SCHOOL OF BUILDING AND ENVIRONMENT
DEPARTMENT OF CIVIL ENGINEEERING

UNIT - I - FUNDAMENTAL KNOWLEDGE OF CONVENTIONAL SURVEYING - SCIA1303

## Fundamental Knowledge of Conventional Surveying

## Definition of Survey

Surveying is the art of determining the relative positions of different objects on the surface of the earth by measuring the horizontal distances between them, and by preparing a map to any suitable scale. Thus, in this discipline, the measurements are taken only in the horizontal plane.

Different methods and instruments are being used to facilitate the work of surveying.

## The primary aims of field surveying are:

To measure the Horizontal Distance between points.
To measure the Vertical elevation between points.
To find out the Relative direction of lines by measuring horizontal angles with reference to any arbitrary direction and
To find out Absolute direction by measuring horizontal angles with reference to a fixed direction. These parameters are utilized to find out the relative or absolute coordinates of a point / location.

## Objectives of Surveying

To collect field data;
To prepare plan or map of the area surveyed;
To analyze and to calculate the field parameters for setting out operation of actual engineering works.
To set out field parameters at the site for further engineering works.

## Division of Surveying

The approximate shape of the earth can best be defined as an oblate tri-axial ovaloid. But, most of the civil engineering works, concern only with a small portion of the earth which seems to be a plane surface. Thus, based upon the consideration of the shape of the earth, surveying is broadly divided into two types.
1.Geodetic Surveying
2. Plane Surveying

## Geodetic Surveying

In this branch of surveying, the true shape of the earth is taken into consideration. This type of surveying is being carried out for highly precise work and is adopted for surveying of large area.

## Plane Surveying

In this method of surveying, the mean surface of the earth is considered to be a plane surface. This type of survey is applicable for small area (less than 200 square kilometer). Thus for most of the Civil Engineering projects, methods of plane surveying are valid. This course is restricted to the different aspects of plane surveying. Henceforth, in this course work, the word surveying implies plane surveying.

## Classification of surveying

Based on the purpose (for which surveying is being conducted),
Surveying has been classified into:

Control surveying: To establish horizontal and vertical positions of controlpoints.
Land surveying: To determine the boundaries and areas of parcels of land, alsoknown as property survey, boundary survey or cadastral survey.

Topographic survey: To prepare a plan/ map of a region which includes naturalas well as and man-made features including elevation.

Engineering survey: To collect requisite data for planning, design and executionof engineering projects. Three broad steps are

Reconnaissance survey: To explore site conditions and availability ofinfrastructures.
Preliminary survey: To collect adequate data to prepare plan / map of areato be used for planning and design.

Location survey: To set out work on the ground for actual construction /execution of the project.

Route survey: To plan, design, and laying out of route such as highways,railways, canals, pipelines, and other linear projects.

Construction surveys: Surveys which are required for establishment of points,lines, grades, and for staking out engineering works (after the plans have been prepared and the structural design has been done).

Astronomic surveys: To determine the latitude, longitude (of the observationstation) and azimuth (of a line through observation station) from astronomical observation.

Mine surveys: To carry out surveying specific for opencast and undergroundmining purposes.

## Based on instruments:

Chain Surveying
Compass Surveying
Plane Table Surveying
Theodolite Surveying
Tachometric Surveying
Photographic Surveying

## Based on methods

Triangulation Surveying and,
Traverse Surveying

## Triangulation Surveying

The selected survey stations are connected with survey lines in such a way resulting in the formation of network of triangles. This survey is useful in surveying larger areas with
uneven site boundaries.The various formulas of the triangle are used to determine and area and various dimensions of the site.

Traverse Surveying
The whole area is divided into various transverses for the easy surveying. Since this method involves division of survey area into various transverse, it is termed as Transverse Survey.

## Based on Object:

Geodetic Surveying
Mine Surveying
Archaeological Surveying, and
Military Surveying.
Based on nature of field
Land Surveying
Marine Surveying and,
Astronomical Surveying

## Land Survey is further classified into following categories

1. Topographical Survey
2. Cadastral Survey
3. City Survey
4. Engineering Survey

## Land Survey

This involved survey of objects in earth's surface.

## Topographical Survey

This survey is performed or determining the natural features of the country such as rivers, mountains, hills, valleys etc. and also artificial features such as canals, railways, roads, towns, villagesetc.This helps to keep a record of existing natural features of the country.

## Cadastral Survey

Cadastral Survey is similar to topographical survey. It only involves a little more precision and detailing in the process. Apart from determining the natural features, this method of survey also helps to determine boundaries of fields, houses and other properties.Cadastral Survey maps are drawn to a more larger scale as compared to topographical survey maps.This is performed in connection with town planning schemes such as drainage, water supply etc and for laying plots, roads, streets etc.

## City Survey

City Survey is carried out in connection with various engineering works such as roads, railways etc and it also involves various works related to water supply sector such as reservoirs, wells etc

## Principle of Surveying

The fundamental principles upon which the surveying is being carried out are working from

1. To Work from the Whole to the Part and
2. To work from Part to the Whole

## Work from the Whole to the Part

The whole area is enclosed by main station (Controlling Station) and main survey lines (Controlling lines). Then the area is divided into a number of parts by forming well conditioned triangles. The survey lines are measured very accurately by using tape or chain or any linear measurement equipments. Then the sides of the triangle measured. The purpose of working from whole to part is

To localize the errors and
To control the accumulation of errors.

This is being achieved by establishing a hierarchy of networks of control points. The less precise networks are established within the higher precise network and thus restrict the errors.

To minimize the error limit, highest precise network (primary network)
To locate a new station by at least two measurements (linear or angular) from fixed reference point

The new stations should always be fixed by at least two measurements from fixed reference points. Linear and angular measurements are measured by using theodolite and prismatic compass. Thus the positions of main stations and direction of main survey lines are fixed.


Figure: 1

## Plan and Maps

## Plan

A plan is the graphical representation to some scale, of the features on, near or below the surface of the earth as projected on a horizontal plane. The horizontal plane is represented by plane of drawing sheets on which the plan is drawn to some scale However the surface of the earth is curved it cannot be truly represented on a plane without distortion. In plain surveying the area involved are small, theearth"s surface may be considered as plane and hence plan is constructed by orthographic projections. A plan is drawn on a relatively large scale.

## Map

If the scale of the graphical projection on a horizontal plane is small, the plan is called a map. Thus graphical representation is called a plan if the scale is large while it is called a map if the scale is small.

On plan, generally only horizontal distances and directions or angles are shown. On topographical map, however the vertical distances (elevations) are also represented by contour lines.

## Scale

It is basic requirement for the preparation of plan or map Scale is used to represent large distances on paper. The ratio by which the actual length of the object is reduced or increased in the drawing is known as the „Scale" for example., if 1 cm on a map represents a distance of 10 metres on the ground, the scale of the map is said to be $1 \mathrm{~cm}=10 \mathrm{~m}$.

## Full-Size Scale

Actual length of the object is shown on the drawing.

## Reducing Scale

Actual length of the object is reduced in order to accommodate on the drawing sheet.

## Increasing or Enlarging Scale

Actual length of an object is enlarged so as to bring out its details more clearly on the drawing.

## Representative Fraction

- The ratio of the distance on the drawing to the corresponding actual length of the object on the ground is known as the representative fractions. i.e..
- R.F= Distance of the Object on Drawing/Corresponding actual distance of object on ground.


## Types of Scales

a) Plain Scale
(b) Diagonal Scale
(c) Chord Scale
(d) Vernier Scale

## Units of Measure

The system of units in India in the recent years in M.K.S. andS.I.but all the records available in surveying done in the past are in F.P.S. units therefore, for an engineer it becomes necessary to know the conversion of units from one system to another, a few are listed below.

## Table -1

| Length | Area |
| :--- | :--- |
| 1 inch $=2.54 \mathrm{~cm}$ | 1 are $=100^{2}$ |
| 1 foot $=0.3048 \mathrm{~m}$ | 100 are $=1$ hectare |
| 1 yard $=3$ feet | 1 hectare $=10,000 \mathrm{~m}^{2}$ |
| 1 mile $=1.609 \mathrm{~km}$ | 1 hectare $=2.471$ acres |
| 1 nauticalmile $=1.852 \mathrm{~km}$ | 100 hectares $=1 \mathrm{~km}^{2}$ |
| 1 GROUND $=2400 \mathrm{sq.ft}$ | 1 cent $=435.6 \mathrm{sq.ft}$ |

## Representative Fraction

- The ratio of the distance on the drawing to the corresponding actual length of the object on the ground is known as the representative fractions. i.e..

Distance of the Object on Drawing
R.F =

Corresponding Actual distance of object on Ground
Both the distances are in same unit.
Scale is a fixed ratio that every distance on the plan bears with corresponding distance on the ground.

## Scale can be represented by the following method

## Representative Fraction

For example 1,
If a scale is $1 \mathrm{~cm}=10 \mathrm{~m}$
R.F. $=1 / 10 \times 100=1 / 1000$ or 1: 1000

For example 2,
If a scale is $1 \mathrm{~cm}=1000 \mathrm{~m} \mathrm{or} 1 \mathrm{~km}$
R.F. $=1 / 1 \times 1000 \times 100=1 / 100000$ or 1: 100000

This type of scale is called engineering scale.
There are four main types of scales which are used for different purposes for measurements

```
1. Plain Scale
2. Diagonal Scale
3. Vernier Scale
```


## Plain Scale

Plain Scale is one on which it is possible to measure two dimensions only. For example, measurements such as units and lengths, metres and decimetres etc.


## Diagonal Scale

On diagonal scale, it is possible to measure three dimensions such as metres, decimetres and centimetres, units, tens and hundreds; yards, feet and inches etc.

A short length is divided into number of parts using the principle of similar triangle in which sides are proportional.



1-1 represent $1 / 10 \mathrm{PQ}$
2-2 represent 2/10 PQ
9-9 represent 9/10 PQ

## Vernier Scale

A device used for measuring the fractional part of one of the smallest divisions of a graduated scale.

It usually consists of a small auxiliary scale which slides alongside of the main scale.
Least count of the vernier $=$ the difference between smallest division on the main division and smallest division on the vernier scale.

## Choice of Scale of a Map

Scales are generally classified as large, medium and small as under.

- Large Scale: $1 \mathrm{~cm}=10 \mathrm{~m}$ or less than 10 m
- Medium Scale: $1 \mathrm{~cm}=10 \mathrm{~m}$ to 100 m
- Small Scale: $1 \mathrm{~cm}=100$ or more than 100 m


## Choice of Scale of a Map

For most of engineering projects, the scale varies from $1 \mathrm{~cm}=2.5 \mathrm{~m}$ to 100 m Small scale topographical maps are usually drawn to scale $1 \mathrm{~cm}=1 \mathrm{~km}$, a scale of $1 \mathrm{~cm}=5 \mathrm{~m}$ to 50 m is generally used for plans prepared for subdivisions of land

Conventional Signs :

| Object | Symbol | Colour |
| :---: | :---: | :---: |
| Lake or pond |  | Prussian blue |
| Open well | (1-2) | Prussian blue |
| Tube well |  | Black |
| Footpath |  | Black |
| Metalled road |  | Burnt sienna |
| Unmetalled road |  | Burnt sienna |
| Railway line (single) | $+$ | Black |
| Railway line (double) |  | Black |
| Road bridge or culvert |  | Black |
| Railway bridge or culvert |  | Black |
| Level-crossing |  | Black and sienna |
| Wall with gate | - | Black |
| Boundary line |  | Black |

## Methods of Linear Measurement

The following methods are generally employed for the linear, measurement are as follows,

By Pacing or Stepping

By Passometer

By speedometers
By perambulator and,

By Chaining

- The walking step of a man is considered as 2.5 Ft or 80 cm .
- A small instrument just like a stop watch, The passometer is used for counting the number of steps automatically by some mechanical devices.
- $\quad$ This is used for recording distances
- It is a wheel fitted with a fork and handle. The wheel is graduated and shows a distance per revolution. There is a dial which records the number or revolutions.
- This is an accurate method of measuring Distance.


## Accessories for Linear Measurements

Ranging Rods - Generally circular in section, of Diameter 25 mm and length 2 m .

Tapes - Cloth or linen tape, Metallic tape, Steel tape and Invar tape.

Arrows - Arrows made up of tempered steel wire of diameter 4 mm . One end of the arrow is bent into ring of $\emptyset$

50 mm and other end is pointed. Length 400 mm
ChainN - A Chain is prepared with 100 or 150 pieces of Galvanized mild steel wire of diameter 4 mm . Two Brass handles are provided at the two ends of the chain.

## Direct Measurements

When the distance between points / stations are measured directly, usually by using tape, is known as direct method.

Ranging: When the distance to be measured is more than a tape length, astraight line is required to be laid between the points/ stations along which measurements are to be carried out. The process of laying out a straight line between points is known as ranging.

There are two types of Ranging

1. Direct Ranging
2. Indirect Ranging

## Direct Ranging



The intermediate points are found by moving a ranging pole in transverse direction and thus, points are selected in such as way that the end points and the intermediate points lie in a straight line (Figure) In this method, two flags, one ranging pole and a bunch of pegs are required in a team of at least one surveyor and one assistant.

## Indirect Ranging

When the end stations between which a straight line is to be laid, are not inter visible, indirect method of ranging is being adopted. It is being carried out either by reciprocal method or by random line method.

There are two types of Indirect Ranging
1.
2.

## Reciprocal Ranging

Random Line Method

## Reciprocal Ranging

Figure shows the field operations involved in reciprocal ranging. Let $A$ and $B$ are the two end points whose distance is required to be found and are not inter visible. To fix the intermediate points in a straight line between these points, two more points say C and


Figure :3Reciprocal Ranging
D are chosen in such a way that $\mathrm{D} \& \mathrm{~B}$ are visible from C and $\mathrm{C} \& \mathrm{~A}$ from D . Then, direct ranging is being carried out alternatively along DCA and CDB for a number of times so that ACDB lie in a straight line.

## Random Line Method

In this method, (Figure) a random line such as BQ is being laid such that R is visible from Q . A perpendicular QR is being erected at Q and measure its distance. Then the desired distance $B R$ is being calculated using distance formula. In laying the line BR, intermediate points are established first. These are laid by taking offset from the random line at distances calculated by using the method of similar triangle such as


## Ranging by Random Line Method

After locating the intermediate points on the line BR, the obstructions get cleared to make the end points intervisible. Then, direct ranging is being carried out to obtain a extended continous straight line.

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## SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT - II -SURVEYING-I- SCIA1303

# Applications of various instruments in plane surveying 

## CHAIN



Figure :1

There Five types of chain

- Metric chain -
- Steel chain
- Engineer's chain
- Gunter's chain
- Revenue chain.


## Metric chain

Metric chains are generally available in 5, 10, 20 and 30 meters. Tallies are fixed at very meter length for chains of 5 m and 10 m length and at every five meters.

## Steel chain

The steel band consists of a long narrow strip of blue steel of uniform width of 12 to 16 mm and thickness of 0.3 to 0.6 mm . metric steel bands are available in lengths of 20 or 30 m .

## Gunter's chain

Gunter's chain is a measuring device used for land survey, Gunter divided the chain into 100 links, marked off into groups of 10 by brass rings which simplified intermediate measurement. On the face of it, the dimensions make no sense: Each link is a fraction
under 8 inches long; 10 links make slightly less than 6 feet, 8 inches and a full length of 66 feet.

## Engineer's chain

The engineer's chain is 100 ft long and consists of 100 links, each link being 1 ft long. At every 10 links, brass tags are fastened.

## Revenue chain

The revenue Chain is 33 ft long and consists of 16 links, each link being 2.0625 ft long. This chain is mainly used for measuring fields in cadastral survey.

## Principles of Chain Surveying

It is one of the oldest methods of surveying. It is used in horizontal measurement on the earth's surface. The principle of chain surveying is the triangulation. It is used for measuring small areas of land when the level of accuracy required is not high.
Chain surveying is recommended where:

- The ground surface is more or less level.
- A small area is to be surveyed.
- A small-scale map is to be prepared
- The formation of well-conditioned triangle is easy.


## Chain survey is unsuitable when

Area is crowded with many details.
Area consists of too many undulations.
Area is very large.
The formation of well-conditioned triangle becomes difficult due to obstacles
It means that the area to be surveyed is divided into number of small triangles.
The angles should not be too acute or too obtuse. The angle: shouldn't less than $30^{\circ}$ and shouldn't greater than $120^{\circ}$ - Well conditioned triangles

## Survey Station

Survey stations are of two kinds
$\checkmark$ Main Stations
$\checkmark$ Subsidiary or tie

## Main Stations

Main stations are the end of the lines, which command the boundaries of the survey, and the lines joining the main stations re called the main survey line or the Chain lines.

## Subsidiary or the tie stations

Subsidiary or the tie stations are the point selected on the main survey lines, where it is necessary to locate the interior detail such as fences, hedges, building etc.

## Tie or subsidiary lines

A tie line joints two fixed points on the main survey lines. It helps to checking the accuracy of surveying and to locate the interior details. The position of each tie line should be close to some features, such as paths, building etc.

## Base Lines

It is main and longest line, which passes approximately through the center of the field. All the other measurements to show the details of the work are taken with respect of this line.

## Check Line

A check line also termed as a proof line is a line joining the apex of a triangle to some fixed points on any two sides of a triangle. A check line is measured to check the accuracy of the framework. The length of a check line, as measured on the ground should agree with its length on the plan.

## OBSTACLES IN CHAINING

Though it is desirable to select stations so as to avoid obstacles, occasionally the obstacles are unavoidable.

During measurement of distance, various obstacles may be encountered in the field. Depending upon the type of obstacle, a suitable geometrical figure has to be framed and an equivalent distance has to measured or computed. Obstacles encountered in the field can be divided into three broad categories.

Various obstacles to chaining may be grouped into:
Obstacles to ranging (chaining free-vision obstructed)
Obstacles to chaining (chaining obstructed-vision free)
Obstacles to both ranging and chaining.

## Obstacles to Ranging

These obstacles can be further classified into the followicategories: (a) Both ends of the line are visible from some intermediate points. Intervening ground is an example of such obstacle. By resorting to reciprocal ranging this difficulty can be overcome.
(b) Both ends of the line may not be visible from intermediate points on the line, but may be visible from a point slightly away from the line. Intervening trees and bushes are the examples of such obstacles. This obstacle to chaining may be overcome by measuring along a random line as shown in Figure. In this case required length

$$
\mathrm{EB}=\sqrt{\mathrm{EC}^{2}+\mathrm{CB}^{2}}
$$



Figure:2 Obstacles to Ranging

## Obstacles to Chaining

In this type the ends of lines are visible but chaining is obstructed. Examples of such obstructions are ponds, lakes, marshy land etc. Various geometric properties may be used to find obstructed length.


Figure 3: Measurement Obstructed Ranging round the Obstacle


Figure 4: Measurement Obstructed Ranging round the Obstacle Obstacles to Both Chaining and Ranging

Building is a typical example of this obstacle.


## Mistakes in Tapping

During taping, mistakes generally made by individuals (usually inexperienced)
are:
i. Adding or dropping a full length of tape
ii. Adding or dropping a part of the length of tap.
iii. Other points incorrectly taken as 0 or 30 meter marks on tape
iv. Reading numbers incorrectly
v. Calling numbers incorrectly or not clearly

## Problems in Obstacles in Chaining

1. A survey line CD intersects a building. To overcome the obstacle a perpendicular DE, 107 m long, is set out at D. From E, two lines EF and EG are set out at angles $50^{\circ}$ and $67^{\circ}$ respectively with ED. Find the lengths EF and EG such that points F and G fall on the prolongation with of CD. Also find the obstructed distance DF.
Solution


From DEF

$$
\begin{aligned}
\frac{\mathrm{DE}}{E F} & =\operatorname{Cos} 50^{\circ} \\
\mathrm{EF} & =\frac{\mathrm{DE}}{\operatorname{Cos} 50^{\circ}}=\frac{107}{\operatorname{Cos} 50^{\circ}}=\mathbf{1 6 6 . 4 6} \mathbf{~ m}
\end{aligned}
$$

and

$$
\frac{\mathrm{DF}}{D E}=\tan 50^{\circ}
$$

$$
\mathrm{DF}=\mathrm{DE} \tan 50^{\circ}=107 \times 1.1917=\quad \mathbf{1 2 7 . 5 2} \mathbf{~ m}
$$

## From DEG

$$
\begin{aligned}
\frac{\mathrm{DE}}{E G} & =\operatorname{Cos} 65^{\circ} \\
\mathrm{EG} & =\frac{\mathrm{DE}}{\operatorname{Cos} 65^{\circ}}=\frac{107}{\operatorname{Cos} 65^{\circ}}=\mathbf{2 5 3 . 1 8} \mathbf{~ m}
\end{aligned}
$$

2. During measurement of distance, a pond had been come across the path. Let $P$ and Q are the stations selected on the opposite side of the pond. A line $\mathrm{PC}=900 \mathrm{~m}$, was set out on one side of PQ , and a line $\mathrm{PD}=1100 \mathrm{~m}$ was set out on the other side, such that CQD was in a straight line. The length of the lines CQ and QD are 500 m and 600 m respectively. Determine the desired distance PQ.

1

$$
\begin{aligned}
& \cos \mathrm{C}=\frac{\left(500^{2}+900^{2}-1100^{2}\right)}{2 \times 500 \times 900} \\
& \mathrm{PQ}=\sqrt{900^{2}+500^{2}-2 \times 500 \times 900 \cos \mathrm{C}}
\end{aligned}
$$



## Errors in Chaining

* Erroneous length of chain or tape
* Bad Ranging
* Careless holding and marking
* Bad Straightening
* Non- Horizontality
* Sag in Chain
* Variation in Temperature
* Variation in Pull
* Personal Mistakes.


## Sources of Errors in Measurement

Depending on Sources of Origin, Errors in measurement fall into three classes, they are
> Natural Errors
> Instrumental Errors
> Personal Errors

## Natural Errors

These are caused due to variations in nature i.e., variations in wind, temperature, humidity, refraction, gravity and magnetic field of the earth

## Instrumental Errors

These result from imperfection in the construction or adjustment of surveying instruments, and movement of their individual parts.

## Personal Errors

These arise from limitations of the human senses of sight, hearing and touch in manipulating instruments.

## Kinds of Errors

Ordinary errors met with in all chances of survey work may be classified as Mistakes

Mistakes are errors which arise from inattention, inexperience, carelessness and poor judgment or confusion in the observer mind.

## Systematic Errors

A systematic error or cumulative error is an error that, under the same condition, will always be of the same size and sign.

## Accidental Errors

Accidental errors or compensating errors are those which remain after mistakes and systematic errors have been eliminated and are caused by a combination of reasons beyond the ability of the observer to control.

## Error in Measurement of Distance

Depending on the nature, errors present in the measurement of distance have been classified into two types: Systematic error and random error.

1. Systematic Errors
2. Random Errors

## Systematic Errors

Systematic errors (in taping) are caused due to: non-standard length of tape, slope in terrain, and variations in temperature during measurement, variations in tension, sag, incorrect alignment of the tape etc.

## Slope Correction

Let s represents the slope distance between two points A and B , h be the difference in elevation and $H$ the horizontal distance between the points, all in the same units. Let q be the slope of the terrain. Then, the horizontal distance is


Figure 5: Distance Measurement on Sloping Ground

And thus, slope correction for measured distance s is
$\mathrm{Ch}=\mathrm{H}-\mathrm{s}=(\mathrm{s} 2-\mathrm{h} 2)-\mathrm{s}($ always subtractive $)$

## Correction for Temperature

If the temperature of standardization is T0 degree and measurements are taken at a temperature of T degree, there is an error in length of the tape, due to difference in standardized temperature and temperature during observation. The corresponding correction Ct is given by

$$
\mathrm{Ct}=\mathrm{a} \mathrm{~L}(\mathrm{~T}-\mathrm{T} 0)
$$

Where,
a is the coefficient of thermal expansion of the material of the tape and Lis the measured distance

## Correction for Tension

If a tape is standardized at a tension of Po and measurements are taken at a tension of P , the correction Cp for change in length per distance L due to difference in standarised pull and actual pull during observation is given by

$$
C_{p}=\frac{\left(P-P_{0}\right) L}{a E}
$$

Where,
a is the cross sectional area of the tape and
$E$ is modulus of elasticity of the material of the tape.

## Correction for Sag

(Figure 9.2) When the tape sags between points of support, it takes the form of a catenary. The correction for sag Cs is always subtracted from observed distance and is given by

$$
c_{s}=\frac{w^{2} L}{24 P^{2}}
$$



Figure 6: Sagging in Distance Measurement

Where,
W is the total weight of tape between supports,
L is the distance between supports and
P is the applied tension.

## Correction for Incorrect Alignment

(Figure) If the intermediate points are not in correct alignment with the ends of the line or they are not on the line to be measured, a correction, Ch known as correction for alignment has to be subtracted to measured distance. It is given by

$$
C_{h}=\frac{d^{2}}{2 L}
$$



Figure 7: Misalignment in Distance Measurement
Where,
$d$ is the perpendicular distance by which the other end of the tape of length $L$ is out of alignment.

1. A tape was exactly 30 m long at $20^{\circ} \mathrm{C}$ when place on the flat under a pull of 75 N . a survey line was measured with this tape under a pull of 120 N and found to be 810 m . the average temperature during the measurements was $30^{\circ} \mathrm{C}$. If the tape was supported in spans of one tape length each time, determine the corrected length of the tape is $7.8 \times 10^{-5}$ $\mathrm{n} / \mathrm{mm}^{3}$. The modulus of elasticity of the material of the tape is $2.1 \times 10^{-5} \mathrm{n} / \mathrm{mm}^{2}$. The coefficient of thermal expansion of the materials of the tape is $11.7 \times 10^{-6} /{ }^{\circ} \mathrm{C}$.

