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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – I – LIMIT STATE METHOD OF DESIGN AND COLLAPSE FLEXURE – SCIA1203

Introduction to Objectives and Methods of Analysis and Design

Reinforced concrete, as a composite material, has occupied a special place in the modern construction of different types of structures due to its several advantages. Italian architect Ponti once remarked that concrete liberated us from the rectangle. Due to its flexibility in form and superiority in performance, it has replaced, to a large extent, the earlier materials like stone, timber and steel. Further, architect's scope and imaginations have widened to a great extent due to its mouldability and monolithicity. Thus, it has helped the architects and engineers to build several attractive shell forms and other curved structures. However, its role in several straight line structural forms like multistoried frames, bridges, foundations etc. is enormous.

The design of these modern reinforced concrete structures may appear to be highly complex. However, most of these structures are the assembly of several basic structural elements such as beams, columns, slabs, walls and foundations. Accordingly, the designer has to learn the design of these basic reinforced concrete elements. The joints and connections are then carefully developed.

Design of reinforced concrete structures started in the beginning of last century following purely empirical approach. Thereafter came the so called rigorous elastic theory where the levels of stresses in concrete and steel are limited so that stress-deformations are taken to be linear. However, the limit state method, though semi-empirical approach, has been found to be the best for the design of reinforced concrete structures . The constraints and applicabilities of both the methods will be discussed later.

Objectives of the Design of Reinforced Concrete Structures

Every structure has got its form, function and aesthetics. Normally, we consider that the architects will take care of them and the structural engineers will be solely responsible for the strength and safety of the structure. However, the roles of architects and structural engineers are very much interactive and a unified approach of both will only result in an "Integrated" structure, where every material of the total structure takes part effectively for form, function, aesthetics, strength as well as safety and durability. This is possible when architects have some basic understanding of structural design and the structural engineers also have the basic knowledge of architectural requirements.

Both the engineer and the architect should realize that the skeletal structure without architecture is barren and mere architecture without the structural strength and safety is disastrous. Safety, here, includes consideration of reserve strength, limited deformation and durability. However, some basic knowledge of architectural and structural requirements would facilitate to appreciate the possibilities and limitations of exploiting the reinforced concrete material for the design of innovative structures.

Before proceeding to the design, one should know the objectives of the design of concrete structures. The objectives of the design are as follows:

The structures so designed should have an acceptable probability of performing satisfactorily during their intended life.

This objective does not include a guarantee that every structure must perform satisfactorily during its intended life. There are uncertainties in the design process both in the estimation of the loads likely to be applied on the structure and in the strength of the material. Moreover, full guarantee would only involve more cost. Thus, there is an acceptable probability of performance of structures as given in standard codes of practices of different countries.

The designed structure should sustain all loads and deform within limits for construction and use.

Adequate strengths and limited deformations are the two requirements of the designed structure. The structure should have sufficient strength and the deformations must be within prescribed limits due to all loads during construction having insufficient strength of concrete which fails in bending compression with the increase of load, though the deformation of the structure is not alarming. On the other hand, another situation where the structure, having sufficient strength, deforms excessively. Both are undesirable during normal construction and use.

However, sometimes structures are heavily loaded beyond control. The structural engineer is not responsible to ensure the strength and deformation within limit under such situation. The staircases in residential buildings during festival like marriage etc., roof of the structures during flood in the adjoining area or for buildings near some stadium during cricket or football matches are some of the examples when structures get overloaded. Though, the structural designer is not responsible for the strength and deformations under these situations, he, however, has to ensure that the failure of the structures should give sufficient time for the occupants to vacate. The structures, thus, should give sufficient warning to the occupants and must not fail suddenly.

The designed structures should be durable.

The materials of reinforced concrete structures get affected by the environmental conditions. Thus, structures having sufficient strength and permissible deformations may have lower strength and exhibit excessive deformations in the long run. The designed structures, therefore, must be checked for durability. Separate checks for durability are needed for the steel reinforcement and concrete. This will avoid problems of frequent repairing of the structure.

The designed structures should adequately resist to the effects of misuse and fire.

Structures may be misused to prepare fire works, store fire works, gas and other highly inflammable and/or explosive chemicals. Fire may also take place as accidents or as secondary effects during earthquake by overturning kerosene stoves or lantern, electrical short circuiting etc. Properly designed structures should allow sufficient time and safe route for the persons inside to vacate the structures before they actually collapse.

Method of Design

Three methods of design are accepted in cl. 18.2 of IS 456:2000 (Indian Standard Plain and Reinforced Concrete - Code of Practice, published by the Bureau of Indian Standards, New Delhi). They are as follows:

Limit state method

The term “Limit states” is of continental origin where there are three limit states - serviceability / crack opening / collapse. For reasons not very clear, in English literature limit state of collapse is termed as limit state.

As mentioned in the semi-empirical limit state method of design has been found to be the best for the design of reinforced concrete members. However, because of its superiority to other two methods , IS 456:2000 has been thoroughly updated in its fourth revision in 2000 taking into consideration the rapid development in the field of concrete technology and incorporating important aspects like durability etc. This standard has put greater emphasis to limit state method of design by presenting it in a full section while the working stress method has been given in Annex B of the same standard. Accordingly, structures or structural elements shall normally be designed by limit state method.

Working stress method

This method of design, considered as the method of earlier times, has several limitations. However, in situations where limit state method cannot be conveniently applied, working stress method can be employed as an alternative. It is expected that in the near future the working stress method will be completely replaced by the limit state method. Presently, this method is put in Annex B of IS 456:2000.

Method based on experimental approach

The designer may perform experimental investigations on models or full size structures or elements and accordingly design the structures or elements. However, the four objectives of the structural design must be satisfied when designed by employing this approach. Moreover, the engineer-in- charge has to approve the experimental details and the analysis connected therewith.

Though the choice of the method of design is still left to the designer as per cl. 18.2 of IS 456:2000, the superiority of the limit state method is evident from the emphasis given to this method by presenting it in a full section (Section 5), while accommodating the working stress method in Annex B of IS 456:2000, from its earlier place of section 6 in IS 456:1978. It is expected that a gradual change over to the limit state method of design will take place in the near future after overcoming the inconveniences of adopting this method in somesituations.

Analysis of Structures

Structures when subjected to external loads (actions) have internal reactions in the form of bending moment, shear force, axial thrust and torsion in individual members. As a result, the structures develop internal stresses and undergo deformations. Essentially, we analyse a structure elastically replacing each member by a line (with EI values) and then design the section using concepts of limit state of collapse. Figure 1.1.1 explains the internal and external reactions of a simply supported beam under external loads. The external loads to be applied on the structures are the design loads and the analyses of structures are based on linear elastic theory (vide cl. 22 of IS 456:2000).

Design Loads

The design loads are determined separately for the two methods of design as mentioned below after determining the combination of different loads.

In the limit state method, the design load is the characteristic load with appropriate partial safety factor (vide sec. 2.3.2.3 for partial safety factors).

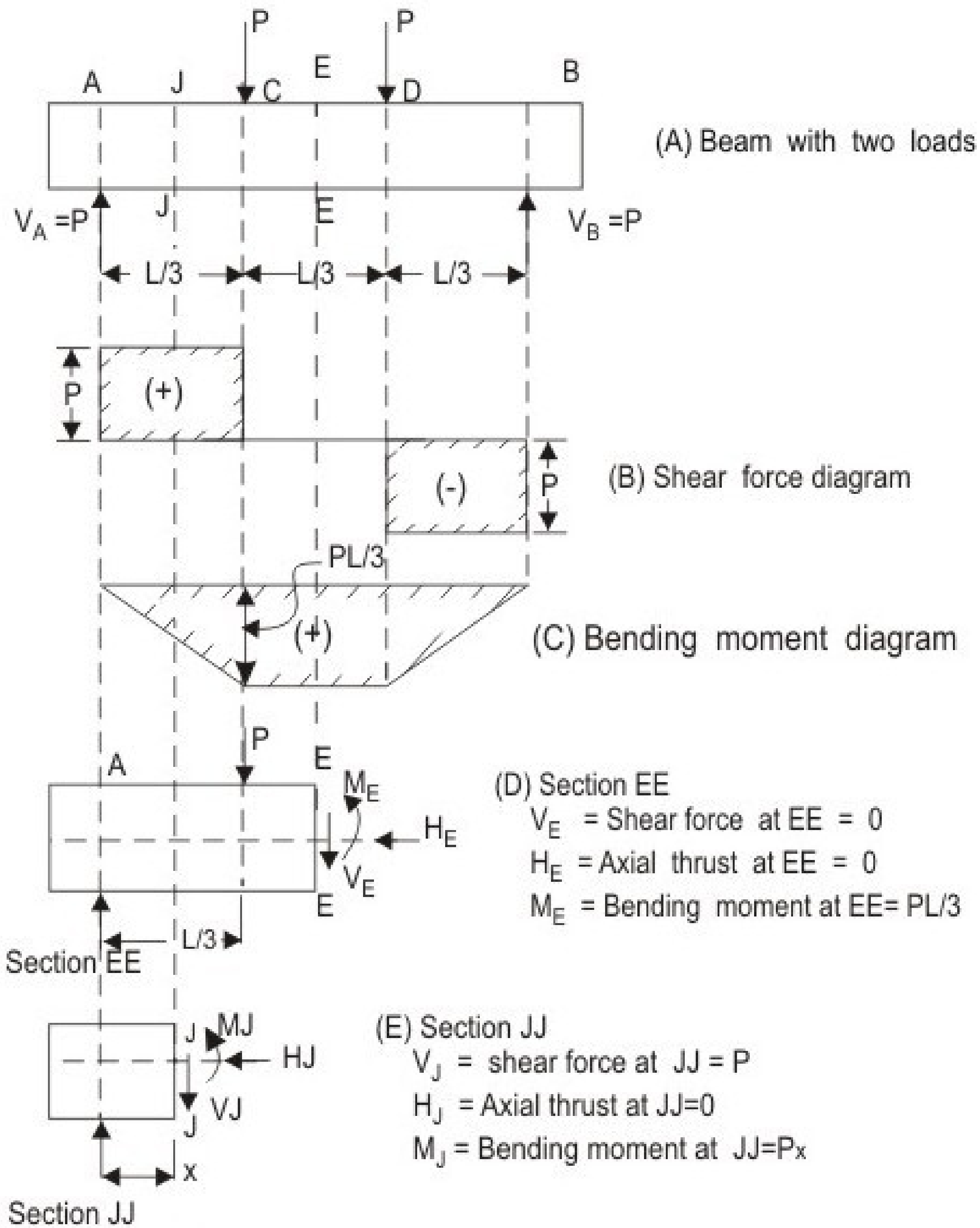


Fig.1.1.1: Simply supported beam

1.1.1.1 In the working stress method, the design load is the characteristic load only.

What is meant by characteristic load?

Characteristic load (cl. 36.2 of IS 456:2000) is that load which has a ninety-five per cent probability of not being exceeded during the life of the structure.

The various loads acting on structures consist of dead loads, live loads, wind or earthquake loads etc. These are discussed in sec. 1.1.6. However, the researches made so far fail to estimate the actual loads on the structure. Accordingly, the loads are predicted based on statistical approach, where it is assumed that the variation of the loads acting on structures follows the normal distribution (Fig. 1.1.2). Characteristic load should be more than the average/mean load. Accordingly,

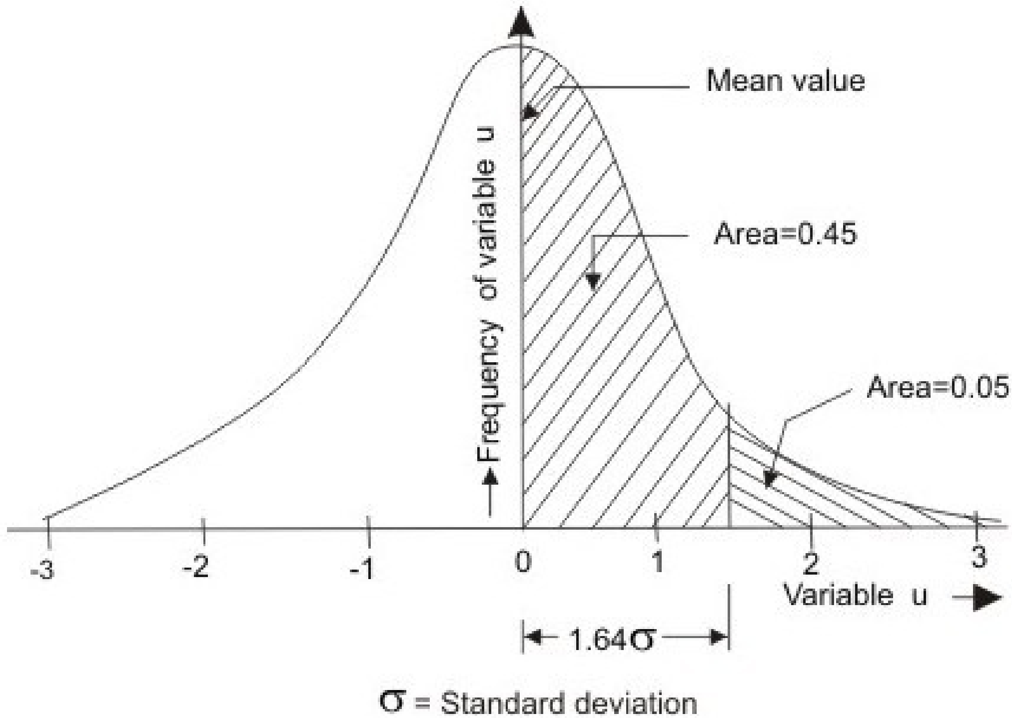


Fig.1.1.2. Normal distribution curve

Characteristic load = Average/mean load + *K* (standard deviation for load)

The value of *K* is assumed such that the actual load does not exceed the characteristic load during the life of the structure in 95 per cent of the cases.

Loads and Forces

The following are the different types of loads and forces acting on the structure. As mentioned , their values have been assumed based on earlier data and experiences. It is worth mentioning that their assumed values as stipulated in IS 875 have been used successfully.

Dead loads

These are the self weight of the structure to be designed (see Anim. 1.1.5a). Needless to mention that the dimensions of the cross section are to be assumed initially which enable to estimate the dead loads from the known unit weights of the materials of the structure. The accuracy of the estimation thus depends on the assumed values of the initial dimensions of the cross section. The values of unit weights of the materials are specified in Part 1 of IS 875.

Imposed loads

They are also known as live loads (Anim. 1.1.5a) and consist of all loads other than the dead loads of the structure. The values of the imposed loads depend on the functional requirement of the structure. Residential buildings will have comparatively lower values of the imposed loads than those of school or office buildings. The standard values are stipulated in Part 2 of IS 875.

Wind loads

These loads (Anim. 1.1.5a) depend on the velocity of the wind at the location of the structure, permeability of the structure, height of the structure etc. They may be horizontal or inclined forces depending on the angle of inclination of the roof for pitched roof structures. They can even be suction type of forces depending on the angle of inclination of the roof or geometry of the buildings (Anim. 1.1.5b). Wind loads are specified in Part 3 of IS 875.

Snow loads

These are important loads for structures located in areas having snow fall, which gets accumulated in different parts of the structure depending on projections, height, slope etc. of the structure (Anim. 1.1.6). The standard values of snow loads are specified in Part 4 of IS 875.

Earthquake forces

Earthquake generates waves which move from the origin of its location (epicenter) with velocities depending on the intensity and magnitude of the earthquake. The impact of earthquake on structures depends on the stiffness of the structure, stiffness of the soil media, height and location of the structure etc. (Anim. 1.1.7). Accordingly, the country has been divided into several zones depending on the magnitude of the earthquake. The earthquake forces are prescribed in IS 1893. Designers have adopted equivalent static load approach or spectral method.

Shrinkage, creep and temperature effects

Shrinkage, creep and temperature (high or low) may produce stresses and cause deformations like other loads and forces (Anim. 1.1.8, 9 and 10). Hence, these are also considered as loads which are time dependent. The safety and serviceability of structures are to be checked following the stipulations of cls. 6.2.4, 5 and 6 of IS 456:2000 and Part 5 of IS 875.

