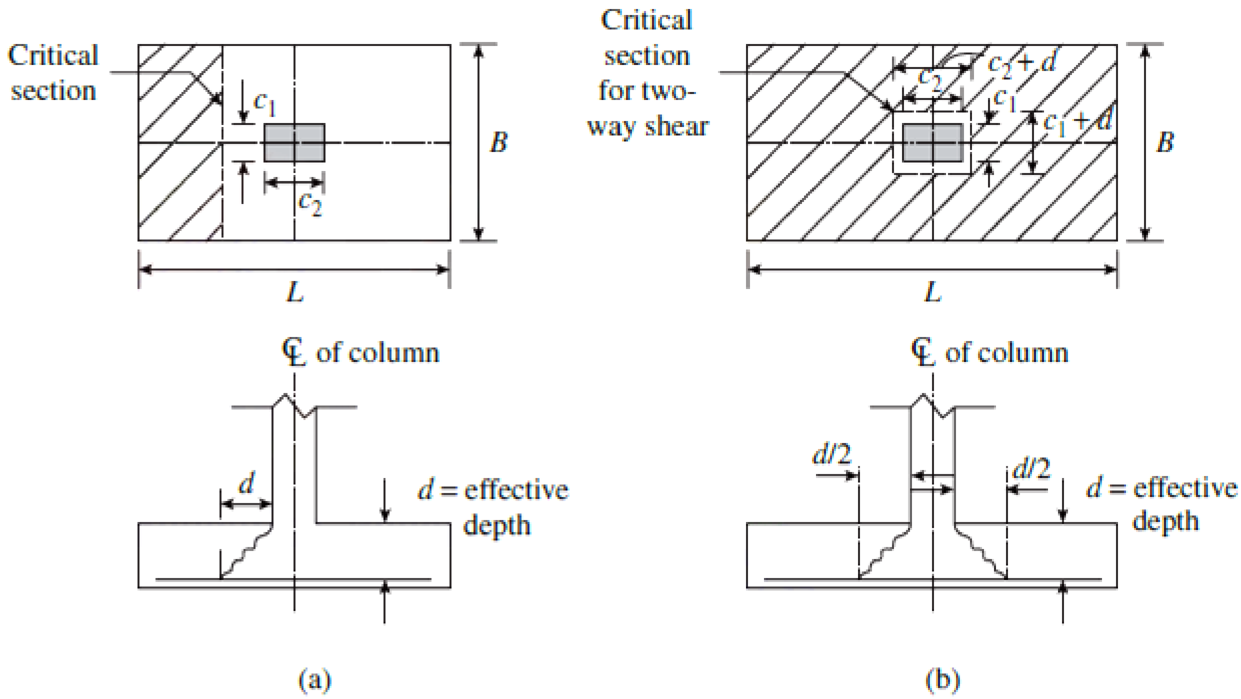


*Fig 3 Critical sections of (a) One-way shear and (b) Two-way punching shear*

**DESIGN PROBLEMS**

Design a reinforced concrete footing for rectangular column section 300 X 500 mm supporting an axial factored load of 1500 KN, the safe bearing capacity of soil at site is 185 $KN/m^2$. Adopt M20 grade concrete and Fe415 steel bars.

**Step 1: Given data**

b= 300 mm

d= 500 mm

$P_u$ = 1500 KN

p= 185 $KN/m^2$ [Safe bearing capacity of the soil]

$f_{ck}$ = 20 $N/mm^2$

$f_y$ = 415 $N/mm^2$

**Step 2: Size of Footing**

Load on the column, $P_u$ = 1500 KN

| | | |
|---|---|---|
| Self weight of footing | = | 10 % of the total load on column |
| [Assumed as 10%] | = | 1500*[10/100] |
| | = | 150 KN |
| Total factored load, W | = | 1500+150 |
| | = | 1650 KN |
| Footing area | = | ? |
| [Note: Stress | = Load /Area which is then | |
| Area | = Load /Stress] | |
| Therefore, Footing area | = | 1650 / [1.5*185] |
| | = | 5.95 $m^2$ say 6 $m^2$ |