## Solution,

From the measured length of 810 m , the following results can be calculated

## Correction to the temperature

$$
\begin{aligned}
\mathrm{C}_{\mathrm{t}}=\mathrm{L} \alpha\left(\mathrm{t}_{\mathrm{m}}-\mathrm{t}_{0}\right) & =810 \times 11.7 \times 10^{-6} \times(30-20) \\
& =0.09477 \mathrm{~m}(+\mathrm{ve})
\end{aligned}
$$

Correction for Pull

$$
\begin{aligned}
\mathrm{C}_{\mathrm{p}}=\frac{P . L}{A . E} & =\frac{(120-75) \times 810}{4 \times 2.1 \times 10-5} \\
& =0.043392857 \mathrm{~m}(+\mathrm{ve})
\end{aligned}
$$

## Corrections

for Sag

$$
\begin{aligned}
\mathrm{C}_{\mathrm{S}}=\quad \frac{L}{24} \times\left(\frac{W}{P} \quad\right)^{2} & = & \frac{810}{24} \times\left(\frac{30 \times 1000 \times 7.8 \times 10-5}{120}\right)^{2} \\
& = & 0.205335 \mathrm{~m}(-\mathrm{ve})
\end{aligned}
$$

Hence the corrected length of the survey line is $=0.09477 \quad+0.043392857-0.205335$

$$
801+0.09477+0.043392857
$$

$$
=\quad 0.205335 \quad=\quad 809.933 \mathrm{~m}
$$

## Error Due to Incorrect Chain

If the length of the chain used in measuring length of the line is not equal to the true length or the designated lengths, the measured length of the line will not be correct and suitable correction will have to be applied.

## 1) Correction to measured length

Let $l^{\prime} \quad-\quad$ Measured length of the line
L - Actual or true length if the line
L - True or designated length of the chain or tape
L' - Incorrect length of the chain or tape
Then the true length of the line $\quad \mathbf{l}=\mathbf{l}^{\prime}\left(\frac{L^{\prime}}{L}\right)$

## 2) Correction to Area

Let $\quad A^{\prime} \quad-\quad$ Measured area of the ground
A - Actual Area of the ground
Then the true length of the line $\mathbf{A}=\mathbf{A}^{\prime}{\left(\frac{L^{\prime}}{L}\right.}^{2}$

## Problem

1. The length of a line measured with a 30 meters chain was found to be 425 meters.

Calculate the true length of the line if the chain was 10 Cm too long.

## Solution

Incorrect length of the chain $=\quad L^{\prime}=30+\left(\frac{10}{100}\right)=\mathbf{3 0 . 1 ~ m}$
Measured length $=\quad l^{\prime}=425 \mathrm{~m}$

$$
\text { Hence true length of the line }=1^{\prime}=425\left(\frac{30.1}{30}\right)=426.4167 \mathrm{~m}
$$

From the relation
Hence true length of the line $=\mathbf{l}=\mathbf{l}^{\prime}\left(\frac{L^{\prime}}{L}\right)$

$$
\begin{aligned}
1506 & =1495\left(\frac{L^{\prime}}{30}\right) \\
& =\mathbf{3 0 . 2 2} \mathbf{~ m}
\end{aligned}
$$

Hence $L^{\prime}$ is greater than $L$. so the chain is too long.
Amount of error $\mathrm{e}=30.22-30=+\mathbf{0 . 2 2} \mathbf{~ m}$
3. On a map draw to a scale of 50 m to 1 Cm , a surveyor measured the distance between two stations as $3,500 \mathrm{~m}$. But it found that by mistake he had used a scale of 200 m to 1 Cm . Find the true distance between the stations.

Solution
As the surveyor used the scale of 100 m to 1 Cm
Distance between stations on map $=\frac{3500^{\prime}}{100}=35 \mathrm{Cm}$
As the actual scale of map is 50 m to 1 Cm
True Distance on the ground $=35 \times 50=1,750 \mathrm{~m}$

## Compass Surveying

## Compass Surveying

It is one of the methods used in traversing
In Compass survey chain or tape is used for linear measurements and compass is used for fixing direction.

In compass freely suspended magnetic needle directs to north- south and the bearing of line is obtained by line of sight.

## Traversing

Traversing is that type of survey in which a number of connecting survey lines form the frame work and the directions and lengths of the survey lines are measured with the help of an angle measuring instrument and a tape respectively.

When the lines form a circuit which ends at the starting point, it is known as a closed traverse.

If the circuit ends elsewhere, it is said to be an open traverse

## Types of Compass

The surveying compass can be classified in to two types,
Prismatic Compass
Surveyor's Compass
Trough Compass
Tubular Compass

## Prismatic Compass



The prismatic compass consists of circular box about 100 mm in diameter. The magnetic needle is pivoted on a hard steel pivot, an aluminum ring, graduated to degrees and half degrees is attached to the needle. The object vane carries a vertical hair of the sink thread attached to a suitable frame. The sight vane consists of vertical slit cut into the upper assembly of the prism.

The object vane provided with a hinge mirror which can be raised upwards or lowered downwards, to sight the objects too high or too low.

## Surveyor's Compass

Surveyor compass acquires its name from its extensive use by surveyors. But the prismatic compass has now replaced as it is light, compact and handy. It is similar in construction to the prismatic compass except for a few differences.

## Principles of Prismatic compass

The prismatic of compass surveying is traversing, which involves a series of connected lines.

Compass surveying is recommended
A large area to be surveyed
$>$ The course of river or coast line is to be surveyed and
The area is crowded with many details and triangulation is not possible.

## Terminology

Magnetic Meridian


## True Meridian or Geographical meridian (GN)

The line passing through the geographical North Pole, geographical South Pole and any point on the surface of the earth is known as the true meridian or geographical meridian.

## Magnetic Meridian

When a magnetic needle is suspended freely and balanced properly, unaffected by magnetic substances, it indicates a direction. This direction is known as magnetic meridian.

## Arbitrary meridian

A convenient direction is assumed as a meridian is known as arbitrary meridian.

## Whole circle bearing

The magnetic bearing of a line measured clockwise From the North Pole towards the line is known as the whole circle bearing

WCB
$0^{\circ}$ to $\mathbf{9 0}^{\circ} \quad$ - First Quadrant
$90^{\circ}$ to $180^{\circ}$ - Second Quadrant West $\mathbf{1 8 0}^{\circ}$
to $\mathbf{2 7 0}{ }^{\circ}$ - Third Quadrant $270^{\circ}$ to $\mathbf{3 6 0}^{\circ}$ -
Fourth Quadrant


When the Whole circle bearing of line is converted to quadrilateral, it is termed as Quadrilateral Bearing. It is termed the reduced bearing

| Bearing | Converting W.C.B in to R.B | Quadrant |
| :---: | :---: | :---: |
| $0^{\circ}$ to 90 ${ }^{\circ}$ | R.B = W.C.B | NE |
| $90^{\circ}$ to $180^{\circ}$ | R.B $=180^{\circ}$ - W.C.B | SE |
| $180^{\circ}$ to $270^{\circ}$ | R.B = W.C.B - $180^{\circ}$ | SW |
| $270{ }^{\circ}$ to $360^{\circ}$ | R.B $=360^{\circ}$ - W.C.B | NW |
| Bearing | Converting R.B in to W.C.B | Quadrant |
| $0^{\circ}$ to $90{ }^{\circ}$ | W.C.B = R.B | NE |
| $90^{\circ}$ to $180^{\circ}$ | W.C.B $=180^{\circ}-\mathrm{R} . \mathrm{B}$ | SE |
| $180^{\circ}$ to $270^{\circ}$ | W.C.B $=180^{\circ}+\mathrm{R} . \mathrm{B}$ | SW |
| $270^{\circ}$ to $360^{\circ}$ | W.C.B $=360^{\circ}-\mathrm{R} . \mathrm{B}$ | NW |

## Fore Bearing

The bearing of a line measured in the direction of the progress of survey line is called Fore Bearing

## Back Bearing

The bearing of a line measured in the direction opposite to the survey line is called the Back bearing

## Magnetic Declination



The horizontal angle between the magnetic meridian and true meridian is known as magnetic declination.

## Local Attraction

A magnetic needle indicates the north direction when freely suspended or pivoted. But if the needle comes near some magnetic substances such as iron, steel structures, electric cable, etc. it is found to be deflected from its true direction and does not show the actual north. This disturbance influence of magnetic substances is known as local attraction.

## Difference between Surveyor's and Prismatic Compass

| S.No | Item | Prismatic Compass | Surveyor Compass |
| :---: | :--- | :--- | :--- |
| 1 | Magnetic Needle | The needle is of broad type, <br> does not acts as index | The needle is of edge bar <br> type, acts as index. |
| 2 | Graduated Card | Graduated card ring <br> is attached with the <br> needle and the ring <br> does not rotate along <br> with the lien of sight. | 1. Graduated card ring is <br> attached with the box needle <br> and not to the needle. The <br> card rotates along with the <br> lien of sight. <br> 2. Graduations are in R.B |
| 3 | Sighting Vanes | The object vane consist of <br> metal vane with vertical hair | The object vane consist of <br> metal vane with vertical hair |
| 4 | Reading | The reading is taken with <br> help of Prism. Sighting and <br> reading are <br> and done | The reading is taken with <br> top of the glass. Sighting <br> and reading are not done <br> simultaneously. |
| 5 | Tripod | Tripod may or may not be <br> used | It cannot be used without <br> tripod stand. |

## Temporary adjustment of any instrument

Setting Up
Leveling Up
Elimination of Parallax
Temporary adjustment of prismatic compass
Fixing the compass with tripod stand
Centering
Leveling
Adjustment of prism
Observation of bearing

## Problems

- Convert the following W.C.B into RB
i. W.C.B of AB
$=65^{\circ} 30^{\prime}$
ii. W.C.B of BC $=145^{\circ} 35^{\prime}$
iii. W.C.B of CD $=226^{\circ} 30^{\prime}$
iv. W.C.BOF DE $=310^{\circ} 20^{\prime}$

Answers

1. QB of $\mathrm{AB}=65^{\circ} 30^{\prime}$
2. QB of $\mathrm{BC}=180^{\circ}-145^{\circ} 35^{\prime}=34^{\circ} 25^{\prime}$
3. QB of $\mathrm{CD}=$ W.C.B. $-180^{\circ}=46^{\circ} 30^{\prime}$
4. QB of $\mathrm{DE}=360^{\circ}-310^{\circ} 20^{\circ}=49^{\circ} 40^{\prime}$

- Convert the following RB into W.C.B

1. QB of $\mathrm{AB}=\mathrm{N} 35^{\circ} 30^{\prime} \mathrm{E}$
2. QB of $\mathrm{BC}=\mathrm{S} 52^{\circ} 35^{\prime} \mathrm{E}$
3. QB of $\mathrm{CD}=\mathrm{S}_{2} 6^{\circ} 30^{\prime} \mathrm{W}$
4. QB of $\mathrm{DE}=\mathrm{N} 79^{\circ} 40^{\prime} \mathrm{W}$

## Answers

i. W.C.B of $\mathrm{AB}=$ Same $=35^{\circ} 30^{\prime}$
ii. W.C.B of $\mathrm{BC}=180^{\circ}-52^{\circ} 35^{\prime}=127^{\circ} 25^{\prime}$
iii. W.C.B of CD $=180^{\circ}+26^{\circ} 30^{\prime}=206^{\circ} 30^{\prime}$
iv. W.C.BOF DE $=360^{\circ}-79^{\circ} 40^{\prime}=280^{\circ} 20^{\prime}$

1. The magnetic bearing of a line is $58^{\circ} 30^{\prime}$. Calculate the rue bearing, if the magnetic declination is $5^{\circ} 30^{\prime}$.

## Solution

Declination $=5^{\circ} 30^{\prime}$.
Therefore true bearing is $=58^{\circ} 30^{\prime}+5^{\circ} 30^{\prime}=64^{\circ} 0,0^{\prime \prime}$

1. The magnetic bearing of a line AB is $\mathrm{S} 55^{\circ} 30^{\prime} \mathrm{E}$. Calculate the true bearing of the line, if the declination is $4^{\circ} 30^{\prime} \mathrm{W}$ respectively.

The positions of true meridian, magnetic and MM the lines have been shown in figure. $4^{\circ} 30^{\prime}$ Hence the true bearing

1. $55^{\circ} 30^{\prime}+4^{\circ} 30^{\prime}$
$=60^{\circ} 0^{\prime} 0^{\prime \prime}$

## Included Angle Problem



- The following are the bearings of a closed traverse using a prismatic compass.

Compute the included angle and the declination angles,

| Length | Fore Bearing | Back Bearing |
| :---: | :---: | :---: |
| AB | $37^{\circ} 30^{\prime}$ | $217^{\circ} 30^{\prime}$ |
| BC | $92^{\circ} 00^{\prime}$ | $272^{\circ} 00^{\prime}$ |
| CD | $151^{\circ} 30^{\prime}$ | $331^{\circ} 30^{\prime}$ |
| DE | $220^{\circ} 15^{\prime}$ | $40^{\circ} 15^{\prime}$ |
| EF | $283^{\circ} 00^{\prime}$ | $103^{\circ} 00^{\prime}$ |
| FA | $330^{\circ} 15^{\prime}$ | $150^{\circ} 15^{\prime}$ |



First Method to calculate included angle

| Line | Angle | Included Angle |
| :---: | :---: | :---: |
| AB | $\begin{aligned} & \mathrm{BB} \text { of } \mathrm{FA}-\mathrm{FB} \text { of } \mathrm{AB} \\ & 150^{\circ} 15^{\prime}-37^{\circ} 30^{\prime}=182^{\circ} 45^{\prime} \end{aligned}$ | $112^{\circ} 45^{\prime}$ |
| BC | BB of AB-FB of BC $217^{\circ} 30^{\prime}-92^{\circ} 00^{\prime}=125^{\circ} 30^{\prime}$ | $125^{\circ} 30^{\prime}$ |
| CD | $\begin{aligned} & \mathrm{BB} \text { of } \mathrm{BC}-\mathrm{FB} \text { of CD } \\ & 272^{\circ} 00^{\prime}-151^{\circ} 30^{\prime}=120^{\circ} 30^{\prime} \end{aligned}$ | $120^{\circ} 30^{\prime}$ |
| DE | BB of CD - FB of DE <br> $331^{\circ} 30^{\prime}-220^{\circ} 15^{\prime}=111^{\circ} 15^{\prime}$ | $111^{\circ} 15^{\prime}$ |
| EF | BB of DE-FB of EF $40^{\circ} 15^{\prime}-283^{\circ} 0^{\prime}+360^{\circ}=117^{\circ} 15^{\prime}$ | $117^{\circ} 15^{\prime}$ |
| FA | $\begin{aligned} & \text { BB of EF- FB of FA } \\ & 103^{\circ} 00^{\prime}-330^{\circ} 15^{\prime}+360^{\circ}=132^{\circ} 45^{\prime} \end{aligned}$ | $132^{\circ} 45^{\prime}$ |
|  | Sum of included Angles | $720^{\circ} 00^{\prime} 0^{\prime \prime}$ |

For the check, the sum of all angles is obtained as $720^{\circ} 00^{\prime}$ since this is an six sides traverse, the correct sum is $(2 \mathrm{n}-4) 90^{\circ}=720^{\circ} 00^{\prime}$. Therefore there does not have any error to be corrected.

- The bearing of the side AB of regular pentagon ABCDE was found to be $54^{\circ}$.

Compute the bearings of the remaining sides if the pentagon is run counterclockwise. Calculate the Fore bearing of the given traverse.

| Line | AB | BC | CD | DE | EA |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bearing | $54^{\circ}$ | $342^{\circ}$ | $270^{\circ}$ | $198^{\circ}$ | $126^{\circ}$ |


$\square$ The following are the fore and back bearing of lines observed in an unclosed traverse ABCDE.

| Line | FB | BB |
| :---: | :---: | :---: |
| AB | $65^{\circ} 30^{\prime}$ | $245^{\circ} 00^{\prime}$ |
| BC | $106^{\circ} 00^{\prime}$ | $286^{\circ} 00^{\prime}$ |
| CD | $220^{\circ} 45^{\prime}$ | $40^{\circ} 30^{\prime}$ |
| DE | $210^{\circ} 20^{\prime}$ | $30^{\circ} 00^{\prime}$ |

Locate the position of local attraction and find the corrected
bearings BC is free from Local Attraction

| BB of $\mathrm{AB}-180^{\circ}\left(245^{\circ} 00^{\prime}-180^{\circ}\right)$ | $=65^{\circ}$ |  |
| :--- | :--- | :--- |
| Corrected Bearing | $=65^{\circ} 30^{\prime}-65^{\circ}$ | $=0^{\circ} 30^{\prime}$ |
| Fore Bearing of CD will be correct $\left(220^{\circ} 45^{\prime}-180^{\circ}\right)$ | $=40^{\circ} 45^{\prime}$ |  |
| BB of CD - Corrected Bearing $\left(40^{\circ} 30^{\prime}-40^{\circ} 45^{\prime}\right)$ | $=-0^{\circ} 15^{\prime}$ |  |
| Corrected bearing of DE | $=210^{\circ} 20+0^{\circ} 15^{\prime}$ | $=210^{\circ} 35^{\prime}$ |
| Corrected Bearing | $=210^{\circ} 35^{\prime}-180^{\circ}$ | $=30^{\circ} 35^{\prime}$ |


| Line | FB | BB |
| :---: | :---: | :---: |
| AB | $65^{\circ} 00^{\prime}$ | $245^{\circ} 00^{\prime}$ |
| BC | $106^{\circ} 00^{\prime}$ | $286^{\circ} 00^{\prime}$ |
| CD | $220^{\circ} 45^{\prime}$ | $40^{\circ} 45^{\prime}$ |
| DE | $210^{\circ} 35^{\prime}$ | $30^{\circ} 35^{\prime}$ |

The following are the fore and back bearing of lines observed in an unclosed traverse ABCDE

| Line | Fore Bearing | Back Bearing |
| :---: | :---: | :---: |
| AB | $\mathrm{N} 45^{\circ} 45^{\prime} \mathrm{E}$ | $\mathrm{S} 46^{\circ} 10^{\prime} \mathrm{W}$ |
| BC | $\mathrm{S} 83^{\circ} 05^{\prime} \mathrm{E}$ | $\mathrm{N} 82^{\circ} 55^{\prime} \mathrm{W}$ |
| CD | $\mathrm{N} 29^{\circ} 45^{\prime} \mathrm{E}$ | $\mathrm{S} 29^{\circ} 10^{\prime} \mathrm{W}$ |
| DE | $\mathrm{N} 35^{\circ} 12^{\prime} \mathrm{W}$ | ${\mathrm{S} 35^{\circ}} 12^{\prime} \mathrm{E}$ |

At what station the local attraction is suspended? Determine the corrected bearing in the quadrilateral system.

## Solution

Converting reduced bearing to Whole circle bearing

| Line | Fore Bearing | Back Bearing |
| :---: | :---: | :---: |
| AB | $45^{\circ} 45^{\prime}$ | $226^{\circ} 10^{\prime}$ |
| BC | $96^{\circ} 55^{\prime}$ | $277^{\circ} 5^{\prime}$ |
| CD | $29^{\circ} 45^{\prime}$ | $209^{\circ} 10^{\prime}$ |
| DE | $324^{\circ} 48^{\prime}$ | $144^{\circ} 48^{\prime}$ |

We find that local attraction will be in DE line

## Solution

Hence line DE is Free from Local Attraction

| Calculation | Observed Bearing | Correction | Correction to be applied |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline \text { BB of line CD }-180^{\circ} \\ & 209^{\circ} 10^{\prime}-180^{\circ}=29^{\circ} 10^{\prime} \end{aligned}$ | $29^{\circ} 45^{\prime}$ | $\begin{gathered} 29^{\circ} 45^{\prime}-29^{\circ} 10 \\ =0^{\circ} 35^{\prime} \end{gathered}$ | $-0^{\circ} 35^{\prime}$ |
| BB of line $\mathrm{BC}-0^{\circ} 35^{\prime} \quad 277^{\circ} 5^{\prime}-0^{\circ} 35^{\prime}=276^{\circ} 30^{\prime}$ |  |  |  |
| $\begin{aligned} & \text { Corrected BB of line BC } \\ & 180^{\circ} \\ & 276^{\circ} 30^{\circ}-180^{\circ}=96^{\circ} 30^{\prime} \end{aligned}$ | $96^{\circ} 55^{\prime}$ | $\begin{gathered} 96^{\circ} 55^{\prime}-96^{\circ} 30^{\prime} \\ =0^{\circ} 25^{\prime} \end{gathered}$ | $-0^{\circ} 25^{\prime}$ |
| BB of line $\mathrm{AB}-0^{\circ} 25^{\prime} \quad 226^{\circ} 10^{\prime}-0^{\circ} 25^{\prime}=225^{\circ} 45^{\prime}$ |  |  |  |
| $\begin{array}{\|l\|} \hline \mathrm{BB} \text { of line AB }-180^{\circ} \\ 225^{\circ} 45^{\prime} 180^{\circ}=45^{\circ} 45^{\prime} \end{array}$ | $45^{\circ} 45^{\prime}$ | Hence Solved |  |

## Answer

Hence the bearings are

| Line | Fore Bearing | Back Bearing |
| :---: | :---: | :---: |
| AB | $45^{\circ} 45^{\prime}$ | $225^{\circ} 45^{\prime}$ |
| BC | $96^{\circ} 30^{\prime}$ | $276^{\circ} 30^{\prime}$ |
| CD | $29^{\circ} 10^{\prime}$ | $209^{\circ} 10^{\prime}$ |
| DE | $324^{\circ} 48^{\prime}$ | $144^{\circ} 48^{\prime}$ |

## Methods of traversing

Chain Traversing
Chain and Compass Traversing
$>$ Transit tape Traversing
By Fast needle method
By measurement of angles between the lines
$>$ Plane table Traversing

## Chain Traversing

In this method the whole of the work is done with the chain and tape. No angle measuring instrument is used and the directions of the lines are fixed entirely by linear measurements. Angle fixed by linear or tie measurements are known as chain lines.

## Chain and Compass Traversing (loose needle method or free needle method)

In chain and compass traversing the magnetic bearing of the survey lines are measured by a compass and the lengths of the lines are measured either by chain or tape. The direction of the magnetic meridian is established at each traverse station independently. This method is also known as loose needle method or free needle method

## Check in the Close Traverse

The sum of the measured interior angles should be equal to $(2 \mathrm{~N}-4) * 90^{\circ}$
The sum of the measured Exterior angles should be equal to $(2 \mathrm{~N}+4) * 90^{\circ}$
The algebraic sum of deflection angles should be equal to $360^{\circ}$
The sum of the northing should be equal to that of southing, and sum of Easting should be equal to westing.

## Check in the Open Traverse

In the open traverse measurement cannot be checked in the field.

## Instrumental Errors

They are those which arise due to the faulty adjustments of the instruments. They may be due to the following reason,

- The needle not being perfectly straight
- Pivot being bent
- Sluggish needle
- Blunt pivot point
- Improper balancing weight
- Plane of sight not being vertical
- Line of sight not passing through the centers of the right


## Personal Errors

p

## Natural Errors

Variation in declination
Local attraction due to proximity of local attraction forces
Magnetic changes in the atmosphere due to clouds and storms.
Irregular variations due to magnetic storms etc.

## Plane Table Surveying

## Basics of Plane Table Surveying

A plane table surveying is a graphical method of surveying. In this method of surveying, field observation and plotting are done simultaneously thus helps surveyor to compare the plotted details with actual features of the ground.

## Advantages of Plane Table Surveying

It is suitable for location of details as well as contouring for large scale maps directly in the field.

- As surveying and plotting are done simultaneously in the field, chances of getting omission of any detail get less.
- The plotting details can immediately get compared with the actual objects present in the field. Thus errors as well as accuracy of the plot can be ascertained as the work progresses in the field.

Contours and specific features can be represented and checked conveniently as the whole area is in view at the time of plotting.
-
Only relevant details are located because the map is drawn as the survey progresses. Irrelevant details get omitted in the field itself.

- The plane Table survey is generally more rapid and less costly than most other types of survey.
- As the instruments used are simple, not much skill for operation of instruments is required. This method of survey requires no field book.


## Disadvantages of Plane Table Surveying

The plane table survey is not possible in unfavorable climates such as rain, fog etc. This method of survey is not very accurate and thus unsuitable for large scale or precise work.

As no field book is maintained, plotting at different scale require full exercise.

- The method requires large amount of time to be spent in the field.

Quality of the final map depends largely on the drafting capability of the surveyor.

This method is effective in relatively open country where stations can be sighted easily

## Instruments

A plane table mounted on a tripod stand and a number of accessories are being used during plane table survey. The accessories consist of alidade, spirit level, trough compass, plumbing fork, plumb bob, drawing sheet.

## Plane Tables

The plane table consists of a drawing board with arrangement for fixing on a tripod stand. There are various types of plane tables, depending upon the arrangement of fixing the boards to the tripod, leveling of the table and rotating arrangement in a horizontal plane. Figure shows a simple plane table.



#### Abstract

Alidade Alidade, Telescopic Alidade and digital alidade.

Three Types of Alidade Plain Alidade

^[ $\checkmark$ Telescopic Alidade $\checkmark$ Digital Alidade ]


An alidade is a device in which the vertical plane of the line of sight is maintained parallel to a straight-edge ruler on which the sighting arrangement is kept. It is used to draw a line parallel to the line of sight and thus provides the direction of the object to be plotted. Depending on the type of sighting arrangement, alidades are classified as Plain

## Plain Alidade

It consists of a straight-edge ruler, made of a metal or wood, with one of the edges is bevelled and graduated known as fiducial edge. It consists of two vanes in perpendicular as its ends, fitted with hinges at their bases, known as sight vanes. These are kept folded down on the ruler when not in use. One of the sight vanes is provided with a narrow slit having three holes.


## Plain Alidade

This is used as eye vane. The other, used as object vane, is open and carries a hair or thin wire at its centre. Thus, the line passing through the slit of the eye vane joining the thin wire of the object vane and passing beyond is known as the line of sight of a plane alidade. A string is fitted at the top of the sight vanes and is used for inclined sight. In some alidade, a compass needle as well as a spirit level gets fitted in a box engraved at its base. However, the plain alidade is not very accurate. Figure show a plain alidade.

## Telescopic Alidade

It consists of a telescope as an arrangement for sighting (similar to that present in the upper part of a theodolite). The telescope is fitted with a stadia diaphragm and can be used as tacheometer also for computations of horizontal distance and vertical elevations. The line of sight of the telescope is aligned along the fiducial edge. In this instrument, the object is sighted through the telescope and the distance is scaled off in that direction along the fiducial edge. Figure shows a telescopic alidade.


Telescopic Alidade

The telescopic alidade is designed for greater precision and longer range of sights. It can be used with advantages for contouring and plotting of details during topographic surveying.

## Digital Alidade

It consists of an EDM, with a built-in telescope for sighting, an automatic angle sensor for registering vertical angle and a microcomputer for yielding horizontal distance and difference in elevation. It also consists of a liquid crystal display which is used to display and thus read and retrieve the observer and calculated parameters. Digital alidade is particularly useful for accurate plotting of detail and for the long line of sight.

