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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – I – STRUCTURAL ANALYSIS 1 – SCIA1501

1.0 Structural system

Structural system or **structural frame** in structural engineering refers to load-resisting sub-system of a structure. The structural system transfers loads through interconnected structural components or members.

Tensile structures: Members of tensile structures are subjects to pure tension under the action of external loads.

Compressive structures: Compression structures develop mainly compressive stresses under the action of axial loads. Because compressive structures are susceptible to buckling or instability.

Trusses: Trusses are composed of straight members connected at their ends by hinged connections to form a stable configuration.

Shear structures: These are structures such as reinforced concrete or wooden shear walls, which are used in multistory buildings to reduce lateral movements due to wind loads and earthquake excitations.

Bending structures: Bending structures develop mainly bending stresses under the action of external loads.

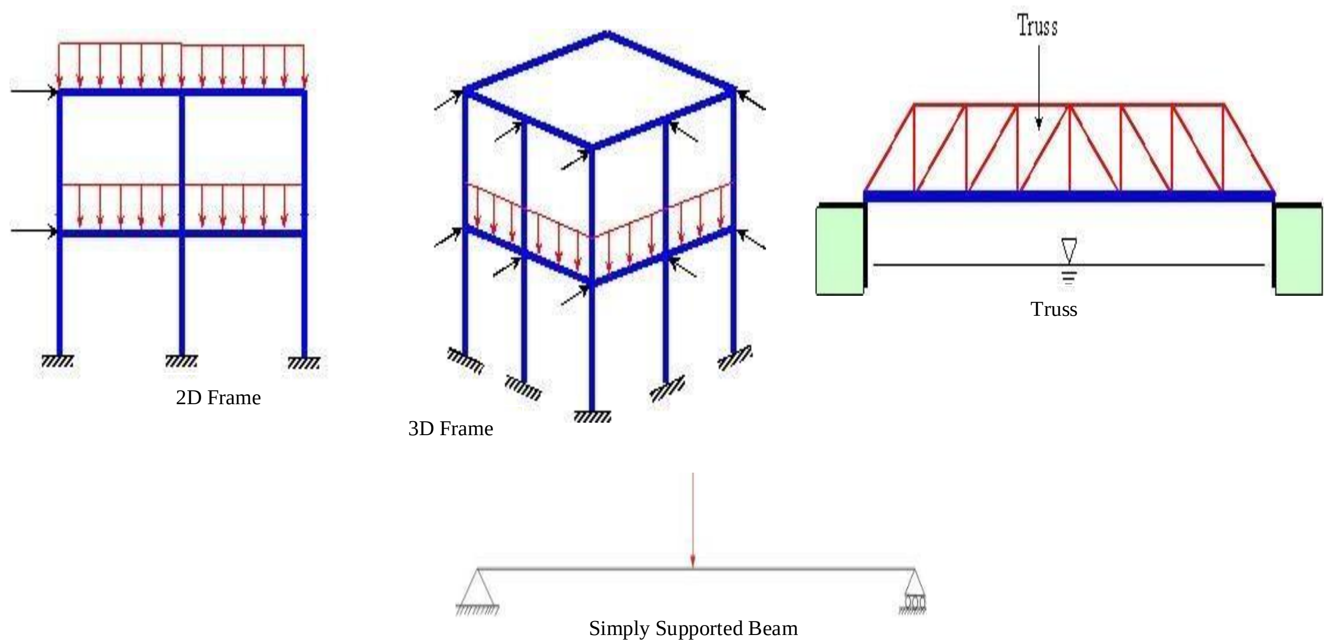


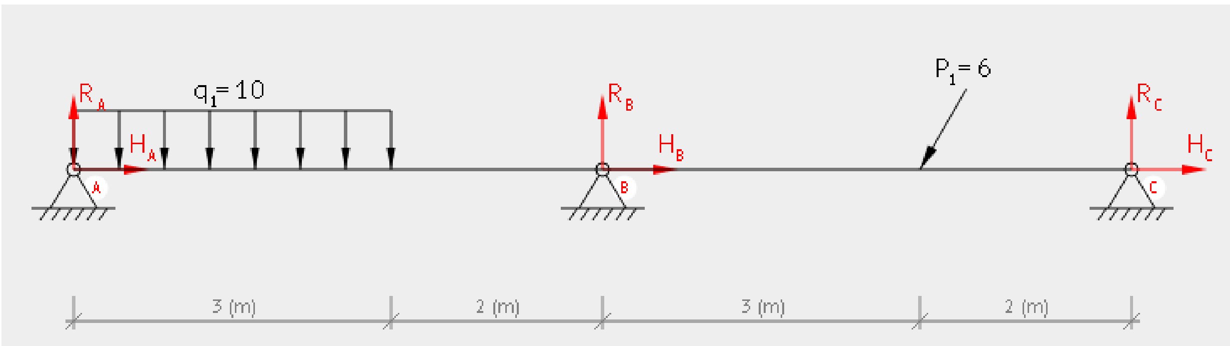
Fig 1.0 Determinate structure

Equilibrium Equations

Equilibrium equations: The static equilibrium of a particle is an important concept in statics. A particle is in equilibrium only if the resultant of all forces acting on the particle is equal to zero.

$\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma F_z = 0$;$\Sigma M_x = 0$;$\Sigma M_y = 0$; $\Sigma M_z = 0$

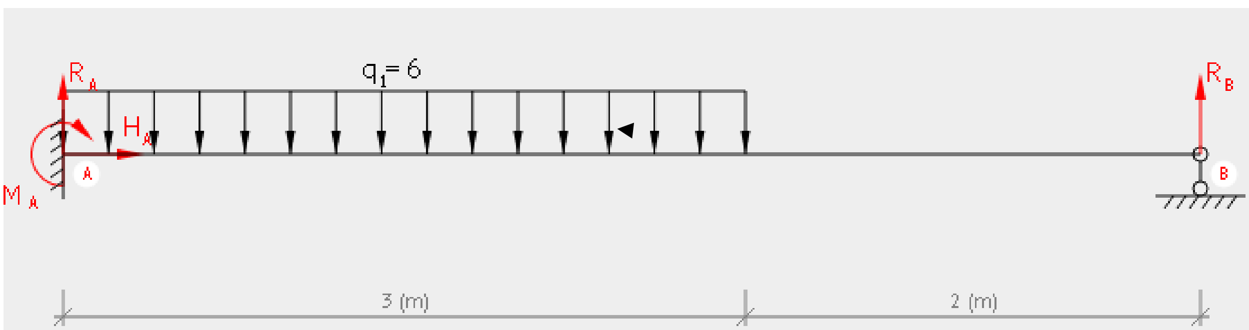
Static Indeterminacy



Number of unknown reactions (NR) = 6

Number of Equilibrium Equations (NE) = 3

Static Indeterminacy = NR – NE = 6-3 =3 (3^{rd} Degree of Indeterminacy)



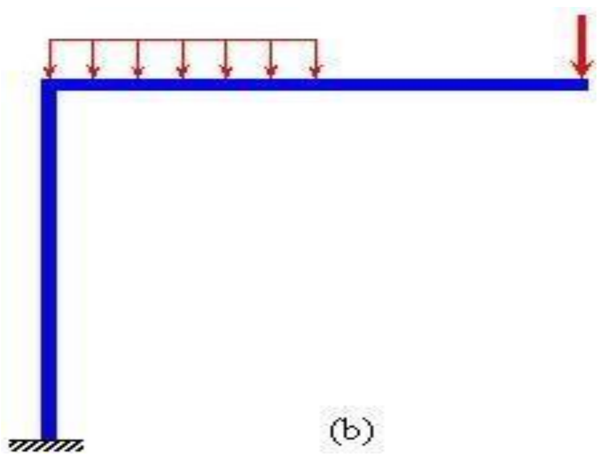
Number of unknown reactions (NR) = 4

Number of Equilibrium Equations (NE) = 3

Static Indeterminacy = NR – NE = 4-3 =1 (1^{st} Degree of Indeterminacy) Statically Indeterminate

Number of Equilibrium Equations (NE) = 3;
Number of Reactions (NR) = 3

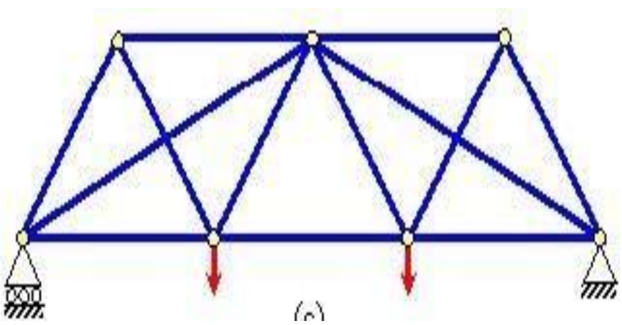
Static Indeterminacy= NR-NE = 3-3 =0,
Statically Determinate



Number of Unknown Reactions = 3

Number of Equilibrium Equation= 3

Determinacy= NR- NE = 0 Statically Determinate

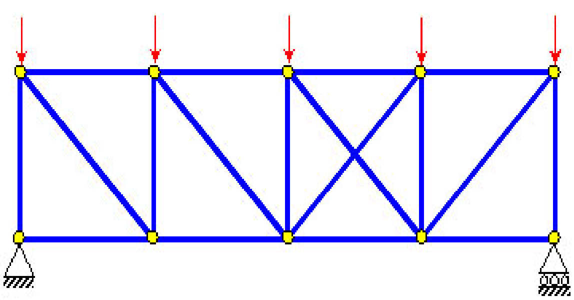


Number of Unknown Forces (Internal) = 13

Number of Unknown Forces (External) = 3

Number of Equilibrium Equations (NE) = 2j = 2 x7

Indeterminacy = N.U.F (Int) + N.U.F (Ext) – NE = 16-14 = 2 (2^{nd} Degree of Static Indeterminacy) Statically Indeterminate.

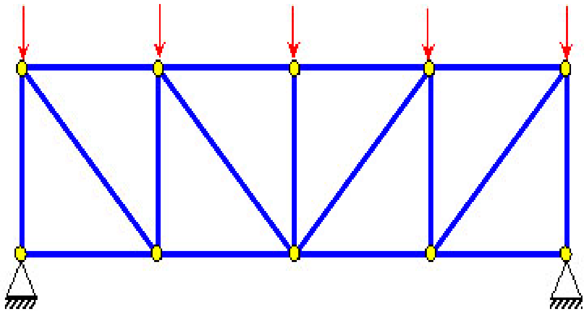


Number of Unknown Forces (Internal) = 18

Number of Unknown Forces (External) = 3

Number of Equilibrium Equations (NE) = 2j = 2 x10 = 20

Indeterminacy = N.U.F (Int) + N.U.F (Ext) – NE = 18+3-20= 1 (Internally Indeterminate)



Number of Unknown Forces (Internal) = 17

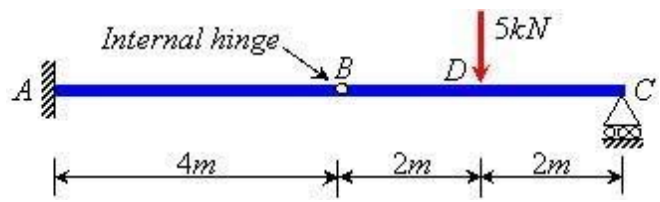
Number of Unknown Forces (External) = 4

Number of Equilibrium Equations (NE) = 2j = 2 x10 = 20

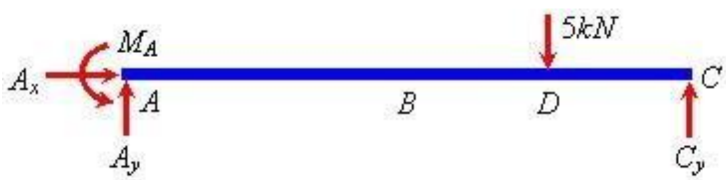
Indeterminacy = N.U.F (Int) + N.U.F (Ext) – NE = 18+3- 20= 1 (Externally Indeterminate)

Free Body Diagram

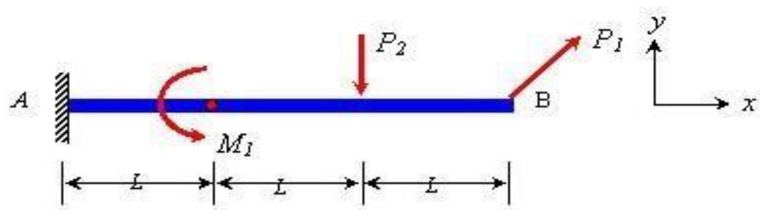
The free body diagram of a body (or its part, or a connected system of bodies) is obtained by isolating it from the all *other* surrounding bodies. The diagram detaches the system in consideration from all mechanical contacts with *other* bodies and sets it *free.*

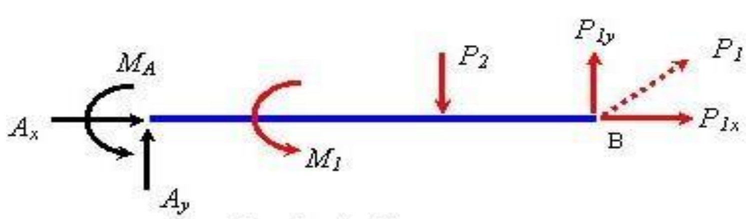


Whole Structure



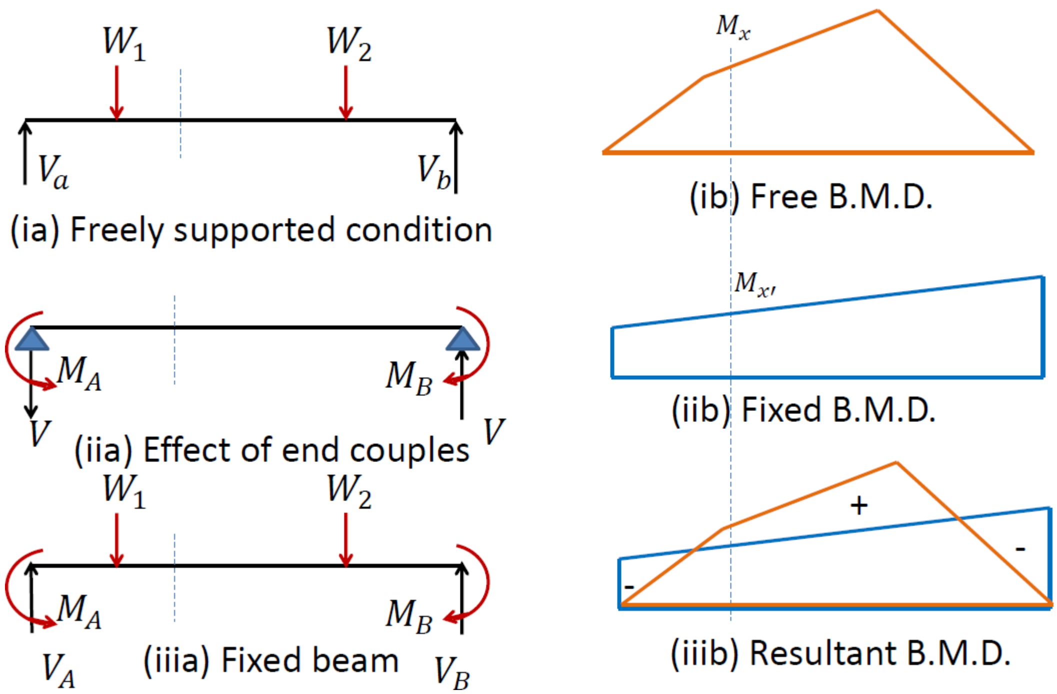
Free Body Diagram





Free body diagram

2.0 Fixed beam



$$M_A + M_B = \frac{Wab}{l} ----(1)$$

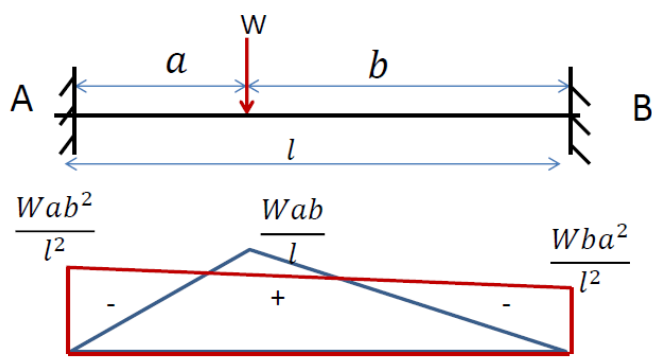
$$M_B = M_A \times \frac{a}{b} ---(2)$$

By substituting (2) in (1),

$$M_A = \frac{Wab^2}{l^2}$$

From (2),

$$M_B = \frac{Wba^2}{l^2}$$



Resultant BMD

3.0 Continuous beam with supports at different levels

A beam is generally supported on a hinge at one end and a roller bearing at the other end. The reactions are determined by using static equilibrium equations. Such as beam is a statically determinate structure. If the ends of the beam are restrained/clamped/encastre/fixed then the moments are included at the ends by these restraints and this moments make the structural element to be a statically indeterminate structure or a redundant structure. These restraints make the slopes at the ends zero and hence in a fixed beam, the deflection and slopes are zero at the supports.

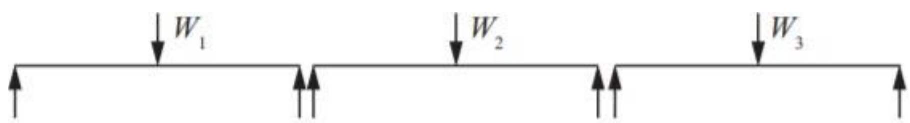


FIG. 11a Simply supported beam

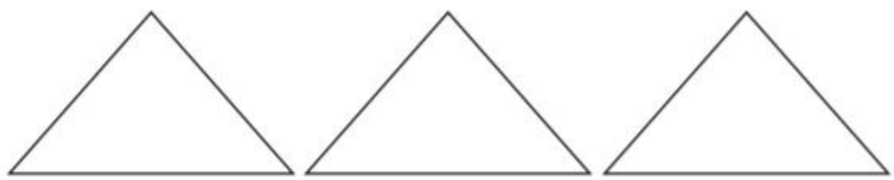


FIG. 11b Bending moment diagrams

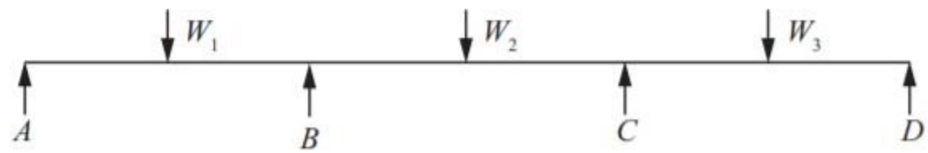


FIG. 11c Continuous beam

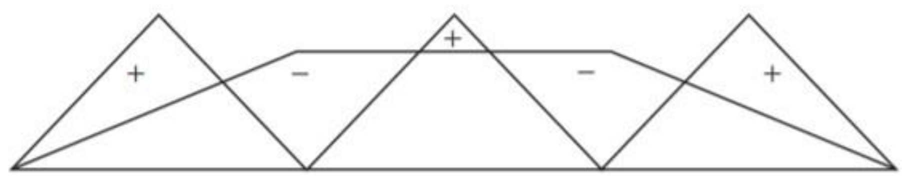


FIG. 11d Bending moment diagram

EXAMPLE .1: A continuous beam ABC is simply supported at A and C and continuous over support B with AB = 4m and BC = 5m. A uniformly distributed load of 10 kN/m is acting over the beam. The moment of inertia is I throughout the span. Analyse the continuous beam and draw SFD and BMD

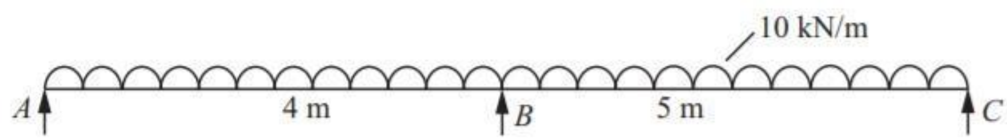


FIG. 11.1a

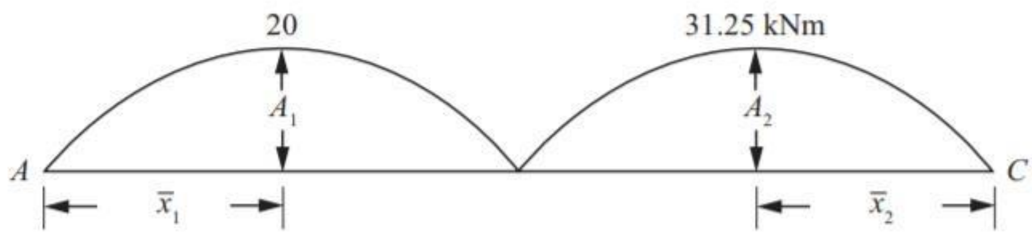


FIG. 11.1b Simple beam moment diagram

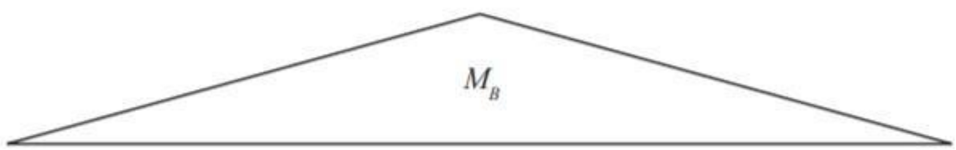


FIG. 11.1c Pure moment diagram

Properties of the simple beam BMD

$A_1 = \frac{2}{3} \times 4 \times 20 = 53.33 \text{ kNm}^2$	$A_2 = \frac{2}{3} \times 5 \times 31.25 = 104.17 \text{ kNm}^2$
$\bar{x_1} = 2\text{m}$	$\bar{x_2} = 2.5\text{m}$
$l_1 = 4\text{m}$	$l_2 = 5.0\text{m}$

Applying three moment equation for the span *ABC*

$$\cancel{M_A l_1} + 2M_B(l_1 + l_2) + \cancel{M_C l_2} = -6\left(\frac{A_1\bar{x_1}}{l_1} + \frac{A_2\bar{x_2}}{l_2}\right)$$

$$2M_B(4+5) = -6\left(\frac{53.33 \times 2}{4} + \frac{104.17 \times 2.5}{5}\right)$$

$$18M_B = -6(26.67 + 52.1)$$

$$M_B = -26.26 \text{ kNm.}$$

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UNIT – II – STRUCTURAL ANALYSIS 1 –SCIA1501

UNIT 2 - Moment Distribution Method

1.0 Introduction

As pointed out earlier, there are two distinct methods of analysis for statically indeterminate structures depending on how equations of equilibrium, load displacement and compatibility conditions are satisfied: 1) force method of analysis and (2) displacement method of analysis. In the last module, force method of analysis was discussed. In this module, the displacement method of analysis will be discussed. In the force method of analysis, primary unknowns are forces and compatibility of displacements is written in terms of pre-selected redundant reactions and flexibility coefficients using force displacement relations. Solving these equations, the unknown redundant reactions are evaluated. The remaining reactions are obtained from equations of equilibrium.

As the name itself suggests, in the displacement method of analysis, the primary unknowns are displacements. Once the structural model is defined for the problem, the unknowns are automatically chosen unlike the force method. Hence this method is more suitable for computer implementation. In the displacement method of analysis, first equilibrium equations are satisfied. The equilibrium of forces is written by expressing the unknown joint displacements in terms of load by using load displacement relations. These equilibrium equations are solved for unknown joint displacements. In the next step, the unknown reactions are computed from compatibility equations using force displacement relations. In displacement method, three methods which are closely related to each other will be discussed.

1) Slope-Deflection Method
2) Moment Distribution Method
3) Direct Stiffness Method

The Slope-deflection and moment distribution methods were extensively used for many years before the compute era. After the revolution occurred in the field of computing only direct stiffness method is preferred.

1.1 Degrees of freedom

In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends. For example, a propped cantilever beam (see Fig.14.01a) under the action of load P will undergo only rotation at B if axial deformation is neglected. In this case kinematic degree of freedom of the beam is only one i.e. $\square_B$ as shown in the figure.