Other forces and effects

It is difficult to prepare an exhaustive list of loads, forces and effects coming onto the structures and affecting the safety and serviceability of them. However, IS 456:2000 stipulates the following forces and effects to be taken into account in case they are liable to affect materially the safety and serviceability of the structures. The relevant codes as mentioned therein are also indicated below:

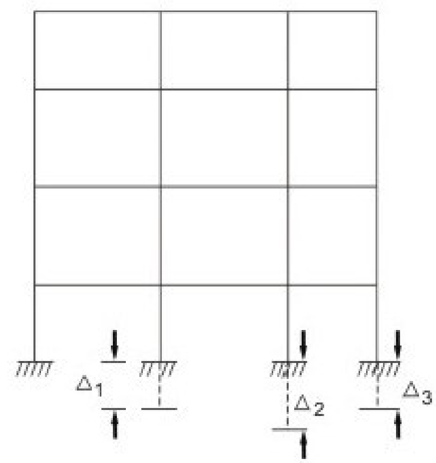


Fig.1.1.3 : Foundation movement

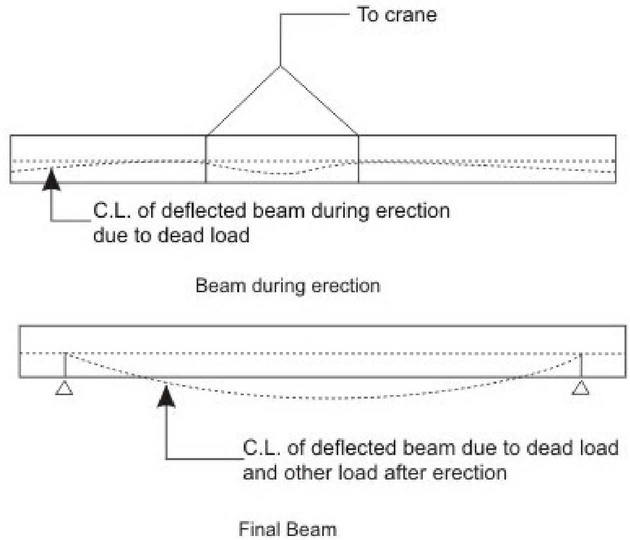


Fig. 1.1.4: Erection loads

- Foundation movement (IS 1904) (Fig. 1.1.3)
- Elastic axial shortening
- Soil and fluid pressures (vide IS 875 - Part 5)
- Vibration
- Fatigue
- Impact (vide IS 875 - Part 5)
- Erection loads (Please refer to IS 875 - Part 2) (Fig. 1.1.4)
- Stress concentration effect due to point of application of load and the like.

Combination of loads

Design of structures would have become highly expensive in order to maintain their serviceability and safety if all types of forces would have acted on all structures at all times. Accordingly, the concept of characteristic loads has been accepted to ensure that in at least 95 per cent of the cases, the characteristic loads considered will be higher than the actual loads on the structure. However, the characteristic loads are to be calculated on the basis of average/mean load of some logical combinations of all the loads mentioned in sec. 1.1.6.1 to 7. These logical combinations are based on (i) the natural phenomena like wind and earthquake do not occur simultaneously, (ii) live loads on roof should not be present when wind loads are considered; to name a few. IS 875 Part 5 stipulates the combination of loads to be considered in the design of structures.

Introduction to properties

It is essential that the designer has to acquire a fair knowledge of the materials to be used in the design of reinforced concrete structure. This lesson summarises the characteristic properties of concrete and steel, the two basic materials used for the design. This summary, though not exhaustive, provides the minimum information needed for the design.

Properties of Concrete

Plain concrete is prepared by mixing cement, sand (also known as fine aggregate), gravel (also known as coarse aggregate) and water with specific proportions. Mineral admixtures may also be added to improve certain properties of concrete. Thus, the properties of concrete regarding its strength and deformations depend on the individual properties of cement, sand, gravel, water and admixtures. Clauses 5 and 6 of IS 456:2000 stipulate the standards and requirements of the individual material and concrete, respectively. Plain concrete after preparation and placement needs curing to attain strength. However, plain concrete is very good in compression but weak in tension. That is why steel is used as reinforcing material to make the composite sustainable in tension also. Plain concrete, thus when reinforced with steel bars in appropriate locations is known as reinforced concrete.

The strength and deformation characteristics of concrete thus depend on the grade and type of cement, aggregates, admixtures, environmental conditions and curing. The increase of strength with its age during curing is considered to be marginal after 28 days. Blended cements (like fly ash cement) have slower rate of strength gain than ordinary Portland cement as recognized by code, Depending on several factors during its preparation, placement and curing, concrete has a wide range of compressive strength and the material is graded on the basis of its compressive strength on 28th day also known as "characteristic strength" as defined below while discussing various strength and deformation properties.

(a) Characteristic strength property

Characteristic strength is defined as the strength below which not more than five per cent of the test results are expected to fall. Concrete is graded on the basis of its characteristic compressive strength of 150 mm size cube at 28 days and expressed in N/mm^2. The grades are designated by one letter M (for mix) and a number from 10 to 80 indicating the characteristic compressive strength (f_{ck}) in N/mm^2. As per IS 456 (Table 2), concrete has three groups as (i) ordinary concrete (M 10 to M 20), (ii) standard concrete (M 25 to M 55) and (iii) high strength concrete (M 60 to M 80). The size of specimen for determining characteristic strength may be different in different countries.

(b) Other strengths of concrete

In addition to its good compressive strength, concrete has flexural and splitting tensile strengths too. The flexural and splitting tensile strengths are obtained as described in IS 516 and IS 5816, respectively. However, the following expression gives an estimation of flexural strength (f_{cr}) of concrete from its characteristic compressive strength (cl. 6.2.2)

$$f_{cr} = 0.7 \sqrt{f_{ck}} \quad \text{in N/mm}^2 \qquad (1.1)$$

(c) Elastic deformation of concrete

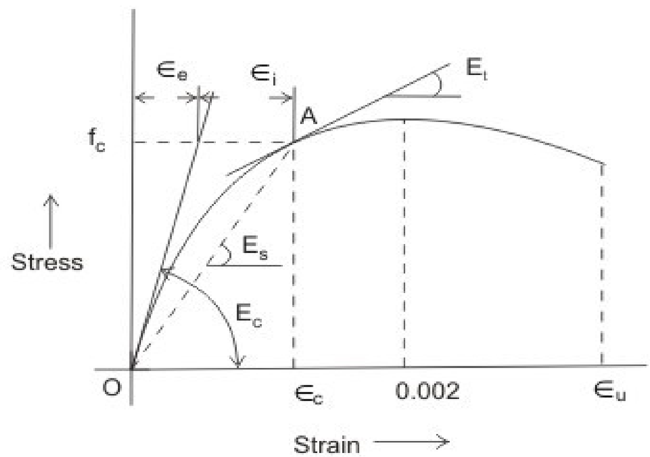


Fig. 1.2.1 : Stress - Strain curve of concrete

Figure 1.2.1 shows a typical stress-strain curve of concrete in compression, where

E_c = initial tangent modulus at the origin, also known as short term static modulus

E_s = secant modulus at *A* E_t =

tangent modulus at *A*

ε_e = elastic strain at *A*

ε_i = inelastic strain at *A*

It is seen that the initial tangent modulus is much higher than E_t (tangent modulus at *A*). Near the failure, the actual strain consists of both ε_e and ε_i (elastic and inelastic respectively) components of strain. The initial tangent modulus E_c in N/mm^2 is estimated from

$$E_c = 5000 \sqrt{f_{ck}} \quad (1.2)$$

where f_{ck} = characteristic compressive strength of concrete at 28 days

The initial tangent modulus E_c is also known as short term static modulus of elasticity of concrete in N/mm^2 and is used to calculate the elastic deflections.

(d) Shrinkage of concrete

Shrinkage is the time dependent deformation, generally compressive in nature. The constituents of concrete, size of the member and environmental conditions are the factors on which the total shrinkage of concrete depends. However, the total shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing for a given humidity and temperature. The cement content, however, influences the total shrinkage of concrete to a lesser extent. The approximate value of the total shrinkage strain for design is taken as 0.0003 in the absence of test data (cl. 6.2.4.1).

(e) Creep of concrete

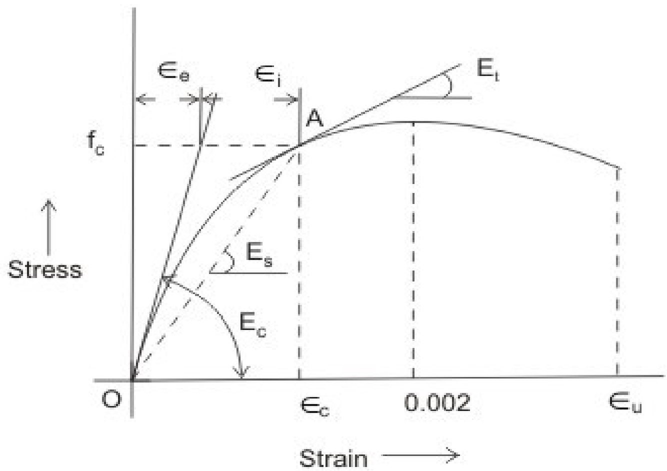


Fig. 1.2.1 : Stress - Strain curve of concrete

Creep is another time dependent deformation of concrete by which it continues to deform, usually under compressive stress. The creep strains recover partly when the stresses are released. Figure 1.2.2 shows the creep recovery in two parts. The elastic recovery is immediate and the creep recovery is slow in nature.

Thus, the long term deflection will be added to the short term deflection to get the total deflection of the structure. Accordingly, the long term modulus E_{ce} or the effective modulus of concrete will be needed to include the effect of creep due to permanent loads. The relationship between E_{ce} and E_c is obtained as follows:

$$\varepsilon_c = f_c / E_c \quad (1.3)$$

where ε_c = short term strain at the age of loading at a stress value of f_c

ε_{cr} = ultimate creep strain

$$\theta = \text{creep coefficient} = \frac{\varepsilon_{cr}}{\varepsilon_c} \quad \text{(cl. 6.2.5.1 of IS 456)} \quad (1.4)$$

The values of θ on 7th, 28th and 365th day of loading are 2.2, 1.6 and 1.1 respectively.

Then the total strain = $\varepsilon_c + \varepsilon_{cr} = \dfrac{f_c}{E_{ce}}$

(1.5)

where E_{ce} = effective modulus of concrete From the

above Eq. (1.5), we have

$$E_{ce} = \frac{f_c}{\varepsilon_c + \varepsilon_{cr}} = \frac{\varepsilon_c E_c}{\varepsilon_c + \varepsilon_{cr}} = \frac{E_c}{1+\theta}$$

(1.6)

The effective modulus of E_{ce} of concrete is used only in the calculation of creep deflection.

It is seen that the value of creep coefficient θ (Eq. 1.4) is reducing with the age of concrete at loading. It may also be noted that the ultimate creep strain ε_{cr} does not include short term strain ε_c. The creep of concrete is influenced by

- Properties of concrete
- Water/cement ratio
- Humidity and temperature of curing
- Humidity during the period of use
- Age of concrete at first loading
- Magnitude of stress and its duration
- Surface-volume ratio of the member

(f) Thermal expansion of concrete

The knowledge of thermal expansion of concrete is very important as it is prepared and remains in service at a wide range of temperature in different countries having very hot or cold climates. Moreover, concrete will be having its effect of high temperature during fire. The coefficient of thermal expansion depends on the nature of cement, aggregate, cement content, relative humidity and size of the section. IS 456 stipulates (cl. 6.2.6) the values of coefficient of thermal expansion for concrete / °C for different types of aggregate.

Workability and Durability of Concrete

Workability and durability of concrete are important properties to be considered. The relevant issues are discussed in the following:

(a) Concrete mix proportioning

The selected mix proportions of cement, aggregates (fine and coarse) and water ensure:

- the workability of fresh concrete,
- required strength, durability and surface finish when concrete is hardened.

Recently more than forty per cent of concrete poured world over would contain admixtures.

(b) Workability

It is the property which determines the ease and homogeneity with which concrete can be mixed, placed, compacted and finished. A workable concrete will not have any segregation or bleeding. Segregation causes large voids and hence concrete becomes less durable. Bleeding results in several small pores on the surface due to excess water coming up. Bleeding also makes concrete less durable. The degree of workability of concrete is classified from very low to very high with the corresponding value of slump in mm (cl. 7 of IS 456).

(c) Durability of concrete

A durable concrete performs satisfactorily in the working environment during its anticipated exposure conditions during service. The durable concrete should have low permeability with adequate cement content, sufficient low free water/cement ratio and ensured complete compaction of concrete by adequate curing. For more information, please refer to cl. 8 of IS 456.

(d) Design mix and nominal mix concrete

In design mix, the proportions of cement, aggregates (sand and gravel), water and mineral admixtures, if any, are actually designed, while in nominal mix, the proportions are nominally adopted. The design mix concrete is preferred to the nominal mix as the former results in the grade of concrete having the specified workability and characteristic strength (vide cl. 9 of IS 456).

(e) Batching

Mass and volume are the two types of batching for measuring cement, sand, coarse aggregates, admixtures and water. Coarse aggregates may be gravel, grade stone chips or other man made aggregates. The quantities of cement, sand, coarse aggregates and solid admixtures shall be measured by mass. Liquid admixtures and water are measured either by volume or by mass (cl. 10 of IS 456).

Properties of Steel

As mentioned earlier in sec. 1.2.2, steel is used as the reinforcing material in concrete to make it good in tension. Steel as such is good in tension as well as in compression. Unlike concrete, steel reinforcement rods are produced in steel plants. Moreover, the reinforcing bars or rods are commercially available in some specific diameters. Normally, steel bars up to 12 mm in diameter are designated as bars which can be coiled for transportation. Bars more than 12 mm in diameter are termed as rods and they are transported in standard lengths.

Like concrete, steel also has several types or grades. The four types of steel used in concrete structures as specified in cl. 5.6 of IS 456 are given below:

(i) Mild steel and medium tensile steel bars conforming to IS 432 (Part 1)

(ii) High yield strength deformed (HYSD) steel bars conforming to IS 1786

(iii) Hard-drawn steel wire fabric conforming to IS 1566

(iv) Structural steel conforming to Grade A of IS 2062.

Mild steel bars had been progressively replaced by HYSD bars and subsequently TMT bars are promoted in our country. The implications of adopting different kinds of blended cement and reinforcing steel should be examined before adopting.

Stress-strain curves for reinforcement

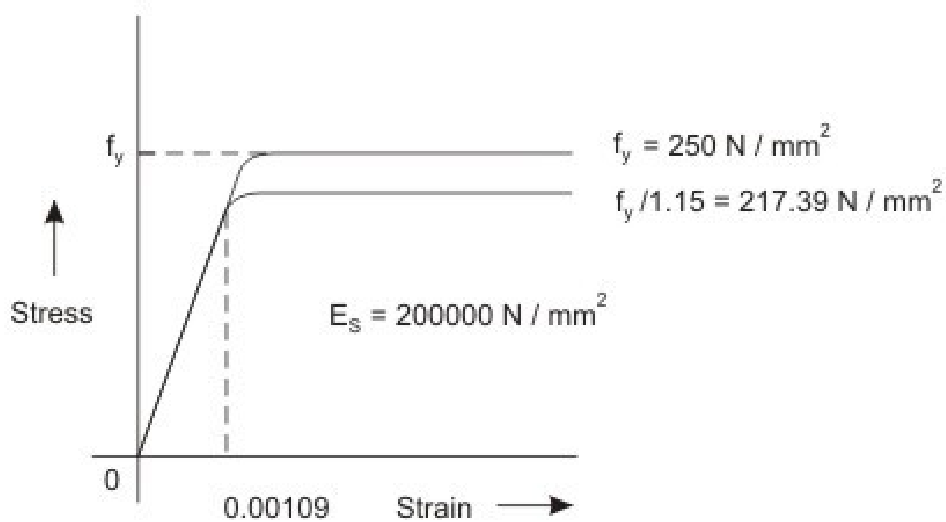


Fig. 1.2.3 : Stress - Strain curve for mild steel (idealised) (Fe 250) with definite yield point.

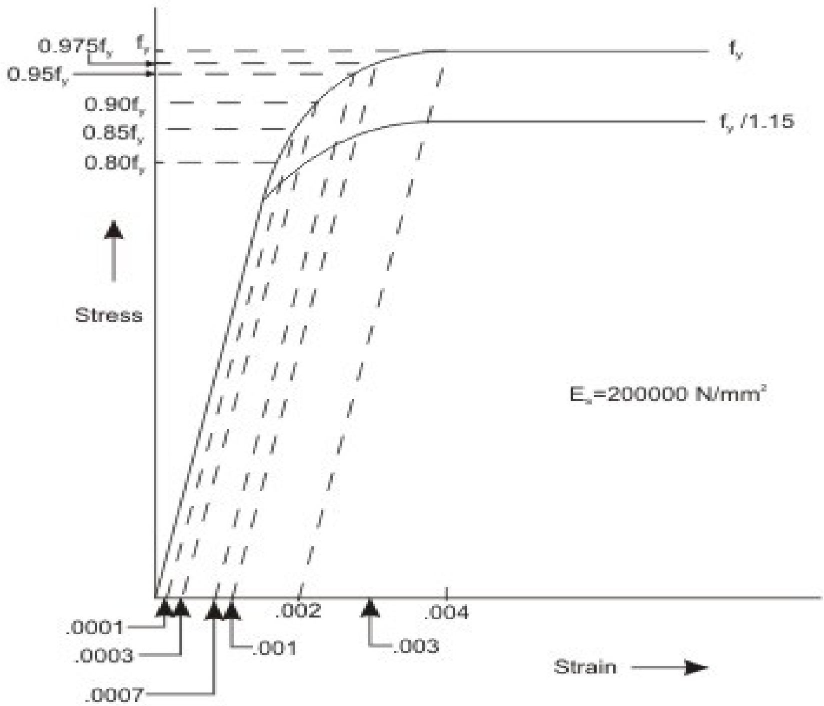


Fig. 1.2.4 : Stress-Strain Curve for cold worked deformed bar

Figures 1.2.3 and 1.2.4 show the representative stress-strain curves for steel having definite yield point and not having definite yield point, respectively. The characteristic yield strength f_y of steel is assumed as the minimum yield stress or 0.2 per cent of proof stress for steel having no definite yield point. The modulus of elasticity of steel is taken to be 200000 N/mm^2.