## Plumbing Fork

A plumbing fork is a U-shaped piece of metal or wooded frame (Figure 33.4). The end of one of its arm is pointed and the other arm is having an arrangement for hanging a plumb bob. The frame is constructed in such away that the tip of the pointed arm and the plumb line lie in the same vertical line.


At the time of use, the pointed arm is placed on the table and the other arm, with a plumb bob attached, is kept below the table. Plumbing fork with a plumb bob is used in large scale surveying for Centering of plane table and for Transferring of ground point.

## Spirit Level

It consists of flat based tube with a small bubble either a circular or tubular in shape. It is used to check the level of plane table by placing it on the board in two positions at right angles to each other. When the bubble tube remains in the centre all over the table, the table is considered to be properly leveled.


Spirit level

## Trough Compass

A trough compass consists of a long, narrow rectangular box, covered with glass. Inside the box, at its centre, there is a magnetic needle resting on the pivot. At the extremities of the trough compass, there are graduated scales with zero at the centre and marking up to $5^{\circ}$ on either side of the zero line. Figure 33.6 shows a Trough compass.


## Trough Compass

The trough compass is used for marking the magnetic north line on the drawing sheet of the plane table. In this case, the magnetic needle point to $0^{\circ}-0^{\circ}$ of the graduated scale and a line drawn parallel to the edge of the trough compass is along the magnetic meridian. A trough compass is also used to orient the plane table with respect to the magnetic meridian.

## Drawing Arrangements

A drawing paper of good quality and well-seasoned is to be used for plotting. It should be able to withstand the rubbing of the alidade. The effect of changes in humidity and temperature of the atmosphere should be minimum thus reduction in the expansion or contraction of the paper, and alter the scale of the map and distort the plan.

A sharp, hard $(4 \mathrm{H}$ or 6 H$)$ pencil is to be used for drawing lines. Soft pencils are used for lines to be erased.
$>$
A good quality rubber eraser is used for erasing the lines.

## Basic Principle of Plane Table Surveying

Plane table surveying is based on the principle that lines drawn during plotting always lie parallel to the corresponding lines actually present on the ground.

For example, let us consider four ground stations A, B, C and D which on joining provides a rectangle ABCD . This has been plotted on a sheet of paper at a scale by plane table surveying. Here, the sides $\mathrm{AB}, \mathrm{BC}, \mathrm{CD}$ and DA are plotted in such a way that they are parallel to the sides actually available on the ground.

## Setting of Instrument

At each station, the plane table is required to get set up before carrying out any plotting work. It basically consists of the three operations: fixing, centering and leveling.

## Fixing

In this operation, first the top of the tripod stand is fixed in level by eye estimation at convenient height with its legs uniformly spread and shoes fixed firmly into the ground. The board is fixed to the tripod head by tightening the clamping screw.

## > Centering

There are two types of operations involved in centering of plane table.
. The centering with reference to already plotted position of the station. In this case, the pointed end of the upper arm of a U -fork is hold at the plotted position of the station and the table is shifted and adjusted in such a way that the tip of the plumb bob points exactly over the ground station.

The centering with reference to ground location of the station. In this case, the tip of the plumb bob attached to a U-frame is placed exactly over the ground point which is required to be transferred. The point corresponding to the tip of the pointed arm of the U-frame provides the transferred position on the table.However, the accuracy with which centring is being carried out depends upon the scale of plotting.

* Levelling

The top of the table is leveled by moving the legs of the tripod. The level of plane table is first judged by eye estimation. Further, it is checked by keeping spirit level at different positions on the table and if required, legs are further adjusted.

## Temporary Adjustment of Plane Table

It is necessary to check whether the accessories satisfies some basic conditions and if required, necessary adjustments are to be done before starting any plane table surveying work. The operations involved in this are known as temporary adjustment of plane table. The conditions needs to be tested and subsequent adjustments are as follows:

The surface of the board should be a perfectly plane.
Test: It is tested by placing a straight edge on the top surface of the plane table in different directions. If there is no gap between the base of the straight edge and the surface of the plane table then the surface is perfectly plane. Otherwise, the surface is not perfectly plane.

Adjustment: If the gaps are minute, those are removed by rubbing with sand paper and for more gaps, the table should be replaced.

The fiducial edge of the alidade should be straight.
Test: It is tested by drawing a fine line on the paper along the fiducial edge of the alidade. Then, by reversing the alidade, end for end, and placing against the line drawn, a line is to be drawn again along the fiducial edge. If the two lines coincide, the edge is straight. Otherwise, the edge is not straight.

Adjustment: The fiducial edge of the alidade is too made straight by filing and then test is repeated till satisfactory outcome.
$\square$ In fully opened condition, the sight vanes of the alidade should be perpendicular to its base.

Test: Hang a plumb bob at a distance of about 5 to 10 m from the plane table. Bisect the string of the plumb bob through the alidade placed on properly leveled plane table. If the sighting slit, the object vane hair and the plumb bob string lie the same vertical line, the vanes of the alidade are perpendicular to the base of the alidade. Otherwise, it requires adjustment.

Adjustment: is being carried out by inserting packing under the base of the sight vanes or by filing the base, as required. The test and adjustment get repeated till satisfactory outcome is achieved.
4. The telescopic alidade if used should be in perfect adjustment.

## Orientation of Plane Table

The objective of this operation is to maintain the orientation of the table constant at all the stations in any particular plane table surveying i.e., the four edges of the plane table will always be in the same directions at all the stations. Thus, all lines plotted on the plane table sheet will maintain parallel to their corresponding lines on the ground.

During orientation, the leveling of the plane table generally gets disturbed so it is usually carried out with leveling simultaneously iteratively. The orientation of plane table can be carried out by:

Using a
$\square$ Trough compass;
$\square$ Back sighting;
$\square$ Resection.

## Orientation of Trough compass

In this method, the edge of the trough compass is placed along the magnetic meridian (drawn at the starting station) and the plane table is rotated till the needle points to zero-zero of the scale. Once it is achieved, the table is said to be oriented and thus clamped. This method of orientation is not very accurate and also may get affected by local attraction and is generally used for small-scale survey.

## Orientation of Back sighting

In this method, the fiducial edge of alidade is laid along a ray drawn from previous station to the present station and the plane table is then rotated till the line of sight of alidade bisects exactly the ranging rod placed at previous station. The plane table is then clamped and said to be oriented. In this method, the level of the plane table has to be maintained identical in both the stations.

## Orientation of Resection

This method of plane table surveying is employed to locate and plot the position of the plane table during surveying. This also results in the orientation of the plane table. The basic principle of resection is opposite to that of the method of intersection. In this method, the position of the plane table is determined by drawing resectors from already
plotted points. There are different methods for locating plane table by method of resection and are primarily based on the type of orientation which precedes resection.

## Methods of Plane Table

Different operations are involved during the location of details through plane table surveying. To carry out the operations under different field conditions, different methods of plane table surveying have been evolved - method of radiation, method of intersection, method of traversing and the method of resection. The method radiation and the method of intersection are employed to locate objects and features present in the area of survey; the method of traversing is used to plot the network of stations and the method of resection is employed to determine and to plot the location of the plane table as well as to orient the table simultaneously.

## Radiation Method

In the radiation method of plane table surveying, the direction of the objects or points to be located are obtained by drawing radial lines along fiducial edge of alidade after getting the objects or points bisected along the line of sight of the alidade. The horizontal distances are then measured and scaled off on the corresponding radial lines to mark their positions on the drawing.


## Radiation Method of plane table surveying

## Procedure

Let Figure 34.1 be considered in which a plane table is set up at station X and details in the area got plotted by method of radiation. In order to carry out this, first the plane table is set over the station X, clamped and its position is plotted on paper as x . Now, with alidade pivoted at x , salient object points present in the area such as $\mathrm{A}, \mathrm{B}, \mathrm{C}$, D etc of a building around the plane table got bisected and radial lines are drawn showing their directions. The corresponding field distances XA, XB, XC, XD are measured and scale off on respective radial lines. Thus, plotted their positions as $a, b, c, d$ etc. The plotted positions are then joined to represent the object. In this way, points e, f, g, h are
also plotted and joined to get another building corner. The location of a telephone line pillar T plotted as t .

The radiation method of plane table surveying is suitable for locating objects which are within a single tape length (from the location of plane table). The method is convenient if telescopic or digital alidade is used. Otherwise, it is effective when associated with tacheometer or EDM for measurement of horizontal distance.

## Intersection Method

In intersection method of plane table surveying, the objects or points to be located are obtained at the point of intersection of radial lines drawn from two different stations.


## Intersection method of plane table surveying

## Procedure

Let us consider plane table stations Z and Y Figure from which details got plotted by method of intersection. In this, first the plane table is set over the station Z , clamped and its position is plotted on paper as z . If the position of the station is not already plotted, then the station Y is bisected and plotted by method of radiation. Now, with alidade pivoted at z , salient object points on the surface of earth such as $1,2,3$ etc of a building, 4 a telephone pillar, 5 a tree etc around the plane table got bisected and radial lines are drawn showing their directions. The table is then shifted to the station Y nd get it set and after orienting by backsighting to station Z get it clamped. With alidade pivoted at y , same objects i.e., $1,2,3,4,5$ etc. get bisected and rays are drawn. The intersection of the respective rays provides the plotted positions of the respective details. The plotted positions of 1,2,3 are then joined to represent the building corner. Telephone pillar and tree are represented by their conventional symbols.

In this method, the plotting of plane table stations are to be carried out accurately. Checking is important and thus done by taking third sight from another station.

The intersection method is suitable when distances of objects are large or cannot be measured properly. Thus, this method is preferred in small scale survey and for mountainous regions.

## Plane Table Traversing

This method of plane table surveying is used to plot a traverse in cases stations have not been previously plotted by some other methods. In this method, traverse stations are first selected.

The stations are plotted by method of radiation by taking back sight on the preceding station and a fore sight to the following station. Here distances are generally measured by tachometric method and surveying work has to be performed with great care.


Plane table traversing

## Procedure

Let us consider the stations $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and S Figure which are to plotted by method of traversing. Stations are to be chosen in such a way that adjoining stations are visible. First, the plane table is to be set at station P and then plotted as p . The orientation of the table and scale of plotting should be such that all other stations will be accommodated within the boundary of the sheet. With the alidade pivoted at p , draw the rays to Q and S . Distances PQ and PS are measured and plotted on the respective rays, pq and ps respectively.

The plane table is then shifted to station Q , get it set and then oriented by back sighting to station P. With the alidade pivoted at q, draw a ray to R. Distance QR is measured and plotted on the ray as qr. In this way, plane table is shifted to stations R and S and corresponding rays are drawn to obtain the plotting of the traverse pqrs.

Check lines. To check the accuracy of the plane table traverse, a few check lines are taken by sighting back to some preceding station. In this example, a check line rp is drawn from the station R to P when the plane table is occupying the station R . If the traverse is correct, the check line rp would pass through p , the plotted position of station P. Likewise, a check line sq is drawn from $S$ to $Q$. In case there is no suitable preceding station visible, any well-defined point, such as a corner of a building or a tree, which has been previously plotted, can be used for checking.

Error of closure. If the traverse to be plotted is a closed traverse, the foresight from the terminating station should pass through the first station. Otherwise the amount by which plotted position of the first station on the foresight fails to close is designated as the error of closure. It is adjusted graphically, if the error is within permissible limits, before any further plotting works are done.

## Three Point Problem

In this method, three well defined points, having locations already being plotted on the drawing are involved. These are used to find and subsequently plot the location of the plane table station.


Principle of Three - point problem Solution
The method is based on the fact that, in a correctly oriented plane table, resectors through well defined points get intersected at a point which represents the location of the plane table station on the drawing. For example, as shown in Figure 35.1, if X, Y and Z are well defined objects present in the field whose plotted positions are $\mathrm{x}, \mathrm{y}$ and z . Now, if the plane table is oriented correctly, the three resectors $\mathrm{Xx}, \mathrm{Yy}$ and Zz get intersected at p which represents the location of the plane table station, P on the drawing sheet. Thus, through solution of three-point problem, both orientation and resection of plane table gets accomplished simultaneously.

There are several methods for solution of the three point problem: (i) trial and Error method, (ii) mechanical method, (iii) graphical method, (iv) Analytical method and 1. geometrical construction method. Of these, the trial and error method is easy, quick and accurate. It is commonly used in practice and hence, has been discussed in detail.

## Trial and Error method

In three point problem, if the orientation of the plane table is not proper, the intersection of the resectors through the three points will not meet at a point but will form a triangle, known as triangle of error Figure. The size of the triangle of error depends upon the amount of angular error in the orientation.


Triangle of Error

The trial and error method of three point problem, also known as Lehman's method minimises the triangle of error to a point iteratively. The iterative operation consists of drawing of resectors from known points through their plotted position and the adjustment of orientation of the plane table.

The estimation of location of the plane table depends on its position relative to the well defined points considered for this purpose. Depending on their relative positions, three cases may arise:

1. The position of plane table is inside the great triangle;
2. The position of plane table is outside the great triangle;
3. Tinelp.osition of plane table lies on or near the circumference of the great

In case of (iii), the solution of the three-point problem becomes indeterminate or unstable. But for the cases (i) and (ii), Lehmann,s rules are used to estimate the location of plane table.

## Leveling

The aim of leveling is to determine the relative heights of different objects on or below the surface of the earth and to determine the undulation of the ground surface. Uses of leveling

- To prepare a contour map for fixing sites for reservoirs, dams, barrages, etc. and to fix the alignment of roads, railways, irrigation canals, and so on.
- To determine the altitudes of different important points on a hill or to know the reduced levels of different points on or below the surfaces of the earth.
- To prepare a longitudinal section and cross sections of a project in order to determine the volumes of earth.
- To prepare a layout map for water supply, sanitary or drainage scheme



## Level Surface

A level surface is defined as a curve surface which at each point is perpendicular to the direction of gravity the point.

## Level Line

A line lying in a level surface is known as level line.

## Horizontal line

A horizontal plane through a point is the plane normal to the direction of gravity at that point. It is tangential to the level surface at the point.

## Vertical line

The vertical line at a point is the direction of gravity (i.e., towards the centre of the earth) as indicated by a plumb line.

## Datum

Datum is any surface to which elevations are referred the mean sea level afford a conveinient datum world over and elevations are commonly given as so much above or below sea level.

## Mean Sea Level

Mean sea level is the average height of the sea for all stages of the tides. At any particular place it is derived by averaging the hourly tide heights over a long period of 19 years.

## Reduced Level (RL)

The vertical distance of a point above or below the datum line is known as the reduced level of the point

## Line of collimation

It is the imaginary line passing through the intersection of the cross hair at the diaphragm and the optical centre of the object glass and its continuation. It is also known as the line of sight.

## Height of instrument

When the leveling instrument is properly leveled. The RL of the line of collimation is known as the height of instrument. This is obtained by adding the BS reading to the RL of the BM

## Bench Mark

Bench mark is a relatively permanent point reference whose elevation with respect to some assumed datum is known.
There are four type of bench mark
GTS (Great trigonometrical Survey)
$\checkmark$ Permanent Bench Mark
$\checkmark$ Arbitrary Bench Mark
$\checkmark$ Temporary Bench Mark

## GTS (Great trigonometrical Survey)

GTS bench marks were established throughout the country to a high degree of precision by the survey of India during their Great trigonometrical Survey. Their plan positions and elevations with respect to the standard datum- mean sea level at Karachi. Are tabulated in the catalogue published by them.

## Permanent Bench Mark

Permanent bench marks are fixed reference points established by the government organization or agencies such as the Public Works Department in certain states from the standards bench mark. Conspicuous points on culverts, bridges, and buildings are normally chosen.

## Arbitrary Bench Mark

Arbitrary Bench Mark are reference points whose elevations are Arbitrary assumed.

## Temporary Bench Mark

Temporary Bench Mark is established at the end of a day's work so that it can be continued later with reference o them. Firm points such as parts of permanent objects are used. (Man Made)

## Fore Sight

The last sight taken before shifting the instrument to a different station is foresight.

## Back Sight

The first sight taken after setting up the instrument at a point is the back sight.

## Intermediate sight

Once the back sight is taken at any instrument station, the surveyor is bale to determine the level of any number of points of interest within range before the fore sight is taken on a chosen point and the instrument is shifted.

## Change Points

A staff station on which a fore sight before shifting the instrument is called a change Points or turning point.

## Leveling instrument

The instruments used for leveling are as follows

1. Dumpy Level

The telescope of the dumpy level is rigid fixed to its supports, it cannot be removed from its support nor can it be rotated about its longitudinal axis. The instrument is stable and retains its permanent adjustment for a long time.
2. Wye level

The telescope is held in two ' Y ' supports. It can remove from the support and reversed from one end of the telescope to be other end. The ' $Y$ ' support consists of two curved clips which may be raised. Thus the telescope can be rotated about its longitudinal axis.

## 3. Modern Tilting Level

The telescope can be tilted slightly about its horizontal axis with the help of a tilting screw. In this instrument the line of collimation is made horizontal for each observation by means of the tilting screw.
4. Automatic level

This is also known as the self-aligning level. This instrument is leveled automatically within a certain tilt range by means of a compensating device

## Description of Dumpy Level



## Tripod Stand

The tripod stand consists of three legs which may be solid or framed. The legs are made of light and hard wood. The lower ends of the legs are fitted with steel shoes.

## Diaphragm

The diaphragm is a brass ring fitted inside the telescope just in front of the eye piece. The ring carries the cross hair, which get magnified when viewed through eye piece.

## Leveling Head

The leveling head consists of two parallel triangular plates having three grooves to support the foot screw.


Patterns of cross-hairs used in telescopes

## Bubble tube

Two bubble tubes, one called the longitudinal bubble tube and other the cross bubble tube, are placed at right angles to each other. These tubes contain spirit bubble. The bubble is brought to the centre with the help of foot screws.


## Compass

A compass is provided just below the telescope for taking the magnetic bearing of a line when required.

## Leveling Staff

It is a self-reading graduated wooden rod having rectangular cross section. The lower end of the rod is shod with metal to protect it from wear and usually point of zero measurement from which the graduations are numbered. Staff are either solid (having single piece of 3 meter height) or folding staff (of 4 meter height into two or three pieces) The least count of a leveling staff is 5 mm .

## Fundamental Lines of a Level

There are three fundamental lines in a level instrument figure. These are
$\neg$ Vertical axis
$\neg$ Axis of the level tube
$\neg$ Line of sight


Fundamental Lines in a level

Temporary adjustment
Each surveying adjustment needs two types of adjustments.
a) Temporary adjustments or station adjustment

The adjustment mad easy at ever set up of the level before the staff reading are taken are known as temporary adjustment.
The temporary adjustments for a level consist of the following

## Setting Up

1) Fixing the instrument on the stand
2) Leveling the instrument approximately by leg adjustment.

Leveling Up
The purpose of leveling is to make the vertical axis truly vertical. The manner of leveling the instrument by the plate levels depends upon whether there are three leveling screws.
3) Elimination of Parallax

Parallax is a condition arising when the image formed by the objective is not in the plane of the cross hair. It is by focusing the eye piece and by focusing the objective to bring the image of the object in the plane of the cross hair.
b) Permanent Adjustment of Dumpy Level

If any fundamental relation is found to be disturbed in a dumpy level, the crosshairs and level tube are adjusted so that the fundamental relations get satisfied. The reference line for the adjustments in dumpy level is the vertical line which remains fixed in direction, as it depends upon the direction of gravity.
$\square$ Axis of the Level Tube is Perpendicular to the Vertical axis
$\square$ Horizontal Cross Hair Should Lie in a Plane Perpendicular to the Vertical axis
$\square$ The Line of Sight is Parallel to the axis of the Bubble Tube

## Calculation of Levels

The observed staff readings as noted in a level book are further required to be manipulated to find out the elevation of points. The operation is known as reduction of level. There are two methods for reduction of levels:

- Height of instrument method.
- Rise and Fall method and


## Height of collimation method

In any particular set up of an instrument height of instrument, which is the elevation of the line of sight, is constant. The elevation of unknown points can be obtained by subtracting the staff readings at the desired points from the height of instrument. This is the basic behind the height of instrument method for reduction of level.


## Rise and fall method

For the same set up of an instrument, Staff reading is more at a lower point and less for a higher point. Thus, staff readings provide information regarding relative rise and fall of terrain points. This provides the basics behind rise and fall method for finding out elevation of unknown points.

## Booking and Reduced Levels

There are two methods of booking levels
Height of collimation method

- Rise \& Fall method


## Height of collimation

In this method, the height of the instrument is calculated for the each setting of the instrument by adding back sight to the elevation of the BM.

| Station | B.S | I.S | F.S | H.I | R.L | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First | 3.415 |  |  | 515.415 | 512.00 |  |
|  |  | 2.725 |  |  | 512.69 |  |
| Second | 0.975 |  | 1.855 | 514.535 | 513.560 | Change Point |
| Third | 1.365 |  | 2.450 | 513.450 | 512.085 | Change Point |
|  |  | 0.475 |  |  | 512.975 |  |
| Fourth | 2.805 |  | 2.405 | 513.85 | 511.045 | Change Point |
| Fifth | 3.065 |  | 1.685 | 515.23 | 512.165 | Change Point |
| Sixth | 1.500 |  | 1.400 | 515.33 | 513.83 | Change Point |
|  |  |  | 2.750 |  | 512.58 |  |


| $\sum$ B.S | - | $\sum$ F.S | $=$ | Last R.L | - | First R.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 13.125 | - | 12.545 | $=$ | 512.58 | - | 512.00 |
|  | 0.58 |  | $=$ |  | 0.58 |  |

2. A level book page is ruled as shown below to suit the height of collimation method, and the readings entered in it. The first reading from every set up is a back sight, the last reading as a fore sight, and all the other in between are intermediate sights. Also consecutive readings just before and after shifting the instrument are recorded as foresight and back sight in the same line as they refer to a change point.

| Back Sight | Inter Sight | Fore sight | Height of Co | RL | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.900 |  |  | 100.90 | 100.000 |  |
| 2.400 |  | 1.250 | 102.05 | 099.650 | Change Point |
| 2.945 |  | 1.375 | 103.620 | 100.675 | Change Point |
|  | 3.125 |  |  | 100.495 |  |
| 0.100 |  | 3.725 | 99.995 | 099.895 | Change Point |
|  | 1.975 |  |  | 098.020 |  |
|  | 2.025 |  |  | 097.970 |  |
|  |  |  |  |  |  |
| 1.775 |  |  |  |  |  |
|  |  | 098.220 |  |  |  |
| 6.345 | 8.125 |  |  | 1.78 |  |

The arithmetical checks are ok as shown below
$\sum \mathrm{B} . \mathrm{S}-\sum \mathrm{FS}=1.78$
The fall from first to last RL
$\sum$ First RL $-\sum$ Last RL $=1.78$
Hence Proved
2. The following consecutive readings were taken with a dumpy level, 3.864, 3.346, $2.932,1.952,0.854,37.796,2.639,1.542,1.934,0.864$, and 0.665 . The level was shifted after the fifth reading and eight readings. The first reading was taken on a bench mark of RL 150.250. Calculate the reduced levels of the change points and difference of level between the first and last point.

