SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – I - STRESS AND STRAIN – SCIA1301

STRESS & STRAIN - HOOKE'S LAW

STRESS:

The force of resistance per unit area offered by a body against deformation is known as stress.

Stress is "force per unit area" - the ratio of applied force P to cross section area.

$$\sigma = \frac{P}{A}$$

Where, σ is stress in N/mm^2.

P is load in N.

A is area in mm^2.

UNITS :

The basic units of stress in S.I units i.e. (International system) are N/m^2 (or) Pa.

MPa = 10^6 Pa, GPa = 10^9 Pa, KPa = 10^3 Pa

Sometimes N/mm^2 units are also used, because this is an equivalent to MPa.

TYPES OF STRESSES:

Tensile Stress:

The stress induced in a body, when subjected to two equal and opposite pulls.

Stress that tends to stretch or lengthen the material - acts normal to the stressed area.

Compressive Stress:

The stress induced in a body, when subjected to two equal and opposite pushes.

Stress that tends to compress or shorten the material - acts normal to the stressed area.

Shearing Stress:

The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section.

Stress that tends to shear the material - acts in plane to the stressed area at right-angles to compressive or tensile stress.

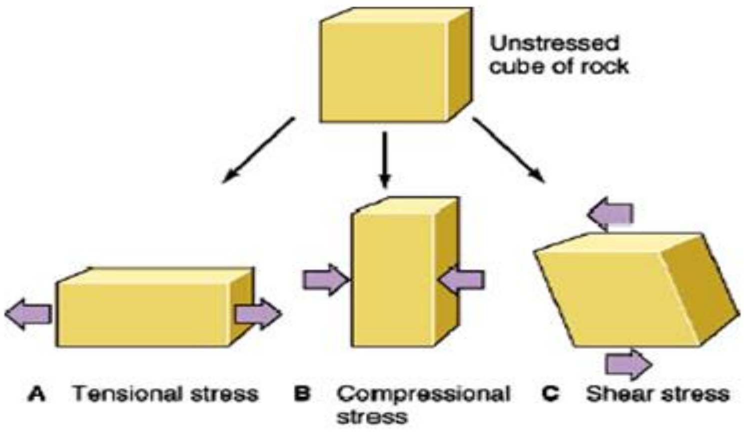


Figure No: 1

STRAIN:

It is defined as the ratio of change in dimension to the original dimension.

It is represented by 'e'

It has no unit.

TYPES OF STRAIN:

Linear Or Longitudinal Strain:

It is defined as the ratio of change in linear dimensions (length) to the original dimensions (length).

It is represented by e_L.

$$e_L = \frac{\delta L}{L}$$

Linear Tensile Strain:

It is defined as the ratio of change in increase in length to the original length.

It is represented by e_L.

$$e_L = \frac{\delta L}{L}$$

Linear Compressive Strain:

It is defined as the ratio of change in decrease in length to the original length.

It is represented by e_L.

$$e_L = \frac{\delta L}{L}$$

It has no unit.

Lateral Strain:

It is defined as the ratio of change in lateral dimensions to the original lateral dimensions. It is represented by e_b, ed, e_t, e_h.

$e_b = \delta b / b$

Shear Strain:

It is defined as the ratio of transverse displacement to the distance from the lower face. It is represented by '$\emptyset$'.

$\emptyset = \delta l / h$

Volumetric Strain:

It is defined as the ratio of change in lateral dimensions to the original lateral dimensions. It is represented by e_v.

$e_v = \delta v / v$

ELASTICITY:

The property of material by virtue of which it returns to its original shape and size upon removal of load is known as elasticity.

ELASTIC LIMIT:

The maximum extent to which a solid may be stretched without permanent alteration of size or shape.

HOOKE'S LAW:

It states that within elastic limit stress is proportional to strain.

Stress α Strain (or) Stress/ Strain = a Constant (E)

Mathematically, E= Stress/ Strain N/mm^2.

FACTOR OF SAFETY:

It is defined as the ratio of ultimate stress to the working or permissible stress.

Factor of safety = ultimate stress / working stress

ELASTIC CONSTANTS

ELASTIC CONSTANTS OF A MATERIAL:

Young's modulus

Bulk modulus

Shear modulus

Poisson's ratio.

MODULUS OF ELASTICITY (OR) YOUNG'S MODULUS:

Young's modulus is defined as the ratio of stress to strain within elastic limit.

It is represented by 'E'. Its units are N/mm^2.

Mathematically, $E= \sigma / e$

MODULUS OF RIGIDITY (OR) SHEAR MODULUS:

It is the ratio of shear stress (τ) to shear strain ($\emptyset$).

It is represented by 'C', 'N' or 'G'.

Its unit is N/mm^2.

C, N or $G = \tau / \emptyset$

BULK MODULUS:

It is defined as the ratio of applied pressure to volumetric strain.

It is represented by 'K'.

Its unit is N/mm^2.

$K = \sigma / e_v$

POISSON'S RATIO:

The ratio of lateral strain to longitudinal strain produced by a single stress is known as Poisson's ratio.

It is represented by μ or $1/m$.

The value of 'μ' varies from 0.25 to 0.50 depending upon the material.

RELATIONSHIP BETWEEN THE ELASTIC CONSTANTS

Relation between E and C:

$E = 2C [1+ \mu]$

Relation between E and K:

$E = 3K (1-2\mu)$

Relation between E, C and K:

$E=9KC/ (3K+C)$

UNIT CONVERSIONS:

$1 \text{ m} = 10^3 \text{ mm}$

$1 \text{ m}^2 = 10^6 \text{ mm}^2$

$1 \text{ m}^3 = 10^9 \text{ mm}^3$

1 KN = 10^3 N

1 MN = 10^6 N

1 GN = 10^9 N

1 Pa = 1 N/m^2

1 Mpa = 1 X 10^6 / 10^6 N/ mm^2

1 Mpa = 1 N/ mm^2

PROBLEMS

1.A steel rod 1 m long and 20 mm X 20 mm in cross section is subjected to a tensile force of 40 KN. Determine the elongation of the rod, if modulus of elasticity for the rod material is 200 GPa.

Given Data:

Length, L = 1 m = 1 X 10^3 mm.

Size of rod = 20 mm x 20 mm = 400 mm^2.(A)

Tensile Force, P = 40 KN = 40 X 10^3 N.

Modulus of Elasticity,E = 200 GPa = 200 X 10^9 N/m^2 = 200 X 10^9/ 10^6 mm^2

= 200 X 10^3 N/mm^2

To Find:

Elongation of the rod

Solution:

1. Elongation of the rod,

$$\delta L = \frac{PL}{AE} = \frac{40 \times 10^3 \times 1 \times 10^3}{400 \times 200 \times 10^3} = 0.5 \text{ mm.}$$

2. A steel specimen of 13 mm diameter was found to extend to elongate 0.2 mm in a 200 mm gauge length when it was subjected to a tensile force of 26.8 KN. If the specimen was tested within the elastic range, what is the value of Young's modulus for the steel specimen?

Given Data:

Length, L = 200 mm.

Diameter, d = 13 mm.

Elongation, δL = 0.2 mm.

Tensile Force, P = 26.8 KN = 26.8 X 10^3 N.

To Find:

Young's modulus.

Solution:

1.Young's modulus,

$$\delta L = \frac{PL}{AE}$$

$$E = \frac{PL}{A \times \delta L} = \frac{26.8 \times 10^3 \times 200}{132.73 \times 0.2}$$

$$A = \frac{\pi \times d^2}{4}$$

$$E = 2.019 \times 10^5 \text{ N/mm}^2.$$

$$= 132.73 \text{ mm}^2.$$

3. A steel bar 2 m long, 40 mm wide and 20 mm thick is subjected to an axial pull of 160 KN in the direction of its length. Find the changes in length, width and thickness of the bar. Take E = 200 GPa. and poisson's ratio = 0.3.

Given Data:

Length, $L = 2 \text{ m} = 2 \times 10^3$ mm.

Width, $b = 40$ mm.

Thick, $t = 20$ mm.

Axial pull, $P = 160 \text{ KN} = 160 \times 10^3$ N.

$E = 200 \text{ GPa} = 2 \times 10^5 \text{ N/mm}^2$.

poisson's ratio = 0.3.

To Find:

change in length.

change in width.

change in thickness.

1. Change in length,

$$\delta L = \frac{PL}{AE} = \frac{160 \times 10^3 \times 2 \times 10^3}{40 \times 20 \times 2 \times 10^5}$$

$$\delta L = 2 \text{ mm}.$$

2. Change in width,

- ✓ $\mu = \dfrac{e_b}{e_l}$

$$e_L = \frac{\delta L}{L} = \frac{2}{2 \times 10^3} = 0.001$$

$$e_b = e_l \times \mu = 0.001 \times 0.3$$

$= 0.0003$

$$e_b = \frac{\delta b}{b}$$

$\delta b = e_b \times b = 0.0003 \times 40$

$\delta b = 0.012$ mm.

3.Change in thickness,

$$e_t = \frac{\delta t}{t}$$

$\delta t = e_t \times t = 0.0003 \times 20$

$\delta t = 0.006$ mm.

4. A bar of 30 mm diameter is subjected to a pull of 60 KN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039 mm. Calculate the poisson's ratio and the values of the three moduli.

Given Data:

Diameter, d = 30 mm.

Length, L = 200 mm.

Change in length, δL = 0.09 mm.

change in diameter, δd = 0.0039 mm

Axial pull, P = 60 KN = 60 X 10^3 N.

To Find:

Poisson's ratio

Three moduli. (E, C and K)

Solution:

1.Poisson's ratio,

$$\mu = \frac{e_d}{e_L}$$

$$e_d = \frac{\delta d}{d} = \frac{0.0039}{30}$$

$= 0.00013$

✓ $$e_L = \frac{\delta L}{L} = \frac{0.09}{200}$$

$$\mu = \frac{0.00013}{0.00045}$$

$\mu = 0.28$

2.Young's modulus,

$$\delta L = \frac{PL}{AE}$$

$$E = \frac{PL}{A \times \delta L} = \frac{60 \times 10^3 \times 200}{706.9 \times 0.09}$$

$A = \pi \times d^2/4$

$= 706.9\ mm^2$.

$E = 1.88 \times 10^5\ N/mm^2$.

3. Shear modulus,

$E = 2C\ [1 + \mu]$

$$C = \frac{E}{2\ [1+\mu]} = \frac{1.88 \times 10^5}{2\ (1 + 0.28)}$$

$C = 73.3 \times 10^3\ N/mm^2$.

4.Bulk Modulus,

$E = 3K\ (1-2\mu)$

$$K = \frac{E}{3\ (1-2\mu)} = \frac{1.88 \times 10^5}{3\ (1 - 2 \times 0.28)} = 149.2 \times 10^3\ N/mm^2.$$

5. A rod 150 cm long and of diameter 2 cm is subjected to an axial pull of 20 KN. If the modulus of elasticity of the material of the rod is 2 X 10^5 N/mm^2.Determine the stress, strain and the elongation of the rod.

Given Data:

Diameter, d = 2 cm = 20 mm.

Length, L = 150 cm = 1500 mm.

Axial pull, $P = 20\ KN = 20 \times 10^3\ N$.

$E = 2 \times 10^5\ N/mm^2$.

$A = \pi \times d^2/4 = 314.\ 15\ mm^2$

1. Stress,

$$\sigma = \frac{P}{A} = \frac{20 \times 10^3}{314.15}$$

$\sigma = 63.66\ N/mm^2$.

2. Strain,

$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E} = \frac{63.66}{2 \times 10^5}$$

$e = 0.000318$

3. Elongation,

$$\delta L = \frac{PL}{AE} = \frac{20 \times 10^3 \times 1500}{314.15 \times 2 \times 10^5} \qquad \delta L = 0.477\ mm.$$

6. The safe stress for a hollow steel column which carries an axial load of 2.1 X 10^3 KN is 125 MN/m^2. If the external diameter of the column is 30 cm, determine the internal diameter.

Given Data:

External diameter, D = 30 cm = 300 mm.

Axial load, P = 2.1 X 10^3 KN = 2.1 X 10^6 N.

$\sigma = 125\ MN/m^2 = 125\ N/mm^2$.

Solution:

1. Internal diameter,

$$\sigma = \frac{P}{A} = \frac{2.1 \times 10^6}{\pi (D^2 - d^2)/4}$$

$$125 = \frac{2.1 \times 10^6}{\pi (300^2 - d^2)/4}$$

$$(300^2 - d^2) = \frac{2.1 \times 10^6 \times 4}{125 \times \pi}$$

$d = 261.9\ mm.$

7. The ultimate stress for a hollow steel column which carries an axial load of 1.9 MN is 480 N/mm^2. If the external diameter of the column is 200 mm, determine the internal diameter. Take the factor of safety as 4.

Given Data:

External diameter, D = 200 mm.

Axial load, P = 1.9 MN = 1.9 X 10^6 N.

ultimate stress = 480 N/mm^2.

Factor of safety = 4.

Solution:

1. **Internal diameter,**

Factor of safety = ultimate stress / working stress

4 = 480 / working stress

working stress = 480 / 4

$$\sigma = 120 \text{ N/mm}^2.$$

$$\sigma = \frac{P}{A} = \frac{1.9 \times 10^6}{\pi (200^2 - d^2)/4}$$

d = 140.85 mm.

STRESS STRAIN CURVE FOR MILD STEEL

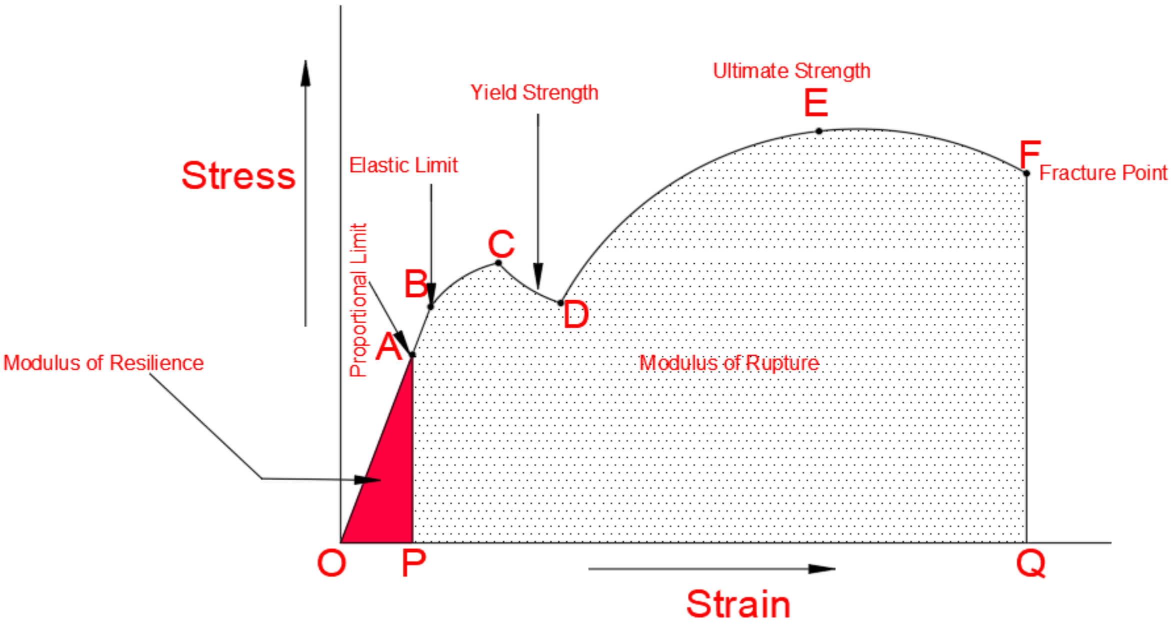


Figure No.2

- **Stress strain curve is a behavior of material when it is subjected to load.**
- **when a ductile material like mild steel is subjected to tensile test, then it passes various stages before fracture.**
- **These stages are;**
  - ✓ **Proportional Limit**
  - ✓ **Elastic Limit**

- ✓ **Yield Point**
- ✓ **Ultimate Stress Point**
- ✓ **Breaking Point**

➢ **PROPORTIONAL LIMIT**

- ✓ **Proportional limit is point on the curve up to which the value of stress and strain remains proportional.**
- ✓ **From the diagram point A is the called the proportional limit point or it can also be known as limit of proportionality.**
- ✓ **The stress up to this point can be also be known as proportional limit stress.**
- ✓ **Hook's law of proportionality from diagram can be defined between point OA. It is so, because OA is a straight line which shows that Hooke's law of stress strain is followed up to point A.**

➢ **ELASTIC LIMIT**

- ✓ **Elastic limit is the limiting value of stress up to which the material is perfectly elastic.**
- ✓ **From the curve, point B is the elastic limit point.**
- ✓ **Material will return back to its original position, if it is unloaded before the crossing of point B.**
- ✓ **This is so, because material is perfectly elastic up to point B.**

➢ **YIELD STRESS POINT**

- ✓ **Yield stress is defined as the stress after which material extension takes place more quickly with no or little increase in load.**
- ✓ **Point C is the yield point on the graph and stress associated with this point is known as yield stress.**

➢ **MODULUS OF RUPTURE**

- ✓ **Rapture strength is the strength of the material at rupture and is represented by point D.**
- ✓ **ULTIMATE STRESS POINT**
- ✓ **Ultimate stress point is the maximum strength that material have to bear stress before breaking.**
- ✓ **It can also be defined as the ultimate stress corresponding to the peak point on the stress strain graph.**
- ✓ **On the graph point E is the ultimate stress point.**
- ✓ **After point E material have very minute or zero strength to face further stress.**

- **BREAKING STRESS (POINT OF RUPTURE)**

- ✓ **Breaking point or breaking stress is point where strength of material breaks.**
- ✓ **The stress associates with this point known as breaking strength or rupture strength.**
- ✓ **On the stress strain curve, point F is the breaking stress point.**

8. A tensile was conducted on a mild steel bar. The following data was obtained from the test.

Diameter of the steel bar = 3 cm

Gauge length of the bar = 20 cm

Load at elastic limit = 250 KN

Extension at a load of 150 KN = 0.21 mm

Maximum load = 380 KN

Total extension = 60 mm

Diameter of the rod at the failure = 2.25 cm.

Determine the Young's modulus, stress at the elastic limit, percentage elongation and percentage decrease in area.

Solution:

1. Young's modulus,

$$E = \frac{\sigma}{e} \qquad A = \pi \times D^2/4 = \pi \times 30^2/4$$

$$= 706.85\ mm^2$$

$$\sigma = \frac{P}{A} = \frac{150 \times 10^3}{706.85} = 212.2\ N/mm^2.$$

$$e_L = \frac{\delta L}{L} = \frac{0.21}{200} = 0.00105$$

$$E = \frac{212.2}{0.00105} = 2.02 \times 10^5\ N/mm^2.$$

2. Stress at the elastic limit,

Stress = Load at elastic limit / Area

$$= \frac{250 \times 10}{706.85} = 353.68\ N/mm^2.$$

3. Percentage Elongation,

$$\text{Percentage Elongation} = \frac{\text{Total increase in length}}{\text{Original length}} \times 100$$

$$= \frac{60}{200} \times 100 = 30\ \%.$$

4. Percentage decrease in area,

$$\text{Percentage decrease in area} = \frac{(\text{Original area} - \text{area at the failure})}{\text{Original area}} \times 100$$

$$= \frac{(\pi \times D^2/4) - (\pi \times d^2/4)}{\pi \times D^2/4} \times 100$$

$$= \frac{(\pi \times 30^2/4) - (\pi \times 22.5^2/4)}{\pi \times 30^2/4} \times 100$$

$$= 43.75\ \%.$$

COMPOSITE SECTIONS (OR) COMPOSITE BARS

- **A bar made up of two or more different materials joined together is called a composite bar.**
- **The bars are joined in such manner that the system extends or contracts as one unit equally when subject to tension or compression.**
- **The following two points should always be kept in view while solving the problems.**
  - ✓ **Extension or contraction of the bar being equal that is deformation per unit length is equal.**
  - ✓ **The total external load on the bar is equal to the sum of the loads carried by the different materials.**
- **Total external load which will be acting over the composite bar will be shared by each bar of composite bar and hence we can say that total external load on composite bar will be equal to the addition of the load shared by each bar of composite bar.**
- **Let,**

A_1 and A_2 = Area of cross section of bar 1 and bar 2 respectively.

E_1 and E_2 = Young's modulus of elasticity for material of bar 1 and material of bar 2 respectively.

P_1 and P_2 = Load shared by bar 1 and bar 2 respectively.

σ_1 and σ_2 = Stress induced in bar 1 and bar 2 respectively.

- **As we have already discussed that total external load which will be acting over the composite bar will be shared by each bar of composite bar and therefore, we will have following equation.**

$P = P_1 + P_2$

Stress induced in bar 1, $\sigma_1 = P_1 / A_1$

Stress induced in bar 1, $\sigma_2 = P_2 / A_2$

$P = \sigma_1 A_1 + \sigma_2 A_2$

- **We have also discussed above that composite bar will have two or more than two bars of similar length and these bars will be rigidly fixed with each other and therefore change in length will be similar for each bar or we can say that strains will be same for each bar of composite bar.**

Strain in bar 1, $e1 = \sigma 1 / E1$

Strain in bar 2, $e2 = \sigma 2 / E2$

- **From above statement that strains will be same for each bar of composite bar, we will have following equation.**

$\sigma_1 / E_1 = \sigma_2 / E_2$

MODULAR RATIO:

The ratio of young's modulus of one material to the young's modulus of another material (E_1 / E_2) is called modular ratio.

9.A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter of 4cm. The composite bar is ten subjected to an axial pull of 45000 N. If the length of each bar is equal to 15cm, determine the stresses in the rod and tube, and load carried by each bar. Take E for steel = 2.1×10^5 N/mm^2 and for copper = 1.1×10^5 N/mm^2.

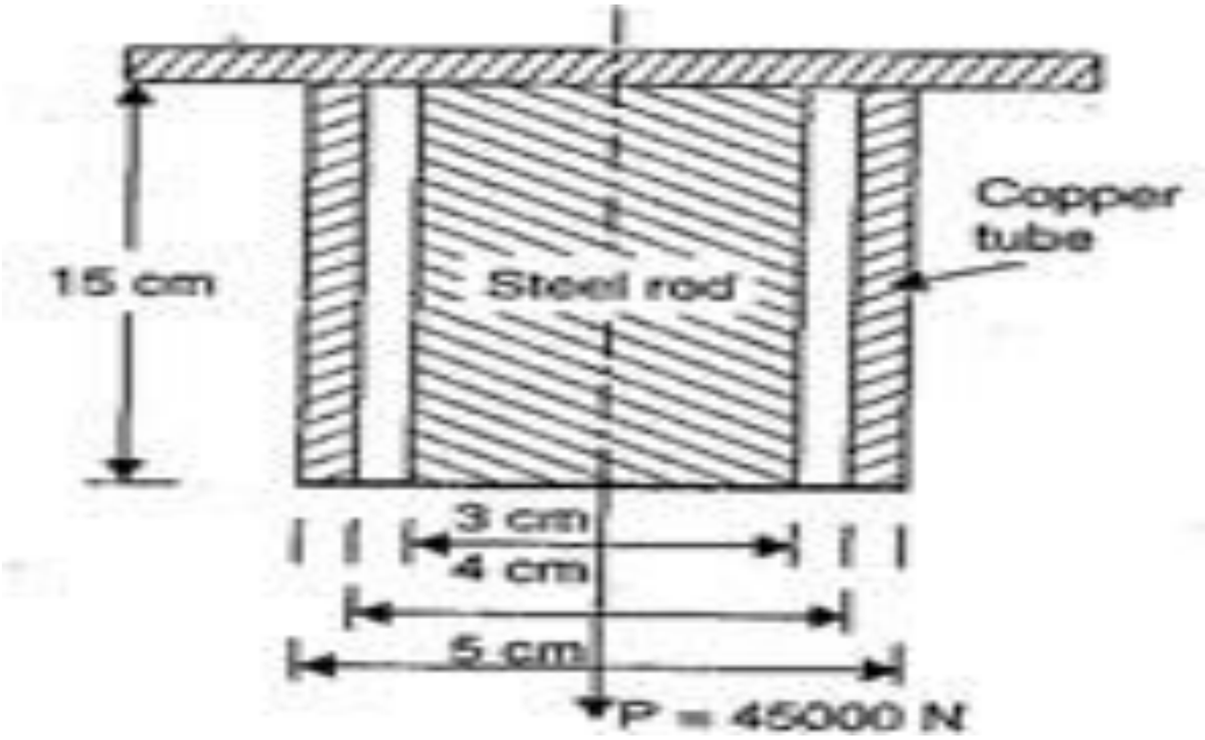


Figure No.3

Solution:

1.The stresses in the rod and tube,

Diamater of steel rod = 3cm = 30mm

Area of steel rod, $A_s = \frac{\pi \times 30^2}{4} = 706.85$ mm^2

Area of copper tube, $A_c = \pi (50^2 - 40^2) / 4 = 706.85$ mm^2

Now strain in steel = Strain in copper

$$\frac{\sigma_S}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_S = \frac{E_s}{E_c} \times \sigma_c$$

$$\sigma_S = \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c$$

$$\sigma_S = 1.909\, \sigma_c \text{ .... } (1)$$

Stress = Load / Area

Load = Stress X Area

Load on steel + load on copper = Total load

σ_S x A_S + σ_c x A_c = P

1.909 σ_c x 706.86 + σ_c X 706.86 = 45000

σ_c (1.909 x 706.86 + 706.86) = 45000

2056.25 σ_c = 45000

$$\sigma_c = \frac{45000}{2056.25}$$

σ_c = 21.88 N/mm^2

Substituting the value of σ_c in equation (1), we get

σ_c = 1.909 x 21.88

σ_c = 41.77 N/mm^2.

2. Load carried by each bar,

Load = Stress x Area

Load carried by steel rod,

P_s = σ_S x A_S = 41.77 x 706.86 = 29525.5 N.

Load Carried by copper tube, P = P_s + P_c

P_c = P - P_s = 45000 – 29525.5 or σ_c x A_c = 15474.5 N.

10.A load of 2 MN is applied on a short concrete column 500 mm X 500 mm. the column is reinforced with four steel bars of 10 mm diameter, one in each corner. Find the stresses in the concrete and steel bars. Take E for steel = 2.1 x 10^5 N/mm^2 and for concrete = 1.4 x 10^4 N/mm^2.

Solution:

1.The stresses in the concrete and steel bars,

Now strain in steel = Strain in concrete

$$\frac{\sigma_S}{E_s} = \frac{\sigma_{conc}}{E_{conc}}$$

$$\sigma_S = \frac{E_s}{E_{conc}} X \ \sigma_{conc} = \frac{2.1 \times 10^5 X \ \sigma_{conc}}{1.4 \times 10^4}$$

$$\sigma_S = 15\ \sigma_{conc} \quad \ldots\ldots\ldots\ldots\ldots\ldots.. \quad (1)$$

Area of column = 500 X 500 = 250000 mm^2

Area of 4 steel bars, A_s = π X 10^2 X 4= 314.159 mm^2

Area of concrete = Area of column - Area of 4 steel bars

$$= 250000 - 314.159 = 249685.841 \text{ mm}^2$$

Load on steel + load on copper = Total load

$$\sigma_S \times A_S + \sigma_{conc} \times A_{conc} = P$$

$$15\,\sigma_{conc} \times 314.159 + \sigma_{conc} \times 249685.841 = 2000000$$

$$\sigma_{conc} = 7.86 \text{ N/mm}^2.$$

Substitute the value of σ_{conc} in equ. 1, $\sigma_S = 15\,\sigma_{conc}$

$$\sigma_S = 117.92 \text{ N/mm}^2.$$

11.Three bars made of copper, zinc and aluminium are of equal length and have cross section 500, 750 and 1000 mm respectively. They are rigidly connected at their ends. If this compound member is subjected to a longitudinal pull of 250 KN, estimate the load carried on each rod and the induced stresses. Take E for copper = 1.3×10^5 N/mm^2 , for zinc = 1.0×10^5 N/mm^2 and for aluminium = 0.8×10^5 N/mm^2.

Solution:

1.The stresses in the copper, zinc and aluminium rods,

Now strain in copper = Strain in zinc = strain in aluminium

$$\frac{\sigma_c}{E_c} = \frac{\sigma_z}{E_z} = \frac{\sigma_{al}}{E_{al}}$$

$$\sigma_c = \frac{E_c}{E_{al}} \times \sigma_{al} = \frac{1.3 \times 10^5}{0.8 \times 10^5} \times \sigma_{al}$$

$$= 1.625\,\sigma_{al} \;\ldots\ldots\ldots\ldots\ldots\; (1)$$

$$\sigma_z = \frac{E_z}{E_{al}} \times \sigma_{al} = \frac{1.0 \times 10^5}{0.8 \times 10^5} \times \sigma_{al}$$

$$= 1.25\,\sigma_{al} \;\ldots\ldots\ldots\ldots\; (2)$$

Now,

Total load = Load on copper + load on zinc + Load on aluminium

$$= \sigma_c \times A_c + \sigma_z \times A_z + \sigma_{al} \times A_{al}$$

$$250 \times 10^3 = 1.625\,\sigma_{al} \times 500 + 1.25\,\sigma_{al} \times 750 + \sigma_{al} \times 1000$$

$$250 \times 10^3 = 2750\,\sigma_{al}$$

$$\sigma_{al} = \frac{250 \times 10^3}{2750}$$

$\sigma_{al} = 90.9$ N/mm^2.

Substitute the value of σ_{al} in equ. 1, $\sigma_c = 1.625\ \sigma_{al}$

$= 1.625 \times 90.9$

$\sigma_c = 147.7$ N/mm^2.

Substitute the value of σ_{al} in equ. 2, $\sigma_z = 1.25\ \sigma_{al}$

$= 1.25 \times 90.9$

$\sigma_z = 113.625$ N/mm^2.

2. Load carried by each rod,

Load carried by copper rod, $P_c = \sigma_c \times A_c = 147.7 \times 500$

$P_c = 73850$ N.

Load carried by zinc rod, $P_z = \sigma_z \times A_z = 113.625 \times 750$

$P_z = 85218$ N.

Load carried by aluminium rod, $P_{al} = \sigma_{al} \times A_{al} = 90.9 \times 1000$

$P_{al} = 90900$ N.

THERMAL STRESSES

Thermal stresses are the stresses induced in a body due to change in temperature.

Thermal strain, $e = \dfrac{\delta L}{L} = \dfrac{\alpha\, T\, L}{L} = \alpha\, T$

Thermal stress, σ **= Thermal strain X E**

$= \alpha\, T \times E.$

Stress & Strain when the supports yield,

If the supports yield by an amount equal to δ, then the actual expansion = Expansion due to rise in temperature – δ

$= \alpha\, T\, L - \delta$

Actual Strain = Actual Expansion / Original Length

$= (\alpha\, T\, L - \delta) / L$

Actual Stress = Actual Strain X E = $(\alpha\, T\, L - \delta) / L \times E.$

11.A rod is 2 m long at a temperature of 10°C. Find the expansion of the rod, when the temperature is raised to 80°C. If this expansion is prevented, find the stress induced in the material of the rod. Take E = 1.0 x 10^5 MN/m^2 and α = 0.000012 per degree centigrade.

Solution:

1. Expansion of the rod, T = 80°C - 10°C = 70°C

Expansion of the rod = α T L = 0.000012 X 70 X 2000

= 1.68 mm.

2. The stress induced in the material of the rod,

Thermal stress, σ = Thermal strain X E

= α T X E = 0.000012 X 70 X 1.0 x 10^5

σ = 84 N/mm^2

12.A steel rod of 3 cm diameter and 5 m long is connected to two grips and the rod is maintained at a temperature of 95°C. Determine the stress and pull exerted when the temperature falls to 30°C, if

i) the ends do not yield and

ii) the ends yield by 0.12 cm. Take E = 2.0 x 10^5 MN/m^2 and α = 0.000012 per degree centigrade.

Solution:

1.Stress and pull when the ends do not yield,

Stress = α T X E = 0.000012 X 65 X 2.0 x 10^5 **T = 95°C - 30°C**

σ = 156 N/mm^2 **= 65°C**

Pull in the rod, P = Stress X Area = 156 X 225 X π

P = 110269.9 N

Area of the rod, A = π X d^2 / 4 = π X 30^2 / 4 = 225 π mm^2.

2.Stress and pull when the ends yield by 0.12 cm, δ = 0.12 cm

= 1.2 mm

Actual Stress = (α T L – δ) / L X E.

$$= \frac{(0.000012 \times 65 \times 5000 - 1.2)}{5000} \times 2.0 \times 10^5$$

σ = 108 N/mm^2

Pull in the rod, P = Stress X Area = 108 X 225 X π

P = 76340.7 N

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – II - PRINCIPAL STRESSES AND STRAIN– SCIA1301

INTRODUCTION

- **PRINCIPAL PLANES:**
- ✓ **The planes which have no shear stress are known as principal planes.**
- ✓ **Hence principal planes are the planes of zero shear stress.**
- ✓ **These planes carry only normal stresses.**
- **PRINCIPAL STRESSES:**
- ✓ **The normal stresses acting on a principal plane are known as principal stresses.**

METHODS FOR DETERMINING STRESSES ON OBLIQUE SECTION:

- **Analytical Method**
- **Graphical method**
- **MAJOR PRINCIPAL STRESS:**
- ✓ **The plane carrying the maximum normal stress is called the major principal plane and normal stress is called major principal stress.**
- **MINOR PRINCIPAL STRESS**
- ✓ **The plane carrying the minimum normal stress is known as minor principal plane and normal stress is called minor principal stress.**

ANALYTICAL METHOD FOR DETERMINING STRESSES ON OBLIQUE SECTION

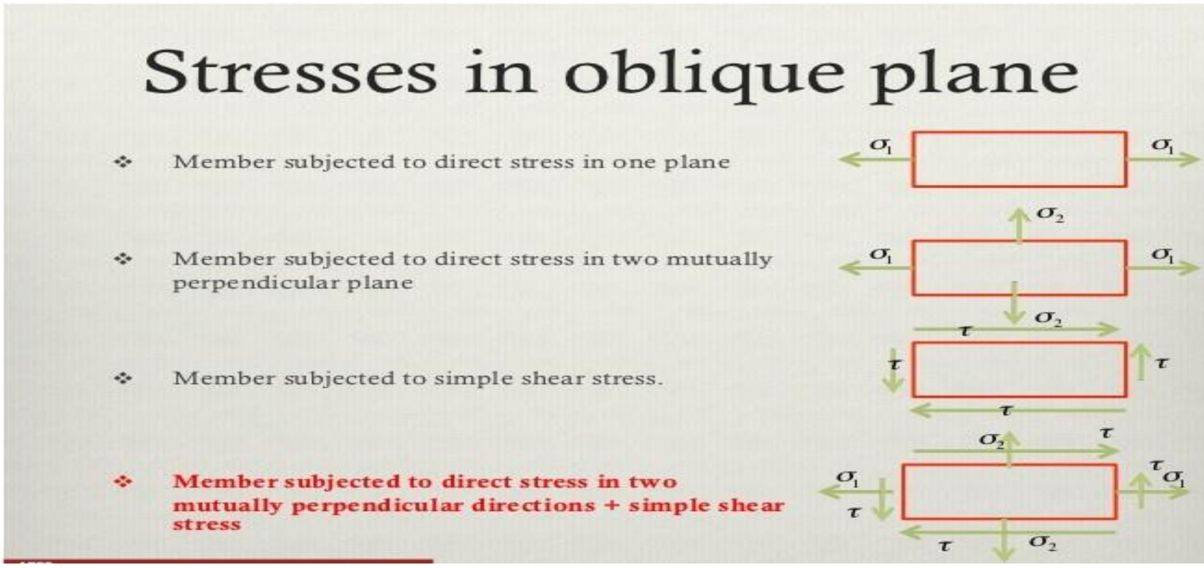


Figure No.1

- **Induced stress is divided into two components which are given as:**
- ✓ **Normal stress**

- **Normal Stress on an inclined section.**
- ✓ **Tangential stress**
- **Shear Stress on an inclined section.**

MEMBER SUBJECTED TO A DIRECT STRESS IN ONE PLANE

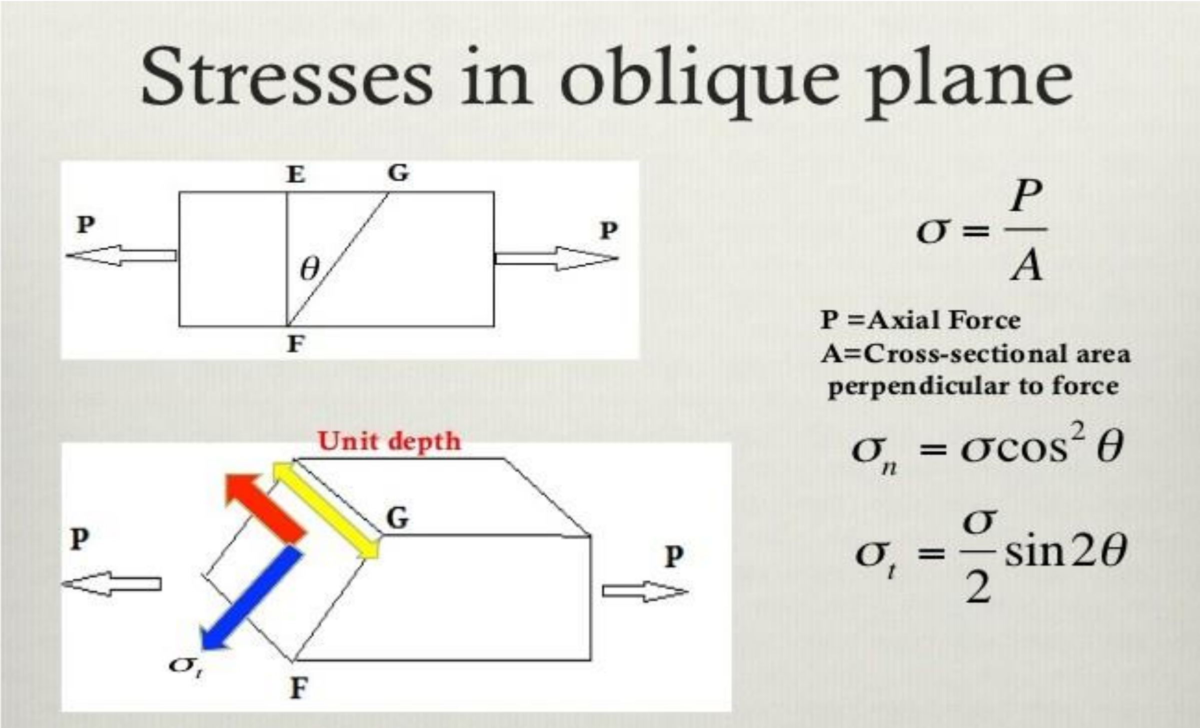


Figure No.2

MEMBER SUBJECTED TO DIRECT STRESSES IN TWO MUTUALLY PERPENDICULAR DIRECTIONS

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

MEMBER SUBJECTED TO A SIMPLE SHEAR STRESS

$$\sigma_n = \tau \sin 2\theta$$

$$\sigma_t = -\tau \cos 2\theta$$

MEMBER SUBJECTED TO DIRECT STRESSES IN TWO MUTUALLY PERPENDICULAR DIRECTIONS ACCOMPANIED BY A SIMPLE SHEAR STRESS

Stresses in oblique plane

- **Member subjected to direct stress in two mutually perpendicular directions + simple shear stress**

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - \tau \cos 2\theta$$

$$\text{Major principal Stress} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$\text{Minor principal Stress} = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

MAX SHEAR STRESS

$$\frac{d}{d\theta}(\sigma_t) = 0$$

$$\frac{d}{d\theta}\left[\frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta\right] = 0 \Longrightarrow \tan 2\theta = \frac{\sigma_1 - \sigma_2}{2\tau}$$

MAXIMUM SHEAR STRESS

$$(\sigma_t)_{\max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

OBLIQUITY:

The angle made by the resultant stress with the normal of the oblique plane is known as Obliquity.

It is denoted by φ.