For mild steel (Fig. 1.2.3), the stress is proportional to the strain up to the yield point. Thereafter, post yield strain increases faster while the stress is assumed to remain at constant value of f_y.

For cold-worked bars (Fig. 1.2.4), the stress is proportional to the strain up to a stress of 0.8 f_y. Thereafter, the inelastic curve is defined as given below:

Stress	**Inelastic strain**
0.80 f_y	Nil
0.85 f_y 0.90 f_y	0.0001 0.0003
0.95 f_y 0.975 f_y	0.0007 0.0010
1.00 f_y	0.0020

Linear interpolation is to be done for intermediate values. The two grades of cold- worked bars used as steel reinforcement are Fe 415 and Fe 500 with the values of f_y as 415 N/mm^2 and 500 N/mm^2, respectively.

Considering the material safety factor γ_m (vide sec. 2.3.2.3 of Lesson 3) of steel as 1.15, the design yield stress (f_{yd}) of both mild steel and cold worked bars is computed from

$$f_{yd} = f_y / \gamma_m$$

(1.7)

Accordingly, the representative stress-strain curve for the design is obtained by substituting f_{yd} for f_y in Figs. 1.2.3 and 1.2.4 for the two types of steel with or without the definite yield point, respectively.

Other Important Factors

The following are some of the important factors to be followed properly as per the stipulations in IS 456 even for the design mix concrete with materials free from impurities in order to achieve the desired strength and quality of concrete. The relevant clause numbers of IS 456 are also mentioned as ready references for each of the factors.

(a) Mixing (cl. 10.3)

Concrete is mixed in a mechanical mixer at least for two minutes so as to have uniform distribution of the materials having uniform colour and consistency.

(b) Formwork (cl. 11)

Properly designed formwork shall be used to maintain its rigidity during placing and compaction of concrete. It should prevent the loss of slurry from the concrete. The stripping time of formwork should be such that the concrete attains strength of at least twice the stress that the concrete may be subjected at the time of removing the formwork. As a ready reference IS 456 specifies the minimum period before striking formwork.

There is a scope for good design of formwork system so that stripping off is efficient without undue shock to concrete and facilitating reuse offormwork.

(c) Assembly of reinforcement (cl. 12)

The required reinforcement bars for the bending moment, shear force and axial thrust are to be accommodated together and proper bar bending schedules shall be prepared. The reinforcement bars should be placed over blocks, spacers, supporting bars etc. to maintain their positions so that they have the required covers. High strength deformed steel bars should not be re-bent. The reinforcement bars should be assembled to have proper flow of concrete without obstruction or segregation during placing, compacting and vibrating.

(d) Transporting, placing, compaction and curing (cl. 13)

Concrete should be transported to the formwork immediately after mixing to avoid segregation, loss of any of the ingredients, mixing of any foreign matter or loss of workability. Proper protections should be taken to prevent evaporation loss of water in hot weather and loss of heat in cold weather.

To avoid rehandling, concrete should be deposited very near to the final position of its placing. The compaction should start before the initial setting time and should not be disturbed once the initial setting has started. While placing concrete, reinforcement bars should not be displaced and the formwork should not be moved.

The compaction of concrete using only mechanical vibrators is very important, particularly around the reinforcement, embedded fixtures and the corners of the formwork to prevent honeycomb type of concreting. Excessive vibration leads to segregation.

Proper curing prevents loss of moisture from the concrete and maintains a satisfactory temperature regime. In moist curing, the exposed concrete surface is kept in a damp or wet condition by ponding or covering with a layer of sacking, canvas, hessian etc. and kept constantly wet for a period of 7-14 days depending on the type of cement and weather conditions. Blended cement needs extended curing. In some situations, polyethylene sheets or similar impermeable membranes may be used to cover the concrete surface closely to prevent evaporation.

(e) Sampling and strength of designed concrete mix (cl. 15)

Random samples of concrete cubes shall be cast from fresh concrete, cured and tested at 28 days as laid down in IS 516. Additional tests on beams for modulus of rupture at 3 or 7 days, or compressive strength tests at 7 days shall also be conducted. The number of samples would depend on the total quantity of concrete as given in cl. 15.2.2 and there should be three test specimens in each sample for testing at 28 days, and additional tests at 3 or 7 days.

(f) Acceptance criteria (cl. 16)

Concrete should be considered satisfactory when both the mean strength of any group of four consecutive test results and any individual test result of compressive strength and flexural strength comply the limits prescribed in IS 456.

(g) Inspection and testing of structures (cl. 17)

Inspection of the construction is very important to ensure that it complies the design. Such inspection should follow a systematic procedure covering materials, records, workmanship and construction.

All the materials of concrete and reinforcement are to be tested following the relevant standards. It is important to see that the design and detailing are capable of execution maintaining a standard with due allowance for the dimensional tolerances. The quality of the individual parts of the structure should be verified. If needed, suitable quality assurance schemes should be used. The concrete should be inspected immediately after the removal of formwork to remove any defective work before concrete has hardened.

Standard core tests (IS 516) are to be conducted at three or more points to represent the whole concrete work in case of any doubt regarding the grade of concrete during inspection either due to poor workmanship or unsatisfactory results on cube strength obtained following the standard procedure. If the average equivalent cube strength of cores is equal to at least 85 per cent of the cube strength of that grade of concrete at that age and each of the individual cores has strength of at least 75 per cent, then only the concrete represented by the core test is considered acceptable. For unsatisfactory core test results, load tests should be conducted for the flexural members and proper analytical investigations should be made for non-flexural members.

Such load tests should be done as soon as possible after expiry of 28 days from the date of casting of the flexural members subjected to full dead load and 1.25 times the imposed load for 24 hours and then the imposed load shall be removed. The maximum deflection of the member during 24 hours under imposed load in mm should be less than 40 l^2/D, where l is the effective span in m and D is the overall depth of the member in mm. For members showing more deflection, the recovery of the deflection within 24 hours of removal of the imposed load has to be noted. If the recovery is less than 75 per cent of the deflection under imposed load, the test should be repeated after a lapse of 72 hours. The structure is considered unacceptable if the recovery is less than 80 per cent.

There are further provisions of conducting non-destructive tests like ultrasonic pulse velocity (UPV), rebound hammer, probe penetration, pull out and maturity, as options to core tests or to supplement the data obtained from a limited number of cores. However, it is important that the acceptance criteria shall be agreed upon prior to these non-destructive testing. There are reports that UPV tests conducted three days after casting after removal of side formwork give very dependable insight about the quality of concrete.

Concluding Remarks

The reinforced concrete consisting of plain concrete and steel reinforcement opened a new vista fulfilling the imaginations of architect with a unified approach of the architect and structural engineer. This has been made possible due to mouldability and monolithicity of concrete in addition to its strength in both tension and compression when reinforced with steel. However, concrete is produced by mixing cement, sand, gravel, water and mineral admixtures, if needed. Therefore, the final strength of concrete depends not only on the individual properties of its constituent materials, but also on the proportions of the material and the manner in which it is prepared, transported, placed, compacted and cured. Moreover, durability of the concrete is also largely influenced by all the steps of its preparation.

Steel reinforcement though produced in steel plants and made available in form of bars and rods of specific diameter also influences the final strength of reinforced concrete by its quality and durability due to environmental effects.

Concrete cover provides the protective environment to embedded steel from rusting that would need presence of both oxygen and moisture. Not only the extent of cover but the quality of cover is important for this reason.

Accordingly, inspection of concrete work, sample testing of specimens, core tests, load tests and non-destructive tests are very important to maintain the quality, strength and durability of reinforced concrete structures. Moreover, it is equally important to remove small defects or make good of it after removing the formwork before it has thoroughly hardened.

Thus, starting from the selection of each constitutive material to the satisfactory construction of the structure, the designer's responsibility will only produce the desired concrete structure which will satisfy the functional requirements as well as will have its aesthetic values exploiting all the good properties of this highly potential material.

Introduction to Limit State Method

In any method of design, the following are the common steps to be followed:

(i) To assess the dead loads and other external loads and forces likely to be applied on the structure,

(ii) To determine the design loads from different combinations of loads,

(iii) To estimate structural responses (bending moment, shear force, axial thrust etc.) due to the design loads,

(iv) To determine the cross-sectional areas of concrete sections and amounts of reinforcement needed.

Many of the above steps have lot of uncertainties. Estimation of loads and evaluation of material properties are to name a few. Hence, some suitable factors of safety should be taken into consideration depending on the degrees of such uncertainties.

Limit state method is one of the three methods of design as per IS 456:2000. The code has put more emphasis on this method by presenting it in a full section (Section 5), while accommodating the working stress method in Annex B of the code (IS 456). Considering rapid development in concrete technology and simultaneous development in handling problems of uncertainties, the limit state method is a superior method where certain aspects of reality can be explained in a better manner.

Limit State Method

What are limit states?

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended. It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded.

Limit state of collapse was found / detailed in several countries in continent fifty years ago. In 1960 Soviet Code recognized three limit states: (i) deformation, (ii) cracking and (iii) collapse.

How many limit states are there?

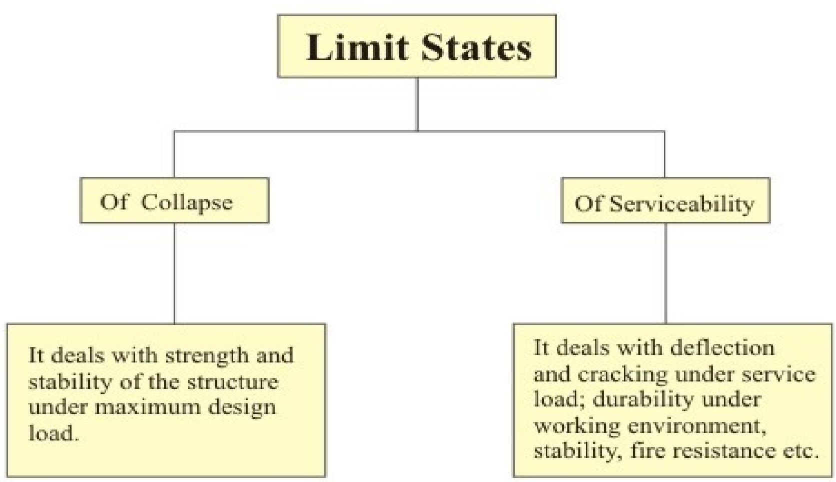


Fig. 2.3.1: Two main limit states

There are two main limit states: (i) limit state of collapse and (ii) limit state of serviceability (see Fig. 2.3.1).

(i) Limit state of collapse deals with the strength and stability of structures subjected to the maximum design loads out of the possible combinations of several types of loads. Therefore, this limit state ensures that neither any part nor the whole structure should collapse or become unstable under any combination of expected overloads.

(ii) Limit state of serviceability deals with deflection and cracking of structures under service loads, durability under working environment during their anticipated exposure conditions during service, stability of structures as a whole, fire resistance etc.

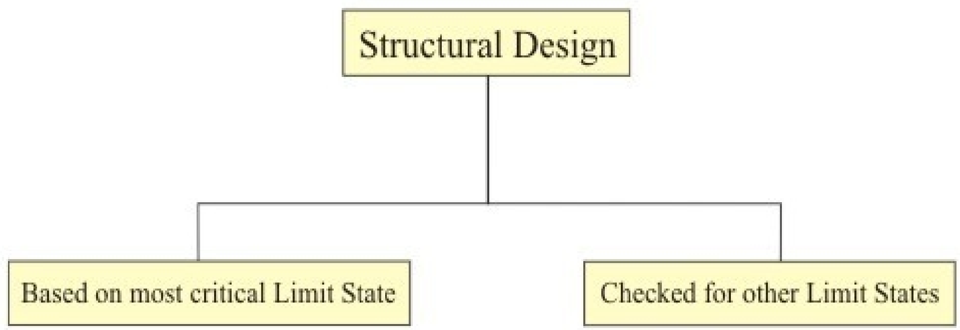


Fig. 2.3.2 : Structural Design

All relevant limit states have to be considered in the design to ensure adequate degree of safety and serviceability. The structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states (see Fig. 2.3.2).

Partial safety factors

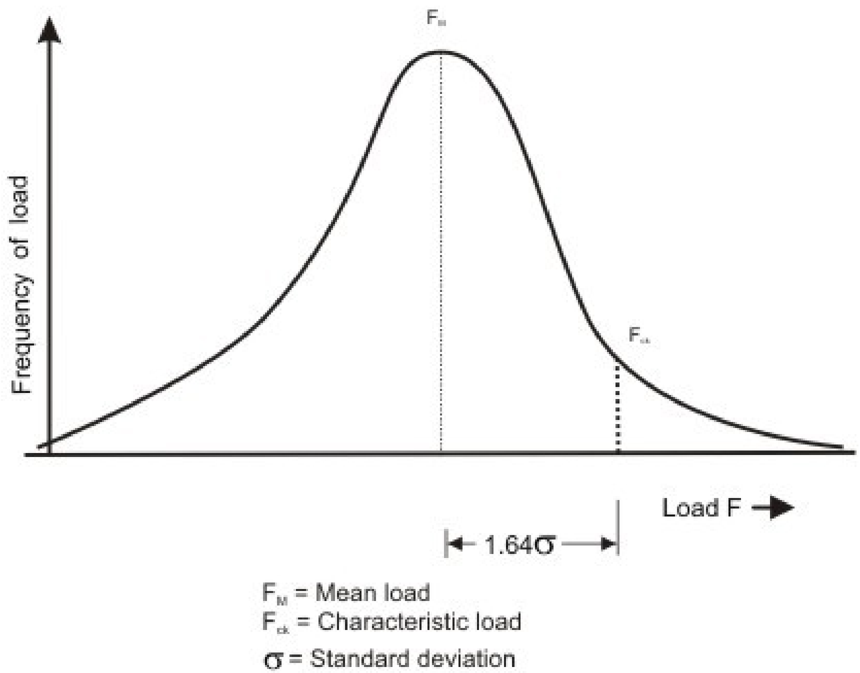


Fig. 2.3.3: Characteristic loads F_{ck}

The characteristic values of loads as discussed in sec. 1.1.5 are based on statistical data. It is assumed that in ninety-five per cent cases the characteristic loads will not be exceeded during the life of the structures (Fig. 2.3.3). However, structures are subjected to overloading also. Hence, structures should be designed with loads obtained by multiplying the characteristic loads with suitable factors of safety depending on the nature of loads or their combinations, and the limit state being considered. These factors of safety for loads are termed as partial safety factors (γ_f) for loads. Thus, the design loads are calculated as

(Design load F_d) = (Characteristic load F) (Partial safety factor for load γ_f) (2.1)

Respective values of γ_f for loads in the two limit states as given in Table 18 of IS 456 for different combinations of loads are furnished in Table 2.1.

Table 2.1 Values of partial safety factor γ_f for loads

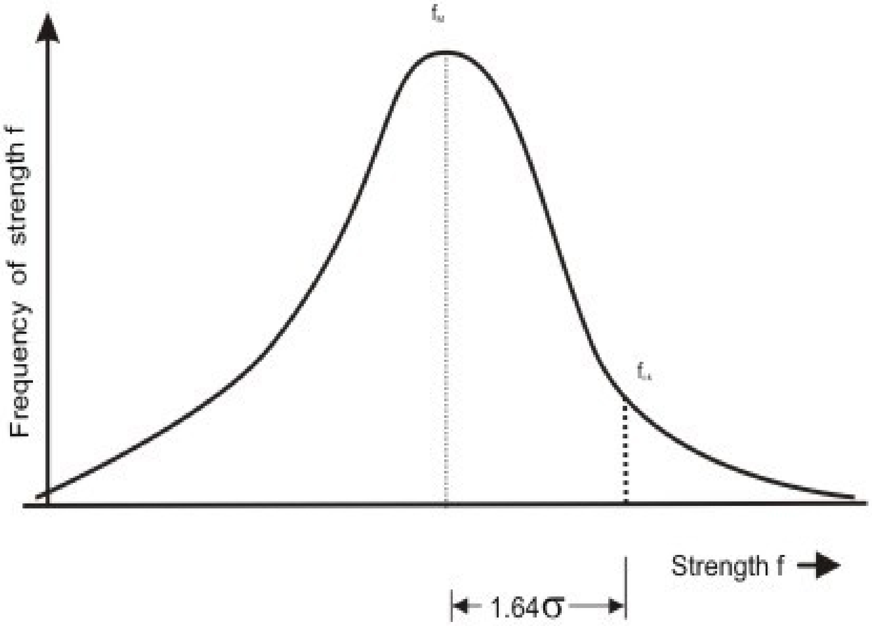
Load combinations	Limit state of collapse			Limit state of serviceability (for short term effects only)		
	DL	*IL*	*WL*	*DL*	*IL*	*WL*
DL + IL	1.5		1.0	1.0	1.0	-
DL + WL	1.5 or 0.9[1)]	-	1.5	1.0	-	1.0
DL + IL + WL	1.2			1.0	0.8	0.8

NOTES:

1 While considering earthquake effects, substitute *EL* for *WL.*

2 For the limit states of serviceability, the values of γ_f given in this table are applicable for short term effects. While assessing the long term effects due to creep the dead load and that part of the live load likely to be permanent may only be considered.