Solution

| Back Sight | Inter Sight | Fore sight | Height of Co | RL | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3.864 |  |  | 154.114 | 150.250 |  |
|  | 3.346 |  |  |  |  |
|  | 2.932 |  |  |  |  |
|  | 1.952 |  |  |  |  |
| 3.796 |  | 0.854 | 157.056 | 153.260 | Change Point |
|  | 2.639 |  |  |  |  |
| 1.934 |  | 1.542 | 157.448 | 155.514 | Change Point |
|  | 0.864 |  |  |  |  |
|  |  | 0.665 |  | 156.783 | Last Point |
| 9.594 |  | 3.061 |  | 6.533 |  |

The arithmetical checks are ok as shown below
$\sum$ B. S- $\sum \mathrm{FS}$
$=9.594-3.061$
$=6.533$

The fall from first to last RL
$\sum$ First RL $-\sum$ Last RL $=156.783-150.250=6.533$

## Hence Proved

2. The following consecutive readings were taken with a dumpy level, the instruments having been shifted after the second, fourth and seventh readings: $0.900,1.250$, $2.400,1.375,2.945,3.125,3.725,0.100,1.975,2.025$ and 1.775 . the first reading was taken with a staff held on a bench mark of elevation 100.00. Enter the readings in a level book and reduce the level by the rise and fall method. Apply the usual check.

| Back Sight | Inter Sight | Fore Sight | Rise | Fall | Reduced Level | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.900 |  |  |  |  | 100.00 | B.M: <br> Reduced <br> Level <br> 100.00 |
| 2.400 |  | 1.250 |  | 0.350 | 99.650 |  |
| 2.945 |  | 1.375 | 1.025 |  | 100.675 |  |
|  | 3.125 |  |  | 0.180 | 100.495 |  |
| 0.100 |  | 3.725 |  | 0.600 | 99.895 |  |
|  | 1.975 |  |  | 1.875 | 98.020 |  |
|  | 2.025 |  |  | 0.050 | 97.970 |  |
|  |  | 1.775 | 0.250 |  | 98.220 |  |
| 6.345 |  | 8.125 | 1.275 | 3.055 | Fall 1.780 |  |

The arithmetical checks are
ok $\sum$ fore sight - $\sum$ back sight
The fall from the first to last
RL $\sum$ Fall- $\sum$ Rise

- $8.125-6.345$
$=100.00-98.220=1.780$
$=3.055-1.275=1.780$
- Complete the leveling table given tables. If an even gradient of 1 vertical in every horizontal starts 1 m above peg 0 . What is the height of the gradient above or its depth below peg?

| Station | Back Sight | Inter Sight | Fore Sight | Rise | Fall | Reduced <br> Level | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B.M | 3.10 |  |  |  |  | 193.62 |  |
| 0 |  | 2.56 |  | 0.54 |  | 194.16 |  |
| 1 |  | 1.07 |  | 1.49 |  | 195.65 |  |
| 2 | 1.92 |  | 3.96 |  | 2.89 | 192.76 | Change Point |
| 3 | 1.20 |  | 0.67 | 1.25 |  | 194.01 | Change Point |
| 4 |  | 4.24 |  |  | 3.04 | 190.97 |  |
| 5 | 0.22 |  | 1.87 | 2.37 |  | 193.34 | Change Point |
| 6 |  | 3.03 |  |  | 2.81 | 190.53 |  |
| 7 |  |  | 1.41 | 1.62 |  | 192.15 |  |
|  | 6.44 |  | 7.91 | 7.27 | 8.74 |  |  |

The arithmetical checks are ok
$\sum$ fore sight $-\sum$ back sight $=6.44-7.91=-1.47$
The fall from the first to last RL $=100.00-98.220=-1.47$
$\sum$ Fall- $\sum$ Rise $=7.91-8.74=-1.47$
Height of gradient above peg $7=193.16-192.15=1.01 \mathrm{~m}$ above
$>$ Given below is page from a level book. Furnish the missing entries and complete the field book with the usual check

| S.no | Back <br> Sight | Inter Sight | Fore Sight | Rise | Fall | Reduced Level | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B.M | X |  |  |  |  | 150.00 | B.M: <br> Reduced <br> Level <br> 150.00 |
| 1 |  | 2.457 |  |  | 0.827 | X |  |
| 2 |  | 2.400 |  | 0.057 |  | X |  |
| 3 | 2.697 |  | X |  | X | 148.070 |  |
| 4 | X |  | 2.051 | 0.646 |  | 148.716 |  |
| 5 |  | 2.500 |  | 1.068 |  | 149.784 |  |
| 6 |  | 2.896 |  |  | X | 149.388 |  |
| 7 |  | X |  |  | 0.124 | X |  |
| 8 |  |  | 2.672 | 0.348 |  | 149.784 |  |

Remembering the rules of reduction the missing entries may be easily deduced and entered as follows (reduced entries are underlined). It can be easily seen that the arithmetical checks are satisfied.

| Back Sight | Inter Sight | Fore Sight | Rise | Fall | Reduced <br> Level | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.630 |  |  |  |  | 150.00 |  |
|  | 2.457 |  |  | 0.827 | 149.173 |  |
|  | 2.400 |  | 0.057 |  | 149.230 | B.M: |
| 2.697 |  | 3.560 |  | 1.160 | 148.070 |  |
| 3.568 |  | 2.051 | 0.646 |  | 148.716 |  |
|  | 2.500 |  | 1.068 |  | 149.784 |  |
|  | 2.896 |  |  | 0.396 | 149.388 |  |
|  | 3.020 |  |  | 0.124 | 149.264 |  |
|  |  | 2.672 | 0.348 |  | 149.612 |  |
|  |  | 8.283 | 2.119 | 2.507 |  |  |

## Home Work

Given below is page from a level book. Furnish the missing entries and complete the field book with the usual check. Calculate the missed data. (Home Work)

| S.no | Back <br> Sight | Inter Sight | Fore Sight | Rise | Fall | Reduced <br> Level | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B.M | 2.150 |  |  |  |  | 450.00 |  |
| 1 | 1.645 |  | X | 0.500 |  |  |  |
| 2 |  | 2.345 |  |  | X |  | B.M: <br> Reduced <br> Level <br> Len |
| 3 | X |  | 1.965 | X |  |  |  |
| 4 | 2.050 |  | 1.825 |  | 0.400 |  |  |
| 5 | X |  | X | X |  | 451.500 |  |
| 6 | 1.690 |  | 1.570 | 0.120 |  |  |  |
| 7 | 2.865 |  | 2.100 |  | X |  |  |
| 8 |  |  | X | X |  | 451.250 |  |
| 9 |  |  |  |  |  |  |  |

Table 1: Compare between Collimation and Rise \& Fall System

| S.no | Collimation System | Rise \& Fall System |
| :---: | :--- | :--- | :--- |
| 1 | It is rapid as it involves few <br> calculation | It is laborious, involving several <br> calculations |
| 2 | There is no check on the RL on <br> intermediate points | There is a check on the RL of <br> intermediate points |
| 3 | Errors in intermediate RL's cannot be <br> detected | Errors in intermediate RL's can be <br> detected as all the points are corrected |
| 4 | There are two checks on the accuracy <br> of RL calculation | There are three checks on the accuracy <br> of RL calculation |
| 5 | This system is suitable for <br> longitudinal leveling where there are <br> a number of intermediate sights | This system is suitable for fly leveling <br> where there are no intermediate sights, |

## Methods of leveling

There are various leveling techniques each used for specific purpose.
Simple leveling
> Differential Leveling
> Fly Leveling
> Check Leveling
> Profile Leveling or Longitudinal Leveling
> Cross Sectional Leveling
> Reciprocal Leveling
> Precise Leveling
> Trigonometrically Leveling
> Barometry Leveling or Barometric Leveling
> Hypsometry Leveling

The last three leveling is indirect approach.

## Simple Leveling

Simple Leveling is a straightforward technique in which the difference in level between two points is found with just one set up of the level approximately midway between them.

## Differential Leveling

Differential Leveling is when points whose difference in elevation is required are too far apart, or when the difference in level is too large. For this one set up of level will not bet adequate.
Fly Leveling
Low precision, to find/check approximate level, generally used during reconnaissance survey. A level line run at the close of a working day to check the results of an extended line run in one direction only. Longer sights and fewer setups are used as the purpose is to detect large mistakes. Also called fly levelling. Check Leveling

Check leveling is employed to check the accuracy of a part or whole of a leveling job. At the end of the day's work, a line of levels is run back to the starting point, merely to check the accuracy of the work.


If the closing error is within permissible limits, the work is considered satisfactory, otherwise the job is repeated. A simpler way to check any leveling job is to take staff readings before chaining the level station twice, and checking for consistency at every stage. Sometimes two different leveling instruments are employed simultaneously, at every stage of the work, to get the difference of level between any two consecutive points. The work proceeds further only after agreement of the values obtained by both surveyors. This is a kind of fly leveling, but is also called check leveling by some. Profile Leveling or longitudinal sectioning

Profile Leveling or longitudinal sectioning is used in route surveys for highways and railways. The reduced levels of points along the proposed centre line, a known distance apart, are obtained by the principles of differential and fly leveling. The object is to determine the profile of the ground in a vertical longitudinal section.

## Cross sectional leveling:

Cross sectioning is the determination of levels at specified points to obtain an idea of the variation of the
 ground level transverse
to the centre line of a route on either side of it for a desired distance. Taking longitudinal sections (LS) and cross sections (CS) helps in the estimation of the earth work involved in a route.

## Reciprocal leveling

Reciprocal leveling is the accurate determination of the difference in level between two points by setting up the instrument near each of the points in succession and taking observation of staff readings on both. It is useful when it is not feasible to set up the level midway between the points.

## Precise leveling

Precise leveling establishes bench mark fro future public use with a high degree of precision. Refined instruments and procedure enhance the accuracy of the work, which is usually conducted by government agencies like the survey of India.

## Trigonometric leveling

Trigonometric leveling enables one to compute the elevation of highly separated points from horizontal distances and vertical angles measured in the field.

## Barometric Leveling

In this method, altitudes of points are related to changes in atmospheric pressure. It is used to determine the rough levels of points at widely differing altitudes, as often encountered in reconnaissance and preliminary surveys.

## Hypsometry or Hypsometric leveling

Here the changes in the boiling point of water at points with differing altitudes yield the difference of level approximately. The method is not reliable fro accurate work. It is used for reconnaissance surveys.

Errors, Mistakes and Precautions in Leveling Types of Errors

$\square$ Instrumental Errors
$\square$ Personal Errors
$\square$ Errors due to Natural Causes
$\square$ Error due to Earths Curvature \& Refraction

## Instruments Error:

Error in permanent adjustment of level: For any major surveying work, instrument needs to be tested and if required, gets to be adjusted. For small works, bubble of the level tube should be brought to the centre before each reading and balancing of sights are to be maintained.

Staff defective and/or of non-standard quality: The graduation in staff may lack standard distance and thus may cause error in reading. In an ordinary leveling, the error may be negligible but in the case of precise leveling, the graduations are to be standardized with invar tape.

Error due to defective level tube: The bubble of the level tube may remain central even though the bubble axis is not horizontal due to its sluggishness or it may take considerable time to occupy central position, if it is very sensitive. Also, there may be irregularity in the curvature of the tube causing delirious effect.

Error due to defective tripod: The tripod stand should be strong and stable otherwise it causes setting of the instrument unstable and considerable time is required to make it level. The nuts provided at the joints of the legs to the tripod head should be well-tightened before mounting the instrument. The tripod should be set up on a stable, firm ground.

## Personal Errors

Due to imperfection in temporary adjustment of the instrument
These errors are caused due to careless setting up of the level, improper leveling of the instrument, lack in focus of eyepiece or/and objective and error in sighting of the staff.
$\square$ Careless set-up of the instrument: If the instrument is not set up firmly, it gets disturbed easily. If the ground is not firm, it may settled down and on hard ground, it may get slipped.
$\square$ Imperfect leveling of the instrument: Due to improper leveling of the instrument, bubble does not remain at the centre when the sights are taken resulting error in reading. To avoid the error, the bubble should be brought to the centre before each reading.

1. Imperfect focusing: If either the eye-piece or the objective or both are not properly focused, parallax and thus error in the staff readings occur. Due to
2. movement of eyes if there is any apparent change in the staff reading the eyepiece and objective need proper focusing.

Errors in sighting: This occurs when the horizontal cross-hair does not exactly coincide with the staff graduation or it is difficult to see the exact coincidence of the cross hairs and the staff graduations. The error can be minimised by keeping the small sight distance.

Error due to staff held Non-vertical: If the staff is not held vertical, the staff reading obtained is greater than the correct reading. To reduce the error, the staff should be held exactly vertical or the staff man should be asked to waive the staff towards the instrument and then away from the instrument and the lowest reading should be taken.

Errors in reading the staff: These errors occur if staff is read upward, instead of downwards, read against the top or bottom hair instead of the central hair, mistakes in reading the decimal part and reading the whole meter wrongly.

Errors in recording: The common errors are entering a wrong reading (with digits interchanged or mistaking the numerical value of a reading called by the level man), recording in wrong column, e.g., B.S. as I.S., omitting an entry, entering the inverted staff reading without a minus sign etc.

Errors in computing: adding the fore sight reading instead of subtracting it and or subtracting a back sight reading instead of adding.

## Errors due to Natural Causes

Error due to curvature: In case of small sight distance error due to the curvature are negligible, but if the sight distances are large, the error should be estimated and accounted for, as discussed below. However, the error can be minimized through balancing of sight or reciprocal observation.


## Error due to Curvature of the earth (ec) and Refraction of light(er)

With reference to figure, the horizontal line of sight through an instrument set at $L$ is $L^{\prime} x$ $h$. The level line passing through $L^{\prime}$ is $L^{\prime} x_{1}$. The correct staff reading at $X$ is $x_{1}$. Thus, horizontal staff reading at station $\mathrm{X}, \mathrm{x}_{\mathrm{h}}$ is associated with an error $\mathrm{x}_{\mathrm{h}} \mathrm{x}_{1}$ due to curvature of the earth.

In Figure, PH is a horizontal line tangent at P to the level line along the mean radius, $\mathrm{R}_{\mathrm{m}}$ of the earth. At station L, LH is the amount of departure of the horizontal line from level line and thus the error due to curvature of the earth $\left(\mathrm{e}_{\mathrm{c}}\right)$. This can be calculated from the triangle OPH in which
$\mathrm{OH}^{2}=\mathrm{OP}^{2}+\mathrm{PH}^{2}$
Or, $\left(\mathrm{R}_{\mathrm{m}}+\mathrm{e}_{\mathrm{c}}\right)^{2}=\mathrm{R}_{\mathrm{m}}{ }^{2}+\mathrm{PH}^{2}$
Or, $e_{c}=\frac{P H^{2}}{2 R_{m}+e_{c}}$
Or, $e_{c}=\frac{P H^{2}}{2 R_{m}}$
(Neglecting $\mathrm{e}_{\mathrm{c}}$ in the denominator as it is very small in comparison to $\mathrm{R}_{\mathrm{m}}$ ).
Assuming, mean radius of the earth as 6367 Km , and D is the distance in Km from the instrument position to the staff station, the error due to the curvature of the earth is
$\mathrm{e}_{\mathrm{c}}=0.0785 \mathrm{D}^{2}$
It is subtractive in nature as curvature of the earth always provides increase in staff reading.

Error due to refraction: It varies with temperature, terrain and other atmospheric conditions. It is usually considered to be one seventh times but in opposite nature to the error due to curvature. To minimize this error, reciprocal observation at the same instant of time is required to be adopted.


In actual field condition, the line of sight through a level is not straight but it bends downward due to the refraction of rays of light as it passes through the intervening medium. Thus, reduces the error due to curvature of the earth by approximately $14 \%$. With reference to Figure, the actual line of sight of the instrument set at L is thus $\mathrm{L}^{\prime} \mathrm{x}$ a. The observed staff reading at station X is $\mathrm{x}_{\mathrm{a}}$. Thus, the compensation due to refraction is thus $\mathrm{x}_{\mathrm{h}} \mathrm{x}$ a which is error due to refraction ( $\mathrm{e}_{\mathrm{r}}$ ) through intervening atmosphere. In figure, HA is the error due to refraction ( $\mathrm{e}_{\mathrm{r}}$ ).

## Error due to Earths Curvature \& Refraction

The combined error due to curvature and refraction ( $\mathrm{e}_{\mathrm{comb}}$ ) is thus given by $\mathrm{e}_{\text {comb }}=0.0675 \mathrm{D}^{2} \mathrm{~m}$ where D is the distance in km

It is finally subtractive in nature as the combined effect provides increase in staff reading. In Figure, $\mathrm{x}_{1} \mathrm{x}$ a represents the combined error due to curvature and refraction and in Figure, it is AL.


## Error due to Curvature of the earth (ec) and Refraction of light(er)

In most ordinary leveling operation, the line of sight is rarely more than 2 meter above the ground (where the variation in temperature causes substantial uncertainties in the refraction index of air). Fortunately, most lines of sights in leveling are relatively short ( $<30 \mathrm{~m}$ ) and B.S. \& F.S. are balanced. Consequently, curvature and refraction corrections are relatively small thus insignificant except for precise leveling.

## Contouring

## Contour

A contour is defined as an imaginary line of constant elevation on the ground surface. It can also be defined as the line of intersection of a level surface with the ground surface. For example, the line of intersection of the water surface of a still lake or pond with the surrounding ground represents a contour line.

## Contour Interval

The difference in elevation between successive contour lines on a given map is fixed. This vertical distance between any two contour lines in a map is called the contour interval (C.I.) of the map. Figure (a) shows contour interval of 1 m whereas Figure (b) shows 10 m .


## Contour lines (a) $\mathbf{1} \mathbf{m}$ (b) $\mathbf{1 0} \mathbf{m}$

The choice of suitable contour interval in a map depends upon four principal considerations. These are:

- Nature of the Terrain
- Scale of the Map
- Accuracy
- Time of Cost


## Horizontal Equivalent

The horizontal distance between two points on two consecutive contour lines for a given slope is known as horizontal equivalent. For example, in Figure (b) having contour interval 10 m , the horizontal equivalent in a slope of 1 in 5 would be 50 m . Thus, horizontal equivalent depends upon the slope of the ground and required grade for construction of a road, canal and contour interval.


Contour lines 10 m

## Characteristics of contour

The principal characteristics of contour lines which help in plotting or reading a contour map are as follows:

- The variation of vertical distance between any two contour lines is assumed to be uniform.
- The horizontal distance between any two contour lines indicates the amount of slope and varies inversely on the amount of slope. Thus, contours are spaced equally for uniform slope (Figure); closely for steep slope contours (Figure) and widely for moderate slope (Figure).
- The steepest slope of terrain at any point on a contour is represented along the normal of the contour at that point (Figure). They are perpendicular to ridge and valley lines where they cross such lines.
- Contours do not pass through permanent structures such as buildings (Figure)
- Contours of different elevations cannot cross each other (caves and overhanging cliffs are the exceptions). (Figure)
- Contours of different elevations cannot unite to form one contour (vertical cliff is an exception). (Figure)
- Contour lines cannot begin or end on the plan.
- A contour line must close itself but need not be necessarily within the limits of the map.
- A closed contour line on a map represents either depression or hill (Figure (a)). A set of ring contours with higher values inside, depicts a hill whereas the lower value inside, depicts a depression (without an outlet) Figure (b).
- Contours deflect uphill at valley lines and downhill at ridge lines. Contour lines in U-shape cross a ridge and in V-shape cross a valley at right angles. The concavity in contour lines is towards higher ground in the case of ridge and towards lower ground in the case of valley (Figure).
- Contours do not have sharp turnings.


## Methods of Contouring

- Direct Method
- Indirect Method


## Direct Method

In the direct method, the contour to be plotted is actually traced on the ground. Points which happen to fall on a desired contour are only surveyed, plotted and finally joined to obtain the particular contour. This method is slow and tedious and thus used for large scale maps, small contour interval and at high degree of precision. Direct method of contouring can be employed using Level and Staff as follows:

Vertical control: In this method, a benchmark is required in the project area. The level is set up on any commanding position and back sight is taken on the bench mark. Let the back sight reading on the bench mark be 1.485 m . If the reduced level of the bench mark is 100 m , the height of instrument would be $100+1.485=101.485 \mathrm{~m}$. To locate the contour of 100.5 m value, the staff man is directed to occupy the position on the ground where the staff reading is $101.485-100.500=0.985 \mathrm{~m}$. Mark all such positions on the ground where the staff reading would be 0.985 m by inserting pegs. Similarly locate the points where the staff reading would be $101.485-101=0.485 \mathrm{~m}$ for 101 m contour. The contour of 101.5 m cannot be set from this setting of the instrument because the height of instrument for this setting of the instrument is only 101.485 m . Therefore, locating contours of higher value, the instrument has to be shifted to some other suitable position. Establish a forward station on a firm ground and take fore sight on it. This point acts as a point of known elevation, for shifting the position of the instrument to another position, from where the work proceeds in the similar manner till the entire area is contoured.

Horizontal control: The horizontal control is generally provided by method of plane table surveying or locating the positions of points by other details of which will be discussed in later module (Figure).


## Direct Method of contouring

## Indirect Method

In this method, points are located in the field, generally as comers of well-shaped geometrical figures such as squares, rectangles, and spot levels are determined. Elevations of desired contours are interpolated in between spot levels and contour lines are drawn by joining points of equal elevation.

Indirect methods are less expensive, less time consuming and less tedious as compared to the direct method. These methods are commonly employed in small scale surveys of large areas or during mapping of irregular surface or steep slope. There are two different ways usually employed for indirect method of contouring:
$\neg$ Grid method and
$\neg$ Radial line method

## Gird Method

In this method, the area to be surveyed is divided into a grid or series of squares (Figure). The grid size may vary from $5 \mathrm{~m} \times 5 \mathrm{~m}$ to $25 \mathrm{~m} \times 25 \mathrm{~m}$ depending upon the nature of the terrain, the contour interval required and the scale of the map desired. Also, the grids may not be of the same size throughout but may vary depending upon the requirement and field conditio

The grid corners are marked on the ground and spot levels of these comers are determined by leveling. The grid is plotted to the scale of the map and the spot levels of the grid corners are entered. The contours of desired values are then located by interpolation. Special care should be taken to give the spot levels to the salient features of the ground such as hilltops, deepest points of the depressions, and their measurements from respective comers of the grids, for correct depiction of the features. The method is used for large scale mapping and at average precision.

Table 2: A Comparison between Direct and Indirect Methods of Contouring

|  | Direct Method | Indirect Method |
| :---: | :---: | :---: |
| 1 | Very accurate but slow and tedious | Not very accurate but quicker and less <br> tedious. |
| 2 | Expensive | Reasonable cost |
| 3 | Appropriate for small projects <br> requiring high accuracy, e.g., layout <br> of building, factory, structural <br> foundations, etc. | Suitable for large projects requiring <br> moderate to low accuracy, e.g., layout of <br> highway, railway, canal, etc. |
| 4 | More suitable for low undulating <br> terrain. | Suitable for hilly terrain. |
| 5 | Calculations need to be carried out in <br> the field | Calculation in the field is not mandatory. |
| 6 | After contouring, calculation cannot <br> be checked. | Calculations can be checked as and when |
| needed |  |  |

## Contouring Gradient

An imaginary line on the surface of the earth having a constant inclination with the horizontal (slope) is called contour gradient. The inclination of a contour gradient is generally given either as rising gradient or falling gradient, and is expressed as ratio of the vertical height to a specified horizontal distance. If the inclination of a contour gradient is 1 in 50 , it means that for every 50 m horizontal distance, there is a rise (or fall) of 1 m .

When the inclination of a contour gradient is given its direction from a point may be easily located either on the map or on the ground by the methods discussed below.
$\checkmark$ Location Contour Gradient on a Map
$\checkmark$ Location Contour Gradient on the Ground

## Uses of Contours

$\square$ Nature of Grounds
$\square$ To Locate Route
$\square$ Intervisibility between Stations
$\square$ To Determine Catchments Area or Drainage Area
$\square$ Storage capacity of a Reservoir

| Trapezoidal Rule | Simpson's Rule |
| :--- | :--- |
| The boundary between the ordinates is <br> considered to be straight | The boundary between the ordinates is <br> considered to be an Arc of a Parabola. |
| There is no limitation. it can be applied for <br> any number of ordinates. | To apply this rule, the number of ordinates <br> must be odd. That is the number of <br> divisions should be even. |
| It gives an approximate results | It gives a more accurate result. |

## Problem

$\square$ The following perpendicular offsets were taken at 10 m interval from a chain line to an irregular boundary: 3.06, 4.14, 5.60, 6.00, 6.60, 7.80, 6.30 and 7.20 m . Calculate the are enclosed between the chain line, the irregular boundary using,

Average Ordinate Rule
Trapezoidal Rule
Simpson's Rule

## Average Ordinate Rule

Area $=\frac{\text { Sum of Ordinates }}{\text { No.of Ordinates }} \times l$
Area $=\frac{8 \times 10}{9} x(3.06+4.14+5.60+4.86+6.00+6.60+7.80+6.30+7.20)=458.31$
sq.m

## Trapezoidal Rule

Area $=\frac{\text { Common Ordinates }}{2} \times(1$ st Ordinate + Last Ordinate $+2($ Sumof other ordinates $))$
Area $=\frac{10}{2} \times(3.06+7.20+2(4.14+5.60+4.86+6.00+6.60+7.80+6.30))=464.30$

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## SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

## Application of Theodolite

## Theodolite Survey

The Theodolite is the most accurate instrument used by Surveyor for the measurement of horizontal and vertical angles. It is wide applicable in Survey such as,

- Laying off horizontal angles
- Locating points on line
- Prolonging Survey lines
- Establishing grades
- Determining difference in elevation
- Setting out Curves


## Types of Theodolite

Theodolite can be classified as

- Transit Theodolite
- Non - Theodolite


## Transit Theodolite

A transit theodolite is one in which the telescope can be revolved through a complete revolution about $180^{\circ}$, its horizontal axis in a vertical plane.

## Non - Theodolite

The non - theodolite is one in which cannot be transited. In non transit theodolite the telescope is mounted in such a manner that the line of sight cannot be revolved by revolving the telescope. Nowadays only transit theodolite can used.

Theodolite can be further classified in to two types

- Vernier theodolite
- Microptic theodolite


## Vernier theodolite

Vernier theodolite is placed on the rounded plate to measure the reading clearly quickly.

## Microptic theodolite

Micrometer theodolite is placed on the rounded plate to measure the reading clearly quickly.

Theodolite are made of various sizes. The diameter of the horizontal graduated circle (lower plate) defines the size of the theodolite. The sizes rangefrom 8 cm to $25 \mathrm{~cm}, 8 \mathrm{~cm}$ to 12 cm size instruments are used for general survey, while 14 cm to 25 cm size instruments are used for triangulation.

## Modern Theodolite

The optical Theodolite has optical system to read both horizontal and vertical angles with greater accuracy. The electronic theodolites have optoelectronic system to scan both horizontal vertical circles and display them digitally on a screen. Apart from this angle measuring instrument, electronic distance measurement equipment (EDM) such as distomats is also used to measure the distance automatically.

Various modern Theodolites are as follows
Electronic theodolite
Wild T 1000
Wild T 1600
Wild T 2002
Wild T 3000

## Transit Theodolite

The Main features of all theodolites are the same, the instruments of different markers and the different models of each mark vary in many of their details, but the essential parts are the same in each.Component parts of transit theodolite are

| 1. | Telescope | 13. Altitude level |
| :--- | :--- | :--- |
| 2. | Trunnion axis | 14. Leveling Head |
| 3. | Vernier frame | 15. Levelingscrew |
| 4. | Vertical Circle | 16. Plumb Bob |
| 5. | Plate levels | 17. Arm of vertical Circle clamp |
| 6. | Standard frames | 18. Foot plate |
| 7. | Upper Plate | 19. Tripod Head |
| 8. | Horizontal Plate Vernier | 20. Upper clamp |
| 9. | Horizontal Circle | 21. Lower Clam |
| 10. | Lower Plate | 22. Vertical circle clamp |
| 11. | Inner Axis | 23. Tripod |

## Leveling Head

It consists of two parallel plates separated by leveling screw, the upper plate is called tribrach and the lower plate is trivet. The trivet has a central aperture through with a plumb can be supported. It supports the entire instruments \& also serves to mount the instrument on a tripod.

## Shifting Head

It is self centering device placed below the lower plate, but above the tribrach to center the instrument is first approximately centered over the station by moving the tripod legs, then exact centering is done by using the shifting head.

## Lower Plate

It is a graduated horizontal circular plate graduated in clock-wise direction from $0^{\circ}$ to $360^{\circ}$, each fifth degree is numbered. It is provided, lower clamp screw is tightened and the outer spindle is fixed to tribrach of the lower plate is fixed.

## Plate level

A level tube is mounted on an upper plate in the plate level to make the vertical axis truly vertical.

## Upper Plate

Upper plate is attached to inner solid spindle. It carries two vernier A and B diametrically opposite to each other at $180^{\circ}$. The upper plate is provided with upper screw and tangent screws. The upper screw is clamped, both the plate move together as one unit.

## Standards or 'A' frame

The two standards are ' A ' frame shape is mounted on the upper plate.

## Compass

It is a circular/trough/tubular to measure magnetic bearing of survey line.

## Altitude Bubble

It is a sensitive level tube, attached to top of ' T ' frame for measuring vertical angles transit is leveled with reference to altitude bubble.

## Tripod

It is a stand to support the theodolite for using in the field. The legs are made up of a solid frame. It is pointed with steel shoes pointed down. Its has a tripod head has external screws to which trivet of the leveling head is screwed.