Mathematically, tan φ = σ_t / σ_n

MAXIMUM SHEAR STRESS:

$$(\sigma_t)_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

PROBLEMS:

1.A rectangular bar of cross sectional area 10000 mm^2 is subjected to an axial load of 20 KN. Determine the normal and shear stresses on a section which is inclined at an angle of 30° with normal cross section of the bar.

Solution:

Normal Stress:

$\sigma_n = \sigma \cos^2 \Theta$

$\sigma = P/A = 20 X10 / 10000$

$\sigma = 2$ N/mm^2

$\sigma_n = \sigma \cos^2 \Theta = 2 X \cos^2 30°$

$\sigma_n = 1.5$ N/mm^2.

$\sigma_t = \sigma/2 X \sin 2\Theta$

$= 2/2 X \sin 2 X 30°$

$\sigma_t = 0.866$ N/mm^2.

2.Find the diameter of a circular bar which is subjected to an axial pull of 160 KN, if the maximum allowable shear stress on any section is 65 N/mm^2.

Solution:

Diameter of a circular bar,

Direct stress, $\sigma = P/A = \dfrac{160000}{\pi X d^2/4} = \dfrac{640000}{\pi X d^2}$

Maximum shear stress $= \sigma /2 = \dfrac{640000/ \pi X d^2}{2}$

$65 = \dfrac{640000/ \pi X d^2}{2}$

$d^2 = 1567$

d = 39.58 mm.

3. A rectangular bar of cross-sectional area of 11000 mm is subjected to a tensile load P as shown in fig. The permissible normal and shear stresses on the oblique plane BC are given as 7 N/mm^2 and 3.5 N/mm^2 respectively. Determine the safe value of P.

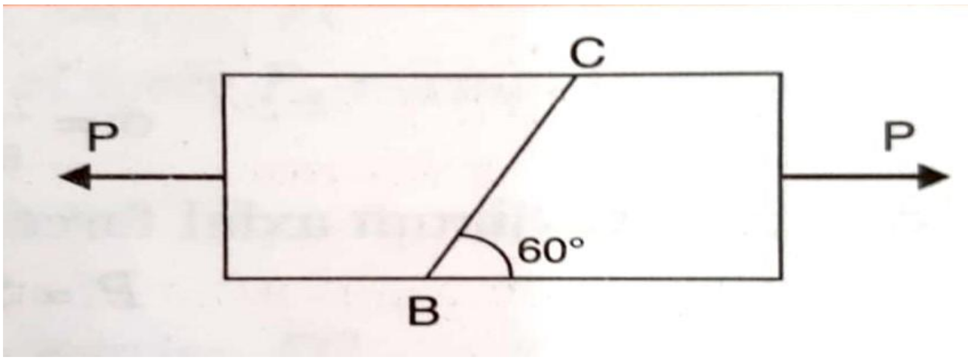


Figure No.3

Solution:

Angle of oblique plane with the axis of the bar = 60°

Angle of oblique plane BC with the normal cross section of the bar,

$\Theta = 90° - 60° = 30°$

$\sigma_n = \sigma \cos^2 \Theta$

$7 = \sigma \cos^2 \Theta$

$\sigma = 7 / \cos^2 \Theta = 7 / \cos^2 30°$

$\sigma = 9.334 \text{ N/mm}^2$.

$\sigma_t = \sigma/2 \sin 2\Theta$

$3.5 = \sigma/2 \times \sin 2\Theta$

$\sigma = 3.5 \times 2 / \sin 2\Theta = 3.5 \times 2 / \sin 2 \times 30°$

$\sigma = 8.083 \text{ N/mm}^2$.

The safe stress is the least value of the above two, $\sigma = 8.083 \text{ N/mm}^2$.

Safe Value of axial pull, P = Safe stress X Area of cross section

$P = 8.083 \times 11000 = 88913 \text{ N}$

$P = 88.913 \text{ KN}$

4.Two wooden pieces 10 cm X 10 cm in cross section are joined together along line AB as shown in fig. What maximum axial force P can be applied if the allowable shearing stress along AB is 1.2 N/mm².

Solution:

Angle of line AB with the axis of axial force = 30°

Angle of line AB with the normal cross-section,

$\Theta = 90° - 30° = 60°$

Let P – maximum axial force

$\sigma_t = \sigma/2 \times \sin 2\Theta$

$1.2 = \sigma/2 \times \sin 2 \times 60°$

$\sigma = 1.2 \times 2 / \sin 2 \times 60° = 1.2 \times 2 / \sin 120°$

$\sigma = 2.771 \text{ N/mm}^2$.

$P = \sigma \times A = 2.771 \times (100 \times 100) = 27710 \text{ N or } 27.710 \text{ KN}$

5. The tensile stresses at a point across two mutually perpendicular planes are 120 N/mm² and 60 N/mm². Determine the normal, tangential and resultant stresses on a plane inclined at 30°to the axis of minor stress.

Solution:

Angle of oblique plane with the axis of minor principal stress, ϴ = 30°

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

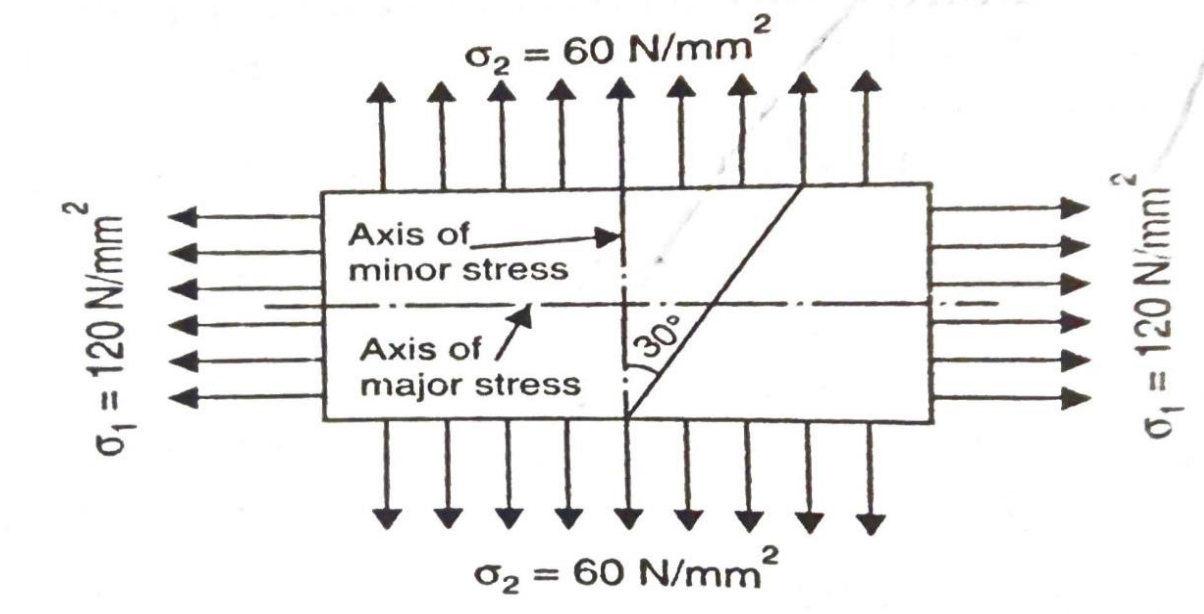


Figure No.4

Normal stress:

$$\sigma_n = \frac{120 + 60}{2} + \frac{120 - 60}{2}\cos 2 \times 30°$$

$$\sigma_n = 105\ \text{N/mm}^2.$$

The tangential (or shear stress) σ_t is given by equation (3.7).

$$\therefore \quad \sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

$$= \frac{120 - 60}{2} \sin (2 \times 30°)$$

$$= 30 \times \sin 60° = 30 \times 0.866$$

$$= \mathbf{25.98\ N/mm^2.} \quad \textbf{Ans.}$$

Resultant stress

The resultant stress (σ_R) is given by equation (3.8)

$$\therefore \quad \sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{105^2 + 25.98^2}$$

$$= \sqrt{11025 + 674.96} = \mathbf{108.16\ N/mm^2}$$

6.The stresses at a point in a bar are 200 N/mm^2 (T) and 100 N/mm^2(C). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major stress. Also determine the intensity of shear stress in the material at the point.

Solution:

Angle of plane with the major principal stress = 60°

Angle of plane with the normal cross-section,

θ = 90° - 60° = 30°

1. Normal stress:

$$\sigma_n = \frac{200 + (-100)}{2} + \frac{200 - (-100)}{2} \times \cos 2 \times 30°$$

$$\sigma_n = 125\ N/mm^2.$$

2. Tangential stress:

$$\sigma_t = \frac{200 - (-100)}{2} \sin 2 \times 30°$$

$$\sigma_t = 129.9\ N/mm^2.$$

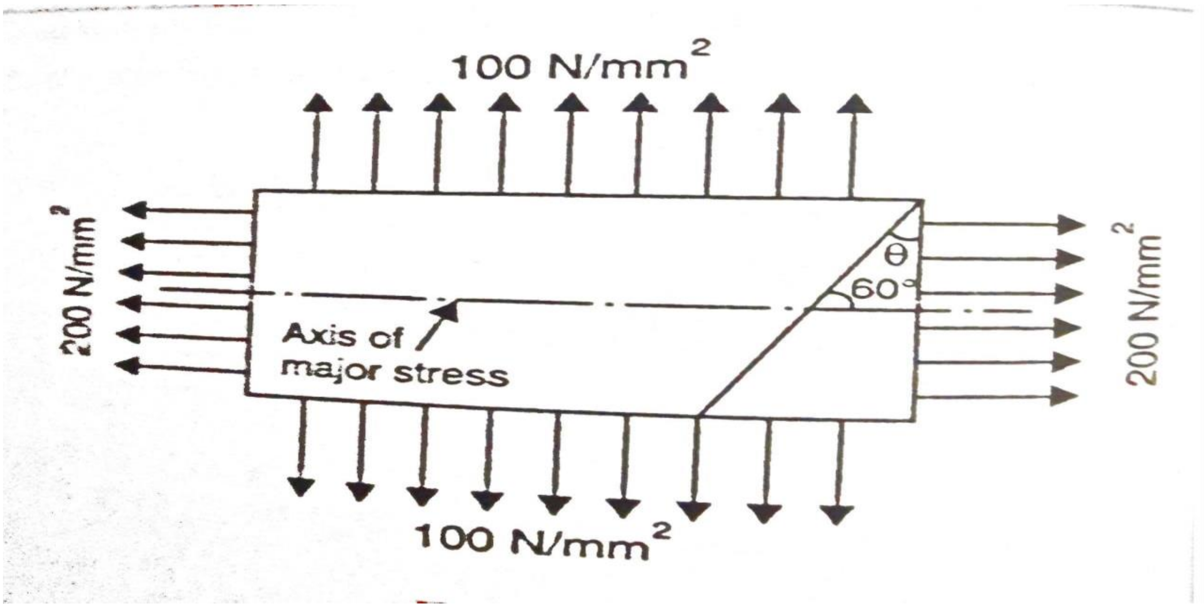


Figure No.5

Using equation (3.8) for resultant stress,

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{125^2 + 129.9^2}$$

$$= \sqrt{15625 + 16874} = \mathbf{180.27\ N/mm^2}. \quad \textbf{Ans.}$$

The inclination of the resultant stress with the normal of the inclined plane is given by equation [3.8 (A)] as

$$\tan\phi = \frac{\sigma_t}{\sigma_n} = \frac{129.9}{125} = 1.04$$

$$\therefore \qquad \phi = \tan^{-1} 1.04 = \mathbf{46° \ 6'}. \quad \textbf{Ans.}$$

Maximum shear stress

Maximum shear stress is given by equation (3.9)

$$\therefore \qquad (\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{200 - (-100)}{2} = \frac{200 + 100}{2} = \mathbf{150\ N/mm^2}. \quad \textbf{Ans.}$$

Problem 3.7. *At a point in a strained material the principal tensile stresses across two perpendicular planes, are 80 N/mm² and 40 N/mm². Determine normal stress, shear stress and the resultant stress on a plane inclined at 20° with the major principal plane. Determine also the obliquity. What will be the intensity of stress, which acting alone will produce the same maximum strain if Poisson's ratio = $\frac{1}{4}$.*

Sol. Given :

Major principal stress, $\sigma_1 = 80$ N/mm²

Minor principal stress, $\sigma_2 = 40$ N/mm²

The plane CE is inclined at angle 20° with major principal plane (*i.e.*, plane BC).

$\therefore \quad \theta = 20°$

Poisson's ratio, $\mu = \dfrac{1}{4}$

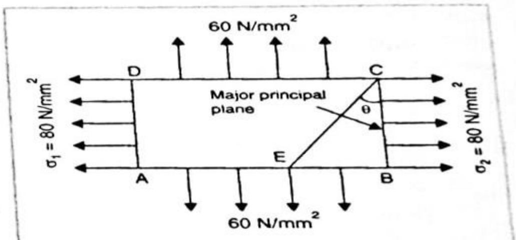


Fig. 3.7

Let σ_n = Normal stress on inclined plane CE

σ_t = Shear stress and

σ_R = Resultant stress.

Using equation (3.6), we get

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta = \frac{80 + 40}{2} + \frac{80 - 40}{2} \cos (2 \times 20°)$$

$$= 60 + 20 \times \cos 40° = \mathbf{75.32\ N/mm^2}.\quad \textbf{Ans.}$$

The shear stress is given by equation (3.7)

$$\therefore \quad \sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta = \frac{80 - 40}{2} \sin (2 \times 20°) = 20 \sin 40°$$

$$= \mathbf{12.865\ N/mm^2}.\quad \textbf{Ans.}$$

The resultant stress is given by equation (3.8)

$$\therefore \quad \sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$$= \sqrt{75.32^2 + 12.856^2} = \mathbf{76.4\ N/mm^2}.\quad \textbf{Ans.}$$

Obliquity (ϕ) is given by equation [3.8 (A)]

$$\tan \phi = \frac{\sigma_t}{\sigma_n} = \frac{12.856}{75.32}$$

$$\therefore \quad \phi = \tan^{-1} \frac{12.856}{75.32} = \mathbf{9° 41'}.\quad \textbf{Ans.}$$

Let σ = stress which acting alone will produce the same maximum strain. The maximum strain will be in the direction of major principal stress.

$$\therefore \quad \text{Maximum strain} = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} = \frac{1}{E}(\sigma_1 - \mu\sigma_2)$$

$$= \frac{1}{E}\left(80 - \frac{40}{4}\right) = \frac{70}{E}$$

The strain due to stress $\quad \sigma = \dfrac{\sigma}{E}$

Equating the two strains, we get $\quad \dfrac{70}{E} = \dfrac{\sigma}{E}$

$$\therefore \quad \sigma = \mathbf{70\ N/mm^2}.\quad \textbf{Ans.}$$

Problem 3.8. *At a point in a strained material the principal stresses are 100 N/mm² (tensile) and 60 N/mm² (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50° to the axis of major principal stress. Also determine the maximum shear stress at the point.*

Sol. Given :

Major principal stress, $\sigma_1 = 100 \text{ N/mm}^2$

Minor principal stress, $\sigma_2 = -60 \text{ N/mm}^2$ (Negative sign due to compressive stress)

Angle of the inclined plane with the axis of major principal stress = 50°

∴ Angle of the inclined plane with the axis of minor principal stress,

$$\theta = 90 - 50 = 40°.$$

Normal stress (σ_n)

Using equation (3.6),

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$= \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2}\cos(2 \times 40°)$$

$$= \frac{100 - 60}{2} + \frac{100 + 60}{2}\cos 80°$$

$$= 20 + 80 \times \cos 80° = 20 + 80 \times .1736$$

$$= 20 + 13.89 = \mathbf{33.89 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Shear stress (σ_t)

Using equation (3.7), $\sigma_t = \dfrac{\sigma_1 - \sigma_2}{2}\sin 2\theta$

$$= \frac{100 - (-60)}{2}\sin(2 \times 40°)$$

$$= \frac{100 + 60}{2}\sin 80° = 80 \times 0.9848 = \mathbf{78.785 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Resultant stress (σ_R)

Using equation on (3.8),

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{33.89^2 + 78.785^2}$$

$$= \sqrt{1148.53 + 6207.07} = \mathbf{85.765 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Maximum shear stress

Using equation (3.9),

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100 - (-60)}{2}$$

$$= \frac{100 + 60}{2} = \mathbf{80 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Problem 3.9. *At a point in a strained material, the principal stresses are 100 N/mm² tensile and 40 N/mm² compressive. Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major principal stress. What is the maximum intensity of shear stress in the material at the point ?*

Sol. Given :

The major principal stress, $\sigma_1 = 100$ N/mm²

The minor principal stress, $\sigma_2 = -40$ N/mm² (Minus sign due to compressive stress)

Inclination of the plane with the axis of major principal stress = 60°

∴ Inclination of the plane with the axis of minor principal stress,

$$\theta = 90 - 60 = 30°.$$

Resultant stress in magnitude

The resultant stress (σ_R) is given by equation (3.8) as

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

where σ_n = Normal stress and is given by equation (3.6) as

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$= \frac{100 + (-40)}{2} + \frac{100 - (-40)}{2} \cos (2 \times 30°)$$

$$= \frac{100 - 40}{2} + \frac{100 + 40}{2} \cos 60° \qquad (\because \cos 60° = 0.5)$$

$$= 30 + 70 \times 0.5$$

$$= 65 \text{ N/mm}^2$$

and σ_t = Shear stress and is given by equation (3.7) as

$$= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta = \frac{100 - (-40)}{2} \sin (2 \times 30°)$$

$$= \frac{100 + 40}{2} \sin 60° = 70 \times .866 = 60.62 \text{ N/mm}^2$$

$$\sigma_R = \sqrt{65^2 + 60.62^2} = \mathbf{88.9\ N/mm^2}. \quad \textbf{Ans.}$$

Direction of resultant stress

Let the resultant stress is inclined at an angle ϕ to the normal of the oblique plane. Then using equation [3.8 (*A*)].

$$\tan \phi = \frac{\sigma_t}{\sigma_n} = \frac{60.62}{65}$$

$$\therefore \qquad \phi = \tan^{-1} \frac{60.62}{65} = \mathbf{43°}. \quad \textbf{Ans.}$$

Maximum shear stress

Using equation (3.9), $(\sigma_t)_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$

$$= \frac{100 - (-40)}{2} = \frac{100 + 40}{2} = \mathbf{70\ N/mm^2}. \quad \textbf{Ans.}$$

Problem 3.11. *At a point within a body subjected to two mutually perpendicular directions, the stresses are 80 N/mm² tensile and 40 N/mm² tensile. Each of the above stresses is accompanied by a shear stress of 60 N/mm². Determine the normal stress, shear stress and resultant stress on an oblique plane inclined at an angle of 45° with the axis of minor tensile stress.*

Sol. Given :

Major tensile stress, $\sigma_1 = 80$ N/mm²

Minor tensile stress, $\sigma_2 = 40$ N/mm²

Shear stress, $\tau = 60$ N/mm²

Angle of oblique plane, with the axis of minor tensile stress,

$\theta = 45°$.

(i) Normal stress (σ_n)

Using equation (3.12),

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{80+40}{2} + \frac{80-40}{2}\cos(2\times 45°) + 60\sin(2\times 45°)$$

$$= 60 + 20\cos 90° + 60 \sin 90°$$

$$= 60 + 20\times 0 + 60\times 1 \qquad (\because \cos 90° = 0)$$

$$= 60 + 0 + 60 = \mathbf{120\ N/mm^2}. \quad \textbf{Ans.}$$

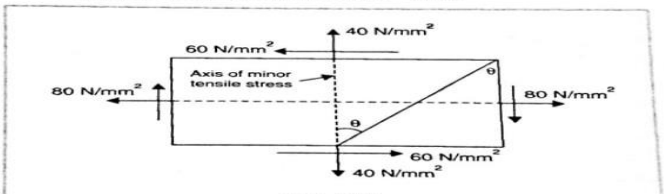


Fig. 3.13

(ii) Shear (or tangential) stress (σ_t)

Using equation (3.13),

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$$

$$= \frac{80-40}{2}\sin(2\times 45°) - 60\times\cos(2\times 45°)$$

$$= 20\times \sin 90° - 60\cos 90°$$

$$= 20\times 1 - 60\times 0$$

$$= \mathbf{20\ N/mm^2}. \quad \textbf{Ans.}$$

(iii) Resultant stress (σ_R)

Using equation, $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

121.655 N/mm²

11. A rectangular block of material is subjected to a tensile stress of 110 N/mm² on one plane and a tensile stress of 47 N/mm² on the plane at right angles to the former. Each of the above stresses is accompanied by a shear stress of 63 N/mm² and that associated with the former tensile stress tends to rotate the block anticlockwise. Find the direction and magnitude of each of the principal stress and magnitude of the greatest shear stress.

Solution:

Sol. Given :

Major tensile stress, $\sigma_1 = 110$ N/mm²

Minor tensile stress, $\sigma_2 = 47$ N/mm²

Shear stress, $\tau = 63$ N/mm²

(i) Major principal stress is given by equation (3.15).

$$\therefore \text{Major principal stress} = \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

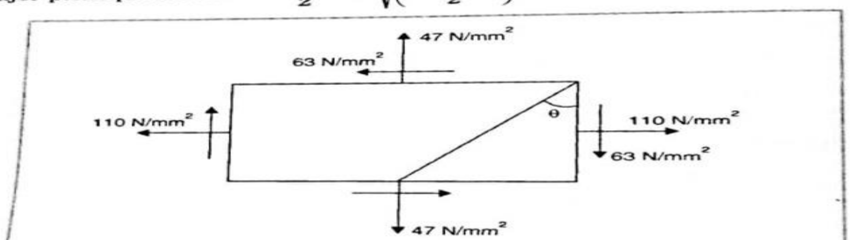


Fig. 3.14

$$= \frac{110+47}{2} + \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2}$$

$$= \frac{157}{2} + \sqrt{\left(\frac{63}{2}\right)^2 + (63)^2}$$

$$= 78.5 + \sqrt{31.5^2 + 63^2} = 78.5 + \sqrt{992.25 + 3969}$$

$$= 78.5 + 70.436 = \mathbf{148.936\ N/mm^2}. \quad \textbf{Ans.}$$

Minor principal stress is given by equation (3.16).

$$\therefore \textit{Minor principal stress}, = \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{110 + 47}{2} - \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2} = 78.5 - 70.436$$

$$= \mathbf{8.064\ N/mm^2.}\quad \textbf{Ans.}$$

The directions of principal stresses are given by equation (3.14).

∴ Using equation (3.14),

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 63}{110 - 47}$$

$$= \frac{2 \times 63}{63} = 2.0$$

∴ $2\theta = \tan^{-1} 2.0 = 63° 26'$ or $243° 26'$

∴ $\theta = \mathbf{31° 43'}$ **or** $\mathbf{121° 43'}$. **Ans.**

(*ii*) *Magnitude of the greatest shear stress*

Greatest shear stress is given by equation (3.18).

Using equation (3.18),

$$(\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(100 - 47)^2 + 4 \times 63^2}$$

$$= \frac{1}{2}\sqrt{63^2 + 4 \times 63^2} = \frac{1}{2} \times 63 \times \sqrt{5}$$

$$= \mathbf{70.436\ N/mm^2.}\quad \textbf{Ans.}$$

Problem 3.13. *Direct stresses of 120 N/mm² tensile and 90 N/mm² compression exist on two perpendicular planes at a certain point in a body. They are also accompanied by shear stress on the planes. The greatest principal stress at the point due to these is 150 N/mm².*

(*a*) *What must be the magnitude of the shearing stresses on the two planes ?*

(*b*) *What will be the maximum shearing stress at the point ?*

Sol. Given :

Major tensile stress, $\sigma_1 = 120$ N/mm²

Minor compressive stress, $\sigma_2 = -90$ N/mm² (Minus sign due to compression)

Greatest principal stress $= 150$ N/mm²

(*a*) Let τ = Shear stress on the two planes.

Using equation (3.15) for greatest principal stress, we get

$$\text{Greatest principal stress} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

or

$$150 = \frac{120 + (-90)}{2} + \sqrt{\left(\frac{120 - (-90)}{2}\right)^2 + \tau^2}$$

$$= \frac{120 - 90}{2} + \sqrt{\left(\frac{120 + 90}{2}\right)^2 + \tau^2}$$

$$= 15 + \sqrt{105^2 + \tau^2}$$

or $$150 - 15 = \sqrt{105^2 + \tau^2}$$

or $$135 = \sqrt{105^2 + \tau^2}$$

Squaring both sides, we get

$$135^2 = 105^2 + \tau^2$$

or $$\tau^2 = 135^2 - 105^2 = 18225 - 11025 = 7200$$

$$\therefore \quad \tau = \sqrt{7200} = \mathbf{84.853\ N/mm^2.} \quad \textbf{Ans.}$$

(*b*) *Maximum shear stress at the point*

Using equation (3.18) for maximum shear stress,

$$(\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{[120 - (-90)]^2 + 4 \times 7200} \qquad (\because \quad \tau^2 = 7200)$$

$$= \frac{1}{2}\sqrt{210^2 + 28800} = \frac{1}{2}\sqrt{44100 + 28800} = \frac{1}{2} \times 270$$

$$= \mathbf{135\ N/mm^2.} \quad \textbf{Ans.}$$

13. The normal stress in two mutually perpendicular directions are 600 N/mm^2 and 300 N/mm^2 both tensile. The complimentary shear stress in these directions are of intensity 450 N/mm$^{2.}$ Find the normal and tangential stresses on the two planes which are equally inclined to the planes carrying the normal stresses mentioned above.

Solution:

1.Normal stress which is equally inclined to the plane,

The normal and tangential stresses are to be calculated on the two planes which are equally inclined to the planes of major tensile stress and of minor tensile stress. This means ϴ = 45° and 135°.

When ϴ = 45°, normal stress is given by

$$\sigma_n = \frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2 \times \Theta + \tau \sin 2 \times \Theta$$

$$\sigma_n = \frac{600 + 300}{2} + \frac{600 - 300}{2} \cos 2 \times 45° + 450 \times \sin 2 \times 45°$$

$$\sigma_n = 450 + 150 \times 0 + 450 \times 1$$

$$\sigma_n = 900\ N/mm^2$$

When ϴ = 135°, normal stress is given by

$$\sigma_n = \frac{600 + 300}{2} + \frac{600 - 300}{2} \cos 2 \times 135° + 450 \times \sin 2 \times 135°$$

$$\sigma_n = 450 + 150 \times 0 + 450 \times (-1) = 450 - 450$$

$$\sigma_n = 0$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\Theta - \tau \cos 2\Theta$$

2. Tangential stress which is equally inclined to the plane,

When $\Theta = 45°$, tangential stress is given by

$$\sigma_t = \frac{600 - 300}{2} \sin 2 \times 45° - 450 \times \cos 2 \times 45°$$

$$= 150 \times 1 - 450 \times 0$$

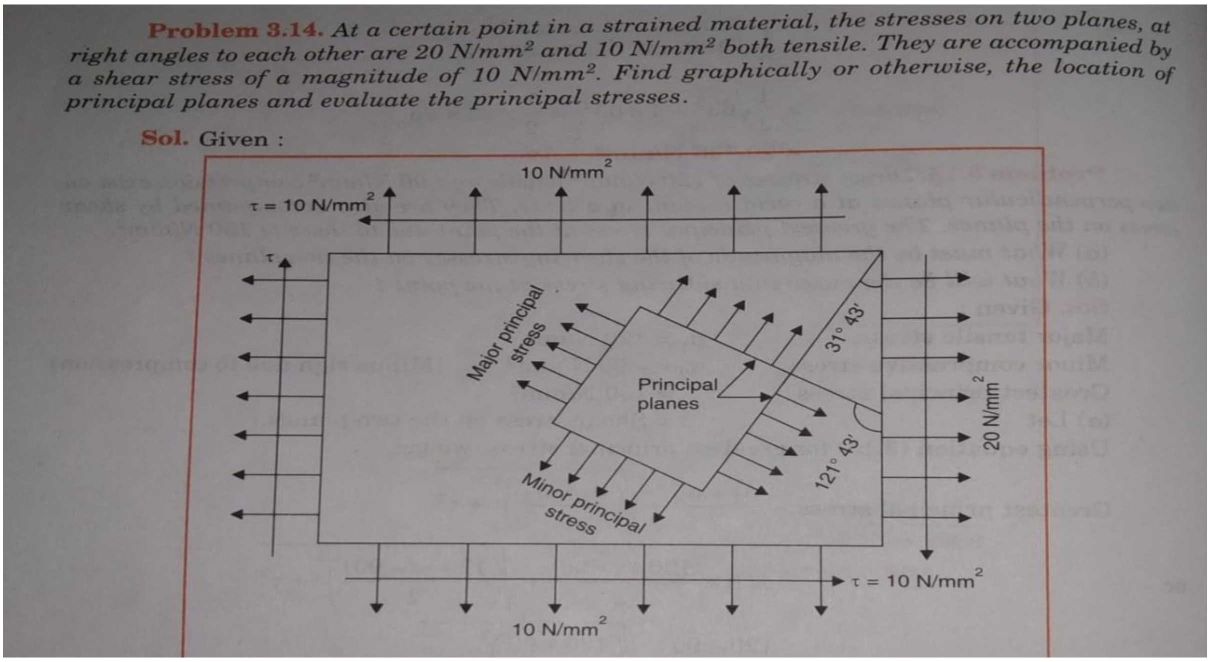
$$\sigma_t = 150 \text{ N/mm}^2$$

When $\Theta = 135°$, tangential stress is given by

$$\sigma_t = \frac{600 - 300}{2} \sin 2 \times 135° - 450 \times \cos 2 \times 135°$$

$$= 150 \times (-1) - 450 \times 0$$

$$\sigma_t = -150 \text{ N/mm}^2$$



Problem 3.14. *At a certain point in a strained material, the stresses on two planes, at right angles to each other are 20 N/mm^2 and 10 N/mm^2 both tensile. They are accompanied by a shear stress of a magnitude of 10 N/mm^2. Find graphically or otherwise, the location of principal planes and evaluate the principal stresses.*

Sol. Given :

Major tensile stress, $\sigma_1 = 20$ N/mm²
Minor tensile stress, $\sigma_2 = 10$ N/mm²
Shear stress, $\tau = 10$ N/mm²

Location of principal planes

The location of principal planes is given by equation (3.14).
Using equation (3.14),

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 10}{20 - 10} = \frac{2 \times 10}{10} = 2.0$$

$$\therefore \quad 2\theta = \tan^{-1} 2.0 = 63° \, 26' \text{ or } 243° \, 26'$$

or $\quad \theta = \mathbf{31° \, 43' \text{ or } 121° \, 43'}$. **Ans.**

Magnitude of principal stresses

The major principal stress is given by equation (3.15)

$\therefore$ Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} = \frac{20 + 10}{2} + \sqrt{\left(\frac{20 - 10}{2}\right)^2 + 10^2}$$

$$= 15 + \sqrt{5^2 + 100} = 15 + \sqrt{25 + 100} = 15 + \sqrt{125} = 15 + 11.18$$

$$= \mathbf{26.18 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

The minor principal stress is given by equation (3.16).

$\therefore$ Minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{20 + 10}{2} - \sqrt{\left(\frac{20 - 10}{2}\right)^2 + 10^2}$$

$$= 15 - 11.18 = \mathbf{3.82 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

15. At a point in a strained material, on plane BC there are normal and shear stresses are 560 N/mm² and 140 N/mm² respectively. On plane AC, perpendicular to plane BC, there are normal and shear stresses are 280 N/mm² and 140 N/mm² respectively as shown in fig. Determine the,

- ✓ **Principal stresses and location of the planes on which they act.**
- ✓ **Maximum shear stress and the plane on which it acts.**

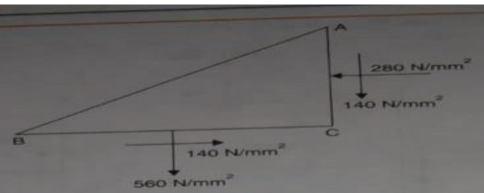


Fig. 3.16 (e)

Sol. Given :

On plane *AC*, $\sigma_1 = -280$ N/mm² (– ve sign due to compressive stress)
$\tau = 140$ N/mm²

On plane *BC*, $\sigma_2 = 560$ N/mm²
$\tau = 140$ N/mm²

(i) Principal stresses and location of the planes on which they act.

Principal stress are given by equations (3.15) and (3.16)

$$\therefore \quad \text{Principal stresses} = \frac{\sigma_1 + \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{-280 + 560}{2} \pm \sqrt{\left(\frac{-280 - 560}{2}\right)^2 + 140^2}$$

$$= 140 \pm 442.7$$

$$= 582.7 \text{ and } (140 - 442.7) \text{ N/mm}^2$$

$$= 582.7 \text{ and } -302.7 \text{ N/mm}^2$$

$\therefore$ Major principal stress $= \mathbf{582.7 \text{ N/mm}^2}$ **(Tensile). Ans.**

$\therefore$ Minor principal stress $= \mathbf{-302.7 \text{ N/mm}^2}$. **Ans.**

The planes on which principal stresses act, are given by equation (3.14) as

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 140}{-280 - 560} = \frac{280}{-840} = -0.33$$

$$\therefore \quad 2\theta = \tan^{-1} - 0.33 = -18.26°$$

– ve sign shows that 2θ is lying in 2nd and 4th quadrant

$$\therefore \quad 2\theta = (180 - 18.26°) \text{ or } (360 - 18.26°)$$

$$= 161.34° \text{ or } 341.34°$$

$\therefore \quad \theta = \mathbf{80.67° \text{ and } 170.67°}$. **Ans.**

(ii) Maximum shear stress and the plane on which it acts.

Maximum shear stress is given by equation (3.18).

$$\therefore \quad (\sigma_t)_{max} = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{-280 - 560}{2}\right)^2 + 140^2}$$

$$= \sqrt{420^2 + 140^2} = \textbf{442.7 N/mm}^2. \quad \textbf{Ans.}$$

The plane on which maximum shear stress acts is given by equation (3.17) as

$$\tan 2\theta = \frac{\sigma_2 - \sigma_1}{2\tau}$$

$$= \frac{560 - (-280)}{2 \times 140} = \frac{840}{280} = 3.0$$

$$\therefore \quad 2\theta = \tan^{-1} 3.0 = 71.56° \text{ or } 251.56°$$

$$\therefore \quad \theta = \textbf{35.78° or 125.78°}. \quad \textbf{Ans.}$$

Problem 3.20. *On a mild steel plate, a circle of diameter 50 mm is drawn before the plate is stressed as shown in Fig. 3.17. Find the lengths of the major and minor axes of an ellipse formed as a result of the deformation of the circle marked.*

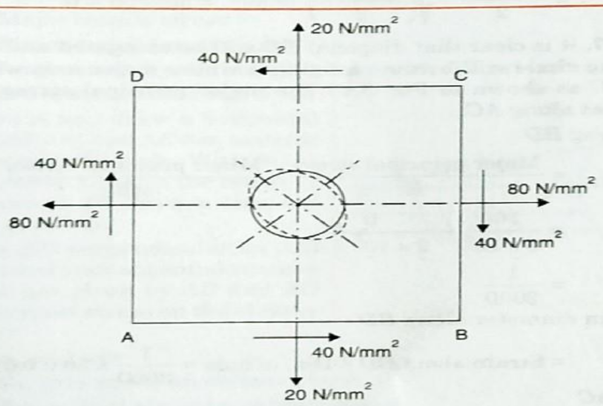


Fig. 3.17

Take $E = 2 \times 10^5$ *N/mm²* *and* $\frac{1}{m} = \frac{1}{4}$.

Sol. Given :

Major tensile stress, $\sigma_1 = 80$ N/mm²

Minor tensile stress, $\sigma_2 = 20$ N/mm²
Shear stress, $\tau = 40$ N/mm²
Value of E $= 2 \times 10^5$ N/mm²

Major principal stress is given by equation (3.15).

$\therefore$ Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{80 + 20}{2} + \sqrt{\left(\frac{80 - 20}{2}\right)^2 + 40^2}$$

$$= 50 + \sqrt{30^2 + 40^2} = 50 + 50 = 100 \text{ N/mm}^2 \text{ (tensile)}$$

Minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{80 + 20}{2} - \sqrt{\left(\frac{80 - 20}{2}\right)^2 + 40^2} = 50 - 50 = 0.$$

From Fig. 3.17, it is clear that diagonal BD will be elongated and diagonal AC will be shortened. Hence the circle will become an ellipse whose major axis will be along BD and minor axis along AC as shown in Fig. 3.17.The major principal stress acts along BD and minor principal stress along AC.

$\therefore$ Strain along BD

$$= \frac{\text{Major principal stress}}{E} - \frac{\text{Minor principal stress}}{mE}$$

$$= \frac{100}{2 \times 10^5} - \frac{0}{2 \times 10^5 \times 4} \qquad \left(\because \frac{1}{m} = \frac{1}{4}\right)$$

$$= \frac{1}{2000}$$

$\therefore$ Increase in diameter along BD

$$= \text{Strain along } BD \times \text{Dia. of hole} = \frac{1}{2000} \times 50 = 0.025 \text{ mm}$$

Strain along AC

$$= \frac{\text{Minor principal stress}}{E} - \frac{\text{Major principal stress}}{mE}$$

$$= \frac{0}{2 \times 10^5} - \frac{100}{4 \times 2 \times 10^5}$$

$$= -\frac{1}{8000}$$ (– ve sign shows that there is a decrease in length)

$\therefore$ Decrease in length of diameter along AC

$$= \text{Strain along } AC \times \text{Dia. of hole} = \frac{1}{8000} \times 50 = 0.00625 \text{ mm}$$

$\therefore$ The circle will become an ellipse whose major axis will be $50 + 0.025 = 50.025$ mm and minor axis will be

$$50 - 0.00625 = 49.99375 \text{ mm.}$$

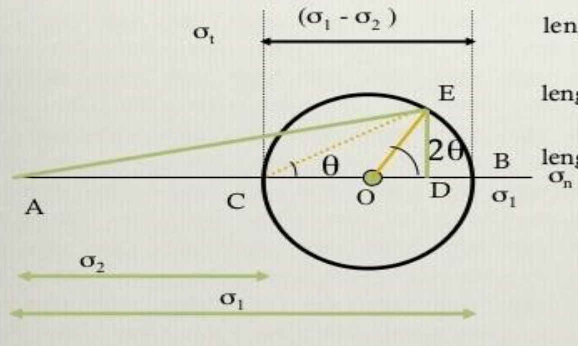
MOHR'S CIRCLE

Mohr Stress Circle

- Graphical method to determine stresses.
  - Body subjected to two mutually perpendicular principal stresses of unequal magnitude.
  - Body subjected to two mutually perpendicular principal stresses of unequal magnitude and unlike (one tensile and other compressive).
  - Body subjected to two mutually perpendicular principal stresses + simple shear stress.