In Fig.14.01b, we have nodes at A,B,C and D. Under the action of lateral loads and , this continuous beam deform as shown in the figure. Here axial deformations are neglected. For this beam we have five degrees of freedom 21,P P3 P θ_C θ_B θ_A,, , and as indicated in the figure. In Fig.14.02a, a symmetrical plane frame is loaded symmetrically. In this case we have only two degrees of freedom Dθ D ΔB θ and C θ. Now consider a frame as shown in Fig.14.02b. It has three degrees of freedom viz. Bθ,Cθ and DΔ as shown. Under the action of horizontal and vertical load, the frame will be displaced as shown in the figure. It is observed that nodes at B and C undergo rotation and also get displaced horizontally by an equal amount. Hence

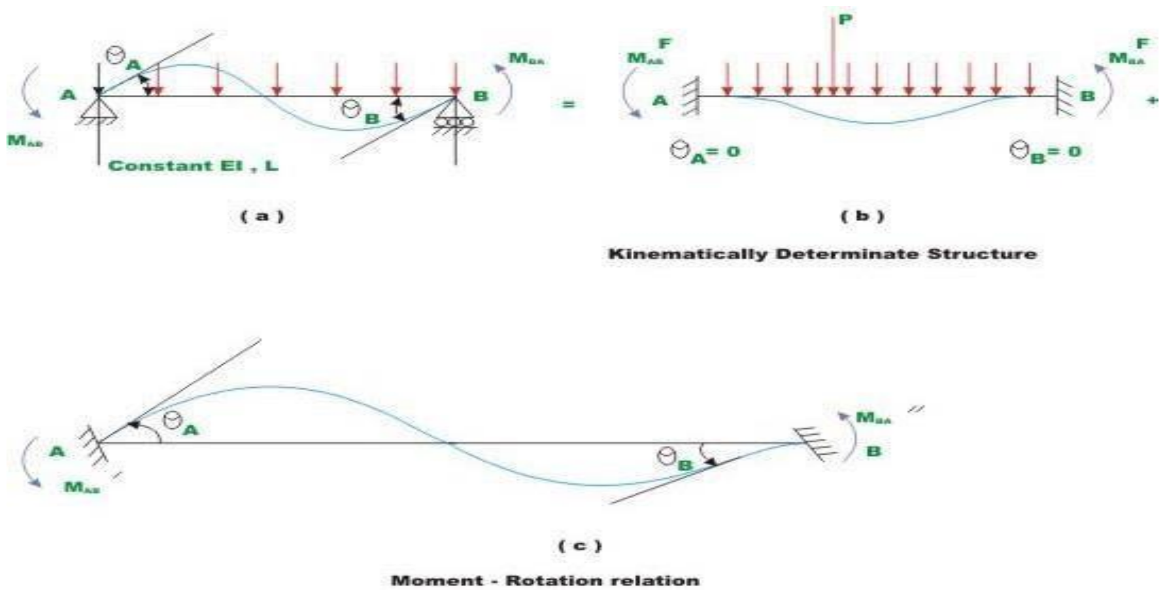


Fig 1. Slope deflection method

Objectives

After reading this chapter the student will be able to

1. Calculate kinematic degrees of freedom of continuous beam.
2. Derive slope-deflection equations for the case beam with unyielding supports.
3. Differentiate between force method and displacement method of analyses.
4. State advantages of displacement method of analysis as compared to force method of analysis.
5. Analyse continuous beam using slope-deflection method.

In this lesson the slope-deflection equations are derived for the case of a beam with unyielding supports .In this method, the unknown slopes and deflections at nodes are related to the applied loading on the structure. As introduced earlier, the slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames. In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. As discussed earlier in the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope-deflection equations are not that lengthy in comparison.

The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

Slope-Deflection Equations

Consider a typical span of a continuous beam *AB* as shown in Fig.14.1.The beam has constant flexural rigidity *EI* and is subjected to uniformly distributed loading and concentrated loads as shown in the figure. The beam is kinematically indeterminate to second degree. In this lesson, the slope-deflection equations are derived for the simplest case i.e. for the case of continuous beams with unyielding supports. In the next lesson, the support settlements are included in the slope-deflection equations.

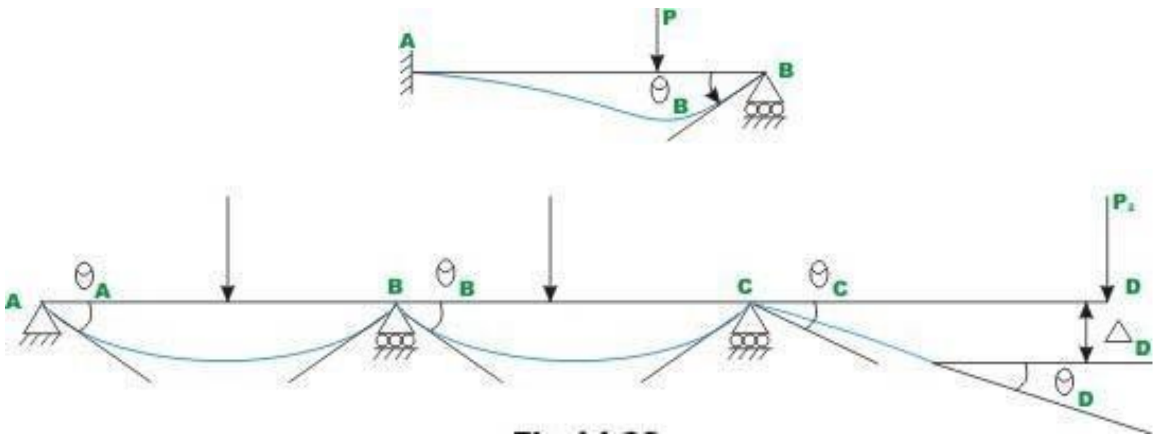


Fig 2. Slope deflection method

For this problem, it is required to derive relation between the joint end moments M_{AB} and M_{BA} in terms of joint rotations θ_A and θ_B and loads acting on the beam .Two subscripts are used to denote end moments. For example, end moments denote moment acting at joint A of the member AB. Rotations of the tangent to the elastic curve are denoted by one subscript. Thus, θ_A denotes the rotation of the tangent to the elastic curve at A. The following sign conventions are (3)
used in the slope-deflection equations (1) Moments acting at the ends of the member in counterclockwise direction are taken to be positive. (2) The rotation of the tangent to the elastic curve is taken to be positive when the tangent to the elastic curve has rotated in the counterclockwise direction from its original direction. The slope-deflection equations are derived by superimposing the end moments developed due to (1) applied loads (2) rotation θ_A

rotation θ_B . This is shown in Fig.14.2 (a)-(c). In Fig. 14.2(b) a kinematically determinate structure is obtained. This condition is obtained by modifying the support conditions to fixed so that the unknown joint rotations become zero. The structure shown in Fig.14.2 (b) is known as kinematically determinate structure or restrained structure. For this case, the end moments are denoted by M^F_{AB} and M^F_{BA}. The fixed end moments are evaluated by force–method of analysis as discussed in the previous module. For example for fixed- fixed beam subjected to uniformly distributed load, the fixed-end moments are shown in Fig.14.3.

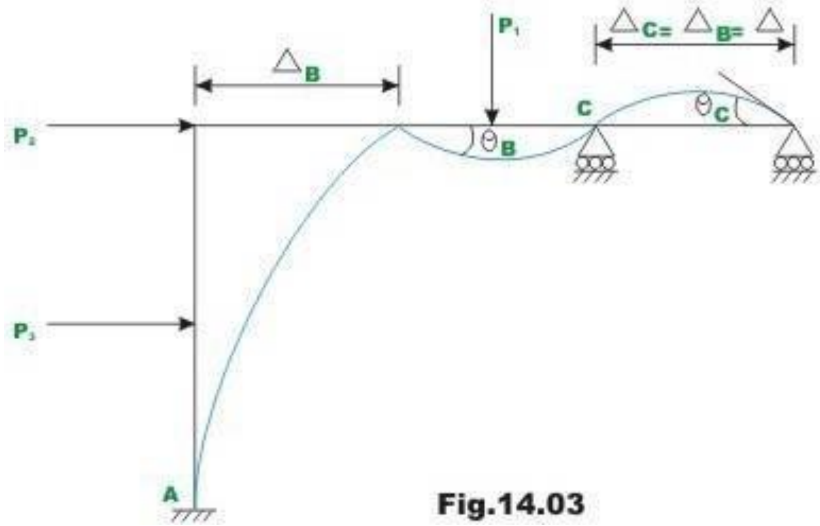


Fig.14.03

The fixed end moments are required for various load cases. For ease of calculations, fixed end forces for various load cases are given at the end of this lesson. In the actual structure end A rotates by θ_A and end B rotates by θ_B . Now
it is required to derive a relation relating θ_A and θ_B with the end moments Ma and

$$\theta'_A = \frac{M'_{AB}L}{3EI} \quad (14.1a)$$

$$\theta'_B = -\frac{M'_{AB}L}{6EI} \quad (14.1b)$$

Now a similar relation may be derived if only M'_{BA} is acting at end B (see Fig. 14.4).

$$\theta''_B = \frac{M'_{BA}L}{3EI} \text{ and} \quad (14.2a)$$

$$\theta''_A = -\frac{M'_{BA}L}{6EI} \quad (14.2b)$$

Now combining these two relations, we could relate end moments acting at A and B to rotations produced at A and B as (see Fig. 14.2c)

$$\theta_A = \frac{M'_{AB}L}{3EI} - \frac{M'_{BA}L}{6EI} \quad (14.3a)$$

Application of Slope-Deflection Equations to Statically Indeterminate Beams.

The procedure is the same whether it is applied to beams or frames. It may be summarized as follows:

1. Identify all kinematic degrees of freedom for the given problem. This can be done by drawing the deflection shape of the structure. All degrees of freedom are treated as unknowns in slope-deflection method.
2. Determine the fixed end moments at each end of the span to applied load. The table given at the end of this lesson may be used for this purpose.
3. Express all internal end moments in terms of fixed end moments and near end, and far end joint rotations by slope-deflection equations.
4. Write down one equilibrium equation for each unknown joint rotation. For example, at a support in a continuous beam, the sum of all moments corresponding to an unknown joint rotation at that support must be zero. Write down as many equilibrium equations as there are unknown joint rotations.
5. Solve the above set of equilibrium equations for joint rotations.
6. Now substituting these joint rotations in the slope-deflection equations evaluate the end moments.
7. Determine all rotations.

Example 1

A continuous beam *ABC* is carrying uniformly distributed load of 2 kN/m in

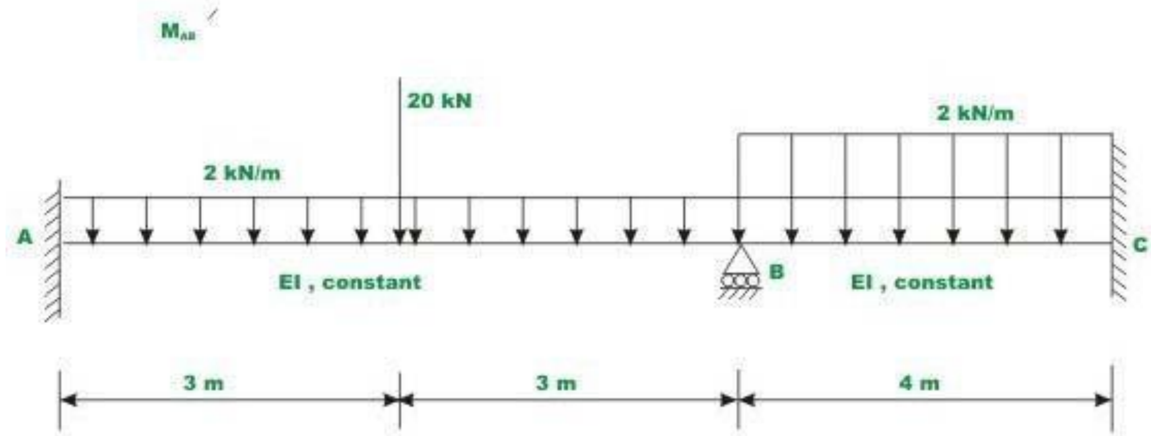


Fig. 14.5(a) Example 14.1

addition to a concentrated load of 20 kN as shown in Fig.14.5a. Draw bending moment and shear force diagrams. Assume *EI* to be constant.

(a). Degrees of freedom

It is observed that the continuous beam is kinematically indeterminate to first degree as only one joint rotation □ *B* is unknown. The deflected shape /elastic

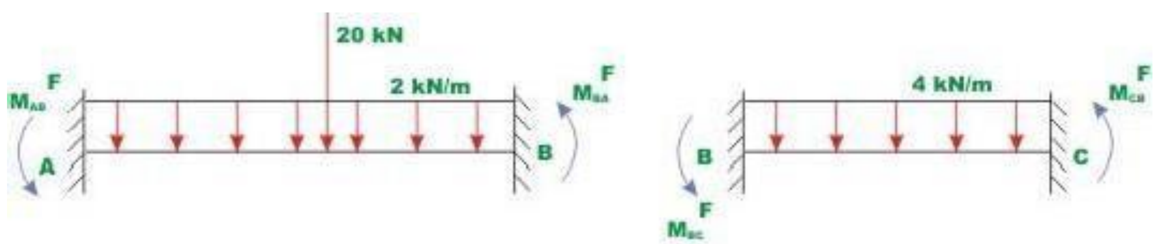
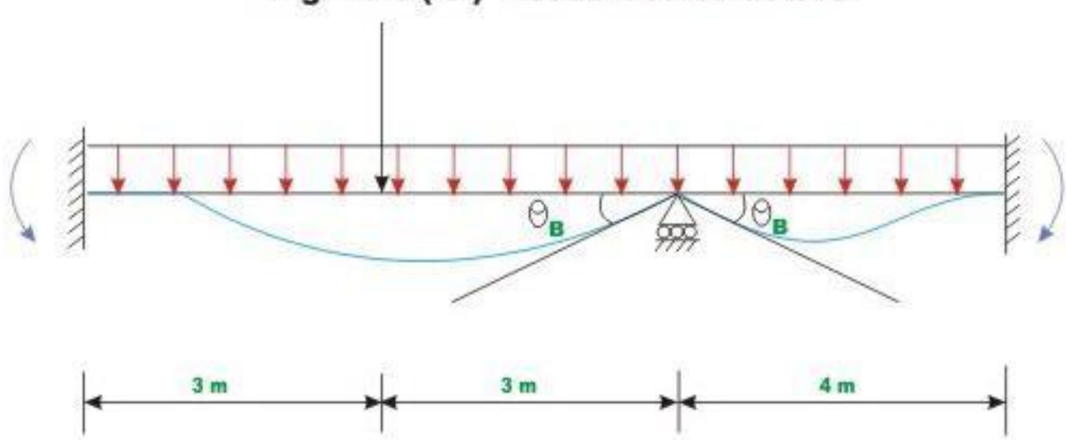


Fig. 14.5 (c) Restrained Structure.



curve of the beam is drawn in Fig.14.5b in order to identify degrees of freedom.

By fixing the support or restraining the support beams *B* against rotation, the fixed-fixed area obtained as shown in Fig.14.5c.

(c) Slope-deflection equations

Since ends *A* and *C* are fixed, the rotation at the fixed supports is zero,

Only one non-zero rotation is to be evaluated for this problem. Now, write slope-deflection equations for span *AB* and *BC*.

(b). Fixed end moments $M^F_{AB}, M^F_{BA}, M^F_{BC}$ and M^F_{CB} are calculated referring to the Fig. 14. and following the sign conventions that counterclockwise moments are positive.

$$M^F_{AB} = \frac{2\times 6^2}{12} + \frac{20\times 3\times 3^2}{6^2} = 21 \text{ kN.m}$$

$$M^F_{BA} = -21 \text{ kN.m}$$

$$M^F_{BC} = \frac{4\times 4^2}{12} = 5.33 \text{ kN.m}$$

$$M^F_{CB} = -5.33 \text{ kN.m} \qquad (1)$$

(c) Slope-deflection equations

Since ends *A* and *C* are fixed, the rotation at the fixed supports is zero, $\theta_A = \theta_C = 0$. Only one non-zero rotation is to be evaluated for this problem. Now, write slope-deflection equations for span *AB* and *BC*.

$$M_{AB} = M^F_{AB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$M_{AB} = 21 + \frac{2EI}{6}\theta_B \qquad (2)$$

$$M_{BA} = -21 + \frac{2EI}{l}(2\theta_B + \theta_A)$$

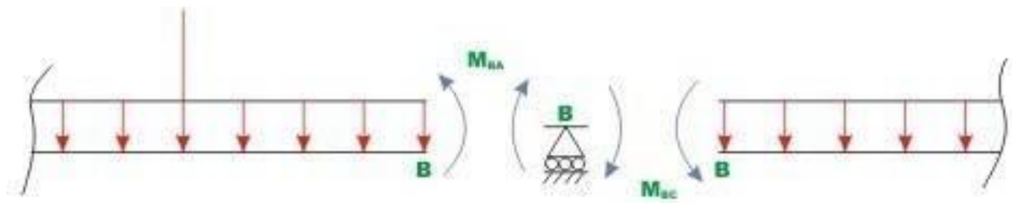
$$M_{BA} = -21 + \frac{4EI}{6}\theta_B \qquad (3)$$

$$M_{BC} = 5.33 + EI\theta_B \qquad (4)$$

$$M_{CB} = -5.33 + 0.5EI\theta_B \qquad (5)$$

(d) Equilibrium equations

In the above four equations (2-5), the member end moments are expressed in terms of unknown rotation □B . Now, the required equation to solve for the rotation □B is the moment equilibrium equation at support B . The free body diagram of support B along with the support moments acting on it is shown in Fig. 14.5d. For, moment equilibrium at



support B , one must have, $\sum M_B = 0$ $\quad M_{BA} + M_{BC} = 0$ (6)

Substituting the values of M_{BA} and M_{BC} in the above equilibrium equation,

$$-21 + \frac{4EI}{6}\theta_B + 5.33 + EI\theta_B = 0$$

$$\Rightarrow 1.667\theta_B EI = 15.667$$

$$\theta_B = \frac{9.398}{EI} \cong \frac{9.40}{EI} \qquad (7)$$

(e) Reactions

Now, reactions at supports are evaluated using equilibrium equations (vide Fig. 14.5e)

(e) End moments

After evaluating θ_B, substitute it in equations (2-5) to evaluate beam end moments. Thus,

$$M_{AB} = 21 + \frac{EI}{3}\theta_B$$

$$M_{AB} = 21 + \frac{EI}{3} \times \frac{9.398}{EI} = 24.1331$$

$$M_{BA} = -21 + \frac{EI}{3}(2\theta_B)$$

$$M_{BA} = -21 + \frac{EI}{3} \times \frac{2 \times 9.4}{EI} = -14.7$$

$$M_{BC} = 5.333 + \frac{9.4}{EI}EI = 14.733kN$$

$$M_{CB} = -5.333 + \frac{9.4}{EI} \times \frac{EI}{2} = -0.63$$

(9)

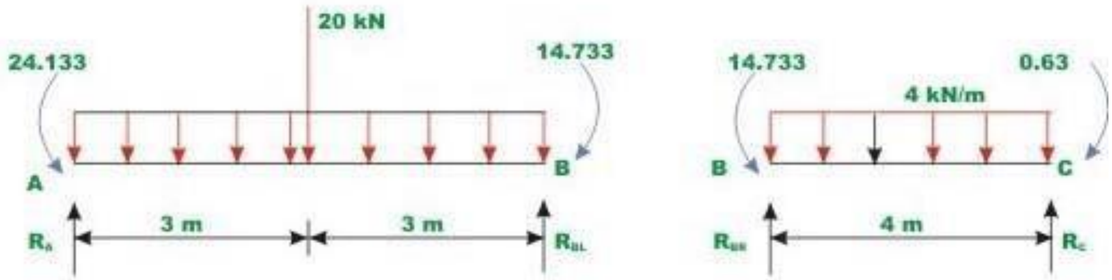
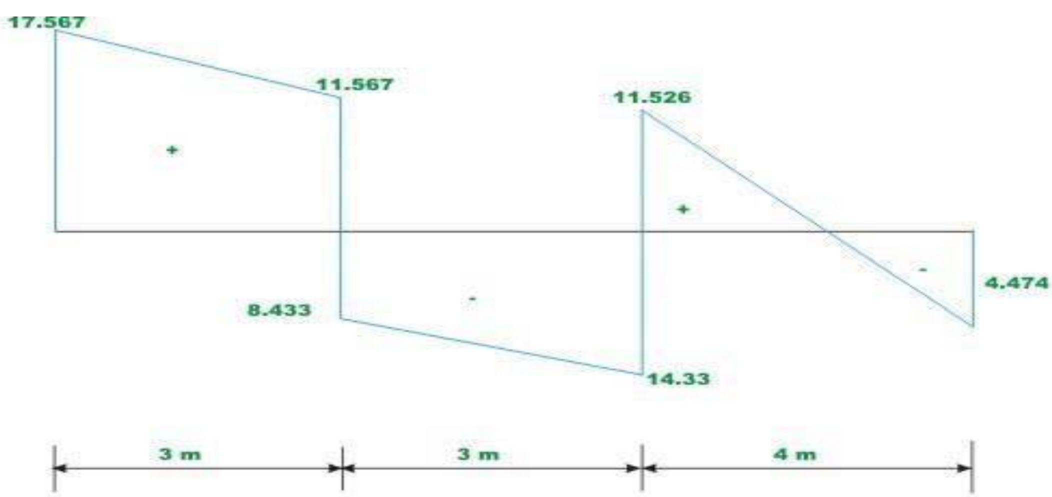
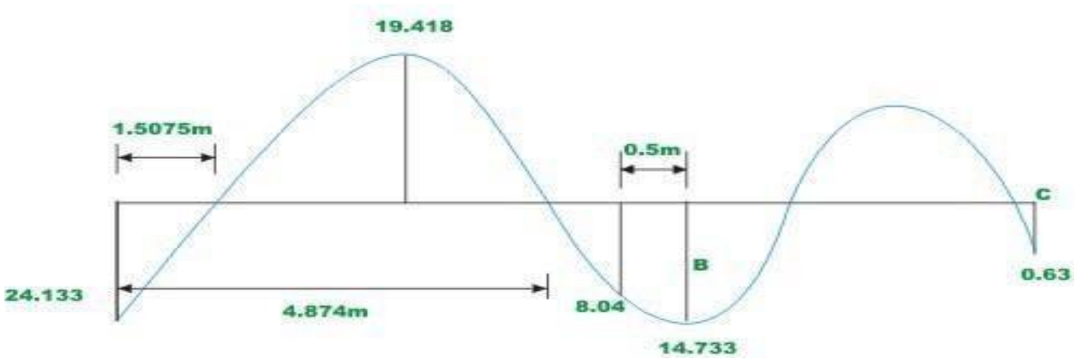


Fig. 14.5 (e) Free - body diagram of two members

The shear force and bending moment diagrams are shown in Fig. 14.5f.