## Axis of telescope

The line joining the optical cetre of the object glass to the centre of the eye piece.

## Face left and Face right observation

The observation of the horizontal and vertical angles with the face vertical either to the left or to the right of an observer are known as Face left and Face right.

## Swinging

Turning the telescope in a horizontal plane about vertical axis is generally called swinging.


Figure 1: Theodolite

Fundamental Lines of a theodolite and the desired relationship between them
The fundamental lines of theodolite are as follows,

- The vertical Axis
- The horizontal Axis
- The line of Collimation
- Axis of plate level
* Axis of altitude Level
$\Rightarrow$ Axis of Striding level


## Temporary Adjustment

## Setting Up

Leveling Up
Elimination of parallax

## Setting Up

$\checkmark$ Centering of the instrument over the station mark by a plumb bob.
$\checkmark$ Approximate levelling with the help of both tripod legs.
$\checkmark$ By moving the leg radially, the plum bob is shifted in the direction of legs while by moving by legs circumferentially or side way Considerer able change in the inclination is effected without disturbing the tripodlegs.
$\checkmark$ The approximate levelling is done either with the reference to a small Circular bubble provided on tribrach.

## Levelling Up

$\checkmark$ repeatTurn the upper plate until the longitudinal axis of the plate level is roughly parallel to a line joining any two of the levelling screws.
$\checkmark$ Hold these two levelling screws between the thumb and first finger of Each hand and turn them uniformly so that the thumb moves either towards eachother.
$\checkmark$ Turn the upper plate through $90^{\circ}$.
$\checkmark$ Turn the levelling screws until the bubble is central.
$\checkmark$ Return the upper plate through $90^{\circ}$ to its original position till the bubble at centre.
$\checkmark$ Turn back again through $90^{\circ}$ and the second again.
$\checkmark$ Repeat the $2^{\text {nd }}$ and $4^{\text {th }}$ step, till the bubble comes at central position.

## Elimination of Parallax

Parallax is a condition arising when the image formed by the objective is not in the plane of the cross-hair. Unless parallax is eliminated, accurate sighting is impossible. Parallax can be terminated by using followings two steps,
$\checkmark$ Focusing the eye-piece.
$\checkmark$ Focusing the objective.

## Measurement of Horizontal Angles

General Method

Repetition method
Reiteration Method

## General Method

$\checkmark$ Set up the instrument at Q and level it accurately.
$\checkmark$ Release all the clamps turn the upper and lower plates in opposite Directions till the zero of one vernier is against zero of the scale and the vertical circle is to the left.
$\checkmark$ Loose the lower clamp and turn the instrument towards the signal at P. Since the both plates are clamped together the instrument will rotate about the outer axis.
$\checkmark$ Bisect the P accurately by using lower tangents screws.
$\checkmark$ Unclamp the upper clamp and rotate the instrument clockwise about the inner axis to bisect the point R , and clamp the upper plate.
$\checkmark$ Read both vernier, the readings A gives the $\triangle \mathrm{PQR}$ directly while the vernier B gives by deducting $180^{\circ}$.
$\checkmark$ Change the face by transiting the telescope and repeat the whole Process.
$\checkmark$ The average horizontal angle is then obtained.

## Repetition method

$\checkmark$ Set up the instrument at Q and level it accurately.
$\checkmark$ Release all the clamps turn the upper and lower plates in opposite Directions till the zero of one vernier is against zero of the scale and the vertical circle is to the left.
$\checkmark$ Loose the lower clamp and turn the instrument towards the signal at P .
$\checkmark$ Since the both plates are clamped together the instrument will rotate about the outer axis. Bisect the P accurately by using lower tangents screws.
$\checkmark$ Unclamp the upper clamp and rotate the instrument clockwise about the inner axis to bisect the point R , and clamp the upper plate.
$\checkmark$ Read both vernier, the readings A gives the $\triangle \mathrm{PQR}$ directly while the vernier B gives by deducting $180^{\circ}$.
$\checkmark$ Unclamp the lower clamp screw and again sight P and clamp it. Unclamp the upper clamp and rotate the instrument clockwise about the inner axis to bisect the point R , and clamp the upper plate. Note down thereadings
$\checkmark$ Repeat the process until the angles is repeated the required number of Times (Usually3). The average angle with face left will be equal to final reading.

## Reiteration Method

$\checkmark$ Set the instrument over O and level it. Set one vernier to zero and bisect point A accurately.
$\checkmark$ Loose the upper clamp and turn the telescope clockwise to point B. Bisect B accurately using the upper tangent screw.
$\checkmark$ Read both the verniers. The mean of the readings will give angles.
$\checkmark$ Similarly, bisects successively, C, D, etc thus closing the circle. Read both the vernierat each bisection. Since the graduated circle remains in a fixed position throughout the entire process, each included angle is obtained by taking the difference between two consecutive readings.
$\checkmark$ On final sight to A, the reading of the vernier should be the same as the original setting. It not, notes the readings and finds the error dueto slips etc, and if the error is small, distribute it equally to all angles. If large, repeat the procedure and take a fresh set a reading.
$\checkmark$ Repeat $2^{\text {nd }}$ and $4^{\text {th }}$ steps with the other face.

## Measurement of Vertical Angles

$\checkmark$ Level the instrument with reference to the plate level, as already explained.
$\checkmark$ Keep the altitude level parallel to any two foot screws and bring The bubble central. Rotate the telescope through $90^{\circ}$ till the altitude.
$\checkmark$ Bubble is on the third screw. Bring the bubble to the centre with the third foot screw. Repeat the procedure till the bubble is central in both the positions. If the bubble is in adjustment it will remain central for all pointing of the telescope.
$\checkmark$ Loose the vertical circle clamp and rotate the telescope in vertical Plane to sight the object. Use vertical circle tangent screw for accurate Bisection.
$\checkmark$ Read both verniers (i.e.C and D) of vertical circle. The mean of the two gives the vertical circle. Similarly observation may be made with another face. The average of the two gives the required angle.

## Miscellaneous Operations with

## Theodolite

To measure the magnetic bearing of a line.
To measure direct angles
To measure deflection angles
To prolong a straight line
To run a straight line between two points
To locate the point of intersection of two straight lines
To lay off a horizontal angle
To lay off an angle by repetition

## Sources of Error in Theodolite Work

$\checkmark$ The sources of error in transit work are:
$\checkmark$ Instrumental Errors
$\checkmark$ Non Adjustment of plate
$\checkmark$ Line of collimation not being perpendicular to horizontal axis
$\checkmark$ Horizontal axis not being perpendicular to vertical axis
$\checkmark$ Line of collimation not being parallel to axis of telescope
$\checkmark$ Eccentrically of inner and outer axis
$\checkmark$ Graduations not being uniform
$\checkmark$ Vernier being eccentric
$\checkmark$ Personal Errors, andNatural Errors

## OMITTED MEASUREMENTS

## Co-ordinate Method

The latitude of a survey line may be defined as its co-ordinate length measured parallel to an assumed meridian direction. The departure of survey line may be defined as its co-ordinate length measured at right angles to the meridian direction. The latitude (L) and departure (D) are tabulated below.
$\mathrm{L}=1 \cos \varnothing$
$\mathrm{D}=1 \sin \varnothing$
$\mathbf{L}=1 \cos \varnothing$
$D=1 \sin \varnothing$


The following tables give signs of latitude and departures
TABLE 1

| Quadrant | WCB |  | RB | Ratitude | Departure |
| :--- | :--- | :--- | :--- | :--- | :--- |
| II |  |  |  |  |  |

## Independent Co-ordinate

The co-ordinate of any point with respect to a common origin are said to be independent co ordinates of that point.

## Consecutive co-ordinate

The latitude and departure of a point calculated with reference to the preceding point for what are called consecutive co-ordinate

## Closing and limitation

If a closed traverse is plotted according to the field measurements, the end point of the traverse will not coincide exactly with the starting point; such error is known as closing error. In a closed traverse the algebraic sum of latitudes should be zero. The algebraic sum of departure should be zero.

## Adjustment of angular errors

Before calculating latitude and departure, the traverse angles should be adjusted to satisfy geometric conditions.

In closed traverse, the sum of al interior angles should be equal to ( $2 \mathrm{~N}-4$ ) right angles, or the algebraic sum of deflection angles should be 360

## Adjustment of bearings

In a closed traverse in which the bearing are observed, the closing error in bearing may be determined by comparing the two bearing of the last line as observed by the first land last station of the traverse. Let e be the closing error bearing of last line of a closed traverse having N sides

## The following are common methods of adjusting the traverse

- Bow ditch's method
- Transit method
- Third Rule


## Bow ditch's method

- It is also termed as compass rule, is mostly used to balance the traverse where linear and angular measurements are equal precision.

Correction of the latitude of any side $=\frac{\text { lengthof that side }}{\text { perimeter of the traverse }} *_{\text {total error in Latitude }}$
Correction of the departure of any side $=\frac{\text { lengthof that side }}{\text { perimeter of the traverse }} * *$ total error in Deaparture

## Transit Rule

The transit rule may be employed where the angular measurement are more precise that the linear measurement.

Correction of the latitude of any side $=\frac{\text { latitude of } \quad \text { that side }}{\text { arithmetical sum of all latitudes }} \quad *$ total error in Latitude
Correction of the departure of any side $=\frac{\text { latitude of that side }}{\text { arithmetical sum of all Deaparture } *} *$ total error in Deaparture

In a closed traverse, the algebraic sum of latitude and departure must be equal to zero.

$$
\sum \mathrm{L}=1_{1} \operatorname{Cos} \Theta_{1}+1_{2} \operatorname{Cos} \Theta_{2}+1_{3} \operatorname{Cos} \Theta_{3}+1_{4} \operatorname{Cos} \Theta_{4}+\ldots \ldots \ldots . .=0
$$

## When the length of one side and the bearing of another side are omitted

The following records are obtained in a traverse survey, when the length and bearing of the last line is not recorded

| .Line | Length(m) | Bearings |
| :---: | :---: | :---: |
| AB | 204.0 | $?$ |
| BC | 226.0 | $20^{\circ} 20^{\prime}$ |
| CD | 187.0 | $280^{\circ} 0^{\prime}$ |
| DA | $?$ | $210^{\circ} 80^{\prime}$ |

Compute the length and bearing of the line DA.
Solution,

| Line | Length(m) | Bearing | WCB | Northing <br> $(+)$ | Southing <br> $(-)$ | Easting <br> $(+)$ | Westing <br> $(-)$ |
| :---: | :---: | :---: | :--- | :---: | :---: | :---: | :---: |
| AB | 204.0 | $?$ | - | - |  | - |  |
| BC | 226.0 | $20^{\circ} 20^{\prime}$ | $90^{\circ}-180^{\circ}$ |  | $\mathrm{L} \operatorname{Cos} \Theta$ | $\mathrm{L} \operatorname{Sin} \Theta$ |  |
| CD | 187.0 | $280^{\circ} 0^{\prime}$ | $270^{\circ}-360^{\circ}$ | $\mathrm{L} \operatorname{Cos} \Theta$ |  |  | $\mathrm{L} \operatorname{Sin} \Theta$ |
| DA | $?$ | $210^{\circ} 80^{\prime}$ | $180^{\circ}-270^{\circ}$ |  | $\mathrm{L} \operatorname{Cos} \Theta$ |  | $\mathrm{L} \operatorname{Sin} \Theta$ |


| Line | Latitude $(\mathrm{L} \operatorname{Cos} \Theta)$ | Departure(L Sin $\theta)$ |
| :--- | :--- | :--- |
| AB | $204.0 \times \operatorname{Cos} 0=+204.0$ | $204.0 \times \operatorname{Sin} 0=0$ |
| BC | $226.0 \times \operatorname{Cos} 20^{\circ} 20^{\prime}=+211.091$ | $226.0 \times \operatorname{Sin} 20^{\circ} 20^{\prime}=+78.53$ |
| CD | $187.0 \times \operatorname{Cos} 280^{\circ} 0^{\prime}=+32.47$ | $187.0 \times \operatorname{Sin} 280^{\circ} 0^{\prime}=-184.15$ |
| DA | $\mathrm{L}_{1} \operatorname{Cos} 210^{\circ} 80=-0.854 \mathrm{~L}_{1}$ | $\mathrm{~L}_{1} \operatorname{Sin} 210^{\circ} 80=-0.52 \mathrm{~L}_{2}$ |

In a closed traverse, the algebraic sum of latitude and departure must be equal to zero.

$$
\begin{array}{lllll}
\sum \mathrm{L}=1_{1} \operatorname{Cos} \Theta_{1}+1_{2} \operatorname{Cos} \Theta_{2}+1_{3} \operatorname{Cos} \Theta_{3}+1_{4} \operatorname{Cos} \Theta_{4}+\ldots \ldots \ldots \ldots & = & 0 \\
\sum \mathrm{D}= & 1_{1} \operatorname{Sin} \Theta_{1}+l_{2} \operatorname{Sin} \Theta_{2}+1_{3} \operatorname{Sin} \Theta_{3}+1_{4} \operatorname{Sin} \Theta_{4}+\ldots \ldots \ldots \ldots & = & 0
\end{array}
$$

$$
+204+211.0913+32.47-0.854 \mathrm{~L}_{1}=0
$$

$$
\mathrm{L}_{1} \operatorname{Cos} \Theta=-524.1
$$

$$
+65.96+168.87-24.43+\mathrm{L}_{1} \operatorname{Sin} \Theta=210.4
$$

$$
\mathrm{L}_{1} \operatorname{Sin} \theta=203.11
$$

Since the latitude $(\mathrm{L} \operatorname{Cos} \theta)$ is positive and departure $(\mathrm{L} \operatorname{Sin} \theta)$ is in Negative, hence it is in North West Quadrant.
Bearing of line DA $=D$

\[

\]

## When the length of two side are omitted

A closed traverse was conducted round an obstacle and the following observations were made.

| Line | Length(m) | Bearings |
| :---: | :---: | :---: |
| AB | 300 | $98^{\circ} 30^{\prime}$ |
| BC | 620 | $30^{\circ} 20^{\prime}$ |
| CD | 468 | $298^{\circ} 30^{\prime}$ |
| DA | $?$ | $230^{\circ} 0^{\prime}$ |
| EA | $?$ | $150^{\circ} 10^{\prime}$ |

Compute the length and bearing of the line DA length and EA length.

## Solution,

L1 = Latitude of DA,
L2 = Latitude of DA,

| Line | Bearing | WCB | Northing (+) | Southing <br> (-) | $\begin{gathered} \text { Eastin } \\ g \\ (+) \end{gathered}$ | Westing <br> (-) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AB | $98^{\circ} 30^{\prime}$ | $90^{\circ}-180^{\circ}$ |  | $\mathrm{L} \operatorname{Cos} \theta$ | $\begin{gathered} \mathrm{L} \operatorname{Sin} \\ \Theta \end{gathered}$ |  |
| BC | $30^{\circ} 20^{\prime}$ | $0^{\circ}-90^{\circ}$ | $\mathrm{L} \operatorname{Cos} \theta$ |  | $\begin{gathered} \mathrm{L} \operatorname{Sin} \\ \Theta \end{gathered}$ |  |
| CD | $298{ }^{\circ} 3{ }^{\prime}$ | $270^{\circ}-360^{\circ}$ | L $\operatorname{Cos} \theta$ |  |  | $\mathrm{L} \operatorname{Sin} \theta$ |
| DA | $230^{\circ} 0{ }^{\prime}$ | $180^{\circ}-270^{\circ}$ |  | $\mathrm{L} \operatorname{Cos} \theta$ |  | $\mathrm{L} \operatorname{Sin} \theta$ |
| EA | $150^{\circ} 10^{\prime}$ | $90^{\circ}-180^{\circ}$ |  | $\mathrm{L} \operatorname{Cos} \Theta$ | $\begin{array}{\|c} \mathrm{L} \operatorname{Sin} \\ \Theta \end{array}$ |  |

In a closed traverse, the algebraic sum of latitude and departure must be equal to zero.

$$
\begin{align*}
& \sum \mathrm{L}=l_{1} \operatorname{Cos} \Theta_{1}+l_{2} \operatorname{Cos} \Theta_{2}+l_{3} \operatorname{Cos} \Theta_{3}+l_{4} \operatorname{Cos} \Theta_{4}+\ldots \ldots \ldots . .=0 \\
& \sum \mathrm{D}=1_{1} \operatorname{Sin} \Theta_{1}+l_{2} \operatorname{Sin} \Theta_{2}+1_{3} \operatorname{Sin} \Theta_{3}+1_{4} \operatorname{Sin} \Theta_{4}+ \\
& \text {............ } \\
& =0 \\
& -73.90+535.17+223.31-0.64 \mathrm{~L}_{1}-0.87 \mathrm{~L}_{2}=0 \\
& +494.51+313.12-411.28-0.76 \mathrm{~L}_{1}+0.48 \mathrm{~L}_{2}=0 \\
& -0.64 \mathrm{~L}_{1}-0.87 \mathrm{~L}_{2}=-684.58  \tag{1}\\
& -0.76 \mathrm{~L}_{1}+0.48 \mathrm{~L}_{2}=-396.35 \text {. }  \tag{2}\\
& 0.64 \mathrm{~L}_{1}+0.87 \mathrm{~L}_{2}=684.58  \tag{1}\\
& 0.76 \mathrm{~L}_{1}-0.48 \mathrm{~L}_{2}=396.35 \text {. } \tag{2}
\end{align*}
$$

Multiplying the Equation (1) with 0.76 and Multiplying the Equation (2) with 0.64
$0.4864 \mathrm{~L}_{1}+0.6612 \mathrm{~L}_{2}=$
$0.4864 \mathrm{~L}_{1}+0.3072 \mathrm{~L}_{2}=$

## (-) <br> (-)

520.28 $\qquad$ (3).
(-)
253.664

| $0.354 \mathrm{~L}_{2}$ |  | $=$ | 266.64 |
| :--- | :--- | :--- | :--- |
| $0.354 \mathrm{~L}_{2}$ |  | $=$ | 266.64 |
|  | $\mathrm{~L}_{2}$ | $=$ | 753.22 m |

Substituting the value of $\mathrm{L}_{2}$ in equation (3)

$$
\begin{array}{rll}
0.4864 \mathrm{~L}_{1}+0.6612 \times 753.22 & = & 520.28 \\
0.4864 \mathrm{~L}_{1} & = & -498.03 \\
\mathrm{~L}_{1} & = & 1023.9 \mathrm{~m}
\end{array}
$$

## Definition of Curves:

Curves are regular bends provided in the lines of communication like roads, railways etc. and also in canals to bring about the gradual change of direction. They are also used in the vertical plane at all changes of grade to avoid the abrupt change of grade at the apex.

Curves provided in the horizontal plane to have the gradual change in direction are known as Horizontal curves, whereas those provided in the vertical plane to obtain the gradual change in grade are known as vertical curves. Curves are laid out on the ground along the centre line of the work. They may be circular or parabolic.

## (i) Simple Curve:

A simple curve consists of a single arc of a circle connecting two straights. It has radius of the same magnitude throughout. In fig. 11.1 $\mathrm{T}_{1} \mathrm{D} \mathrm{T}_{2}$ is the simple curve with $\mathrm{T}_{1} \mathrm{O}$ as its radius.


Fig 11.1

## (ii) Compound Curve:

A compound curve consists of two or more simple curves having different radii bending in the same direction and lying on the same side of the common tangent. Their centres lie on the same side of the curve. In fig. 11.2, $\mathrm{T}_{1} \mathrm{P} \mathrm{T}_{2}$ is the compound curve with $\mathrm{T}_{1} \mathrm{O}_{1}$ and $\mathrm{PO}_{2}$ as its radii.


Fig 11.2

## (iii) Reverse (or Serpentine) Curve:

A reverse or serpentine curve is made up of two arcs having equal or different radii bending in opposite directions with a common tangent at their junction. Their centres lie of opposite sides of the curve. In fig. $11.3 \mathrm{~T}_{1} \mathrm{P} \mathrm{T}_{2}$ is the reverse curve with $\mathrm{T}_{1} \mathrm{O}_{1}$ and $\mathrm{PO}_{2}$ as its radii.


Fig 113

## Names of Various Parts of a Curve: (Fig. 11.5)



Fig 11.5
The two straight lines AB and BC , which are connected by the curve are called the tangents or straights to the curve.
(ii) The points of intersection of the two straights (B) is called the intersection point or the vertex.
(iii) When the curve deflects to the right side of the progress of survey as in fig. 11.5, it is termed as right handed curve and when to the left, it is termed as left handed curve.
(iv) The lines AB and BC are tangents to the curves. AB is called the first tangent or the rear tangent BC is called the second tangent or the forward tangent.
(v) The points ( $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ ) at which the curve touches the tangents are called the tangent points. The beginning of the curve $\left(\mathrm{T}_{1}\right)$ is called the tangent curve point and the end of the curve (T2) is called the curve tangent point.
(vi) The angle between the tangent lines AB and $\mathrm{BC}(\mathrm{ABC})$ is called the angle of intersection (I)
(vii) The angle by which the forward tangent deflects from the rear tangent is called the deflection angle ( $\phi$ ) of the curve.
(viii) The distance the two tangent point of intersection to the tangent point is called the tangent length $\left(\mathrm{BT}_{1}\right.$ and $\left.\mathrm{BT}_{2}\right)$.
(ix) The line joining the two tangent points ( $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ ) is known as the long-chord
(x) The arc $\mathrm{T}_{1} \mathrm{FT}_{2}$ is called the length of the curve.
(xi) The mid-point ( F ) of the arc $\left(\mathrm{T}_{1} \mathrm{FT}_{2}\right)$ in called summit or apex of the curve.
(xii) The distance from the point of intersection to the apex of the curve BF is called the apex distance.
(xiii) The distance between the apex of the curve and the midpoint of the long chord (EF) is called the versed sine of the curve.
(xiv) The angle subtended at the centre of the curve by the arc $\mathrm{T}_{1} \mathrm{FT}_{2}$ is known as the Central angle and is equal to the deflection angle ( $\phi$ ).

## Elements of a curve

(i) Angle of intersection + Deflection angle $=180^{\circ}$
or $\quad \mathrm{I}+\phi=180^{\circ} \quad \ldots \quad \ldots \quad \ldots$ (Eqn. 11.1)
(ii) $\angle \mathrm{T}_{1} \mathrm{OT}_{2}=180^{\circ}-\mathrm{I}=\phi \quad \ldots \quad \ldots \quad \ldots$ (Eqn. 11.2.) (i.e. the central angle $=$ the deflection angle).
(iii) Tangent length $=\mathrm{BT}_{1}=\mathrm{BT}_{2}=\mathrm{OT}_{1}$ tan $\frac{\phi}{2}$

$$
=R \tan \frac{\phi}{2} \quad \ldots \cdot \ldots \quad \ldots(\text { Eqn. 11.3 })
$$

(iv) Length of Long Chord $=2 \mathrm{~T}_{1} \mathrm{E}=2 \times \mathrm{OT}_{1} \sin \left(\frac{\phi}{2}\right)$

$$
\begin{equation*}
=2 \mathrm{R} \sin \frac{\phi}{2} \tag{Eqn.11.4}
\end{equation*}
$$

(v) Length of the curve $=$ Length of the arc $\mathrm{T}_{1} \mathrm{FT}_{2}$
$=\mathrm{R}_{\phi}$ (in radians)
$=\frac{\pi \mathrm{R} \phi}{180^{\circ}}$
$\mathrm{BF}=\mathrm{BO}-\mathrm{OF}$

$$
\begin{aligned}
& =\mathrm{R} \sec \frac{\phi}{2}-\mathrm{R} \\
& =\mathrm{R}\left(\sec \frac{(\phi}{2}-1\right) \ldots \quad \ldots(\text { Eqn. 11.6) }
\end{aligned}
$$

(vii) Versed sine of the curve $=\mathrm{EF}=\mathrm{OF}-\mathrm{OE}$

$$
\begin{aligned}
& =R-R \cos \frac{\phi}{2} \\
& =R\left(1=\cos \frac{\phi}{2}\right)=R \text { versine } \frac{\phi}{2} \ldots \ldots(\text { Eqn. 11.7) }
\end{aligned}
$$

## Methods of Curve Ranging:

## A curve may be set out:

1. By linear methods, where chain and tape are used.
2. By angular or instrumental methods, where a theodolite with or without a chain is used.

Before starting setting out a curve by any method, the exact positions of the tangent points between which the curve lies, must be determined.

## For this, proceed as follows: (Fig. 11.5)

(i) Having fixed the directions of the straights, produce them to meet at point (B).
(ii) Set up a theodolite at the intersection point (B) and measure the angle of intersection (I). Then find the deflection angle ( $\phi$ ) by subtracting (I) from $180^{\circ}$. i.e., $\phi=180^{\circ}$ - I
(iii) Calculate the tangent length from the Eqn. 11.3:
$\left(\tan\right.$ lenght $\left.=R \tan \frac{\phi}{2}\right)$
(iv) Measure the tangent length $\left(\mathrm{BT}_{1}\right)$ backward along the rear tangent BA from the intersection point B , thus locating the position of $\mathrm{T}_{1}$.
(v) Similarly, locate the position of $\mathrm{T}_{2}$ by measuring the same distance forward along the forward tangent BC from B ,

Having located the positions of the tangent points $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$; their changes may be determined. The change of $\mathrm{T}_{1}$ is obtained by subtracting the tangent length from the known change of the intersection point B . And the change of $\mathrm{T}_{2}$ is found by adding the length of the curve to the change to $\mathrm{T}_{1}$.

Then the pegs are fixed at equal intervals on the curve. The interval between the pegs is usually 30 m or one chain length. This distance should actually be measured along the arc, but in practice it is measured along the chord, as the difference between the chord and the corresponding arc is small and hence negligible. In order that this difference is always small and negligible, the length of the chord should not be more than $1 / 20$ th of the radius of the curve. The curve is then obtained by joining all these pegs.

The distances along the centre line of the curve are continuously measured from the point of beginning of the line upto the end, i.e., the pegs along the centre line of the work should be at equal interval from the beginning of the line to the end. There should be no break in the regularity of their spacing in passing from a tangent to a curve or from a curve to a tangent.

For this reason, the first peg on the curve is fixed at such a distance from the first tangent point $\left(\mathrm{T}_{1}\right)$ that its change becomes the whole number of chains i.e. the whole number of peg interval. The length of the first chord is thus less than the peg interval and is called as a sub- chord. Similarly there will be a sub chord at the end of the curve. Thus a curve usually consists of two-chords and a number of full chords. This is made clear from the following example.