Mohr Stress Circle

- Body subjected to two mutually perpendicular principal stresses of unequal magnitude



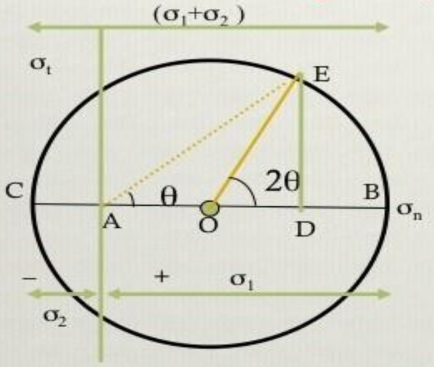
length AD = Normal stress on oblique plane $= \sigma_n$

length ED = Tangential stress on Oblique plane $= \sigma_t$

length AE = Resultant stress on Oblique plane $= \sqrt{\sigma_t^2 + \sigma_n^2}$

Mohr Stress Circle

- Body subjected to two mutually perpendicular principal stresses of unequal magnitude and unlike (one tensile and other compressive).



length AD = Normal stress on oblique plane

$$= \sigma_n$$

length ED = Tangential stress on Oblique plane

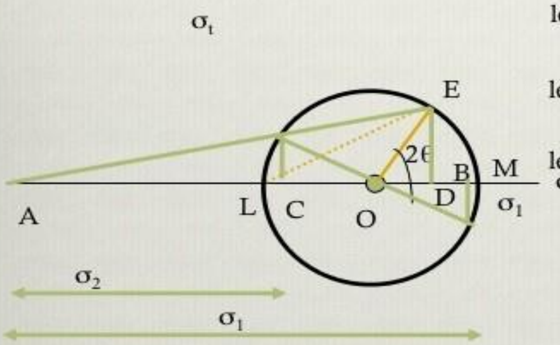
$$= \sigma_t$$

length AE = Resultant stress on Oblique plane

$$= \sqrt{\sigma_t^{\ 2} + \sigma_n^{\ 2}}$$

Mohr Stress Circle

- Body subjected to two mutually perpendicular principal stresses + simple shear stress.



length AD = Normal stress on oblique plane

$$= \sigma_n$$

length ED = Tangential stress on Oblique plane

$$= \sigma_t$$

length AE = Resultant stress on Oblique plane

$$= \sqrt{\sigma_t^{\ 2} + \sigma_n^{\ 2}}$$

3.5. MOHR'S CIRCLE

Mohr's circle is a graphical method of finding normal, tangential and resultant stresses on an oblique plane. Mohr's circle will be drawn for the following cases :

(*i*) A body subjected to two mutually perpendicular principal tensile stresses of unequal intensities.

(*ii*) A body subjected to two mutually perpendicular principal stresses which are unequal and unlike (*i.e.*, one is tensile and other is compressive).

(*iii*) A body subjected to two mutually perpendicular principal tensile stresses accompanied by a simple shear stress.

3.5.1. Mohr's Circle when a Body is Subjected to two Mutually Perpendicular Principal Tensile Stresses of Unequal Intensities. Consider a rectangular body subjected to two mutually perpendicular principal tensile stresses of unequal intensities. It is required to find the resultant stress on an oblique plane.

Let σ_1 = Major tensile stress

σ_2 = Minor tensile stress, and

θ = Angle made by the oblique plane with the axis of minor tensile stress.

Mohr's circle is drawn as : (See Fig. 3.18).

Take any point A and draw a horizontal line through A. Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right from A to some suitable scale. With BC as diameter describe a circle. Let O is the centre of the circle. Now through O, draw a line OE making an angle 2θ with OB.

From E, draw ED perpendicular on AB. Join AE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE.

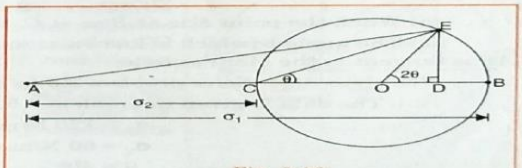


Fig. 3.18

From Fig. 3.18, we have

Length AD = Normal stress on oblique plane

Length ED = Tangential stress on oblique plane

Length AE = Resultant stress on oblique plane.

Radius of Mohr's circle $= \dfrac{\sigma_1 - \sigma_2}{2}$

Angle ϕ = obliquity.

Proof. (See Fig. 3.18)

$$CO = OB = OE = \text{Radius of Mohr's circle} = \frac{\sigma_1 - \sigma_2}{2}$$

The tensile stresses at a point across two mutually perpendicular planes are 120 N/mm^2 and 60 N/mm^2. Determine the normal, tangential and resultant stresses on a plane inclined at 30°to the axis of minor stress.

$$\therefore \quad AO = AC + CO$$

$$= \sigma_2 + \frac{\sigma_1 - \sigma_2}{2} = \frac{2\sigma_2 + \sigma_1 - \sigma_2}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

$$OD = OE \cos 2\theta$$

$$= \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta \qquad \left(\because \; OE = \frac{\sigma_1 - \sigma_2}{2}\right)$$

$$\therefore \quad AD = AO + OD$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$= \sigma_n$ or Normal stress

and $$ED = OE \sin 2\theta$$

$$= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

$= \sigma_t$ or Tangential stress.

Important points. (See Fig. 3.18)

(*i*) Normal stress is along the line ACB. Hence maximum normal stress will be when point E is at B. And minimum normal stress will be when point E is at C. Hence maximum normal stress $= AB = \sigma_1$ and minimum normal stress $= AB = \sigma_2$.

(*ii*) Tangential stress (or shear stress) is along a line which is perpendicular to line CB. Hence maximum shear stress will be when perpendicular to line CB is drawn from point O. Then maximum shear stress will be equal to the radius of the Mohr's circle.

$$\therefore \quad (\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2}.$$

(*iii*) When the point E is at B or at C, the shear stress will be zero.

(*iv*) The angle ϕ (which is known as angle of obliquity) will be maximum, when the line AE is tangent to the Mohr's circle.

Problem 3.21. *Solve problem 3.5 by using Mohr's circle method.*

Sol. The data is given in problem 3.5, is

σ_1 = 120 N/mm^2 (tensile)

σ_2 = 60 N/mm^2 (tensile)

θ = 30°.

Scale. Let 1 cm = 10 N/mm^2

Then $\sigma_1 = \dfrac{120}{10} = 12$ cm

and $\sigma_2 = \dfrac{60}{10} = 6$ cm

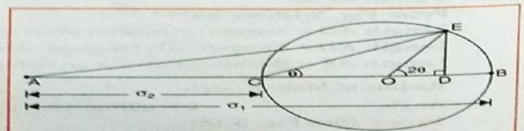


Fig. 3.19

Mohr's circle is drawn as : (See Fig. 3.19).

Take any point A and draw a horizontal line through A. Take $AB = \sigma_1 = 12$ cm and $AC = \sigma_2 = 6$ cm. With BC as diameter (*i.e.*, $BC = 12 - 6 = 6$ cm) describe a circle. Let O is the centre of the circle. Through O, draw a line OE

making an angle 2θ (*i.e.*, $2 \times 30 = 60°$) with OB. From E, draw ED perpendicular to CB. Join AE. Measure lengths AD, ED and AE.

By measurements :

Length $AD = 10.50$ cm

Length $ED = 2.60$ cm

Length $AE = 10.82$ cm

Then normal stress $= \text{Length } AD \times \text{Scale}$
$= 10.50 \times 10 = $ **105 N/mm²**. **Ans.**

Tangential or shear stress $= \text{Length } ED \times \text{Scale}$
$= 2.60 \times 10 = $ **26 N/mm²**. **Ans.**

Resultant stress $= \text{Length } AE \times \text{Scale}$
$= 10.82 \times 10 = $ **108.2 N/mm²**. **Ans.**

3.5.2. Mohr's Circle when a Body is Subjected to two Mutually Perpendicular Principal Stresses which are Unequal and Unlike (i.e., one is Tensile and other is Compressive). Consider a rectangular body subjected to two mutually perpendicular principal stresses which are unequal and one of them is tensile and the other is compressive. It is required to find the resultant stress on an oblique plane.

Let σ_1 = Major principal tensile stress,

σ_2 = Minor principal compressive stress, and

θ = Angle made by the oblique plane with the axis of minor principal stress.

Mohr's circle is drawn as : (See Fig. 3.20)

Take any point A and draw a horizontal line through A on both sides of A as shown in Fig. 3.20. Take $AB = \sigma_1(+)$ towards right of A and $AC = \sigma_2(-)$ towards left of A to some suitable scale. Bisect BC at O. With O as centre and radius equal to CO or OB, draw a circle. Through O draw a line OE making an angle 2θ with OB.

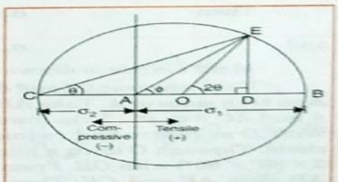


Fig. 3.20

From E, draw ED perpendicular to AB. Join AE and CE. Then normal and shear stress (*i.e.*, tangential stress) on the oblique plane are given by AD and ED. Length AE represents the resultant stress on the oblique plane.

$\therefore$ From Fig. 3.20, we have

Length AD = Normal stress on oblique plane,

Length ED = Shear stress on oblique plane,

Length AE = Resultant stress on oblique plane, and

Angle ϕ = Obliquity.

Radius of Mohr's circle $= CO$ or $OB = \dfrac{\sigma_1 + \sigma_2}{2}$.

Proof. (See Fig. 3.20).

$CO = OB = OE$ = Radius of Mohr's circle

$= \dfrac{\sigma_1 + \sigma_2}{2}$

The stresses at a point in a bar are 200 N/mm² (T) and 100 N/mm²(C) . Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major stress. Also determine the intensity of shear stress in the material at the point.

$$AO = OC - AC$$
$$= \frac{\sigma_1 + \sigma_2}{2} - \sigma_2 = \frac{\sigma_1 + \sigma_2 - 2\sigma_2}{2} = \frac{\sigma_1 - \sigma_2}{2}$$
$$\therefore \quad AD = AO + OD$$
$$= AO + OB \cos 2\theta \qquad (\because \ OD = OE \cos 2\theta)$$
$$= \frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta \qquad \left(\because \ OE = \text{Radius} = \frac{\sigma_1 + \sigma_2}{2}\right)$$
$= \sigma_n$ or Normal stress

and
$$ED = OE \sin 2\theta$$
$$= \frac{\sigma_1 + \sigma_2}{2} \sin 2\theta \qquad \left(\because \ OE = \frac{\sigma_1 + \sigma_2}{2}\right)$$
$= \sigma_t$ or Tangential (or shear) stress.

Problem 3.22. *Solve problem 3.6 by using Mohr's circle method.*

Sol. Given : The data given in problem 3.6, is

$\sigma_1 = 200$ N/mm²

$\sigma_2 = -100$ N/mm² (compressive)

$\theta = 30°$.

It is required to determine the resultant stress and the maximum shear stress by Mohr's circle method. First choose a suitable scale.

Let 1 cm represents 20 N/mm².

Then
$$\sigma_1 = \frac{200}{20} = 10 \text{ cm}$$
and
$$\sigma_2 = \frac{-100}{20} = -5 \text{ cm}$$

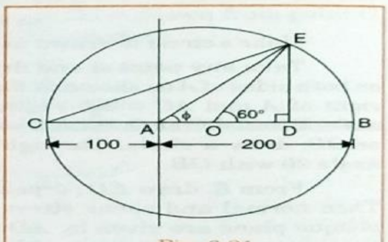


Fig. 3.21

Mohr's circle is drawn as given in Fig. 3.21.

Take any point *A* and draw a horizontal line through *A* on both sides of *A*. Take $AB = \sigma_1 = 10$ cm towards right of *A* and $AC = \sigma_2 = -5$ cm towards left of *A*. Bisect *BC* at *O*. With *O* as centre and radius equal to *CO* or *OB*, draw a circle. Through *O* draw a line *OE* making an angle 2θ (*i.e.*, $2 \times 30° = 60°$) with *OB*. From *E*, draw *ED* perpendicular to *AB*. Join *AE* and *CE*. Then *AE* represents the resultant stress and angle ϕ represents the obliquity.

By measurement from Fig. 3.21, we have

Length $AE = 9.0$ cm

Length $AD = 6.25$ cm and length $ED = 6.5$ cm

Angle $\phi = 46°$

$\therefore$ Resultant stress $=$ Length $AE \times$ Scale

$= 9.0 \times 20 =$ **180 N/mm².** **Ans.**

Angle made by the resultant stress with the normal of the inclined plane $= \phi =$ **46°.** **Ans.**

Normal stress $=$ Length $AD \times 20$

$= 6.25 \times 20 = 125$ N/mm²

Shear stress $=$ Length $ED \times 20$

$= 6.5 \times 20 = 130$ N/mm².

Maximum shear stress. Shear stress is along a line which is perpendicular to the line *AB*. Hence maximum shear stress will be when perpendicular to line *AB* is drawn from point *O*. Then maximum shear stress will be equal to the radius of Mohr's circle.

$\therefore$ Maximum shear stress $=$ Radius of Mohr's circle
$$= \frac{\sigma_1 + \sigma_2}{2} = \frac{200 + 100}{2} = \mathbf{150 \ N/mm^2.} \quad \textbf{Ans.}$$

3.5.3. Mohr's Circle when a Body is Subjected to two Mutually Perpendicular Principal Tensile Stresses Accompanied by a Simple Shear Stress.

Consider a rectangular body subjected to two mutually perpendicular principal tensile stresses of unequal intensities accompanied by a simple shear stress. It is required to find the resultant stress on an oblique plane as shown in Fig. 3.22.

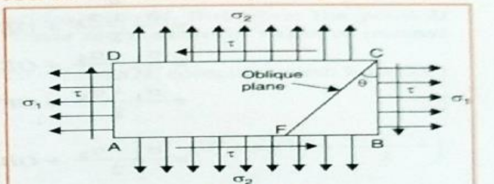


Fig. 3.22

Let $\sigma_1 =$ Major tensile stress

$\sigma_2 =$ Minor tensile stress

$\tau =$ Shear stress across face *BC* and *AD*

$\theta =$ Angle made by the oblique plane with the plane of major tensile stress.

According to the principle of shear stress, the faces *AB* and *CD* will also be subjected to a shear stress of τ.

Mohr's circle is drawn as given in Fig. 3.23.

Take any point *A* and draw a horizontal line through *A*.

Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right of *A* to some suitable scale. Draw perpendiculars at *B* and *C* and cut off *BF* and *CG* equal to shear stress τ to the same scale. Bisect *BC* at *O*. Now with *O* as centre and radius equal to *OG* or *OF* draw a circle. Through *O*, draw a line *OE* making an angle of 2θ with *OF* as shown in Fig. 3.23. From *E*, draw *ED* perpendicular to *CB*. Join *AE*. Then length *AE* represents the resultant stress on the given oblique plane. And lengths *AD* and *ED* represents the normal stress and tangential stress respectively.

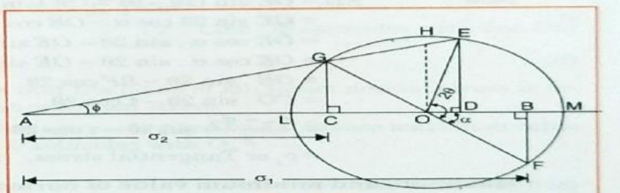


Fig. 3.23

Hence from Fig. 3.23, we have

Length $AE =$ Resultant stress on the oblique plane

Length $AD =$ Normal stress on the oblique plane

Length $ED =$ Shear stress on the oblique plane.

Proof. (See Fig. 3.23).
$$CO = \frac{1}{2} CB = \frac{1}{2} [\sigma_1 - \sigma_2] \qquad (\because \ CB = \sigma_1 - \sigma_2)$$
$$AO = AC + CO = \sigma_2 + \frac{1}{2} [\sigma_1 - \sigma_2]$$

$$= \frac{2\sigma_2 + \sigma_1 - \sigma_2}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

$$AD = AO + OD$$

$$= \frac{\sigma_1 + \sigma_2}{2} + OE \cos (2\theta - \alpha) \qquad [\because \quad OD = OE \cos (2\theta - \alpha)]$$

$$= \frac{\sigma_1 + \sigma_2}{2} + OE [\cos 2\theta \cos \alpha + \sin 2\theta \sin \alpha]$$

$$= \frac{\sigma_1 + \sigma_2}{2} + OE \cos 2\theta \cos \alpha + OE \sin 2\theta \sin \alpha$$

$$= \frac{\sigma_1 + \sigma_2}{2} + OE \cos \alpha \,.\, \cos 2\theta + OE \sin \alpha \,.\, \sin 2\theta$$

$$= \frac{\sigma_1 + \sigma_2}{2} + OF \cos \alpha \,.\, \cos 2\theta + OF \sin \alpha \,.\, \sin 2\theta \qquad (\because \quad OE = OF = \text{Radius})$$

$$= \frac{\sigma_1 + \sigma_2}{2} + OB \cos 2\theta + BF \sin 2\theta \qquad (\because \quad OF \cos \alpha = OB,\ OF \sin \alpha = BF)$$

$$= \frac{\sigma_1 + \sigma_2}{2} + CO \cos 2\theta + \tau \sin 2\theta \qquad (\because \quad OB = CO,\ BF = \tau)$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \qquad \left(\because \quad CO = \frac{\sigma_1 - \sigma_2}{2}\right)$$

$= \sigma_n$ or Normal stress

Now $\quad ED = OE \sin (2\theta - \alpha) = OE (\sin 2\theta \cos \alpha - \cos 2\theta \sin \alpha)$

$$= OE \sin 2\theta \cos \alpha - OE \cos 2\theta \sin \alpha$$

$$= OE \cos \alpha \,.\, \sin 2\theta - OE \sin \alpha \,.\, \cos 2\theta$$

$$= OE \cos \alpha \,.\, \sin 2\theta - OE \sin \alpha \,.\, \cos 2\theta \qquad (\because \quad OE = OF = \text{Radius})$$

$$= OB \,.\, \sin 2\theta - BF \cos 2\theta \qquad (\because \quad OF \cos \alpha = OB,\ OF \sin \alpha = BF)$$

$$= CO \,.\, \sin 2\theta - \tau \cos 2\theta \qquad (\because \quad OB = CO,\ BF = \tau)$$

$$= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - \tau \cos 2\theta \qquad \left(\because \quad CO = \frac{\sigma_1 - \sigma_2}{2}\right)$$

$= \sigma_t$ or Tangential stress.

Maximum and minimum value of normal stress. In Fig. 3.23, the normal stress is given by AD. Hence the maximum value of AD will be when D coincides with M and minimum value of AD will be when D coincides with L.

$\therefore \quad$ Maximum value of normal stress,

$$(\sigma_n)_{max} = AM = AO + OM$$

$$= \frac{\sigma_1 + \sigma_2}{2} + OF \qquad \left(\because \quad AO = \frac{\sigma_1 + \sigma_2}{2},\ OM = OF = \text{Radius}\right)$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{OB^2 + BF^2} \qquad (\because \quad \text{In triangle } OBF,\ OF = \sqrt{OB^2 + BF^2}\,)$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} \qquad \left(\because \quad OB = \frac{\sigma_1 - \sigma_2}{2},\ BF = \tau\right)$$

Minimum value of normal stress,

$$(\sigma_n)_{min} = AL = AO - LO$$

$$= \frac{\sigma_1 + \sigma_2}{2} - OF \qquad (\because \quad LO = OF = \text{Radius})$$

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

(*i*) For maximum normal stress, the point D coincides with M. But when the point D coincides with M, the point E also coincides with M. Hence for maximum value of normal stress,

Angle $\quad 2\theta = \alpha \qquad (\because \quad \text{Line } OE \text{ coincides with line } OM)$

$$\therefore \quad \theta = \frac{\alpha}{2} \qquad \ldots(i)$$

Also $\quad \tan 2\theta = \tan \alpha = \dfrac{BF}{OB} = \dfrac{\tau}{\frac{\sigma_1 - \sigma_2}{2}} \qquad \left(\because \quad BF = \tau, OB = \dfrac{\sigma_1 - \sigma_2}{2}\right)$

$$= \frac{2\tau}{\sigma_1 - \sigma_2}.$$

(*ii*) For maximum and minimum normal stresses, the shear stress is zero and hence the planes, on which maximum and minimum normal stresses act, are known as *principal planes* and the stresses are known as *principal stresses*.

(*iii*) For minimum normal stress, the point D coincides with point L. But when the point D coincides with L, the point E also coincides with L. Then

Angle $\quad 2\theta = \pi + \alpha \qquad (\because \quad \text{Line } OE \text{ coincides with line } OL)$

$$\therefore \quad \theta = \frac{\pi}{2} + \frac{\alpha}{2} \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), it is clear that the plane of minimum normal stress is inclined at an angle 90° to the plane of maximum normal stress.

Maximum value of shear stress. Shear stress is given by ED. Hence maximum value of ED will be when E coincides with G, and D coincides with O.

$\therefore \quad$ Maximum shear stress,

$$(\sigma_t)_{max} = OH = OF \qquad (\because \quad OH = OF = \text{radius})$$

$$= \sqrt{OB^2 + BF^2} \qquad (\because \quad \text{In triangle } OBF,\ OF = \sqrt{OB^2 + BF^2}\,)$$

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} \qquad \left(\because \quad OB = \frac{\sigma_1 - \sigma_2}{2},\ BF = \tau\right)$$

Problem 3.23. *A point in a strained material is subjected to stresses shown in Fig. 3.24. Using Mohr's circle method, determine the normal and tangential stresses across the oblique plane. Check the answer analytically.*

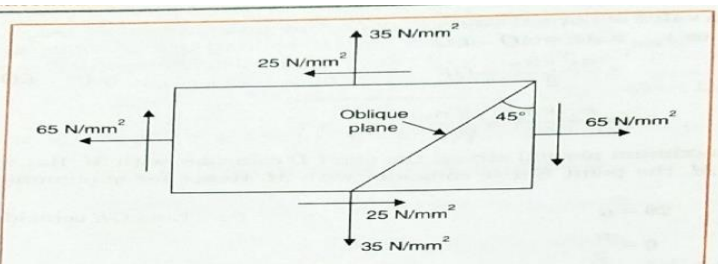


Fig. 3.24

Sol. Given :

Major principal stress, $\sigma_1 = 65\text{ N/mm}^2$

Minor principal stress, $\sigma_2 = 35\text{ N/mm}^2$

Shear stress, $\tau = 25\text{ N/mm}^2$

Angle of oblique plane, $\theta = 45°$.

Mohr's circle method

Let $1\text{ cm} = 10\text{ N/mm}^2$

Then $\sigma_1 = \dfrac{65}{10} = 6.5\text{ cm},$

$$\sigma_2 = \frac{35}{10} = 3.5\text{ cm and } \tau = \frac{25}{10} = 2.5\text{ cm}$$

Mohr's circle is drawn as given in Fig. 3.25.

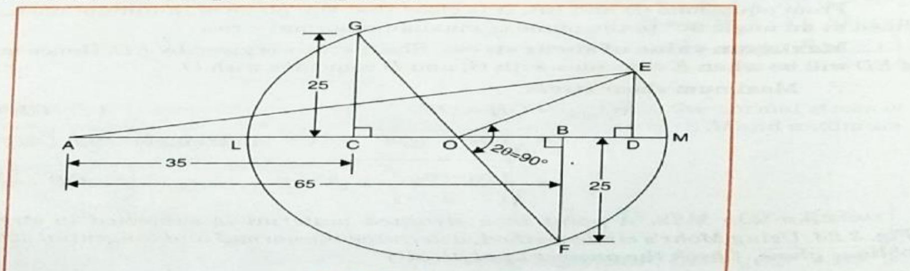


Fig. 3.25

Take any point A and draw a horizontal line through A. Take $AB = \sigma_1 = 6.5$ cm and $AC = \sigma_2 = 3.5$ cm towards right of A. Draw perpendicular at B and C and cut off BF and CG equal to shear stress $\tau = 2.5$ cm. Bisect BC at O. Now with O as centre and radius equal to OF (or OG) draw a circle. Through O, draw a line OE making an angle of 2θ (*i.e.*, $2 \times 45° = 90°$) with OF as shown in Fig. 3.25. From E, draw ED perpendicular to AB produced. Join AE. Then length AD represents the normal stress and length ED represents the shear stress.

By measurements, length $AD = 7.5$ cm and

length $ED = 1.5$ cm.

$\therefore$ Normal stress (σ_n) $=$ Length $AD \times$ Scale $= 7.5 \times 10 = \mathbf{75\text{ N/mm}^2}$. **Ans.**

($\because$ 1 cm = 10 N/mm²)

And tangential stress (σ_t) $=$ Length $ED \times$ Scale $= 1.5 \times 10 = \mathbf{15\text{ N/mm}^2}$. **Ans.**

Analytical Answers

Normal stress (σ_n) is given by equation (3.12).

$\therefore$ Using equation (3.12),

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{65 + 35}{2} + \frac{65 - 35}{2}\cos(2 \times 45°) + 25 \sin(2 \times 45°)$$

$$= 50 + 15 \cos 90° + 25 \sin 90°$$

$$= 50 + 15 \times 0 + 25 \times 1 \qquad (\because \cos 90° = 0, \sin 90° = 1)$$

$$= 50 + 0 + 25 = \mathbf{75\text{ N/mm}^2}. \text{ Ans.}$$

Tangential stress is given by equation (3.13)

$\therefore$ Using equation (3.13),

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

$$= \frac{65 - 35}{2}\sin(2 \times 45) - 25\cos(2 \times 45)$$

$$= 15 \sin 90° - 25 \cos 90° = 15 \times 1 - 25 \times 0 = 15 - 0$$

$$= \mathbf{15\text{ N/mm}^2}. \text{ Ans.}$$

Problem 3.24. *At a certain point in a strained material, the intensities of stresses on two planes at right angles to each other are 20 N/mm² and 10 N/mm² both tensile. They are accompanied by a shear stress of magnitude 10 N/mm². Find graphically or otherwise, the location of principal planes and evaluate the principal stresses.*

Sol. Given :

Major tensile stress, $\sigma_1 = 20\text{ N/mm}^2$

Minor tensile stress, $\sigma_2 = 10\text{ N/mm}^2$

Shear stress, $\tau = 10\text{ N/mm}^2$

This problem may be solved analytically or graphically. Here we shall solve it graphically (*i.e.*, by Mohr's circle method).

Scale. Take $1\text{ cm} = 2\text{ N/mm}^2$

Then $\sigma_1 = \dfrac{20}{2} = 10\text{ cm}, \quad \sigma_2 = \dfrac{10}{2} = 5\text{ cm}$

and $\tau = \dfrac{10}{2} = 5\text{ cm}.$

Fig. 3.26

Mohr's circle is drawn as given in Fig. 3.26.

Take any point A and draw a horizontal line through A. Take $AB = \sigma_1 = 10$ cm and $AC = \sigma_2 = 5$ cm towards right side of A. Draw perpendiculars at B and C and cut off $BF = CG = \tau = 5$ cm. Bisect BC at O. Now with O as centre and radius equal to OG (or OF), draw a circle cutting the horizontal line through A, at L and M as shown in Fig. 3.26. Then AM and AL represent the major principal and minor principal stresses.

By measurements, we have

Length $AM = 13.1$ cm and Length $AL = 1.91$ cm

$\angle FOB$ (or 2θ) $= 63.7°$.

$\therefore$ Major principal stress $=$ Length $AM \times$ Scale

$= 13.1 \times 2$ N/mm^2 $\qquad (\because \; 1 \text{ cm} = 2 \text{ N/mm}^2)$

$=$ **26.2 N/mm²**. **Ans.**

Minor principal stress $=$ Length $AL \times$ Scale

$= 1.91 \times 2 =$ **3.82 N/mm²**. **Ans.**

Location of principal planes

$$2\theta = 63.7°$$

$$\therefore \quad \theta = \frac{63.7°}{2} = \mathbf{31.85°}. \quad \textbf{Ans.}$$

The second principal plane is given by

$\theta + 90°$ or $31.85° + 90°$ or **121.85°**. **Ans.**

Problem 3.25. *An elemental cube is subjected to tensile stresses of 30 N/mm² and 10 N/mm² acting on two mutually perpendicular planes and a shear stress of 10 N/mm² on these planes. Draw the Mohr's circle of stresses and hence or otherwise determine the magnitudes and directions of principal stresses and also the greatest shear stress.*

Sol. Given :

Major tensile stress, $\sigma_1 = 30$ N/mm^2

Minor tensile stress, $\sigma_2 = 10$ N/mm^2

Shear stress, $\tau = 10$ N/mm^2

Scale. Take 1 cm $= 2$ N/mm^2

Then $\sigma_1 = \dfrac{30}{2} = 15$ cm

$\sigma_2 = \dfrac{10}{2} = 5$ cm and $\tau = \dfrac{10}{2} = 5$ cm

Mohr's circle of stresses is drawn as given in Fig. 3.27.

Take any point A and draw a horizontal line through A.

Take $AB = \sigma_1 = 15$ cm and $AC = \sigma_2 = 5$ cm towards right side of A. Draw perpendiculars at B and C and cut off $BF = CG = \tau = 5$ cm. Bisect BC at O. Now with O as centre and radius equal to OG (or OF), draw a circle cutting the horizontal line through A at L and M as shown in Fig. 3.27. Then AM and AL represents the major and minor principal stresses respectively. And OH represents the maximum shear stress.

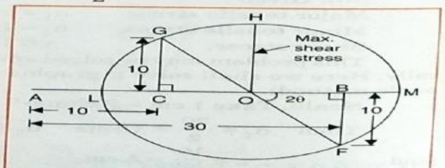


Fig. 3.27

By measurements, we have

Length $AM = 17.1$ cm

Length $AL = 2.93$ cm

Length $OH =$ Radius of Mohr's circle

$= 7.05$ cm

$\angle FOB$ (or 2θ) $= 45°$.

$\therefore$ Major principal stress

$=$ Length $AM \times$ Scale

$= 17.1 \times 2$ $\qquad (\because \; 1 \text{ cm} = 2 \text{ N/mm}^2)$

$=$ **34.2 N/mm²**. **Ans.**

Minor principal stress $=$ Length $AL \times$ Scale

$= 2.93 \times 2$ $\qquad (\because \; 1 \text{ cm} = 2 \text{ N/mm}^2)$

$=$ **5.86 N/mm²**. **Ans.**

$\angle FOB$ or $2\theta = 45°$

$$\therefore \quad \theta = \frac{45}{2} = \mathbf{22.5°}. \quad \textbf{Ans.}$$

The second principal plane is given by $\theta + 90°$.

$\therefore$ Second principal plane $= 22.5 + 90 =$ **112.5°**. **Ans.**

The greatest shear stress $=$ Length $OH \times$ Scale

$= 7.05 \times 20 =$ **14.1 N/mm²**. **Ans.**

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – III –BENDING MOMENTS AND SHEAR FORCE – SCIA1301

BEAM

- **Beams are horizontal structural elements that withstand vertical loads, shear forces, and bending moments.**
- **They transfer loads that imposed along their length to their endpoints such as walls, columns, foundations, etc.**

TYPES OF BEAMS

  - ✓ **Simply Supported Beam.**
  - ✓ **Fixed Beam.**
  - ✓ **Cantilever Beam.**
  - ✓ **Continuous Beam.**
  - ✓ **Overhanging Beam**

- ✓ **SIMPLY SUPPORTED BEAM:**
  - ✓ **A beam supported or resting freely on the supports at its both ends is known as simply supported beam.**
- ✓ **FIXED BEAM:**
  - ✓ **A beam whose both ends are fixed or built-in walls is known as fixed beam.**
  - ✓ **Fixed beam is also known as a built-in or encasted beam.**
- ✓ **CANTILEVER BEAM:**
  - ✓ **A beam which is fixed at one end and free at the other end is known as cantilever beam.**
- ✓ **CONTINUOUS BEAM:**
  - ✓ **A beam which is provided more than two supports is known as continuous beam.**
- ✓ **OVERHANGING BEAM:**
  - ✓ **If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam.**

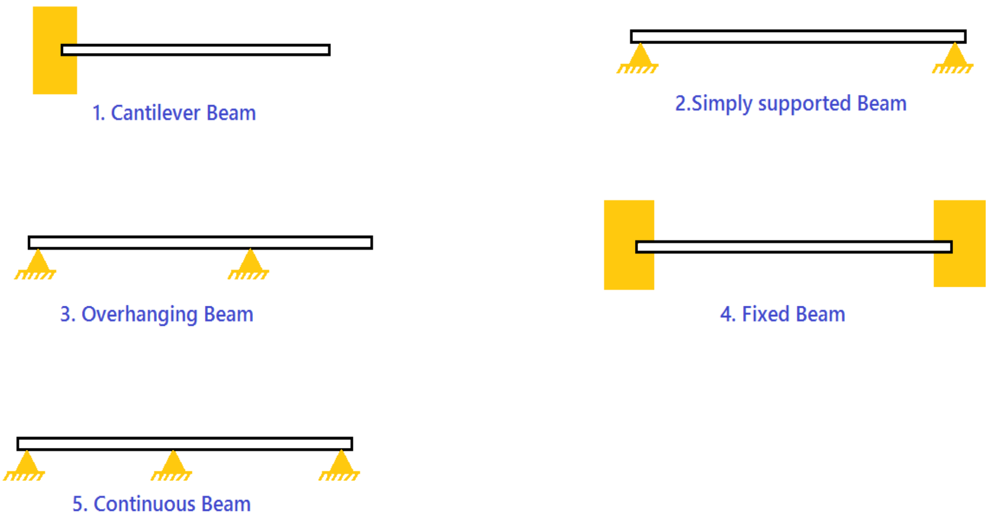


Figure No.1

TYPES OF SUPPORTS

- ➢ **Fixed Support**
- ➢ **Simple Support**
- ✓ **Pinned Support**
- ✓ **Roller Support**

FIXED SUPPORT:

- ➢ **A fixed support is the most rigid type of support or connection.**
- ➢ **It constrains the member in all translations and rotations (i.e. it cannot move or rotate in any direction).**
- ➢ **The easiest example of a fixed support would be a pole or column in concrete.**

SIMPLE SUPPORT:

- ➢ **Simple support is just a support on which structural member rests.**

PINNED SUPPORT:

- ➢ **A pinned support can resist both vertical and horizontal forces but not a moment.**
- ➢ **A pinned support is a very common type of support.**
- ➢ **A pinned support is same as hinged support.**

ROLLER SUPPORT:

- ➢ **Roller supports are free to rotate and translate along the surface upon which the roller rests.**
- ➢ **Roller supports are commonly located at one end of long bridges in the form of bearing pads.**

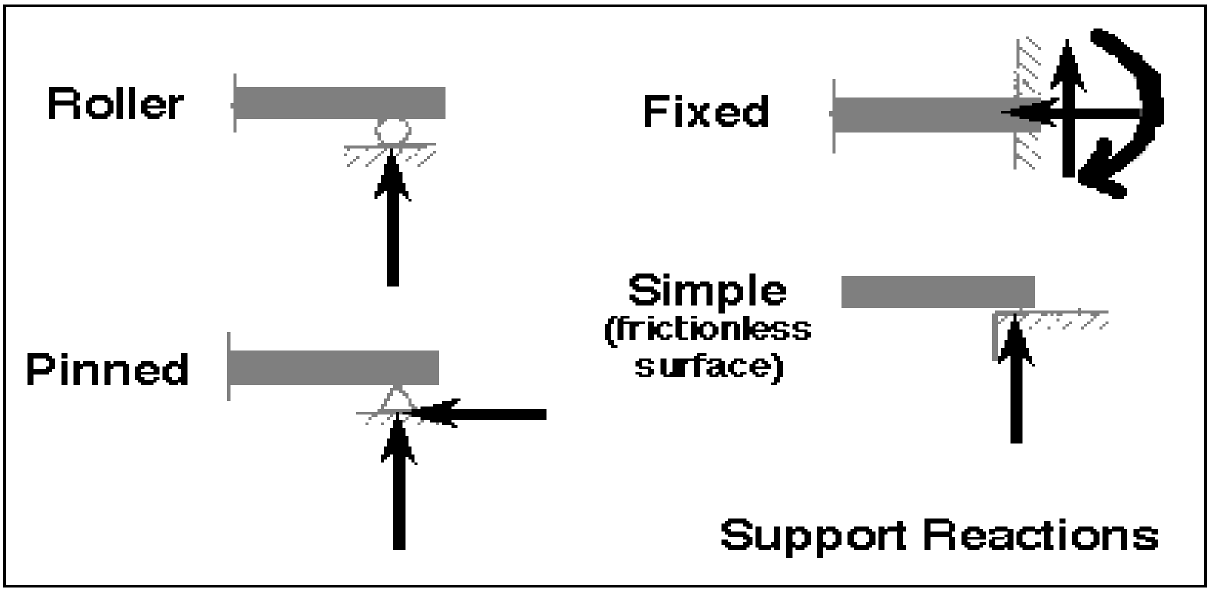


Figure No.2

TYPES OF LOADS

- **Point load or concentrated load:**
  - ✓ **It is one which is considered to act at a point.**
  - ✓ **The load is expressed as W N.**
- **Uniformly distributed load:**
  - ✓ **It is one which is spread over a beam in such a manner that rate of loading w is uniform along the length. The rate of loading is expressed as w N/m.**
- **Uniformly varying load:**
  - ✓ **It is one which is spread over a beam in such a manner that rate of loading varies from point to point along the length.**

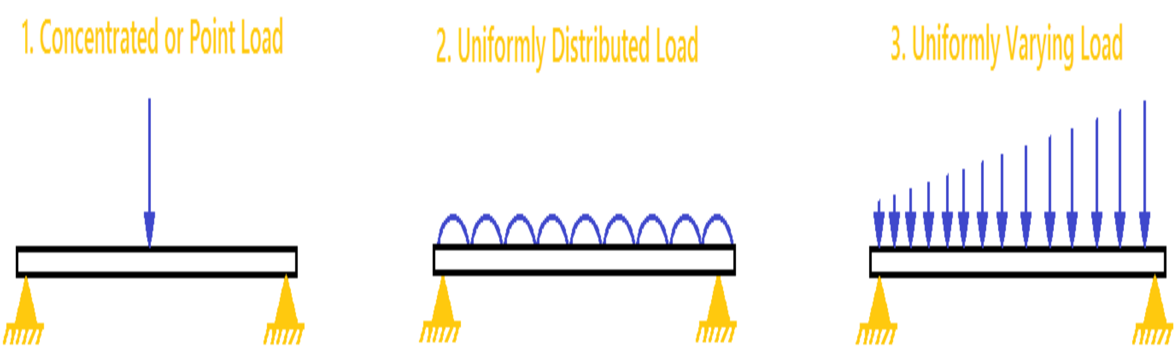
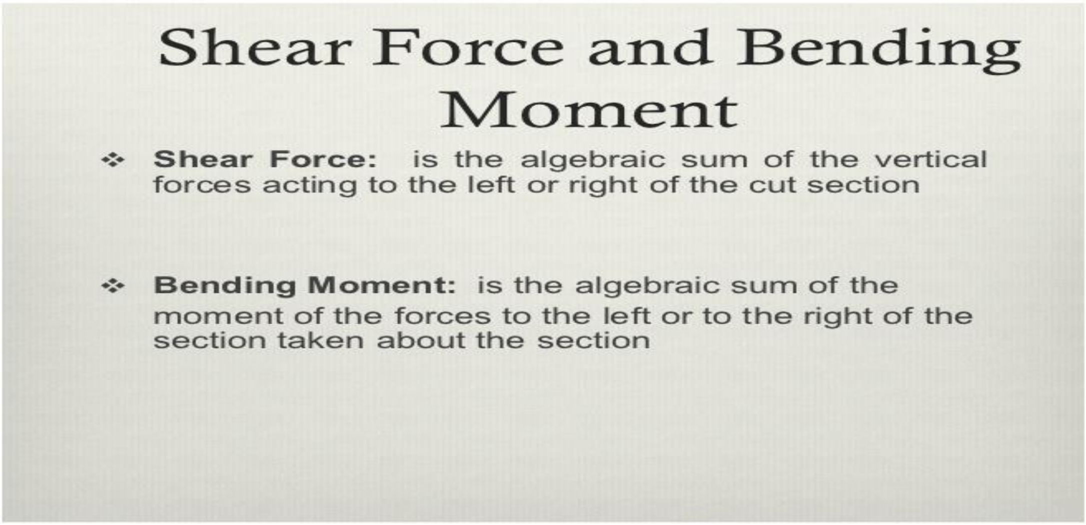


Figure No.3

SHEAR FROCE AND BENDING MOMENT:



SIGN CONVENTIONS FOR SHEAR FORCE AND BENDING MOMENT:

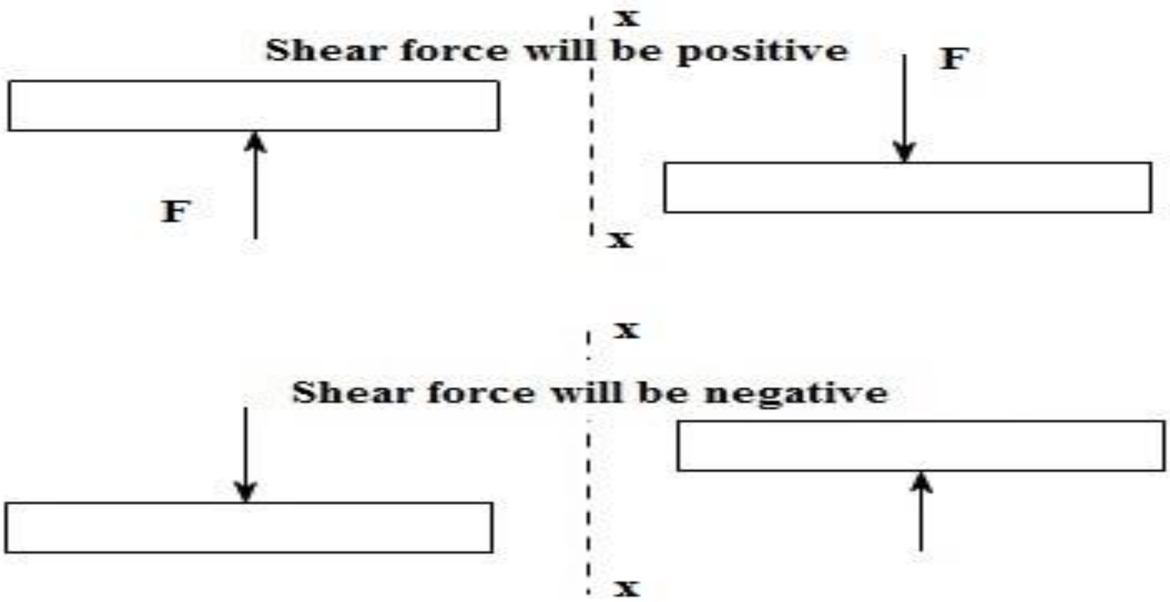


Figure No.4

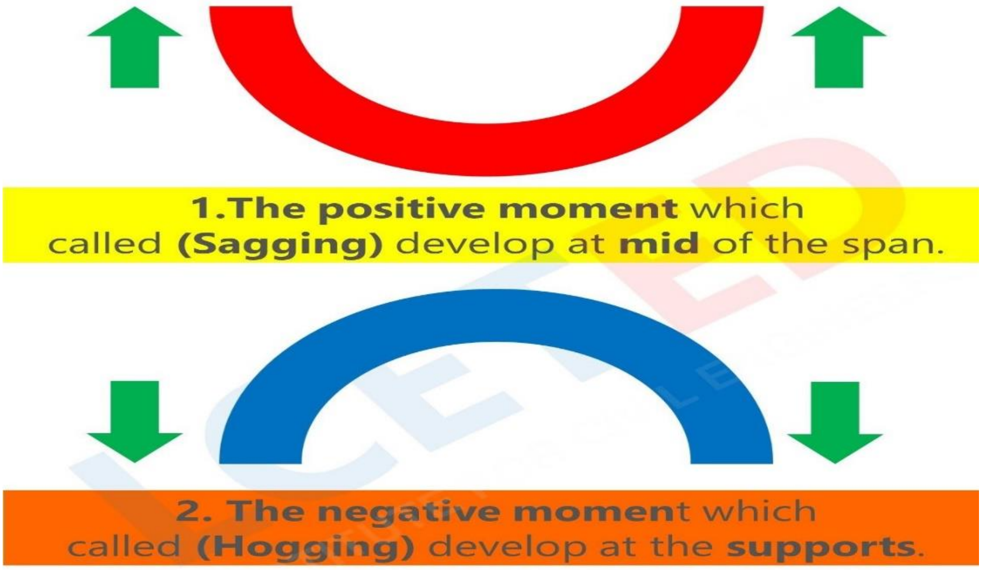


Figure No.5

SAGGING BENDING MOMENT:

- **The moment that makes beam to concave upward is called sagging moment.**
- **It is generally taken as positive moment.**
- **In this moment, the upward of beam compresses and downward of the beam extends. So, stresses are developed called bending stress.**

HOGGING BENDING MOMENT:

- **The moment that makes beam to concave downward is called Hogging moment.**
- **It is taken as negative moment.**
- **The upward point is under tension and below point is at tension.**

RULES FOR DRAWING SFD & BMD

- **Consider the left or right portion of section.**
- **If the left portion of section is choosen, a force on the left portion acting upwards is +ve and acting downwards is –ve.**
- **If right portion is choosen, a force on the right portion acting downwards is +ve and acting upwards is –ve.**
- **The +ve values of shear force and bending moment are plotted above the base line and –ve values below the base line.**
- **The Bending Moment at two supports of a simply supported beam and at the free end of a cantilever beam are zero.**

CANTILEVER BEAM

CANTILEVER WITH POINT LOAD AT THE FREE END:

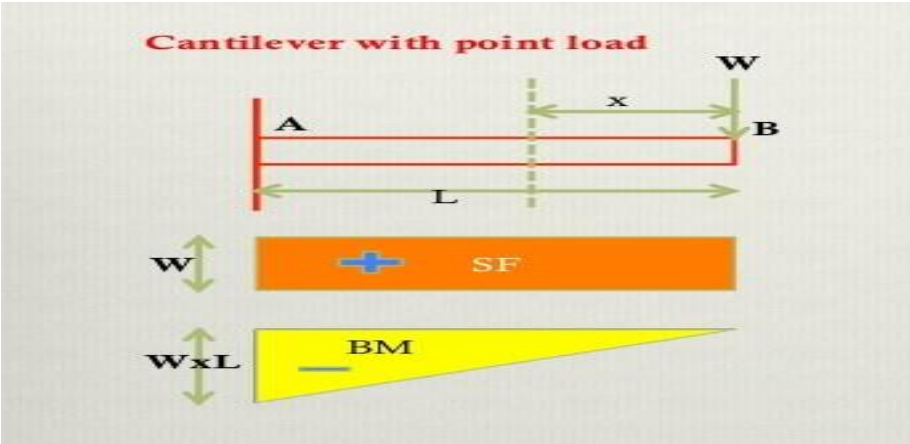


Figure No.6

SHEAR FORCE:

V_B (r) = 0 KN

V_B (l) = W KN

V_{xx} = W KN

V_A (r) = W KN

V_A (l) = 0 KN

BENDING MOMENT:

Bending Moment = Load X Distance

BM at free end B, M_B = 0 KN.m

BM at section XX, M_{xx} = -W X x = -Wx KN.m

BM at fixed end A, M_A = -W X L = -WL KN.m

CANTILEVER WITH UDL THROUGHOUT THE LENGTH:

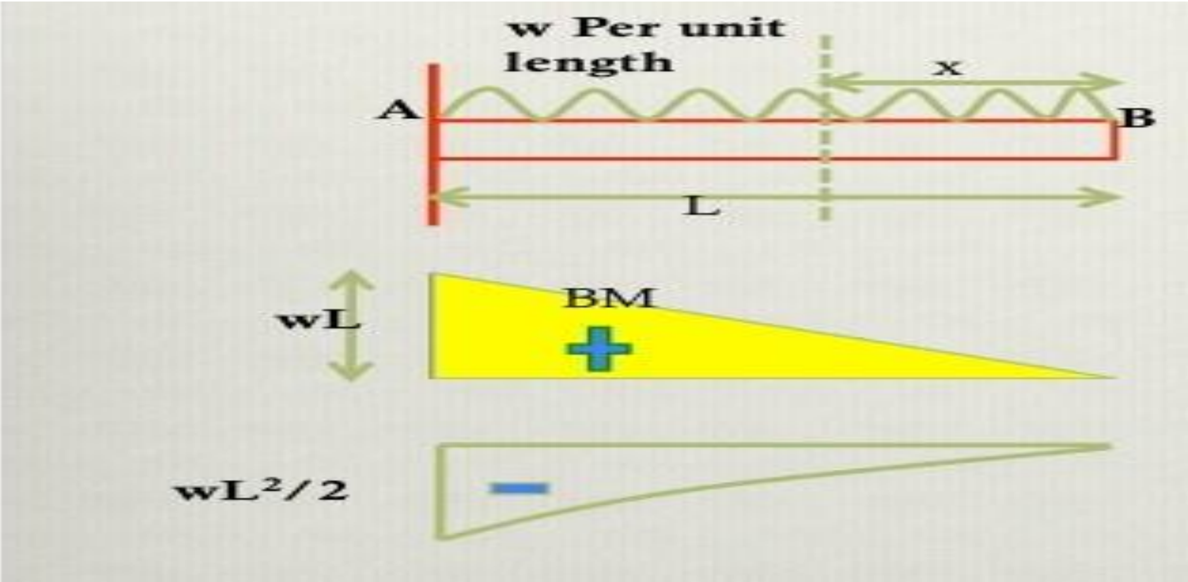


Figure No.7

SHEAR FORCE:

V_B = 0 KN

V_{xx} = wx KN

V_A (r) = wL KN

V_A (l) = 0 KN

BENDING MOMENT:

Bending Moment = Load X Distance X Distance/2

BM at free end B, M_B = 0 KN.m

BM at section XX, M_{xx} = -W X (x) X (x/2) = -$Wx^2/2$ KN.m

BM at fixed end A, M_A = -W X L X L/2= -$WL^2/2$ KN.m

PROBLEMS:

1.A cantilever beam of length 2 m carries the point loads as shown in fig. Draw SFD and BMD.

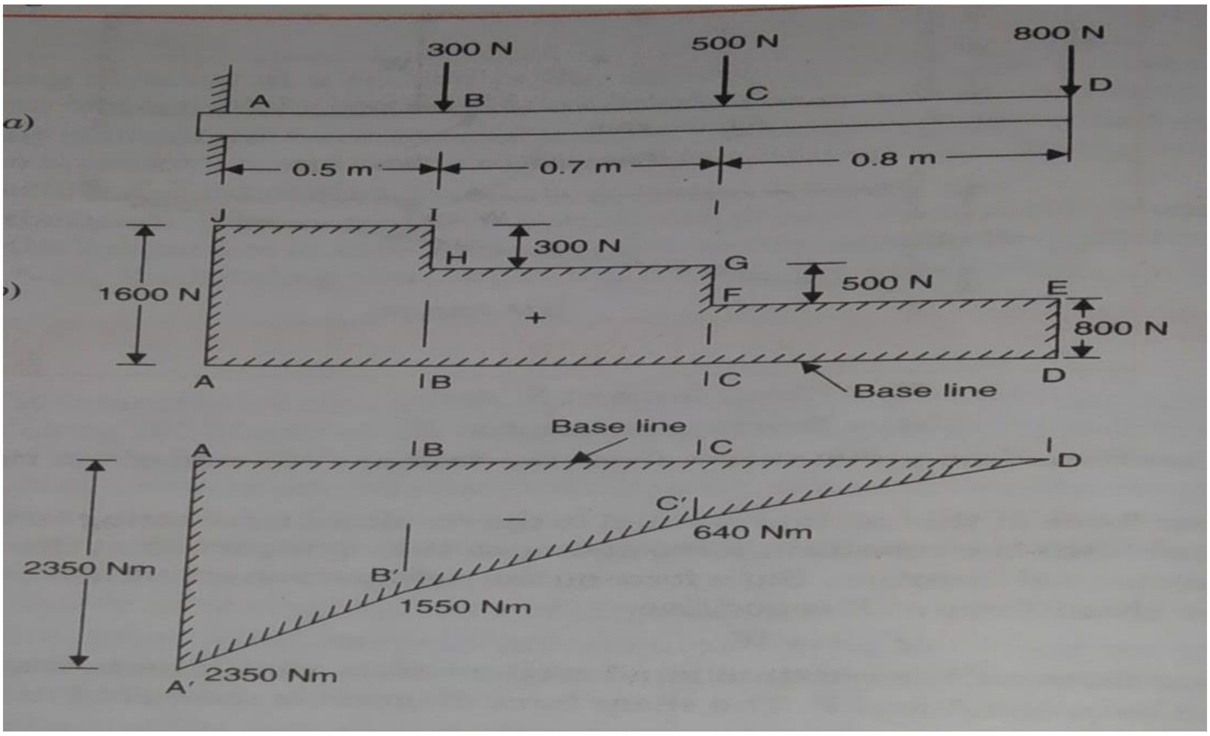


Figure No.8

SHEAR FORCE:

V_D (r) = 0 KN

V_D (l) = 800 N

V_C (r) = 800 N

V_C (l) = 800 + 500 = 1300 N

V_B (r) = 1300 N

V_B (l) = 1300 + 300 = 1600 N

V_A (r) = 1600 N

V_A (l) = 0 KN

BENDING MOMENT:

Bending Moment = Load X Distance

BM at the free end, M_D = 0 N.m

BM at the point C, M_C = -800 X 0.8 – 500 X 0 = -640 N.m

BM at the point B, M_B = -800 X 1.5 – 500 X 0.7 = -1550 N.m

BM at the fixed end, M_A = -800 X 2 – 500 X 1.2 – 300 X 0.5 = -2350 N.m

2.A cantilever of length 2 m carries a uniformly distributed load of 1 KN/m run over a length of 1.5 m from the free end. Draw SFD and BMD

SHEAR FORCE:

V_B = 0 KN

V_C = 1 X 1.5 = 1.5 KN

V_A (r) = 1.5 KN

V_A (l) = 0 KN

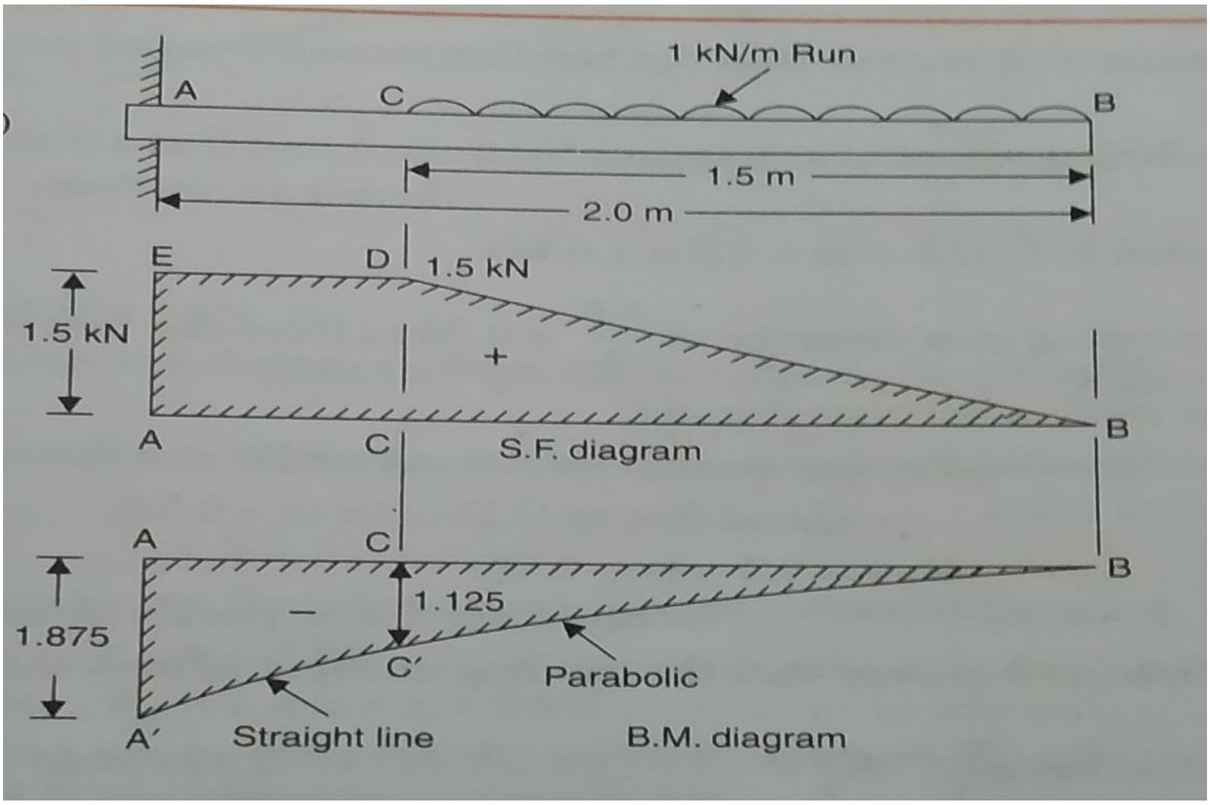


Figure No.9

BENDING MOMENT:

Bending Moment = Load X Distance X Distance/2

BM at free end B, M_B = 0 KN.m

BM at the point C, M_C = -1 X 1.5 X 1.5/2 = -1.125 KN.m

BM at fixed end A, M_A = -1 X 1.5 X (1.5/2 + 0.5) = -1.875 KN.m

3.A cantilever of length 2 m carries a uniformly distributed load of 2 KN/m length over the whole length and a point load of 3 KN at the free end. Draw SFD and BMD.

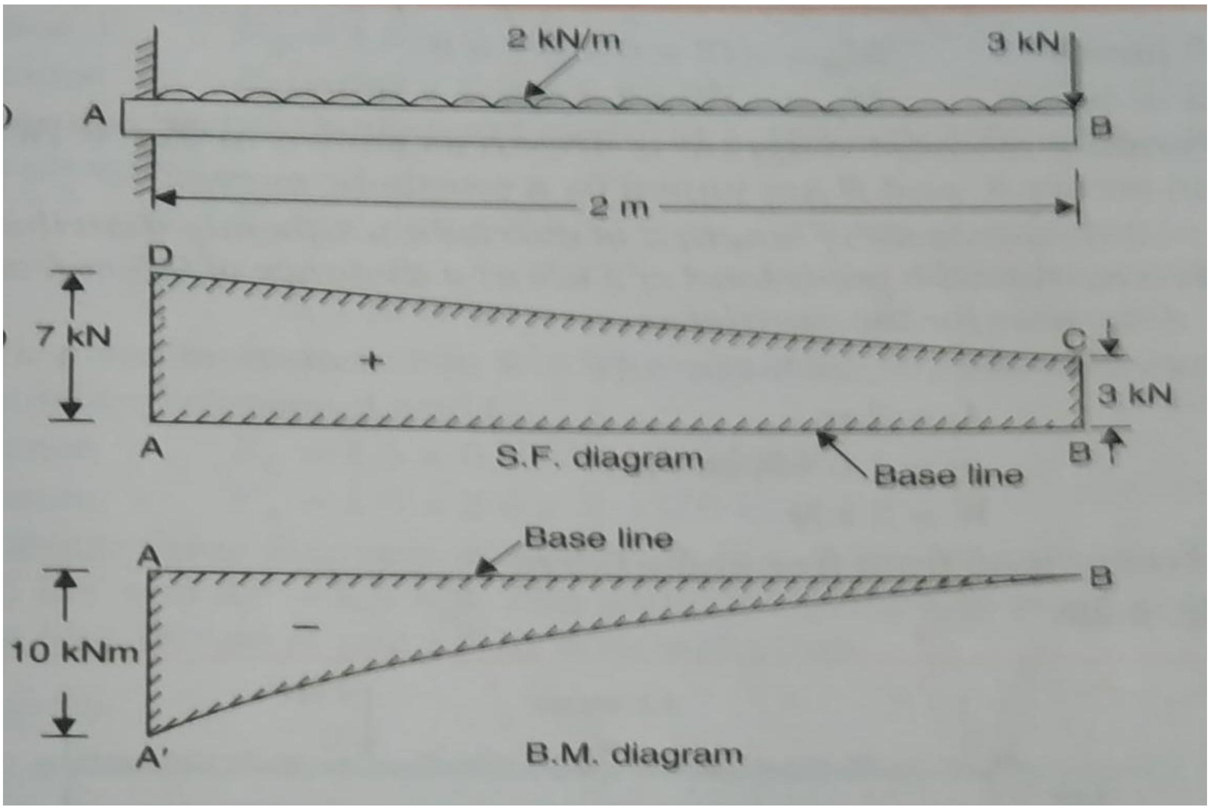


Figure No.10

SHEAR FORCE:

V_B (r) = 0 KN

V_B (l) = 3 KN

V_A (r) = 3 + 2 X 2 = 7 KN

V_A (l) = 0 KN

BENDING MOMENT:

BM at free end B, M_B = 0 KN.m

BM at fixed end A, M_A = -3 X 2 – 2 X 2 X 2/2 = -10 KN.m

4. A cantilever of length 2 m carries a uniformly distributed load of 1.5 KN/m run over the whole length and a point load of 2 KN at a distance of 0.5 m from the free end. Draw SFD and BMD.

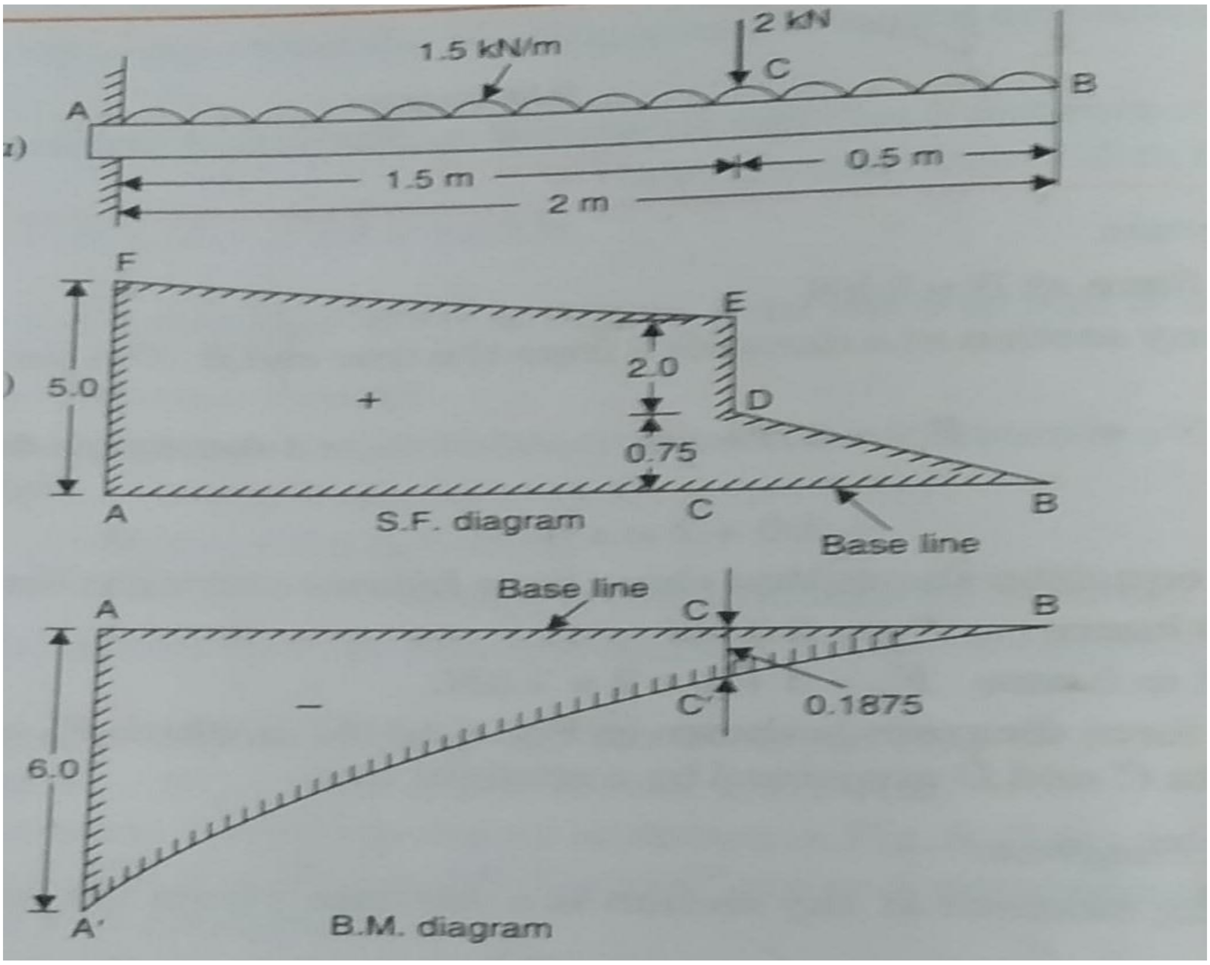


Figure No.11

SHEAR FORCE:

V_B = 0 KN

V_C (r) = 1.5 X 0.5 = 0.75 KN

V_C (l) = 0.75 + 2 = 2.75 KN

V_A (r) = 2.75 + 1.5 X 1.5 = 5 KN

V_A (l) = 0 KN

BENDING MOMENT:

BM at free end B, M_B = 0 KN.m

BM at point C, M_C = -1.5 X 0.5 X 0.5/2 = -0.1875 KN.m

BM at fixed end A, M_A = -1.5 X 2 X 2/2 – 2 X 1.5 = -6 KN.m

5. A cantilever 1.5 m long is loaded with a uniformly distributed load of 2 KN/m run over a length of 1.25 m from the free end. It also carries a point load of 3 KN at a distance of 0.25 m from the free end. Draw SFD and BMD.

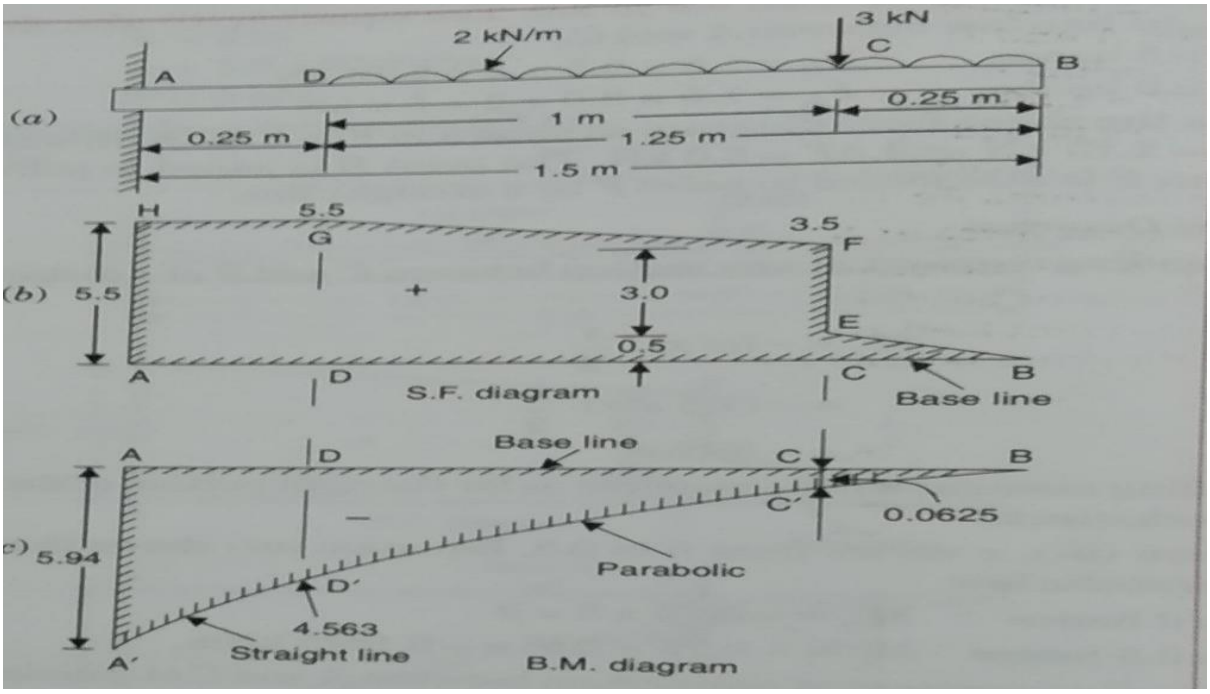


Figure No.12

SHEAR FORCE:

V_B = 0 KN

V_C (r) = 2 X 0.25 = 0.5 KN

V_C (l) = 0.5 + 3 = 3.5 KN

V_D (r) = 3.5 + 2 X 1 = 5.5 KN

V_D (l) = 5.5 KN

V_A (r) = 5.5 KN

V_A (l) = 0 KN

BENDING MOMENT:

BM at free end B, M_B = 0 KN.m

BM at point C, M_C = -2 X 0.25 X 0.25/2 = -0.0625 KN.m

BM at point D, M_D = -2 X 1.25 X 1.25/2 - 3 X 1 = -4.563 KN.m

BM at fixed end A, M_A = -2 X 1.25 X (1.25/2 + 0.25) – 3 X 1.25 = -5.94 KN.m

6. A cantilever of length of 5 m is loaded as shown in fig. Draw SFD and BMD.

SHEAR FORCE:

V_B (r) = 0 KN

V_B (l) = 2.5 KN

V_C = 2.5 KN

V_D = 2.5 + 1 X 2 = 4.5 KN

V_E (r) = 4.5 KN

V_E (l) = 4.5 + 3 = 7.5 KN

V_A (r) = 7.5 KN

V_A (l) = 0 KN

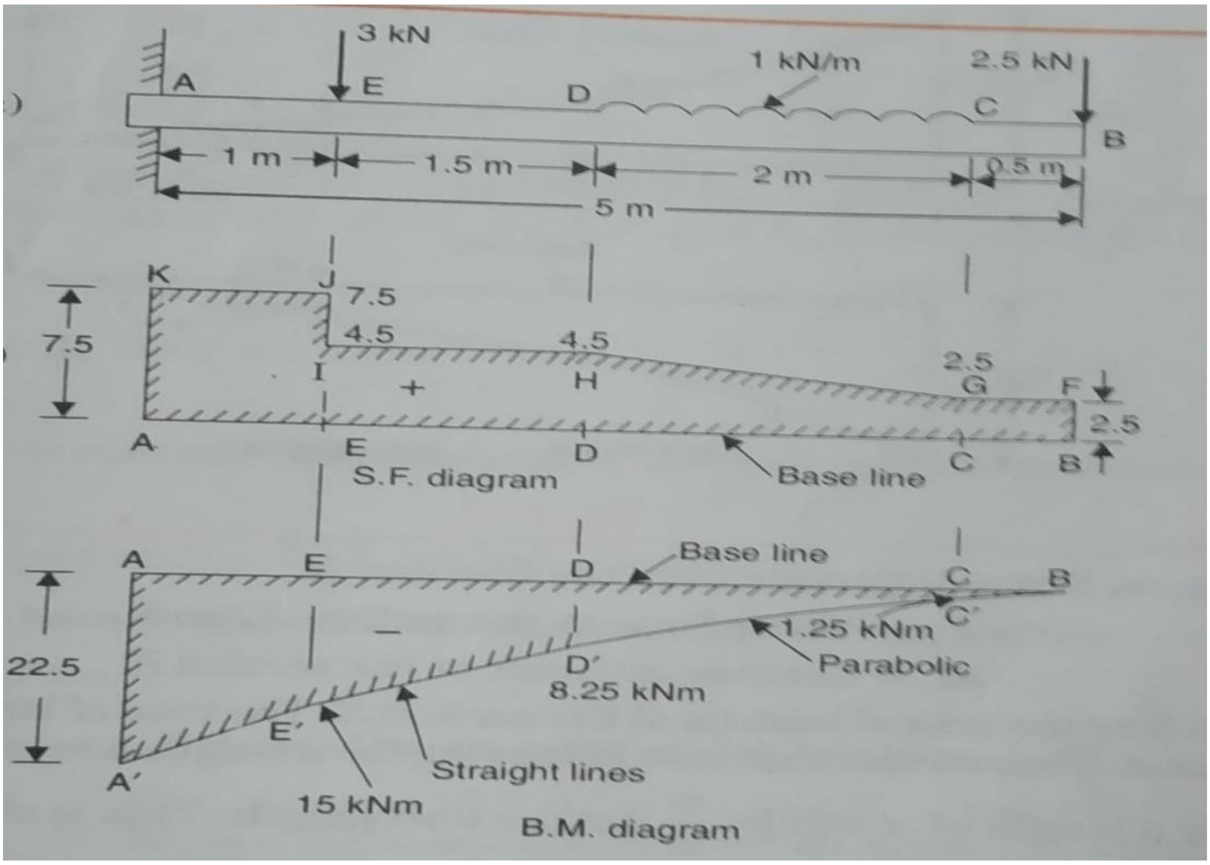


Figure No.13

BENDING MOMENT:

BM at free end B, M_B = 0 KN.m

BM at point C, M_C = -2.5 X 0.5 = -1.25 KN.m

BM at point D, M_D = -2.5 X 2.5 – 1 X 2 X 2/2 = -8.25 KN.m

BM at point E, M_E = -2.5 X 4 - 1 X 2 X (2/2 + 1.5) = -15 KN.m

BM at fixed end A, M_A = -2.5 X 5 – 1 X 2 X (2/2 + 2.5) – 3 X 1 = -22.5 KN.m

SIMPLY SUPPORTED BEAM

SIMPLY SUPPORTED BEAM WITH MID POINT LOAD

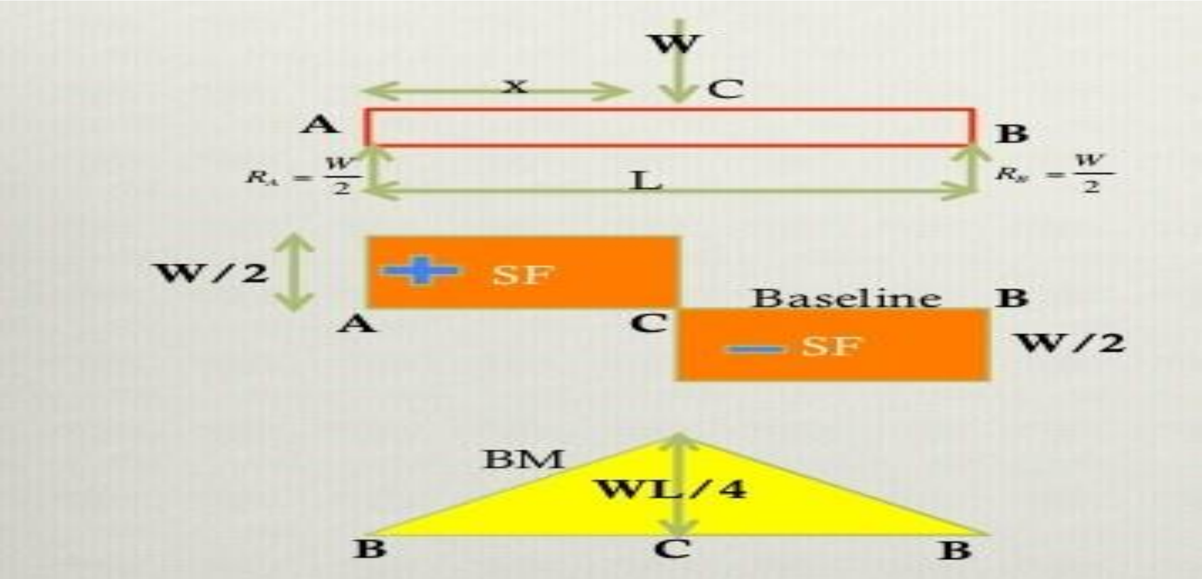


Figure No.14

REACTIONS:

$R_A + R_B$ = Total Load = W KN.

$R_A = R_B$ (symmetrical Loading)

$R_A + R_A$ = W

2 R_A = W

R_A = W/2 KN and R_B = W - R_A = W – W/2 = W/2 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = W/2 KN

V_c (l) = W/2 KN

V_c (r) = W/2 - W = - W/2 KN

V_B (l) = - W/2 KN

V_B (r) = - W/2 + W/2 = 0 KN

BENDING MOMENT:

Bending Moment = Load X Distance

BM at end supports A & B, $M_A = M_B$ = 0 KN.m

BM at point C, M_C = W/2 X L/2 = WL/4 KN.m

SIMPLY SUPPORTED BEAM WITH UDL THROUGHOUT THE LENGTH:

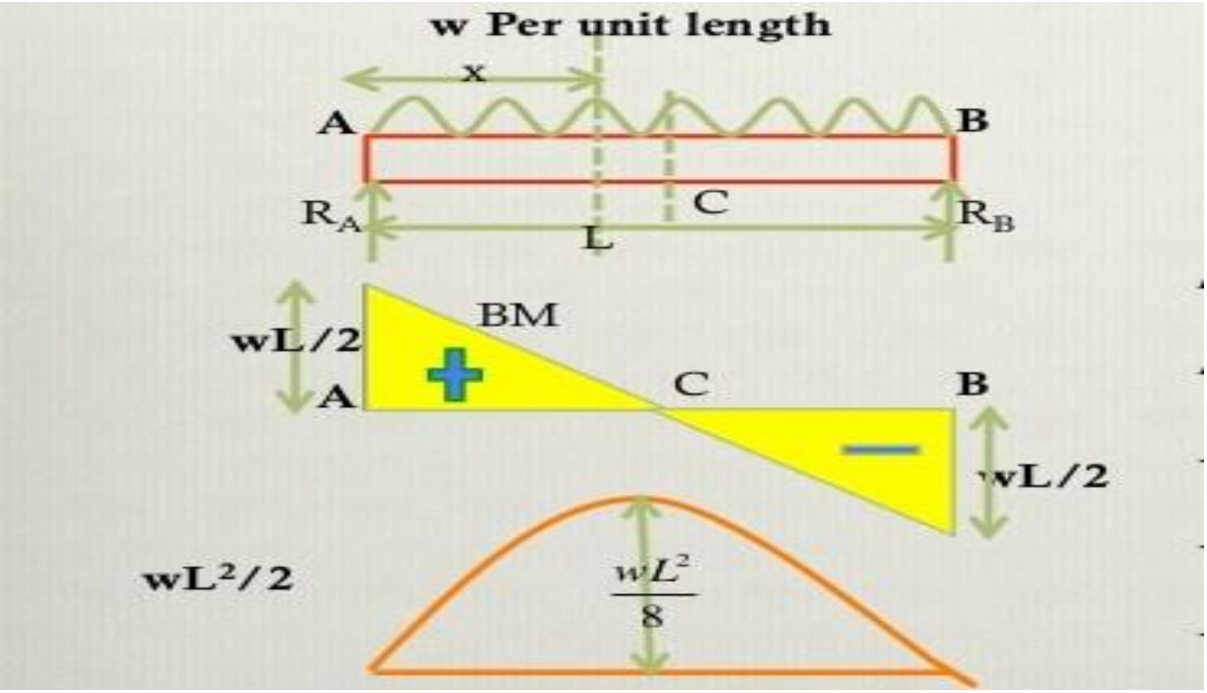


Figure No.15

REACTIONS:

$R_A + R_B$ = Total Load = wL

$R_A = R_B$ (symmetrical Loading)

$R_A + R_A$ = wL

2 R_A = wL

R_A = wL/2 KN and R_B = wL - wL/2 = wL/2 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = WL/2 KN

V_c = WL/2 - WL/2 = 0 KN

V_B (l) = 0 - wL/2 = - wL/2 KN

V_B (r) = - wL/2 + wL/2 = 0 KN

BENDING MOMENT:

Bending Moment = Load X Distance X Distance/2

BM at end supports A & B, $M_A = M_B$ = 0 KN.m

BM at point C, M_C = wL/2 X L/2 – w X L/2 X L/4 = $wL^2/4$ - $wL^2/8$

= $wL^2/8$ KN.m

7. A simply supported beam of length 6 m, carries point loads of 3 KN and 6 KN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 9 KN.

Taking moment reactions about A,

R_B X 6 – 3 X 2 – 6 X 4 = 0

6 R_B - 6 – 24 = 0

6 R_B = 30, R_B = 30/6 = 5 KN

$R_A + R_B$ = 9

R_A = 9 – 5 = 4 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = 4 KN

V_C (l) = 4 KN

V_C (r) = 4 – 3 = 1 KN

V_D (l) = 1 KN

V_D (r) = 1 – 6 = -5 KN

V_B (l) = -5 KN

V_B (r) = -5 + 5 = 0 KN.