[1)] This value is to be considered when stability against overturning or stress reversal is critical.



f_M = Mean strength
f_{ck} = Characteristic strength
σ = Standard deviation

Fig. 2.3.4: Characteristic strength f_{ck}

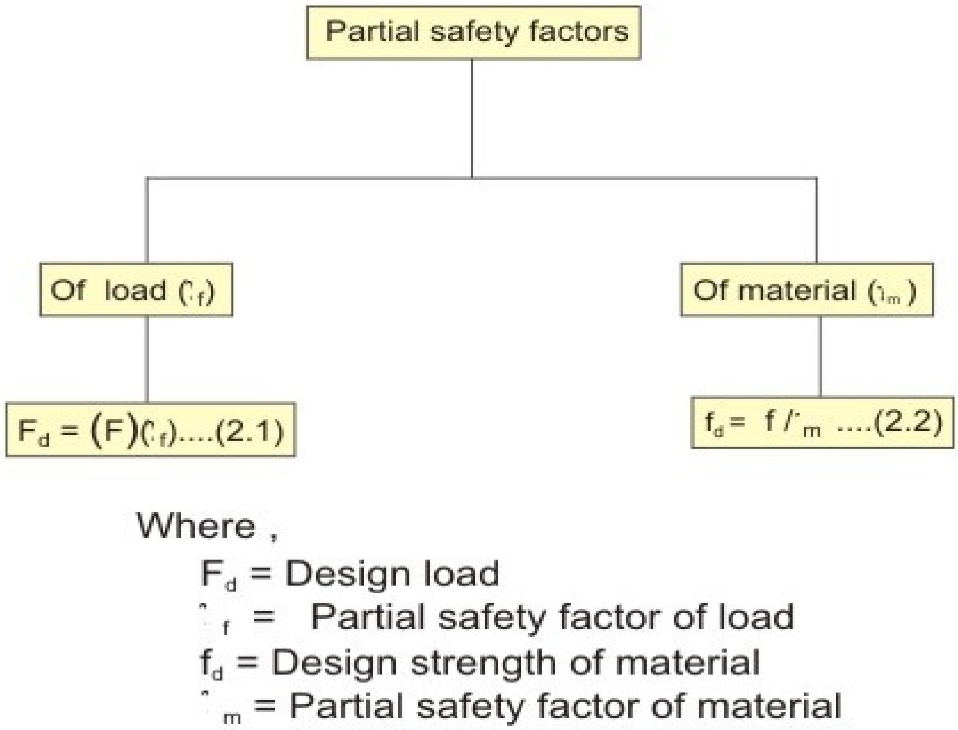


Fig 2.3.5 : Partial safety factors

Similarly, the characteristic strength of a material as obtained from the statistical approach is the strength of that material below which not more than five per cent of the test results are expected to fall (see Fig. 2.3.4). However, such characteristic strengths may differ from sample to sample also. Accordingly, the design strength is calculated dividing the characteristic strength further by the partial safety factor for the material (γ_m), where γ_m depends on the material and the limit state being considered. Thus,

$$\text{Design strength of the material } f_d = \frac{\text{Characteristic strength of the material } f}{\text{Partial safety factor of the material } \gamma_m}$$

(2.2)

Both the partial safety factors are shown schematically in Fig. 2.3.5.

Clause 36.4.2 of IS 456 states that γ_m for concrete and steel should be taken as 1.5 and 1.15, respectively when assessing the strength of the structures or structural members employing limit state of collapse. However, when assessing the deflection, the material properties such as modulus of elasticity should be taken as those associated with the characteristic strength of the material. It is worth mentioning that partial safety factor for steel (1.15) is comparatively lower than that of concrete (1.5) because the steel for reinforcement is produced in steel plants and commercially available in specific diameters with expected better quality control than that of concrete.

Further, in case of concrete the characteristic strength is calculated on the basis of test results on 150 mm standard cubes. But the concrete in the structure has different sizes. To take the size effect into account, it is assumed that the concrete in the structure develops a strength of 0.67 times the characteristic strength of cubes. Accordingly, in the calculation of strength employing the limit state of collapse, the characteristic strength (f_{ck}) is first multiplied with 0.67 (size effect) and then divided by 1.5 (γ_m for concrete) to have 0.446 f_{ck} as the maximum strength of concrete in the stress block.

Analysis

Analysis of structure has been briefly mentioned in sec. 1.1.4 earlier. Herein, the analysis of structure, in the two limit states (of collapse and of serviceability), is taken up. In the limit state of collapse, the strength and stability of the structure or part of the structure are ensured. The resistances to bending moment, shear force, axial thrust, torsional moment at every section shall not be less than their appropriate values at that section due to the probable most unfavourable combination of the design loads on the structure. Further, the structure or part of the structure should be assessed for rupture of one or more critical sections and buckling due to elastic or plastic instability considering the effects of sway, if it occurs or overturning.

Linear elastic theory is recommended in cl. 22 of IS 456 to analyse the entire structural system subjected to design loads. The code further stipulates the adoption of simplified analyses for frames (cl. 22.4) and for continuous beams (cl. 22.5). For both the limit states the material strengths should be taken as the characteristic values in determining the elastic properties of members. It is worth mentioning that the statically indeterminate structures subjected to design loads will have plastic hinges at certain locations as the loads increase beyond the characteristic loads. On further increase of loads, bending moments do not increase in the locations of plastic hinges as they are already at the full capacities of bending moments. However, these plastic hinges undergo more rotations and the moments are now received by other sections which are less stressed. This phenomenon continues till the plastic hinges reach their full rotation capacities to form a mechanism of collapse of the structure. This is known as the redistribution of moments (Figs. 2.3.6 and 2.3.7). The theory and numerical problems of “Redistribution of moments” are presented elaborately in Lesson 38.

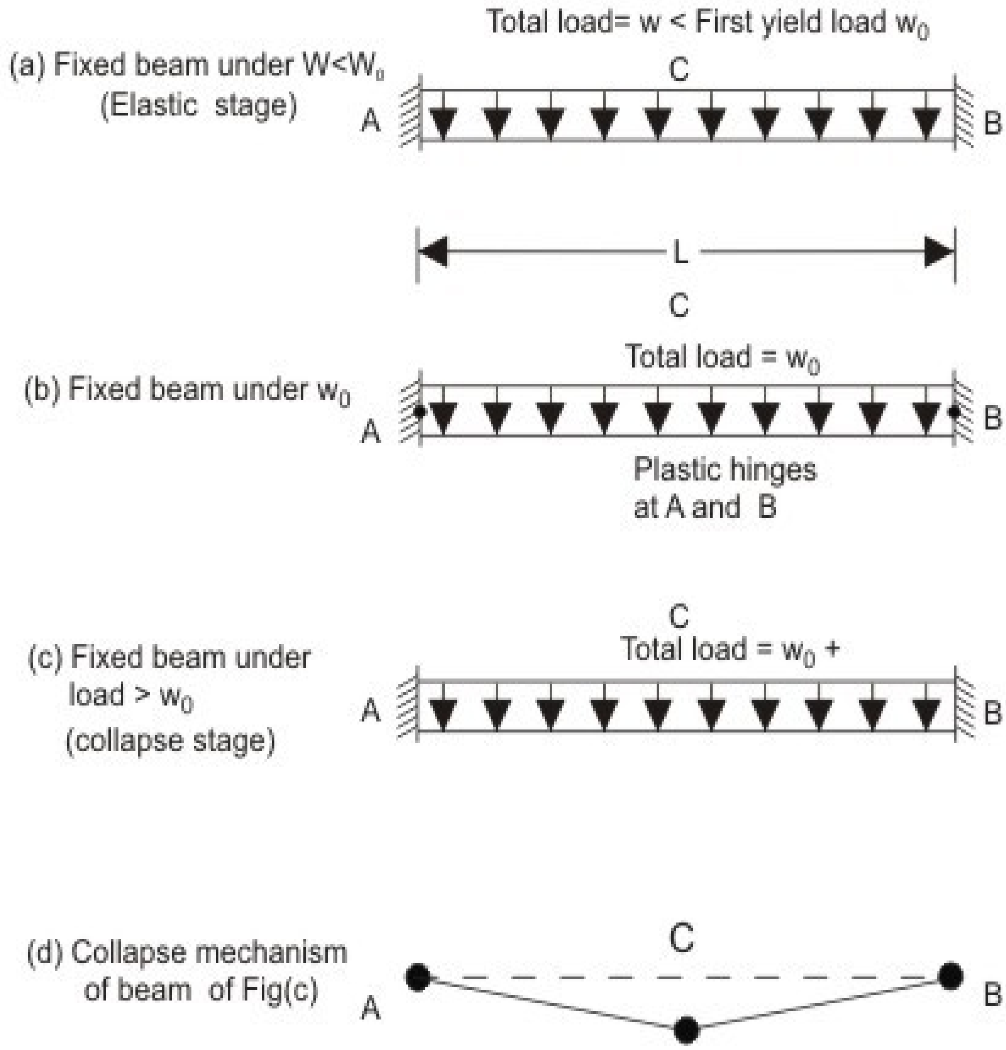


Fig 2.3.6 : Redistribution of moments

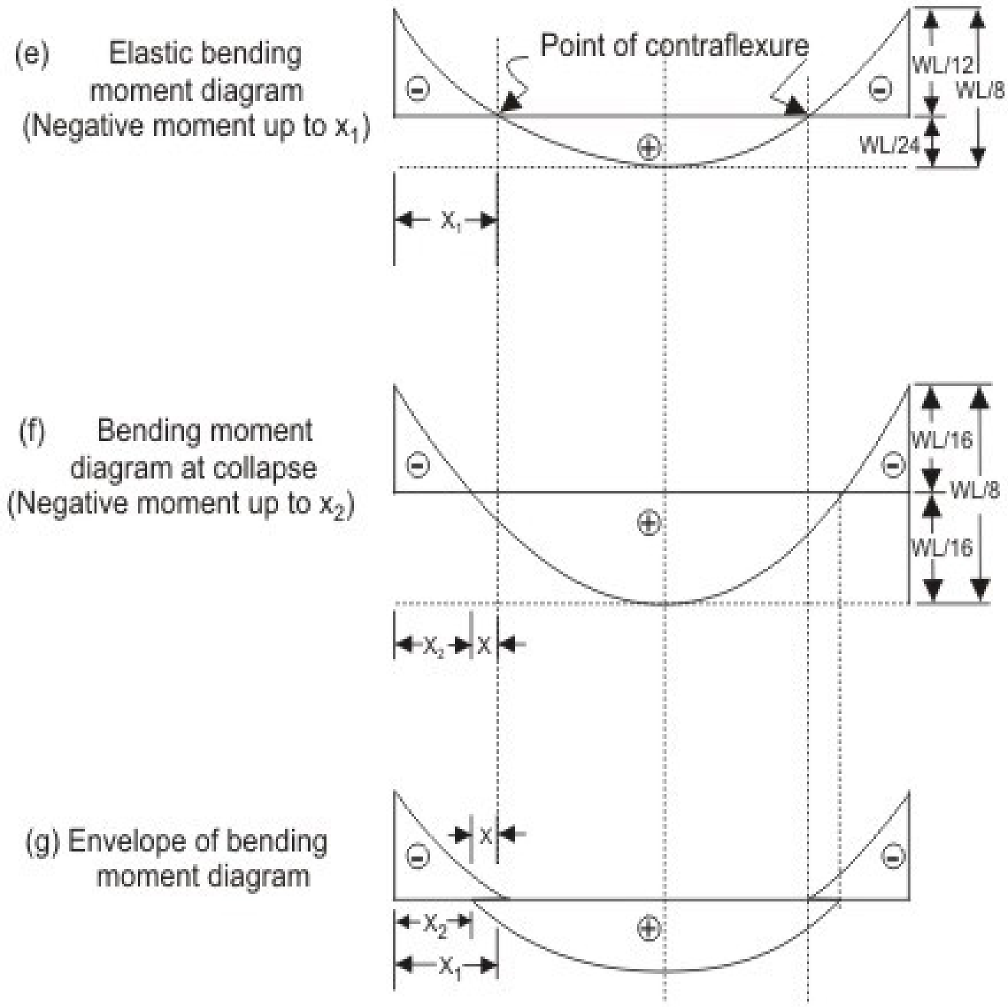


Fig 2.3.7 : Redistribution of moments

The design of structure, therefore, should also ensure that the less stressed sections can absorb further moments with a view to enabling the structure to rotate till their full capacities. This will give sufficient warming to the users before the structures collapse. Accordingly, there is a need to redistribute moments in continuous beams and frames. Clause 37.1.1 stipulates this provision and the designer has to carry out the redistribution by satisfying the stipulated conditions there.

The analysis of slabs spanning in two directions at right angles should be performed by employing yield line theory or any other acceptable method. IS 456:2000 has illustrated alternative provisions for the simply supported and restrained slabs spanning in two directions in Annex D along with Tables 26 and 27 giving bending moment coefficients of these slabs for different possible boundary conditions. These provisions enable to determine the reinforcement needed for bending moments in two directions and torsional reinforcement wherever needed.

Concluding Remarks

The limit state method is based on a stochastic process where the design parameters are determined from observations taken over a period of time. The concept of separate partial safety factors for loads and material strengths are based on statistical and probabilistic grounds. These partial safety factors for the material strengths are determined on the basis of reliability of preparations of concrete and reinforcement. The overloading of structure has been kept in mind while specifying the partial safety factors of loads.

The stress block of structures or parts of structure designed on the basis of limit state method subjected to the designed loads or collapse loads represents the stress-strain diagram at the defined states of collapse and satisfy the requirements of strength and stability. Simultaneous checking of these structures or parts of them for the limit state of serviceability ensures the deflection and cracking to remain within their limits. Thus, this method is more rational and scientific.

Introduction to parameters

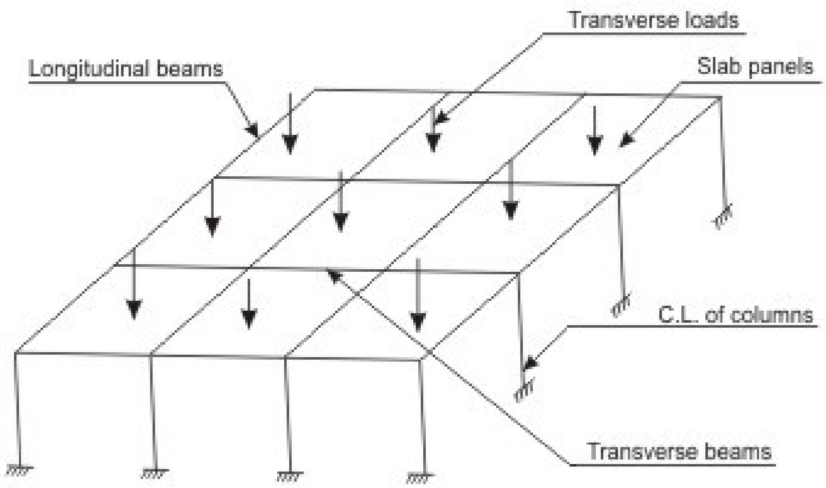


Fig. 3.4.1: Slab-beam-column system under transverse loads

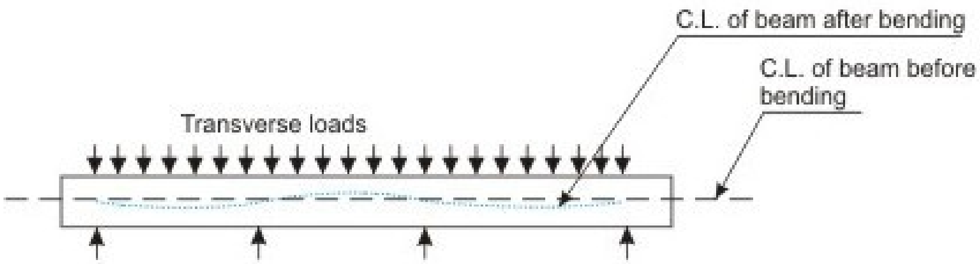


Fig. 3.4.2 : Beam under bending

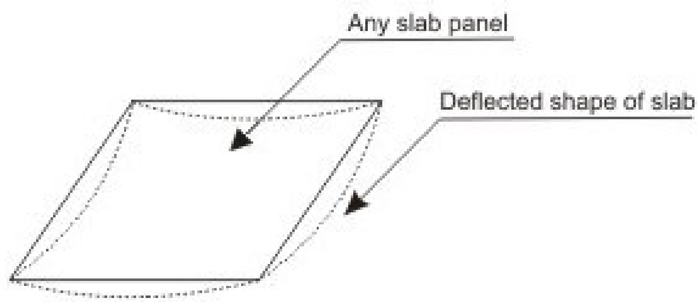


Fig. 3.4.3: Slab under bending (supported at corners)

Reinforced concrete beams and slabs carry loads primarily by bending (Figs. 3.4.1 to 3). They are, therefore, designed on the basis of limit state of collapse in flexure. The beams are also to be checked for other limit states of shear and torsion. Slabs under normal design loadings (except in bridge decks etc.) need not be provided with shear reinforcement. However, adequate torsional reinforcement must be provided wherever needed.