## Transition Curves:

A non-circular curve of varying radius introduced between a straight and a circular curve for the purpose of giving easy changes of direction of a route is called a transition or easement curve. It is also inserted between two branches of a compound or reverse curve.

## Advantages of providing a transition curve at each end of a circular curve:

(i) The transition from the tangent to the circular curve and from the circular curve to the tangent is made gradual.
(ii) It provides satisfactory means of obtaining a gradual increase of super-elevation from zero on the tangent to the required full amount on the main circular curve.
(iii) Danger of derailment, side skidding or overturning of vehicles is eliminated.
(iv) Discomfort to passengers is eliminated.

## Conditions to be fulfilled by the transition curve:

(i) It should meet the tangent line as well as the circular curve tangentially.
(ii) The rate of increase of curvature along the transition curve should be the same as that of increase of super-elevation.
(iii) The length of the transition curve should be such that the full super-elevation is attained at the junction with the circular curve.
(iv) Its radius at the junction with the circular curve should be equal to that of circular curve.

## There are three types of transition curves in common use:

(1) A cubic parabola,
(2) A cubical spiral, and
(3) A lemniscate, the first two are used on railways and highways both, while the third on highways only.

When the transition curves are introduced at each end of the main circular curve, the combination thus obtained is known as combined or Composite Curve.

## Super-Elevation or Cant:

When a vehicle passes from a straight to a curve, it is acted upon by a centrifugal force in addition to its own weight, both acting through the centre of gravity of the vehicle. The centrifugal force acts horizontally and tends to push the vehicle off the track.

In order to counteract this effect the outer edge of the track is super elevated or raised above the inner one. This raising of the outer edge above the inner one is called super elevation or cant. The amount of super-elevation depends upon the speed of the vehicle and radius of the curve.


## Let:

$\mathrm{W}=$ the weight of vehicle acting vertically downwards.
$\mathrm{F}=$ the centrifugal force acting horizontally,
$\mathrm{v}=$ the speed of the vehicle in metres $/ \mathrm{sec}$.
$\mathrm{g}=$ the acceleration due to gravity, 9.81 metres $/ \mathrm{sec}^{2}$.
$\mathrm{R}=$ the radius of the curve in metres,
$h=$ the super-elevation in metres.
$\mathrm{b}=$ the breadth of the road or the distance between the centres of the rails in metres.
Then for equilibrium, the resultant of the weight and the centrifugal force should be equal and opposite to the reaction perpendicular to the road or rail surface.

$$
\begin{aligned}
& \text { The centrifugal force, } \mathrm{F}=\frac{\mathrm{W} v^{2}}{g \mathrm{R}} \\
& \therefore \quad \frac{\mathrm{~F}}{\mathrm{~W}} \equiv \frac{v^{2}}{g \mathrm{R}}
\end{aligned}
$$

If $\theta$ is the inclination of the road or rail surface, the inclination of the vertical is also $\theta$

$$
\tan \theta=\frac{d c}{a c}=\frac{\mathrm{F}}{\mathrm{~W}}=\frac{v^{2}}{g \mathrm{R}}
$$

uper-elevation $=b \tan \theta$.

$$
=\frac{b v^{2}}{g R} \quad \ldots \quad \ldots \quad \text { (Eqn. 11.28) }
$$

## Characteristics of a Transition Curve (Fig 11.25):

Here two straights AB and BC make a deflection angle $\Delta$, and a circular curve EE ' of radius R , with two transition curves TE and E'T' at the two ends, has been inserted between the straights.

It is clear from the figure that in order to fit in the transition curves at the ends, a circular imaginary curve $\left(\mathrm{T}_{1} \mathrm{~F}_{1} \mathrm{~T}_{2}\right)$ of slightly greater radius has to be shifted towards the centre as $\left(\mathrm{E}_{1} \mathrm{EF} E \mathrm{E}_{1}\right.$. The distance through which the curve is shifted is known as shift ( S ) of the curve, and is equal to $\frac{\mathrm{L}^{2}}{24 \mathrm{R}}$,
where L is the length of each transition curve and R is the radius of the desired circular curve (EFE'). The length of shift ( $\mathrm{T}_{1} \mathrm{E}_{1}$ ) and the transition curve (TE) mutually bisect each other.

(ii) The tangent length for the combined curve

$$
\begin{aligned}
& =\mathrm{O}_{1} \tan \frac{\Delta}{2}+\frac{\mathrm{L}}{2} \\
& =(\mathrm{R}+\mathrm{S}) \tan \frac{\Delta}{2}+\frac{\mathrm{L}}{2}
\end{aligned}
$$

(iii) The spiral angle $\varphi_{1}=\frac{\frac{L}{2}}{R}=\frac{L}{2 R}$ radians

## (iv) The central angle for the circular curve:

$\angle E O E=\Delta 2 \phi_{1}$
(v) Length of the circular curve EFE'
$=\frac{\pi \mathrm{R}\left(\Delta-2 \phi_{1}\right)}{180^{\circ}}$, where $\Delta$ and $\phi_{1}$ are in degrees.
(vi) Length of the combined curve TEE'T"
$=\mathrm{TE}+\mathrm{EE}^{\prime}+\mathrm{E}^{\prime} \mathrm{T}^{\prime}$
$=\mathrm{L}+\frac{\pi \mathrm{R}\left(\Delta-2 \phi_{1}\right)}{180^{\circ}}+\mathrm{L}$
$=\frac{\pi R\left(\Delta-2 \phi_{1}\right)}{180^{\circ}}+2 L$
(vii) Change of beginning ( T ) of the combined curve $=$ Change of the intersection point (B)-total tangent length for the combined curve (BT).
(viii) Change of the junction point (E) of the transition curve and the circular curve $=$ Change of T + length of the transition curve (L).
(ix) Change of the other junction point ( $E^{\prime}$ ) of the circular curve and the other transition curvechange of $E+$ length of the circular curve.
(x) Change of the end point ( $T^{\prime}$ ) of the combined curve $=$ change of $E^{\prime}+$ length of the transition curve.

## Check:

The change of T thus obtained should $\mathrm{be}=$ change of $\mathrm{T}+$ length of the combined curve .

## Note:

The points on the combined curve should be pegged out with through change so that there will be sub-chords at each end of the transition curve and of the circular curve.
(xi) The deflection angle for any point on the transition curve distant I from the beginnings of combined curve (T),

$$
\begin{aligned}
\alpha & =\frac{l^{2}}{6 \mathrm{RL}} \text { radians }=\frac{1800 l^{2}}{\pi \mathrm{RL}} \text { minutes. } \\
& =\frac{573 l^{2}}{\mathrm{RL}} \text { minutes. }
\end{aligned}
$$

## Check:

The deflection angle for the full length of the transition curve:

$$
\begin{aligned}
\alpha & =\frac{l^{2}}{6 \mathrm{RL}}=\frac{\mathrm{L}^{2}}{6 \mathrm{RL}} \quad(\because l=\mathrm{L}) \\
& =\frac{\mathrm{L}}{6 \mathrm{R}} \text { radians }=\frac{1}{3} \phi_{1}
\end{aligned}
$$

(xii) The deflection angles for the circular curve are found from:

$$
\delta_{n}=1718.9 \frac{\mathrm{C}_{n}}{\mathrm{R}} \text { minutes. }
$$

## Check:

The deflection angle for the full length of the circular curve:
$\Delta_{\mathrm{n}}=\frac{1}{2} \times$ Central angle
i.e., $\Delta_{\mathrm{n}}=\frac{1}{2} \times\left(\Delta-2 \emptyset_{1}\right)$
(xiii) The offsets for the transition curve are found from:

Perpendicular offset, $y=\frac{x^{-}}{6 R L}$, where $x$ is measured along the tangent TB
Tangentail offset, $y=\frac{l^{3}}{6 R L}$, where $I$ is measured along the curve

Check: (a) The offset at half the length of the transition curve,

$$
\begin{aligned}
y & =\frac{l^{3}}{6 \mathrm{RL}}=\frac{(L / 2)^{3}}{6 \mathrm{RL}}(\because l=\mathrm{L} / 2) \\
& =\frac{\mathrm{L}^{2}}{48 \mathrm{R}}=\frac{1}{2} \mathrm{~S}
\end{aligned}
$$

(b) The offset at junction point on the transition curve,

$$
\begin{aligned}
y=\frac{l^{3}}{6 \mathrm{RL}}=\frac{\mathrm{L}^{3}}{6 \mathrm{RL}} & =\frac{\mathrm{L}^{2}}{6 \mathrm{R}}(\because l=\mathrm{L}) \\
& =4 \mathrm{~S}
\end{aligned}
$$

(xiv) The offsets for the circular curve from chords producers are found from:

$$
\mathrm{O}_{n}=\frac{C_{n}\left(C_{n-1}+C_{n}\right)}{2 R}
$$

## Method of Setting Out Combined Curve by reflection Angles (Fig. 11.25):

The first transition curve is set out from T by the deflection angles and the circular curve from the junction point $E$. The second transition curve is then set out from T' and the work is checked on the junction point E' which has been previously fixed from E.
(i) Assume or calculate the length of the transition curve.
(ii) Calculate the value of the shift by:

$$
\mathrm{S}=\frac{L^{2}}{24 R}
$$

(iii) Locate the tangent point T by measuring backward the total tangent length BT (article 11.14, ii) from the intersection point B along BA , and the other tangent T by measuring forward the same distance from $B$ along $B C$.
(iv) Set up a theodolite at T , set the vernier A to zero and bisect B .
(v) Release the upper clamp and set the vernier to the first deflection angle ( $\mathrm{x}_{1}$ ) As obtained from the table of deflection angles, the line of sight is thus directed along the first point on the transition curve. Place zero end of the tape at T and measure along this line a distance equal to first sub chords, thus locating first point on the transition curve.
(vi) Repeat the process, until the end of the curve E is reached.

## Check:

The last deflection angle should be equal to $\varphi_{1} / 3$, and the perpendicular offset from the tangent TB for the last point E should be equal to 4 S .

## Note:

The distance to each of the successive points on the transition curve is measured from T.
(vii) Having laid the transition curve, shift the theodolite to E and set it up and level it accurately. (viii) Set the vernier to a reading ( $360^{\circ}-2 / 3 \varphi 1$ ) for a right-hand curve (or $2 / 3 \varphi 1$ ) for a left-hand curve and lake a back sight on T. Loosen the upper clamp and turn the telescope clockwise through an angle $2 / 3 \varphi 1$ the telescope is thus directed towards common tangent at E and the vernier reads $360^{\circ}$. Transit the telescope, now it points towards the forward direction of the common tangent at E i.e. towards the tangent for the circular curve.
(ix) Set the vernier to the first tabulated deflection angle for the circular curve, and locate the first point on the circular curve as already explained in simple curves.
(x) Set out the complete circular curve up to E' in the usual way

## Check:

The last deflection angle should be equal to $\frac{1}{2}\left(\Delta-2 \Phi_{1}\right)$
(xi) Set out the other transition curve from T as before. The point E ' to be set from T should be the same as already set out from E.

## Method of Setting Out a Combined Curve by Tangential Offsets (Fig. 11.25):

(i) Assume or calculate the length of the transition curve.
(ii) find the value of the shift train,
(iii) Locate the tangent points T and T as in article (11.15, iii),
(iv) Calculate the offset for the transition curve as in article (11.14 xiv)
(v) Locate die points on the transition curve as well as on the circular curves by setting out the respective offsets.

## SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

## Distance and Elevation by Tacheometry Tacheometric Surveying

Tacheometric is a branch of surveying in which horizontal and vertical distances are determined by taking angular observation with an instrument known as a tachometer. Tacheometric surveying is adopted in rough in rough and difficult terrain where direct leveling and chaining are either not possible or very tedious. The accuracy attained is such that under favorable conditions the error will not exceed $1 / 100$. and if the purpose of a survey does not require accuracy, the method is unexcelled. Tacheometric survey also can be used for Railways, Roadways, and reservoirs etc. Though not very accurate.Tacheometric surveying is very rapid, and a reasonable contour map can be prepared for investigation works within a short time on the basis of such survey.

## Uses of Tachometry

$\checkmark$ Tachometry is used for
$\checkmark$ preparation of topographic map where both horizontal and vertical distances are required to be measured;
$\checkmark$ survey work in difficult terrain where direct methods of measurements are inconvenient;
$\checkmark$ reconnaissance survey for highways and railways etc;
Establishment of secondary control points.

## Instruments used in tachometric surveying

An ordinary transits theodolite fitted with a stadia diaphragm is generally used for tacheometric surveying. The stadia diaphragm essentially consists of one stadia hair above and the other an equal distance below the horizontal cross hair, the stadia hair being mounted in the same ring and in the same vertical plane as the horizontal and vertical cross-hair.

The telescope used in stadia surveying are three kinds,
$\checkmark$ The Simple external focusing telescope.
$\checkmark$ The external focusing anal lactic telescope (porro's telescope).
$\checkmark$ The internal focusing telescope.

The first type is known as stadia theodolite, while the second type is known as tacheometer. The tacheometer has the advantage over the first and third type due to fact that the additive constant of the instrument is zero.


The instruments employed in tachometry are the engineer's transit and the leveling rod or stadia rod, the theodolite and the subtense bar, the self-reducing theodolite and the leveling rod, the distance wedge and the horizontal distance rod, and the reduction tacheometer and the horizontal distance rod.

## Features of tacheometer or Characteristic of tacheometer

The multiple constant ( $\mathrm{f} / \mathrm{i}$ ) should have a normal value of 100 and the error contained in this value should not exceed 1 in 1000.

The axial horizontal lines should be exactly midway between the other two lines. The telescope should be fitted with an anallatic lens to make the additive constant
$(\mathrm{f}+\mathrm{d})$ exactly to zero.
The telescope should be truly analectic.
The telescope should be powerful having a magnification of 20 to 30 diameters. The Aperture of the object should be 35 to 45 mm in diameter.

## Levelling and Stadia Staff Rod

For short distances, ordinary leveling staves are used. The leveling staff normally 4 m long, and it can be folded with here parts. The graduations are so marked that a minimum reading of 0.005 or 0.001 m can be taken.

## Different systems of Tacheometric Measurement

The various systems of tacheometric survey may be classified as follows, The Stadia Method
a) Fixed Hair Method and
b) Movable Hair Method
c) Tangential System

## Measurements by means of special instruments.

The principle is common to all system is to calculate the horizontal distance between two points $A$ and $B$ their deference in elevation, by observing 1) the angle at the instrument at $A$ subtended by known short distance along a staff kept at $B$ and 2) the vertical angle to $B$ from A

## Stadia systems

In this systems staff intercepts, at a pair of stadia hairs present at diaphragm, are considered.
The stadia system consists of two methods:

- Fixed-hair method and
- Movable-hair method


## Fixed-hair method

In this method, stadia hairs are kept at fixed interval and the staff interval or intercept (corresponding to the stadia hairs) on the leveling staff varies. Staff intercept depends upon the distance between the instrument station and the staff.

## Movable- hair method

In this method, the staff interval is kept constant by changing the distance between the stadia hairs. Targets on the staff are fixed at a known interval and the stadia hairs are adjusted to bisect the upper target at the upper hair and the lower target at the lower hair. Instruments used in this method are required to have provision for the measurement of the variable interval between the stadia hairs. As it is inconvenient to measure the stadia interval accurately, the movable hair method is rarely used.

## Non-stadia systems

This method of surveying is primarily based on principles of trigonometry and thus telescopes without stadia diaphragm are used. This system comprises of two methods:

- Tangential method and
- Subtense bar method.


## Tangential method

In this method, readings at two different points on a staff are taken against the horizontal cross hair and corresponding vertical angles are noted.

## Subtense bar method.

In this method, a bar of fixed length, called a subtense bar is placed in horizontal position. The angle subtended by two target points, corresponding to a fixed distance on the subtense bar, at the instrument station is measured. The horizontal distance between the subtense bar and the instrument is computed from the known distance between the targets and the measured horizontal angle.

## Principles of Stadia Method

(Figure 23.1) A tacheometer is temporarily adjusted on the station P with horizontal line of sight. Let $a$ and $b$ be the lower and the upper stadia hairs of the instrument and their actual vertical separation be designated as i. Let f be the focal length of the objective lens of the tacheometer and c be horizontal distance between the optical centre of the objective lens and the vertical axis of the instrument. Let the objective lens is focused to a staff held vertically at Q , say at horizontal distance D from the instrument station.


Figure 1: Principle of Stadia Measurement

By the laws of optics, the images of readings at A and B of the staff will appear along the stadia hairs at a and b respectively. Let the staff interval i.e., the difference between the readings at A and B be designated by s. Similar triangle between the object and image will form with vertex at the focus of the objective lens $(\mathrm{F})$. Let the horizontal distance of the staff from F be d . Then, from the similar Ds ABF and a' b' F,

$$
\begin{aligned}
& \frac{A B}{d}=\frac{a^{\prime} b^{\prime}}{f} \\
& O r, d=\frac{A B}{a^{\prime} b^{\prime}} \times f=\frac{s}{i} \times f \\
& \therefore d=\frac{f}{i} \times s
\end{aligned}
$$

as $a^{\prime} b^{\prime}=a b=i$. The ratio (f $/ i$ ) is a constant orf a particular instrument and is known as stadia interval factor, also instrument constant. It is denoted by K and thus
d = K.s ---------------------- Equation (23.1)
The horizontal distance (D) between the center of the instrument and the station point (Q) at which the staff is held is $d+f+c$. If $C$ is substituted for $(f+c)$, then the horizontal distance D from the center of the instrument to the staff is given by the equation
$\mathrm{D}=\mathrm{Ks}+\mathrm{C}$
Equation (23.2)
The distance C is called the stadia constant. Equation (23.2) is known as the stadia equation for a line of sight perpendicular to the staff intercept.

## Theory of Stadia Tacheometry

The following is the notation used in stadia tacheometry

$$
\begin{array}{ll}
\mathrm{O} & =\text { Optical centre of object glass. } \\
\mathrm{A}_{1}, \mathrm{~A}_{2}, \mathrm{C} & =\text { Readings on staff cut by three hairs } \\
\mathrm{a}_{1, \mathrm{a}_{2}, \mathrm{C}} & =\text { Bottom Top, and Central Hair of diaphragm } \\
\mathrm{a}_{1 \mathrm{a}_{2,}, \mathrm{i}} & =\text { length of image } \\
\mathrm{A}_{1}, \mathrm{~A}_{2},=\mathrm{S} & =\text { Staff Intercept } \\
\mathrm{V} & =\text { Vertical axis of instrument }
\end{array}
$$



| f | $=$ Focal lengh of a object glass |
| :--- | :--- |
| d | $=$ distance between optical centre and vatical axis of instrument |
| u | $=$ distance between optical centre and staff |
| v | $=$ distance between optical centre and inage |

For similar triangles $\mathrm{a}_{1}, \mathrm{Oa}_{2 \mathrm{men}} \mathrm{A}_{1}, \mathrm{OA}_{2_{2}}$
$\frac{i}{s}=\frac{v}{u} \quad$ or $\quad v=\frac{\text { in }}{s}$
From the properties of length.

$$
\begin{equation*}
\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \tag{2}
\end{equation*}
$$

Putting the value of v in Eq. (2)

$$
\frac{1}{\mathrm{in} / 5}+\frac{1}{u}=\frac{1}{\mathrm{f}}
$$

Or

$$
\frac{5}{\text { in }}+\frac{1}{u}=\frac{1}{f}
$$

Or

$$
\frac{1}{u} \quad\left\{\frac{s}{i}+1\right\}=\frac{1}{f}
$$

$$
\begin{aligned}
& u=\left\{\frac{s}{i}+1\right\} f \\
& D=u+d- \\
& D=\left\{\frac{s}{i}+1\right\} f+d \\
& \frac{s}{i} x f+f+d=\left\{\frac{f}{i}\right\} x s+(f+d)
\end{aligned}
$$

The quanta ties ( $\mathrm{f} / \mathrm{i}$ ) and ( $\mathrm{f}+\mathrm{d}$ ) are known as techeometric constants. ( $\mathrm{f} / \mathrm{i}$ ) is called the multiplying constant, as already stated, and ( $\mathrm{f}+\mathrm{d}$ ) the additive constant. by adopting an anal latic lens in the telescope of a tacheometric, the multiplying constant is made 100 , and the additive constant zero. However, in some tacheometers the additive constants are not exactly zero, but vary from 30 cm to 60 cm

## Inclined Stadia Measurements

It is usual that the line of sight of the tacheometer is inclined to the horizontal. Thus, it is frequently required to reduce the inclined observations into horizontal distance and difference in elevation.


Figure 2: Principle of Stadia Measurement

Let us consider a tacheometer (having constants K and C ) is temporarily adjusted on a station, say $P$ (Figure 23.2). The instrument is sighted to a staff held vertically, say at Q . Thus, it is required to find the horizontal distance PP1 $(=\mathrm{H})$ and the difference in elevation P1Q. Let A, R and B be the staff points whose images are formed respectively at the upper, middle and lower cross hairs of the tacheometer. The line of sight, corresponding to the middle cross hair, is inclined at an angle of elevation q and thus, the staff with a line perpendicular to the line of sight. Therefore $A^{\prime} B^{\prime}=A B \cos q=s \cos q$ where $s$ is the staff intercept AB . The distance $\mathrm{D}(=\mathrm{OR})$ is $\mathrm{C}+\mathrm{K} \cdot \operatorname{s\operatorname {cos}q\text {(fromEquation23.2).Butthedistance}}$ OO1 is the horizontal distance $H$, which equals OR $\cos q$. Therefore the horizontal distance $H$ is given by the equation.
$H=(K s \cos q+C) \cos q$

Or $\mathrm{H}=\mathrm{Ks} \cos 2 \mathrm{q}+\mathrm{C} \cos \mathrm{q}$------------------ Equation (23.3)
in which K is the stadia interval factor ( $\mathrm{f} / \mathrm{i}$ ), s is the stadia interval, C is the stadia constant ( f $+c)$, and $q$ is the vertical angle of the line of sight read on the vertical circle of the transit.

The distance RO1, which equals OR $\sin \mathrm{q}$, is the vertical distance between the telescope axis and the middle cross-hair reading. Thus V is given by the equation
$V=(K s \cos q+c) \sin q$

$$
V=K s \sin q \cos q+C \sin q--------------- \text { Equation (23.4) }
$$

Thus, the difference in elevation between P and Q is $(\mathrm{h}+\mathrm{V}-\mathrm{r})$, where h is the height of the instrument at P and r is the staff reading corresponding to the middle hair.

## Uses of Stadia

The stadia method of surveying is particularly useful for following cases:

- In differential leveling, the back sight and foresight distances are balanced conveniently if the level is equipped with stadia hairs.
- In profile leveling and cross sectioning, stadia is a convenient means of finding distances from level to points on which rod readings are taken.
- In rough trigonometric, or indirect, leveling with the transit, the stadia method is more rapid than any other method.
- For traverse surveying of low relative accuracy, where only horizontal angles and distances are required, the stadia method is a useful rapid method.
- On surveys of low relative accuracy - particularly topographic surveys-where both the relative location of points in a horizontal plane and the elevation of these points are desired, stadia is useful. The horizontal angles, vertical angles, and the stadia interval are observed, as each point is sighted; these three observations define the location of the point sighted.


## Errors in Stadia Measurement

Most of the errors associated with stadia measurement are those occur during observations for horizontal angles (Lesson 22) and differences in elevation (Lesson 16). Specific sources of errors in horizontal and vertical distances computed from observed stadia intervals are as follows:

## Error in Stadia Interval factor

This produces a systematic error in distances proportional to the amount of error in the stadia interval factor.

## Error in staff graduations

If the spaces on the rod are uniformly too long or too short, a systematic error proportional to the stadia interval is produced in each distance.

## Incorrect stadia Interval

The stadia interval varies randomly owing to the inability of the instrument operator to observe the stadia interval exactly. In a series of connected observations (as a traverse) the
error may be expected to vary as the square root of the number of sights. This is the principal error affecting the precision of distances. It can be kept to a minimum by proper
focusing to eliminate parallax, by taking observations at favorable times, and by care in observing.

## Error in verticality of staff

This condition produces a perceptible error in measurement of large vertical angles than for small angles. It also produces an appreciable error in the observed stadia interval and hence in computed distances. It can be eliminated by using a staff level.

## Error due to refraction

This causes random error in staff reading.

## Error in vertical angle

Error in vertical angle is relatively unimportant in their effect upon horizontal distance if the angle is small but it is perceptible if the vertical angle is large.

## Tangential Method

The tangential method of tacheometry is being used when stadia hairs are not present in the diaphragm of the instrument or when the staff is too far to read.

In this method, the staff sighted is fitted with two big targets (or vanes) spaced at a fixed vertical distances. Vertical angles corresponding to the vanes, say q1 and q2 are measured. The horizontal distance, say D and vertical intercept, say V are computed from the values s (pre-defined known) q1 and q2. This method is less accurate than the stadia method.

Depending on the nature of vertical angles i.e, elevation or depression, three cases of tangential methods are there.

When the Both Angles are Angles of Elevation
When one of the Vertical Angles is Elevation Angle and the other is Depression Angle When Both Vertical Angles are Depression Angles

When the Both Angles are Angles of Elevation


Figure 3

$$
\mathrm{V}=\mathrm{D} \tan \mathrm{q} 1
$$

and $\mathrm{V}+\mathrm{s}=\mathrm{D} \tan \mathrm{q} 2$

Thus, $\mathrm{s}=\mathrm{D}(\tan \mathrm{q} 2-\tan \mathrm{q} 1)$

$$
\begin{aligned}
& \text { and } V=\frac{8}{\left(\tan \theta_{2} \cdot \tan \theta_{1}\right)} \tan \theta_{1} \\
& \text { Equation (24.2) }
\end{aligned}
$$

Therefore RL. of $\mathrm{Q}=(\mathrm{RL}$. of $\mathrm{P}+\mathrm{h})+\mathrm{V}-\mathrm{I}-\ldots-\mathrm{B}-\mathrm{B}$ - Equation (24.3)
where, $h$ is the height of the instrument, $r$ is the staff reading corresponding to lower vane.