Shear force diagram



Bending Moment diagram

$$R_A \times 6 + 14.733 - 20 \times 3 - 2 \times 6 \times 3 - 24.133 = 0$$

$$R_A = 17.567 \text{ kN}(\uparrow)$$

$$R_{BL} = 16 - 1.567 = 14.433 \text{ kN}(\uparrow)$$

$$R_{BR} = 8 + \frac{14.733 - 0.63}{4} = 11.526 \text{ kN}(\uparrow)$$

$$R_C = 8 + 3.526 = 4.47 \text{ kN}(\uparrow) \qquad (9)$$

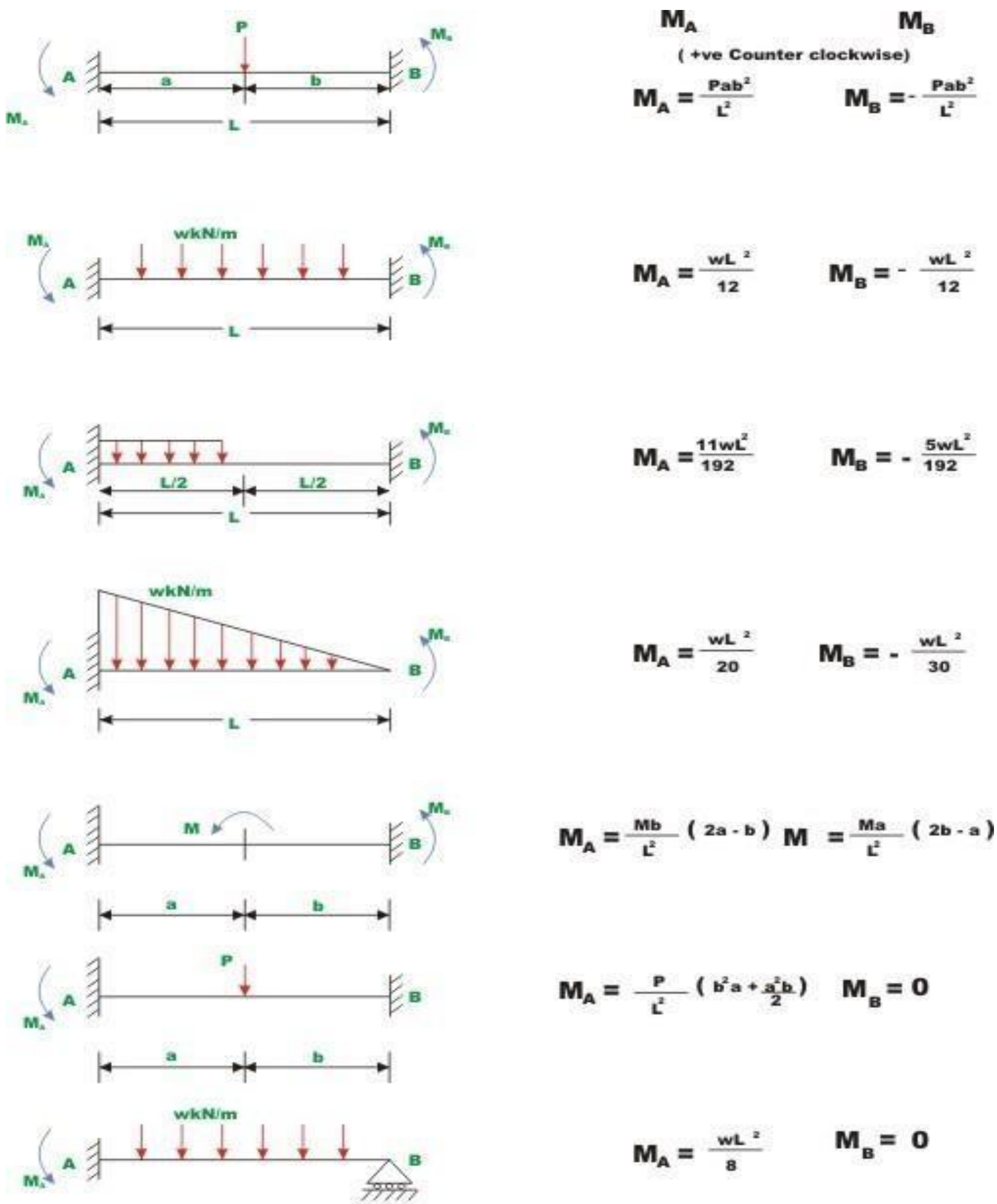
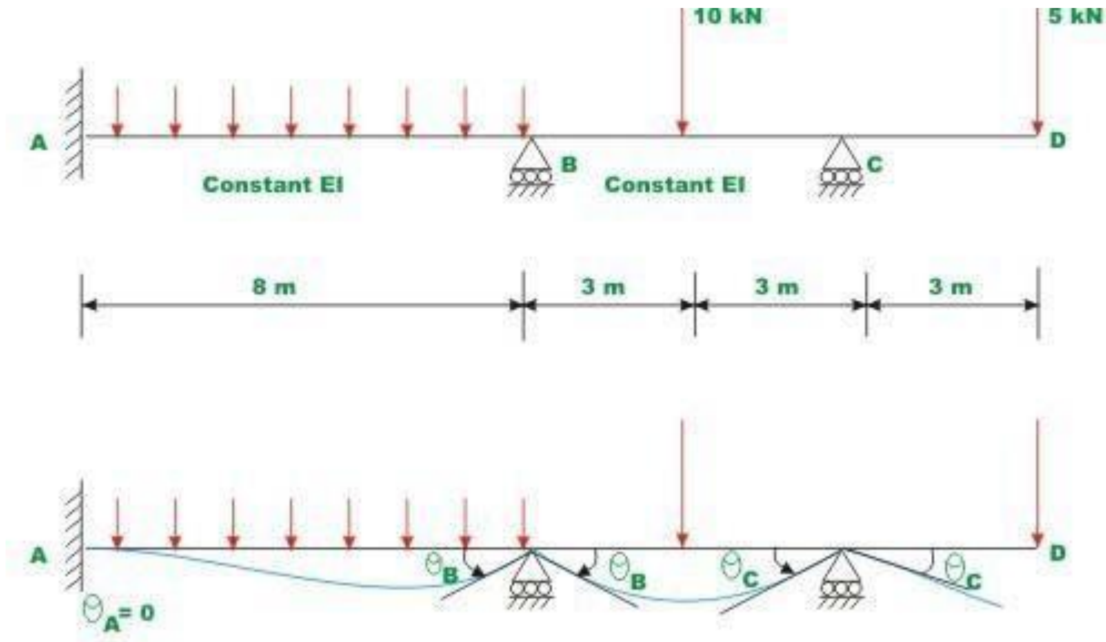


Fig. 14.7 Table of fixed end moments

Example 14.2

Draw shear force and bending moment diagram for the continuous beam

ABCD loaded as shown in Fig.14.6a.The relative stiffness of each span of the beam is also shown in the figure.



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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – III – STRUCTURAL ANALYSIS 1 – SCIA1501

UNIT 3 - Moment Distribution Method

1.0 Introduction

- Moment Distribution is an iterative method of solving an indeterminate Structure.
- Moment distribution method was first introduced by Hardy Cross in 1932.
- Moment distribution is suitable for analysis of all types of indeterminate beams and rigid frames.
- It is also called a ‘relaxation method’ and it consists of successive approximations using a series of cycles, each converging towards final result.
- It is comparatively easier than slope deflection method. It involves solving number of simultaneous equations with several unknowns, but in this method does not involve any simultaneous equations.
- It is very easily remembered and extremely useful for checking computer output of highly indeterminate structures.
- It is widely used in the analysis of all types of indeterminate beams and rigid frames.
- The moment-distribution method was very popular among engineers.
- It is very simple and is being used even today for preliminary analysis of small structures.
- The primary concept used in this methods are,
  - Fixed End Moments
  - Relative or Beam Stiffness or Stiffness factor
  - Distribution factor
  - Carry over moment or Carry over factor

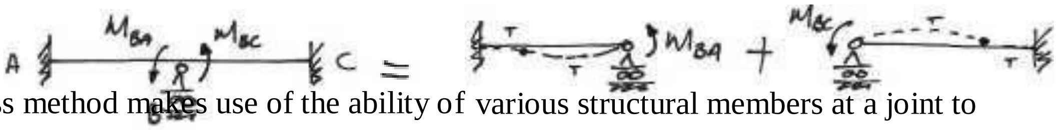
Basic Concepts

- In moment-distribution method, counterclockwise beam end moments are taken as positive.
- The counterclockwise beam end moments produce clockwise moments on the joint.
- Note the sign convention:

  Anti-clockwise is positive (+)
  Clockwise is negative (-)

2.0 Assumptions in moment distribution method

- All the members of the structures are assumed to be fixed and fixed end moments due to external loads are obtained.
- All the hinged joints are released by applying an equal and opposite moment.
- The joints are allowed to deflect (rotate) one after the other by releasing them successively.
- The unbalanced moment at the joint is shared by the members connected at the joint when it isreleased.
- The unbalanced moment at a joint is distributed in to the two spans with their distribution factor.

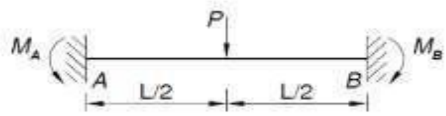
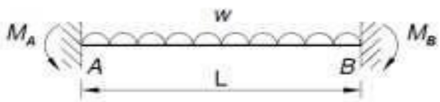
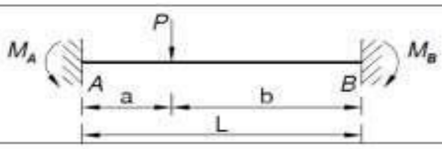
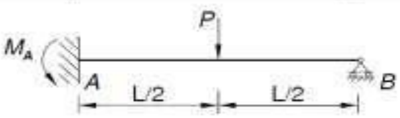
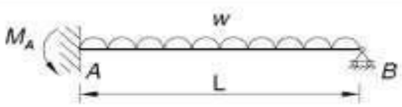
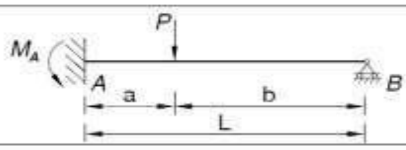


- Hardy cross method makes use of the ability of various structural members at a joint to sustain moments in proportional to their relative stiffness.

Fixed End Moments

- All members of a given frame are initially assumed fixed at both ends.
- The loads acting on these fixed beams produce fixed end moments at the ends.
- FEM are the moments exerted by the supports on the beam ends.
- These (non-existent) moments keep the rotations at the ends of each member zero.

Table .1: Fixed End moment

M_A	Configuration	M_B
$+\frac{PL}{8}$		$-\frac{PL}{8}$
$+\frac{wL^2}{12}$		$-\frac{wL^2}{12}$
$+\frac{Pab^2}{L^2}$		$-\frac{Pa^2b}{L^2}$
$+\frac{3PL}{16}$		-
$+\frac{wL^2}{8}$		-
$+\frac{Pab(2L-a)}{2L^2}$		-

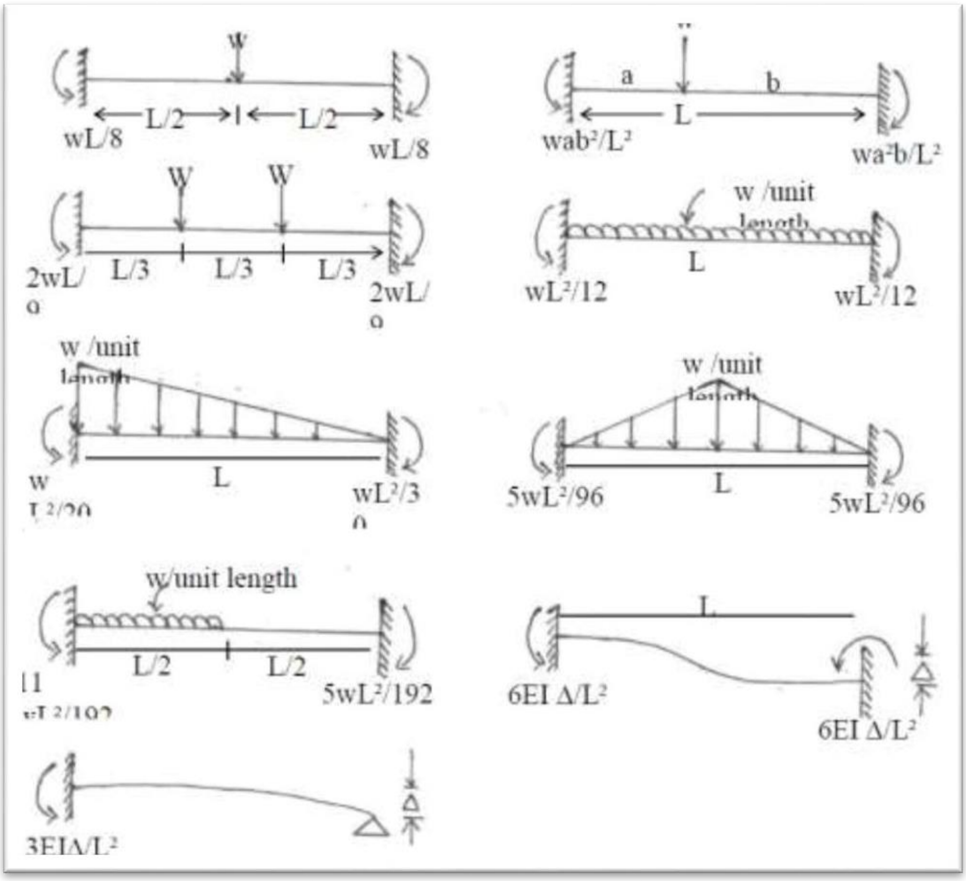


Fig 1: Fixed end moment

Relative or Beam Stiffness or Stiffness factor

- When a structural member of uniform section is subjected to a moment at one end, then the moment required so as to rotate that end to produce unit slope is called the stiffness of the member.
- Stiffness is the member of force required to produce unit deflection.
- It is also the moment required to produce unit rotation at a specified joint in a beam or a structure. It can be extended to denote the torque needed to produce unit twist.
- It is the moment required to rotate the end while acting on it through a unit rotation, without translation of the far end being
    - ✓ Beam is hinged or simply supported at both ends

      k = 3 EI / L
    - ✓ Beam is hinged or simply supported at one end and fixed at other end

      k = 4 EI / L
    - ✓ Stiffness of members in continuous beams and rigid frames
        - Stiffness of all intermediate members k = 4 EI / L
        - Stiffness of edge members,
            - If edge support is fixed k = 4 EI / L
            - If edge support is hinged or roller k = 3 EI / L

- Where, E = Young"s modulus of the beam material

  I = Moment of inertia of the beam

  L = Beam"s span length

Distribution factor

- When several members meet at a joint and a moment is applied at the joint to produce rotation without translation of the members, the moment is distributed among all the members meeting at that joint proportionate to their stiffness.
- Distribution factor = Relative stiffness / Sum of relative stiffness at the joint
- If there is 3 members,

  Distribution factors = $k_1 / (k_1+k_2+k_3)$, $k_2/ (k_1+k_2+k_3)$, $k_3/ (k_1+k_2+k_3)$

Carry over moment

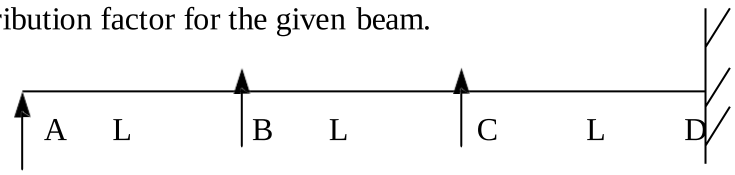
- Carry over moment: It is defined as the moment induced at the fixed end of the beam by the action of a moment applied at the other end, which is hinged.
- Carry over moment is the same nature of the applied moment.

Carry over factor (C.O):

- A moment applied at the hinged end B "carries over" to the fixed end „ A"', a moment equal to half the amount of applied moment and of the same rotational sense. C.O =0.5

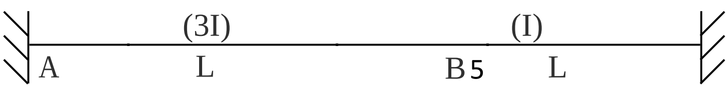
Problem:

1. Find the distribution factor for the given beam.



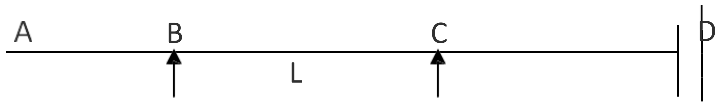
Joint	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
A	AB	4EI / L	4EI / L	(4EI / L) / (4EI / L) = 1
B	BA BC	3EI /L 4EI / L	3EI /L + 4EI / L = 7EI / L	(3EI / L) / (7EI / L)= 3/7 (4EI / L) / (7EI / L) = 4/7
C	CB CD	4EI / L 4EI / L	4EI / L + 4EI / L =8EI / L	(4EI / L) / (8EI / L) =4/8 (4EI / L) / (8EI / L)= 4/8
D	DC	4EI / L	4EI / L	(4EI / L)/ (4EI / L) = 1

2. Find the distribution factor for the given beam.



Joint	Member	Relative stiffness	Sum of Relative stiffness	Distribution factor
A	AB	4E (3I) / L	12EI / L	(12EI / L) / (12EI / L) = 1
B	BA BC	4E(3I) /L 4EI / L	12EI /L + 4EI / L = 16EI / L	(12EI / L) / (16EI / L)= 3/4 (4EI / L) / (16EI / L) = 1/4
C	CB	4EI / L	4EI / L	(4EI / L) / (4EI / L) =1

3. Find the distribution factor for the given beam.



Joint	Member	Relative stiffness	Sum of Relative	Distribution factor
B	BA BC	0 (no support) 3EI / L	3EI / L	0 (3EI / L) / (3EI / L) =1
C	CB CD	3EI / L 4EI / L	3EI /L + 4EI / L = 7EI / L	(3EI / L) / (7EI / L)= 3 / 7 (4EI / L) / (7EI / L) = 4 / 7
D	DC	4EI / L	4EI / L	(4EI / L) / (4EI / L) =1

Flexural Rigidity of Beams:

- The product of young‟s modulus (E) and moment of inertia (I) is called Flexural Rigidity (EI) of Beams. The unit is N.mm^2.

Constant strength beam:

- If the flexural Rigidity (EI) is constant over the uniform section, it is called Constant strength beam.

Sway:

- Sway is the lateral movement of joints in a portal frame due to the unsymmetrical dimensions, loads, moments of inertia, end conditions, etc.

What are the situations where in sway will occur in portal frames?

- Eccentric or unsymmetrical loading
- Unsymmetrical geometry
- Different end conditions of the columns
- Non-uniform section of the members
- Unsymmetrical settlement of supports
- A combination of the above

What are symmetric and antisymmetric quantities in structural behaviour?

- When a symmetrical structure is loaded with symmetrical loading, the bending moment and deflected shape will be symmetrical about the same axis.
- Bending moment and deflection are symmetrical quantities

Steps involved in Moment Distribution Method:

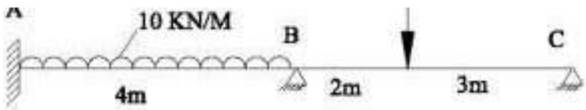
1. Calculate fixed end moments due to applied loads following the same sign convention and procedure, which was adopted in the slope-deflection method.
2. Calculate relative stiffness.
3. Determine the distribution factors for various members framing into a particular joint.
4. Distribute the net fixed end moments at the joints to various members by multiplying the net moment by their respective distribution factors in the first cycle.
5. In the second and subsequent cycles, carry-over moments from the far ends of the same member (carry-over moment will be half of the distributed moment).
6. Consider this carry-over moment as a fixed end moment and determine the balancing moment. This procedure is repeated from second cycle onwards till convergence

Advantages of Fixed Ends or Fixed Supports

1. Slope at the ends is zero.
2. Fixed beams are stiffer, stronger and more stable than SSB.
3. In case of fixed beams, fixed end moments will reduce the BM in each section.
4. The maximum defection is reduced.

Problem:

1. Analyse the frame given in figure by moment distribution method and draw the B.M.D & S.F.D



Step: 1 - Fixed end moment

M^F_{AB} =	$-WL^2/12$	=	$-10\times4^2/12$	=	-13.33 KNM
M^F_{BA} =	$WL^2/12$	=	$10\times4^2/12$	=	-13.33 KNM
M^F_{BC} =	$-Wab^2/L^2$	=	$-50\times2\times3^2/5^2$	=	-36 KNM
M^F_{CB} =	Wa^2b/L^2	=	$50\times2^2\times3/5^2$	=	24 KNM

Step: 2 - Stiffness

K_{AB} = K_{BA} = 4EI/L = EI

K_{BC} = K_{CB} = 3EI/L = 0.6EI

Step: 3 - Distribution factor

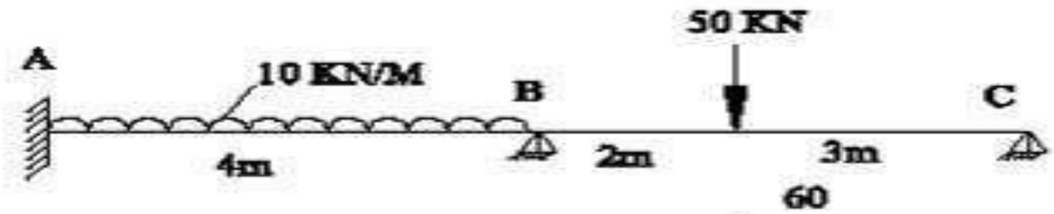
Joint B

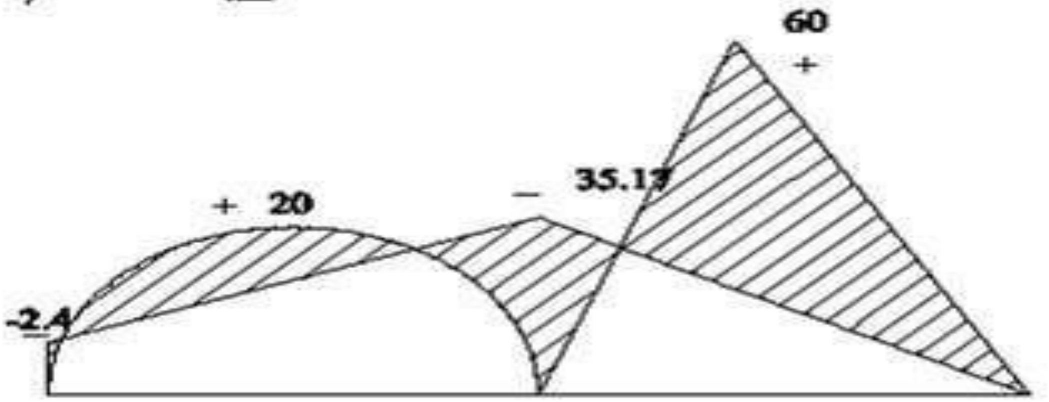
$D^F_{BA} = K_{BA}/(K_{BA}+K_{BC}) = 0.63$

$D^F_{BC} = K_{BC}/(K_{BA}+K_{BC}) = 0.37$

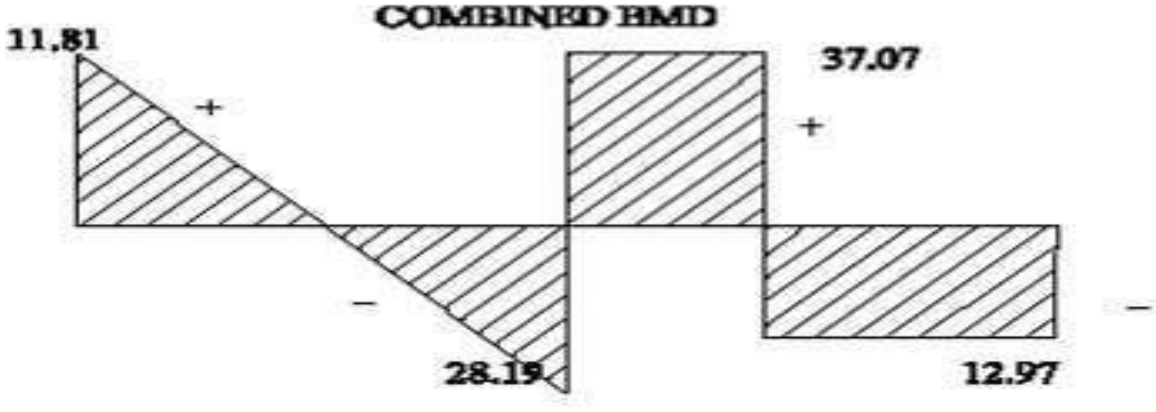
Step: 4 - Moment distribution

MEMBER	AB	B		CB
		BA	BC	
DF	0	0.67	0.33	0
FEM	-13.33	+13.33	-36	+24
BALANCING	0	0	0	-24
CF	0	0	-12	0
M	-13.33	+13.33	-48	0
BALANCING	0	21.84	12.83	0
CF	10.92	0	0	0
M-FINAL	-2.4	35.17	-35.17	0





COMBINED BMD



SFD

Step: 5 - Reactions

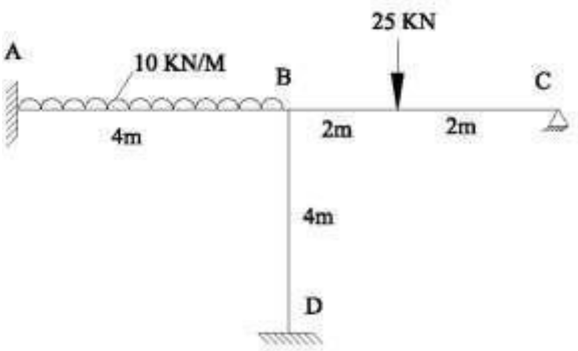
Span AB:

R_A	=	11.81 KN
R_{B1}	=	28.19 KN

Span BC:

R_{B2}	=	37.03 KN
R_C	=	12.97 KN

2. Analyse the frame given in figure by moment distribution method and draw the B.M.D&S.F.D



Step: 1 - Fixed end moment

M^F_{AB}	=	$-WL^2/12$	=	$-10\times4^2/12$	=	-13.33 KNM
M^F_{BA}	=	$WL^2/12$	=	$10\times4^2/12$	=	-13.33 KNM
M^F_{BC}	=	$-WL/8$	=	$-25\times4/8$	=	-12.5 KNM
M^F_{CB}	=	$WL/8$	=	$25\times4/8$	=	12.5 KNM
M^F_{BD}	=	0				
M^F_{DB}	=	0				

Step: 2 - Stiffness

K_{AB}	=	K_{BA}	=	4EI/L	=	EI
K_{BC}	=	K_{CB}	=	3EI/L	=	0.75EI
K_{BD}	=	K_{DB}	=	4EI/L	=	EI

Step: 3 - Distribution factor Joint B

D F BA = KBA/ (KBA+KBC+KBD) = 0.36

D F BC = KBC/ (KBA+KBC+KBD) = 0.28

D F BD = KBD/ (KBA+KBC+KBD) = 0.36

Step: 4 - Moment distribution

MEMBER	AB	B			DB	CB
		BA	BC	BD		
DF	0	0.36	0.28	0.36	0	
FEM	-13.33	+13.33	-12.5	0	0	+12.5
CF	0	0	-6.25	0	0	-12.5
M(initial)	-13.33	+13.33	-18.75	0	0	0
BALANCING	0	+1.95	1.52	1.95	0	0
MF	0.98	0	0	0	0.98	0
M-FINAL	-12.35	15.28	-17.23	1.95	0.98	0

Step: 5 - Find reactions:

Span AB:

R_A = 19.27 KN

R_{B1} = 20.73 KN

Span BC:

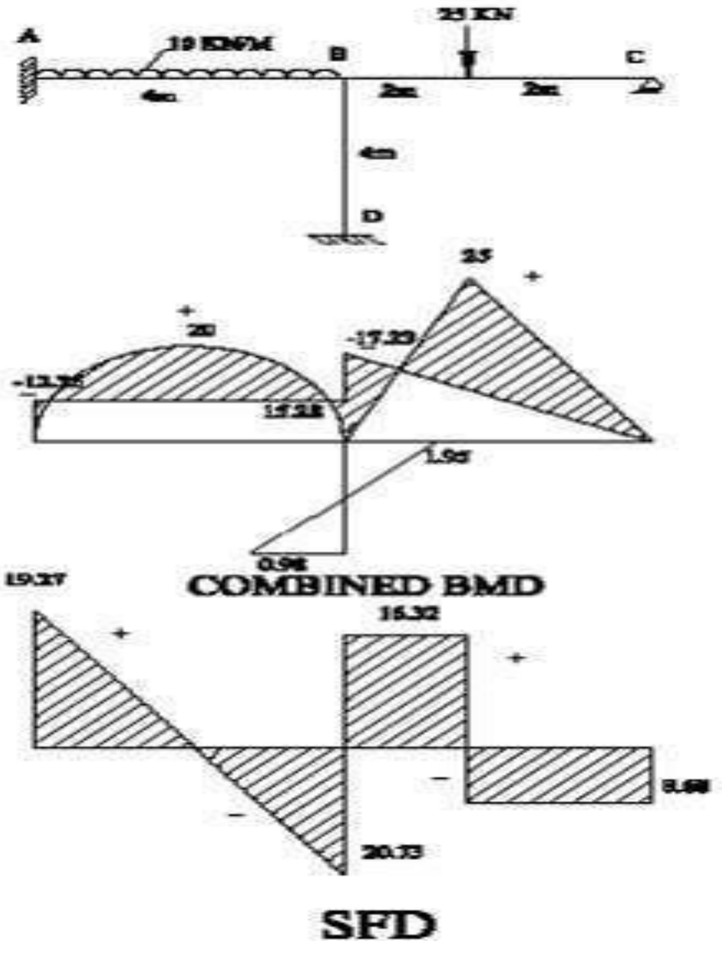
R_{B2} = 16.32 KN

R_C = 8.68 KN

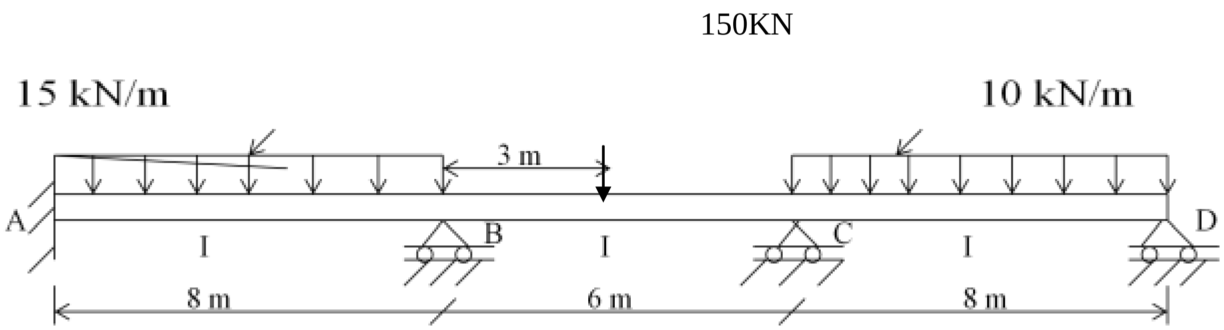
Span BD:

R_{B3} = -0.73 KN

R_D = 0.73 KN



3. The continuous beam ABCD, subjected to the given loads, as shown in Figure below. Assume that only rotation of joints occurs at B, C and D, and that no support displacements occur at B, C and D. Due to the applied loads in spans AB, BC and CD, rotations occur at B, C and D using moment distribution method.



Step: 1 - Fixed end moments

$$M_{AB} = -M_{BA} = -\frac{wl^2}{12} = -\frac{(15)(8)^2}{12} = -80\ kN.m$$

$$M_{BC} = -M_{CB} = -\frac{wl}{8} = -\frac{(150)(6)}{8} = -112.5\ kN.m$$

$$M_{CD} = -M_{DC} = -\frac{wl^2}{12} = -\frac{(10)(8)^2}{12} = -53.333\ kN.m$$

Step: 2 - Stiffness Factors (Unmodified Stiffness)

$$K_{AB} = K_{BA} = \frac{4EI}{L} = \frac{(4)(EI)}{8} = 0.5EI$$

$$K_{BC} = K_{CB} = \frac{4EI}{L} = \frac{(4)(EI)}{6} = 0.667EI$$

$$K_{CD} = \left[\frac{4EI}{8}\right] = \frac{4}{8}EI = 0.5EI$$

$$K_{DC} = \frac{4EI}{8} = 0.5EI$$

Step: 3 - Distribution Factors

$$DF_{AB} = \frac{K_{BA}}{K_{BA} + K_{wall}} = \frac{0.5EI}{0.5 + \infty\ (wall\ stiffness)} = 0.0$$

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{0.5EI}{0.5EI + 0.667EI} = 0.4284$$

$$DF_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{0.667EI}{0.5EI + 0.667EI} = 0.5716$$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{0.667EI}{0.667EI + 0.500EI} = 0.5716$$

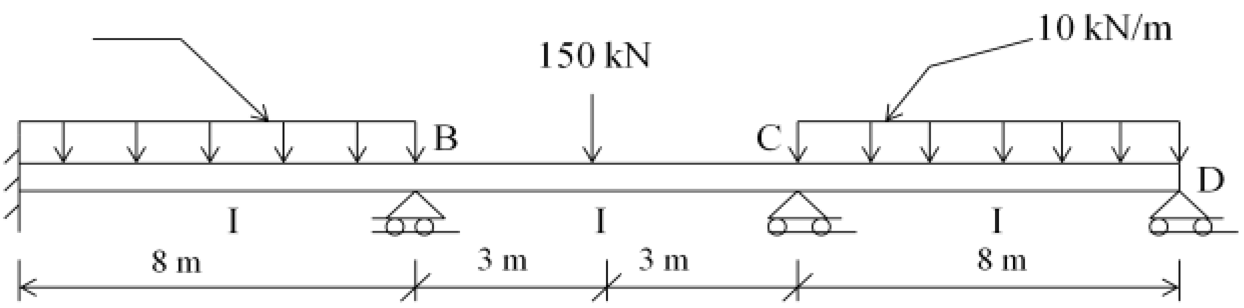
$$DF_{CD} = \frac{K_{CD}}{K_{CB} + K_{CD}} = \frac{0.500EI}{0.667EI + 0.500EI} = 0.4284$$

$$DF_{DC} = \frac{K_{DC}}{K_{DC}} = 1.00$$

Step: 4 - Moment Distribution

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.4284	0.5716	0.64	0.36	1
FEM	-80	80	-112.50	112.50	-53.33	53.33
Ist Distribution		13.923	18.577	-37.87	-21.3	-53.33
Carry over Moment	6.962		-18.93	9.289	-26.67	-10.65
2nd Distribution		8.111	10.823	11.122	6.256	10.65
Carry over Moment	4.056		5.561	5.412	5.325	3.128
3rd Distribution		-2.382	-3.179	-6.872	-3.865	-3.128
Carry over Moment	-1.191		-3.436	-1.59	-1.564	-1.933
4th Distribution		1.472	1.964	2.019	1.135	1.933
Carry over Moment	0.736		1.01	0.982	0.967	0.568
5th Distribution		-0.433	-0.577	-1.247	-0.702	-0.568
Carry over Moment						
M-FINAL	-69.44	100.69	-100.7	-93.748	93.75	0

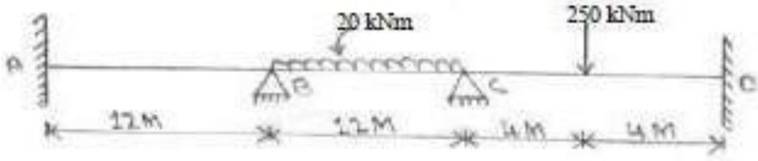
Step: 5 - Computation of Shear Forces



Simply-supported reaction	60	60	75	75	40	40
End reaction due to left hand FEM	8.726	-8.726	16.665	-16.67	12.079	-12.08
End reaction due to right hand FEM	-12.5	12.498	-16.1	16.102	0	0
Summed-up moments	56.228	63.772	75.563	74.437	53.077	27.923

5. Analyse the beam as shown in figure by moment distribution method and draw the BMD.

Assume EI is constant



Step: 1 - Fixed end moments

M^F_{AB} = 0

M^F_{BA} = 0

M^F_{BC} = $-WL^2/12$ = $-20\times12^2/12$ = -240 KNM

M^F_{CB} = $WL^2/12$ = $-20\times12^2/12$ = 240 KNM

M^F_{CD} = $-WL/8$ = $-250\times8/8$ = -250 KNM

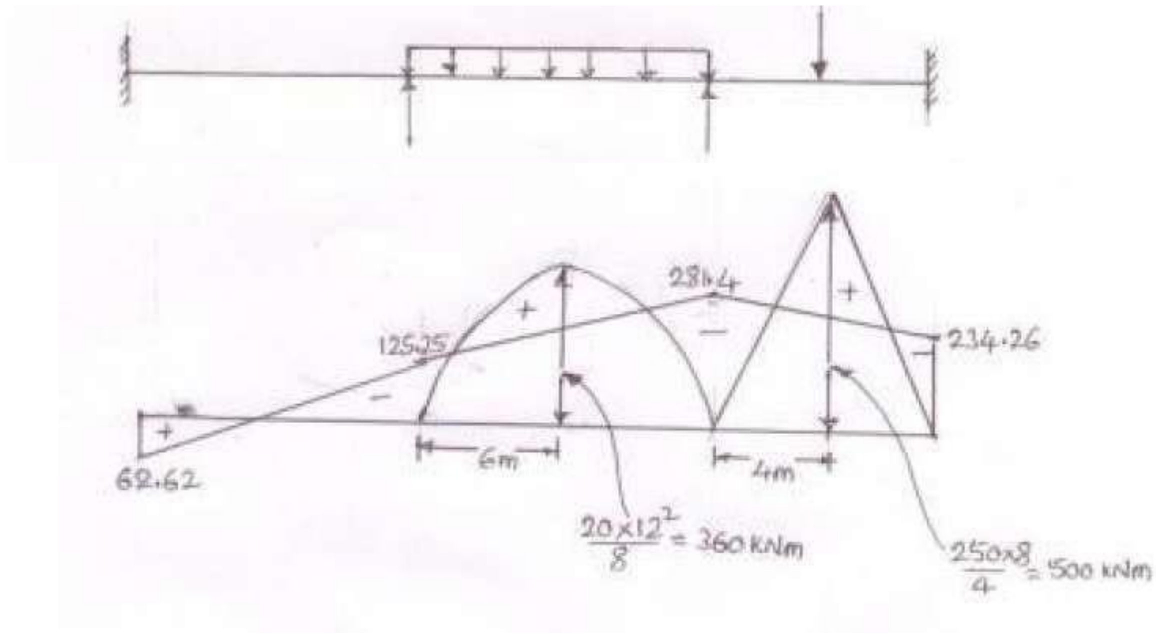
M^F_{DC} = $WL/8$ = $250\times8/8$ = 250 KNM

Step:2 - Distribution factor

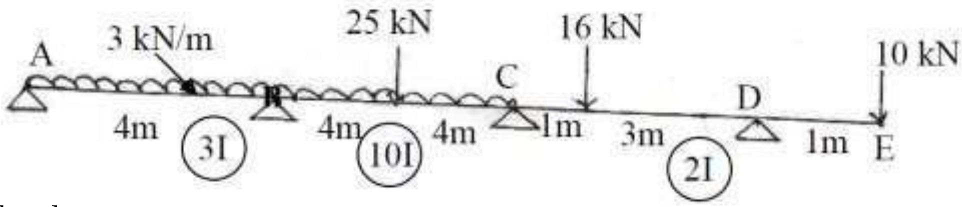
Joint	Member	Relative Stiffness (K)	ΣK	D.F = (K / ΣK)
B	BA	I / L = (I /12)	I / 6	0.50
	BC	I / L = (I /12)		0.50
C	CB	I / L = (I /12)	5I / 24	0.40
	CD	I / L = (I /8)		0.60

Step:3 – Moment Distribution

Jt	A	B		C		D
Member	AB	BA	BC	CB	CD	α
D.F	0	0.5	0.5	0.4	0.6	0
FEM	0	0	-240	+240	-250	+250
Balance		+120	+120	4	6	
C.O	60		2	60		3
Balance		-1	-1	-24	-36	
C.O	-0.5		-12	-0.5		-18
Balance		+6	+6	0.2	0.3	
C.O	3		0.1	3		0.15
Balance		-0.05	-0.05	-1.2	-1.8	
C.O	-0.03		-0.6	-0.03		-0.9
Balance		+0.3	+0.3	0.01	0.02	
C.O	0.15					0.01
Final moments	**62.62**	**125.25**	**-125.25**	**281.48**	**-281.48**	**234.26**



5. Analyze the continuous beam as shown in fig by moment distribution method and draw BMD & SFD



Step: 1 - Fixed end moments

$$\text{FEM:} \quad M_{FAB} = -\frac{3x4^2}{12} = -4\,\text{kNm}; \; M_{FBA} = 4\,\text{kNm}$$

$$M_{FBC} = -\frac{3x8^2}{12} - \frac{25\,x\,8}{8} = -41\text{ kNm} \quad M_{FAB} = +\frac{3x8^2}{12} + \frac{25\,x\,8}{8} = +41\text{kNm}$$

$$M_{FDC} = \frac{16\,x\,1^2\,x\,3}{4^2} = +3\text{ kNm} \qquad M_{DE} = -10 \text{ x } 1 = -10\text{kNm}$$

$$M_{FCD} = \frac{-16 \times 1 \times 3^2}{4^2} = -9\text{ kNm}$$

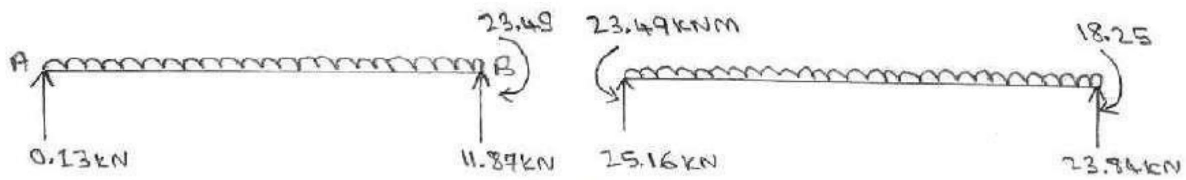
Step:2 - Distribution factor

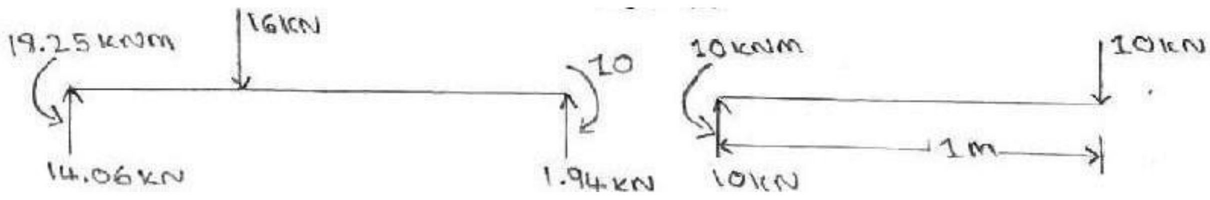
Jt.	Member	Relative stiffness (K)	ΣK	$DF = \frac{K}{\sum K}$
B	BA	$\frac{3}{4} x \frac{3I}{4} = 0.56I$	1.81I	0.31
	BC	10I/8 = 1.25I		0.69
C	CB	10I/8 = 1.25I	1.63I	0.77
	CD	$\frac{3}{4} x \frac{2I}{4} = 0.38I$		0.23

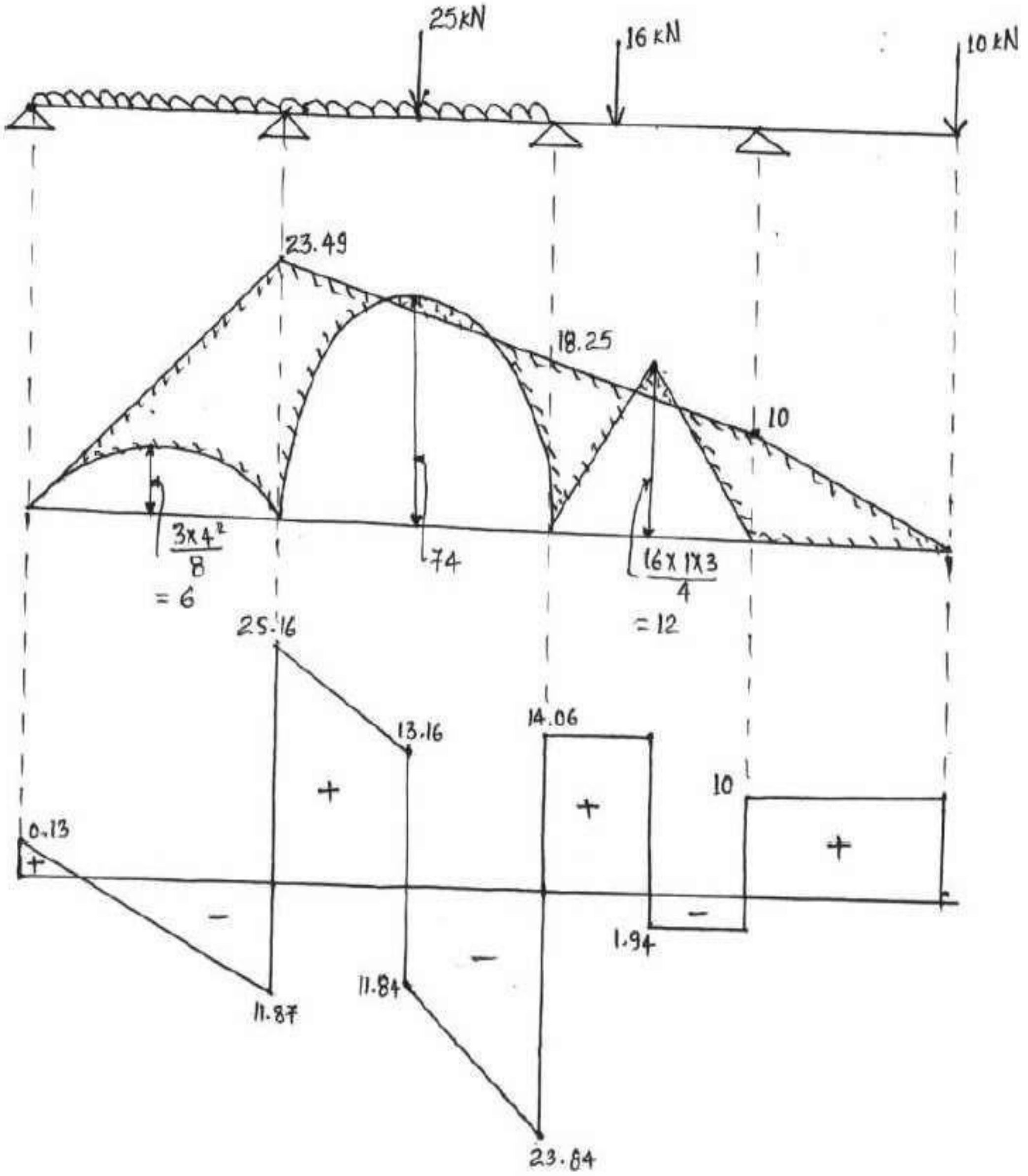
Step: 3 - Moment Distribution

Jt	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	DE
D.F	1	0.31	0.69	0.77	0.23	1	01
FEM	-4	4	-41	+41	-9	3	-10
Release of joint A and adjusting moment at 'D'	+4					+7	
		2			3.5		
Initial moments	0	6	-41	41	-5.5	+10	-10
Balance		10.9	24.1	-27.3	-8.2		
C.O			-13.7	12.1			
Balance		4.2	9.5	-9.3	-2.8		
C.O			-4.7	4.8			
Balance		1.5	3.2	-3.7	-1.1		
C.O			-1.9	1.6			
Balance		0.6	1.3	-1.2	-0.4		
C.O			-0.6	0.7			
Balance		0.2	0.4	-0.5	-0.2		
C.O			-0.3	0.2			
Balance		0.09	0.21	-0.15	-0.05		
Final moments	**0**	**23.49**	**-23.49**	**18.25**	**-18.25**	**10**	**-10**

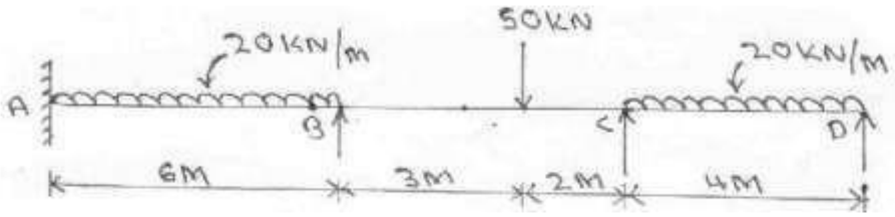
Step: 4 – BMD & SFD




19.25 kNm
16kN
10
14.06kN
1.94kN
10kNm
10kN
1m
10kN


25kN
16 kN
10 kN
23.49
18.25
10
3x4²/8 = 6
74
16x1x3/4 = 12
25.16
13.16
14.06
10
0.13
11.87
11.84
1.94
23.84

6. Analyze the continues beam as shown in figure by moment distribution method and draw the B.M. diagrams



Support B sinks by 10mm, and take E = 2 x 10^5 N/mm², I = 1.2 x 10^{-4} m^4

Step: 1 - Fixed end moments

M_{FAB} = FEM due to load + FEM due to sinking

$$= \frac{-wl^2}{12} + \left[\frac{-6EI\Delta}{l^2}\right]$$

$$= \frac{-20 x 6^2}{12} - \frac{6 x 2 x 10^5 x 1.2 x 10^{-4} x 10^{12} x 10}{(6000)^2 x 10^6}$$

$= -60 - 40$

M_{FAB} = -100 kNm

M_{FBA} = FEM due to load + FEM due to sinking

$= +60 - 40$

M_{FBA} = +20 kNm

M_{FBC} = FEM due to loading + FEM due to sinking

$$= \frac{-Wab^2}{l^2} + \frac{6EI\Delta}{l^2}$$

$$= \frac{-50 x 3 x 2^2}{5^2} + \frac{6 x 2 x 10^5 1.2 x 10^{-4} x 10^{12} x 10}{(5000)^2 x 10^6}$$

$= -24 + 57.6$

M_{FBA} = + 33.6 kNm

$$M_{FCB} = + \frac{Wa^2b}{l^2} + \frac{6EI\Delta}{l^2}$$

$$= \frac{50 x 3^2 x 2}{5^2} + 57.6$$

M_{FCB} = 93.6kNm

M_{FCD} = due to load only (∵ C & D are at some level)

$$M_{FCD} = \frac{-wl^2}{12} = \frac{-20 x 4^2}{12} = -26.67 kNm$$

M_{FDC} = + 26.67 kNm

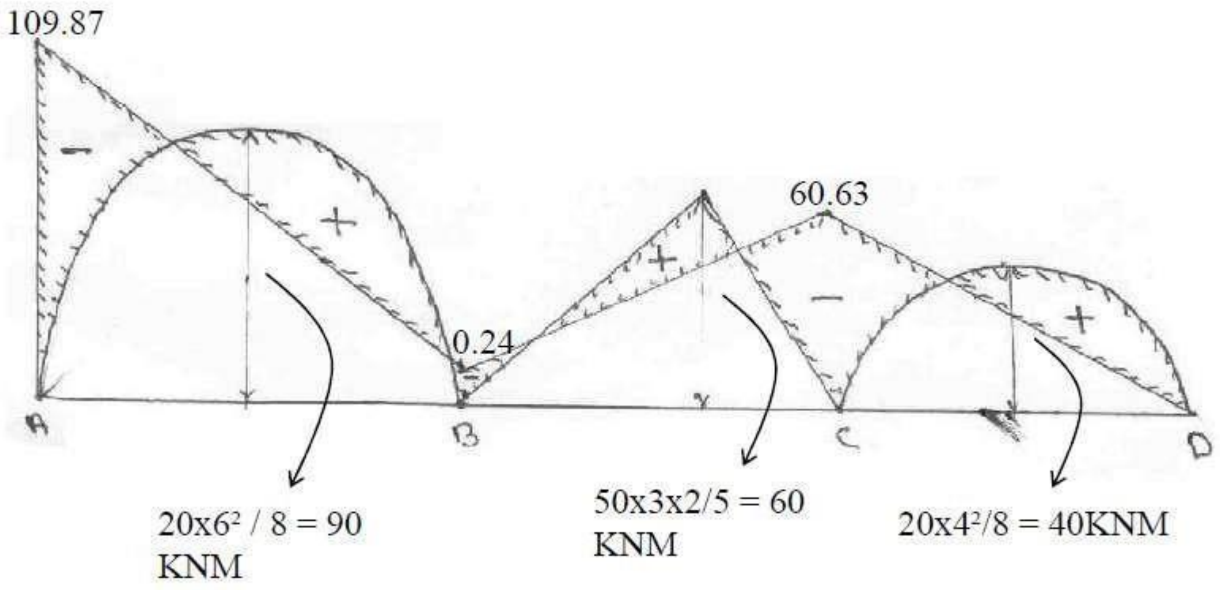
Step:2 - Distribution factor

Jt.	Member	Relative stiffness (K)	ΣK	DF = $\frac{K}{\sum K}$
B	BA BC	I/6 I/5	0.36I	0.46 0.54
C	CB CD	I/5 $\frac{3}{4}x\frac{I}{4}=0.19I$	0.39I	0.51 0.49