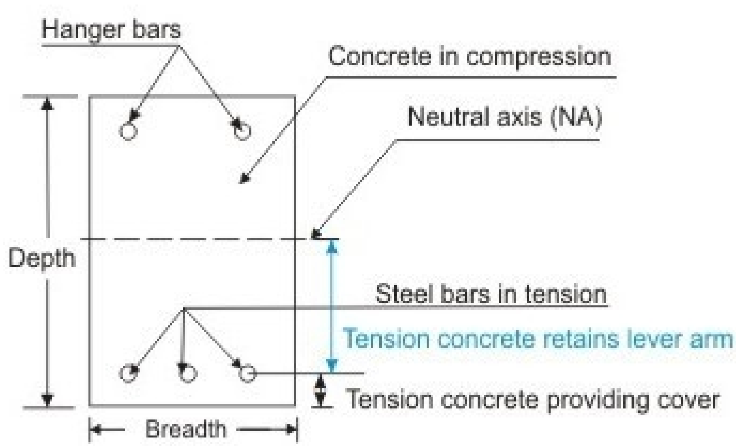


Fig 3.4.4: Singly reinforced rectangular beam under positive bending moment (near midspan)

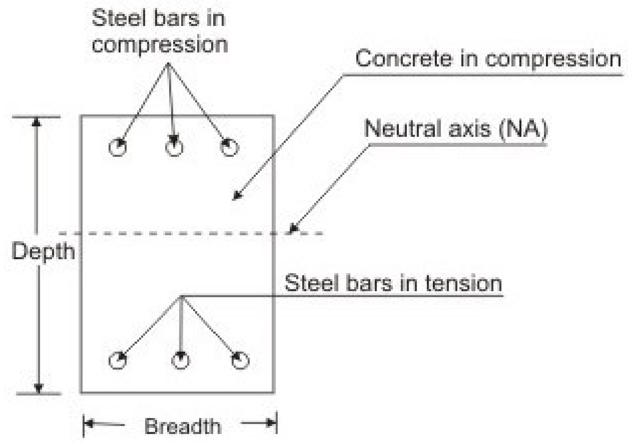


Fig 3.4.5: Doubly reinforced rectangular beam under positive bending moment (near midspan)

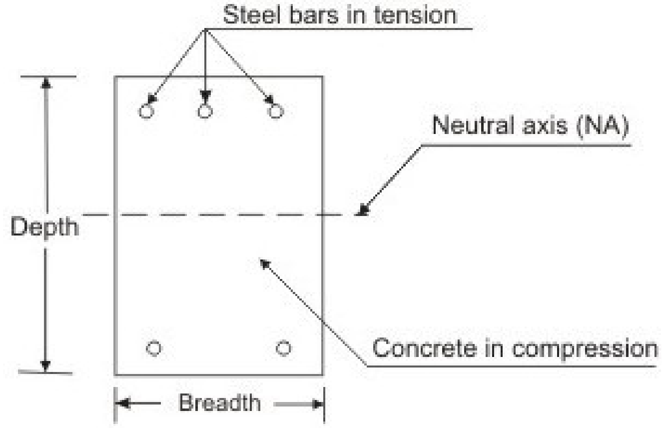


Fig. 3.4.6: Singly reinforced rectangular beam under negative bending moment (over the support)

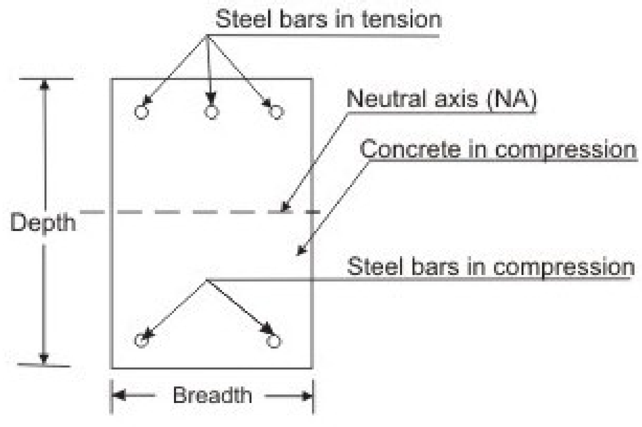


Fig. 3.4.7: Doubly reinforced rectangular beam under negative bending moment (over the support)

This lesson explains the basic governing equations and the computation of parameters required for the design of beams and one-way slabs employing limit state of collapse in flexure. There are three types of reinforced concrete beams:

(i) Singly or doubly reinforced rectangular beams (Figs. 3.4.4 to 7)
(ii) Singly or doubly reinforced *T*-beams (Figs. 3.4.8 to 11)
(iii) Singly or doubly reinforced *L*-beams (Figs. 3.4.12 to 15)

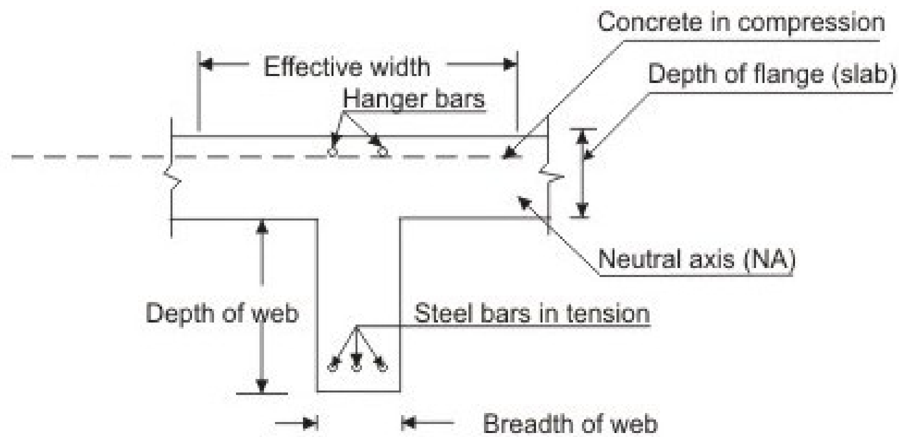


Fig. 3.4.8: Singly reinforced T- beam under positive bending moment (near midspan)

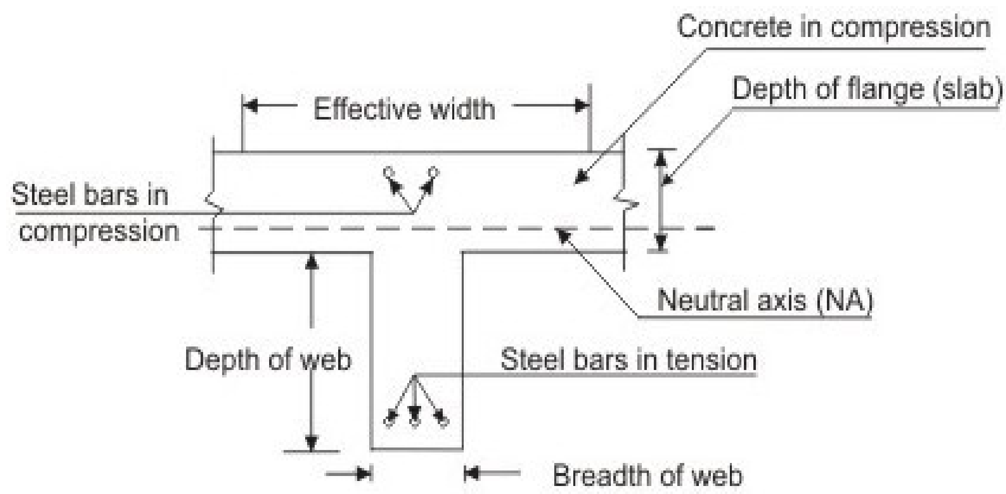
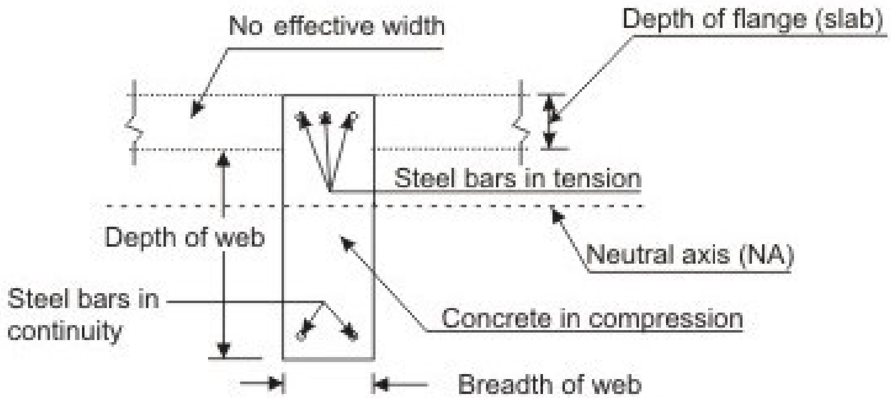
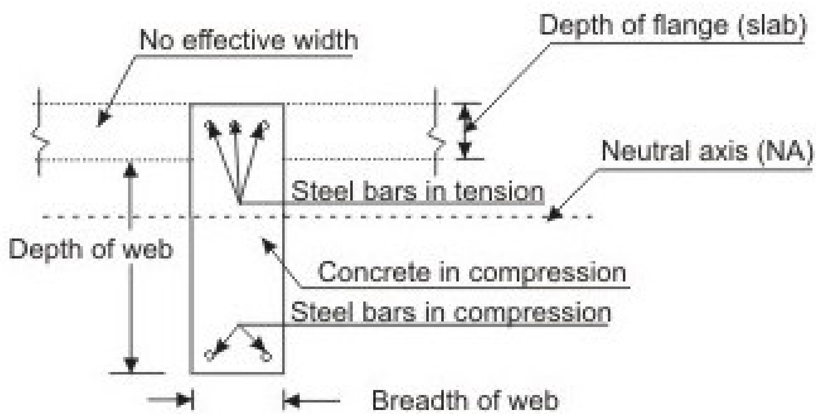


Fig. 3.4.9: Doubly reinforced T- beam under positive bending moment (near midspan)



T-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 3.4.10: Singly reinforced T-beam under negative bending moment (over the support)



T-beam ≡ Rectangular beam (no flange action as concrete above NA is in tension)

Fig. 3.4.11: Doubly reinforced T-beam under negative bending moment (over the support)

During construction of reinforced concrete structures, concrete slabs and beams are cast monolithic making the beams a part of the floor deck system. While bending under positive moments near midspan, bending compression stresses at the top are taken by the rectangular section of the beams above the neutral axis and the slabs, if present in *T* or *L*-beams (Figs. 3.4.4, 5, 8, 9, 12 and 13). However, under the negative moment over the support or elsewhere, the bending compression stresses are at the bottom and the rectangular sections of rectangular, *T* and *L*-beams below the neutral axis only resist that compression (Figs. 3.4.6, 7, 10, 11, 14 and 15). Thus, in a slab-beam system the beam will be considered as rectangular for the negative moment and *T* for the positive moment. While for the intermediate spans of slabs the beam under positive moment is considered as *T*, the end span edge beam is considered as *L*-beam if the slab is not projected on both the sides of the beam. It is worth mentioning that the effective width of flange of these *T* or *L*-beams is to be determined which depends on:

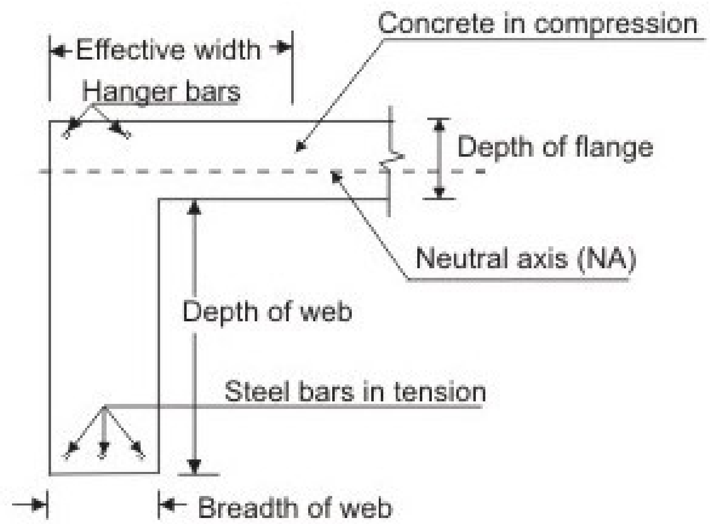


Fig. 3.4.12: Singly reinforced L-beam under positive bending moment (near mid-span)

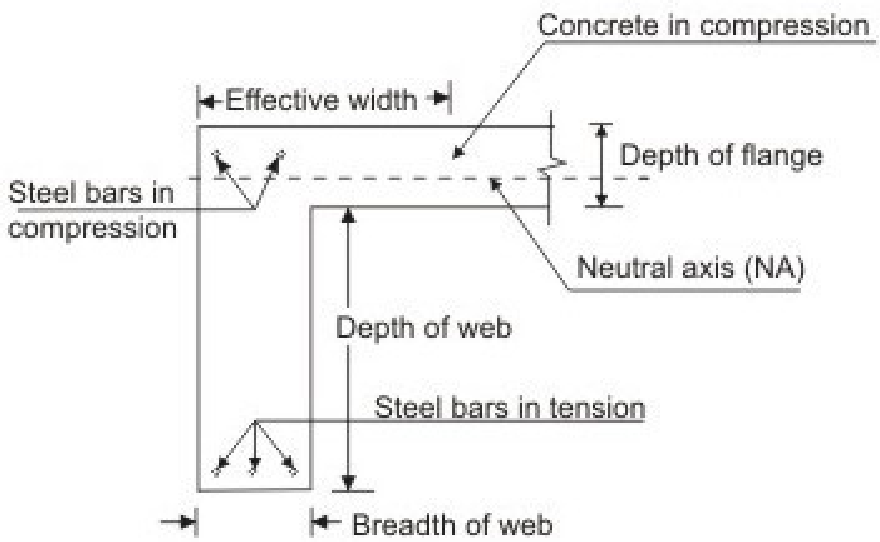


Fig. 3.4.13: Doubly reinforced L-beam under positive bending moment (near mid-span)

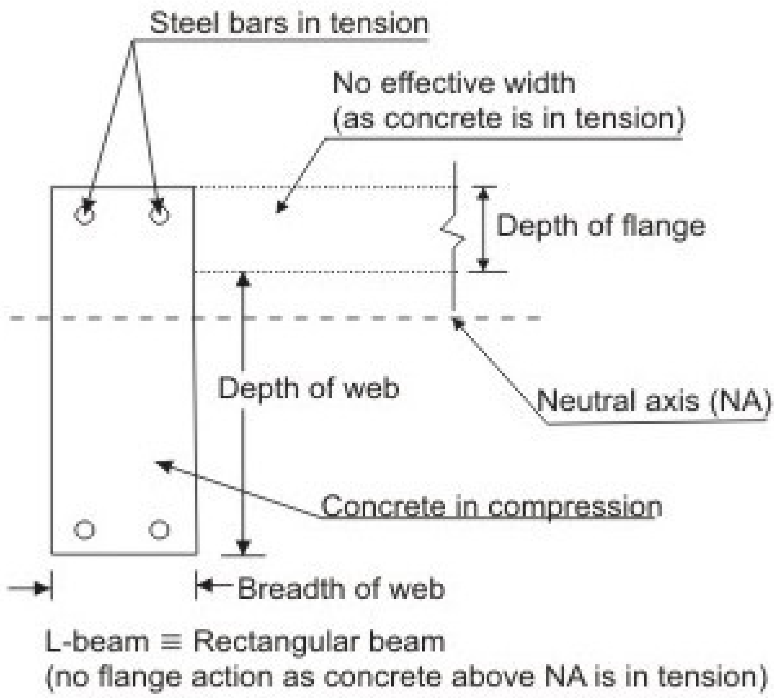


Fig. 3.4.14: Singly reinforced L-beam under negative bending moment (over the support)