When Both Vertical Angles are Depression Angles


Figure 4
$\mathrm{V}=\mathrm{D} \tan \mathrm{q} 1$
and $\mathrm{V}-\mathrm{s}=\mathrm{D} \tan \mathrm{q} 2$

Thus, $\mathrm{s}=\mathrm{D}(\tan \mathrm{q} 1-\tan \mathrm{q} 2)$

$$
\begin{aligned}
& \operatorname{Or} \mathrm{C}=\frac{5}{\left(\tan \theta_{2}-\tan \theta_{1}\right)} \\
& \text { Equation (24.4) } \\
& \text { and } V=\frac{s}{\left(\tan \theta_{1}-\tan \theta_{2}\right)} \tan \theta_{1}
\end{aligned}
$$

Therefore R.L. of $\mathrm{Q}=(\mathrm{R} . \mathrm{L}$. of $\mathrm{P}+\mathrm{h})-\mathrm{V}-\mathrm{r}--------$ - Equation (24.6)
Where, h is the height of the instrument, r is the staff reading corresponding to lower vane.

When one of the Vertical Angles is Elevation Angle and the other is Depression Angle


Figure 5
$\mathrm{V}=\mathrm{D} \tan \mathrm{q} 1$
and $\mathrm{s}-\mathrm{V}=\mathrm{D} \tan \mathrm{q} 2$

Thus, $\mathrm{s}=\mathrm{D}(\tan \mathrm{q} 2+\tan \mathrm{q} 1)$

Or, $D=\frac{s}{\left(\tan \theta_{2}+\tan \theta_{1}\right)}$


Therefore R.L. of $\mathrm{Q}=(\mathrm{R} . \mathrm{L}$. of $\mathrm{P}+\mathrm{h})-\mathrm{V}-\mathrm{r}-\mathrm{-}-\mathrm{-}-\mathrm{-}-\mathrm{-}-\mathrm{-}-\mathrm{-}$ - Equation (24.9)
where, h is the height of the instrument, r is the staff reading corresponding to lower vane.

## Example Problem

In a tangential method of tacheometry two vanes were fixed 2 m apart, the lower vane being 0.5 m above the foot of the staff held vertical at station A . The vertical angles measured are $+1^{\circ} 12^{\prime}$ and $-1^{\circ} 30$ '. find the horizontal distance of A and reduced level of A, if the R.L. of the observation station is 101.365 m and height of instrument is 1.230 m .


## Solution:

Let D be the horizontal distnce between the observation station P and staff point A . Then, from Figure Ex24.1,
$\mathrm{V}=\mathrm{D} \tan \mathrm{a} 1$
$\mathrm{s}-\mathrm{V}=\mathrm{D}$ tana 1
Or, $\mathrm{s}=\mathrm{D} \operatorname{tana} 2+\mathrm{D} \operatorname{tana} 1$
$\therefore \mathrm{D}=\frac{\mathrm{s}}{\tan \alpha_{1}+\tan \alpha_{2}}$
Given, $\mathrm{s}=2 \mathrm{~m} ; \mathrm{a} 1=1^{\circ} 30^{\prime} \& \mathrm{a} 2=1^{\circ} 12^{\prime}$

$$
\begin{aligned}
& \therefore D=\frac{2.0}{\tan \left(1^{\circ} 30^{\prime}\right)+\tan \left(1^{\circ} 12\right)}=42.43 \mathrm{~m} \\
& \text { and } V=\frac{\operatorname{stan} \alpha_{1}}{\tan \alpha_{1}+\tan \alpha_{2}}=\frac{\operatorname{stan} 1^{\circ} 30^{\prime}}{\tan 1^{\circ} 30^{\prime}+\tan 1^{\circ} 12^{\prime}}=1.11 \mathrm{~m}
\end{aligned}
$$

Therefore R.L. of $\mathrm{A}=101.365+1.230-1.11-0.5=100.985 \mathrm{~m}$

## Tacheometric Formula

## Different systems of Tacheometric Measurement

The various systems of tacheometric survey may be classified as follows

- The Stadia Method
I. Fixed Hair Method

When the line of sight is horizontal, with staff held vertical


When the line of sight is horizontal, the general formula for finding distance between instruments station and object station.

D $\quad=\quad \frac{f}{i} \times S+(\mathrm{f}+\mathrm{d})$

Where,

$$
\frac{f}{i}=\text { Multiplying constant }(\mathrm{K})(\mathrm{f}+\mathrm{d})=\text { Additive constant }(\mathrm{C})
$$

$\mathrm{D} \quad=\mathrm{K} x \mathrm{~S}+\mathrm{C}$
Generally in tacheometry surveying, the multiplying constant $\mathrm{i}=100$, and
additive constants $(\mathrm{f}+\mathrm{d})=0$.
RL of Instruments axis $=\quad$ RL of BM +BS
RL of station point $\mathrm{P}=$
RL of Instrument axis-h
Where,
BM =Bench Mark
BS =Back Sight
height of the instrument $\mathrm{HI}=$ Height of Collimation

When the line of sight is Inclined(Angle of Elevation), with staff held vertical


When the line of sight is Inclined (Angle of Elevation), the general formula for finding distance between instruments station and object station.


Generally in tacheometry surveying, the multiplying constant $\mathrm{i}=100$, and additive constants $(\mathrm{f}+\mathrm{d})=0$.
We know that,
V $=\quad \mathrm{L} \operatorname{Sin} \varnothing$
L

$$
=\quad \frac{f}{i} \times \operatorname{Sos} \operatorname{Cos}+(\mathrm{f}+\mathrm{d})
$$

Therefore,

1. $\quad f \times \operatorname{SCos} \varnothing \operatorname{Sin} \emptyset+(\mathrm{f}+\mathrm{d}) \operatorname{Sin} \emptyset$
$i$
$\mathrm{V}=\mathrm{D} \tan \varnothing$

RL of Instruments axis =
RL of station point $\mathrm{P}=$
$R L$ of $B M+B S$
RL of Instrument axis + V -h
c) When the line of sight is Angle of Depression with staff held vertical Line of Axis


When the line of sight is Angle of Depression the general formula for finding distance between instruments station and object station.

$$
\mathrm{D} \quad=\quad \frac{f}{i} \times \operatorname{Sos}^{2} \emptyset+(\mathrm{f}+\mathrm{d}) \cos \emptyset
$$

Generally in tacheometry surveying, the multiplying constant $i=100$, and additive constants $(\mathrm{f}+\mathrm{d})=0$.

We know that,
V
L

$$
\begin{aligned}
& =\quad \mathrm{L} \operatorname{Sin} \varnothing \\
& =\quad \frac{f}{i} \times \operatorname{SCos} \emptyset+(\mathrm{f}+\mathrm{d})
\end{aligned}
$$

Therefore,

$$
f_{-} \mathrm{x} \operatorname{SCos} \emptyset \operatorname{Sin} \emptyset+(\mathrm{f}+\mathrm{d}) \operatorname{Sin} \emptyset
$$

$i$

$$
\begin{array}{ll}
\mathrm{V}= & \frac{f \mathrm{x}}{i 2} \underline{S x \operatorname{Sin} 2 \phi}+(\mathrm{f}+\mathrm{d}) \operatorname{Sin} \emptyset \\
\mathrm{V} & =\mathrm{D} \tan \emptyset
\end{array}
$$

RL of Instruments axis $=$ $R L$ of $B M+B S$
RL of station point $\mathrm{P}=$

RL of Instrument axis $-V-h$

## Problem

1 A tacheometer is set up at a station point P and the following readings were taken

| Instrument <br> Station | Staff <br> Station | Vertical <br> Angle | Staff Reading | Remarks |
| :--- | :--- | :--- | :--- | :--- |
| A | BM | $-6^{\circ} 30^{\prime}$ | $0.98,1.54,2.10$ | RL of the |
| A | P | $+12^{\circ} 24^{\prime}$ | $1.89,2.48,3.07$ | BM <br> 384.5 m |

Calculate the horizontal distance between AB and RL of B , when the constants of the instruments are 100 and 0.5

## Solution

When the line of sight is inclined, the general formula for finding distance between instrument station and the object station.

$$
\mathrm{D}=\frac{f}{i} \times \operatorname{Sos}^{2} \emptyset+(\mathrm{f}+\mathrm{d}) \operatorname{Cos} \emptyset
$$

The tacheometry surveying, the multiplying constant $\mathrm{i}=100$, and additive constants $(\mathrm{f}+\mathrm{d})=$ 0.5

a) First observation (BM to A)

$$
\begin{aligned}
\mathrm{S}_{1} & =2.10-0.98=1.12 \mathrm{~m} \\
\emptyset & =6^{\circ} 30^{\prime} \\
& \\
\mathrm{D}_{1} & =\underline{f} \times \operatorname{Sec}^{2} \emptyset+(\mathrm{f}+\mathrm{d}) \cos \emptyset
\end{aligned}
$$

Second observation (A to P)

$$
\begin{aligned}
& \mathrm{S}_{2}=3.07-1.89=1.18 \mathrm{~m} \\
& \emptyset=12^{\circ} 24^{\prime}
\end{aligned}
$$

$$
\mathrm{D}_{1}=\quad \frac{f}{i} \mathrm{x} \mathrm{~S} \operatorname{Cos}^{2} \emptyset+(\mathrm{f}+\mathrm{d}) \operatorname{Cos} \emptyset
$$

$$
=\quad 100 \times 1.18 \operatorname{Cos}^{2} 12^{\circ} 24^{\prime}+\left(0.5 \operatorname{Cos} 12^{\circ} 24^{\prime}\right.
$$

$$
=\quad 113.05 \mathrm{~m}
$$

$\mathrm{V}_{1}$

$$
=\quad f \times \frac{S x \operatorname{Sin} 2 \emptyset}{2}+(\mathrm{f}+\mathrm{d}) \operatorname{Sin} \emptyset
$$

2

$$
100 \times \underline{1.18 x \operatorname{Sin} 2 \times 12^{\circ} 24}+0.5 \operatorname{Sin} 12^{\circ} 24^{\prime}
$$

### 24.85 m

$$
\begin{array}{ll}
\text { RL of the instruments axis }= & \mathrm{RL} \text { of } \mathrm{BM}+\mathrm{h}_{1}+\mathrm{V}_{1} \\
& =384.5+12.65+1.54 \\
& =398.69 \mathrm{~m} \\
\text { RL of } \mathrm{P} \quad & =\quad \mathrm{RL} \text { of instrument axis }+\mathrm{V}_{2}-\mathrm{h}_{1} \\
& =421.06
\end{array}
$$

2.Atacheometer is set up at a station point P and the following readings were taken

| Instrument <br> Station | Staff <br> Station | Vertical <br> Angle | Staff Reading | Remarks |
| :--- | :--- | :--- | :--- | :--- |
| A | BM | $-7^{\circ} 30^{\prime}$ | $1.300,1.80,2.30$ | RL of the |
| A | P | $+18^{\circ} 24^{\prime}$ | $0.95,1.700,2.450$ | BM |
| B | P | $-9^{\circ} 30^{\prime}$ | $1.915,2.515,3.115$ | 415.5 m |

$$
\begin{aligned}
& =\quad 100 \times 1.12 \operatorname{Cos}^{2} 6^{\circ} 30^{\prime}+\left(0.5 \operatorname{Cos} 6^{\circ} 30^{\prime}\right. \\
& =99.21 \mathrm{~m} \\
& \mathrm{~V}_{1} \quad=\quad f_{-} \frac{S x \operatorname{Sin} 2 \varnothing}{2}+(\mathrm{f}+\mathrm{d}) \operatorname{Sin} \varnothing \\
& =100 \mathrm{x}^{1.12 x \operatorname{Sin} 2 \times 6^{\circ} 30}+0.5 \operatorname{Sin} 6^{\circ} 30^{\prime} \\
& =12.65 \mathrm{~m}
\end{aligned}
$$

Calculate the horizontal distance between AB and RL of B , when the constants of the instruments are 100 and 0

## Solution

When the line of sight is inclined, the general formula for finding distance between instrument station and the object station.

$$
\mathrm{D}=\quad \frac{f}{i} \times \mathrm{S} \operatorname{Cos}^{2} \emptyset+(\mathrm{f}+\mathrm{d}) \operatorname{Cos} \emptyset
$$

The tacheometry surveying, the multiplying constant $\mathrm{i}=100$, and additive constants $(\mathrm{f}+\mathrm{d})=$ 0

a) First observation ( BM to A )

$$
\begin{aligned}
\mathrm{S}_{1} & =2 \cdot 30-1.30=1.0 \mathrm{~m} \\
\emptyset & =6^{\circ} 30^{\prime} \\
\mathrm{D}_{1} & =\frac{f}{i} \times S \operatorname{Cos}^{2} \phi+(\mathrm{ftd}) \cos \phi
\end{aligned}
$$

$$
\begin{aligned}
& =100 \times 1.12 \operatorname{Cos}^{2} 6^{\circ} 30^{\circ}+\left(0.5 \operatorname{Cos} 6^{\circ} 30^{\prime}\right. \\
& =99.21 \mathrm{~m} \\
\mathrm{~V}_{1} & =\frac{f}{i} \times \frac{5 \times \operatorname{Sin} 20}{2}+(\mathrm{f}+\mathrm{d}) \operatorname{Sin} \varnothing \\
& =100 \times \frac{1.12 \times \operatorname{Sin} 2 \times 6^{\circ} 30}{2}+0.5 \operatorname{Sin} 6^{\circ} 30^{\prime} \\
& =12.65 \mathrm{~m}
\end{aligned}
$$

b) Second observation (A to $P$ )

$$
\begin{aligned}
\mathrm{S}_{2} & =3.07-1.89=1.18 \mathrm{~m} \\
\varnothing & =12^{\circ} 24 \\
\mathrm{D}_{1} & =\frac{f}{i} \times \operatorname{SCos}{ }^{2} \varnothing+(\mathrm{f}+\mathrm{d}) \operatorname{Cos} \varnothing \\
& =100 \times 1.18 \operatorname{Cos}^{2} 12^{\circ} 24^{\prime}+\left(0.5 \operatorname{Cos} 12^{\circ} 24^{\prime}\right. \\
& =113.05 \mathrm{~m} \\
\mathrm{~V}_{1} & =\frac{f}{i} \times \frac{5 \times \operatorname{Sin} 2 \varnothing}{2}+(\mathrm{f}+\mathrm{d}) \operatorname{Sin} \varnothing \\
& =100 \times \frac{1.18 \times \operatorname{Sin} 2 \times 12^{\circ} 24}{2}+0.5 \operatorname{Sin} 12^{\circ} 24^{\prime} \\
& =24.85 \mathrm{~m}
\end{aligned}
$$

RL of the instruments axis $=\quad \mathrm{RL}$ of $\mathrm{BM}+\mathrm{h}_{1}+\mathrm{V}_{1}$

$$
\begin{aligned}
& =384.5+12.65+1.54 \\
& =398.69 \mathrm{~m}
\end{aligned}
$$

$R L$ of $P \quad=\quad R L$ of instrument axis $+V_{2}-h_{1}$ $=421.06 \mathrm{~m}$

## Triangulation

Triangulation is the surveying technique in which unknown distances between stations may be determined $b$ trigonometric applications of a triangle or triangles. In triangulation, one side called the baseline and at least two interior angles are measured. When the all the three interior angles are measured, accuracy of the calculated distances is in increased and a check provided against any measurements error.

## Geodetic triangulation

A geodetic triangulation survey, in which stations are miles apart, must consider the earth's size and shape.

## Application of triangulation

Triangulation may be used for
Establishing accurately located control points for plane and geodetic survey of large areas.
Establishing accurately located control points in connection with aerial surveying, Accurate location of engineering projects such as centre lines, terminal points and
shafts for long tunnels and abutments for long bridges.

## Triangulation system

It may be defined as a system consisting of triangulation by a chain of triangles.
The complete figure is called triangulation system or triangulation figure.

## Triangle

A chain of triangles is very rapid and economical when a narrow strip of terrain is to be surveyed.

Angles less than $30^{\circ}$ or more than $120^{\circ}$ are not permitted

## Advantages

Simple, rapid and economical

## Classification of triangulation

The classification of triangulation is based upon the degree of accuracy required, the extent of the area to be surveyed, length of the base, length of the sides, and triangulation misclosure.

The three classification of triangulation are
Primary triangulation (first order triangulation)
Secondary triangulation (Second order triangulation) Tertiary
triangulation (Third order triangulation)
The recommendation regarding the precession to be obtained during triangulation are given by

| s.no | Requirements | Primary | Secondary | Tertiary |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Length of sides of triangle | $\begin{gathered} 30-160 \\ \mathrm{Km} \end{gathered}$ | $8-70 \mathrm{Km}$ | $1.5-10 \mathrm{Km}$ |
| 2 | Length of baseline | $5-20 \mathrm{Km}$ | $2-5 \mathrm{Km}$ | $0.5-3 \mathrm{Km}$ |
| 3 | Average Triangular Closure | $1 "$ | $3 "$ | 6 " |
| 4 | Maximum Triangular Closure | $3 "$ | 8" | 12 " |
| 5 | Instruments | Wild or Zeiss | Wild theodolite | Any theodolite with 5" L.C |
| 6 | Limiting Strength of figure | 25 | 40 | 50 |
| 7 | Sets of observations | 16 | 8 | 3 |
| 8 | Actual Error of base | 1 in 300,000 | 1 in 150,000 | 1 in 750,000 |
| 9 | Probable Error of base | $\begin{gathered} \hline 1 \text { in } \\ 1,000,000 \end{gathered}$ | 1 in 5000,000 | 1 in 250,000 |
| 10 | Discrepancy between station | $\begin{gathered} 10 \mathrm{~mm} \\ \sqrt{\mathrm{Km}} \end{gathered}$ | $20 \mathrm{~mm} \sqrt{ } \mathrm{Km}$ | $25 \mathrm{~mm} \sqrt{ } \mathrm{Km}$ |

It may define as a figure which gives the least error in the calculated length of the last line in the system due to shape of triangles and composition of the figures.

## Distances angles

In computing a given side in any triangle there are always two distance angles, namely, the angle opposite the known side and the angle opposite the unknown side.
c
A
B
B

a

## Criterion of strength

The method is based on an expression for the square of probable error $\mathrm{L}^{2}$

D - C
Calculating strength of figure $\mathrm{R}=------------\sum^{*}\left(\delta \mathrm{~A}^{2}+\delta A \delta B+\delta B^{2}\right)$
D
D - C
$\mathrm{R}=$ * $\Sigma \mathrm{a}$

D
Where,
$\mathrm{R}=$ Strength of figure
$\mathrm{D}=$ Number of direction observed, excluding those of known side of a given fig.
$\square=\delta \mathrm{A}^{2}+\delta \mathrm{A} \delta \mathrm{B}+\delta \mathrm{B}^{2}$ (the values of probable error of an observed direction)
$\mathrm{C}=$ the number of geometric condition (angle condition + side condition)
3) $\left(n^{\prime}-s^{\prime}+1\right)+(n-2 s+3)$
$\mathrm{n}=$ number of lines
$s^{\prime}=$ number of station occupied
$\mathrm{s}=$ total number of station
n'= number of lines observed in both the direction

## Problem

- In a triangle ABC , angles $\mathrm{A}, \mathrm{B}$ and C were observed as $70^{\circ}, 50^{\circ}$ and $60^{\circ}$ respectively. The line $A C$ was used as a base line of the known length. Calculate the strength of the figure.

Solution,

A
c
$50^{\circ}$,

$\mathrm{C}=$ the number of geometric condition (angle condition + side condition)

- $\left(n^{\prime}-s^{\prime}+1\right)+(n-2 s+3)$
- $(3-3+1)+(3-6+3)$

Distance of given triangle
Distance Angle $\mathrm{A}=70^{\circ}$ (Angle opposite to known side)
Distance Angle $\mathrm{B}=50^{\circ}$ (Angle opposite to unknown side, which ever higher)

D - C
Calculating strength of figure $\mathrm{R}=$ $\qquad$ * $\Sigma \mathrm{a}$

D

4-1
$\mathrm{R}=$

* $5=3.75$

4

## TYPES OF TRIANGULATION NETWORKS

In
triangulation there are three types of triangulation networks (or nets). They are the chain of single triangles, chain of polygons, and chain of quadri- laterals.


Chains of triangles


Chain of polygon


Chains of Quadrilaterals

## TRIANGULATION STATIONS, SIGNALS, AND INSTRUMENT SUPPORTS

## Signals

Signals are those survey targets that either are illuminated by natural sunlight or are electrically lighted by use of wet or dry cell batteries. The observations for all first-and second-order triangulation and first-order traverse are usually done at night using signal lights, because of more stable atmospheric conditions, which allow for better pointings. Observations maybe made during daylight hours using lights, but for high-accuracy surveys, this is done only under extreme conditions.

## Towers

Towers must be built on some stations to raise the lines of sight to clear obstructions or to lengthen the lines of sight to increase distances between stations of area surveys. A tower consists of an instrument stand (inner structure) and a platform to support the observer (outer structure). Towers fall roughly into three classes: prefabricated aluminum or steel, wooden, and expedient towers.

## Classification of signals

The signals may be classified under two types,
Luminous Signals Opaque
Signals

## Luminous signals

Luminous are sub-dived in to types

## Sun Signals Night Signals

Those Signals which is reflect the ray's of the sun towards the station of observation are known as heliotropes. Such signals can be used in clear weather.

## Heliotropes

The word "Heliotrope", when examined in detail, means "the sun" and "to turn". It is a fitting name for an instrument which does just that reflects the sun toward a given point.

The heliotrope was utilized by surveyors as a specialized form of target; it was employed during large triangulation surveys where, because of the great distance between stations (usually twenty miles or more), a regular target would appear indistinct.

## Opaque Signals

Opaque or non luminous signals used during day are or various forms and commonly used ones are,

1. Pole Signals
2. Target signals


- (iii) Pole and brush signals


## Requirements of good signals

A good signals should fulfills the following requirements
It should be conspicuous (clearly visible)
It should provide easy and accurate bisection by a telescope It should be capable of being centered over the station marks

It should be exhibits very little phase error of bisection of the signals.

## Buoys

A buoy is a floating device that can have many different purposes. It can be anchored (stationary) or allowed to drift.


## Phase Signals

Phase signal is the error of bisection which arises from the fact that, under lateral illumination, the signal is partly in light and partly in shade. The observer sees only the illuminated portion and bisects it.
$=$ the angle which the direction of sun makes
r $\quad=$ Radius of the signal
$\mathrm{D} \quad=$ Distance between A station and B station

$\mathrm{e} \quad=\quad \frac{r \cos 2(\alpha / 2)}{D} \times 206265$ (Seconds)

## Reconnaissance

The object of the reconnaissance is to plan a system of triangulation in accordance with specification laid down for the type of triangulation. A through reconnaissance of the area contributes to the accuracy, geometrical strength, simplicity and economy.
$>$ Size of the triangles
$>$ Shapes of the triangle
> Selection of stations
> Inter-visibility and height of station
The distance between stations
Relative elevation of stations
Profile of the intervening ground
The following instruments used for reconnaissance survey,
Theodolite and sextantPrismatic compass
$\square$ Aneroid barometer for ascertaining elevations
Steel tapeGood telescopeHeliotropes for testing inter-visibility
Drawing instruments and materialsGuyed ladders, ropes creepers etc,

## Size of the triangles

The main factor influencing the size of the triangle in a framework is the purpose for which the survey is conducted. In topographic survey the station points should be close. In case of geodetic survey, the triangles are quite large having sides of several kilometers, with due allowance being made for the curvature of the earth.

## Shapes of triangles

The shape of the triangles should be such that any error in the measurement of an angle should affect the calculated sides the least. In triangulation, the sides are computed by using sine rule and it should satisfy the geometric condition.

## Selection of stations

The triangulation station should be selected under the following conditions
$\square \quad$ Stations should be inter-visibleStations should be well conditioned
Stations should be easily accessible
$\square \quad$ Stations should be so selected that survey lines are not too long
$\square \quad$ Stations should be so selected that the cost of cleaning, cutting and building towers is minimum

## Inter-visibility and height of station

The most economical condition of triangulation is that the station should be inter-visible. The calculation of the heights of a signal as well as that of instruments depends upon the following condition

## The distance between stations

If there is no obstruction due to interviewing ground, the distance of the visible horizon from a station of known elevation above datum is given by
$\mathrm{h} \quad=\frac{\mathrm{D} 2 \mathrm{x}}{2 R}(1-2 \mathrm{~m})=0.06735 \mathrm{D}^{2}$
Where,
h $\quad=$ Height of the station above datum (in Km)
$\mathrm{D} \quad=$ Distance to the visible horizon (in m )
$\mathrm{R} \quad=$ mean radius of earth
$\square \quad=$ Co-efficient of refraction
0.07 for sights over land
0.08 for sights over sea

## Relative elevation of stations

If there is no obstruction due to interviewing ground, then the required elevation of a signal at the other station can be calculated is given by

$$
\mathrm{h} \quad=\quad \frac{\mathrm{D} 2}{2 R} \mathrm{x}(1-2 \mathrm{~m})=0.06735 \mathrm{D}^{2}
$$

Where,
h $\quad=$ Height of the station above datum (in Km )
D = Distance to the visible horizon (in m)
R
$=\quad$ mean radius of earth
$=$ Co-efficient of refraction
0.07 for sights over land
0.08 for sights over sea

## - Inter-visibility and heights of stations

The most essential condition of triangles is that the Stations should be inter-visible. The calculation of the heights of a signal as well as that of an instrument depends upon the following factors.

## The distance between stations

The distance of visible horizon above the datum is given by
$\mathrm{h}=\frac{\mathrm{D} 2}{2 R} \mathrm{x}(1-2 \mathrm{~m})=0.06735 \mathrm{D}^{2}$
$\mathrm{D}=\sqrt{ } \frac{\mathrm{h}}{0.06735}$
D = Distance of the visible horizon
$\mathrm{h}=$ Height of the station above datum
$\mathrm{R}=$ mean Radius of earth

- =co-efficient of refraction
- $\quad 0.07$ for sight over land
- $\quad 0.08$ for sight over sea


## Relative elevation of stations

If there is no obstruction due
intervening ground, then the elevation of a signal at the other station can be calculated by using the expression.

## Types of triangulation station

The triangulation station may be categorized as under
Main station’
Subsidiary station Satellite station Pivot
station

## Main Station

The triangulation stations which are used to carry forward the net work of triangulation are known as main stations. Observations at each main station are made with every care.

## Subsidiary station

Triangulation stations which are used only to provide additional rays to intersected points are known as subsidiary station. Observations of the horizontal angles for subsidiary station are made two zero's only.