BENDING MOMENT:

Bending Moment = Load X Distance

BM at end supports A & B, $M_A = M_B$ = 0 KN.m

BM at point C, M_C = 4 X 2 = 8 KN.m

BM at point D, M_D = 4 X 4 – 3 X 2 = 10 KN.m

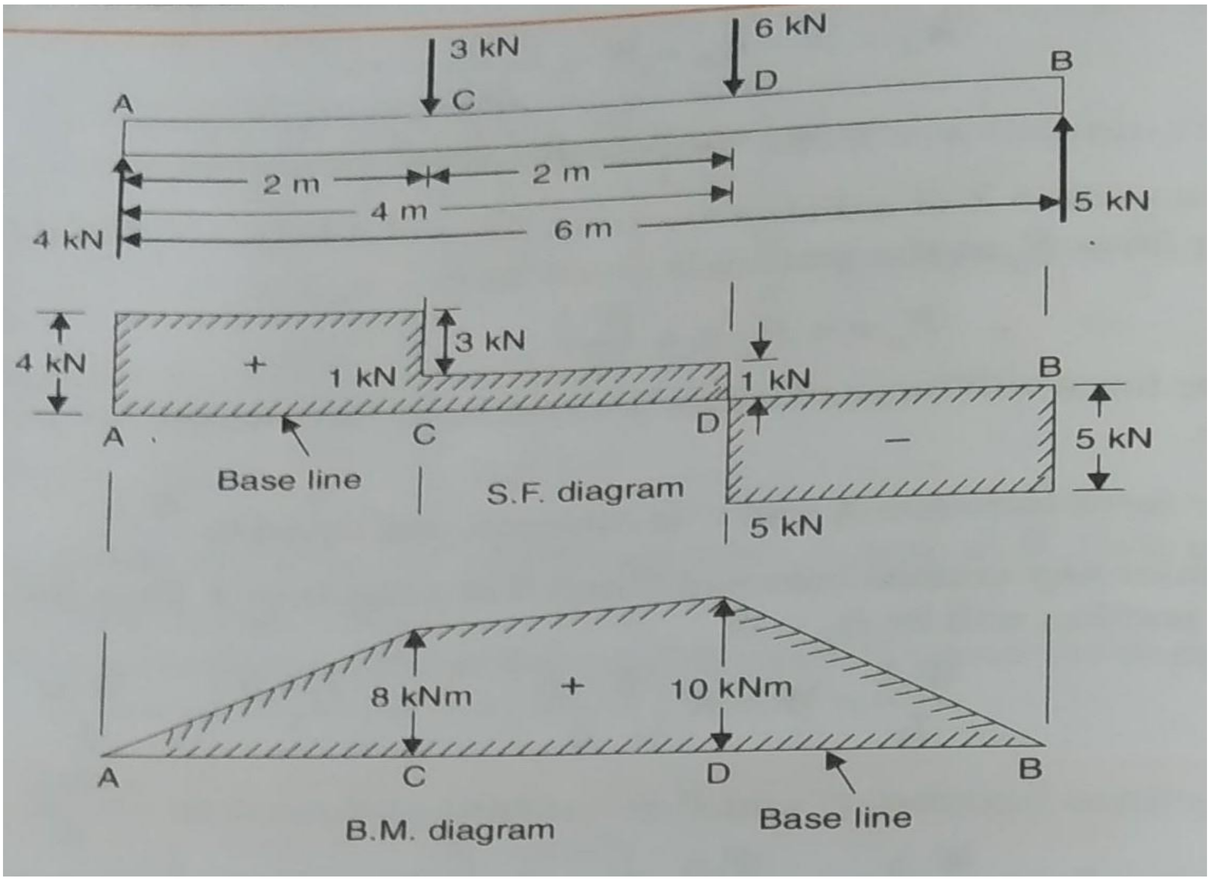


Figure No.16

8. Draw the SFD and BMD for a simply supported beam of length 9 m and carrying a UDL of 10 KN/m for a distance of 6 m from the left end. Also calculate the maximum BM on the section.

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 10 X 6 = 60 KN.

Taking moment reactions about A,

R_B X 9 – 10 X 6 X 6/2 = 0

9 R_B - 180 = 0

R_B = 180 / 9 = 20 KN

$R_A + R_B$ = 60

R_A = 60 – 20 = 40 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = 40 KN

V_C = 40 – 10 X 6 = -20 KN

V_B (l) = -20 KN

V_B (r) = -20 + 20 = 0 KN.

The shear force changes its sign from positive to negative between A and C.

Distance, x = S.F / UDL = V_A (r) / 10 = 40/10 = 4 m.

BENDING MOMENT:

Bending Moment = Load X Distance X Distance/2

BM at end supports A & B, M_A = M_B = 0 KN.m

BM at point C, M_C = 40 X 6 – 10 X 6 X 6/2 = 60 KN.m

BM at point D or Max. BM, M_D or $M_{max.}$ = 40 X 4 – 10 X 4 X 4/2 = 80 KN.m

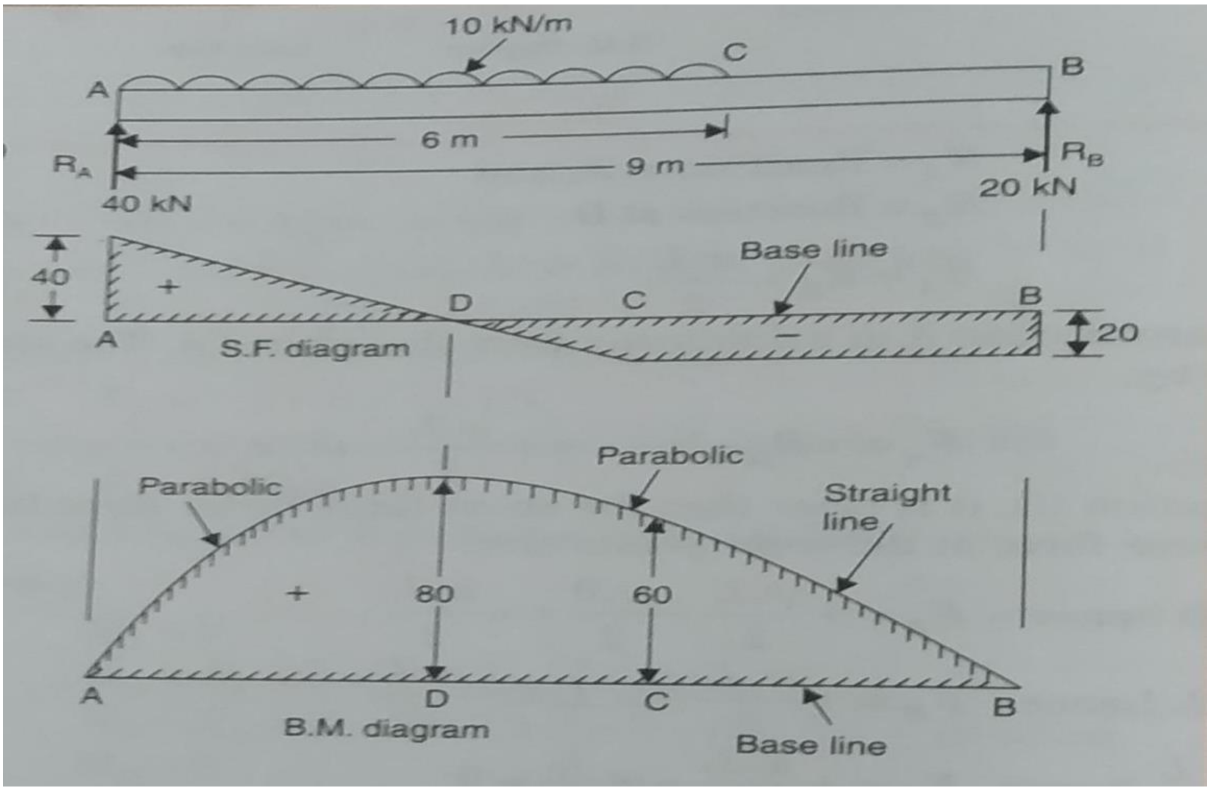


Figure No.17

9. Draw SFD and BMD for a simply supported beam of length 8 m and carrying a UDL of 10 KN/m for a distance of 4 m as shown in fig.

Solution:

REACTIONS:

R_A + R_B = Total Load = 10 X 4 = 40 KN.

Taking moment reactions about A,

R_B X 8 – 10 X 4 X (4/2 + 1) = 0

8 R_B - 120 = 0

R_B = 120 / 8 = 15 KN

R_A + R_B = 40

R_A = 40 – 15 = 25 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = 25 KN

V_C (l) = 25 KN

V_C (r) = 25 KN

V_D (l) = 25 - 40 = -15 KN

V_D (r) = -15 KN

V_B (l) = -15 KN

V_B (r) = -15 + 15 = 0 KN.

The shear force changes its sign from positive to negative between C and D.

Distance, x = S.F / UDL = V_C (r) / UDL = 25/10 = 2.5 m.

BENDING MOMENT:

Bending Moment = Load X Distance X Distance/2

BM at end supports A & B, M_A = M_B = 0 KN.m

BM at point C, M_C = 25 X 1 = 25 KN.m

BM at point D, M_D = 25 X 5 – 10 X 4 X 4/2 = 45 KN.m

BM at point E or Max. BM, M_E or $M_{max.}$ = 25 X 3.5 – 10 X 2.5 X 2.5/2

= 56.25 KN.m

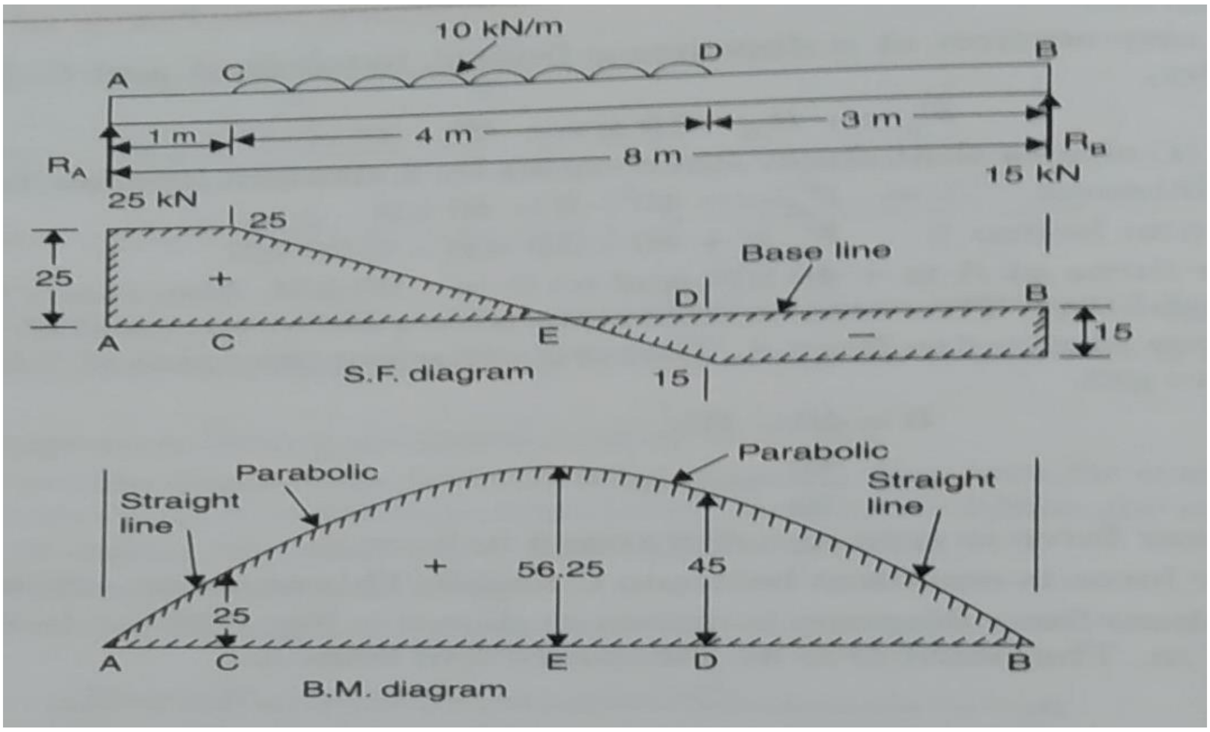


Figure No.18

10. Draw SFD and BMD of a simply supported beam of length 7 m carrying uniformly distributed loads as shown in fig.

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 10 X 3 + 5 x 2 = 40 KN.

Taking moment reactions about A,

R_B X 7 – 5 X 2 X (2/2 + 5) – 10 x 3 x 3/2 = 0

R_B = 15 KN

$R_A + R_B$ = 40

R_A = 40 – 15 = 25 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = 25 KN

V_C = 25 - 10 x 3 = -5 KN

V_D (l) = -5 KN

V_D (r) = -5 – 5 x 2 = -15 KN

V_B (l) = -15 KN

V_B (r) = -15 + 15 = 0 KN.

The shear force changes its sign from positive to negative between A and C.

Distance, x = S.F / UDL = V_A (r) / UDL = 25/10 = 2.5 m.

BENDING MOMENT:

Bending Moment = Load X Distance X Distance/2

BM at end supports A & B, $M_A = M_B = 0$ KN.m

BM at point C, $M_C = 25 \times 3 - 10 \times 3 \times 3/2 = 30$ KN.m

BM at point D, $M_D = 25 \times 5 - 10 \times 3 \times (3/2 + 2) = 20$ KN.m

BM at point E or Max. BM, M_E or $M_{max.} = 25 \times 2.5 - 10 \times 2.5 \times 2.5/2$

$= 31.25$ KN.m

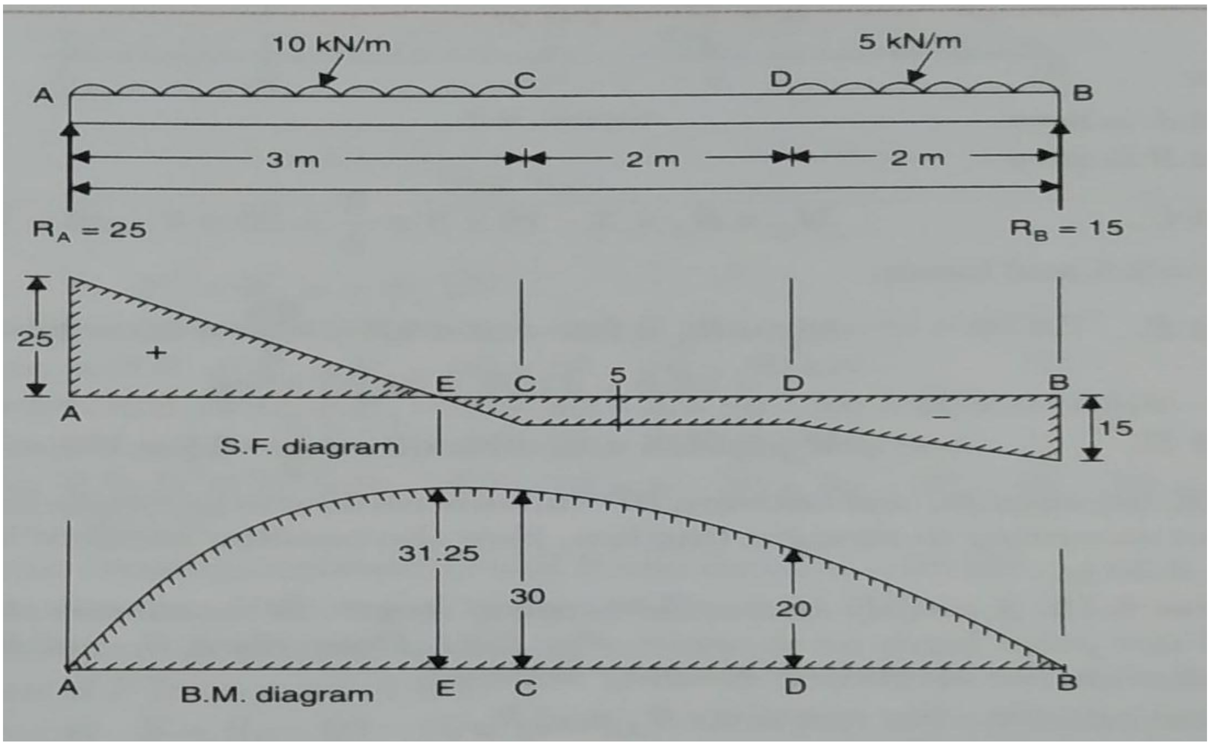


Figure No.19

11. A simply supported beam of length 10 m, carries the UDL and two point loads as shown in fig. Draw SFD and BMD for the beam. Also calculate the maximum bending moment.

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 50 + 40 + 10 X 4 = 130 KN.

Taking moment reactions about A,

R_B X 10 – 50 X 2 – 40 X 6 – 10 X 4 X (4/2 + 2) = 0

10 R_B - 500 = 0

R_B = 500/10 = 50 KN

R_A + R_B = 130 KN

R_A = 130 – 50 = 80 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = 80 KN

V_C (l) = 80 KN

V_C (r) = 80 – 50 = 30 KN

V_D (l) = 30 – 10 X 4 = -10 KN

V_D (r) = - 10 - 40 = - 50 KN

V_B (l) = - 50 KN

V_B (r) = -50 + 50 = 0 KN.

The shear force changes its sign from positive to negative in between C and D.

Distance, x = S.F / UDL = V_C (r) / 10 = 30/10 = 3 m.

BENDING MOMENT:

BM at end supports A & B, M_A = M_B = 0 KN.m

BM at point C, M_C = 80 X 2 = 160 KN.m

BM at point D, M_D = 80 X 6 – 50 X 4 – 10 X 4 X 4/2 = 200 KN.m

BM at point E or Max. BM, M_E or $M_{max.}$ = 80 X 5 – 50 X 3 - 10 X 3 X 3/2 = 205 KN.m

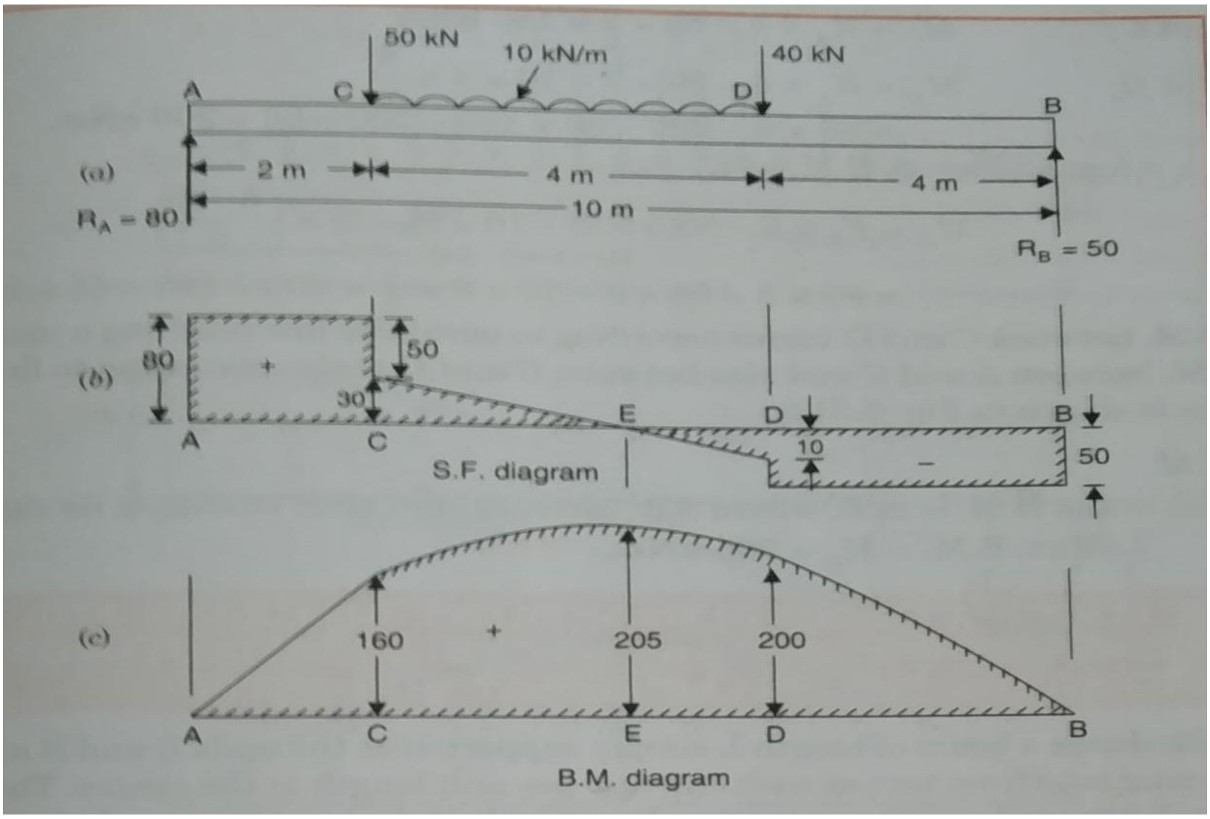


Figure No.20

POINT OF CONTRAFLEXURE

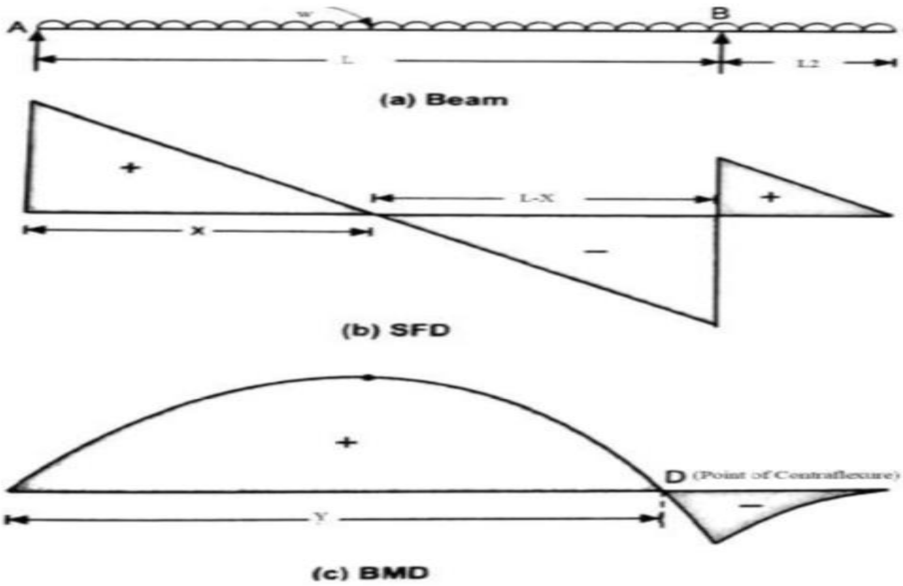


Figure No.21

- **Point of contraflexure is the point where bending moment changes its sign i.e, from positive value to a negative value or vice versa.**
- **The point of contraflexure occurs where bending is zero.**
- **It will occur in the overhanging beam.**
- **It is also known as point of inflexion.**

OVERHANGING BEAM

12. Draw SFD and BMD for the overhanging beam carrying UDL of 2 KN/m over the entire length as shown in fig. Also locate the point of contraflexure.

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 2 x 6 = 12 KN.

Taking moment reactions about A,

R_B X 4 – 2 x 6 x 6/2 = 0

4 R_B – 36 = 0

R_B = 36/4 = 9 KN

$R_A + R_B$ = 12 KN

R_A = 12 – 9 = 3 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = 3 KN

V_B (l) = 3 – 2 x 4 = -5 KN

V_B (r) = -5 + 9 = 4 KN

V_C (l) = 4 – 2 x 2 = 0 KN

V_C (r) = 0 KN

The shear force changes its sign from positive to negative in between A and B.

Distance, x = S.F / UDL = V_A (r) / 2 = 3/2 = 1.5 m.

BENDING MOMENT:

BM at left end support A, M_A = 0 KN.m

BM at free end C, M_C = 0 KN.m

BM at support B, M_B = 3 X 4 – 2 X 4 X 4/2 = 12 – 16 = -4 KN.m

BM at point D, M_D or $M_{max.}$ = 3 X 1.5 – 2 X 1.5 X 1.5/2 = 2.25 KN.m

POINT OF CONTRAFLEXURE:

The BM changes its sign from positive to negative in between the sections A and B.

Let BM is Zero is at a distance of y m from A.

Let point E is at a distance of y m.

$M_E = 3 \times y - 2 \times y \times y/2 = 0$

$= 3y - 2y^2/2 = 0$

$= y(3 - y) = 0$

$= 3 - y = 0$

y = 3 m. Hence point of contraflexure will be at a distance of 3 m from A.

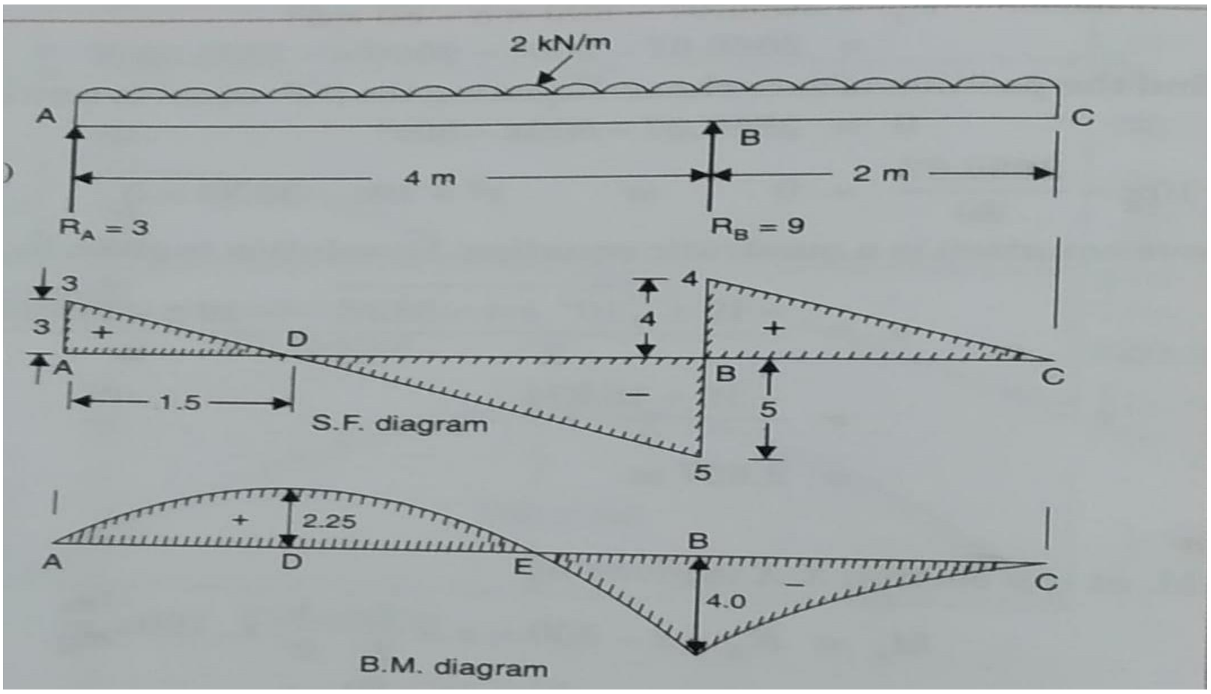


Figure No.22

13. **Draw SFD and BMD for the overhanging beam carrying UDL of 2 KN/m over the entire length and a point load of 2 KN as shown in fig. Locate the point of contraflexure.**

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 2 x 6 + 2 = 14 KN.

Taking moment reactions about A,

$R_B \times 4 - 2 \times 6 \times 6/2 - 2 \times 6 = 0$

$4 R_B - 36 - 12 = 0$

R_B = 48/4 = 12 KN

$R_A + R_B$ = 14 KN

R_A = 14 – 12 = 2 KN

SHEAR FORCE:

V_A (l) = 0 KN

V_A (r) = 2 KN

V_B (l) = 2 – 2 x 4 = -6 KN

V_B (r) = -6 + 12 = 6 KN

V_C (l) = 6 – 2 x 2 = 2 KN

V_C (r) = 2 – 2 = 0 KN

The shear force changes its sign from positive to negative in between A and B.

Distance, x = S.F / UDL = V_A (r) / 2 = 2/2 = 1 m.

BENDING MOMENT:

BM at left end support A, M_A = 0 KN.m

BM at free end C, M_C = 0 KN.m

BM at support B, M_B = 2 X 4 – 2 X 4 X 4/2 = 8 – 16 = -8 KN.m

BM at point D, M_D or $M_{max.}$ = 2 X 1 – 2 X 1 X 1/2 = 1 KN.m

POINT OF CONTRAFLEXURE:

The BM changes its sign from positive to negative in between the sections A and B.

Let BM is Zero is at a distance of y m from A.

Let point E is at a distance of y m.

M_E = 2 X y – 2 X y X y/2 = 0

= 2y – 2y^2/2 = 0

= y (2 – y) = 0

= 2 – y = 0

y = 2 m. Hence point of contraflexure will be at a distance of 2 m from A.

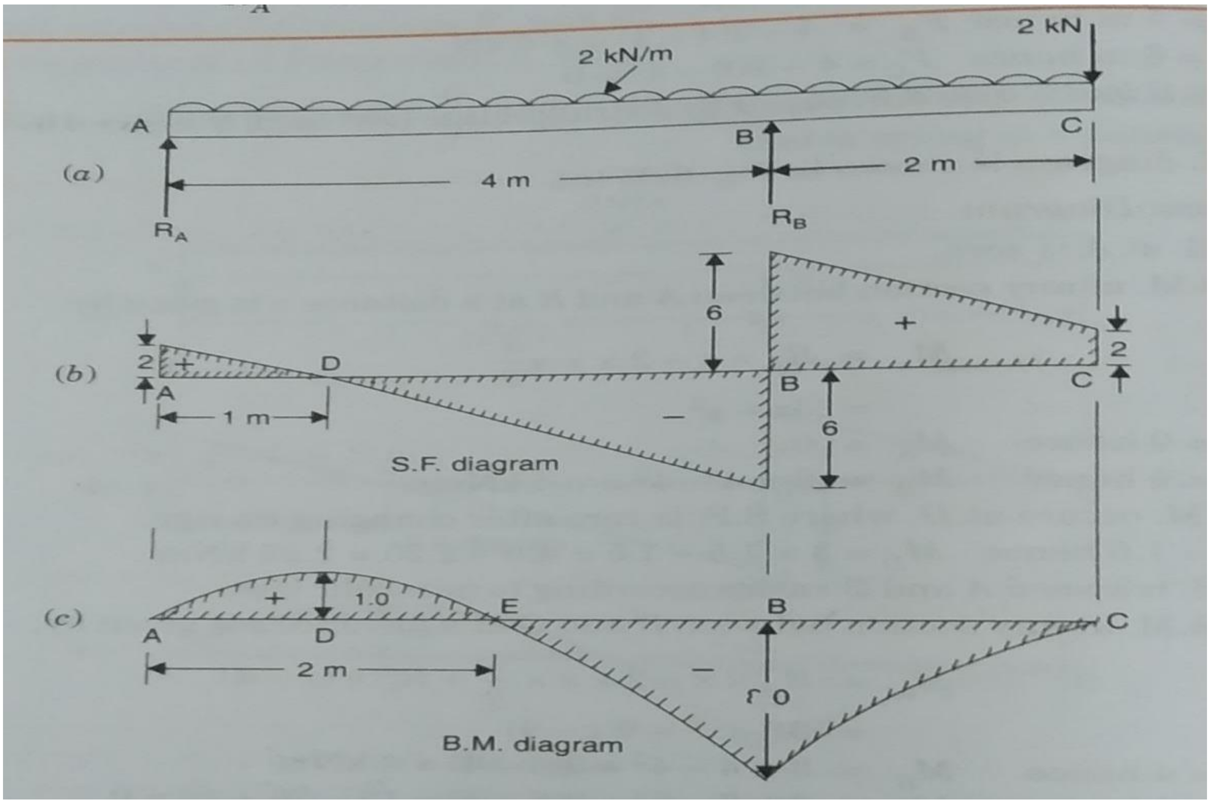


Figure No.23

14. A beam of length 12 m is simply supported at two supports which are 8 m apart, with an overhang of 2 m on each side as shown in fig. The beam carries a concentrated load of 1000 N at each end. Draw SFD and BMD.

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 1000 + 1000 = 2000 N.

As the loading on the beam is symmetrical,

hence $R_A = R_B = 2000/2 = 1000$ N.

SHEAR FORCE:

V_C (l) = 0 N

V_C (r) = -1000 N

V_A (l) = -1000 N

V_A (r) = -1000 + 1000 = 0 N

V_B (l) = 0 N

V_B (r) = 0 + 1000 = 1000 N

V_D (l) = 1000 N

V_D (r) = 1000 – 1000 = 0 N

BENDING MOMENT:

BM at the free end C and D, M_C and M_D = 0 N.m

BM at the left support A, M_A = - 1000 X 2 = -2000 N.m

BM at the right support B, M_B = - 1000 X 2 = -2000 N.m

POINT OF CONTRAFLEXURE:

No point of contraflexure, because of constant bending moment.

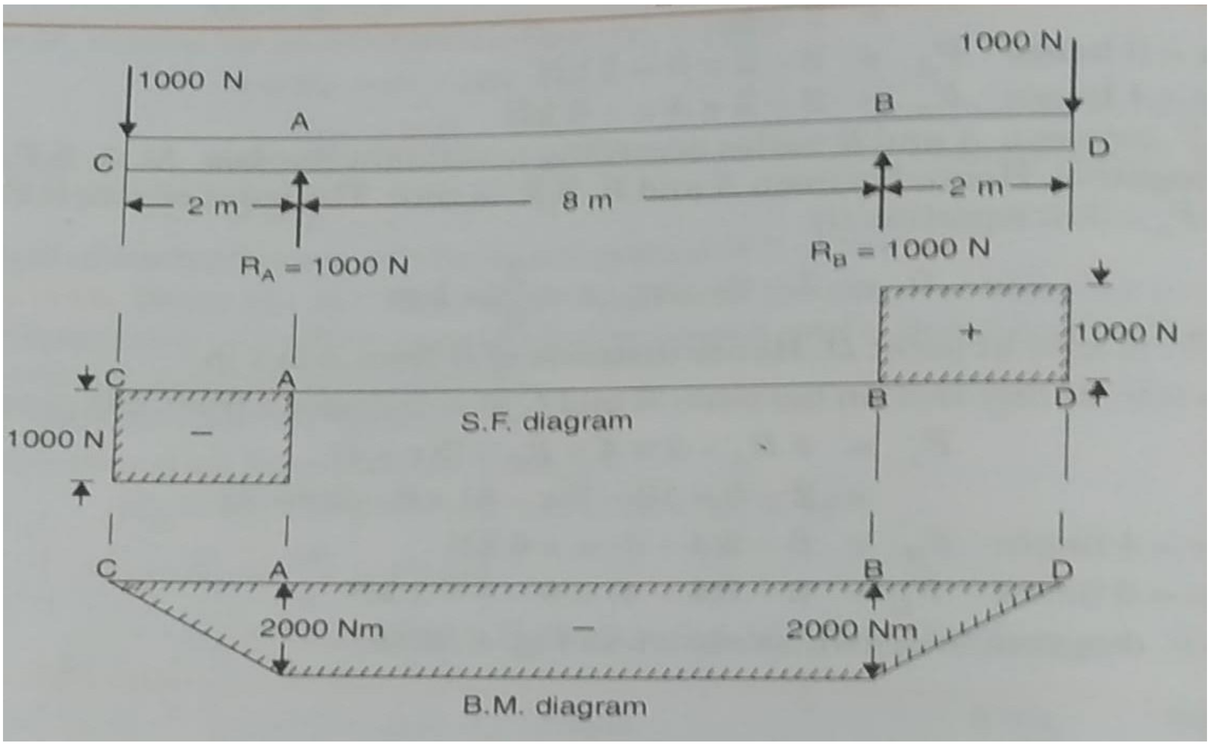


Figure No.24

15. **Draw SFD and BMD for the beam which is shown in fig. Determine the points of contraflexure within the span AB.**

Solution:

REACTIONS:

$R_A + R_B$ = Total Load = 800 + 2000 + 1000 = 3800 N.

Taking moment reactions about A,

R_B X 8 – 2000 x 5 – 1000 X 10 + 800 X 3 = 0

8 R_B – 10000 – 10000 + 2400 = 0

R_B = 17600/4 = 2200 N

R_A + 2200 = 3800 N

R_A = 3800 – 2200 = 1600 N

SHEAR FORCE:

V_C (l) = 0 N

V_C (r) = -800 N

V_A (l) = -800 N

V_A (r) = -800 + 1600 = 800 N

V_D (l) = 800 N

V_D (r) = 800 - 2000 = -1200 N

V_B (l) = -1200 N

V_B (r) = -1200 + 2200 = 1000 N

V_E (l) = 1000 N

V_E (r) = 1000 - 1000 = 0 N

BENDING MOMENT:

BM at the free end C and E, M_C and M_E = 0 N.m

BM at the left support A, M_A = - 800 X 3 = -2400 N.m

BM at the right support B, M_B = - 800 X 11 + 1600 X 8 – 2000 x 3 = -2000 N.m

BM at the point D, M_D = - 800 X 8 + 1600 X 5 = 1600 N.m

POINTS OF CONTRAFLEXURE:

Here, two points of contraflexure O_1 and O_2 where the BM becomes zero.

Point O_1 lies between A and D and point O_2 lies between D and B.

Let point O_1 is at a distance of x_1 m from A,

$M_{O1} = -800 \times (3 + x_1) + 1600 \times x_1 = -2400 – 800\, x_1 + 1600\, x_1$

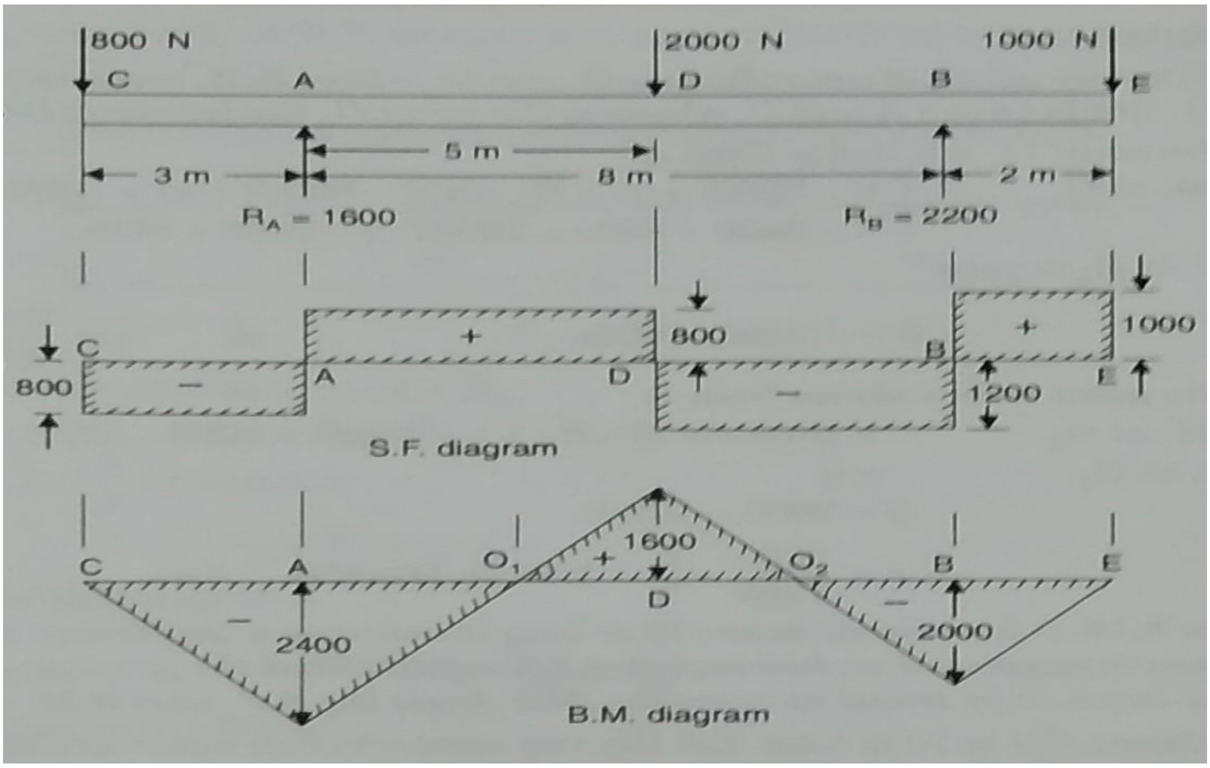
x_1 = 3 m.