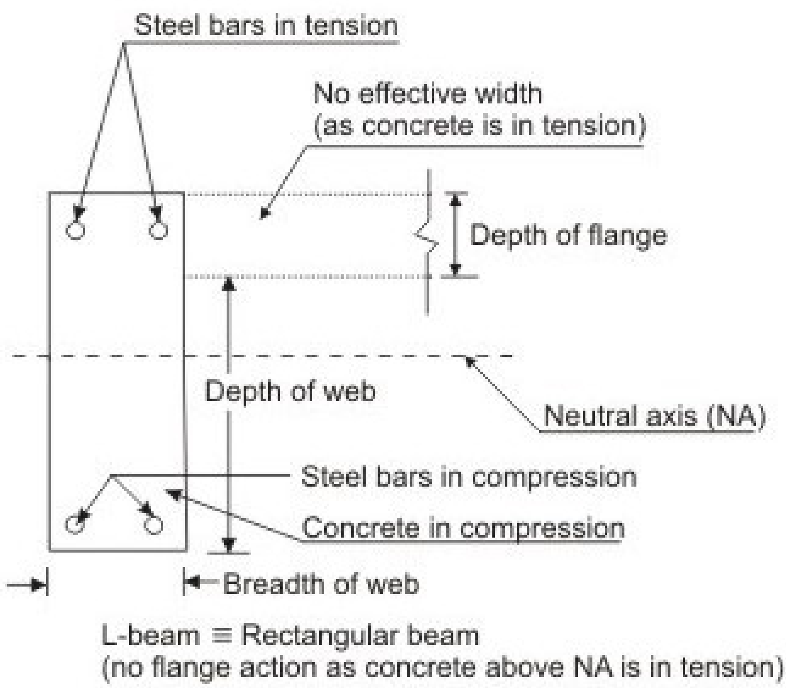


Fig. 3.4.15: Doubly reinforced L-beam under negative bending moment (over the support)

(a) if it is an isolated or continuous beam
(b) the distance between points of zero moments in the beam
(c) the width of the web
(d) the thickness of the flange

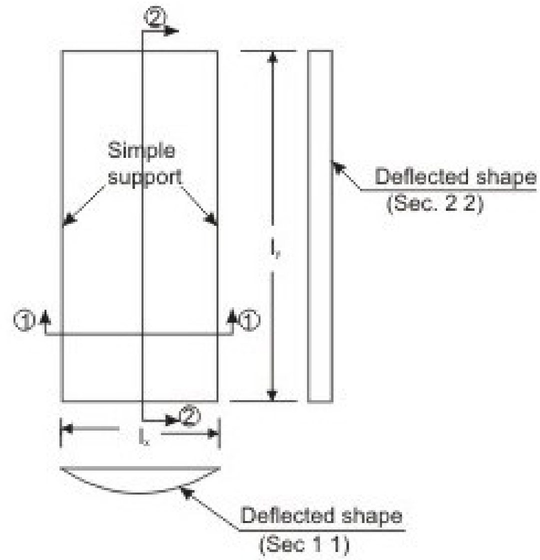


Fig. 3.4.16: One way slab (l_y/l_x >2)

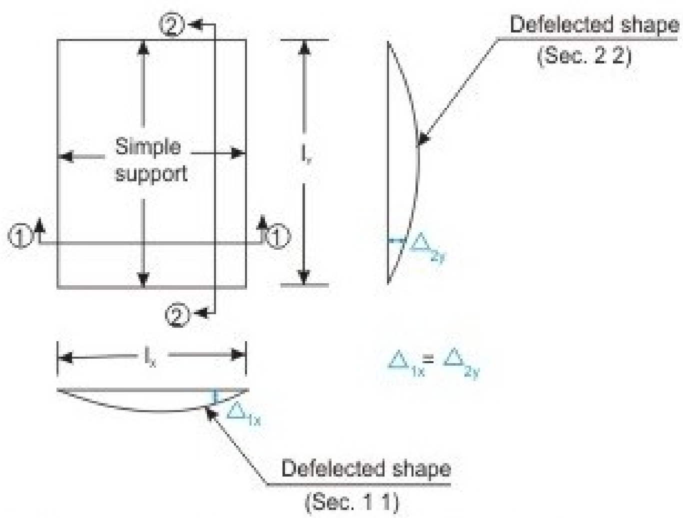


Fig. 3.4.17: Two way slab (ly/l_x <=2)

Reinforced concrete slabs are classified as one-way or two-way depending on if they are spanning in one or two directions (Figs. 3.4.16 and 17). As a guideline, slabs whose ratio of longer span (l_y) to the shorter span (l_x) is more than two are considered as one-way slabs. One-way slabs also can be designed following the procedure of the design of beams of rectangular cross- section. Again, slabs may be isolated or continuous also.

Assumptions

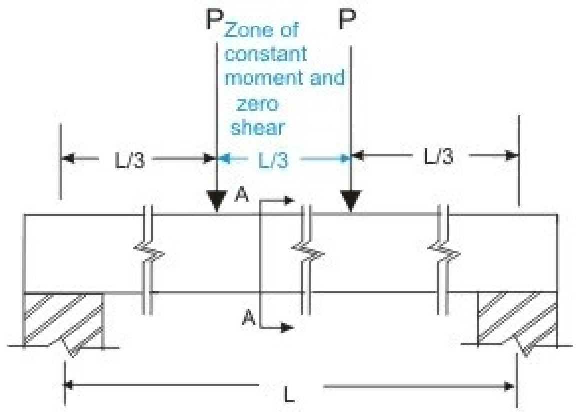


Fig. 3.4.18: Rectangular beam under flexure

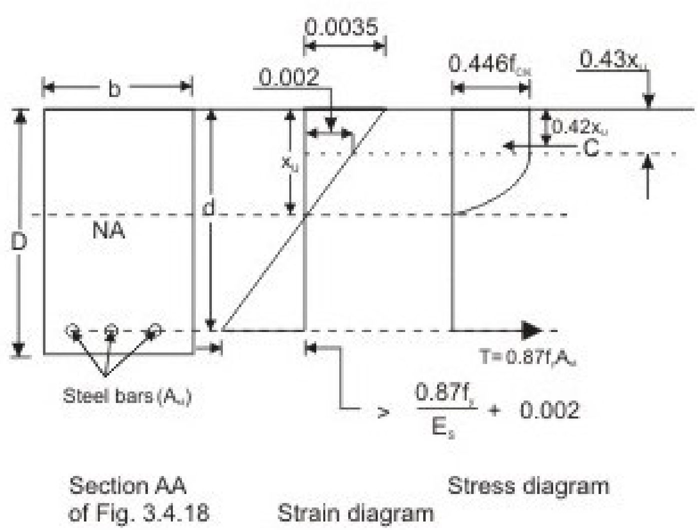


Fig. 3.4.19: Rectangular beam under flexure when $x_u < x_{u,max}$

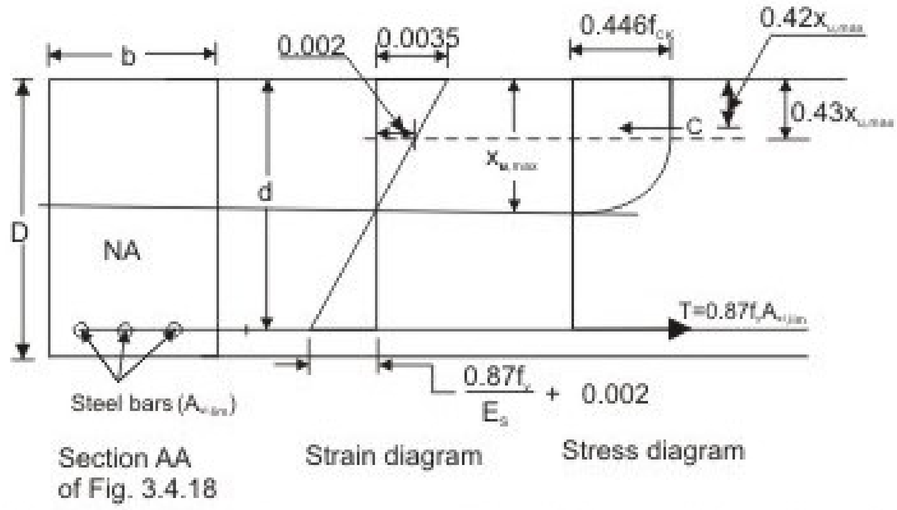


Fig. 3.4.20: Rectangular beam under flexure when $x_u = x_{u,max}$

The following are the assumptions of the design of flexural members (Figs. 3.4.18 to 20) employing limit state of collapse:

(i) Plane sections normal to the axis remain plane after bending.

This assumption ensures that the cross-section of the member does not warp due to the loads applied. It further means that the strain at any point on the cross-section is directly proportional to its distance from the neutral axis.

(ii) The maximum strain in concrete at the outer most compression fibre is taken as 0.0035 in bending (Figs. 3.4.19 and 20).

This is a clearly defined limiting strain of concrete in bending compression beyond which the concrete will be taken as reaching the state of collapse. It is very clear that the specified limiting strain of 0.0035 does not depend on the strength of concrete.

(iii) The acceptable stress-strain curve of concrete is assumed to be parabolic as shown in Fig. 1.2.1 of Lesson 2.

The maximum compressive stress-strain curve in the structure is obtained by reducing the values of the top parabolic curve (Figs. 21 of IS 456:2000) in two stages. First, dividing by 1.5 due to size effect and secondly, again dividing by 1.5 considering the partial safety factor of the material. The middle and bottom curves (Fig. 21 of IS 456:2000) represent these stages. Thus, the maximum compressive stress in bending is limited to the constant value of 0.446 f_{ck} for the strain ranging from 0.002 to 0.0035 (Figs. 3.4.19 and 20, Figs. 21 and 22 of IS 456:2000).

(iv) The tensile strength of concrete is ignored.

Concrete has some tensile strength (very small but not zero). Yet, this tensile strength is ignored and the steel reinforcement is assumed to resist the tensile stress. However, the tensile strength of concrete is taken into account to check the deflection and crack widths in the limit state of serviceability.

(v) The design stresses of the reinforcement are derived from the representative stress-strain curves as shown in Figs. 1.2.3 and 4 of Lesson 2 and Figs. 23A and B of IS 456:2000, for the type of steel used using the partial safety factor γ_m as 1.15.

In the reinforced concrete structures, two types of steel are used: one with definite yield point (mild steel, Figs. 1.2.3 of Lesson 2 and Figs. 23B of IS 456:2000) and the other where the yield points are not definite (cold work deformed bars). The representative stress-strain diagram (Fig. 1.2.4 of Lesson 2 and Fig. 23A of IS 456:2000) defines the points between 0.8 f_y and 1.0 f_y in case of cold work deformed bars where the curve is inelastic.

(vi) The maximum strain in the tension reinforcement in the section at failure shall not be less than $f_y/(1.15\ E_s) + 0.002$, where f_y is the characteristic strength of steel and E_s = modulus of elasticity of steel (Figs. 3.4.19 and 20).

This assumption ensures ductile failure in which the tensile reinforcement undergoes a certain degree of inelastic deformation before concrete fails in compression.

Singly Reinforce Rectangular Beams

Figure 3.4.18 shows the singly reinforced rectangular beam in flexure. The following notations are used (Figs. 3.4.19 and 20):

A_{st} = area of tension steel
b = width of the beam
C = total compressive force of concrete
d = effective depth of the beam
L = centre to centre distance between supports
P = two constant loads acting at a distance of L/3 from the two supports of the beam
T = total tensile force of steel
x_u = depth of neutral axis from the top compression fibre

Equations of Equilibrium

The cross-sections of the beam under the applied loads as shown in Fig. 3.4.18 has three types of combinations of shear forces and bending moments: (i) only shear force is there at the support and bending moment is zero, (ii) both bending moment (increasing gradually) and shear force (constant = *P*) are there between the support and the loading point and (iii) a constant moment (= *PL*/3) is there in the middle third zone i.e. between the two loads where the shear force is zero (Fig. 1.1.1 of Lesson 1). Since the beam is in static equilibrium, any cross- section of the beam is also in static equilibrium. Considering the cross-section in the middle zone (Fig. 3.4.18) the three equations of equilibrium are the following (Figs. 3.4.19 and 20):

(i) Equilibrium of horizontal forces: $\Sigma H = 0$ gives $T = C$
(3.1)

(ii) Equilibrium of vertical shear forces: $\Sigma V = 0$
(3.2)

This equation gives an identity 0 = 0 as there is no shear in the middle third zone of the beam.

(iii) Equilibrium of moments: $\Sigma M = 0$,
(3.3)

This equation shows that the applied moment at the section is fully resisted by moment of the resisting couple $T\,a = C\,a$, where *a* is the operating lever arm between *T* and *C* (Figs. 3.4.19 and 20).

Computations of *C* and *T*

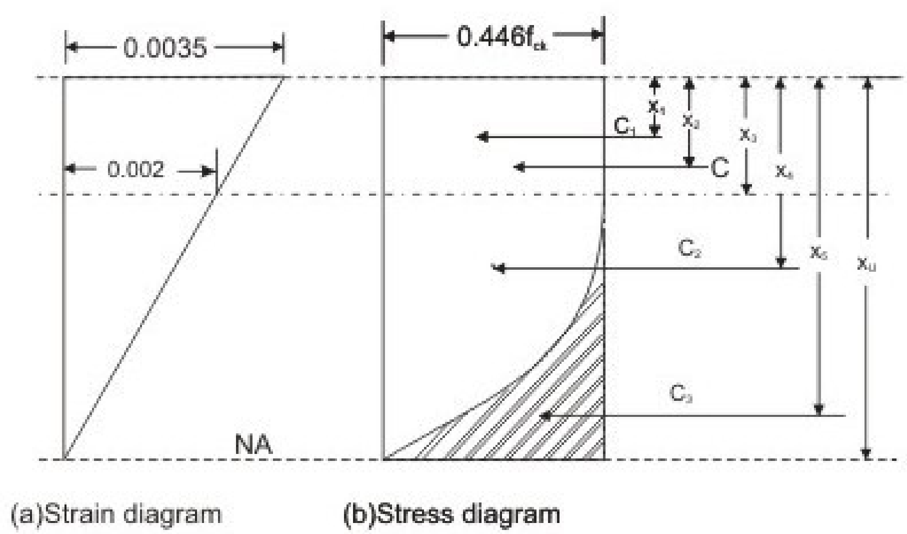


Fig. 3.4.21: Stress and strain diagrams above neutral axis

Figures 3.4.21a and b present the enlarged view of the compressive part of the strain and stress diagrams. The convex parabolic part of the stress block of Fig. 3.4.21b is made rectangular by dotted lines to facilitate the calculations adding another concave parabolic stress zone which is really non-existent as marked by hatch in Fig. 3.4.21b.

The different compressive forces C, C_1, C_2 and C_3 and distances x_1 to x_5 and x_u as marked in Fig. 3.4.21b are explained in the following:

C = Total compressive force of concrete = $C_1 + C_2$

C_1 = Compressive force of concrete due to the constant stress of 0.446 f_{ck} and up to a depth of x_3 from the top fibre

C_2 = Compressive force of concrete due to the convex parabolic stress block of values ranging from zero at the neutral axis to 0.446 f_{ck} at a distance of x_3 from the top fibre

C_3 = Compressive force of concrete due to the concave parabolic stress block (actually non-existent) of values ranging from 0.446 f_{ck} at the neutral axis to zero at a distance of x_3 from the top fibre

x_1 = Distance of the line of action of C_1 from the top compressive fibre

x_2 = Distance of the line of action of C (= $C_1 + C_2$) from the top compressive fibre

x_3 = Distance of the fibre from the top compressive fibre, where the strain = 0.002 and stress = 0.446 f_{ck}

x_4 = Distance of the line of action of C_2 from the top compressive fibre x_5 = Distance of the line of action of C_3 from the top compressive fibre x_u = Distance of the neutral axis from the top compressive fibre.