## Satellite Station

The stations which are selected close to the main triangulation station to avoid intervening obstruction are known as satellite station, or eccentric or false station. Observations are made with the same precision as for the main station

## Pivot station

The station at which no observations are made but, angles at those stations are used for the continuity of a triangulation series are known as pivot station

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## SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

# UNIT - V- CONTROL SURVEYING AND ADJUSTMIENTS 

- SCIA1303


## CONTROL SURVEYING AND ADJUSTMENTS

## HORIZONTAL CONTROLS \& ITS METHODS

The horizontal control consists of reference marks of known plan position, from which salient points of designed structures may be set out. For large structures primary and secondary control points are used. The primary control points are triangulation stations. The secondary control points are reference to the primary control stations. Reference Grid

Reference grids are used for accurate setting out of works of large magnitude. The following types of reference grids are used:
1.
2.
3.
4.

## Survey Grid

Site Grid
tructural Grid
Secondary Grid

Survey grid is one which is drawn on a survey plan, from the original traverse. Original traverse stations form the control points of the grid. The site grid used by the designer is the one with the help of which actual setting out is done. As far as possible the site grid should be actually the survey grid. All the design points are related in terms of site grid coordinates. The structural grid is used when the structural components of the building are large in numbers and are so positioned that these components cannot be set out from the site grid with sufficient accuracy. The structural grid is set out from the site grid points. The secondary grid is established inside the structure, to establish internal details of the building, which are otherwise not visible directly from the structural grid.

## VERTICAL CONTROL \& ITS METHODS:

The vertical control consists of establishment of reference marks of known height relative to some special datum. All levels at the site are normally reduced to the nearby bench mark, usually known as master bench mark.

The setting of points in the vertical direction is usually done with the help of following rods:

1. Boning rods and travelers
2. Sight Rails
3. Slope rails or batter boards
4. Profile boards

A boning rod consist of an upright pole having a horizontal board at its top, forming a ' T 'shaped rod. Boning rods are made in set of three, and many consist of three ' T ' shaped rods, each of equal size and shape, or two rods identical to each other and a third one consisting of longer rod with a detachable or movable ' $T$ ' piece. The third one is called traveling rod or traveler.

## Sight Rails:

A sight rail consist of horizontal cross piece nailed to a single upright or pair of uprights driven into the ground. The upper edge of the cross piece is set to a convenient height above the required plane of the structure, and should be above the ground to enable a man to conveniently align his eyes with the upper edge. A stepped sight rail or double sight rail is used in highly undulating or falling ground. Slope rails or Batter boards:
hese are used for controlling the side slopes in embankment and in cuttings. These consist of two vertical poles with a sloping board nailed near their top. The slope rails define a plane parallel to the proposed slope of the embankment, but at suitable vertical distance above it. Travelers are used to control the slope during filling operation.

## Profile boards:

These are similar to sight rails, but are used to define the corners, or sides of a building. A profile board is erected near each corner peg. Each unit of profile board consists of two verticals, one horizontal board and two cross boards. Nails or saw cuts are placed at the top of the profile boards to define the width of foundation and the line of the outside of the wall

An instrument was set up at P and the angle of elevation to a vane 4 m above the foot of the staff held at Q was $9^{\circ} 30^{\prime}$. The horizontal distance between P and Q was known to be 2000 metres. Determine the R.L. of the staff station Q given that the R.L. of the instrument axis was 2650.38.

Solution:

Height of vane above the instrument axis
$\begin{array}{ll}= & \mathrm{D} \tan \alpha=2000 \tan 9^{\circ} 30^{\prime} \\ = & 334.68 \mathrm{~m}\end{array}$

Correction for curvature and refraction

$$
\begin{aligned}
\mathrm{C} & =0.06735 \mathrm{D}^{2} \mathrm{~m}, \text { when } \mathrm{D} \text { is in } \mathrm{km} \\
& =0.2694 \approx 0.27 \mathrm{~m}(+\mathrm{ve})
\end{aligned}
$$

Height of vane above the instrument axis

$$
=334.68+0.27=334.95
$$

R.L. fo vane $=334.95+2650.38=2985.33 \mathrm{~m}$

$$
\text { R.L. of } Q=2985.33-4=2981.33 \mathrm{~m}
$$

An instrument was set up at P and the angle of depression to a vane 2 m above the foot of the staff held at Q was $5^{\circ} 36^{\prime}$. The horizontal distance between P and Q was known to be 3000
metres. Determine the R.L. of the staff station Q given that staff reading on a B.M. of elevation 436.050 was 2.865 metres.

## Solution:

The difference in elevation between the vane and the instrument axis

$$
\begin{aligned}
& =\mathrm{D} \tan \alpha \\
& =3000 \tan 5^{\circ} 36^{\prime}=294.153
\end{aligned}
$$

Combined correction due to cuvature and refraction
$\mathrm{C}=0.06735 \mathrm{D}^{2}$ metres, when D is in km
$=0.606 \mathrm{~m}$.
Since the observed angle is negative, the combined correction due to curvature and refraction is subtractive.

Difference in elevation between the vane and the instrument axis
$=294.153-0.606=293.547=\mathrm{h}$.
$\begin{aligned} \text { R.L. of instrument axis } & =436.050+2.865=438.915 \\ \text { R.L. of the vane } & =\text { R.L. of instrument aixs }-\mathrm{h}\end{aligned}$

$$
=438.915-293.547=145.368
$$

R.L. of $\mathrm{Q}=145.368-2$

$$
=143.368 \mathrm{~m} .
$$

In order to ascertain the elevation of the top $(\mathrm{Q})$ of the signal on a hill, observations were made from two instrument stations P and R at a horizontal distance 100 metres apart, thestation P and R being in the line with Q . The angles of elevation of Q at P and R were $28^{\circ} 42^{\prime}$ and $18^{\circ} 6^{\prime}$ respectively. The staff reading upon the bench mark of elevation 287.28 were respectively 2.870 and 3.750 when the instrument was at P and at R , the telescope being horizontal. Determine the elevation of the foot of the signal if the height of the signal above its base is 3 metres

Solution:
Elevation of instrument axis at $\mathrm{P}=$ R.L. of B.M. + Staff reading

$$
=287.28+2.870=290.15 \mathrm{~m}
$$

Elevation of instrument axis at $\mathrm{R}=$ R.L. of B.M. + staff reading
$=$

$$
287.28+3.750=291.03 \mathrm{~m}
$$

Difference in level of the instrument axes at the two stations

$$
\mathrm{S}=291.03-290.15=0.88 \mathrm{~m} .
$$

$\alpha=28^{\circ} 42$ and $\alpha----=18^{\circ} 6^{\prime}$

$$
\mathrm{s} \cot \alpha---=0.88 \cot 18^{\circ} 6^{\prime}=2.69 \mathrm{~m}
$$

$$
=152.1 \mathrm{~m} .
$$

$$
\mathrm{h}--=\mathrm{D} \tan \alpha--=152.1 \tan 28^{\circ} 42^{\prime}=83.272 \mathrm{~m}
$$

R.L. of foot of signal $=$ R.L. of inst. aixs at $P+h--$ ht. of signal

$$
\text { Check : (b+D) } \begin{aligned}
& =290.15+83.272-3=370.422 \mathrm{~m} . \\
& =100+152.1 \mathrm{~m}=252.1 \mathrm{~m} \\
\mathrm{~h}-- & =(\mathrm{b}+\mathrm{D}) \tan \alpha--=252.1 \mathrm{x} \tan 18^{\circ} 6^{\prime} \\
& =82.399 \mathrm{~m}
\end{aligned}
$$

R.L. of foot of signal $=$ R.L. of inst. axis at $R+h--+h t$. of signal

$$
=291.03+82.399-3=370.429 \mathrm{~m} .
$$

## CLASSIFICATION OF TRIANGULATION SYSTEM:

The basis of the classification of triangulation figures is the accuracy with which the length and azimuth of a line of the triangulation are determined. Triangulation systems of different accuracies depend on the extent and the purpose of the survey. The accepted grades of triangulation are:

1. First order or Primary Triangulation
2. Second order or Secondary Triangulation
3. Third order or Tertiary Triangulation

## FIRST ORDER OR PRIMARY TRIANGULATION:

The first order triangulation is of the highest order and is employed either to determine the earth's figure or to furnish the most precise control points to which secondary triangulation may be connected. The primary triangulation system embraces the vast area (usually the whole of the country). Every precaution is taken in making linear and angular measurements and in performing the reductions. The following are the general specifications of the primary triangulation:

| 1. Average triangle closure | : Less than 1 second |
| :--- | :--- |
| 2. Maximum triangle closure | $:$ Not more than 3 seconds |
| 3. Length of base line | $: 5$ to 15 kilometers |
| 4. Length of the sides of triangles | $: 30$ to 150 kilometers |
| 5. Actual error of base | $: 1$ in 300,000 |
| 6. Probable error of base | $: 1$ in $1,000,000$ |
| 7.Discrepancy between two | $: 10$ mm kilometers |
| measures of a section | $: 1$ in 60,000 to 1 in 250,000 |
| 8. Probable error or computed distance | $: 0.5$ seconds |
| 9. Probable error in astronomic azimuth |  |

## SECONDARY ORDER OR SECONDARY TRIANGULATION

The secondary triangulation consists of a number of points fixed within the framework of primary triangulation. The stations are fixed at close intervals so that the sizes of the
triangles formed are smaller than the primary triangulation. The instruments and methods used are not of the same utmost refinement. The general specifications of the secondary triangulation are:

| 1. Average triangle closure | $: 3 \mathrm{sec}$ |
| :--- | :--- |
| 2. Maximum triangle closure | $: 8 \mathrm{sec}$ |
| 3. Length of base line | $: 1.5$ to 5 km |
| 4. Length of sides of triangles | $: 8$ to 65 km |
| 5. Actual error of base | $: 1$ in 150,000 |
| 6. Probable error of base | $: 1$ in 500,000 |
| 7. Discrepancy between two | $: 20$ mm kilometers |
| measures of a section | $: 1$ in 20,000 to 1 in 50,000 |
| 8. Probable error or computed distance | $: 2.0 \mathrm{sec}$ |
| 9. Probable error in astronomic azimuth |  |

## THIRD ORDER OR TERTIARY TRIANGULATION:

The third-order triangulation consists of a number of points fixed within the framework of secondary triangulation, and forms the immediate control for detailed engineering and other surveys. The sizes of the triangles are small and instrument with moderate precision may be used. The specifications for a third-order triangulation are as follows:

1. Average triangle closure $: 6 \mathrm{sec}$
2. Maximum triangle closure : 12 sec
3. Length of base line $\quad: 0.5$ to 3 km
4. Length of sides of triangles $: 1.5$ to 10 km

| 5. Actual error of base | $: 1$ in 75,0000 |
| :--- | :--- |
| 6. Probable error of base | $: 1$ in 250,000 |

7. Discrepancy between two

Measures of a section :25 mm kilometers
8. Probable error or computed distance : 1 in 5,000 to 1 in 20,000
9. Probable error in astronomic Azimuth: 5 sec .

Explain the factors to be considered while selecting base line.

The measurement of base line forms the most important part of the triangulation operations. The base line is laid down with great accuracy of measurement and alignment as it forms the basis for the computations of triangulation system. The length of the base line depends upon the grades of the triangulation. Apart from main base line, several other check bases are also measured at some suitable intervals. In India, ten bases were used, the lengths of the nine bases vary from 6.4 to 7.8 miles and that of the tenth base is 1.7 miles.

Selection of Site for Base Line. Since the accuracy in the measurement of the base line depends upon the site conditions, the following points should be taken into consideration while selecting the site:

The site should be fairly level. If, however, the ground is sloping, the slope should be uniform and gentle. Undulating ground should, if possible be avoided.

The site should be free from obstructions throughout the whole of the length. The line clearing should be cheap in both labour and compensation.

The extremities of the base should be intervisible at ground level.
The ground should be reasonably firm and smooth. Water gaps should be few, and if possible not wider than the length of the long wire or tape.

The site should suit extension to primary triangulation. This is an important factor since the error in extension is likely to exceed the error in measurement.

In a flat and open country, there is ample choice in the selection of the site and the base may be so selected that it suits the triangulation stations. In rough country, however, the choice is limited and it may sometimes be necessary to select some of the triangulation stations that at suitable for the base line site.

Standards of Length. The ultimate standard to which all modern national standards are referred is the international meter established by the Bureau International der Poids at Measures and kept at the Pavilion de Breteuil, Sevres, with copies allotted to various national surveys. The meter is marked on three platinum- iridium bars kept under standard conditions. One great disadvantage of the standard of length that are made of metal are that they are subject to very small secular change in their dimensions. Accordingly, the meter has now been standardized in terms of wavelength of cadmium light.

## TYPES OF ERROR

Errors of measurement are of three kinds: (i) mistakes, (ii) systematic errors, and (iii) accidental errors.

Mistakes. Mistakes are errors that arise from inattention, inexperience, carelessness and poor judgment or confusion in the mind of the observer. If mistake is undetected, it produces a serious effect on the final result. Hence every value to be recorded in the field must be checked by some independent field observation.
(ii) Systematic Error. A systematic error is an error that under the same conditions will always be of the same size and sign. A systematic error always follows some definite mathematical or physical law, and a correction can be determined and applied. Such errors are of constant character and are regarded as positive or negative according as they make the result too great or too small. Their effect is therefore, cumulative.

If undetected, systematic errors are very serious. Therefore:

All the surveying equipments must be designed and used so that whenever possible systematic errors will be automatically eliminated and (2) all systematic errors that cannot be surely eliminated by this means must be evaluated and their relationship to the conditions that cause them must be determined. For example, in ordinary levelling, the levelling instrument must first be adjusted so that the line of sight is as nearly horizontal as possible when bubble is centered. Also the horizontal lengths for back sight and foresight from each instrument position should be kept as nearly equal as possible. In precise levelling, every day, the actual error of the instrument must be determined by careful peg test, the length of each sight is measured by stadia and a correction to the result is applied.
(iii) Accidental Error. Accidental errors are those which remain after mistakes and systematic errors have been eliminated and are caused by a combination of reasons beyond the ability of the observer to control. They tend sometimes in one direction and some times in the other, i.e., they are equally likely to make the apparent result too large or too small.

An accidental error of a single determination is the difference between (1) the true value of the quantity and (2) a determination that is free from mistakes and systematic errors. Accidental error represents limit of precision in the determination of a value. They obey the laws of chance and therefore, must be handled according to the mathematical laws of probability.

The theory of errors that is discussed in this chapter deals only with the accidental errors after all the known errors are eliminated and accounted for.

## THE LAW OF ACCIDENTAL ERRORS

Investigations of observations of various types show that accidental errors follow a definite law, the law of probability. This law defines the occurrence of errors and can be expressed in the form of equation which is used to compute the probable value or the probable precision of a quantity. The most important features of accidental errors which usually occur are:

Small errors tend to be more frequent than the large ones; that is they are the most probable. Positive and negative errors of the same size happen with equal frequency ; that is, they are equally probable.

Large errors occur infrequently and are impossible.

## PRINCIPLES OF LEAST SQUARES

It is found from the probability equation that the most probable values of a series of errors arising from observations of equal weight are those for which the sum of the squares is a minimum. The fundamental law of least squares is derived from this. According to the principle of least squares, the most probable value of an observed quantity available from a given set of observations is the one for which the sum of the squares of the residual errors is a minimum. When a quantity is being deduced from a series of observations, the residual errors will be the difference between the adopted value and the several observed values,

Let $V 1, V 2, V 3$ etc. be the observed values $\mathrm{x}=$ most probable value

## LAW OF WEIGHTS

From the method of least squares the following laws of weights are established:
(i) The weight of the arithmetic mean of the measurements of unit weight is equal to the number of observations.

For example, let an angle A be measured six times, the following being the values:
A
Weight
A
Weight

| $30^{\circ} 20^{\prime} 8^{\prime \prime}$ | 1 | $30^{\circ} 20^{\prime} 10^{\prime \prime}$ | 1 |
| :--- | :--- | :--- | :--- |
| $30^{\circ} 20^{\prime} 10^{\prime \prime}$ | 1 | $30^{\circ} 20^{\prime} 9 \prime$ | 1 |
| $30^{\circ} 20^{\prime} 7 \prime$ | 1 | $30^{\circ} 20^{\prime} 10^{\prime \prime}$ | 1 |

Arithmetic mean

$$
\begin{aligned}
& =30^{\circ} 20^{\prime}+1 / 6\left(8^{\prime \prime}+10^{\prime \prime}+7 "+10^{\prime \prime}+9 "+10^{\prime \prime}\right) \\
& =30^{\circ} 20^{\prime} 9 \prime \prime
\end{aligned}
$$

Weight of arithmetic mean $=$ number of observations $=6$.
(2) The weight of the weighted arithmetic mean is equal to the sum of the individual weights.

For example, let an angle A be measured six times, the following being the values :

| A | Weight | A |  | Weight |
| :---: | :---: | :---: | :---: | :---: |
| $30^{\circ} 20^{\prime} 8^{\prime \prime}$ | 2 | $30^{\circ} 20^{\prime} 10^{\prime \prime}$ | 3 |  |
| $30^{\circ} 20^{\prime} 10^{\prime \prime}$ | 3 | $30^{\circ} 20^{\prime} 9 \prime$ | 4 |  |
| $30^{\circ} 20^{\prime} 6^{\prime \prime}$ | 2 | $30^{\circ} 20^{\prime} 10^{\prime \prime}$ | 2 |  |

Sum of weights $\quad=2+3+2+3+4+2=16$
Arithmetic mean $=30^{\circ} 20^{\prime}+1 / 16\left(8 " \mathrm{X} 2+10^{\prime \prime} \mathrm{X} 3+7 " \mathrm{X} 2+10 " \mathrm{X} 3+9 " \mathrm{X} 4+10\right.$ "X2 $)$ $=30^{\circ} 20^{\prime} 9^{\prime \prime}$.

Weight of arithmetic mean $=16$.
(3) The weight of algebric sum of two or more quantities is equal to the reciprocals of the individual weights.

For Example angle $\mathrm{A}=30^{\circ} 20^{\prime} 8{ }^{\prime \prime}$, Weight 2

$$
\mathrm{B}=15^{\circ} 20^{\prime} 8^{\prime \prime} \text {, Weight } 3
$$

Weight of $\quad A+B=$
(4) If a quantity of given weight is multiplied by a factor, the weight of the result is obtained by dividing its given weight by the square of the factor.
(5) If a quantity of given weight is divided by a factor, the weight of the result is obtained by multiplying its given weight by the square of the factor.
(6) If a equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of the equation.
(7) The weight of the equation remains unchanged, if all the signs of the equation are changed or if the equation is added or subtracted from a constant.

## DISTRIBUTION OF ERROR OF THE FIELD MEASUREMENT

Whenever observations are made in the field, it is always necessary to check for the closing error, if any. The closing error should be distributed to the observed quantities. For examples, the sum of the angles measured at a central angle should be $360^{\circ}$, the error should be distributed to the observed angles after giving proper weight age to the observations. The following rules should be applied for the distribution of errors:
(1) The correction to be applied to an observation is inversely proportional to the weight of the observation.
(2) The correction to be applied to an observation is directly proportional to the square of the probable error.
(3) In case of line of levels, the correction to be applied is proportional to the length.

The following are the three angles $\alpha, \beta$ and $y$ observed at a station $P$ closingthe horizon, along with their probable errors of measurement. Determine their corrected values.

Solution.
$\alpha=78^{\circ} 12^{\prime} 12^{\prime \prime} \pm 2^{\prime \prime}$
$\beta=136^{\circ} 48^{\prime} 30^{\prime \prime} \pm 4^{\prime \prime}$
$\mathrm{y}=144^{\circ} 59^{\prime} 08^{\prime \prime} \pm 5^{\prime \prime}$
Sum of the three angles $=359^{\circ} 59^{\prime} 50^{\prime \prime}$
Discrepancy $=10$ "
Hence each angle is to be increased, and the error of $10^{\prime \prime}$ is to be distributed in proportion to the square of the probable error.

Let $\mathrm{c} 1, \mathrm{c} 2$ and c 3 be the correction to be applied to the angles $\alpha, \beta$ and y respectively.

$$
c 1: c 2: c 3=(2)^{2}:(4)^{2}:(5)^{2}=4: 16: 25
$$

Also, $\quad \mathrm{c} 1+\mathrm{c} 2+\mathrm{c} 3=10$ "
From (1), $\quad c 2=16 / 4 \mathrm{c} 1=4 \mathrm{c} 1$
And

$$
\begin{equation*}
c 3=25 / 4 \mathrm{c} 1 \tag{2}
\end{equation*}
$$

Substituting these values of c2 and c3 in (2), we get c1 +

$$
4 \mathrm{c} 1+25 / 4 \mathrm{c} 1=10^{\prime \prime}
$$

or

$$
c 1(1+4+25 / 4)=10 "
$$

$$
\mathrm{c} 1=10 \times 4 / 45=0^{\prime \prime} .89
$$

$$
c 2=4 c 1=3^{\prime \prime} .36
$$

And

$$
\mathrm{c} 3=25 / 4 \mathrm{c} 1=5^{\prime} .55
$$

Check: $\quad \mathrm{c} 1+\mathrm{c} 2+\mathrm{c} 3=0 . " 89+3 " .56+5 " .55=10 "$

Hence the corrected angles are

$$
\begin{aligned}
& \alpha=78^{\circ} 12^{\prime} 12^{\prime \prime}+0^{\prime \prime} .89=78^{\circ} 12^{\prime} 12^{\prime \prime} .89 \\
& \beta=136^{\circ} 48^{\prime} 30^{\prime \prime}+3^{\prime \prime} .56=136^{\circ} 48^{\prime} 33^{\prime \prime} .56
\end{aligned}
$$

and

$$
y=144^{\circ} 59^{\prime} 08^{\prime \prime}+5^{\prime \prime} .55=144^{\circ} 59^{\prime} 13^{\prime \prime} .55
$$

Sum

$$
=360^{\circ} 00^{\prime} 00^{\prime \prime}+00
$$

An angle A was measured by different persons and the following are the values

| Angle |  | Number of measurements |
| :---: | :---: | :---: |
| $65^{\circ} 30^{\prime} 10^{\prime \prime}$ | $\ldots$ | 2 |
| $65^{\circ} 29^{\prime} 50^{\prime \prime}$ | $\ldots$ | 3 |
| $65^{\circ} 30^{\prime} 00^{\prime \prime}$ | $\ldots$ | 3 |
| $65^{\circ} 30^{\prime} 20^{\prime \prime}$ | $\ldots$ | 4 |
| $65^{\circ} 30^{\prime} 10^{\prime \prime}$ | $\ldots$ | 3 |

Find the most probable value of the angle.

## Solution.

As stated earlier, the most probable value of an angle is equal to its weighted arithmetic mean.

$$
\Sigma
$$

$$
\begin{array}{r}
65^{\circ} 30^{\prime} 10^{\prime \prime} \times 2=131^{\circ} 00^{\prime} 20^{\prime \prime} \\
65^{\circ} 29^{\prime} 50^{\prime \prime} \times 3=196^{\circ} 29^{\prime} 30^{\prime \prime} \\
65^{\circ} 30^{\prime} 00^{\prime \prime} \times 3=196^{\circ} 30^{\prime} 00^{\prime \prime} \\
65^{\circ} 30^{\prime} 20^{\prime \prime} \times 4=262^{\circ} 01^{\prime} 20^{\prime \prime} \\
65^{\circ} 30^{\prime} 10^{\prime \prime} \times 3=196^{\circ} 30^{\prime} 30^{\prime \prime} \\
---\ldots-\ldots-----------15
\end{array}
$$

Weighted arithmetic mean

$$
\begin{aligned}
& =982^{\circ} 31^{\prime} 40^{\prime \prime} \\
---- & 65^{\circ} 30^{\prime} 6^{\prime \prime} .67
\end{aligned}
$$

Hence most probable value of the angle $=65^{\circ} 30^{\prime} 6^{\prime \prime} .67$
The telescope of a theodilite is fitted with stadia wires. It is required to find the most probable values of the constants C and K of tacheometer. The staff was kept vertical at three points in the field and with of sight horizontal the staff intercepts observed was as follows.

| Distance of staff <br> from tacheometer $\mathrm{D}(\mathrm{m})$ | Staff intercept $\mathrm{S}(\mathrm{m})$ |
| :---: | :---: |
| 150 | 1.495 |
| 200 | 2.000 |
| 250 | 2.505 |

## Solution:

The distance equation is

$$
\mathrm{D}=\mathrm{KS}+\mathrm{C}
$$

The observation equations are

$$
\begin{aligned}
150 & =1.495 \mathrm{~K}+\mathrm{C} \\
200 & =2.000 \mathrm{~K}+\mathrm{C} \\
250 & =2.505 \mathrm{~K}+\mathrm{C}
\end{aligned}
$$

If K and C are the most probable values, then the error of observations are:

$$
\begin{aligned}
& 150-1.495 \mathrm{~K}-\mathrm{C} \\
& 200-2.000 \mathrm{~K}-\mathrm{C} \\
& 250-2.505 \mathrm{~K}-\mathrm{C}
\end{aligned}
$$

By the theory of least squares
$(150-1.495 \mathrm{~K}-\mathrm{C})^{2}+(200-2.000 \mathrm{~K}-\mathrm{C})^{2}+(250-505 \mathrm{~K}-\mathrm{C})^{2}=$ minimum---(i)
For normal equation in $K$,
Differentiating equation (i) w.r.t. K,

$$
\begin{align*}
& 2(-1.495)(150-1.495 \mathrm{~K}-\mathrm{C})+2(-20 .)(200-2.000 \mathrm{~K}-\mathrm{C}) \\
& +2(-2.505)(250-505 \mathrm{~K}-\mathrm{C})=0 \\
& 208.41667-2.085 \mathrm{~K}-\mathrm{C}=0  \tag{2}\\
& \text { Normal equation in C }
\end{align*}
$$

Differentiating equation (i) w.r.t. C,

$$
\begin{align*}
& 2(-1.0)(150-1.495 \mathrm{~K}-\mathrm{C})+2(-1.0)(200-2.000 \mathrm{~K}-\mathrm{C}) \\
& +2(-1.0)(250-505 \mathrm{~K}-\mathrm{C})=0 \\
& 200-2 \mathrm{~K}-\mathrm{C}=0  \tag{3}\\
& \text { On solving Equations (2) and (3) } \\
& \qquad \mathrm{K}=99.0196 \\
& \qquad \mathrm{C}=1.9608
\end{align*}
$$

The distance equation is:

$$
\mathrm{D}=99.0196 \mathrm{~S}+1.9608
$$