Let point O_2 is at a distance of x_2 m from B,

$M_{O2} = 1000 \times (2 + x_2) - 2200 \times x_2 = 2000 + 1000\, x_2 – 2200\, x_2$

x_2 = 1.67 m.

Hence points of contraflexure is at 3 m from A and also 1.67 m from B.



800 N
2000 N
1000 N
C
A
D
B
E
5 m
3 m
8 m
2 m
R_A = 1600
R_B = 2200
C
A
D
B
E
+
−
−
+
800
800
1000
1200
S.F. diagram
C
A
O_1
1600
+
O_2
B
E
D
−
−
2400
2000
B.M. diagram

Figure No.25

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – IV –BENDING & SHEAR STRESS DISTRIBUTION, TORSION - SCIA1301

INTRODUCTION

- **BENDING STRESS:**

- ✓ **When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam.**
- ✓ **Due to the shear force and bending moment, the beam undergoes certain deformation.**
- ✓ **The material of the beam will offer resistance or stresses against these deformations.**
- ✓ **The stresses introduced by bending moment are known as bending stresses.**

PURE BENDING

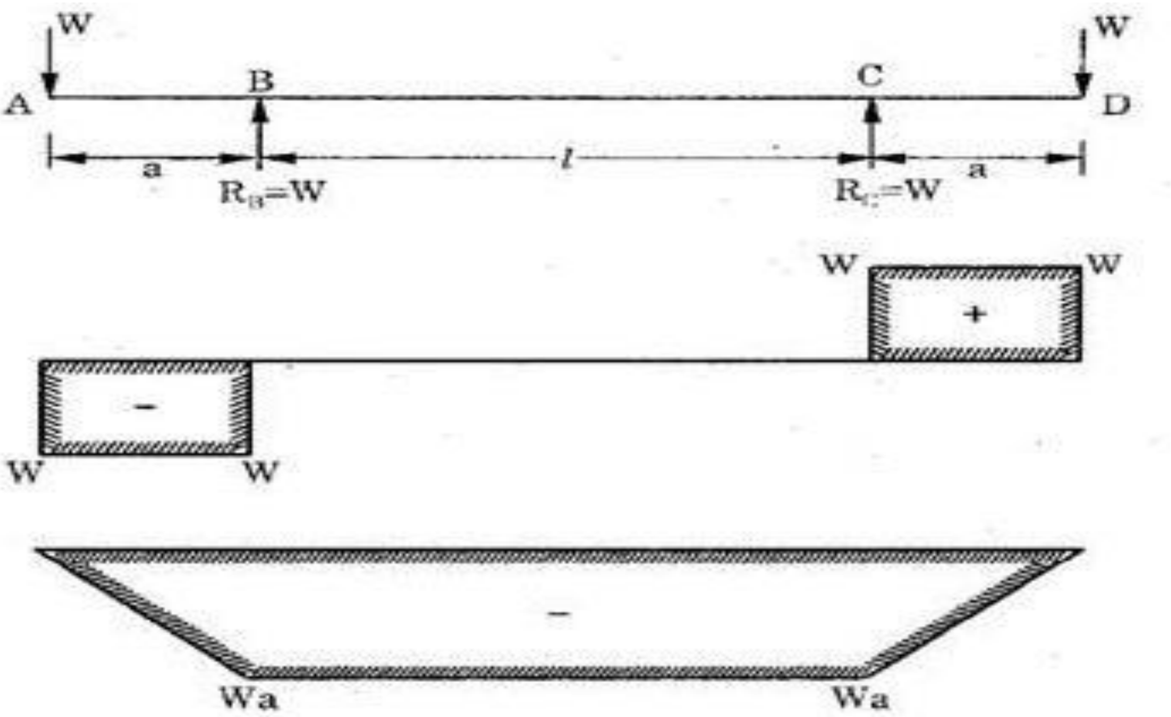


Figure No.1

PURE BENDING:

- ✓ **If a length of a beam is subjected to a constant bending moment and no shear force, then the stresses will be set up in that length of the beam due to bending moment only and that length of the beam is said to be in pure bending or simple bending.**
- ✓ **The stresses set up in that length of beam are known as bending stresses.**

ASSUMPTIONS IN THE THEORY OF SIMPLE BENDING:

- ✓ **The material of the beam is homogeneous and isotropic.**
- ✓ **The value of young's modulus of elasticity is the same in tension and compression.**
- ✓ **The transverse sections which were plane before bending remain plane after bending also.**
- ✓ **The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.**
- ✓ **The radius of curvature is large compared with the dimensions of the cross section.**
- ✓ **Each layer of the beam is free to expand or contract, independently of the layer, above or below it.**
- ➢ **NEUTRAL AXIS:**
- ✓ **It is defined as the line of intersection of the neutral layer with the transverse section.**
- ✓ **It is denoted by 'N.A'.**
- ➢ **MOMENT OF RESISTANCE:**
- ✓ **Due to pure bending, the layers above the N.A are subjected to compressive stresses whereas the layers below the N.A are subjected to tensile stresses.**
- ✓ **Due to these stresses, the forces will be acting on the layers.**
- ✓ **These forces will have moment about the N.A.**
- ✓ **The total moment of these forces about the N.A for a section is known as moment of resistance of that section.**

BENDING EQUATION

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where,

M – bending Moment in N.mm

I - moment of inertia in mm^4

σ - Bending stress in N/mm^2

E – Young's modulus in N/mm^2

R – Radius of curvature in mm.

FLEXURAL RIGIDITY:

- ✓ **The product of young's modulus and the moment of inertia is known as flexural rigidity. (EI)**

- ➢ **SECTION MODULUS:**

- ✓ **It is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.**
- ✓ **It is denoted by Z.**
- ✓ **mathematically, section modulus is given by**

$$Z = I/y_{max.}$$

Where, I - moment of inertia in mm^4

$y_{max.}$ - distance of the outermost layer from the neutral axis in mm.

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max.}}$$

$$M = \frac{I}{y_{max.}} X \sigma_{max}$$

$$M = Z X \sigma_{max}$$

In the above equation, M is the maximum bending moment.

Hence moment of resistance offered the section is maximum when section modulus Z is maximum.

Hence section modulus represents the strength of the section.

STRENGTH OF A SECTION:

The strength of a section means the moment of resistance offered by the section and moment of resistance is given by,

$$M = Z X \sigma$$

SECTION MODULUS FOR VARIOUS SECTIONS

Expressions for section modulus of various standard cross-sections are derived below.

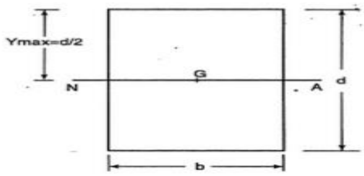
Rectangular section of width *b* and depth *d:*

$$I = \frac{1}{12}bd^3$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{bd^3}{12}}{\frac{d}{2}}$$

$$Z = \frac{1}{6}bd^2$$



Hollow rectangular section with symmetrically placed opening:

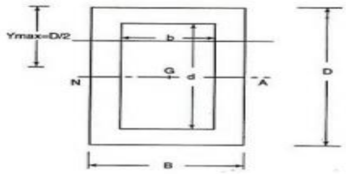
Consider the section of size $B \times D$ with symmetrical opening $b \times d$ as shown in Fig..

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}(BD^3 - bd^3)$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}}$$

$$= \frac{1}{6}\frac{(BD^3 - bd^3)}{D}$$

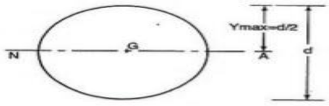


Circular section of diameter *d*

For circular section $= I = \frac{\pi d^4}{64}$

$$y_{max} = \frac{d}{2}$$

$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{\pi d^3}{32}$$



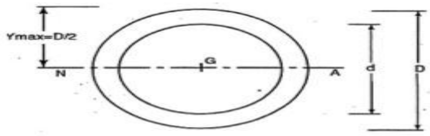
Hollow circular section of uniform thickness:

$$I = \frac{\pi}{64}D^4 - \frac{\pi}{64}d^4$$

$$= \frac{\pi}{64}(D^4 - d^4)$$

Extreme fibre distance $= \frac{D}{2}$

$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{\pi}{32}\frac{(D^4 - d^4)}{D}$$



1. **A steel plate of width 120 mm and of thickness 20 mm is bent into a circular arc of radius 10 m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take E = 2 X 10^5 N/mm^2.**

Given Data:

b = 120 mm

t = 20 mm

R = 10 m = 10 x 10^3 mm

E = 2 X 10^5 N/mm^2.

To Find:

σ_{max} & M.

Solution:

1. **σ_{max}**

$$\sigma_{max} = \frac{E}{R} \times y_{max}$$

$y_{max.}$ = t/2 = 20/2 = 10 mm.

$$= \frac{2 \times 10^5 \times 10}{} = 200 \text{ N/mm}^2.$$

10×10^3

2. M

$$\frac{M}{I} = \frac{E}{R} \qquad I = b \times t^3/12 = 120 \times 20^3/12 = 8 \times 10^4 mm^4$$

$$M = \frac{E}{R} \times I = \frac{2 \times 10^5 \times 8 \times 10^4}{10 \times 10^3} = 16 \times 10^5 \text{ N.mm}$$

2. **Calculate the maximum stress induced in a cast iron pipe of external diameter 40 mm, internal diameter 20 mm and of length 4 m when the pipe is supported at its ends and carries a point load of 80 N at its centre.**

Given Data:

$D = 40$ mm

$d = 20$ mm

$L = 4 \text{ m} = 4 \times 10^3$ mm

$W = 80$ N.

To Find:

σ_{max}

Solution:

In case of simply supported beam carrying a point load at the centre, the maximum bending moment is at the centre of the beam.

Maximum Bending Moment, $M = WL/4 = 80 \times 4 \times 10^3 /4 = 8 \times 10^4$ N.mm

$I = \pi/64 (D^4 - d^4) = \pi/64 (40^4 - 20^4) = 117809.7 \text{ mm}^4$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max.}} \qquad y_{max.} = D/2 = 40/2 = 20 \text{ mm.}$$

$\sigma_{max} = (M/I) \times y_{max.} = (8 \times 10^4 / 117809.7) \times 20 = 13.58 \text{ N/mm}^2.$

3. **A cantilever of length 2 m fails when a load of 2 KN is applied at the free end. If the section of the beam is 40 mm X 60 mm, find the stress at the failure.**

Given Data:

$b = 40$ mm

$d = 60$ mm

$L = 2 \text{ m} = 2 \times 10^3$ mm

$W = 2$ KN.

To Find:

σ_{max}

Solution:

Maximum bending moment for a cantilever is at the fixed end.

$M = W \times L = 2000 \times 2 \times 10^3 = 4 \times 10^6$ N.mm

$\sigma_{max} = M / Z$ **$Z = bd^2/6 = 40 \times 60^2/6$**

$= 24000$ mm^3.

$\sigma_{max} = 4 \times 10^6 / 24000 = 166.67$ N/mm^2.

4. A rectangular beam 200 mm deep and 300 mm wide is simply supported over a span of 8 m. What uniformly distributed load per metre the beam may carry, if the bending stress is not to exceed 120 N/mm^2.

Given Data:

$b = 300$ mm

$d = 200$ mm

$L = 8$ m $= 8 \times 10^3$ mm

$\sigma_{max} = 120$ N/mm^2.

To Find:

w

Solution:

Maximum B.M for a simply supported beam carrying UDL is at the centre of the beam.

$M = w \times L^2/8 = w \times 8^2/8 = 8w$ Nm $= 8w \times 1000$ N.mm

$M = 8000w$ N.mm

$M = \sigma_{max} \times Z$ **$Z = bd^2/6 = 300 \times 200^2/6$**

$= 2000000$ mm^3.

$8000w = 120 \times 2000000$

$w = \dfrac{120 \times 2000000}{8000} = 30 \times 1000$ N/m $= 30$ KN/m.

5. A rectangular beam 300 mm deep is simply supported over a span of 4 m. Determine the uniformly distributed load per metre which the beam may carry, if the bending stress should not exceed 120 N/mm^2.Take $I = 8 \times 10^6$ mm^4.

Solution:

For a simply supported beam carrying UDL, the bending moment is maximum at the centre of the beam.

Max.BM = 2w X 2 – w X 2 X 2/2 = 4w – 2w = 2w N.m = 2w X 1000 N.mm

M = 2000w N.mm

M = σ_{max} X Z **y_{max} = 300/2 = 150 mm.**

Z = I / $y_{max.}$ = 8 X 10^6 /150

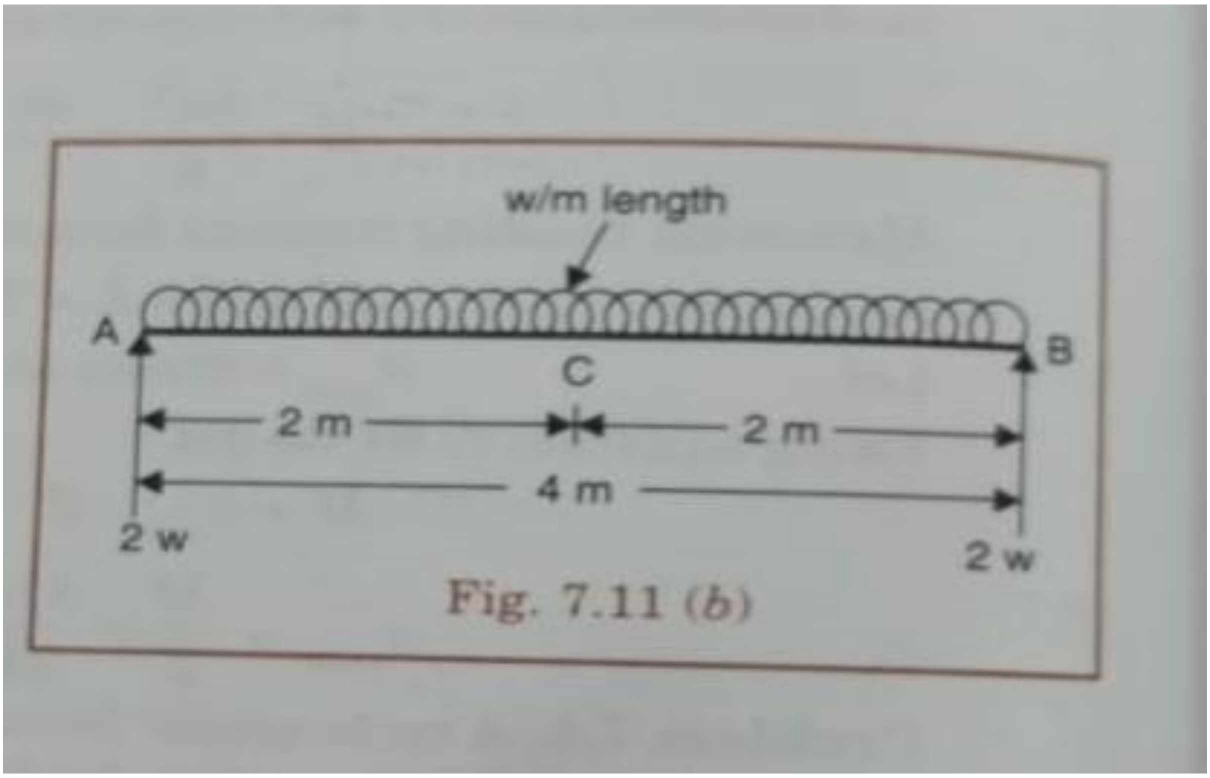


Figure No.2

M = 120 X (8 X 10^6 /150)

2000w = 120 X (8 X 10^6 /150)

$$w = \frac{120 \times 8 \times 10^6}{150 \times 2000} = 3200 \text{ N/m.}$$

6. A square beam 20 mm X 20 mm in section and 2 m long is supported at the ends. The beam fails when a point load of 400 N is applied at the centre of the beam. What uniformly distributed load per metre length will break a cantilever of the same material 40 mm wide, 60 mm deep and 3 m long.

Solution:

In this problem, the maximum stress for the simply supported beam is to be calculated first. As the material of the cantilever is same as that of simply supported beam, hence maximum stress for the cantilever will also be same as that of simply supported beam. [d – 20 mm, b – 20 mm,L – 2 m, W – 400 N]

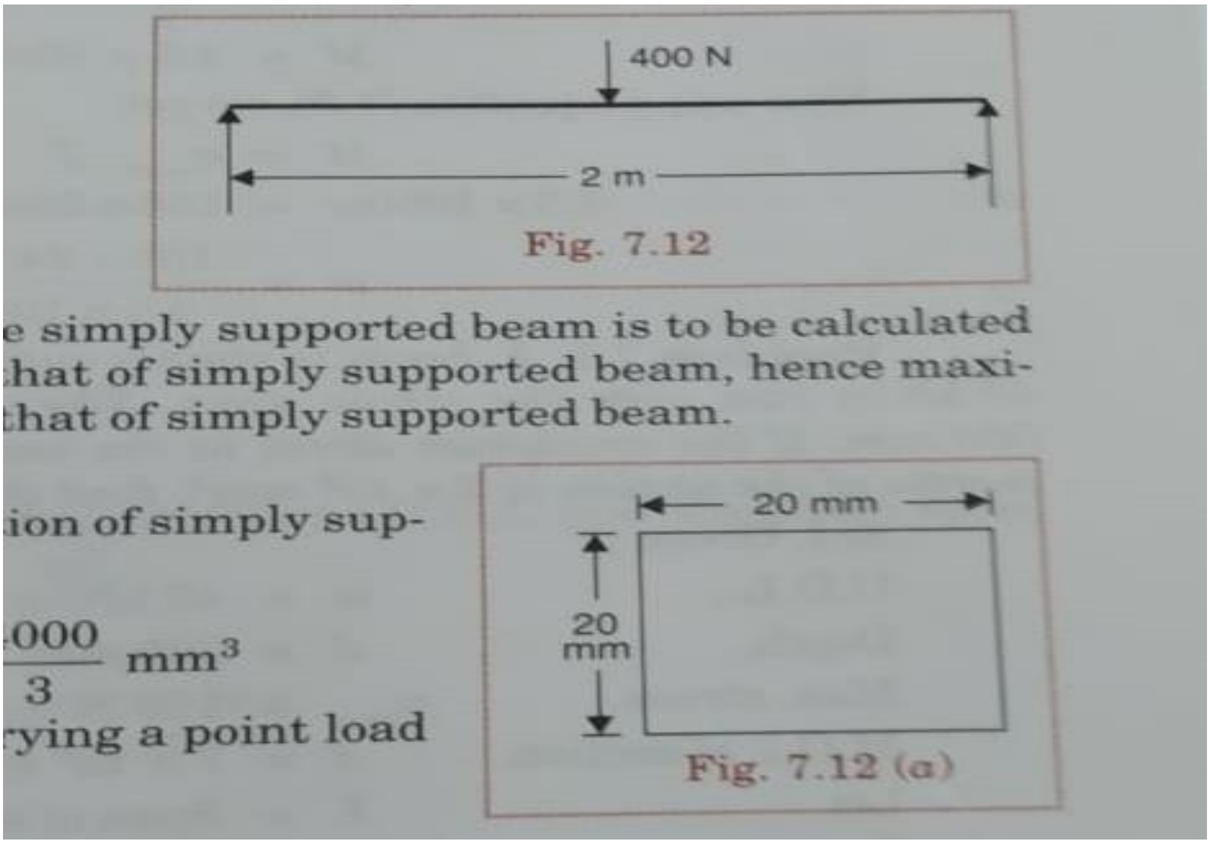


Figure No.3

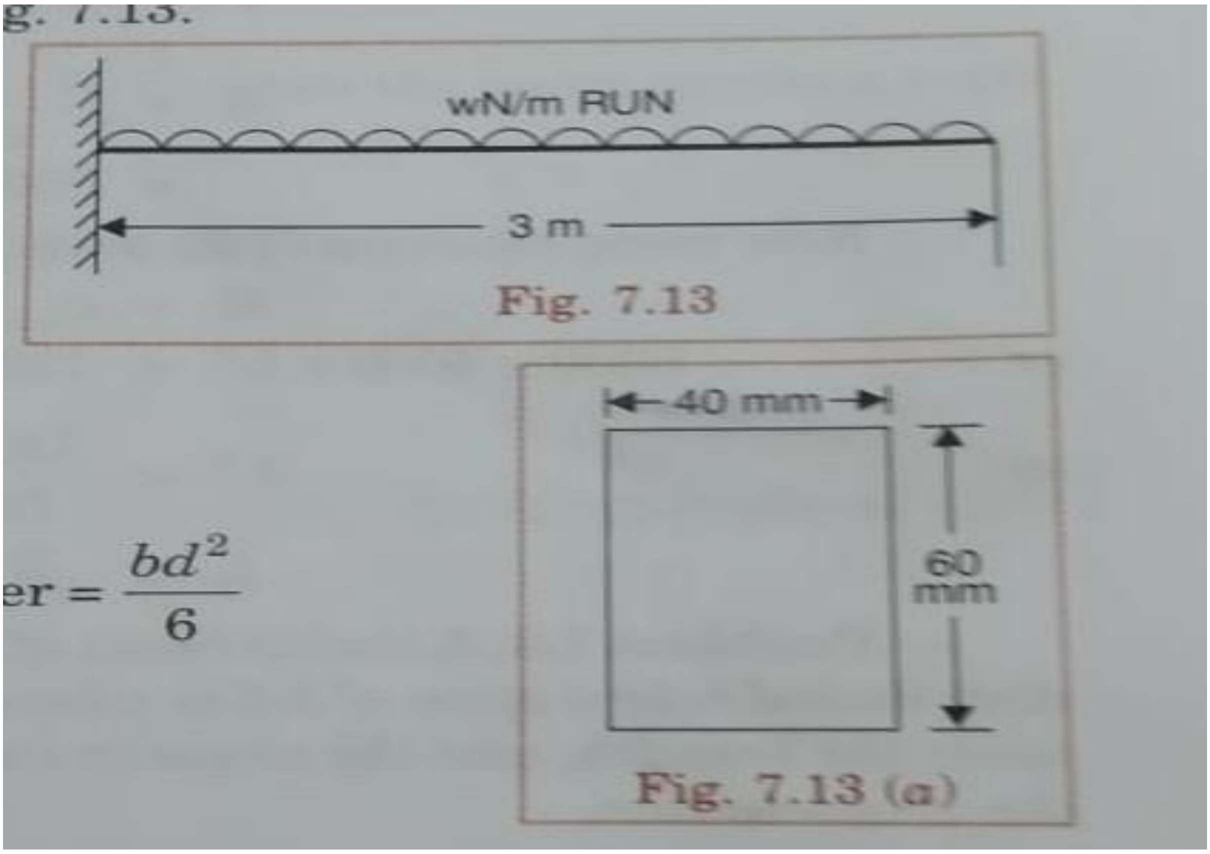


Figure No.4

Max. BM for a simply supported beam carrying a point load at the centre is given by,

$M = W \times L/4 = (400 \times 2 \times 10^3)/4 = 200000$ N.mm

$M = \sigma_{max} \times Z$ **$Z = bd^2/6 = 20 \times 20^2/6$**

$= 4000/3$ mm^3.

$200000 = \sigma_{max} \times 4000/3$

$\sigma_{max} = (200000 \times 3)/4000 = 150$ N/ mm^2.

Now let us consider the cantilever. [b – 40 mm, d – 60 mm, L – 3 m]

Let w – UDL per m run.

Maximum stress will be same as in case of simply supported beam.

$\sigma_{max} = 150$ N/ mm^2.

$Z = bd^2/6 = 40 \times 60^2/6 = 24000$ mm^3.

Maximum BM for a cantilever, $M = w \times L/2 = w \times 3/2$

$= 4.5w$ N.m $= 4.5 \times 1000w$ N.mm

$M = \sigma_{max} \times Z$

$4.5 \times 1000w = 150 \times 24000$

$w = 150 \times 24000 / (4.5 \times 1000)$

$w = 800$ N/m

7. A beam is simply supported and carries a uniformly distributed load of 40 KN/m run over the whole span. The section of the beam is rectangular having depth as 500 mm. If the maximum stress in the material of the beam is 120 N/mm^{2}and moment of inertia of the section is 7×10^8 mm^4. Find the span of the beam.

Solution:

Max. BM for a simply supported beam carrying a UDL over the span is at the centre of the beam is given by,

$M = w \times L^2/8 = (40000 \times L^2)/8 = 5000\ L^2$ N.m $= 5000\ L^2 \times 1000$ N.mm

$M = \sigma_{max} \times Z$ **$y_{max} = d/2 = 500/2 = 250$ mm.**

$Z = I / y_{max.} = 7 \times 10^8 /250 = 28 \times 10^5$ mm^3.

$5000\ L^2 \times 1000 = 120 \times 28 \times 10^5$

$L^2 = 120 \times 28 \times 10^5 / (5000 \times 1000)$

$L = 8.197$ m $= 8.2$ m.

8. A timber beam of rectangular section is to support a load of 20 KN uniformly distributed over a span of 3.6 m when beam is simply supported. If the depth of the section is to be twice the breadth and the stress in the timber is not to exceed 7 N/mm^2. Find the dimensions of the cross section. How would you modify the cross section of the beam, if it carries a concentrated load of 20 KN placed at the centre with the same ratio of breadth to depth?

Solution:

Maximum bending moment, when the simply supported beam carries a UDL over the entire span, is at the centre of the beam is given by,

$M = w \times L/8 = W \times L/8 = 20000 \times 3.6 / 8 = 9000$ N.m $= 9000 \times 1000$ N.mm

$M = \sigma_{max} \times Z$ **$Z = bd^2/6 = b \times (2b)^2/6 = 2b^3/3$ mm^3**

$9000 \times 1000 = 7 \times 2b^3/3$

$b^3 = 9000 \times 1000 \times 3/ (7 \times 2)$

$b = 124.47$ mm $= 124.5$ mm

$d = 2b = 2 \times 124.5 = 249$ mm.

Dimension of the section when the beam carries a point load at the centre.

B.M for simply supported beam carries a point load at the centre will be maximum and is given by,

M = W X L/4 = 20000 X 3.6/4 = 18000 X 1000 N.mm

$M = \sigma_{max} \times Z$

18000 X 1000 = 7 X 2b^3/3

b^3 = 18000 X 1000 X 3 / (7 X 2)

b = 156.82 mm

d = 2b = 2 X 156.82 = 313.64 mm.

9. A rolled steel joist of I section has the dimensions a shown in fig. This beam of I section carries a UDL of 40 KN/m run on a span of 10 m, calculate the maximum stress produced due to bending.

Solution:

Moment of inertia about the neutral axis,

$$= \frac{200 \times 400^3}{12} - \frac{(200-10) \times 360^3}{12}$$

= 327946666 mm^4.

Maximum B.M is given by,

M = w X L^2/8 = 40000 X 10^2/8 = 500000 N.m = 500000 X 1000 N.mm

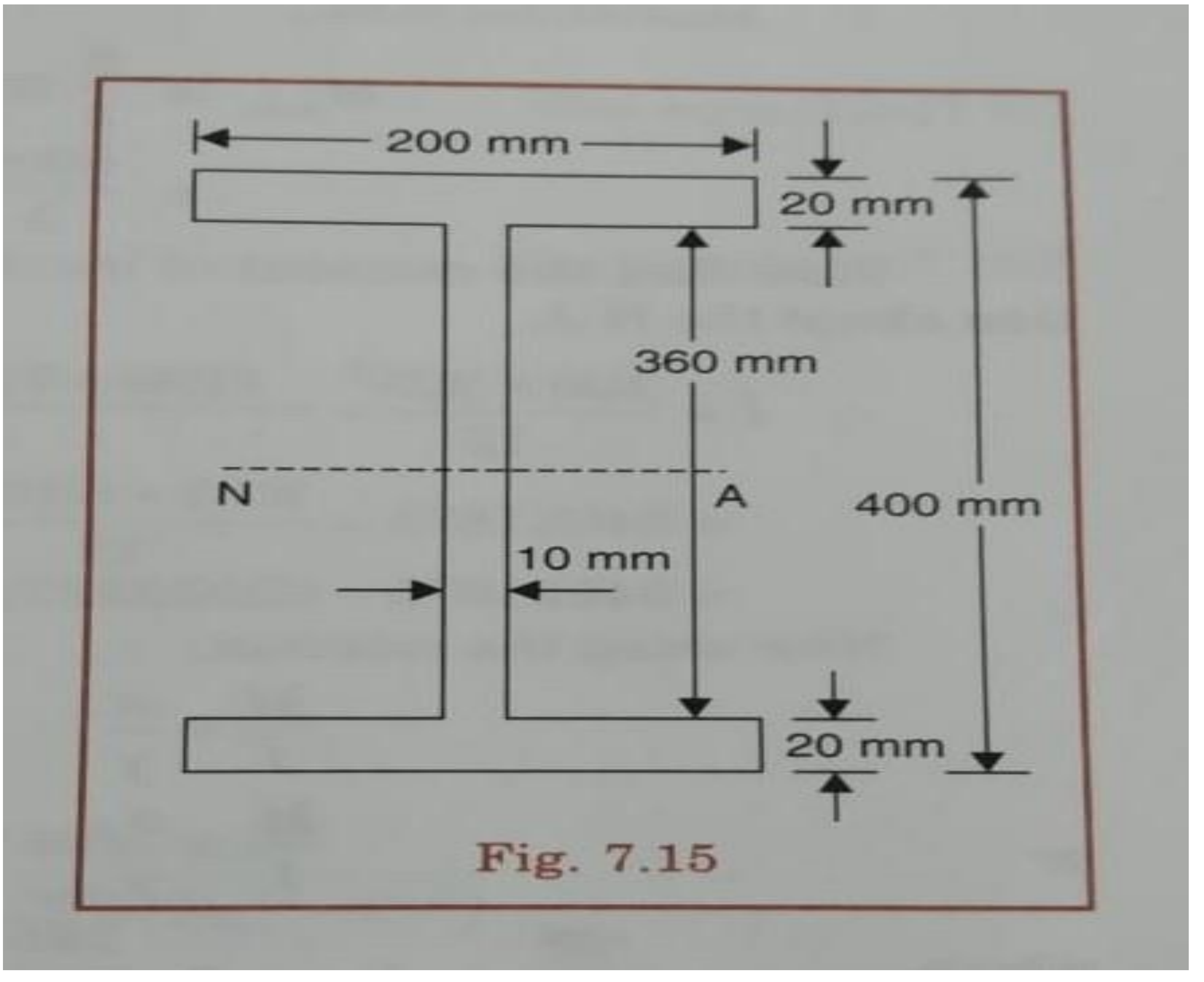


Fig. 7.15

Figure No.5

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$\sigma_{max} = \frac{500000 \times 1000 \times 200}{327946666}$$

$$\sigma_{max} = 304.92 \text{ N/mm}^2$$

10. Two circular beams where one is solid of diameter D and other is a hollow of outer diameter D_o and inner diameter D_i are of same length, same material and of same weight. Find the ratio of section modulus of these circular beams.

Solution:

Let, L – Length of each beam.

W – weight of each beam.

ρ – density of the material of each beam.

Now weight of solid beam = $\rho \times g \times$ Volume = $\rho \times g \times$ Area of cross section $\times L$

$$= \rho \times g \times (\pi/4) \times D^2 \times L$$

weight of hollow beam = $\rho \times g \times$ Volume = $\rho \times g \times$ Area of cross section $\times L$

$$= \rho \times g \times (\pi/4) \times (D_o^2 - D_i^2) \times L$$

But the weights are same.

$$\rho \times g \times (\pi/4) \times D^2 \times L = \rho \times g \times (\pi/4) \times (D_o^2 - D_i^2) \times L$$

$$D^2 = D_o^2 - D_i^2 \quad \text{OR} \quad D_i^2 = D_o^2 - D^2$$

Now section modulus of solid beam, $Z = \pi \times D^3/32$

section modulus of hollow beam, $Z = \pi/32\, D_o \times (D_o^4 - D_i^4)$

$$= \pi/32\, D_o \times (D_o^2 + D_i^2)(D_o^2 - D_i^2)$$

$$\frac{\text{Section modulus of solid beam}}{\text{Section modulus of hollow beam}} = \frac{\pi \times D^3/32}{\pi/32\, D_o \times (D_o^2 + D_i^2)(D_o^2 - D_i^2)}$$

$$\frac{D^3 \times D_o}{(D_o^2 + D_i^2)(D_o^2 - D_i^2)} = \frac{D \times D^2 \times D_o}{(D_o^2 + D_i^2)(D_o^2 - D_i^2)}$$

$$\frac{D \times Do \times (D_o^2 - D_i^2)}{(D_o^2 + D_i^2)(D_o^2 - D_i^2)} = \frac{D \times Do}{(D_o^2 + D_i^2)}$$

$$\frac{\text{Section modulus of solid beam}}{\text{Section modulus of hollow beam}} = \frac{D \times Do}{D_o^2 + D_o^2 - D^2} = \frac{D \times Do}{(2 D_o^2 - D^2)}$$

$$\frac{\text{Section modulus of hollow beam}}{\text{Section modulus of solid beam}} = \frac{2 D_o^2 - D^2}{D \times D_o}$$

$$\frac{2 D_o^2}{D \times D_o} - \frac{D^2}{D \times D_o} = \frac{2 D_o}{D} - \frac{D}{D_o}$$

SHEAR STRESS IN BEAMS

- **The shearing stress in beam is defined as the stress that occurs due to the internal shearing of the beam that results from shear force subjected to the beam.**
- **It is denoted by the symbol τ and is expressed in the unit of psi or N/mm^2.**
- **τ = F X (A y/I.b)**

F – shear force at a section

A – Area of the section

Y – Distance of the C.G of the area A from the neutral axis.

I – Moment of inertia of the section about the neutral axis.

b – Width of the beam.

11. A wooden beam 100 mm wide and 150 mm deep is simply supported over a span of 4 m. If shear force at a section of the beam is 4500 N, find the shear stress at a distance of 25 mm above the N.A.

Solution:

b – 100 mm

d – 150 mm

F – 4500 N

Let τ - shear stress at a distance of 25 mm above the N.A.

τ = F X (A y/I.b)

A = Area of the beam above y_1 (Shaded area)

= 100 X 50 = 5000 mm^2.

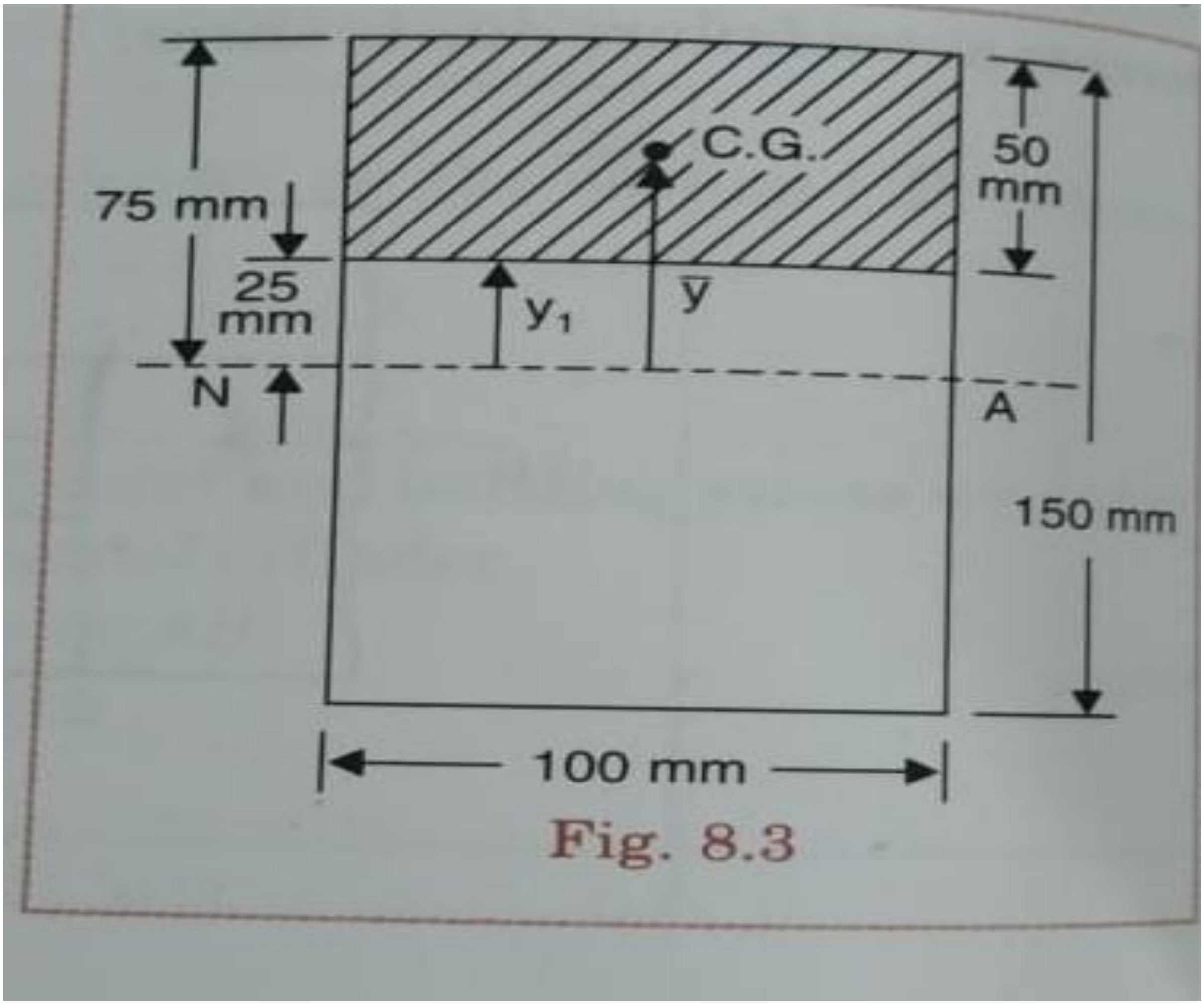


Fig. 8.3

Figure No.6

y = Distance of the C.G of the area A from the neutral axis.

= 25 + 50/2 = 50 mm.

I = M.O.I of the total section.