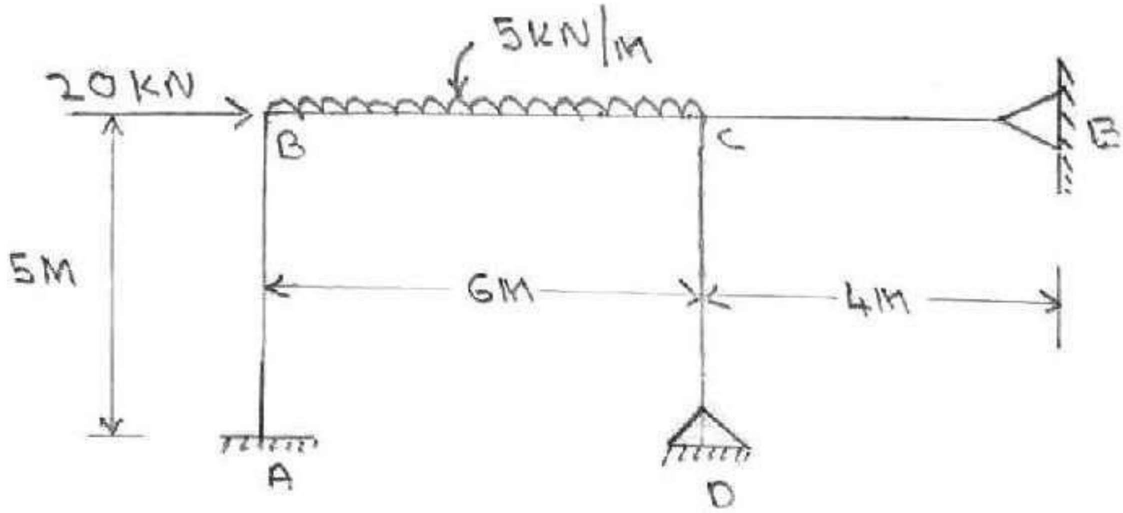
Step: 3 - Moment Distribution

Jt	A		B		C		D
Member	AB	BA	BC	CB	CD	DC	
D.F		0.46	0.54	0.51	0.49		
FEM	-100	+20	+33.6	+93.6	-26.67	+26.67	
Release jt. 'D'						-26.67	
CO					-13.34		
Initial moments	-100	+20	+33.6	+93.6	-40.01	0	
Balance		-24.66	-28.94	-27.33	-26.26		
C.O	-12.33		-13.67	--14.47			
Balance		+6.29	+7.38	+7.38	+7.09		
C.O	+3.15		+3.69	+3.69			
Balance		-1.7	-1.99	-1.88	-1.81		
C.O	-0.85		-0.94	-1			
Balance		+0.43	+0.51	+0.51	+0.49		
C.O	+0.22		+0.26	+0.26			
Balance		-0.12	-0.14	-0.13	-0.13		
C.O	-0.06						
Final moments	**-109.87**	**+0.24**	**-0.24**	**+60.63**	**-60.63**		

Step: 4 – BMD



6. Analysis the frame shown in figure by moment distribution method and draw BMD. Assume EI is constant



Step: 1 - Fixed end moments

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = M_{FCE}$ $M_{FEC} = 0$

$$M_{FBC} = -\frac{5 \times 6^2}{12} = -15\,kNm$$

$M_{FCB} = +15$ kNm

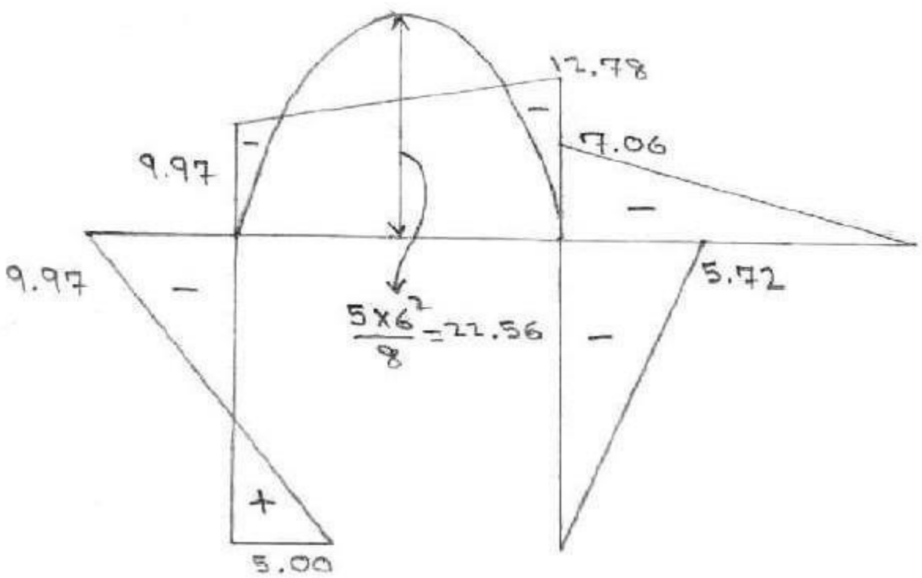
Step:2 - Distribution factor

Jt.	Member	Relative stiffness (K)	ΣK	$DF = \frac{K}{\sum K}$
B	BA	I/5	$\frac{11}{30}I$	0.55
	BC	I/6		0.45
C	CB	I/6 = 0.17 I	0.51 I	0.33
	CD	$\frac{3}{4}$I/5 = 0.15 I		0.3
	CE	$\frac{3}{4}x\frac{I}{4}$= 0.19 I		0.37

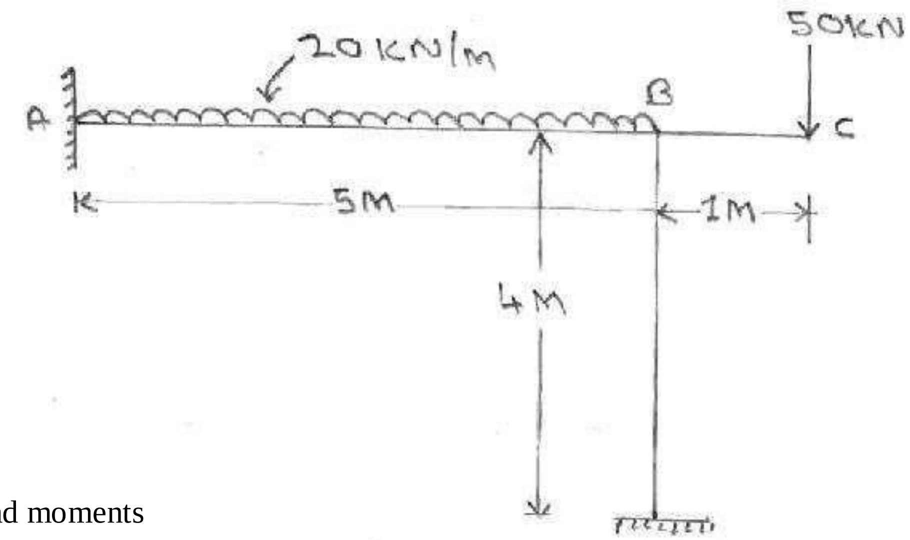
Step: 3 - Moment Distribution

Jt	A	B		C			D	E
Member	AB	BA	BC	CB	CD	CE	DC	EC
D.F	0	0.55	0.45	0.33	0.3	0.37	1	1
FEM	0	0	-15	+15	0	0	0	0
Balance		8.25	6.75	-4.95	-4.5	-5.55		
C.O	4.13		-2.48	3.38				
Balance		1.36	1.12	-1.12	-1.01	-1.25		
C.O	0.68		0.56	0.56				
Balance		0.31	0.25	-0.18	-0.17	-0.21		
C.O	0.16		-0.09	0.13				
Balance		0.05	0.04	-0.04	-0.04	-0.05		
C.O	0.03							
Final moments	**5**	**9.97**	**-9.97**	**12.78**	**-5.72**	**-7.06**	**0**	**0**

Step: 4 – BMD



8. Analyze the frame shown in figure by moment distribution method and draw BMD and SFD



Step: 1 - Fixed end moments

$$M_{FAB} = -\frac{20x5^2}{12} = -41.67 \text{ KNM}$$

$M_{FBA} = +41.67$ KNM

$M_{FBD} = M_{FDB} = 0$

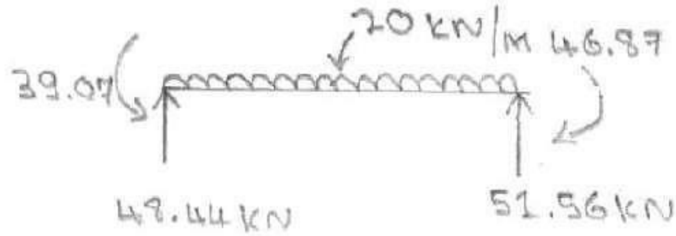
$M_{FBC} = -50 \times 1 = -50$ KNM

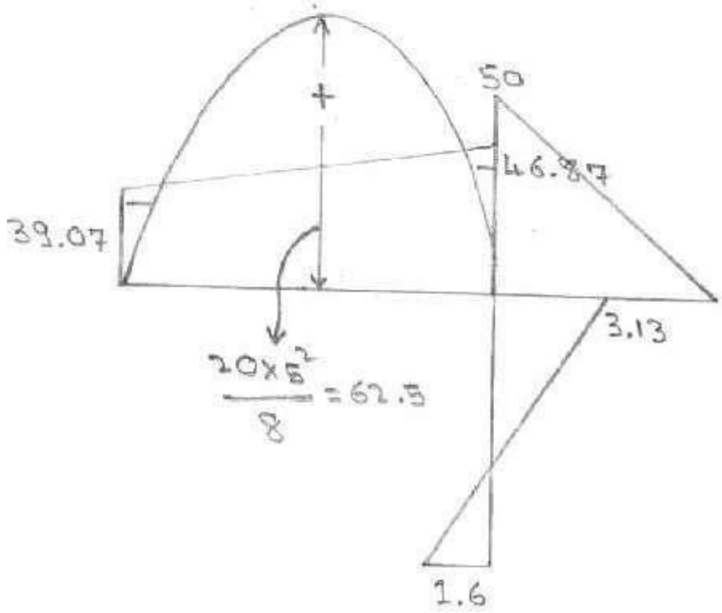
Step:2 - Distribution factor

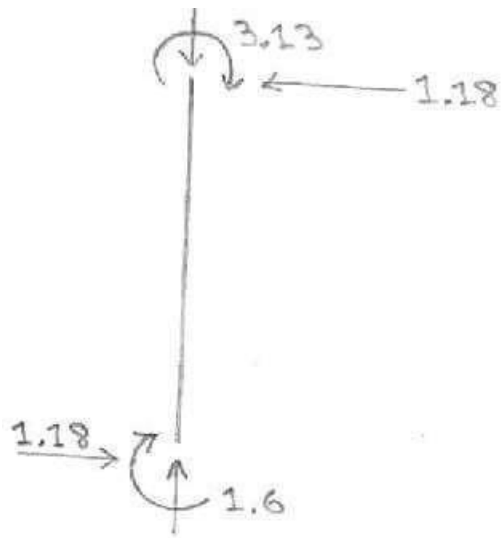
Jt.	Member	K	ΣK	$DF = \frac{K}{\sum K}$
B	BA	2I/5 = 0.4I		0.62
	BC	0	0.65I	0
	BD	I/4 = 0.25 I		0.38

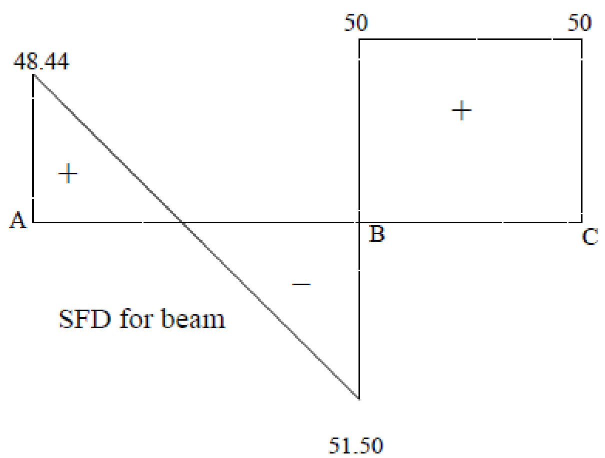
Step: 3 - Moment Distribution

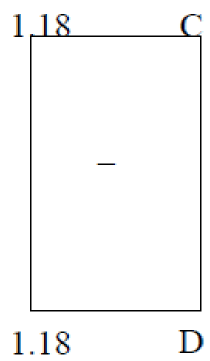
Jt	A	B			D
Member	AB	BA	BC	BD	DB
D.F	01	0.62	0	0.38	0
FEM	-41.67	41.67	-50	0	0
Balance		5.2	0	3.13	
C.O	2.6				1.6
Final moments	**-39.07**	**46.87**	**-50**	**3.13**	**1.6**







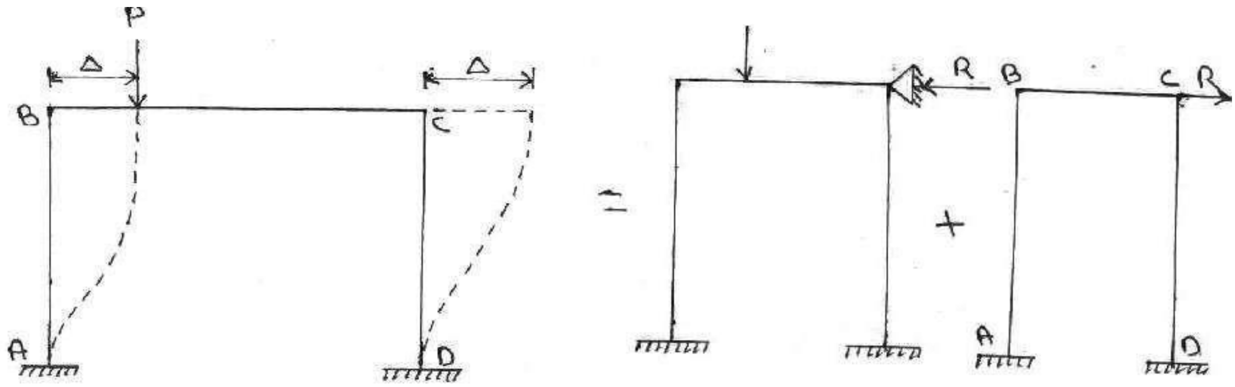




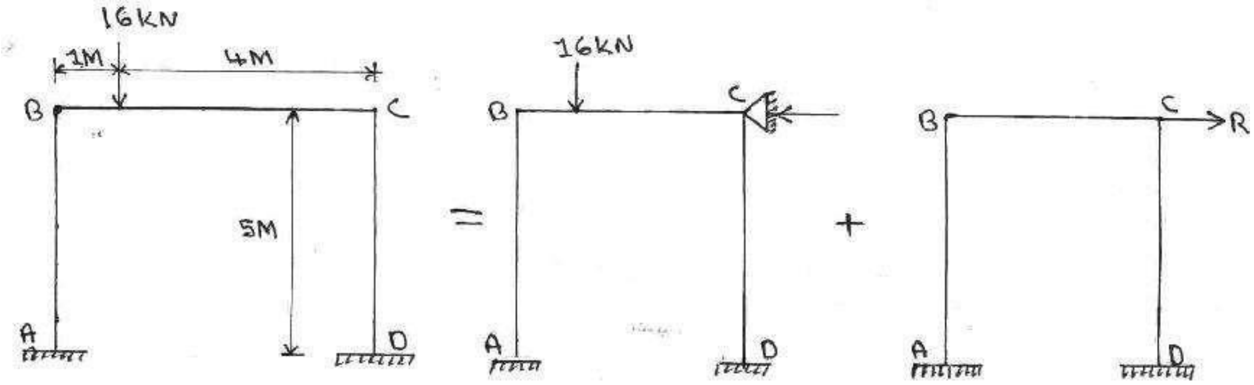
Transvers shear force diagram for

Moment distribution method for frames with side sway:

- Frames that are non symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non symmetrical loading have a tendency to side sway.



9. Analyze the frame shown in figure by moment distribution method. Assume EI is constant.



Non Sway Analysis:

- First consider the frame held from side sway as shown in figure.

Step: 1 - Fixed end moments

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24 \text{ kNm}$$

$$M_{FCB} = \frac{16\times1^2\times4}{5^2} = 2.56 \text{ kNm}$$

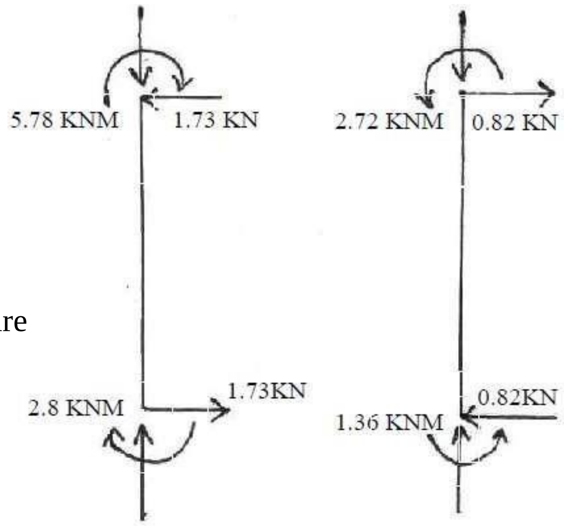
Step:2 - Distribution factor

Jt.	Member	Relative stiffness K	ΣK	DF = $\frac{K}{\sum K}$
B	BA BC	I/5 = 0.2 I I/5 = 0.2 I	0.4 I	0.5 0.5
C	CB CD	I/5 = 0.2I I/5 = 0.2I	0.4I	0.5 0.5

Step: 3 - Moment Distribution

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	0	0.5	0.5	0.5	0.5	0
FEM	0		-10.24	2.56	0	0
Balance		5.12	5.12	1.28	-1.28	
CO	2.56		-0.64	2.56		-0.64
Balance		0.32	0.32	-0.08	-0.08	
CO	0.16		-0.64	0.16		-0.64
Balance		0.32	0.32	-0.08	-0.08	
C.O	0.16		-0.04	0.16		-0.04
Balance		0.02	0.02	-0.08	-0.08	
C.O	0.01					-0.04
Final moments	**2.89**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.36**

FBD of columns:



By seeing of the FBD of columns

R = 1.73 – 0.82

(Using $\Sigma F_x = 0$ for entire frame) = 0.91 KN (←)

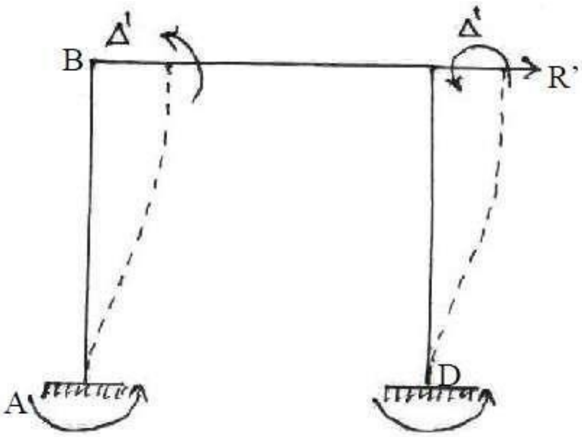
Now apply

R = 0.91 kN acting opposite as shown in figure

for the sway analysis.

ii) Sway analysis:

- For this we will assume a force R□ is applied at C causing the frame to deflect □“ as shown in figure



Since both ends are fixed, columns are of same length & I and assuming joints B & C are temporarily restrained from rotating and resulting fixed end moment are

$$M'_{BA} = M'_{AB} = M'_{CD} = M'_{DC} = \frac{6EI\Delta'}{l^2}$$

Assume $M^1_{BA} = -100\text{ kNm}$

$$\therefore\ M'_{AB} = M'_{CD} = M'_{DC} = -100\text{ kNm}$$

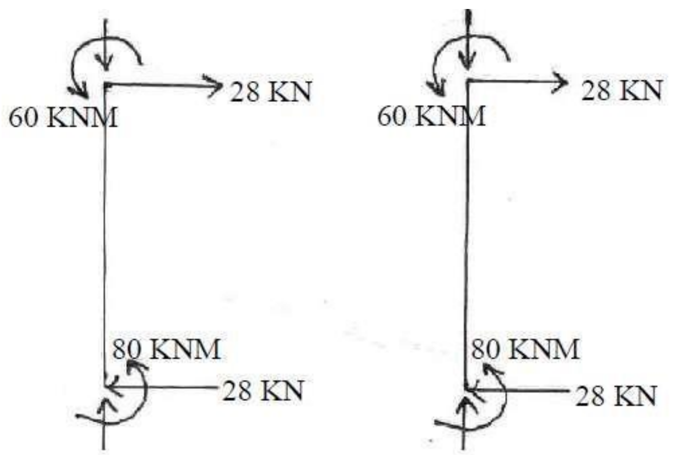
Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	01	0.5	0.5	0.5	0.5	0
FEM	–100	–100	0	0	–100	–100
Balance		50	50	50	50	
CO	25		25	25		25
Balance		–12.5	–12.5	–12.5	–12.5	
CO	-6.25		–6.25	-6.25		–6.25
Balance		3.125	3.125	3.125	3.125	
C.O	1.56		1.56	1.56		1.56
Balance		–0.78	–0.78	–0.78	–0.78	
C.O	-0.39		–0.39	–0.39		0.39
Balance		0.195	0.195	0.195	0.195	
C.O	0.1					0.1
Final moments	**- 80**	**- 60**	**60**	**60**	**– 60**	**– 80**

FBD of columns:

Using $\sum F_x = 0$ for the entire frame

$R' = 28 + 28 = 56\text{ KN } (\rightarrow)$



- Hence R'' = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion.
- Thus final moments are calculated by adding non sway moments and sway moments calculated for R = 0.91KN, as shown below

$$M_{AB} = 2.89 + \frac{0.91}{56}\ (-80) = 1.59\ \text{kNm}$$

$$M_{BA} = 5.78 + \frac{0.91}{56}\ (-60) = 4.81\ \text{kNm}$$

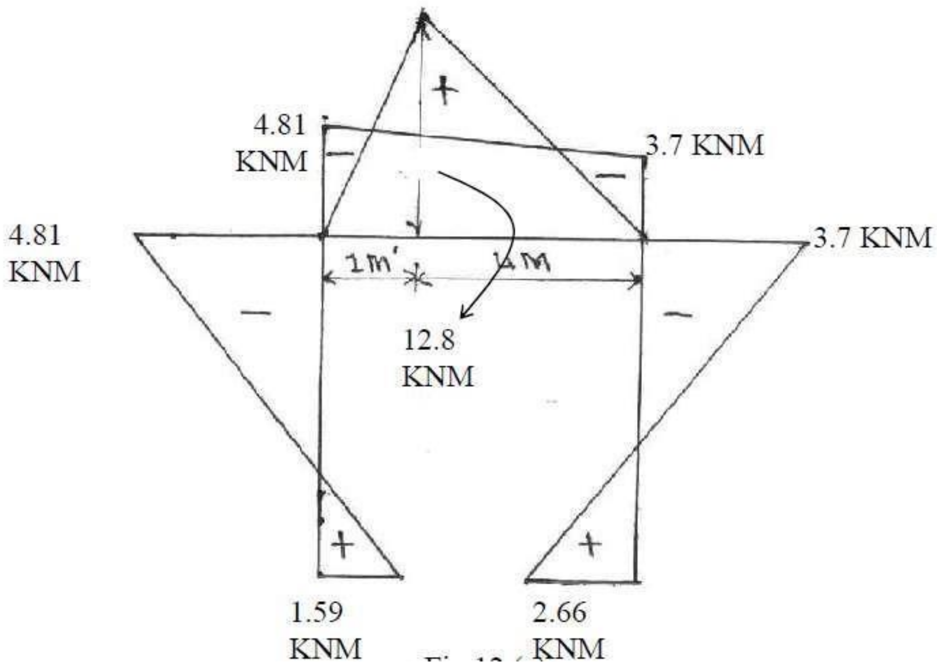
$$M_{BC} = -5.78 + \frac{0.91}{56}\ (60) = -4.81\ \text{kNm}$$

$$M_{CB} = 2.72 + \frac{0.91}{56}\ (60) = 3.7\ \text{kNm}$$

$$M_{CD} = -2.72 + \frac{0.91}{56}\ (-60) = -3.7\ \text{kNm}$$

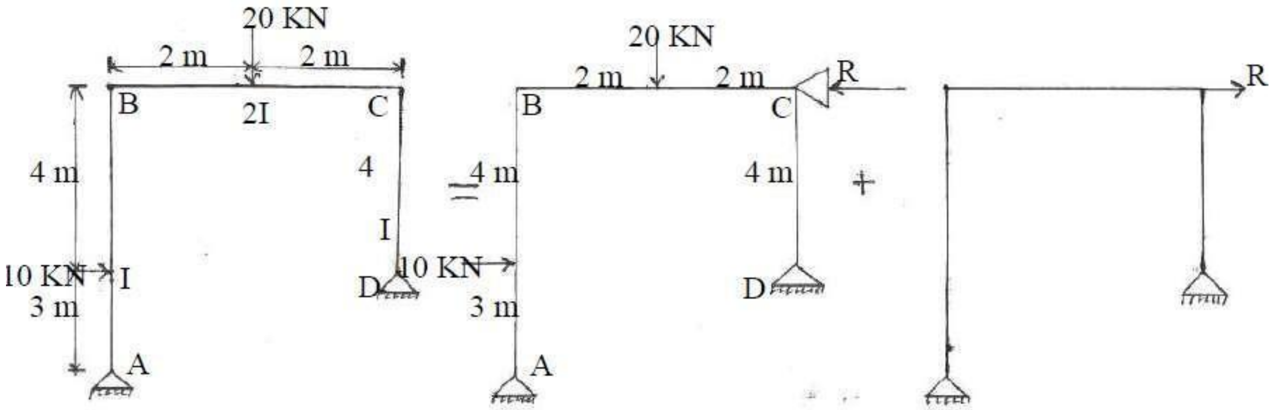
$$M_{DC} = -1.36 + \frac{0.91}{56}\ (-80) = -2.66\ \text{kNm}$$