From the strain triangle of Fig. 3.4.21a, we have

$$\frac{x_u - x_3}{x_u} = \frac{0.002}{0.0035} = \frac{4}{7} = 0.57 \text{, giving}$$

$$x_3 = 0.43\, x_u \qquad (3.4)$$

Since C_1 is due to the constant stress acting from the top to a distance of x_3, the distance x_1 of the line of action of C_1 is:

$x_1 = 0.5\, x_3 = 0.215\, x_u$

(3.5)

From Fig. 3.4.21a:

$$x_5 = x_3 + \frac{3}{4}(x_u - x_3) = 0.43\, x_u + 0.75(0.57\, x_u)$$

or $x_5 = 0.86\, x_u$

(3.6)

The compressive force C_1 due to the rectangular stress block is:

$$C_1 = b\, x_3(0.446\, f_{ck}) = 0.191\, b\, x_u\, f_{ck}$$

(3.7)

The compressive force C_2 due to parabolic stress block is:

$$C_2 = b\,(x_u - x_3)\frac{2}{3}(0.446\, f_{ck}) = 0.17\, b\, x_u\, f_{ck}$$

(3.8)

Adding C_1 and C_2, we have

$$C = C_1 + C_2 = 0.361\, b\, x_u\, f_{ck} = 0.36\, b\, x_u\, f_{ck} \text{ (say)}$$

(3.9)

The non-existent compressive force C_3 due to parabolic (concave) stress block is:

$$C_3 = b\,(x_u - x_3)\frac{1}{3}(0.446\, f_{ck}) = 0.085\, b\, x_u\, f_{ck}$$

(3.10)

Now, we can get x_4 by taking moment of C_2 and C_3 about the top fibre as follows:

$$C_2(x_4) + C_3\,(x_5) = (C_2 + C_3)\left(x_3 + \frac{x_u - x_3}{2}\right)$$

which gives $x_4 = 0.64\ x_u$

(3.11)

Similarly, x_2 is obtained by taking moment of C_1 and C_2 about the top fibre as follows:

$$C_1(x_1) + C_2(x_4) = C(x_2)$$

which gives $x_2 = 0.4153\ x_u$

or $x_2 = 0.42\ x_u$ (say).

(3.12)

Thus, the required parameters of the stress block (Fig. 3.4.19) are

$$C = 0.36\ b\ x_u\ f_{ck} \qquad (3.9)$$

$$x_2 = 0.42\ x_u \qquad (3.12)$$

and lever arm = $(d - x_2) = (d - 0.42\ x_u)$ (3.13)

The tensile force T is obtained by multiplying the design stress of steel with the area of steel. Thus,

$$T = \left(\frac{f_y}{1.15}\right) A_{st} = 0.87\ f_y$$

(3.14)

Introduction to neutral axis

After learning the basic assumptions, the three equations of equilibrium and the computations of the total compressive and tensile forces in Lesson 4, it is now required to determine the depth of neutral axis (NA) and then to estimate the moment of resistance of the beams. These two are determined using the two equations of equilibrium (Eqs. 3.1 and 3.3). It has been explained that the depth of neutral axis has important role to estimate the moment of resistance. Accordingly, three different cases are illustrated in this lesson.

Computation of the Depth of Neutral Axis x_u

From Eqs. 3.1, 9 and 14, we have

$$0.87 f_y A_{st} = 0.36\, b\, x_u f_{ck} \tag{3.15}$$

or

$$x_u = \frac{0.87\, f_y A_{st}}{0.36\, b\, f_{ck}} \tag{3.16}$$

We can also write:

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36\, b\, d f_{ck}} \tag{3.17}$$

Substituting the expression of $\frac{x_u}{d}$ from Eq. 3.17 into Eq. 3.13 of Lesson 4, we have

$$\text{lever arm} = d\left\{1 - 0.42\left(\frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b\, d}\right)\right\}$$

$$= d\left(1 - 1.015\,\frac{A_{st}\, f_y}{f_{ck}\, b\, d}\right) \tag{3.18}$$

Ignoring multiplying factor 1.015 in Eq. 3.18, we have

$$\text{lever arm} = d\left(1 - \frac{A_{st}\, f_y}{f_{ck}\, b\, d}\right) \tag{3.19}$$

Limiting Value of x_u (= $x_{u,\,max}$)

Should there be a limiting or maximum value of x_u?

Equation 3.17 reveals that $\frac{x_u}{d}$ increases with the increase of percentage of steel reinforcement $\frac{A_{st}}{b\,d}$ for fixed values of f_y and f_{ck}. Thus, the depth of the neutral axis x_u will tend to reach the depth of the tensile steel. But, that should not be allowed. However, let us first find out that value of x_u which will satisfy assumptions (ii) and (vi) of sec. 3.4.2 and designate that by $x_{u,\,max}$ for the present, till we confirm that x_u should have a limiting value.

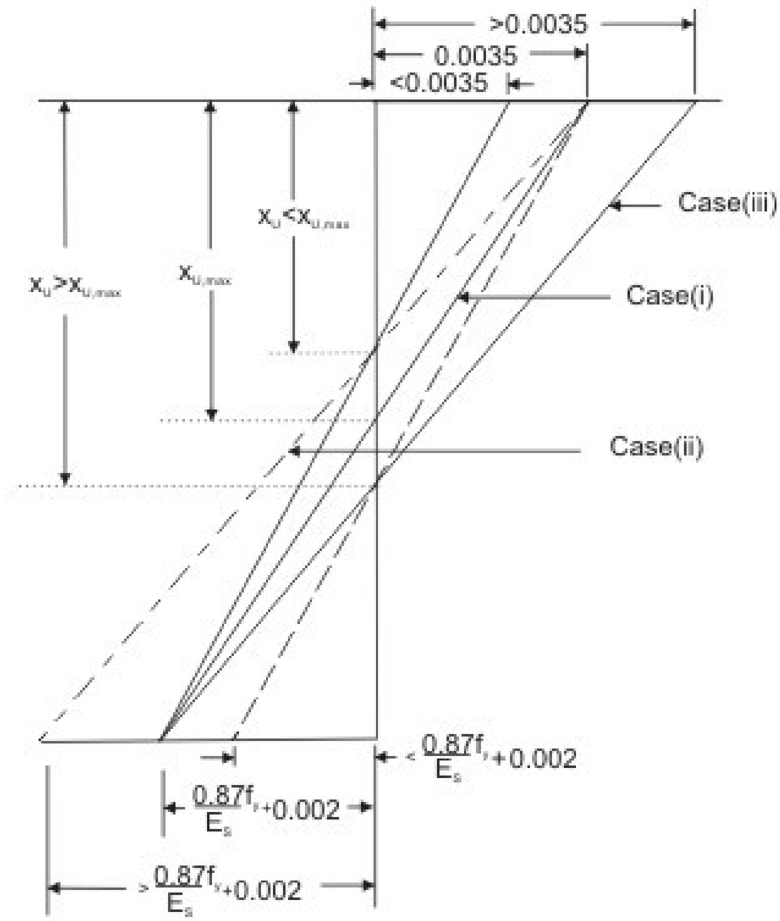


Fig.3.5.1: Strain of steel for three cases $x_u <=> x_{u,max}$

Figure 3.5.1 presents the strain diagrams for the three cases: (i) when $x_u = x_{u,\,max}$; (ii) when x_u is less than $x_{u,\,max}$ and (iii) when x_u is greater than $x_{u,\,max}$. The following discussion for the three cases has the reference to Fig. 3.5.1.

(i) When $x_u = x_{u,\,max}$

The compressive strain at the top concrete fibre = 0.0035 and the tensile strain at the level of steel = $\left(\frac{0.87 f_y}{E_s} + 0.002 \right)$. Thus, it satisfies the assumptions (ii) and (vi) of sec. 3.4.2.

(ii) When x_u is less than $x_{u,\,max}$

There are two possibilities here:

(a) If the compressive strain at the top fibre = 0.0035, the tensile strain is more than $\left(\frac{0.87 f_y}{E_s} + 0.002\right)$. Thus, this possibility satisfies the two assumptions (ii) and (vi) of sec. 3.4.2.

(b) When the steel tensile strain is $\left(\frac{0.87 f_y}{E_s} + 0.002\right)$, the compressive concrete strain is less than 0.0035. Here also, both the assumptions (ii) and (vi) of sec. 3.4.2 are satisfied.

(iii) When x_u is more than $x_{u,\,max}$

There are two possibilities here:

(a) When the top compressive strain reaches 0.0035, the tensile steel strain is less than $\left(\frac{0.87 f_y}{E_s} + 0.002\right)$. It violets the assumption (vi) though assumption (ii) of sec. 3.4.2 is satisfied.

(b) When the steel tensile strain is $\left(\frac{0.87 f_y}{E_s} + 0.002\right)$, the compressive strain of concrete exceeds 0.0035. Thus, this possibility violets assumption (ii) though assumption (vi) is satisfied.

The above discussion clearly indicates that the depth of x_u should not become more than $x_{u,\,max}$. Therefore, the depth of the neutral axis has a limiting or maximum value = $x_{u,\,max}$. Accordingly, if the A_{st} provided yields $x_u > x_{u,\,max}$, the section has to be redesigned.

Since x_u depends on the area of steel, we can calculate $A_{st,\,lim}$ from Eq. 3.17.

From Eq. 3.17 (using $x_u = x_{u,max}$ and $A_{st} = A_{st,\,lim}$), we have

$$\frac{x_{u,\max}}{d} = \frac{0.87 f_y \; A_{st,\lim}}{0.36 \; b\, d\, f_{ck}}$$

or $$\frac{A_{st,lim}}{b\,d}(100) = p_{t,lim} = \left(\frac{0.36\; f_{ck}\; x_{u,\max}}{0.87\, f_y\, d}\right)(100) \qquad (3.20)$$

In the above equation $\frac{x_{u,\max}}{d}$ can be obtained from the strain diagram of Fig. 3.5.1 as follows:

$$\frac{x_{u,\max}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

(3.21)

Values of $\frac{x_{u,\max}}{d}$ and $p_{t,\,lim}$

Equation 3.20 shows that the values of $p_{t,\,lim}$ depend on both the grades of steel and concrete, while Eq. 3.21 reveals that $\frac{x_{u,\max}}{d}$ depends on the grade of steel alone and not on the grade of concrete at all. The respective values of $p_{t,\,lim}$ for the three grades of steel and the three grades of concrete are presented in Table 3.1. Similarly, the respective values of $\frac{x_{u,\max}}{d}$ for three grades of steel are presented in Table 3.2.

Table 3.1 Values of $p_{t,\,lim}$

f_{ck} in N/mm^2	f_y = 250 N/mm^2	f_y = 415 N/mm^2	f_y = 500 N/mm^2
20	1.76	0.96	0.76
25	2.20	1.19	0.94
30	2.64	1.43	1.13

Table 3.2 Values of $\frac{x_{u,\max}}{d}$

f_y in N/mm^2	250	415	500
$\frac{x_{u,\max}}{d}$	0.531 = 0.53 (say)	0.479 = 0.48 (say)	0.456 = 0.46 (say)

A careful study of Tables 3.1 and 3.2 reveals the following:

(i) The $p_{t,\ lim}$ increases with lowering the grade of steel for a particular grade of concrete. The $p_{t,\ lim}$, however, increases with increasing the grade of concrete for a specific grade of steel.

(ii) The maximum depth of the neutral axis $x_{u,max}$ increases with lowering the grade of steel. That is more area of the section will be utilized in taking the compression with lower grade of steel.

Computation of M_u

Equation 3.3 of Lesson 4 explains that M_u can be obtained by multiplying the tensile force T or the compressive force C with the lever arm. The expressions of C, lever arm and T are given in Eqs. 3.9, 3.13 (also 3.19) and

3.14 respectively of Lesson 4. Section 3.5.3 discusses that there are three possible cases depending on the location of x_u. The corresponding expressions of M_u are given below for the three cases:

(i) When $x_u < x_{u,\ max}$

Figure 3.5.1 shows that when concrete reaches 0.0035, steel has started flowing showing ductility (Strain > $\frac{0.87 f_y}{E_s} + 0.002$). So, the computation of M_u is to be done using the tensile force of steel in this case. Therefore, using Eqs. 3.13 and 3.14 of Lesson 4, we have

$$M_u = T \text{ (lever arm)} = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u) \tag{3.22}$$

Substituting the expressions of T and lever arm from Eqs. 3.14 of Lesson 4 and 3.19 respectively we get,

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck}\, b\, d}\right) \tag{3.23}$$

(ii) When $x_u = x_{u,\ max}$

From Fig. 3.5.1, it is seen that steel just reaches the value of $\frac{0.87 f_y}{E_s} +$

0.002

and concrete also reaches its maximum value. The strain of steel can further increase but the reaching of limiting strain of concrete should be taken into consideration to determine the limiting M_u as $M_{u,\ lim}$ here. So, we have

$$M_{u,\ lim} = C \text{ (lever arm)}$$

Substituting the expressions of C and lever arm from Eqs. 3.9 of Lesson 4 and 3.19 respectively, we have

$$M_{u,\lim} = 0.36 \frac{x_{u,\max}}{d} \left(1 - 0.42 \frac{x_{u,\max}}{d}\right) b\, d^2 f_{ck}$$

(3.24)

(iii) When $x_u > x_{u,\ max}$

In this case, it is seen from Fig. 3.5.1 that when concrete reaches the strain of 0.0035, tensile strain of steel is much less than ($\frac{0.87 f_y}{E_s} + 0.002$) and any further increase of strain of steel will mean failure of concrete, which is to be avoided. On the other hand, when steel reaches $\frac{0.87 f_y}{E_s} + 0.002$, the strain of concrete far exceeds 0.0035. Hence, it is not possible. Therefore, such design is avoided and the section should be redesigned.

However, in case of any existing reinforced concrete beam where $x_u > x_{u,\,max}$, the moment of resistance M_u for such existing beam is calculated by restricting x_u to $x_{u,\,max}$ only and the corresponding M_u will be as per the case when $x_u = x_{u,\,max}$.

Computation of Limiting Moment of Resistance Factor

Equation 3.24 shows that a particular rectangular beam of given dimensions of b and d has a limiting capacity of $M_{u,\,lim}$ for a specified grade of concrete. The limiting moment of resistance factor $R_{,lim}$ (= $M_{u,lim}/bd^2$) can be established from Eq. 3.24 as follows:

$$R_{,\lim} = \frac{M_{u,\lim}}{b\,d^2} = 0.36\,\frac{x_{u,\max}}{d}\left(1 - 0.42\,\frac{x_{u,\max}}{d}\right) f_{ck}$$

(3.25)

It is seen that the limiting moment of resistance factor $R_{,lim}$ depends on $\frac{x_{u,\max}}{d}$ and f_{ck}. Since $\frac{x_{u,\max}}{d}$ depends on the grade of steel f_y, we can say that $R_{,lim}$ depends on f_{ck} and f_y. Table 3.3 furnishes the values of $R_{,lim}$ for three grades of concrete and three grades of steel.

Table 3.3 Limiting values of $R_{,\lim} = \frac{M_{u,lim}}{b\,d^2}$ factors (in N/mm^2)

f_{ck} in N/mm^2	f_y = 250 N/mm^2	f_y = 415 N/mm^2	f_y = 500 N/mm^2
20	2.98	2.76	2.66
25	3.73	3.45	3.33
30	4.47	4.14	3.99

A study of Table 3.3 reveals that the limiting moment of resistance factor $R_{,lim}$ increases with higher grade of concrete for a particular grade of steel. It is also seen that this factor increases with lowering the grade of steel for a particular grade of concrete. The increase of this factor due to higher grade of concrete is understandable. However, such increase of the factor with lowering the grade of steel is explained below:

Lowering the grade of steel increases the $\frac{x_{u,\max}}{d}$ (vide Table 3.2) and this enhanced $\frac{x_{u,\max}}{d}$ increases M_u as seen from Eq. 3.24. However, one may argue that Eq. 3.24 has two terms: $\frac{x_{u,\max}}{d}$ and $\left(1 - 0.42\frac{x_{u,\max}}{d}\right)$

With the increase of $\frac{x_{u,\max}}{d}$, $\left(1 - 0.42\frac{x_{u,\max}}{d}\right)$ is decreasing. Then how do we confirm that the product is increasing with the increase of $\frac{x_{u,\max}}{d}$? Actual computation will reveal the fact. Otherwise, it can be further explained from Table 3.1 that as the grade of steel is lowered for a particular grade of concrete, the $p_{t,\,lim}$ gets increased. Therefore, amount of steel needed to have $M_{u,\,lim}$ with lower grade of steel is higher. Thus, higher amount of steel and higher values of $\frac{x_{u,\max}}{d}$ show higher $\frac{M_{u,\lim}}{b\,d^2}$ factor with the lowering of grade of steel for a particular grade of concrete (see Table 3.3).

Types of Problems

Two types of problems are possible: (i) design type and (ii) analysis type. In the design type of problems, the designer has to determine the dimensions b, d, D, A_{st} (Fig. 3.6.1) and other detailing of reinforcement, grades of concrete and steel from the given design moment of the beam. In the analysis type of the problems, all the above data will be known and the designer has to find out the moment of resistance of the beam. Both the types of problems are taken up for illustration in the following two lessons.