Proportioning the following area in the same proportion as the sides of the column

| | | | |
|---|---|---|---|
| Hence | (3x) X (5x) | = | 6 $m^2$ |
| | x | = | 0.63 m |
| Short side of the footing is | | = | 3*0.63 |
| | | = | 1.89 m says 2 m |
| Long side of the footing is | | = | 5*0.63 |
| | | = | 3.15 m says 3 m |

***<u>Adopt a rectangular footing of size 2 m by 3 m [ 2 x 3 m]</u>***

| | | | |
|---|---|---|---|
| Factored soil pressure at base, $q_u$ | = | 1650 / [2*3] | < 1.5*185 |
| | = | 275 | < 277.5 |

**Hence the footing area is adequate since the soil pressure developed at the base is less than the factor bearing capacity of soil [<1.5*185]**

**Step 3: Factored Bending moment**

Cantilever projection from the short side of the column = 1/2*[2-0.3]

= 0.85 m

This the footing dimension [0.85 m]

Cantilever projection from the long side of the column = 1/2*[3-0.5]

= 1.25 m

This the column dimension [1.25 m]

Bending Moment at long side of the column = $[q_u * L^2]/ 2$

= $[275*1.25^2] / 2$

= 214.8 Kn. m

Bending Moment at short side of the column = $[q_u * L^2]/ 2$

= $[275*0.85^2] / 2$

= 99.3 Kn. M

**Step 4: Depth of footing**

**Depth based on moment consideration**

$M_{u, lim}$ = $0.138 * f_{ck} * b * d^2$

$214.8*10^6$ = $0.138 * 230 * 1000 * d^2$

d = 279 mm

**Depth based on shear stress consideration**

Shear force per m in longer direction

Vu * L = qu [L-d] N

= 275 [(300/2)-(500/2)-d] N

= 275 [1250-d] N

Assuming the shear strength of Ʈc = 0.36 $N/mm^2$

M20 grade concrete with Pt = 0.25 %

Ʈc = Vu * L / b * d

Ʈc = Vu * L / b * d

$0.36 = 275 [1250-d] / [1000* d]$

d = 541 mm says 550 mm

Therefore ,

Adopt **effective depth , d = 550 mm**

**Overall depth , D = 600 mm**

**Step 5: Reinforcement in footing**

Longer direction, $M_u$ = 214.8 Kn. m

Shorter direction, $M_u$ = 99.3 Kn. M

***For Longer Direction***

$$Mu, = 0.87 \times f_y \times A_{st} \times d \times (1 - Ast\ f_y / b\ d\ f_{ck})$$

$$214.8 * 10^6 = 0.87 \times 415 \times A_{st} \times 550 \times (1 - Ast * 415 / 20*100*550)$$

$$A_{st} = 1125.5 \text{ mm}^2$$

***For Shorter Direction***

$$M = 0.87 \times f_y \times A_{st} \times d \times (1 - Ast\ f_y / b\ d\ f_{ck})$$

$$99.3 * 10^6 = 0.87 \times 415 \times A_{st} \times 550 \times (1 - Ast * 415 / 20*100*550)$$

$$A_{st} = 510 \text{ mm}^2$$

***For Longer direction:***

Provide 16 mm diameter steel rod with 160 mm C/C spacing

***For Shorter direction:***

Provide 12 mm diameter steel rod with 200 mm C/C spacing

For reinforcement in the short direction a central band equal to the width of footing shall be marked along the length of the footing and the portion of reinforcement is determined in accordance with the equation given below:

**Reinforcement in central band width** = $[2 / (\beta+1)] * A_{st}$

Where,

β = Ratio of long and short side **[Clause 34.2.4.3 (c)]**

= Ly/Lx

= 3 / 2

**β = 1.5**

**Band width of 2 m = [2 / (β+1)]**

= [2 / (1.5+1)] * 2 * 565

Min $A_{st}$ = 0.12*1000*6000 / 100

= 720 mm

$S_v$ = 1000*ast / Ast

= 1000*(π*12$^2$) / 720

= 157 mm says 150 mm

Provide 12 mm diameter bars at 150 mm c/c