= bd/12 = 100 X 150 /12 = 28125000 mm^4.

b – Actual width of section at a distance y_1 from N.A. = 100 mm.

$$\tau = F \times (A\,y/I.b) = \frac{4500 \times 5000 \times 50}{28125000 \times 100} = 0.4\ N/mm^2$$

SHEAR STRESS DISTRIBUTION FOR DIFFERENT SECTIONS

RECTANGULAR SECTION

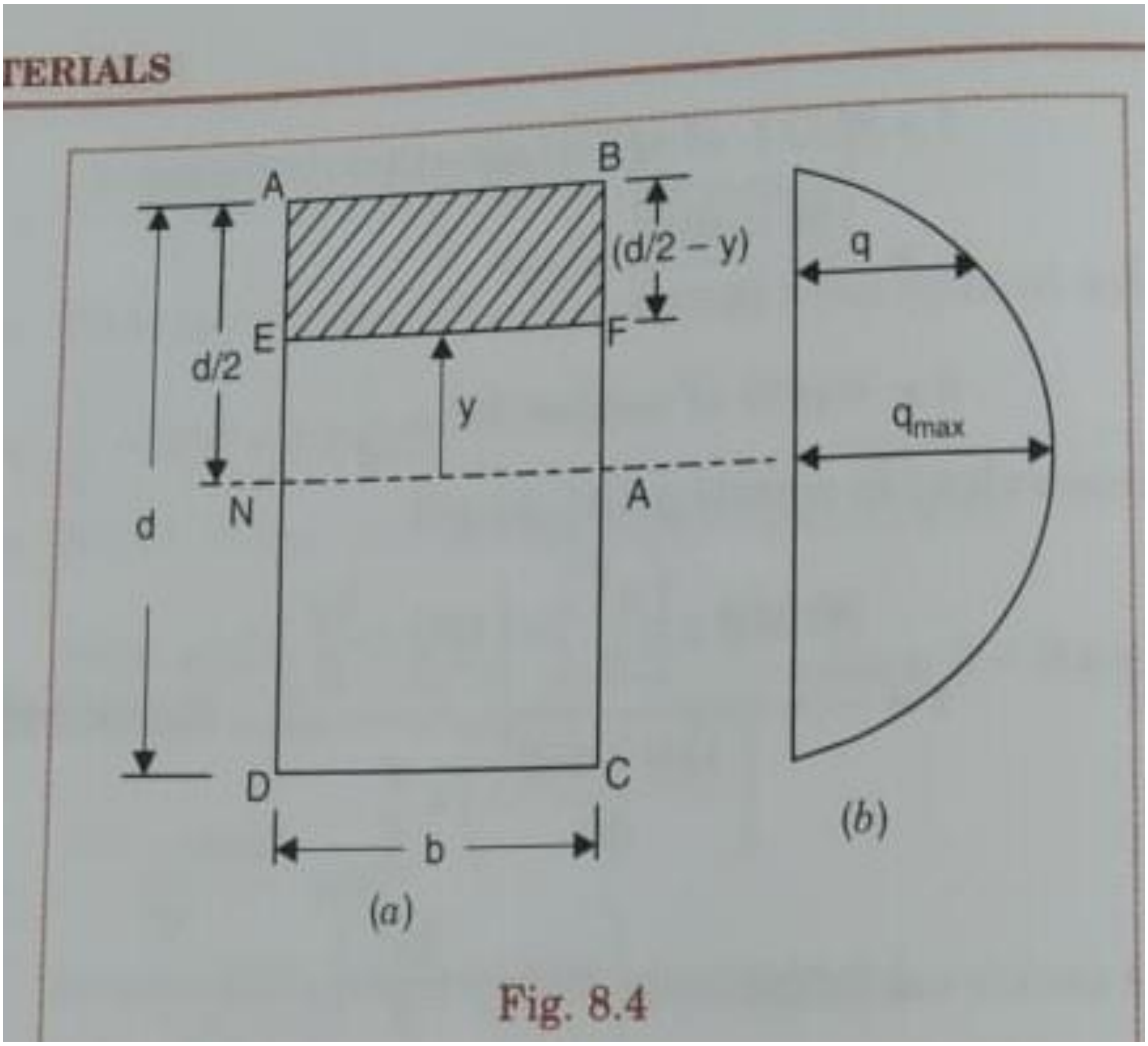


Figure No.7

τ **= F/2I X ($d^2/4 - y^2$)**

12. A rectangular beam 100 mm wide and 250 mm deep is subjected to a maximum shear force of 50 KN. Determine the Average shear stress, Maximum shear stress and Shear stress at a distance of 25 mm above the neutral axis.

Solution:

Average shear stress,

τ_{max} **= F/Area = 50000/ (b X d) = 50000/ (100 X 250)**

= 2 N/mm²

Maximum shear stress,

$\tau_{max} = 1.5 \times \tau_{avg} = 1.5 \times 2 = 3\ N/mm^2$

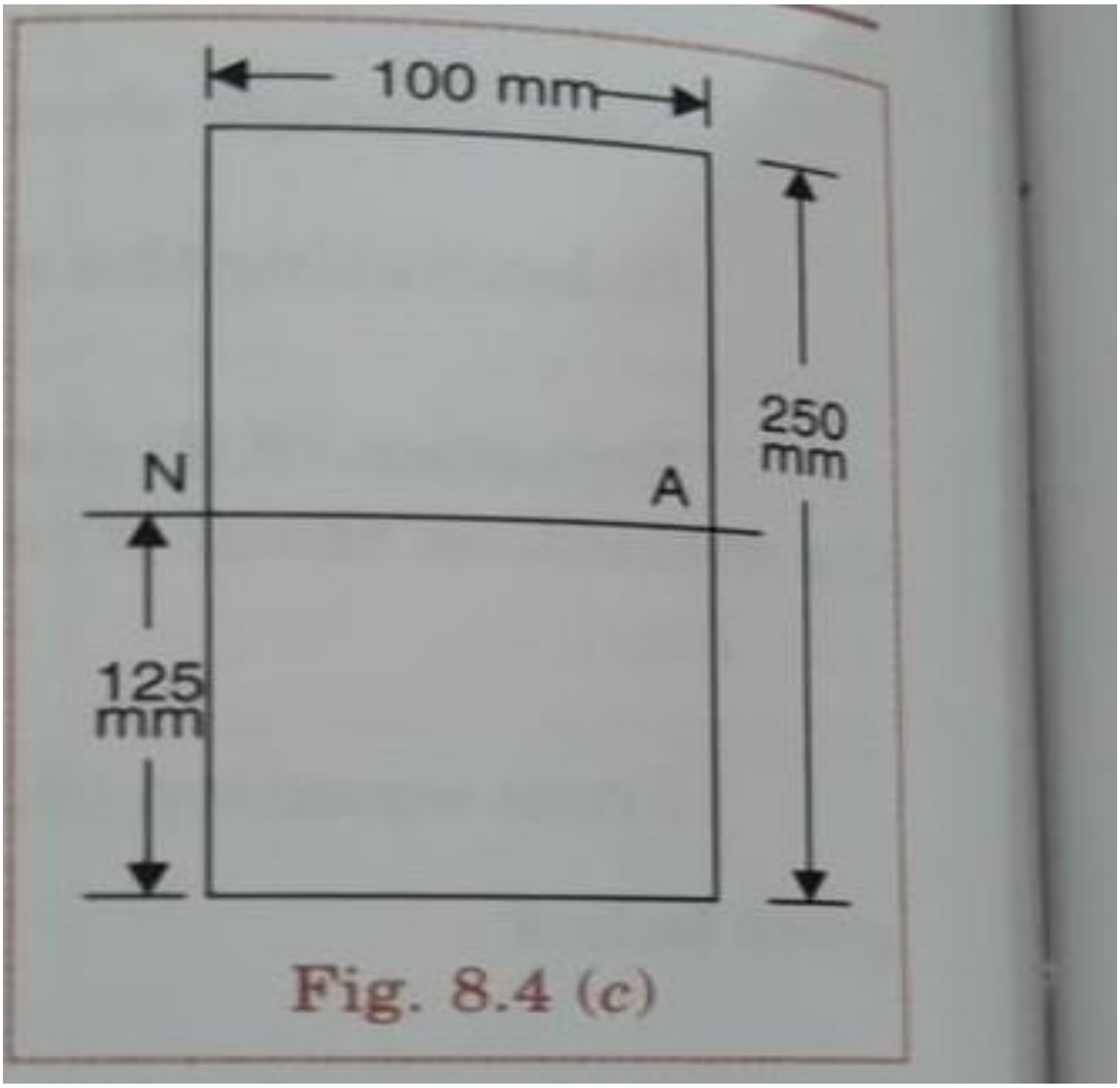


Fig. 8.4 (c)

Figure No.8

3. Shear stress at a distance of 25 mm above the neutral axis,

$$T = F/2I \times (d^2/4 - y^2)$$

$$= 50000/2I\ [\,250^2/4 - 25^2\,]$$

$$= \frac{50000}{2 \times bd^3/12} \times [62500/4 - 625]$$

$$= 2.88\ \text{N/mm}^2$$

13. A timber beam of rectangular section is simply supported at the ends and carries a point load at the centre of the beam. The maximum bending stress is 12 N/mm^2 and maximum shearing stress is 1 N/mm^2. Find the ratio of the span to the depth.

Solution:

Let, b – Width of the beam

d – Depth of the beam

L – Span of the beam

W – Point load at the centre

Maximum shear force, F = W/2

Maximum bending moment, M = W X L/4

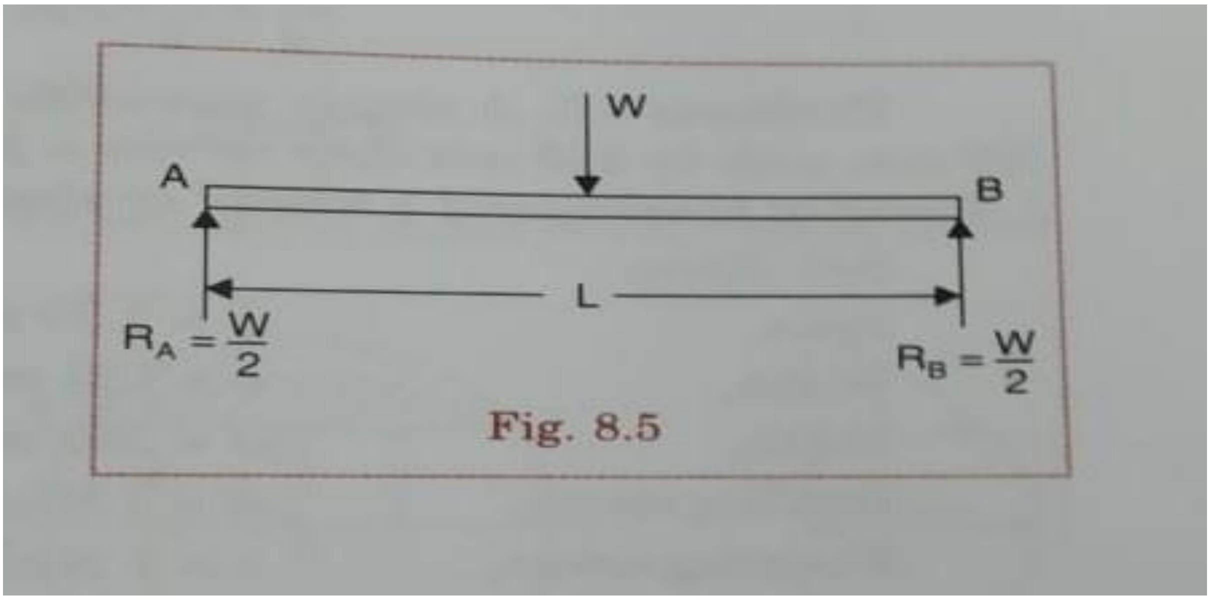


Figure No.9

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area}} = \frac{W/2}{b \times d} = \frac{W}{2bd}$$

$$\tau_{max} = 1.5 \times \tau_{avg} = 1.5 \times \frac{W}{2bd}$$

$$1 = 1.5 \times \frac{W}{2bd}$$

$$\frac{2}{1.5} = \frac{W}{bd}$$

Using Bending Equation,

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$\sigma_{max} = \frac{(W \times L/4) \times (d/2)}{bd^3/12}$$

$$= \frac{12 \times W.L.d}{8 \quad bd^3} = \frac{1.5\ W.\ L}{bd^2}$$

$$12 = \frac{1.5\ W.\ L}{bd^2}$$

$$12 = 1.5 \times \frac{W}{bd} \quad \frac{L}{d}$$

$$12 = 1.5 \times \frac{2}{1.5} \times \frac{L}{d}$$

$$12/2 = L/d = 6$$

CIRCULAR SECTION:

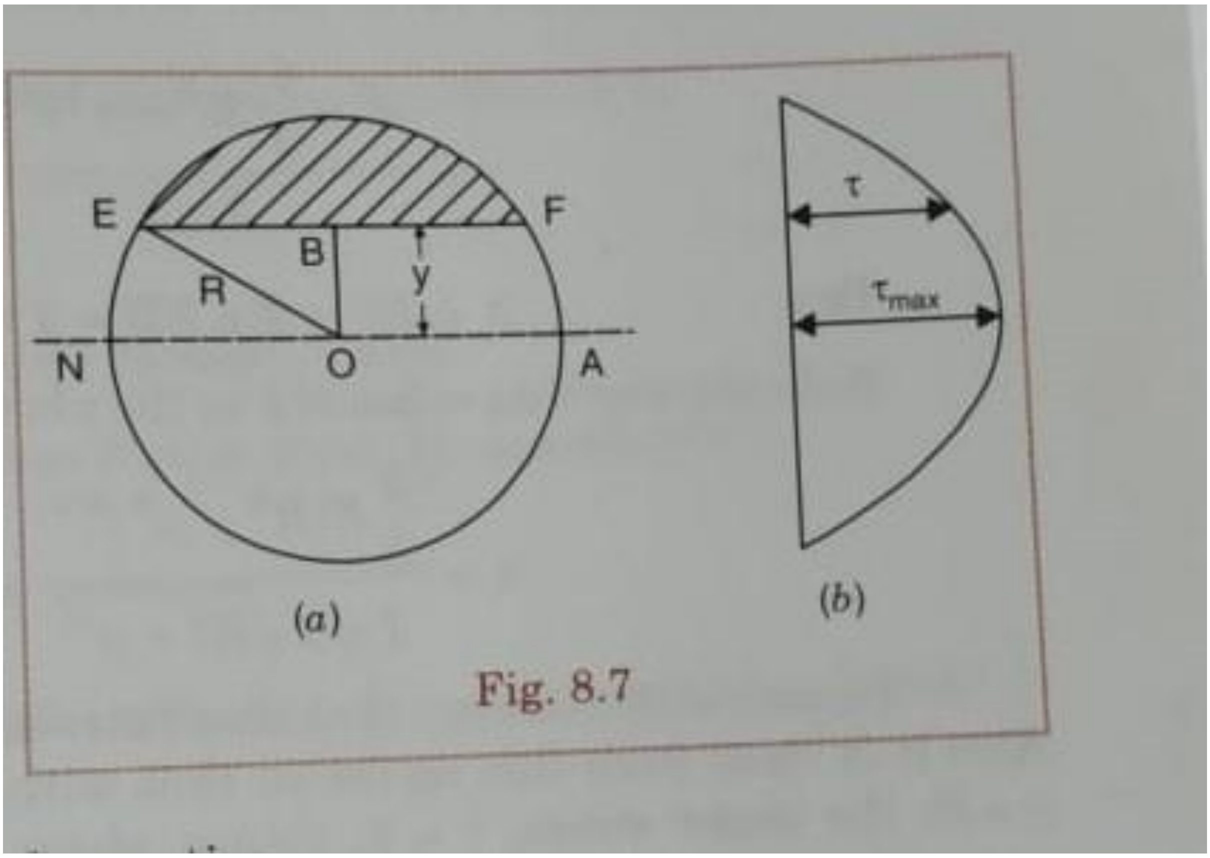


Fig. 8.7

Figure No.10

$$\tau = \frac{F\,(R^2 - y^2)}{3I}$$

$\tau_{avg.}$ = Shear Force / Area of circular section = $F/\pi R^2$

$\tau_{max.}$ = (4/3) X $\tau_{avg.}$ = (4/3) X ($F/\pi R^2$)

14. A circular beam of 100 mm diameter is subjected to a shear force of 5 KN. Calculate Average shear stress, Maximum shear stress and shear stress at a distance of 40 mm from N.A.

Solution:

Average shear stress,

$\tau_{avg.}$ = Shear Force / Area of circular section = $F/\pi R^2$

$$= 5000/ \pi \times 50^2$$

$$= 0.6366 \text{ N/mm}^2$$

Maximum shear stress,

$$\tau_{max.} = (4/3) \times \tau_{avg.} = (4/3) \times 0.6366 = 0.8488 \text{ N/mm}^2$$

Maximum shear stress and shear stress at a distance of 40 mm from N.A.,

$$\tau = \frac{F (R^2 - y^2)}{3I}$$

$$= \frac{5000}{3 \times (\pi d^4/64)} (50^2 - 40^2)$$

$$= \frac{5000}{3 \times (\pi \times 100^4/64)} (50^2 - 40^2)$$

$$\tau = 0.3055 \text{ N/mm}^2$$

I - SECTION

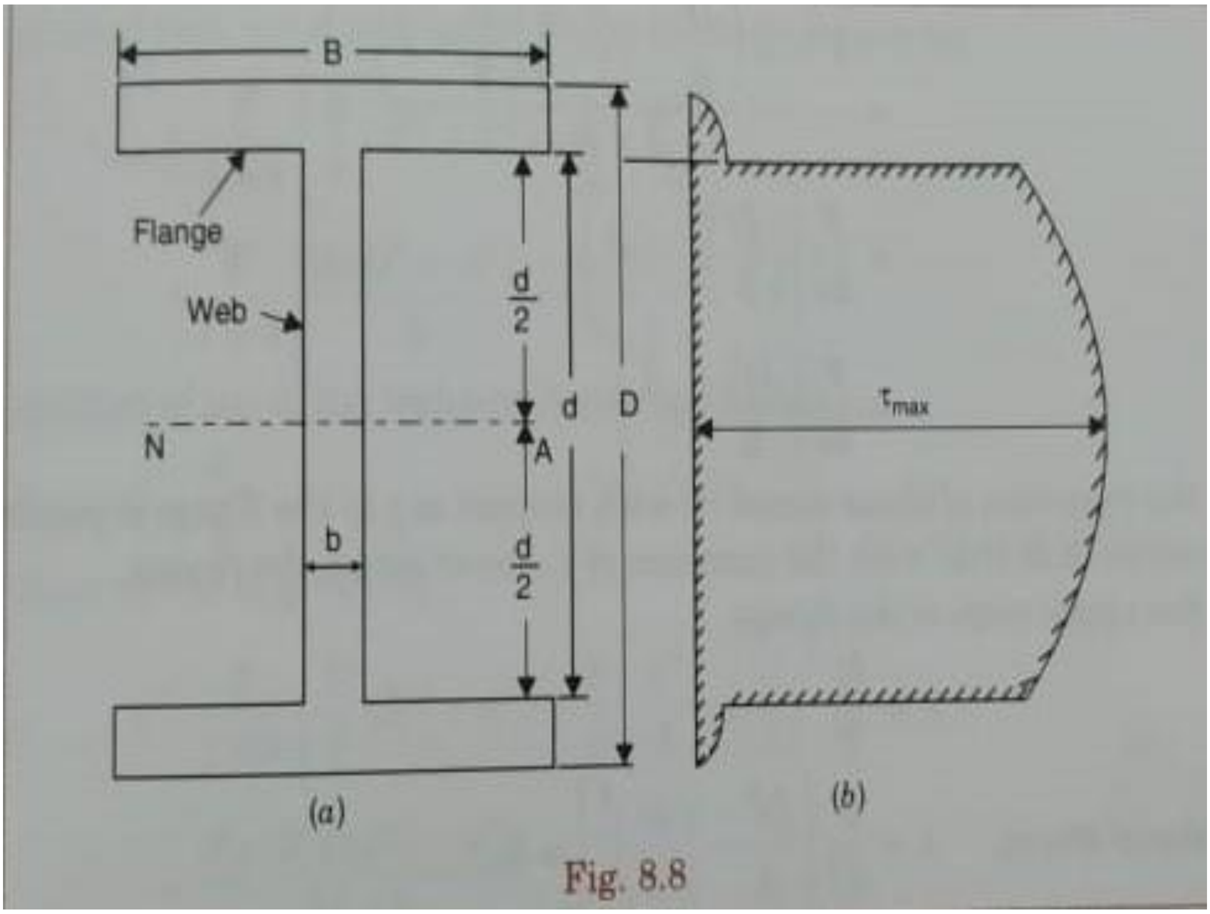


Figure No.11

$$\tau_{max.} = \frac{F}{I \times b}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

15. An I-section beam 350 mm X 150 mm has a web thickness of 10 mm and a flange thickness of 20 mm. If the shear force acting on the section is 40 KN, find the maximum shear stress developed in the I-section.

Given Data:

Overall Depth, D = 350 mm

Overall Width, B = 150 mm

Web Thickness = 10 mm

Flange Thickness = 20 mm

Depth of Web = 350 – (2 X 20) = 310 mm.

Shear Force, F = 40 KN = 40000 N.

To Find:

Maximum Shear stress, τ_{max}

Solution:

$$I = \frac{150 \times 350^3}{12} - \frac{140 \times 310^3}{12}$$

$$= 188375833.4 \text{ mm}^4.$$

$$\tau_{max.} = \frac{F}{I \times b}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

$$= \frac{40000}{188375833.4 \times 10}\left[\frac{150(350^2 - 310^2)}{8} + \frac{10 \times 310^2}{8}\right]$$

$$= 13.06 \text{ N/mm}^2$$

TORSION

- **A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft.**
- **The torque is equal to the product of the force applied and radius of the shaft.**
- **Due to application of the torques at the two ends, the shaft is subjected to a twisting moment.**
- **This causes the shear stresses and shear strains in the material of the shaft.**

TORSION EQUATION

General Torsion Equation (Shafts of circular cross-section)

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\,\theta}{L}$$

1. For Solid Shaft

$$J = \frac{\pi}{2} r^4 = \frac{\pi d^4}{32}$$

2. For Hollow Shaft

$$J = \frac{\pi}{2}(r_1^4 - r_2^4)$$

$$= \frac{\pi}{32}(d_1^4 - d_2^4)$$

T = torque or twisting moment in newton metres
J = polar second moment of area of cross-section about shaft axis.
τ = shear stress at outer fibres in pascals
r = radius of shaft in metres
G = modulus of rigidity in pascals
θ = angle of twist in radians
L = length of shaft in metres
d = diameter of shaft in metres

SHEAR STRESS IN SHAFT

SHEAR STRESS IN SHAFT:(τ)

- When a shaft is subjected to equals and opposite end couples, whose axes coincide with the axis of the shaft, the shaft is said to be in pure torsion and at any point in the section of the shaft stress will be induced.
- That stress is called shear stress in shaft.

ASSUMPTIONS MADE IN THE THEORY OF PURE TORSION

ASSUMPTION IN THE THEORY OF TORSION:

- The following assumptions are made while finding out shear stress in a circular shaft subjected to torsion.

1) The material of shaft is uniform throughout the length.
2) The twist along the shaft is uniform.
3) The shaft is of uniform circular section throughout the length.
4) Cross section of the shaft, which are plane before twist remain plain after twist.
5) All radii which are straight before twist remain straight after twist.

MAXIMUM TORQUE TRANSMITTED BY SOLID CIRCULAR SHAFT

STRENGTH OF SHAFTS

Maximum torque or power the shaft can transmit from one pulley to another, is called strength of shaft.

(a) For solid circular shafts:

Maximum torque (T)is given by :

$$T = \frac{\pi}{16} \times \tau \times D^3$$

where, D = dia. of the shaft

τ=shear stress in the shaft

MAXIMUM TORQUE TRANSMITTED BY HOLLOW CIRCULAR SHAFT

$$T = \tau \text{ x } \frac{\Pi}{16} \text{ x } \left[\frac{Do^4 - Di^4}{Do} \right]$$

Where,

T – Torque transmitted by hollow circular shaft N.mm.

τ - Shear stress in N/mm^2.

D_o – outer diameter in mm.

D_i – Inner diameter in mm.

POWER TRANSMITTED BY SHAFTS

Power, $P = 2\pi N T^*/60$

Where, N – Speed of the shaft in r.p.m.

T^* - Mean Torque in N.m.

POLAR MOMENT OF INERTIA FOR SOLID CIRCULAR SHAFT

- The moment of inertia of a plane area, with respect to an axis perpendicular to the plane of the figure is called polar moment of inertia.
- As per the perpendicular axis theorem.

$$I_{ZZ} = I_{XX} + I_{YY} = J$$

$$= \frac{\pi}{64} \times D^4 + \frac{\pi}{64} \times D^4$$

$$J = \frac{\pi}{32} \times D^4$$

POLAR MOMENT OF INERTIA FOR HOLLOW CIRCULAR SHAFT

$$J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right)$$

POLAR MODULUS

Polar Modulus

Polar modulus is defined as ration of polar moment of inertia to the radius of the shaft.

$$Z_p = \frac{J}{R}$$

For solid shaft => $J = \frac{\pi}{32} D^4$

$$Z_p = \frac{\frac{\pi}{32} D^4}{D/2} = \frac{\pi}{16} D^3$$

For hollow shaft => $J = \frac{\pi}{32}\left[D_0^4 - D_i^4\right]$

$$Z_p = \frac{\frac{\pi}{32}\left[D_0^4 - D_i^4\right]}{D_0/2} = \frac{\pi}{16 D_0}\left[D_0^4 - D_i^4\right]$$

- **STRENGTH OF A SHAFT:**
  - **The strength of a shaft means the maximum torque or maximum power the shaft can transmit.**
- **TORSIONAL RIGIDITY:**
  - **It is defined as the product of modulus of rigidity and polar moment of inertia of the shaft.**
  - **Mathematically, Torsional Rigidity = C X J**
  - **It is also known as stiffness of the shaft.**
  - **Torsional Rigidity is also defined as the torque required to produce a twist of one radian per unit length of the shaft.**

16. A solid shaft of 150 mm diameter is used to transmit torque. Find the maximum torque transmitted by the shaft if the maximum shear stress induced to the shaft is 45 N/mm^2.

Solution:

$$T = \frac{\pi \times D^3 \times \tau}{16}$$

$$= \frac{\pi \times 150^3 \times 45}{16}$$

T = 29820586 N.mm or 29820.586 N.m

17. The shearing stress of a solid shaft is not to exceed 40 N/mm^2, when the torque transmitted is 20000 N.m. Determine the minimum diameter of the shaft.

Solution:

Let D – Minimum diameter of the shaft in mm.

$$T = \frac{\pi \times D^3 \times \tau}{16}$$

$$D^3 = \frac{T \times 16}{\pi \times \tau} = \frac{20000 \times 1000 \times 16}{\pi \times 40}$$

$$D = 136.2 \text{ mm.}$$

18. In a hollow circular shaft of outer and inner diameters of 20 cm and 10 cm respectively, the shear stress is not to exceed 40 N/mm^2. Find the maximum torque which the shaft can safely transmit.

Solution:

$$T = \frac{\pi \times \tau}{16}\left[\frac{D^4_o - D^4_i}{D_o}\right]$$

$$T = \frac{\pi \times 40}{16}\left[\frac{200^4 - 100^4}{200}\right]$$

T = 58904860 N.mm or 58904.860 N.m

19. A hollow shaft of external diameter 120 mm transmits 300 KW power at 200 r.p.m. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm^2.

Solution:

Let D_i - Internal diameter of the shaft.

$$P = 2\pi N T^*/60$$

$$300000 = (2\pi \times 200 \times T^*) / 60$$

$$T^* = \frac{300000 \times 60}{2\pi \times 200}$$

$$T^* = 14323.9 \text{ N.m}$$

$$T = 14323.9 \times 1000 \text{ N.mm}$$

$$T = \frac{\pi \times \tau \times (D_o^4 - D_i^4)}{16 \quad D_o}$$

$$14323.9 \times 1000 = \frac{\pi \times 60 \times (120^4 - D_i^4)}{16 \quad 120}$$

$D_i^4 = 61458000$

$D_i = 88.5$ mm.

Problem 16.4. *Two shafts of the same material and of same lengths are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the shafts.*

Sol. Given :

Two shafts of the same material and same lengths (one is solid and other is hollow) transmit the same torque and develops the same maximum stress.

Let T = Torque transmitted by each shaft

τ = Max. shear stress developed in each shaft

D = Outer diameter of the solid shaft

D_0 = Outer diameter of the hollow shaft

D_i = Inner diameter of the hollow shaft = $\frac{2}{3} D_0$

W_s = Weight of the solid shaft

W_h = Weight of the hollow shaft

L = Length of each shaft

w = Weight density of the material of each shaft.

Torque transmitted by the solid shaft is given by equation (16.4)

$$T = \frac{\pi}{16} \tau D^3 \quad \ldots(i)$$

Torque transmitted by the hollow shaft is given by equation (16.6),

$$T = \frac{\pi}{16} \tau \left[\frac{D_0^4 - D_i^4}{D_0}\right] = \frac{\pi}{16} \tau \left[\frac{D_0^4 - (2/3\, D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16} \tau \left[\frac{D_0^4 - \frac{16}{81} D_0^4}{D_0}\right] = \frac{\pi}{16} \tau \times \frac{65\, D_0^4}{81 \times D_0}$$

$$= \frac{\pi}{16} \tau \times \frac{65\, D_0^3}{81} \quad \ldots(ii)$$

As torque transmitted by solid and hollow shafts are equal, hence equating equations (i) and (ii),

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \times \frac{65}{81} D_0^3$$

Cancelling $\frac{\pi}{16} \tau$ from both sides

or

$$D^3 = \frac{65}{81} D_0^3$$

$$\therefore \quad D = \left[\frac{65}{81} D_0^3\right]^{1/3} = \left(\frac{65}{81}\right)^{1/3} D_0 = 0.929\, D_0 \quad \ldots(iii)$$

Now weight of solid shaft, W_s = Weight density × Volume of solid shaft

$= w \times$ Area of cross-section × Length

$$= w \times \frac{\pi}{4} D^2 \times L \quad \ldots(iv)$$

Weight of hollow shaft,

$W_h = w \times$ Area of cross-section of hollow shaft × Length

$$= w \times \frac{\pi}{4} [D_0^2 - D_i^2] \times L = w \times \frac{\pi}{4} [D_0^2 - (2/3\, D_0)^2] \times L$$

$$= w \times \frac{\pi}{4}\left[D_0^2 - \frac{4}{9} D_0^2\right] \times L = w \times \frac{\pi}{4} \times \frac{5}{9} D_0^2 \times L \quad \ldots(v)$$

Dividing equation (iv) by equation (v),

$$\frac{W_s}{W_h} = \frac{w \times \frac{\pi}{4} D^2 \times L}{w \times \frac{\pi}{4} \times \frac{5}{9} D_0^2 \times L} = \frac{9D^2}{5D_0^2}$$

$$= \frac{9}{5} \times \frac{(0.929 D_0)^2}{D_0^2} \quad [\because \; D = 0.929\, D_0 \text{ from equation (iii)}]$$

$$= \frac{9}{5} \times 0.929^2 \times \frac{D_0^2}{D_0^2} = \frac{1.55}{1}$$

$$\therefore \quad \frac{\text{Weight of solid shaft}}{\text{Weight of hollow shaft}} = \frac{\mathbf{1.55}}{\mathbf{1}}. \quad \textbf{Ans.}$$

Problem 16.5. *A solid circular shaft and a hollow circular shaft whose inside diameter is $\left(\frac{3}{4}\right)$ of the outside diameter, are of the same material, of equal lengths and are required to transmit a given torque. Compare the weights of these two shafts if the maximum shear stress developed in the two shafts are equal.*

Sol. Given :

Dia. of hollow shaft, $D_i = \frac{3}{4}$ Dia. at outside

$= \frac{3}{4} D_0 = 0.75\, D_0$

Let L = Length of both shaft (equal length),

T = Torque transmitted by each shaft (equal torque),

τ = Maximum shear stress developed in each shaft (equal max. shear stress),

D = Dia. of solid shaft,

W_s = Weight of solid shaft, and

W_h = Weight of hollow shaft.

Torque transmitted by a solid shaft is given by equation (16.4) as

$$T = \frac{\pi}{16} \times \tau \times D^3 \quad \ldots(i)$$

Torque transmitted by a hollow shaft is given by equation (16.6) as

$$T = \frac{\pi}{16} \times \tau \times \left[\frac{D_0^4 - D_i^4}{D_0}\right] = \frac{\pi}{16} \times \tau \times \left[\frac{D_0^4 - (0.75\, D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16} \times \tau \times \frac{D_0^4}{D_0}\, [1 - (0.75)^4] = \frac{\pi}{16} \times \tau \times D_0^3 \times [1 - 0.3164]$$

$$= \frac{\pi}{16} \times \tau \times D_0^3 \times 0.6836 \quad \ldots(ii)$$

But torque transmitted by solid shaft

= Torque transmitted by hollow shaft.

Hence equating equations (*i*) and (*ii*), we get

$$\frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times \tau \times D_0^3 \times 0.6836 \text{ or } D^3 = 0.6836\, D_0^3$$

$$\therefore \quad D = (0.6836)^{1/3} \times D_0 \quad \text{or} \quad D = 0.8809\, D_0 \quad \ldots(iii)$$

Now weight of solid shaft,

$$W_s = \rho \times g \times \text{Volume of solid shaft}$$

$$= \rho \times g \times \left(\frac{\pi}{4} D^2 \times L\right) \quad \ldots(iv)$$

Weight of hollow shaft,

$$W_h = \rho \times g \times \text{Volume of hollow shaft}$$

$$= \rho \times g \times \left[\frac{\pi}{4}(D_0^2 - D_i^2) \times L\right] = \rho \times g \times \frac{\pi}{4}\, [D_0^2 - (0.75 D_0)^2] \times L$$

$$= \rho \times g \times \frac{\pi}{4}\, [D_0^2 - 0.5625 D_0^2] \times L = \rho \times g \times \frac{\pi}{4} \times D_0^2 (1 - 0.5625) \times L$$

$$= \rho \times g \times \frac{\pi}{4} \times D_0^2 \times 0.4375 \times L \quad \ldots(v)$$

Dividing equation (*iv*) by equation (*v*),

$$\frac{W_s}{W_h} = \frac{\rho \times g \times \frac{\pi}{4} \times D^2 \times L}{\rho \times g \times \frac{\pi}{4} \times D_0^2 \times 0.4375 \times L} = \frac{D^2}{0.4375\, D_0^2}$$

$$= \frac{(0.8809\, D_0)^2}{0.4375\, D_0^2} \quad [\because \text{ From } (iii),\ D = 0.8809\, D_0]$$

$$= \frac{0.776\, D_0^2}{0.4375\, D_0^2} = 1.7737. \quad \textbf{Ans.}$$

Problem 16.6. *A hollow circular shaft 20 mm thick transmits 300 kW power at 200 r.p.m. Determine the external diameter of the shaft if the shear strain due to torsion is not to exceed 0.00086. Take modulus of rigidity = 0.8 × 10^5 N/mm².*

Sol. Given :

Thickness, $t = 20$ mm

Power transmitted, $P = 300$ kW $= 300{,}000$ W

Problem 16.8. *Find the maximum shear stress induced in a solid circular shaft of diameter 15 cm when the shaft transmits 150 kW power at 180 r.p.m.*

Sol. Given :

Diameter of shaft, $D = 15$ cm $= 150$ mm

Power transmitted, $P = 150$ kW $= 150 \times 10^3$ W

Speed of shaft, $N = 180$ r.p.m.

Let τ = Maximum shear stress induced in the shaft

Power transmitted is given by equation (16.7) as

$$P = \frac{2\pi NT}{60}$$

$$150 \times 10^3 = \frac{2\pi \times 180 \times T}{60}$$

$$\therefore \quad T = \frac{150 \times 10^3 \times 60}{2\pi \times 180} = 7957.7 \text{ Nm} = 7957700 \text{ Nmm}$$

Now using equation (16.4) as,

$$T = \frac{\pi}{16} \tau D^3$$

$$7957700 = \frac{\pi}{16} \times \tau \times 150^3$$

$$\therefore \quad \tau = \frac{16 \times 7957700}{\pi \times 150^3} = 12 \text{ N/mm}^2. \quad \textbf{Ans.}$$

Problem 16.9. *A solid cylindrical shaft is to transmit 300 kW power at 100 r.p.m.*

(a) If the shear stress is not to exceed 80 N/mm², find its diameter.

(b) What percent saving in weight would be obtained if this shaft is replaced by a hollow one whose internal diameter equals to 0.6 of the external diameter, the length, the material and maximum shear stress being the same ?

Sol. Given :

Power, $P = 300$ kW $= 300 \times 10^3$ W

Speed, $N = 100$

Max. shear stress, $\tau = 80$ N/mm²

(*a*) Let D = Dia. of solid shaft

Using equation (16.7),

$$P = \frac{2\pi NT}{60}$$

$$300 \times 10^3 = \frac{2\pi \times 100 \times T}{60}$$

$$\therefore \quad T = \frac{300 \times 10^3 \times 60}{2\pi \times 100} = 28647.8 \text{ N-m} = 28647800 \text{ N-mm}$$

Now using equation (16.4),

$$T = \frac{\pi}{16} \times \tau \times D^3 \quad \text{or} \quad 28647800 = \frac{\pi}{16} \times 80 \times D^3$$

$$\therefore \quad D = \left(\frac{16 \times 28647800}{\pi \times 80}\right)^{1/3} = 121.8 \text{ mm}$$

$$= \textbf{say 122.0 mm.} \quad \textbf{Ans.}$$

(b) *Percent saving in weight*

Let D_0 = External dia. of hollow shaft

D_i = Internal dia. of hollow shaft

$= 0.6 \times D_0$. (given)

The length, material and maximum shear stress in solid and hollow shafts are given the same. Hence torque transmitted by solid shaft is equal to the torque transmitted by hollow shaft. But the torque transmitted by hollow shaft is given by equation (16.6).

∴ Using equation (16.6),

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_0^4 - D_i^4)}{D_0}$$

$$= \frac{\pi}{16} \times 800 \times \frac{[D_0^4 - (0.6\,D_0)^4]}{D_0} \qquad (\because \quad D_i = 0.6\,D_0)$$

$$= \pi \times 50 \times \frac{[D_0^4 - (0.6\,D_0)^4]}{D_0}$$

But torque transmitted by solid shaft

$= 28647800$ N-mm

∴ Equating the two torques, we get

$$28647800 = \pi \times 50 \times \left(\frac{0.8704\,D_0^4}{D_0}\right) = \pi \times 50 \times 0.8704\,D_0^3$$

∴ $$D_0 = \left(\frac{28647800}{\pi \times 50 \times 0.8704}\right)^{1/3} = 127.6 \text{ mm} = \text{say } 128 \text{ mm}$$

∴ Internal dia. $D_i = 0.6 \times D_0 = 0.6 \times 128 = 76.8$ mm

Now let W_s = Weight of solid shaft,

and W_h = Weight of hollow shaft.

Then W_s = Weight density × Area of solid shaft × Length

$= w \times \frac{\pi}{4} D^2 \times L$ (where w = weight density)

Similarly W_h = Weight density × Area of hollow shaft × Length

$= w \times \frac{\pi}{4} [D_0^2 - D_i^2] \times L$

(∵ Both shafts are of same length and of same material)

Now percent saving in weight

$$= \frac{W_s - W_h}{W_s} \times 100$$

$$= \frac{w \times \frac{\pi}{4} D^2 \times L - w \times \frac{\pi}{4} [D_0^2 - D_i^2] \times L}{w \times \frac{\pi}{4} D^2 \times L} \times 100$$

$$= \frac{D^2 - (D_0^2 - D_i^2)}{D^2} \times 100 \qquad \left(\text{Cancelling } w \times \frac{\pi}{4} \times L\right)$$

$$= \frac{122^2 - (128^2 - 76.8^2)}{122^2} \times 100 = \frac{14884 - (16384 - 5898)}{14884} \times 100$$

$$= \frac{14884 - 10486}{14884} \times 100 = \mathbf{29.55\%.} \quad \textbf{Ans.}$$

Problem 16.10. *A solid steel shaft has to transmit 75 kW at 200 r.p.m. Taking allowable shear stress as 70 N/mm², find suitable diameter for the shaft, if the maximum torque transmitted at each revolution exceeds the mean by 30%.*

Sol. Given :

Power transmitted, $P = 75 \text{ kW} = 75 \times 10^3$ W

R.P.M. of the shaft, $N = 200$

Shear stress, $\tau = 70$ N/mm²

Let T = Mean torque transmitted

T_{max} = Maximum torque transmitted = $1.3\,T$

D = Suitable diameter of the shaft

Power is given by the relation,

$$= \frac{2\pi NT}{60}$$

or $$75 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

∴ $$T = \frac{75 \times 10^3 \times 60}{2\pi \times 200} = 3580.98 \text{ N-m} = 3580980 \text{ N-mm}$$

∴ $$T_{max} = 1.3\,T = 1.3 \times 3580980 = 4655274 \text{ N-mm}.$$

Maximum torque transmitted by a solid shaft is given by equation (16.4) as,

$$T_{max} = \frac{\pi}{16} \times \tau \times D^3$$

or $$4655274 = \frac{\pi}{16} \times 70 \times D^3$$

∴ $$D = \left(\frac{16 \times 4655274}{\pi \times 70}\right)^{1/3} = \mathbf{69.57 \text{ mm}} \simeq \mathbf{70 \text{ mm}.} \quad \textbf{Ans.}$$

Problem 16.11. *A hollow shaft is to transmit 300 kW power at 80 r.p.m. If the shear stress is not to exceed 60 N/mm² and the internal diameter is 0.6 of the external diameter, find the external and internal diameters assuming that the maximum torque is 1.4 times the mean.*

Sol. Given :

Power transmitted, $P = 300 \text{ kW} = 300 \times 10^3$ W

Speed of the shaft, $N = 80$ r.p.m.