BMD:



Moment distribution method for frames with side sway:

1. Analysis the rigid frame shown in figure by moment distribution method and draw BMD



i) Non Sway Analysis:

First consider the frame held from side sway as shown in figure 2

FEM calculation:

$$M_{FAB} = -\frac{10\times3\times4^2}{7^2} = -9.8\,KNM$$

$$M_{FBA} = +\frac{10\times3^2\times4}{7^2} = 7.3\,KNM$$

$$M_{FBC} = -\frac{20\times4}{8} = -10\ KNM$$

$M_{FCB} = +\ 10KNM$

$M_{FCD} = M_{FDC} = 0$

Distribution Factor:

Joint	Member	Relative stiffness k	Σk	$DF = \frac{K}{\sum K}$
B	BA BC	$\frac{3}{4}\times\frac{I}{7} = 0.11I$ 2I/4 = 0.5I	0.61 I	0.18 0.82
C	CB CD	2I/4 = 0.5I $\frac{3}{4}\times\frac{I}{4} = 0.19\ I$	0.69 I	0.72 0.28

Moment distribution for non sway analysis:

Joint	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	
D.F	1	0.18	0.82	0.72	0.28	1	
FEM	-9.8	7.3	-10	10	0	0	
Release jt. 'D'	+9.8						
CO		4.9					
Initial moments	0	12.2	-10	10	0	0	
Balance		-0.4	-1.8	-7.2	-2.8		
CO			-3.6	-0.9			
Balance		0.65	2.95	0.65	0.25		
C.O			0.33	1.48			
Balance		-0.06	-0.27	-1.07	-0.41		
C.O			-0.54	-0.14			
Balance		0.1	0.44	0.1	0.04		
Final moments	**0**	**12.49**	**-12.49**	**2.92**	**-2.92**	**0**	

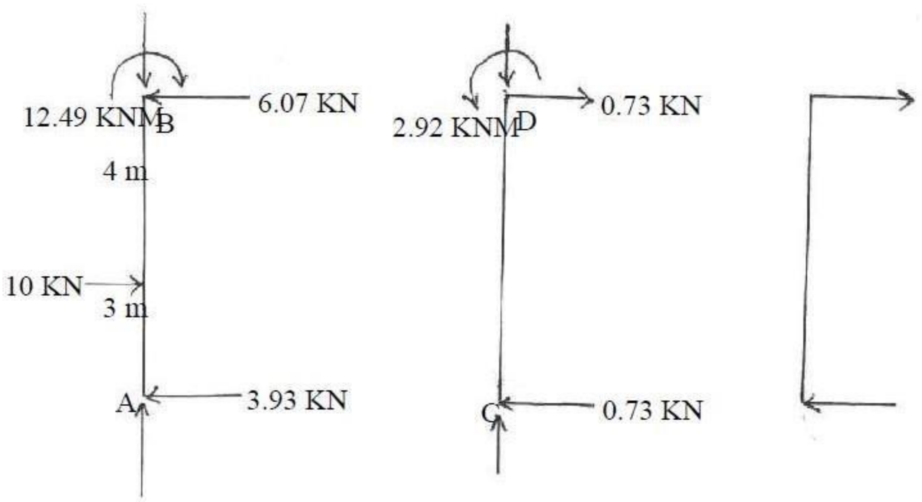
FBD of columns:

- FBD of columns AB & CD for non-sway analysis is shown in figure

Applying $\Sigma F_x = 0$ for frame as a Whole, $R = 10 - 3.93 - 0.73$

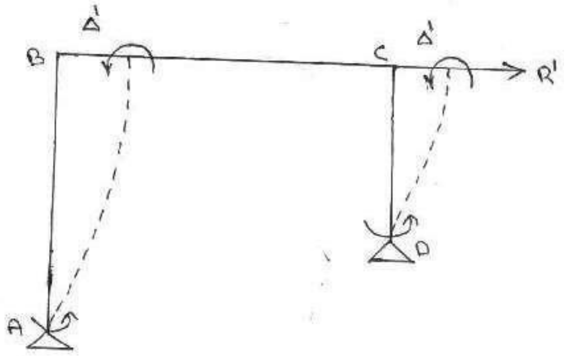
$= 5.34$ kN ($\leftarrow$)

Now apply R = 5.34KN acting opposite as shown in figure for sway analysis



ii) Sway analysis:

- For this we will assume a force R□ is applied at C causing the frame to deflect □□ as shown in figure.



Since ends A & D are hinged and columns AB & CD are of different lengths

$M'_{BA} = -3EI\Delta'/L^2_1$

$M'_{CD} = -3EI\Delta'/L^2_2$

$$\therefore \frac{M^1_{BA}}{M^1_{CD}} = \frac{3EI\ \Delta'/l_1^2}{3EI\ \Delta^1/l_2^2} = \frac{l_2^2}{l_1^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

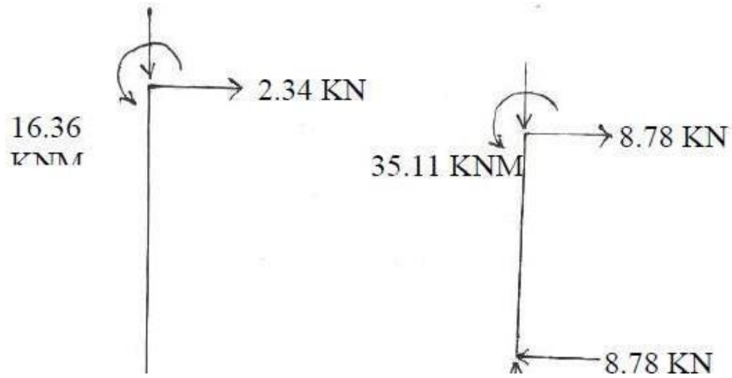
Assume $M^1_{BA} = -16\text{kNm}\ \&\ M'_{AB} = 0$

$\&\ M^1_{CD} = -49\text{kNm}\ \&\ M'_{DC} = 0$

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	0	-16	0	0	-49	0
Balance		2.88	13.12	35.28	13.72	
CO			17.64	6.56		
Balance		-3.18	-14.46	-4.72	-1.84	
CO			-2.36	-7.23		
Balance		0.42	1.94	5.21	2.02	
C.O			2.61	0.97		
Balance		-0.47	-2.14	-0.7	-0.27	
C.O			0.35	-1.07		
Balance		0.06	0.29	0.77	0.3	
C.O			0.39	0.15		
Balance		-0.07	-0.32	-0.11	-0.04	
Final moments	**0**	**-16.36**	**16.36**	**35.11**	**-35.11**	**0**

FBD of columns AB & CD for sway analysis moments is shown in fig.



Using $\Sigma f_x = 0$ for the entire frame
$R' = 11.12$ kN $(\rightarrow)$

Hence $R' = 11.12$ kN creates the sway moments shown in the above moment distribution table. Corresponding moments caused by $R = 5.34$ kN can be determined by proportion.

Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.

$M_{AB} = 0$

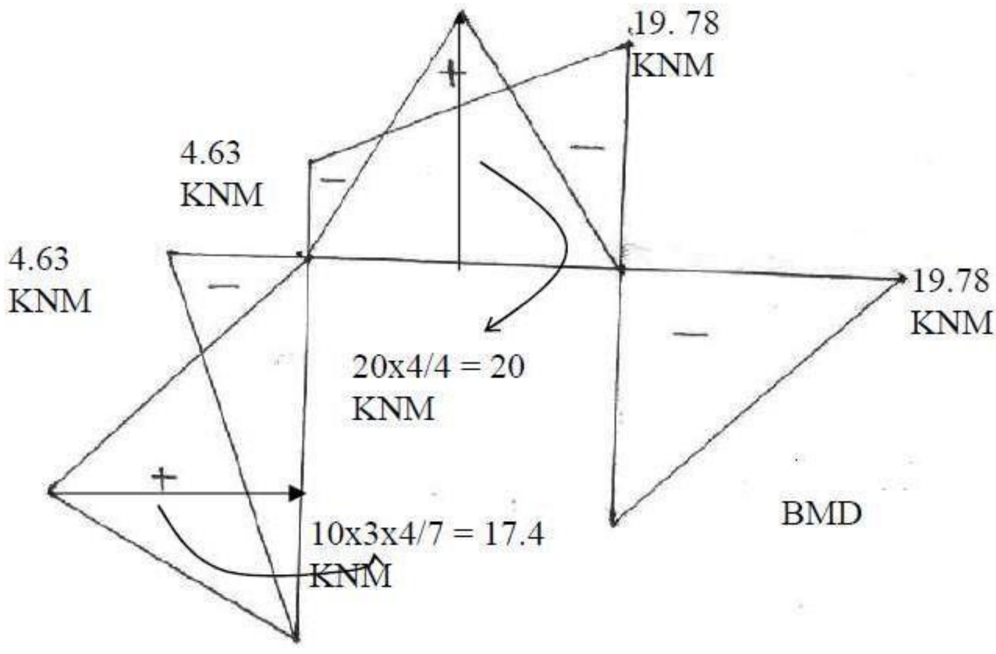
$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63$ kNm

$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63$ KNM

$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78$ kNm

$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78$ KNM

$M_{DC} = 0$



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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – IV– STRUCTURAL ANALYSIS 1 – SCIA1501

UNIT – IV - ARCHES

1.0 INTRODUCTION

An arch could be defined in simple terms as a two-dimensional structure element which is curved in elevation and is supported at ends by rigid or hinged supports which are capable of developing the desired thrust to resist the loads. It could also be defined as a two-dimensional element which resists external loads through its profile. This is achieved by its characteristic horizontal reaction developed at the supports. The horizontal thrust causes hogging bending moment which tend to reduce the sagging moment due to loading and thus, the net bending moment is much smaller. However, at the same time

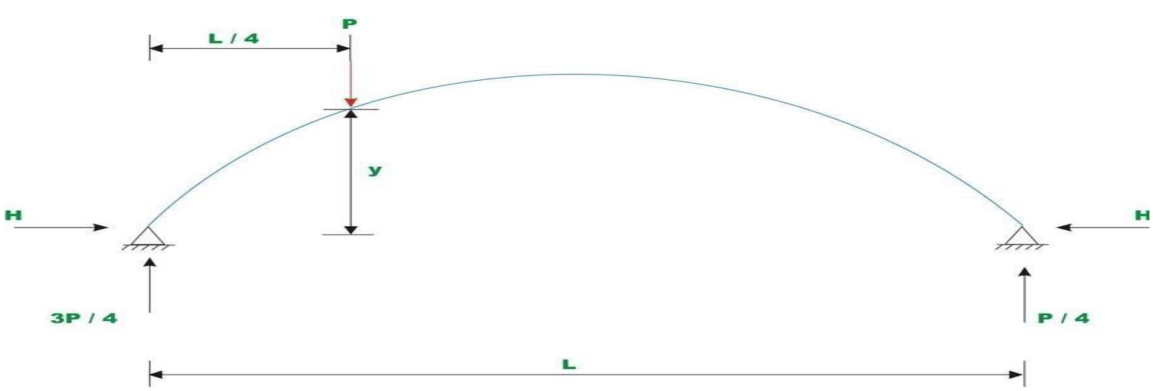
0bjectives

After studying this unit, you should be able to

1. conceptualise the structural behaviour of an arch,
2. determine the internal stress resultants namely, normal thrust, radial shear **and** bending moment for three-hinged arches,
3. obtain influence lines for horizontal thrust and vertical reactions, bending moment, radial shear and normal thrust for a three-hingcd arch, and
4. determine absolute maximum values of these internal stress resultants, or external reactions.

Arch Introduction

In case of beams supporting uniformly distributed load, the maximum bending moment increases with the square of the span and hence they become uneconomical for long span structures. In such situation's arches could be advantageously employed, as they would develop horizontal reactions, which in turn reduce the design bending moment.



(a) Arch

For example, in the case of a simply supported beam shown in Fig. 32.1, the bending moment below the load is $3PL/16$. Now consider a two hinged symmetrical arch of the same span and subjected to similar loading as that of simply supported beam. The vertical reaction could be calculated by equations of statics. The horizontal reaction is determined by the method of least work. Now the bending moment below the load is $3PL/16 - Hy$. It is clear that the bending moment below the load is reduced in the case of an arch as compared to a simply supported beam. It is observed in the last lesson that, the cable takes the shape of the loading and this shape is termed as funicular shape. If an arch were constructed in an inverted funicular shape then it would be subjected to only compression for those loadings for which its shape is inverted funicular.

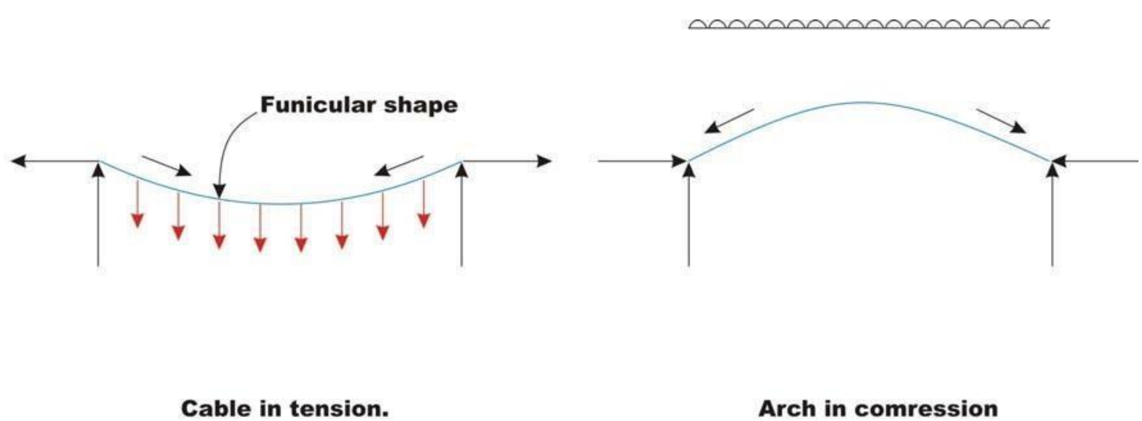


Fig 2. Cable in tension

Since in practice, the actual shape of the arch differs from the inverted funicular shape or the loading differs from the one for which the arch is an inverted funicular, arches are also subjected to bending moment in addition to compression. As arches are subjected to compression, it must be designed to resist buckling.

Until the beginning of the 20th century, arches and vaults were commonly used to span between walls, piers or other supports. Now, arches are mainly used in bridge construction and doorways. In earlier days arches were constructed using stones and bricks. In modern times they are being constructed of reinforced concrete and steel.

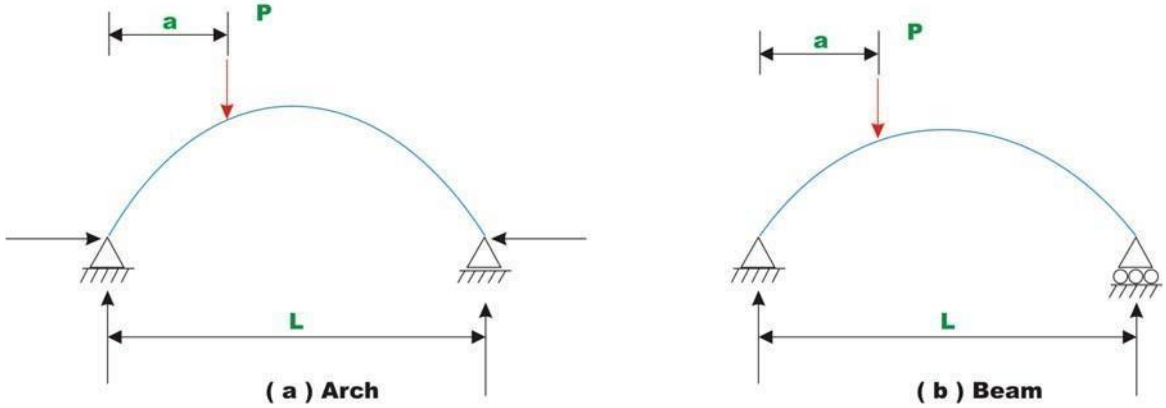


Fig 3. Cable in tension

A structure is classified as an arch not based on its shape but the way it supports the lateral load. Arches support load primarily in compression. For example in Fig 32.3b, no horizontal reaction is developed. Consequently bending moment is not reduced. It is important to appreciate the point that the definition of an arch is a structural one, not geometrical.

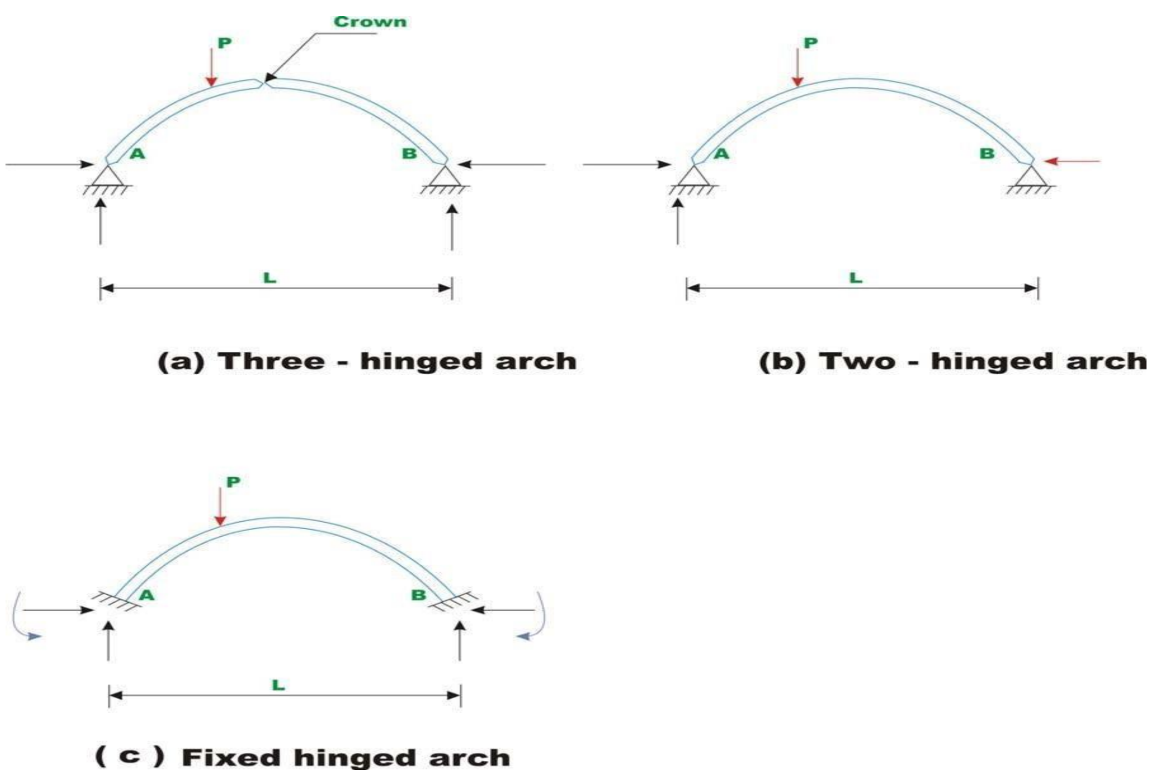


Fig 4. Types of arch

There are mainly three types of arches that are commonly used in practice: three hinged arch, two-hinged arch and fixed-fixed arch. Three-hinged arch is statically determinate structure and its reactions / internal forces are evaluated by static equations of equilibrium. Two-hinged arch and fixed-fixed arch are statically indeterminate structures. The indeterminate reactions are determined by the method of least work or by the flexibility matrix method. In this lesson three- hinged arch is discussed.

Terminology

Following are some commonly used terms relating to arches :

Springing : This is the point where the arch axis meets the supporting structure (column, pier, wall or abutment). In a simple arch, there are two springings. The springings may or may not be at the same level.

Crown : This is the highest point on the curved axis of the arch. In the case of a symmetrical arch with springings at same level, they will be above the midpoint of the arch-span [Figure 3.2 (a)]. In an unsymmetrical arch, the crown is at unequal distances from each support

Soffit: This is the lower surface of the arch which is normally curved in shape. In case of trussed arches, the line joining the nodes form the soffit.

Rise : The vertical height of the crown above the springing is the rise of the arch

Span : The horizontal distance between the springings is called span

Types of Arch

There are various ways in which arches can be classified. Following are some of the classifications of arches :

(a) Arches can be (i) simple, or (ii) multiple.

In the former case of simple arch, the arch consists of a single span structure, whereas in the latter case of multiple arch, it is a multi-span structure

(b) Arches can be classified according to the *materials* used in their construction in following way :

(i) Brick or Stone masonry arches.

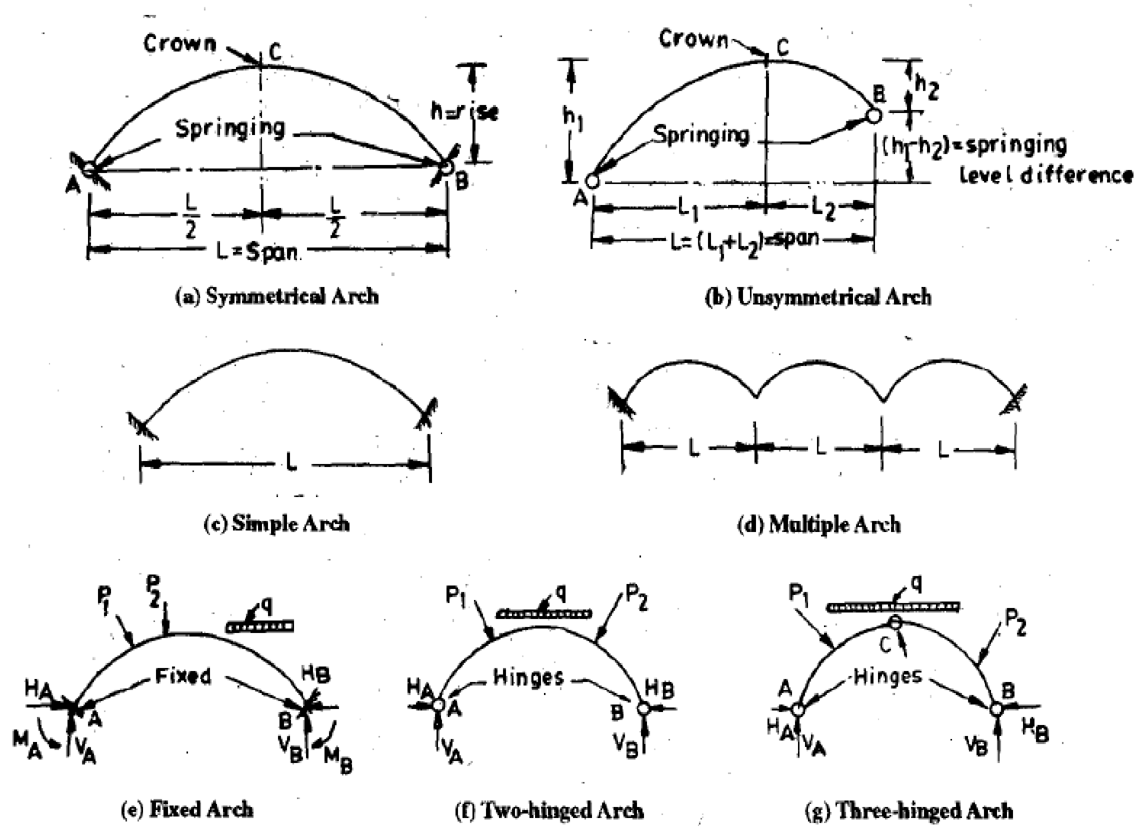
(ii) Reinforced Concrete arches.

(iii) Steel arches.

(iv) Timber arches etc.

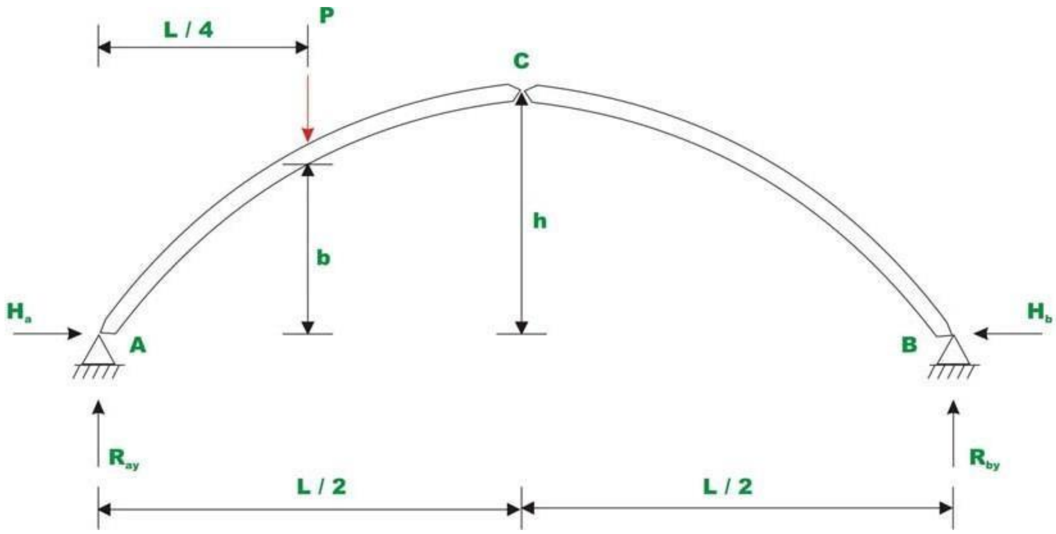
(c) Their classification according to ***structural behaviour*** can ***be*** as follows :

(i) Fixed arches: The arch springings are fixed or clamped (in both position and direction). Here the arch is statically indeterminate to the third degree, as there are six reaction components (three at each support) and only three equations of static equilibrium are available. These arches are also called hingeless arches.



Analysis of three-hinged arch

In the case of three-hinged arch, we have three hinges: two at the support and one at the crown thus making it statically determinate structure. Consider a three hinged arch subjected to a concentrated force *P* as shown in fig.



There are four reaction components in the three-hinged arch. One more equation is required in addition to three equations of static equilibrium for evaluating the four reaction components. Taking moment about the hinge of all the forces acting on either side of the hinge can set up the required equation. Taking moment of all the forces about hinge *A*, yields.

$$R_{by} = \frac{PL}{4L} = \frac{P}{4}$$

$$\sum Fy = 0 \qquad \Rightarrow \qquad R_{ay} = \frac{3P}{4}$$

Taking moment of all forces right of hinge C about hinge C leads to

$$H_b \times h = \frac{R_{by} L}{2}$$

$$\Rightarrow \qquad H_b = \frac{R_{by} L}{2h} = \frac{PL}{8h}$$

Applying $\sum Fx = 0$ to the whole structure gives $H_a = \frac{PL}{8h}$

Now moment below the load is given by ,

$$M_D = \frac{R_{ay} L}{4} - H_a b$$

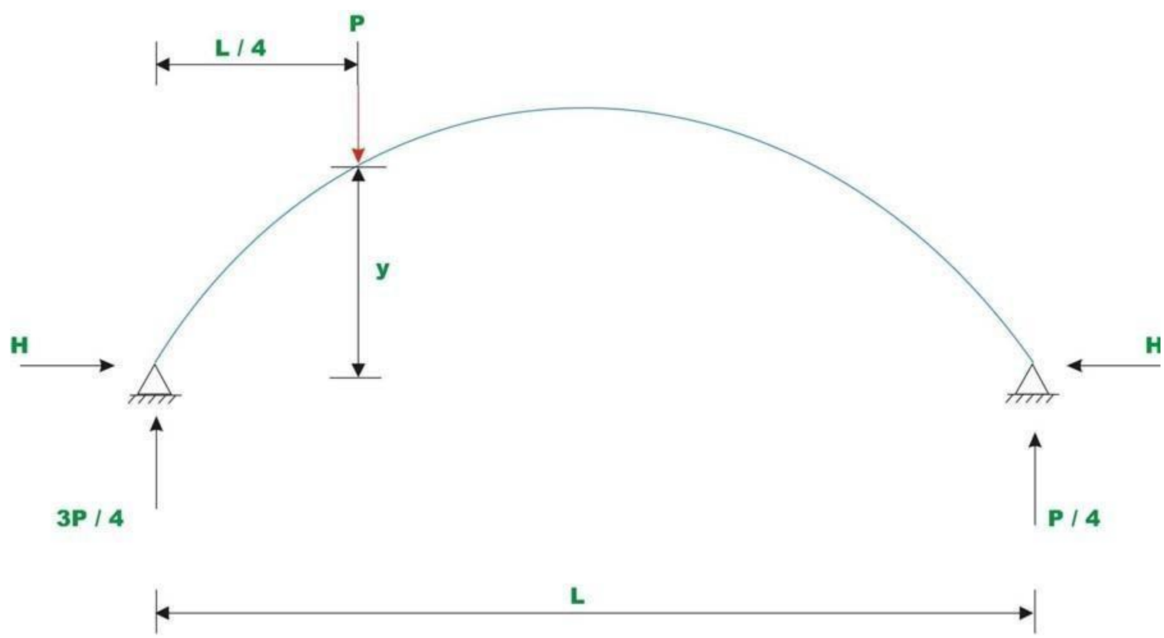
$$M_D = \frac{3PL}{16} - \frac{PLb}{8h} \tag{32.4}$$

If $\frac{b}{h} = \frac{1}{2}$ then $M_D = \frac{3PL}{16} - \frac{PL}{16} = 0.125PL$ (32.5)

For a simply supported beam of the same span and loading, moment under the loading is given by,

$$M_D = \frac{3PL}{16} = 0.375PL \tag{32.6}$$

For the particular case considered here, the arch construction has reduced the moment by 66.66 %.

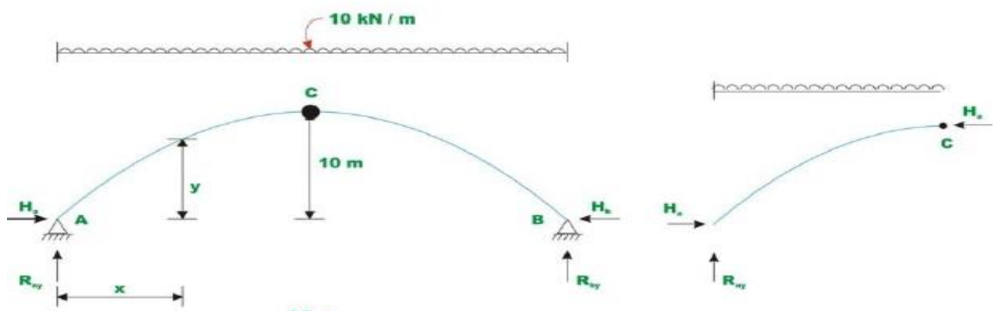


(a) Arch

Example .1

A three-hinged parabolic arch of uniform cross section has a span of 60 m and a rise of 10 m. It is subjected to uniformly distributed load of intensity 10 kN/m as shown in Fig. 32.6 Show that the bending moment is zero at any cross section of the arch.

Solution:



(a)

Free body diagram

(a)

Reactions:

Taking moment of all the forces about hinge A, yields

$$R_{ay} = R_{by} = \frac{10 \times 60}{2} = 300 \text{ kN} \quad (1)$$

Taking moment of forces left of hinge C about C, one gets

$$R_{ay} \times 30 - H_a \times 10 - 10 \times 30 \times \frac{30}{2} = 0$$

$$H_a = \frac{300 \times 30 - 10 \times 30 \times \left(\frac{30}{2}\right)}{10} \quad (2)$$

$$= 450 \text{ kN}$$

From $\sum Fx = 0$ one could write, $H_b = 450$ kN.

The shear force at the mid span is zero.

The bending moment at any section x from the left end is,

$$M_x = R_{ay} x - H_a y - 10\frac{x^2}{2} \quad (3)$$

The equation of the three-hinged parabolic arch is

$$y = \frac{2}{3}x - \frac{10}{30^2}x^2 \qquad (4)$$

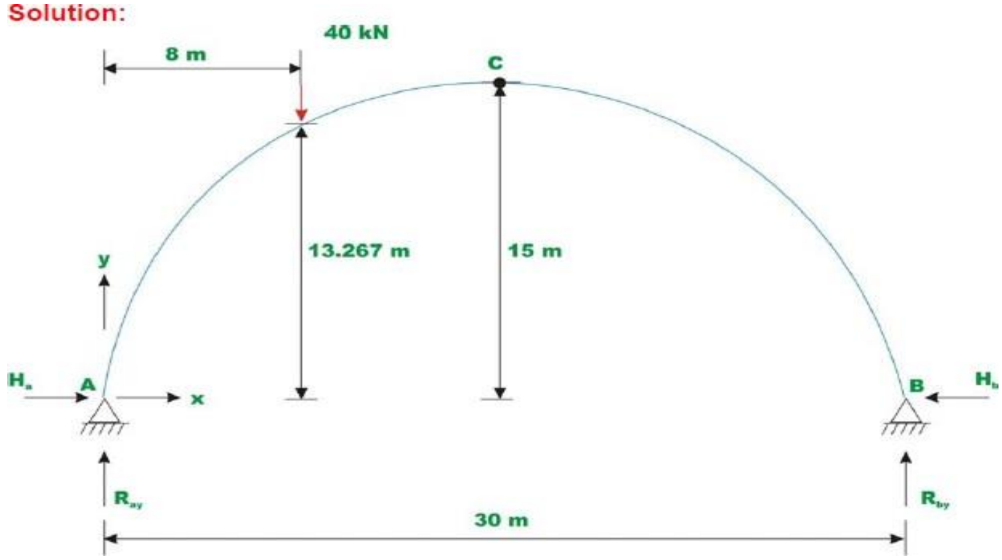
$$M_x = 300x - \left(\frac{2}{3}x - \frac{10}{30^2}x^2\right)450 - 5x^2$$

$$= 300x - 300x + 5x^2 - 5x^2 = 0$$

Example .2

A three-hinged semicircular arch of uniform cross section is loaded as shown in Fig 32.7. Calculate the location and magnitude of maximum bending moment in the arch.

Solution:



Reactions**:**

Taking moment of all the forces about hinge B leads to,

$$R_{ay} = \frac{40 \times 22}{30} = 29.33 \text{ kN } (\uparrow)$$

$$\sum Fy = 0 \qquad \Rightarrow R_{by} = 10.67 \text{ kN } (\uparrow) \qquad (1)$$

Bending Mooment

Now making use of the condition that the moment at hinge C of all the forces left of hinge is zero gives,

$$M_c = R_{ay} \times 15 - H_a \times 15 - 40 \times 7 = 0 \tag{2}$$

$$H_a = \frac{29.33 \times 15 - 40 \times 7}{15} = 10.66 \text{ kN } (\rightarrow)$$

Considering the horizontal equilibrium of the arch gives,

$$H_b = 10.66 \text{ kN } (\leftarrow)$$

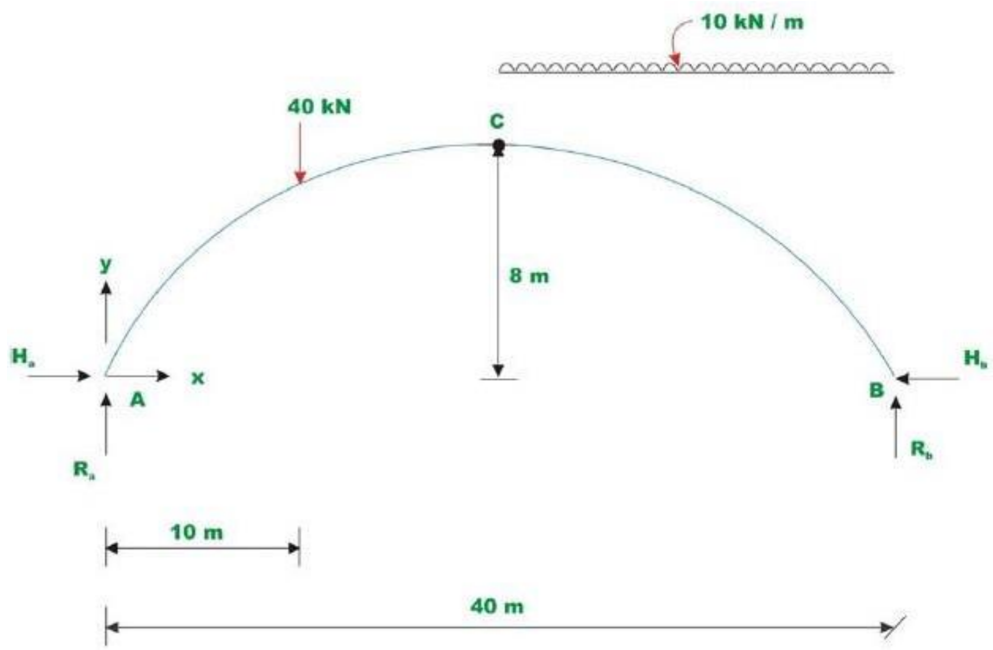
The maximum positive bending moment occurs below D and it can be calculated by taking moment of all forces left of D about D.

$$M_D = R_{ay} \times 8 - H_a \times 13.267 \tag{3}$$

$$= 29.33 \times 8 - 10.66 \times 13.267 = 93.213 \text{ kN}$$

Example .3

A three-hinged parabolic arch is loaded as shown in Fig 32.8a. Calculate the location and magnitude of maximum bending moment in the arch. Draw bending moment diagram.



Taking A as the origin, the equation of the three-hinged parabolic arch is given by,

$$y = \frac{8}{10}x - \frac{8}{400}x^2 \qquad (1)$$

Taking moment of all the forces about hinge B leads to,

$$R_{ay} = \frac{40 \times 30 + 10 \times 20 \times \left(20/2 \right)}{40} = 80 \text{ kN } (\uparrow)$$

$$\sum Fy = 0 \qquad \Rightarrow R_{by} = 160 \text{ kN } (\uparrow) \qquad (2)$$

Now making use of the condition that, the moment at hinge C of all the forces left of hinge C is zero gives,

$$M_c = R_{ay} \times 20 - H_a \times 8 - 40 \times 10 = 0$$

$$H_a = \frac{80 \times 20 - 40 \times 10}{8} = 150 \text{ kN } (\rightarrow) \qquad (3)$$

Considering the horizontal equilibrium of the arch gives,

$$H_b = 150 \text{ kN } (\leftarrow) \qquad (4)$$

Consider a section x from end B. Moment at section x in part CB of the arch is given by (please note that B has been taken as the origin for this calculation),

$$M_x = 160x - \left(\frac{8}{10}x - \frac{8}{400}x^2 \right)150 - \frac{10}{2}x^2 \qquad (5)$$

According to calculus, the necessary condition for extremum (maximum or minimum) is that $\frac{\partial M_x}{\partial x} = 0$.

$$\frac{\partial M_x}{\partial x} = 160 - \left(\frac{8}{10} - \frac{8 \times 2}{400}x \right)150 - 10x \qquad (6)$$

$$= 40 - 4x = 0$$

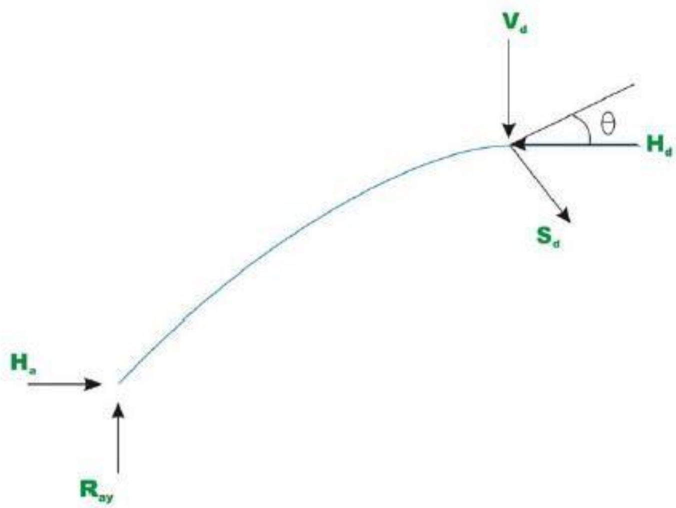
$$x = 10 \text{ m.}$$

Substituting the value of x in equation (5), the maximum bending moment is obtained. Thus,

$$M_{max} = 160(10) - \left(\frac{8}{10}(10) - \frac{8}{400}(10)^2 \right)150 - \frac{10}{2}(10)^2$$

$$M_{max} = 200 \text{ kN.m.} \qquad (7)$$

Shear force at D just left of 40 kN load



The slope of the arch at D is evaluated by,

$$\tan\theta = \frac{dy}{dx} = \frac{8}{10} - \frac{16}{400}x \quad (8)$$

Substituting $x = 10$ m. in the above equation, $\theta_D = 21.8^0$

Shear force (S_d)

$$S_d = H_a \sin\theta - R_{ay} \cos\theta \quad (9)$$

$$S_d = 150\sin(21.80) - 80\cos(21.80)$$

$$= -18.57 \text{ kN.}$$

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UNIT – V – STRUCTURAL ANALYSIS 1 – SCIA1501

1. Introduction

Cables and arches are closely related to each other and hence they are grouped in this course in the same module. For long span structures (for e.g. in case bridges) engineers commonly use cable or arch construction due to their efficiency. In the first lesson of this module, cables subjected to uniform and concentrated loads are discussed. In the second lesson, arches in general and three hinged arches in particular along with illustrative examples are explained. In the last two lessons of this module, two hinged arch and hingeless arches are considered.

Structure may be classified into rigid and deformable structures depending on change in geometry of the structure while supporting the load. Rigid structures support externally applied loads without appreciable change in their shape (geometry). Beams trusses and frames are examples of rigid structures. Unlike rigid structures, deformable structures undergo changes in their shape according to externally applied loads. However, it should be noted that deformations are still small. Cables and fabric structures are deformable structures. Cables are mainly used to support suspension roofs, bridges and cable car system. They are also used in electrical transmission lines and for structures supporting radio antennas. In the following sections, cables subjected to concentrated load and cables subjected to uniform loads are considered.

The shape assumed by a rope or a chain (with no stiffness) under the action of external loads when hung from two supports is known as a funicular shape. Cable is a funicular structure. It is easy to visualize that a cable hung from two supports subjected to external load must be in tension (vide Fig. 31.2a and 31.2b). Now let us modify our definition of cable. A cable may be defined as the structure in pure tension having the funicular shape of the load.

1.1 INTRODUCTION

In many engineering structures, such as suspension bridges, transmission lines, aerial tramway, guy-wires for high towers etc, cables are suspsded between supports and subjected to vertical loads. The load of the bridge floor is transferred to a cable which is stretched over a span to be bridged. The cable is flexible and can adopt any curvature as required by the load. Bridges are of two types (a) unstiffened, and (b) stiffened. In unstiffened bridges the curve of the cable undergoes changes with passage of loads over the

bridge and decking also gets disturbed. The decking of stiffened bridge is stiffened by .girders or trusses which distribute the loads evenly over the entire cable. Various components of the suspension bridge are as follows :

(1) the cable, (6) vertical suspenders,

(2) decking, (7) supporting towers,

(3) main span, (8) side span,

(4) back stay, (9) anchorage, and

(5) saddle or pulley, (10) Stiffening girdet.

The unstiffened bridges subjected to concentrated and uniformly distributed load, shape of cable, tension, length of cable

General

Since flexible cable offers no resistance to bending, the resultant internal force on any cross-section of cable must act along the tangent to the cable at that section. The resist'ulce to bending offered by actual cables is usually relatively smll and can be neglected without serious error. Thus, cable structures carry their load through tension which is most efficient way of re9isting loads.

A cable suspended between two supports at its ends may be subjected to different types of loading and the shape assumed by the cable, therefore, will depend upon the type of loading. Various types of loads are as follows :

(a) Vertically downward concentrated load(s)

(b) Distributed load

(i) The weight of a suspension bridge roadway is an example of this type of load, which is uniformly 'distributed along the horizontal span. Cables loaded uniformly along the span remains in the shape of a parabola.

Miscellaneous Topics

(ii) The weight of homogeneous cable of constant cross-secti'on is an example of load distributed uniformly doing the cable. A cable loaded uniformly along the cable assumes the shape of a catenary.

Assumptions

(a) The **cable** is perfectly flexible. The wire ropes and parallel wire cables are obviously quite flexible and have little flexural stiffness. Eye bar chains cannot carry any moments because of the freedom of rotation at the hinges.

(b) The **stiffening girder** is straight, its moment of inertia is constant and it is tied to the cable throughout its length. This assumption takes into account the deformation of only the chord members of the truss and neglects the deformation of the web members. Also since **suspenders** are closely spaced, their loads may be assumed to be uniformly distributed without the introduction of serious error.

(c) The dead load of the truss (stiffening girder), cable and suspenders is uniform per unit horizontal length; therefore the initial curve of the cable becomes parabolah This is achieved in practice by attaching the girder, after the dead loads have been transferred to the cable, by adjusting the suspender forces during erection by meals of turn-buckles provided in them for this purpose.

(d) The form and magnitude of ordinates of the cable curve remain unchanged even after the application of the live loads. By this assumption, the cable always remains parabolic with the same central sag. Obviously, the stiffening girder must deflect under live loads, pulling the cable downwards along with it. These deflections and resulting errors will increase with larger span and shallower trusses. It is found that with small spans and deep trusses, the errors are small. The errors, however, are found to be always on the safe side.

Concept

To understand the structural behavior of the cable, consider a simply supported beam as shown in Figure 14.1 (a). Beam is subjected to a point load P at mid point. As the cable has zero moment carrying capacity, insert an internal hinge under load so that moment carrying capacity of beam at that point reduces to zero and beam assumes shape as shown in Figure 14.1 (b). Considering the equilibrium point B as shown in Figure 14.1 (c) tension in beam.

T = p/2 sin θ,

and both horizontal components are T cos θ which are equal and opposite at B.

Cable subjected to Concentrated Forces

Consider cable supported on horizontal span of 8 m and length of cable on application of load is 10 m as shown in Figure 14.2 (a). The freebody at B is shown in Figure

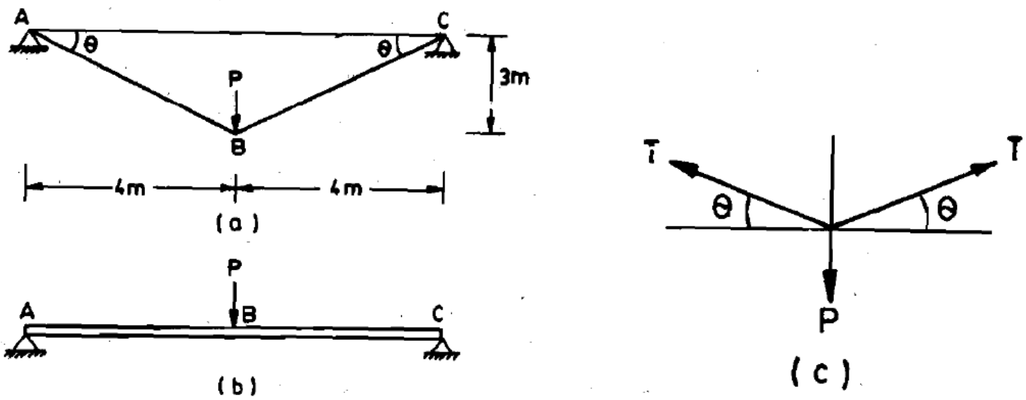


Fig: 1 suspension of cables

Tension in cable at point B

$$T = \frac{P}{2\sin\theta} = \left(\frac{P}{2}\right) \times \left(\frac{5}{3}\right) = \frac{5P}{6}$$

Horizontal component of T

$$H = T\cos\theta = \left(\frac{5P}{6}\right) \times \left(\frac{4}{5}\right) = \frac{2P}{3}$$

Now the moment which is product of horizontal component at B and sag at point B, is

$$\left(\frac{2P}{3}\right) \times 3 = 2P$$

Assuming rigid beam as shown in Figure 14.2 (b). Bending moment at B is calculated as

$$\frac{PL}{4} = \frac{(P \times 8)}{4} = 2P$$

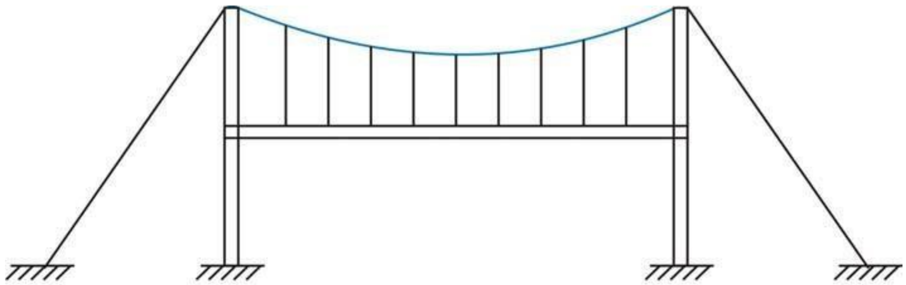


Fig. 31.1 Deformable structure.