Maximum shear stress, $\tau = 60$ N/mm²

Internal diameter, $D_i = 0.6 \times$ External diameter $= 0.6\,D_0$

Maximum torque, $T_{max} = 1.4$ times the mean torque $= 1.4 \times T$

Power is given by the relation,

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60 \times P}{2\pi N} = \frac{60 \times 300 \times 10^3}{2\pi \times 80} = 35809.8 \text{ N-m}$$

or

$$T_{max} = 1.4\, T = 1.4 \times 35809.8 \text{ N-m}$$

$\therefore$

$$= 50133.7 \text{ N-m} = 50133700 \text{ N-mm}.$$

Now maximum torque transmitted by a hollow shaft is given by equation (16.6) as,

$$T_{max} = \frac{\pi}{16} \times \tau \times \left[\frac{D_0^4 - D_i^4}{D_0}\right]$$

or

$$50133700 = \frac{\pi}{16} \times 60 \times \left[\frac{D_0^4 - (0.6D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16} \times 60 \left[\frac{D_0^4 - .1296D_0^4}{D_0}\right] = \frac{\pi}{16} \times 60 \times .8704\, D_0^3$$

$\therefore$

$$D_0 = \left(\frac{16 \times 50133700}{\pi \times 60 \times .8704}\right)^{1/3} = \mathbf{169.2 \simeq 170 \text{ mm.}} \quad \text{Ans.}$$

and

$$D_i = 0.6 \times D_0 = 0.6 \times 170 = \mathbf{102 \text{ mm.}} \quad \text{Ans.}$$

Problem 16.12. *A hollow shaft, having an inside diameter 60% of its outer diameter, is to replace a solid shaft transmitting the same power at the same speed. Calculate the percentage saving in material, if the material to be used is also the same.*

Sol. Given :

Let D_0 = Outer diameter of the hollow shaft

D_i = Inside diameter of the hollow shaft = 60% of D_0

$= \frac{60}{100} \times D_0 = 0.6\, D_0$

D = Diameter of the solid shaft

P = Power transmitted by hollow shaft or by solid shaft

N = Speed of each shaft

τ = Maximum shear stress induced in each shaft. Since material of both shafts is same and hence shear stress will be same.

Power by solid shaft or hollow shaft is given by

$$P = \frac{2\pi NT}{60}$$

$\therefore$

$$T = \frac{60 \times P}{2\pi N} = \text{constant}$$

($\because$ P and N are same for solid and hollow shafts)

$\therefore$ Torque transmitted by solid shaft is the same as the torque transmitted by hollow shaft.

Torque transmitted by solid shaft is given by equation (16.4) as

$$T = \frac{\pi}{16}\, \tau D^3 \qquad \ldots(i)$$

Torque transmitted by hollow shaft is given by equation (16.6) as

$$T = \frac{\pi}{16}\, \tau \left[\frac{D_0^4 - D_i^4}{D_0}\right] = \frac{\pi}{16}\, \tau \left[\frac{D_0^4 - (.6D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16}\,\tau\left[\frac{D_0^{\,4} - 0.1296\,D_0^{\,4}}{D_0}\right] = \frac{\pi}{16}\,\tau \times 0.8704\,D_0^{\,3} \quad ...(ii)$$

Since torque transmitted is the same and hence equating equations (*i*) and (*ii*)

$$\frac{\pi}{16}\,\tau D^3 = \frac{\pi}{16}\,\tau \times 0.8704\,D_0^{\,3}.$$

$$\therefore \quad D = (0.8704)^{1/3}\,D_0 = 0.9548\,D_0.$$

$$\therefore \text{ Area of solid shaft} = \frac{\pi}{4}\,D^2 = \frac{\pi}{4}\,(0.9548\,D_0)^2 = 0.716\,D_0^{\,2}$$

$$\text{Area of hollow shaft} = \frac{\pi}{4}\,[D_0^{\,2} - D_i^{\,2}] = \frac{\pi}{4}\,[D_0^{\,2} - (0.6\,D_0)^2]$$

$$= \frac{\pi}{4}\,[D_0^{\,2} - 0.36\,D_0^{\,2}] = \frac{\pi}{4} \times 0.64\,D_0^{\,2} = 0.502\,D_0^{\,2}.$$

For the shafts of the same material, the weight of the shafts is proportional to the areas.

$\therefore$ Saving in material = Saving in area

$$= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}}$$

$$= \frac{0.716 D_0^{\,2} - 0.502 D_0^{\,2}}{0.716 D_0^{\,2}} = 0.2988.$$

$\therefore$ Percentage saving in material = 0.2988 × 100 = **29.88. Ans.**

Problem 16.14. *Determine the diameter of a solid shaft which will transmit 300 kW at 250 r.p.m. The maximum shear stress should not exceed 30 N/mm² and twist should not be more than 1° in a shaft length of 2 m. Take modulus of rigidity = 1 × 10⁵ N/mm².*

Sol. Given :

Power transmitted, $P = 300 \text{ kW} = 300 \times 10^3$ W

Speed of the shaft, $N = 250$ r.p.m.

Maximum shear stress, $\tau = 30$ N/mm²

Twist in shaft, $\theta = 1° = \frac{\pi}{180} = 0.01745$ radian

Length of shaft, $L = 2$ m = 2000 mm

Modulus of rigidity, $C = 1 \times 10^5$ N/mm²

Let D = Diameter of the shaft.

Power is given by the relation,

$$P = \frac{2\pi NT}{60}$$

or

$$300 \times 10^3 = \frac{2\pi \times 250 \times T}{60}$$

$$\therefore \quad T = \frac{300 \times 10^3 \times 60}{2\pi \times 250} = 11459.1 \text{ N-m} = 11459.1 \times 10^3 \text{ N-mm}$$

(*i*) Diameter of the shaft when maximum shear stress, $\tau = 30$ N/mm²

Maximum torque transmitted by a solid shaft is given by equation (16.4) as

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$\therefore \quad 11459100 = \frac{\pi}{16} \times 30 \times D^3$$

$$\therefore \quad D = \left(\frac{16 \times 11459100}{\pi \times 30}\right)^{1/3} = 124.5 \text{ mm} \quad ...(i)$$

(*ii*) Diameter of shaft when twist should not be more than 1°.

Using equation (16.9),

$$\frac{T}{J} = \frac{C\theta}{L}$$

where J = Polar moment of inertia of solid shaft

$$= \frac{\pi}{32}\,D^4$$

$$\therefore \quad \frac{11459100}{\frac{\pi}{32}\,D^4} = \frac{10^5 \times 0.01745}{2000}$$

$$\therefore \quad D^4 = \frac{32 \times 2000 \times 11459100}{10^5 \times \pi \times 0.01745} = 13377.81 \times 10^4$$

$$\therefore \quad D = (13377.81 \times 10^4)^{1/4} = 107.5 \text{ mm}$$

The suitable diameter of the shaft is the greater* of the two values given by equat (i) and (ii).

$\therefore$ Diameter of the shaft = **124.5 mm say 125 mm. Ans.**

*(If diameter is taken smaller of the two values say 107.5 mm, then from equa $T = \frac{\pi}{16} \tau D^3$, the value of shear stress will be

$$11459100 = \frac{\pi}{16} \tau \times (107.5)^3$$

or $$11459100 = 243920 \, \tau$$

or $$\tau = \frac{11459100}{243920} = 46.978 \text{ N/mm}^2$$

which is more than the given value of 30 N/mm²).

Problem 16.15. *A hollow shaft of diameter ratio $\frac{3}{8}$ (internal dia. to outer dia.) is transmit 375 kW power at 100 r.p.m. The maximum torque being 20% greater than the mea The shear stress is not to exceed 60 N/mm² and twist in a length of 4 m not to exceed Calculate its external and internal diameters which would satisfy both the above conditio Assume modulus of rigidity, $C = 0.85 \times 10^5$ N/mm².*

Sol. Given :

Diameter ratio, $\frac{D_i}{D_0} = \frac{3}{8}$

$\therefore \quad D_i = \frac{3}{8} D_0$

Power, $P = 375 \text{ kW} = 375000 \text{ W}$

Speed, $N = 100$ r.p.m.

Max. torque, $T_{max} = 1.2 \, T_{mean}$

Length, $L = 4 \text{ m} = 4000 \text{ mm}$

Max. twist, $\theta = 2° = 2 \times \frac{\pi}{180}$ radians $= 0.0349$ radians

Modulus of rigidity, $C = 0.85 \times 10^5 \text{ N/mm}^2$

Power is given by, $P = \frac{2\pi NT}{60}$ Here torque is T_{mean}

or $$T = \frac{P \times 60}{2\pi N} = \frac{375000 \times 60}{2\pi \times 100} = 35810 \text{ N-m}$$

or $$T_{mean} = 35810 \text{ N-m}$$

$\therefore$ $$T_{max} = 1.2 \times T_{mean} = 1.2 \times 35810$$

$$= 42972 \text{ N-m} = 42972 \times 1000 \text{ N-mm}.$$

(i) *Diameters of the shaft when shear stress is not to exceed 60 N/mm².*

For the hollow shaft, the torque transmitted is given by

$$T_{max} = \frac{\pi}{16} \times \tau \times \frac{(D_0^4 - D_i^4)}{D_0}$$

or $$42972 \times 1000 = \frac{\pi}{16} \times 60 \times \frac{\left[D_0^4 - \left(\frac{3}{8}D_0\right)^4\right]}{D_0}$$

or $$\frac{42972000 \times 16}{\pi \times 60} = \frac{D_0^4}{D_0}\left(1 - \frac{81}{4096}\right) = D_0^3 \times \frac{4015}{4096}$$

or $$D_0^3 = \frac{42972000 \times 16 \times 4096}{\pi \times 60 \times 4015}$$

$$\therefore \quad D_0 = \left(\frac{42972000 \times 16 \times 4096}{\pi \times 60 \times 4015}\right)^{1/3} = 154.97 \text{ mm say } 155 \text{ mm}$$

$$\therefore \quad D_i = \frac{3}{8} D_0 = \frac{3}{8} \times 155 = 58.1 \text{ mm.}$$

(ii) *Diameters of the shaft when the twist is not to exceed 2 degrees.*

Using equation (16.9) in terms of torque and θ, we get

$$\frac{T}{J} = \frac{C \times \theta}{L}$$

or $$\frac{42972000}{\frac{\pi}{32}[D_0^4 - D_i^4]} = \frac{(0.85 \times 10^5) \times 0.0349}{4000}$$

or $$\frac{42972000 \times 4000 \times 32}{\pi \times 0.85 \times 10^5 \times 0.0349} = D_0^4 - D_i^4 = D_0^4 - \left(\frac{3}{8}D_0\right)^4 = D_0^4 - \frac{81}{4096}D_0^4$$

$$= D_0^4\left[1 - \frac{81}{4096}\right] = \frac{4015}{4096}D_0^4$$

$$\therefore \quad D_0^4 = \frac{42972000 \times 4000 \times 32 \times 4096}{\pi \times 0.85 \times 10^5 \times 0.0349 \times 4015}$$

$$\therefore \quad D_0 = 156.65 \text{ mm say } 157 \text{ mm}$$

and $$D_i = \frac{3}{8} \times 156.65 = 58.74 \text{ mm say } 59 \text{ mm.}$$

The diameters of the shaft, which would satisfy both the conditions are the greater of the two values.

∴ External dia., D_0 = **157 mm. Ans.**

Internal dia., D_i = **59 mm. Ans.**

Problem 16.16. *A solid circular shaft transmits 75 kW power at 200 r.p.m. Calculate the shaft diameter, if the twist in the shaft is not to exceed 1° in 2 metres length of shaft, and shear stress is limited to 50 N/mm². Take C = 1 × 10⁵ N/mm².*

Sol. Given :

Power transmitted, $P = 75 \text{ kW} = 75 \times 10^3 \text{ W}$

Speed of the shaft, $N = 200$

Twist in the shaft, $\theta = 1°$

$$= \frac{\pi}{180} \text{ radians} = 0.01745 \text{ rad.}$$

Length of the shaft, $L = 2 \text{ m} = 2000 \text{ mm}$

Maximum shear stress, $\tau = 50 \text{ N/mm}^2$

Modulus of rigidity, $C = 1 \times 10^5 \text{ N/mm}^2$

$$P = \frac{2\pi NT}{60} \quad \text{or} \quad 75 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

$$\therefore \quad T = \frac{75 \times 10^3 \times 60}{2\pi \times 200} = 3580.98 \text{ N-m} = 3580980 \text{ N-mm.}$$

(i) *Diameter of the shaft when maximum shear stress is limited to 50 N/mm².*

Using equation (16.4),

$$T = \frac{\pi}{16}\tau D^3 \quad \text{or} \quad 3580980 = \frac{\pi}{16} \times 50 \times D^3.$$

$$\therefore \quad D = \left(\frac{16 \times 3580980}{\pi \times 50}\right)^{1/3} = 71.3 \text{ mm}$$

(ii) *Diameter of the shaft when the twist in the shaft is not to exceed 1°.*

Using equation (16.9),

$$\frac{T}{J} = \frac{C\theta}{L}$$

or $$\frac{3580980}{\frac{\pi}{32}D^4} = \frac{10^5 \times 0.01745}{2000} \qquad \left(\because J = \frac{\pi}{32}D^4\right)$$

$$\therefore \quad D = \left(\frac{32 \times 2000 \times 3580980}{\pi \times 10^5 \times 0.01745}\right)^{1/4} = 80.4 \text{ mm.}$$

The suitable diameter of the shaft is the greater value of the two diameters given by equations (i) and (ii) *i.e.*, 80.4 mm say 81 mm. **Ans.**

Problem 16.17. *A hollow shaft, having an internal diameter 40% of its external diameter, transmits 562.5 kW power at 100 r.p.m. Determine the external diameter of the shaft if the shear stress is not to exceed 60 N/mm² and the twist in a length of 2.5 m should not exceed 1.3 degrees. Assume maximum torque = 1.25 mean torque and modulus of rigidity = 9 × 10⁴ N/mm².*

Sol. Given :

Internal diameter, D_i = 40% of external diameter, $(D_0) = 0.40\, D_0$

Power transmitted, $P = 562.5$ kW $= 562.5 \times 10^3$ W

Speed of the shaft, $N = 100$ r.p.m.

Maximum shear stress, $\tau = 60$ N/mm²

Twist in the shaft, $\theta = 1.3° = 1.3 \times \frac{\pi}{180}$ radians $= 0.02269$ rad

Length of shaft, $L = 2.5$ m $= 2500$ mm

Maximum torque, $T_{max} = 1.25 \times T_{mean}$

Modulus of rigidity, $C = 9 \times 10^4$ N/mm²

The horse power transmitted is given by

$$P = \frac{2\pi NT}{60}$$

or $$562.5 \times 10^3 = \frac{2\pi \times 100 \times T_{mean}}{60} \qquad (\because \text{ Here } T = T_{mean})$$

or $$T_{mean} = \frac{60 \times 562.5 \times 10^3}{2\pi \times 100} = 53714.7 \text{ N-m} = 53714700 \text{ N-mm}$$

$\therefore$ $$T_{max} = 1.25 \times T_{mean} = 1.25 \times 53714700 = 67143375 \text{ N-mm}.$$

(*i*) Diameter of the hollow shaft when maximum shear stress is 60 N/mm².

Using equation (16.6) for torque in case of hollow shaft

$$T = \frac{\pi}{16}\tau\left[\frac{D_0^4 - D_i^4}{D_0}\right]$$

where $T = T_{max} = 67143375$ and $D_i = 0.4\, D_0$, $\tau = 60$ N/mm²

$\therefore$ $$67143375 = \frac{\pi}{16} \times 60\left[\frac{D_0^4 - (0.4 D_0)^4}{D_0}\right]$$

$$= \frac{\pi}{16} \times 60\left[\frac{D_0^4 - 0.0256 D_0^4}{D_0}\right]$$

$$= \frac{\pi}{16} \times 60 \times 0.9744 D_0^3 = 11.479 D_0^3$$

$\therefore$ $$D_0 = \left(\frac{67143375}{11.479}\right)^{1/3} = 179.6 \text{ mm} \qquad \ldots(i)$$

(*ii*) Diameter of the shaft when the twist in the hollow shaft is not to exceed 1.3°.

Using equation (16.9), we have

$$\frac{T}{J} = \frac{C\theta}{L} \qquad \ldots(ii)$$

where $T = T_{max} = 67143375$, $\theta = 1.3° = 0.02269$ rad., $L = 2500$ mm, $C = 9 \times 10^4$ N/mm²

and J = Polar moment of inertia for hollow shaft

$$= \frac{\pi}{32}[D_0^4 - D_i^4] \qquad \text{(see equation 16.11)}$$

$$= \frac{\pi}{32}[D_0^4 - (0.4\, D_0)^4] \qquad (\because D_i = 0.4\, D_0)$$

$$= \frac{\pi}{32} \times 0.9744\, D_0^4 = 0.09566\, D_0^4.$$

Substituting these values in equation (*ii*), we get

$$\frac{67143375}{0.09566\, D_0^4} = \frac{9 \times 10^4 \times 0.02269}{2500}$$

$\therefore$ $$D_0^4 = \frac{2500 \times 67143375}{0.09566 \times 9 \times 10^4 \times 0.02269} = 85928.215 \times 10^4$$

$\therefore$ $$D_0 = 85928.215 \times 10^4 = 171.2 \text{ mm} \qquad \ldots(iii)$$

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – V –ANALYSIS OF TRUSSES - SCIA1301

FRAMES

- ✓ A frame is a structure made up of several steel bars which are riveted or welded together.
- ✓ These are made up of angle irons or channel sections and are called members of the frame or framed structure.
- ✓ Although the members are welded or riveted together at their joints, they are considered as hinged or pin- jointed for the purpose of calculations.
- ✓ Determination of forces in a frame is needed in many engineering structures.
- ✓ The forces are determined based on the application of the principles of either statics or graphics.

TYPES OF FRAMES:

1. Perfect frames

2. Imperfect frames

1. PERFECT FRAMES

A perfect frame may be defined as that one which is made up of members just sufficient to keep the frame in equilibrium, when loaded, without any change in the shape.

The simplest example of a perfect frame is a triangle. It is to be noted that the shape will not be distorted when the structure is loaded.

Thus, for three jointed frames, there should be three members to prevent distortion.

2. IMPERFECT FRAMES

An Imperfect frame is one which does not satisfy the relation between the numbers of members and number of joints given by the equation $n = 2j - 3$.

This means that number of members in an imperfect frame will be either more or less than (2j-3) It may be a deficient frame or a redundant frame.

ASSUMPTIONS:

Following are the assumptions made in finding the forces in the members of a frame.

1. The frame is a perfect frame
2. The frame is loaded only at the joints
3. All the members of the frame are pin-jointed
4. Self-weight of the members is neglected.

VARIOUS TYPES OF TRUSSES:

The following five criterions may serve as a basis for the classification of trusses:

1) The shape of the upper and lower chords;
2) The type of the web
3) The conditions of the supports
4) The purpose of the structure
5) The level of the floor (lane, road)

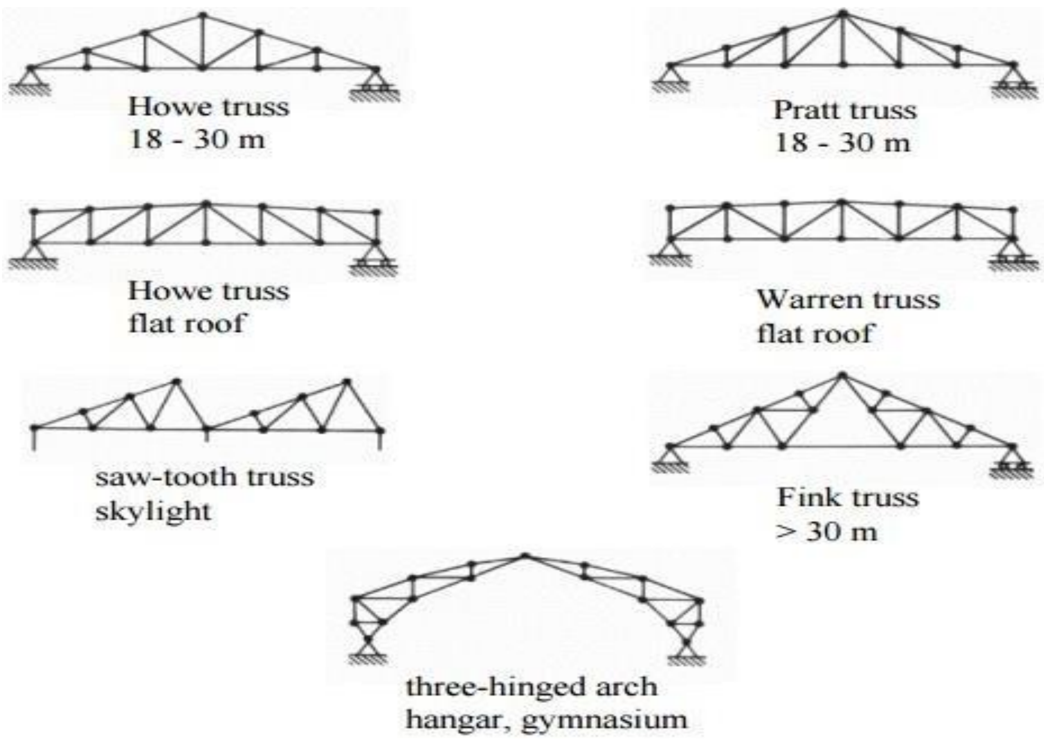


Figure No.1

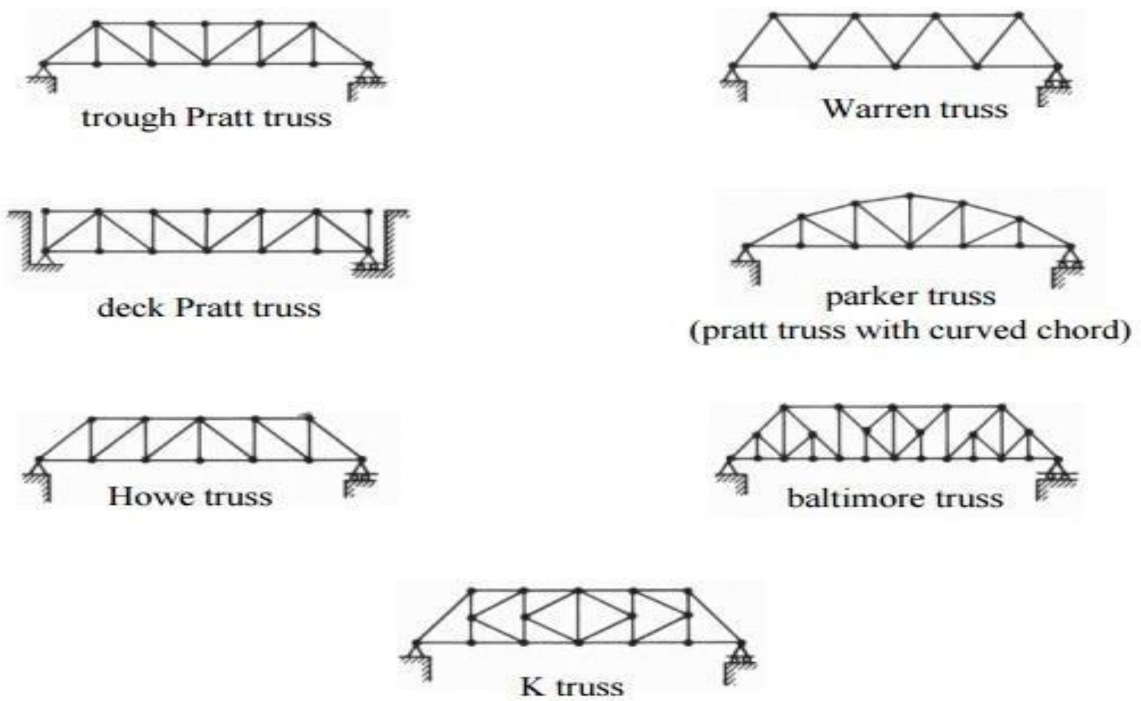


Figure No.2

VARIOUS TYPES OF ANALYTICAL METHODS:

1. Method of joint
2. Method of sections
3. Tension co-efficient method

METHOD OF JOINTS OR METHOD OF RESOLUTION:

- ✓ For a given frame or a truss the support reactions are determined taking moments of the external forces with respect to the support.
- ✓ Then each joint is considered individually as a free body in equilibrium and the forces on the members at that joint are determined by summing up all the vertical component of forces to zero and all the horizontal component of forces to zero.
  - i.e., $\Sigma V = 0$ and $\Sigma H = 0$
- ✓ Joints should be selected such that forces for only two members are unknown in that joint.
- ✓ The force is said to be tensile if it pulls the joint to which it is connected.

METHOD OF SECTIONS OR METHOD OF MOMENTS:

- ✓ When forces in a few members of a truss are to be determined then this method is the
- ✓ simplest one.
- ✓ This method is easy since we do not need the solutions from other joints.
- ✓ Here, we pass a section line passing through the members in which the forces are to be determined.

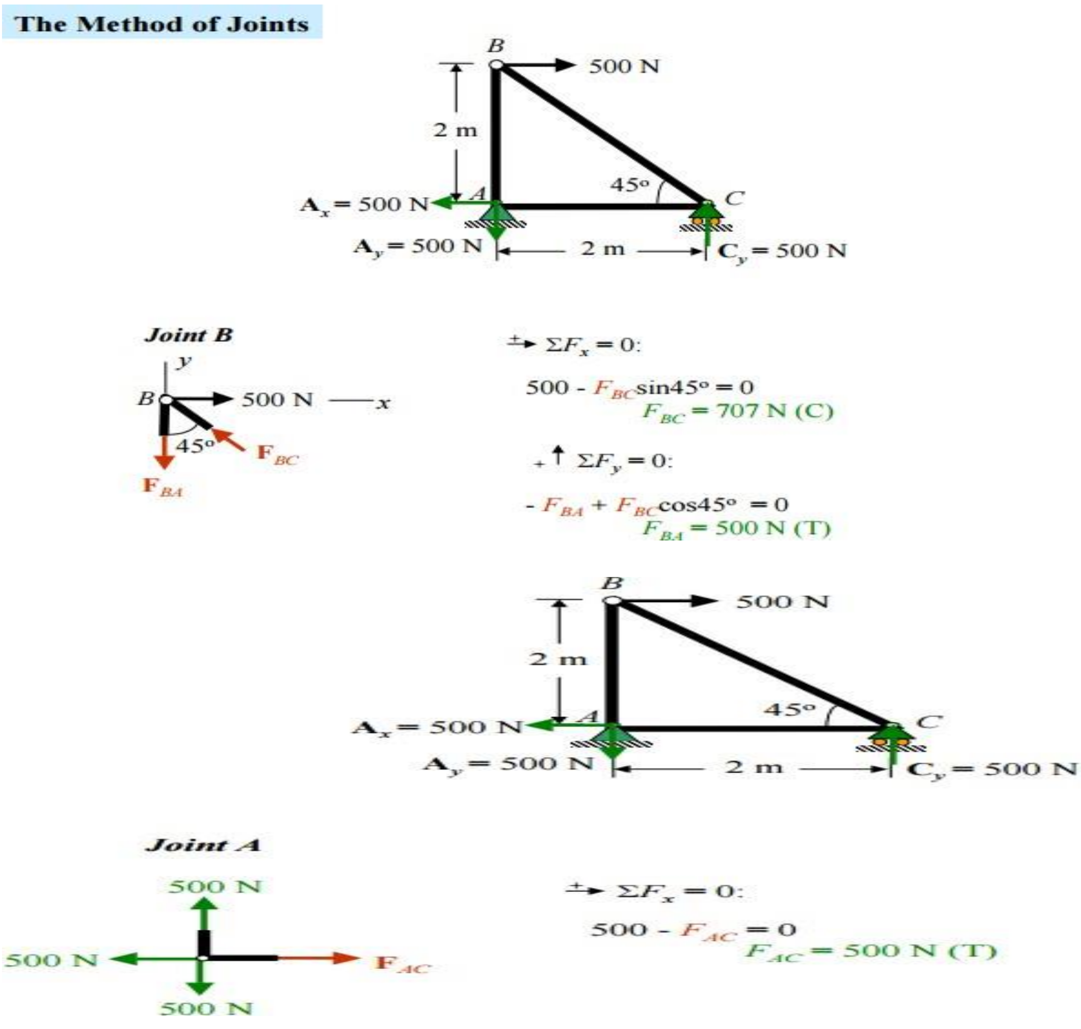
- ✓ The section line should be such that it does not cut more than three members in which the forces are unknown.
- ✓ The truss on one side of the section line is treated as a free body in equilibrium under the action of external forces.
- ✓ The unknown forces are then determined using the equilibrium equations, $\Sigma M = 0$, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
- ✓ When we get a negative value of force in a member then the assumed direction is not correct and it has changed.

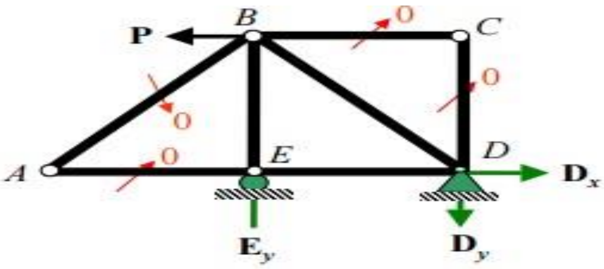
TRUSS – ASSUMPTIONS:

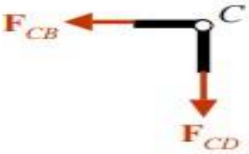
There are four main assumptions made in the analysis of truss:

Truss members are connected together at their ends only.
Truss are connected together by frictionless pins.
The truss structure is loaded only at the joints.
The weights of the members may be neglected.

PROBLEMS:

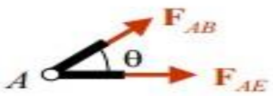






$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CB} = 0$

$+\uparrow \Sigma F_y = 0: \quad F_{CD} = 0$

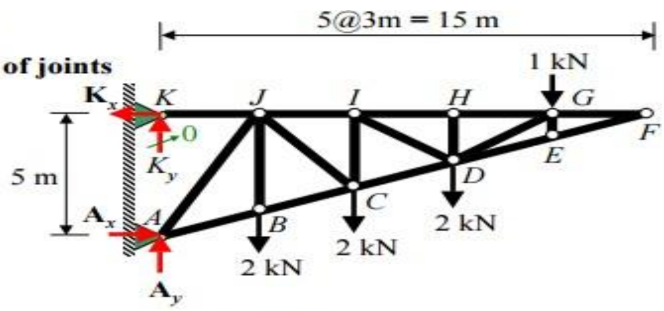


$+\uparrow \Sigma F_y = 0: \quad F_{AB}\sin\theta = 0, \qquad F_{AB} = 0$

$\xrightarrow{+} \Sigma F_x = 0: \quad F_{AE} + 0 = 0, \qquad F_{AE} = 0$

SOLUTION

Use method of joints



$\overset{4}{r} + \overset{18}{b} = \overset{2(11)}{2j}, \longrightarrow$

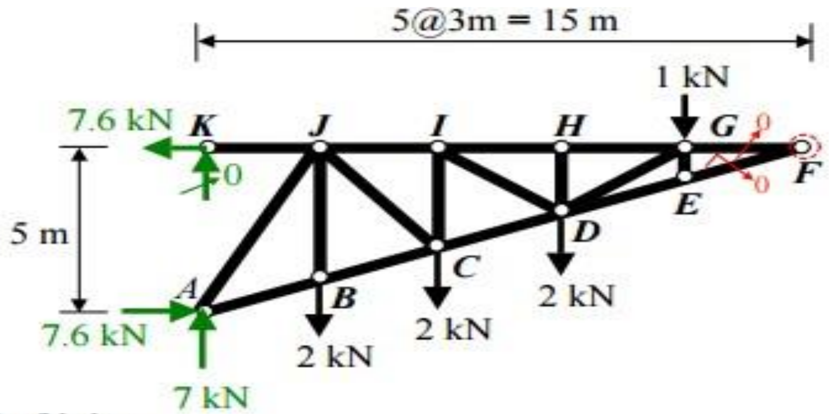
- Determinate
- Stable

$\circlearrowleft + \Sigma M_A = 0: \quad K_x(5) - 2(3) - 2(6) - 2(9) - 1(12) = 0$

$K_x = 7.6 \text{ kN}, \leftarrow$

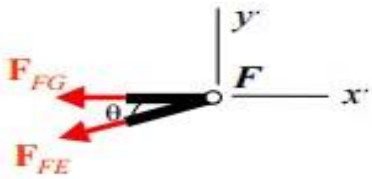
$\xrightarrow{+} \Sigma F_x = 0: \quad -7.6 + A_x = 0, \quad A_x = 7.6 \text{ kN}, \rightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y - 2 - 2 - 2 - 1 = 0, \quad A_y = 7 \text{ kN}, \uparrow$



Use method of joint

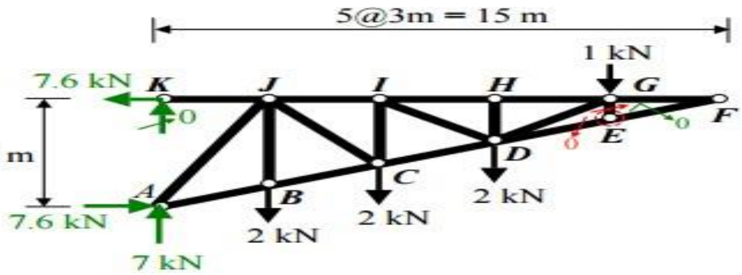
- ***Joint F***



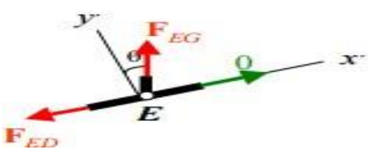
$$+\uparrow \Sigma F_{y'} = 0: \qquad F_{FE}\sin\theta = 0$$

$$F_{FE} = 0$$

$$\xrightarrow{+} \Sigma F_{x'} = 0: \qquad F_{FG} = 0$$



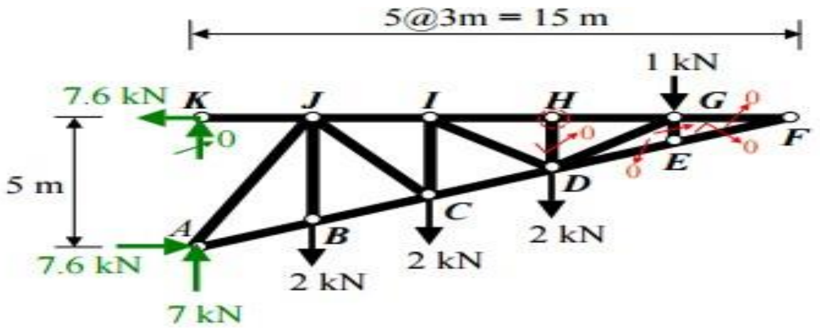
- ***Joint E***



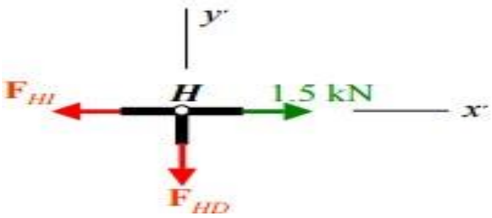
$$+\uparrow \Sigma F_{y'} = 0: \qquad F_{EG}\cos\theta = 0$$

$$F_{EG} = 0$$

$$\xrightarrow{+} \Sigma F_{x'} = 0: \qquad -F_{ED} = 0$$



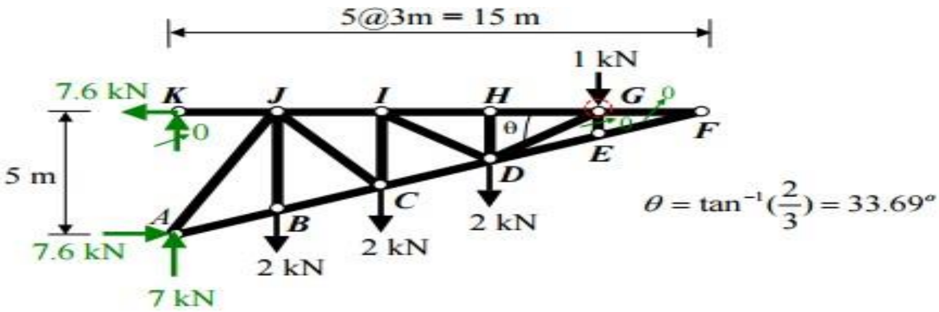
- ***Joint H***



$$+\uparrow \Sigma F_{y'} = 0: \qquad F_{HD} = 0$$

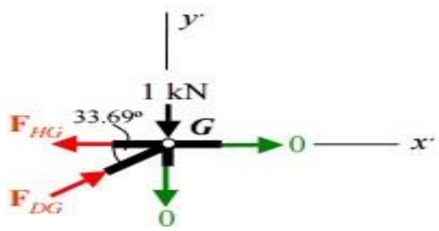
$$\xrightarrow{+} \Sigma F_{x'} = 0: \qquad -F_{HI} + 1.5 = 0$$

$$F_{HI} = 1.5 \text{ kN (T)}$$



$$\theta = \tan^{-1}(\frac{2}{3}) = 33.69^o$$

• ***Joint G***



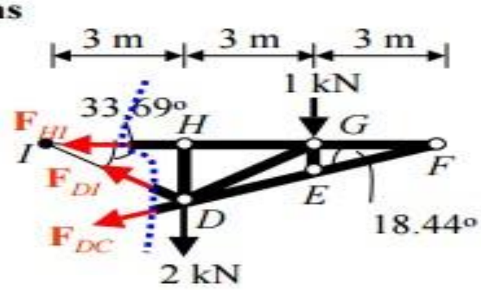
$+\uparrow \Sigma F_{y'} = 0$: $\quad F_{DG} \sin 33.69^o - 1 = 0$

$F_{DG} = 1.803$ kN (C)

$\xrightarrow{+} \Sigma F_{x'} = 0$: $\quad -F_{HG} + 1.803 \cos 33.69 = 0$

$F_{HG} = 1.5$ kN (T)

Use method of sections



$+\circlearrowleft \Sigma M_D = 0$: $\quad F_{HI}(2) - 1(3) = 0$

$F_{HI} = 1.5$ kN (T)

$+\circlearrowleft \Sigma M_F = 0$: $\quad -F_{DI} \sin 33.69(9) + 1(3) + 2(6) = 0$

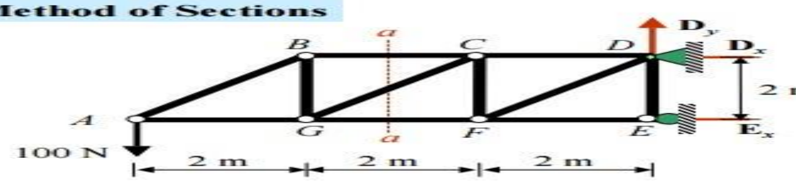
$F_{DI} = 3$ kN (T)

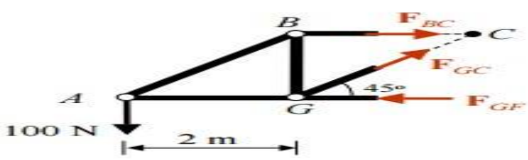
$+\circlearrowleft \Sigma M_I = 0$: $\quad -F_{DC} \sin 18.44(9) - 1(6) - 2(3) = 0$

$F_{DC} = -4.25$ kN (C)

Check : $+\uparrow \Sigma F_y = 0$: $\quad \cancel{F_{DI}}^{3} \sin 33.69 - \cancel{F_{DC}}^{-4.25} \sin 18.44 - 2 - 1 = 0 \qquad$ *O.K.*

The Method of Sections





$+\circlearrowleft \Sigma M_G = 0$:

$100(2) - F_{BC}(2) = 0$

$F_{BC} = 100$ N (T)

$+\uparrow \Sigma F_y = 0$:

$-100 + F_{GC}\sin 45^o = 0$

$F_{GC} = 141.42$ N (T)

$+\circlearrowleft \Sigma M_C = 0$:

$100(4) - F_{GF}(2) = 0$

$F_{GF} = 200$ N (C)