Fig 31.2a Unloaded cable
(when dead load is neglected)

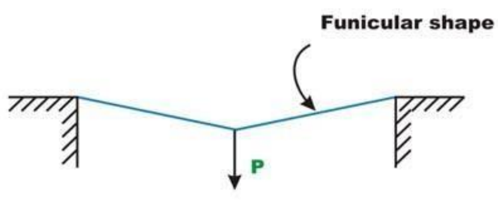


Figure 31.2b Cable in tension.

Cable subjected to Concentrated Loads

As stated earlier, the cables are considered to be perfectly flexible (no flexural stiffness) and inextensible. As they are flexible they do not resist shear force and bending moment. It is subjected to axial tension only and it is always acting tangential to the cable at any point along the length. If the weight of the cable is negligible as compared with the externally applied loads then its self weight is neglected in the analysis. In the present analysis self weight is not considered.

Consider a cable *ACDEB* as loaded in Fig. 31.2. Let us assume that the cable lengths L_1, L_2, L_3, and sag at C, D, E (h_c, h_d, h_e) are known. The four reaction components at A and B, cable tensions in each of the four segments and three sag values: a total of eleven unknown quantities are to be determined. From the geometry, one could write two force equilibrium equations ($\sum F_x = 0, \sum F_y = 0$) at each of the point A, B, C, D and E i.e. a total of ten equations and the required one more equation may be written from the geometry of the cable. For example, if one of the sag is given then the problem can be solved easily. Otherwise if the total length of the cable S is given then the required equation may be written as

$$S = \sqrt{L_1^2 + h_c^2} + \sqrt{L_2^2 + (h_d - h_c)^2} + \sqrt{L_2^2 + (h_d - h_e)^2} + \sqrt{L_2^2 + (h + h_e)^2} \qquad (.1)$$

Cable subjected to uniform load.

Cables are used to support the dead weight and live loads of the bridge decks having long spans. The bridge decks are suspended from the cable using the hangers. The stiffened deck prevents the supporting cable from changing its shape by distributing the live load moving over it, for a longer length of cable. In such cases cable is assumed to be uniformly loaded.

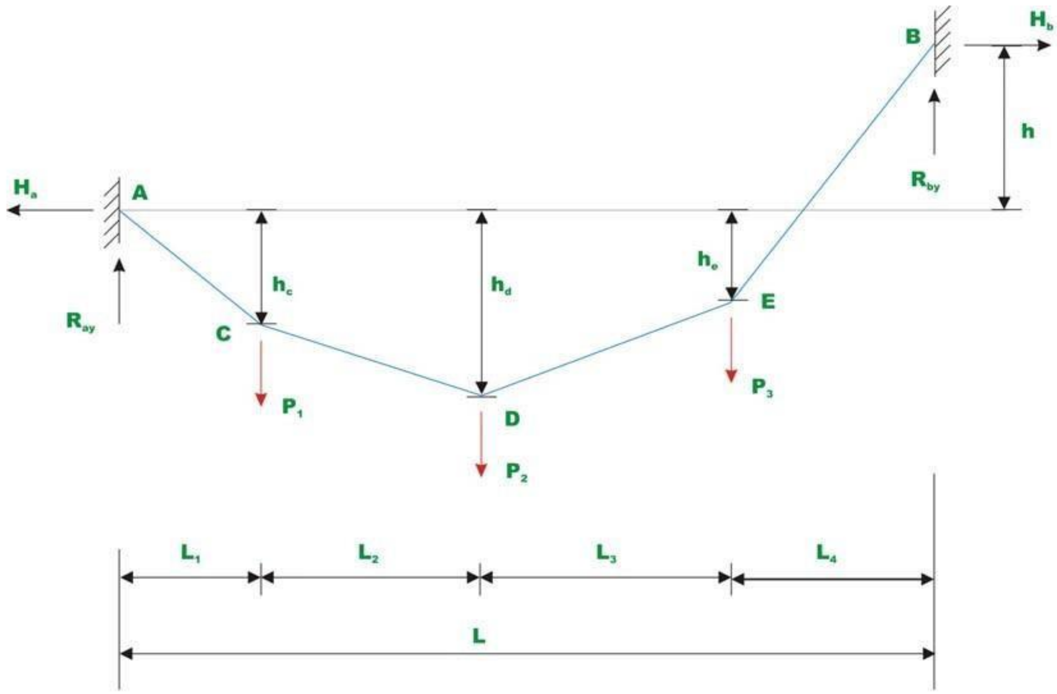


Fig. 31.3a Cable subjected to concentrated load.

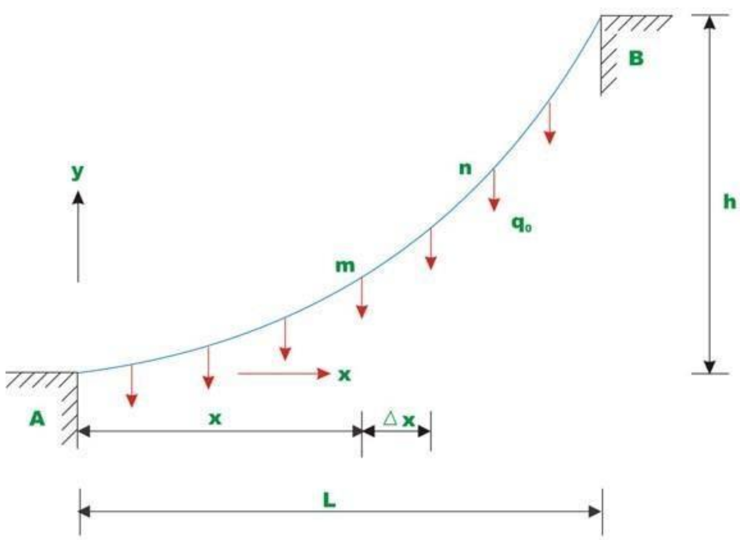


Fig. 31.3b Cable subjected to uniformly distributed load.

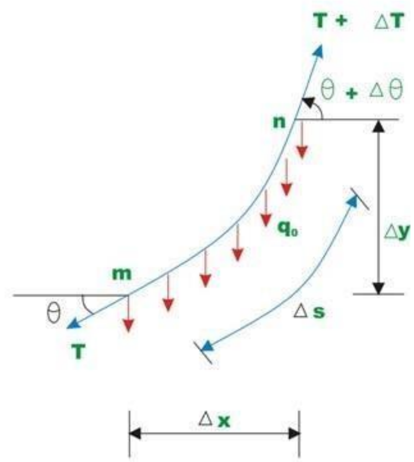


Fig. 31.3c Free-body diagram

Consider a cable which is uniformly loaded as shown in Fig 31.3a. Let the slope of the cable be zero at A . Let us determine the shape of the cable subjected to uniformly distributed load q_0 . Consider a free body diagram of the cable as shown in Fig 31.3b. As the cable is uniformly loaded, the tension in the cable changes continuously along the cable length. Let the tension in the cable at m end of the free body diagram be T and tension at the n end of the cable be $T + \Delta T$. The slopes of the cable at m and n are denoted by θ and $\theta + \Delta\theta$ respectively.

Applying equations of equilibrium, we get

$$\sum Fy = 0 \qquad -T\sin\theta + (T+\Delta T)\sin(\theta+\Delta\theta) - q_0(\Delta x) = 0 \qquad (31.2a)$$

$$\sum Fx = 0 \qquad -T\cos\theta + (T+\Delta T)\cos(\theta+\Delta\theta) = 0 \qquad (31.2b)$$

$$\sum Mn = 0 \qquad -(T\cos\theta)\Delta y + (T\sin\theta)\Delta x + (q_0\Delta x)\frac{\Delta x}{2} = 0 \qquad (31.2c)$$

$$\lim_{\Delta x \to 0} \qquad \frac{\Delta T}{\Delta x}\sin(\theta+\Delta\theta) = q_0$$

$$\frac{d}{dx}(T\sin\theta) = q_0$$

$$\frac{d}{dx}(T\cos\theta) = 0$$

$$\lim_{\Delta x \to 0} \qquad -T\cos\theta\frac{\Delta y}{\Delta x} + T\sin\theta + q_0\frac{x_0}{2} = 0$$

$$\frac{dy}{dx} = \tan\theta$$

Example 31.1

Determine reaction components at A and B, tension in the cable and the sag y_B , and y_E of the cable shown in Fig. 31.4a. Neglect the self weight of the cable in the analysis.

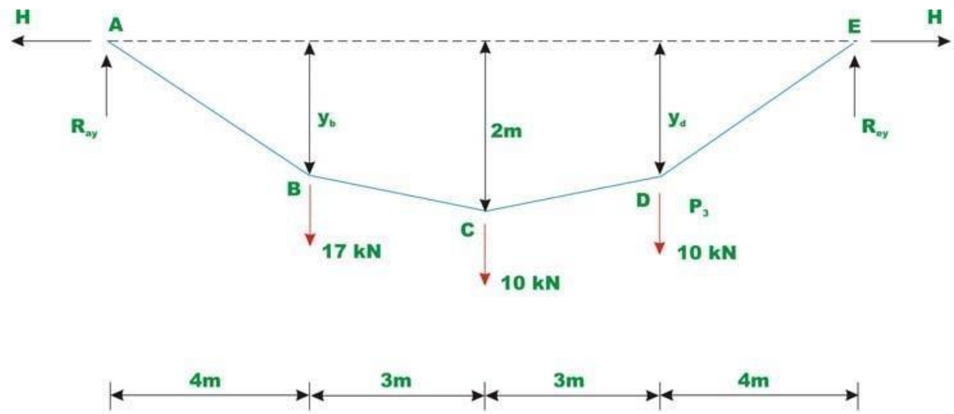
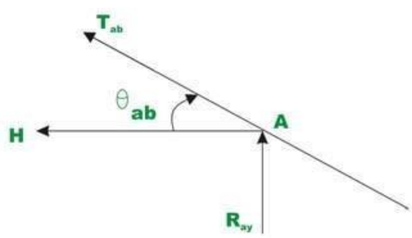
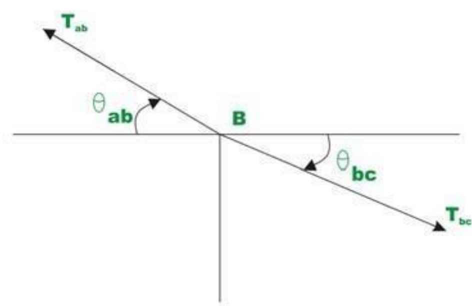


Fig. 31.4 Example 31.1



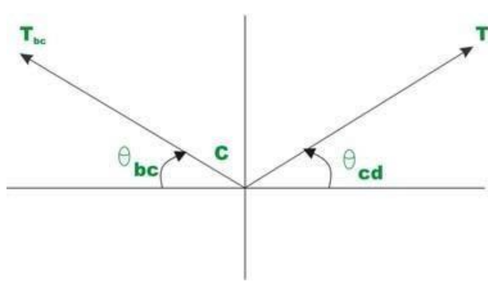
Joint A

Fig. 31.4b



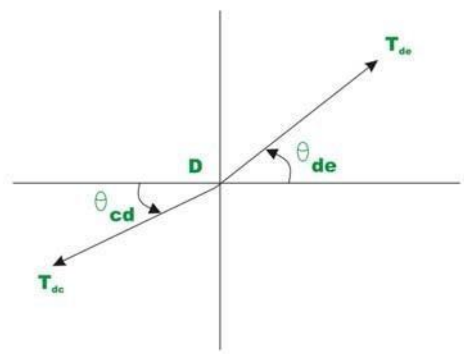
Joint B

Fig. 31.4c



Joint C

Fig. 31.4d



Joint D

Fig. 31.4e

Since there are no horizontal loads, horizontal reactions at A and B should be the same. Taking moment about *E*, yields

$$R_{ay} \times 14 - 17 \times 20 - 10 \times 7 - 10 \times 4 = 0$$

$$R_{ay} = \frac{280}{14} = 20 \quad \text{kN}; \qquad R_{ey} = 37 - 20 = 17\text{kN}.$$

Now horizontal reaction H may be evaluated taking moment about point C of all forces left of C .

$$R_{ay} \times 7 - H \times 2 - 17 \times 3 = 0$$

$$H = 44.5 \text{ kN}$$

Taking moment about B of all the forces left of B and setting $M_B = 0$, we get

$$R_{ay} \times 4 - H \times y_B = 0; \qquad y_B = \frac{80}{44.50} = 1.798\ m$$

Similarly, $y_D = \dfrac{68}{44.50} = 1.528\,m$

To determine the tension in the cable in the segment AB , consider the equilibrium of joint A (vide Fig.31.4b).

$$\sum F_x = 0 \Rightarrow T_{ab} \cos\theta_{ab} = H$$

$$T_{ab} = \frac{44.5}{\left(\dfrac{3}{\sqrt{3^2 + 0.298^2}}\right)} = 48.789 \text{ kN}$$

The tension T_{ab} may also be obtained as

$$T_{ab} = \sqrt{R_{ay}^2 + H^2} = \sqrt{20^2 + 44.5^2} = 48.789 \text{ kN}$$

Now considering equilibrium of joint $B, C,$ and D one could calculate tension in different segments of the cable.

Segment bc

Applying equations of equilibrium,

$$\sum F_x = 0 \Rightarrow T_{ab} \cos\theta_{ab} = T_{bc} \cos\theta_{bc}$$

Segment de

See Fig.31.4

Segment cd

$T_{cd} = \frac{1}{-}$

See Fig.31.4
See Fig.31.4

Segment de

$T_{de} = \frac{1}{-}$

The tension

The tension T_{de} may also be obtained as

$$T_{de} = \sqrt{R_{ey}^2 + H^2} = \sqrt{17^2 + 44.5^2} = 47.636 \text{ kN}$$

Example .2

A cable of uniform cross section is used to span a distance of 40m as shown in Fig 31.5. The cable is subjected to uniformly distributed load of 10 kN/m. run. The left support is below the right support by 2 m and the lowest point on the cable C is located below left support by 1 m. Evaluate the reactions and the maximum and minimum values of tension in the cable.

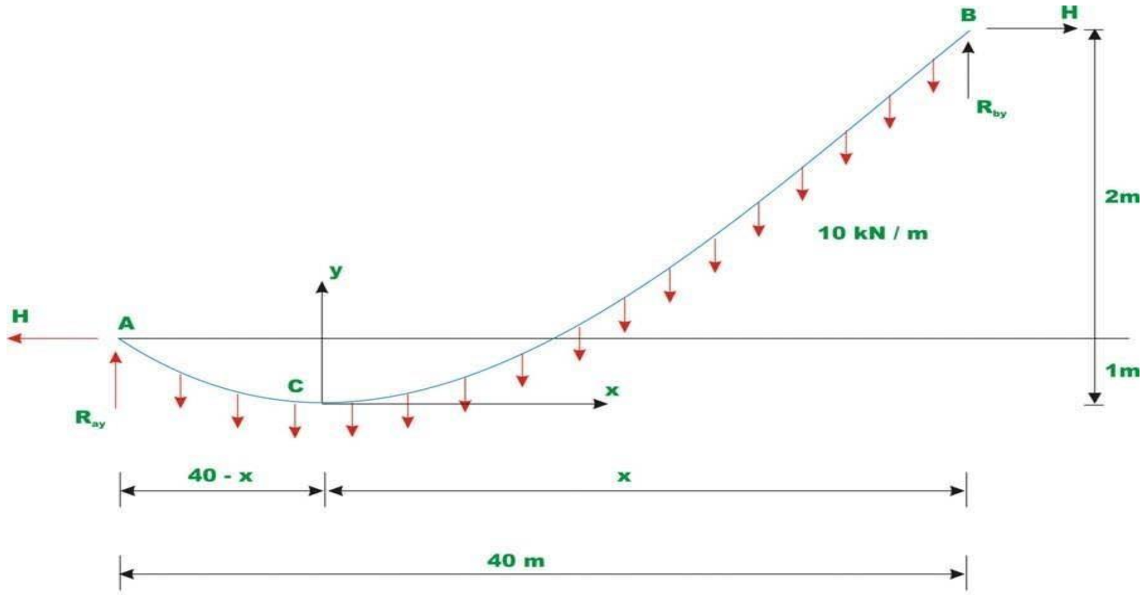


Fig. 31.5 Example 31.2

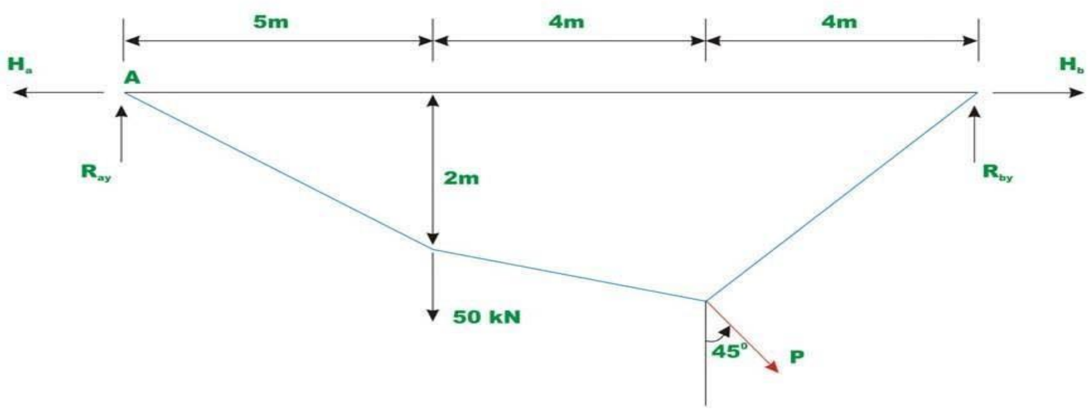


Fig. 31.6 Example 31.3

Assume the lowest point C to be at distance of x m from B . Let us place our origin of the co-ordinate system xy at C . Using equation 31.5, one could write,

$$y_a = 1 = \frac{q_0(40-x)^2}{2H} = \frac{10(40-x)^2}{2H} \tag{1}$$

$$y_b = 3 = \frac{10x^2}{2H} \tag{2}$$

where y_a and y_b be the y co-ordinates of supports A and B respectively. From equations 1 and 2, one could evaluate the value of x . Thus,

$$10(40-x)^2 = \frac{10x^2}{3} \quad \Rightarrow \quad x = 25.359\ m$$

From equation 2, the horizontal reaction can be determined.

$$H = \frac{10 \times 25.359^2}{6} = 1071.80 \text{ kN}$$

Now taking moment about A of all the forces acting on the cable, yields

$$R_{by} = \frac{10 \times 40 \times 20 + 1071.80 \times 2}{40} = 253.59 \text{ kN}$$

Writing equation of moment equilibrium at point B , yields

$$R_{ay} = \frac{40 \times 20 \times 10 - 1071.80 \times 2}{40} = 146.41 \text{ kN}$$

Tension in the cable at supports A and B are

$$T_A = \sqrt{146.41^2 + 1071.81^2} = 1081.76 \text{ kN}$$

$$T_B = \sqrt{253.59^2 + 1071.81^2} = 1101.40 \text{ kN}$$

The tension in the cable is maximum where the slope is maximum as $T\cos\theta = H$. The maximum cable tension occurs at *B* and the minimum cable tension occurs at *C* where

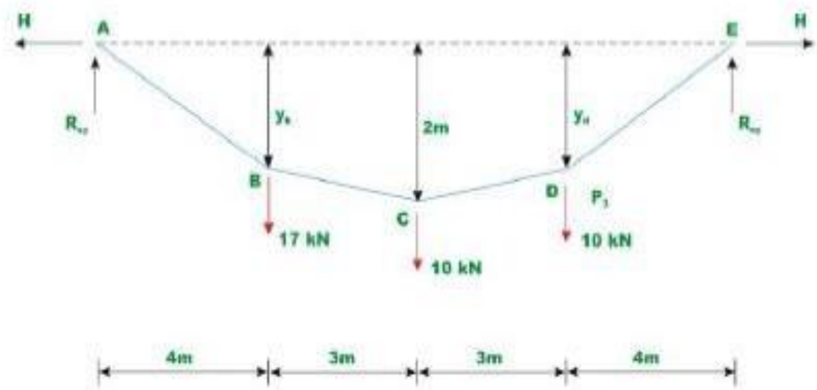
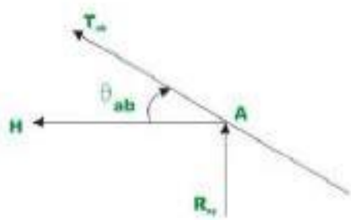
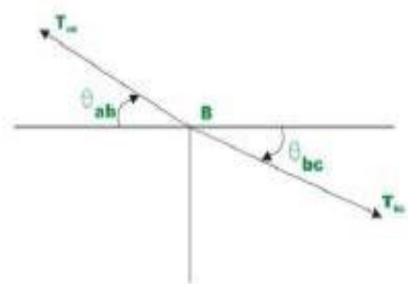


Fig. 31.4 Example 31.1



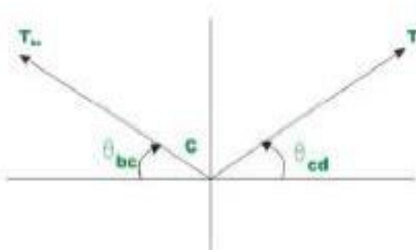
Joint A

Fig. 31.4b



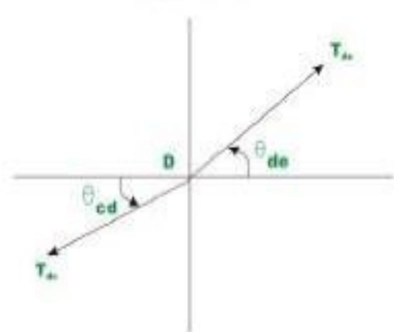
Joint B

Fig. 31.4c



Joint C

Fig. 31.4d



Joint D

Fig. 31.4e

Example .3

A cable of uniform cross section is used to support the loading shown in Fig 31.6. Determine the reactions at two supports and the unknown sag y_C .

Taking moment of all the forces about support B ,

$$R_{ay} = \frac{1}{10}\left[350 + 300 + 100\, y_c\right] \qquad (1)$$

$$R_{ay} = 65 + 10\, y_c$$

Taking moment about B of all the forces left of B and setting M_B □ 0 , we get,

$$R_{ay} \times 3 - H_a \times 2 = 0$$

$$\Rightarrow H_a = 1.5 R_{ay} \qquad (2)$$

Taking moment about C of all the forces left of C and setting M_C □ 0 , we get

$$\sum M_C = 0 \qquad R_{ay} \times 7 - H_a \times y_C - 50 \times 4 = 0$$

Substituting the value of H_a in terms of R_{ay} in the above equation,

$$7R_{ay} - 1.5 R_{ay}\, y_C - 200 = 0 \qquad (3)$$

Using equation (1), the above equation may be written as,

$$y_C^{\,2} + 1.833\, y_C - 17 = 0 \qquad (4)$$

Solving the above quadratic equation, y_C can be evaluated. Hence,

$$y_C = 3.307 m.$$

Substituting the value of y_C in equation (1),

$$R_{ay} = 98.07 \text{ kN}$$

From equation (2),

$$H_a = 1.5R_{ay} \qquad = 147.05 \text{ kN}$$

Now the vertical reaction a D , R_{dy} is calculated by taking moment of all the forces about A ,

$$R_{dy} \times 10 - 100 \times 7 + 100 \times 3.307 - 50 \times 3 = 0$$

$$R_{dy} = 51.93 \text{ kN.}$$

Taking moment of all the forces right of C about C, and noting that $\sum M_C = 0$,

$$R_{dy} \times 3 = H_d \times y_C$$

$$\Rightarrow H_d = 47.109$$