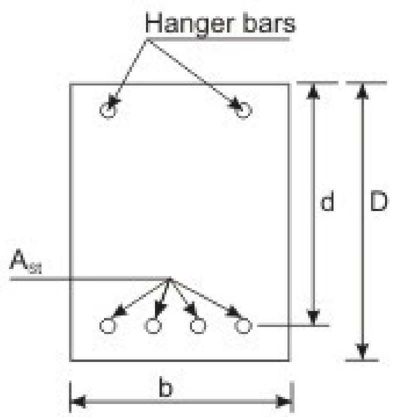


Fig. 3.6.1: Typical section of a beam

Design Type of Problems

The designer has to make preliminary plan lay out including location of the beam, its span and spacing, estimate the imposed and other loads from the given functional requirement of the structure. The dead loads of the beam are estimated assuming the dimensions *b* and *d* initially. The bending moment, shear force and axial thrust are determined after estimating the different loads. In this illustrative problem, let us assume that the imposed and other loads are given. Therefore, the problem is such that the designer has to start with some initial dimensions and subsequently revise them, if needed. The following guidelines are helpful to assume the design parameters initially.

Selection of breadth of the beam *b*

Normally, the breadth of the beam *b* is governed by: (i) proper housing of reinforcing bars and (ii) architectural considerations. It is desirable that the width of the beam should be less than or equal to the width of its supporting structure like column width, or width of the wall etc. Practical aspects should also be kept in mind. It has been found that most of the requirements are satisfied with *b* as 150, 200, 230, 250 and 300 mm. Again, width to overall depth ratio is normally kept between 0.5 and 0.67.

Selection of depths of the beam *d* and *D*

The effective depth has the major role to play in satisfying (i) the strength requirements of bending moment and shear force, and (ii) deflection of the beam. The initial effective depth of the beam, however, is assumed to satisfy the deflection requirement depending on the span and type of the reinforcement. IS 456 stipulates the basic ratios of span to effective depth of beams for span up to 10 m as (Clause 23.2.1)

Cantilever	7
Simply supported	20
Continuous	26

For spans above 10 m, the above values may be multiplied with 10/span in metres, except for cantilevers where the deflection calculations should be made. Further, these ratios are to be multiplied with the modification factor depending on reinforcement percentage and type. Figures 4 and 5 of IS 456 give the different values of modification factors. The total depth *D* can be determined by adding 40 to 80 mm to the effective depth.

Selection of the amount of steel reinforcement A_{st}

The amount of steel reinforcement should provide the required tensile force *T* to resist the factored moment M_u of the beam. Further, it should satisfy the minimum and maximum percentages of reinforcement requirements also. The minimum reinforcement A_s is provided for creep, shrinkage, thermal and other environmental requirements irrespective of the strength requirement. The minimum reinforcement A_s to be provided in a beam depends on the f_y of steel and it follows the relation: (cl. 26.5.1.1a of IS 456)

$$\frac{A_s}{b\,d} = \frac{0.85}{f_y} \tag{3.26}$$

The maximum tension reinforcement should not exceed 0.04 *bD* (cl. 26.5.1.1b of IS 456), where *D* is the total depth.

Besides satisfying the minimum and maximum reinforcement, the amount of reinforcement of the singly reinforced beam should normally be 75 to 80% of $p_{t,\,lim}$. This will ensure that strain in steel will be more than $\left(\frac{0.87 f_y}{E_s} + 0.02\right)$ as the design stress in steel will be 0.87 f_y. Moreover, in many cases, the depth required for deflection becomes more than the limiting depth required to resist $M_{u,\,lim}$. Thus, it is almost obligatory to provide more depth. Providing more depth also helps in the amount of the steel which is less than that required for $M_{u,\,lim}$. This helps to ensure ductile failure. Such beams are designated as under-reinforced beams.

Selection of diameters of bar of tension reinforcement

Reinforcement bars are available in different diameters such as 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36 and 40 mm. Some of these bars are less available. The selection of the diameter of bars depends on its availability, minimum stiffness to resist while persons walk over them during construction, bond requirement etc. Normally, the diameters of main tensile bars are chosen from 12, 16, 20, 22, 25 and 32 mm.

Selection of grade of concrete

Besides strength and deflection, durability is a major factor to decide on the grade of concrete. Table 5 of IS 456 recommends M 20 as the minimum grade under mild environmental exposure and other grades of concrete under different environmental exposures also.

Selection of grade of steel

Normally, Fe 250, 415 and 500 are in used in reinforced concrete work. Mild steel (Fe 250) is more ductile and is preferred for structures in earthquake zones or where there are possibilities of vibration, impact, blast etc.

Design Problem 3.1

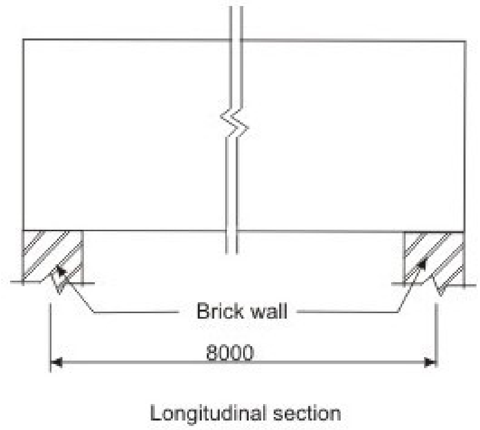


Fig. 3.6.2: Design Problem 3.1

Design a simply supported reinforced concrete rectangular beam (Fig. 3.6.2) whose centre to centre distance between supports is 8 m and supported on brick walls of 300 mm thickness. The beam is subjected to imposed loads of
7.0 kN/m.

Solution by Direct Computation Method

The unknowns are b, d, D, A_{st}, grade of steel and grade of concrete. It is worth mentioning that these parameters have to satisfy different requirements and they also are interrelated. Accordingly, some of them are to be assumed which subsequently may need revision.

Grades of steel and concrete

Let us assume Fe 415 and M 20 are the grades of steel and concrete respectively. As per clause 6.1.2 and Table 5 of IS 456, minimum grade of concrete is M 20 for reinforced concrete under mild exposure (durability requirement).

Effective span L_{eff}

Clause 22.2(a) of IS 456 recommends that the effective span is the lower of (i) clear span plus effective depth and (ii) centre to centre distance between two supports. Here, the clear span is 7700 mm. Thus

(i) Clear span + d = 7700 + 400 (assuming d = 400 from the specified ratio of span to effective depth as 20 and mentioned in the next section)

(ii) Centre to centre distance between two supports = 8000 mm.

Hence, L_{eff} = 8000 mm

Percentage of steel reinforcement p_t

The percentage of steel reinforcement to be provided is needed to determine the modification factor which is required to calculate d. As mentioned earlier in sec. 3.6.2.3, it is normally kept at 75 to 80 per cent of $p_{t,\ lim}$. Here, $p_{t,\ lim}$ = 0.96 (vide Table 3.1 of Lesson 5). So, percentage of steel to be provided is assumed = 0.75 (0.96) = 0.72.

Effective depth d

As per clause 23.2.1 of IS 456, the basic value of span to effective depth ratio here is 20. Further, Fig. 4 of IS 456 presents the modification factor which will be multiplied with the basic span to effective depth ratio. This modification factor is determined on the value of f_s where

$$f_s = 0.58\ f_y\ \frac{\text{Area of cross-section of steel required}}{\text{Area of cross-section of steel provided}}$$

= 0.58 f_y (assuming that the A_{st} provided is the same as A_{st} required)

= 0.58 (415) = 240.7 N/mm^2.

From Fig. 4 of IS 456, the required modification factor is found to be 1.1 for f_s = 240.7 N/mm^2 and percentage of steel = 0.72. So, the span to effective depth ratio = 22 as obtained by multiplying 20 with 1.1. Accordingly, the effective depth = 8000/22 = 363.63 mm, say 365 mm. Since this value of d is different from the d assumed at the beginning, let us check the effective span as lower of (i) 7700 + 365 and (ii) 8000 mm. Thus, the effective span remains at 8000 mm. Adding 50 mm with the effective depth of 365 mm (assuming 50 mm for cover etc.), the total depth is assumed to be 365 + 50 = 415 mm.

Breadth of the beam b

Let us assume b = 250 mm to get b/D = 250/415 = 0.6024, which is acceptable as the ratio of b/D is in between 0.5 and 0.67.

Dead loads, total design loads F_d and bending moment

With the unit weight of reinforced concrete as 25 kN/m^3 (cl. 19.2.1 of IS 456):

Dead load of the beam = 0.25 (0.415) (25) kN/m = 2.59 kN/m Imposed loads = 7.00 kN/m

Thus, total load = 9.59 kN/m, which gives factored load F_d as 9.59 (1.5) (partial safety factor for dead load and imposed load as 1.5) = 14.385 kN/m. We have, therefore, M_u = Factored bending moment = 14.385 (8) = 115.08 kNm.

Checking of effective depth *d*

It is desirable to design the beam as under-reinforced so that the ductility is ensured with steel stress reaching the design value. Let us now determine the limiting effective depth when $x_u = x_{u,\,max}$ and the factored moment $M_u = M_{u,\,lim}$ = 115.08 kNm from Eq. 3.24 of Lesson 5.

$$M_{u,\,\text{lim}} = 0.36 \frac{x_{u,\,\max}}{d}\left\{1 - 0.42\frac{x_{u,\,\max}}{d}\right\} b\,d^2 f_{ck} \tag{3.24}$$

Table 3.2 of Lesson 5 gives $\frac{x_{u,\,\max}}{d} = 0.479$ for f_y = 415 N/mm^2. Thus:

(115.08) 10^6 Nmm = 0.36(0.479) [1 - 0.42(0.479)] $b\,d^2$ (20)

which gives d = 408.76 mm

So, let us revise d = 410 mm from the earlier value of 365 mm to have the total depth = 410 + 50 = 460 mm.

Area of Steel A_{st}

The effective depth of the beam has been revised to 408.76 mm from the limiting moment carrying capacity of the beam. Increasing that depth to 410 also has raised the $M_{u,\,lim}$ of the beam from the design factored moment of 115.08 kNm. Therefore, the area of steel is to be calculated from the moment equation (Eq. 3.23 of Lesson 5), when steel is ensured to reach the design stress f_d = 0.87 (415) = 361.05 N/mm^2.

$$M_u = 0.87\, f_y\, A_{st}\, d \left\{1 - \frac{A_{st}\, f_y}{f_{ck}\, b\, d}\right\} \tag{3.23}$$

Here, all but A_{st} are known. However, this will give a quadratic equation of A_{st} and one of the values, the lower one, will be provided in the beam. The above equation gives:

$$115.08\,(10^6)\text{ Nmm} = 0.87(415)A_{st}(410)\left\{1 - \frac{A_{st}(415)}{20(250)(410)}\right\}\text{ Nmm}$$

$$= 148030.5\,A_{st} - 29.96715\,A_{st}^2$$

or $$A_{st}^2 - 4939.759\,A_{st} + 3840205 = 0$$

which gives

$$A_{st} = 966.5168\text{ mm}^2 \text{ or } 3973.2422\text{ mm}^2$$

The values of x_u determined from Eq. 3.16 of Lesson 5 are 193.87 mm and 796.97 mm respectively, when A_{st} = 966.5168 mm^2 and 3973.2422 mm^2. It is seen that the value of x_u with lower value of A_{st} is less than $x_{u,max}$ (= 216 mm). However, the value of x_u with higher value of A_{st} (= 3973.2422 mm^2) is more than the value of $x_{u,max}$ (= 0.48 d = 216 mm), which is not permissible as it exceeds the total depth of the beam (= 460 mm). In some problems, the value of x_u may be less than the total depth of the beam, but it shall always be more than $x_{u,max.}$ The beam becomes over-reinforced. Therefore, the lower value of the area of steel is to be accepted as the tensile reinforcement out of the two values obtained from the solution of the quadratic equation involving $A_{st.}$

Accepting the lower value of becomes A_{st} = 966.5168 mm^2, the percentage of steel

$$\frac{966.5168\,(100)}{250\,(410)} = 0.9429 \quad \text{per cent}$$

This percentage is higher than the initially assumed percentage as 0.72. By providing higher effective depth, this can be maintained as shown below.

Increase of effective depth and new A_{st}

Increasing the effective depth to 450 mm from 410 mm, we have from Eq. 3.23 of Lesson 5,

$$115.08\,(10^6) = 0.87\,(415)\,A_{st}\,(450)\left[1 - \frac{A_{st}\,(415)}{20\,(250)\,(450)}\right]$$

$$= 162472.5\, A_{st} - 29.967148\, A_{st}^2$$

or $A_{st}^2 - 5421.6871\, A_{st} + 3840205.2 = 0$

or $A_{st} = 0.5\,\{5421.6871 \pm 3746.1808\}$

The lower value of A_{st} now becomes 837.75315 which gives the percentage of A_{st} as

$$\frac{837.75315\,(100)}{250\,(450)} = 0.7446$$, which is close to earlier assumed percentage of 0.72.

Therefore, let us have d = 450 mm, D = 500 m, b = 250 mm and A_{st} = 837.75315 mm^2 for this beam.

For any design problem, this increase of depth is obligatory to satisfy the deflection and other requirements. Moreover, obtaining A_{st} with increased depth employing moment equation (Eq. 3.23 of Lesson 5) as illustrated above, results in under-reinforced beam ensuring ductility.

Further change of A_{st} due to increased dead load

However, increasing the total depth of the beam to 500 mm from earlier value of 415 mm has increased the dead load and hence, the design moment M_u. This can be checked as follows:

The revised dead load = 0.25 (0.5) (25) = 3.125 kN/m Imposed loads =

7.00 kN/m

Total factored load F_d = 1.5(10.125) = 15.1875 kN/m

M_u = 15.1875 (8) = 121.5 kNm

The limiting moment that this beam can carry is obtained from using $M_{u,\,lim}/bd^2$ factor as 2.76 from Table 3.3 of of Lesson 5. Thus,

$M_{u,\,lim} = (2.76)\, bd^2 = (2.76)\,(250)\,(450)^2$ Nmm

$= 139.72$ kNm > (M_u = 121.5 kNm)

Hence, it is under-reinforced beam.

Equation 3.23 of Lesson 5 is now used to determine the A_{st} for M_u = 121.5 kNm

$$M_u = 0.87 f_y A_{st} d \left\{ 1 - \frac{A_{st} f_y}{f_{ck} b d} \right\} \qquad (3.23)$$

or $\quad 121.5\,(10^6) = 0.87\,(415)\, A_{st}\,(450) \left\{ 1 - \frac{A_{st}\,(415)}{20\,(250)\,(450)} \right\}$

$$= 162472.5\, A_{st} - 29.96715\, A_{st}^2$$

or $\quad A_{st} = 0.5\,\{5421.6867 \pm 3630.0038\} = 895.84145\ \text{mm}^2$

The steel reinforcement is of $p_{t,lim}$. $\frac{895.84\,(100)}{250\,(450)}$ = 0.7963 per cent which is 83 per cent

So, we have the final parameters as b = 250 m, d = 450 mm, D = 500 mm, A_{st} = 895.84 mm^2. A selection of 2-20 T bars and 2-14 T bars gives the A_{st} = 935 mm^2 (Fig. 3.6.3). Though not designed, Fig. 3.6.3 shows the holder bars and stirrups also.

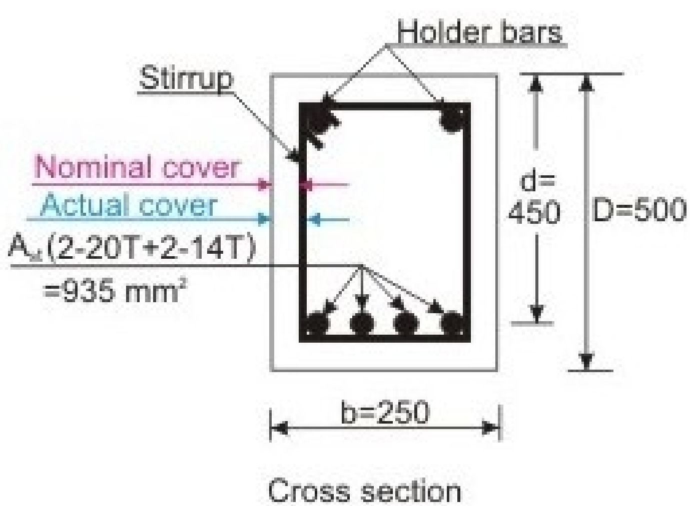


Fig. 3.6.3: Design Problem 3.1

Summary of steps

Table 3.4 presents the complete solution of the problem in eleven steps. Six columns of the table indicate (i) parameters assumed/determined, (ii) if they need revision, (iii) final parameters, (iv) major requirements of the parameter, (v) reference section numbers, and (vi) reference source material.

Table 3.4 Steps of the illustrative problem

Step	Assumed/ determined parameter(s) (i)	If need(s) revision (ii)	Final parameter(s) (iii)	Major requirement of the parameter(s) (iv)	Reference section number (v)	Reference source Material(s) (vi)
1	f_{ck}, f_y	No	f_{ck}, f_y	Durability for f_{ck} and ~~ductility~~ for f_y	3.6.4.1	cl.6.1.2, cl. 8 and Table 5 of IS 456
2	d	Yes	No	$d = \frac{\text{c/c span}}{20}$	3.6.4.2	cl. 23.2 of IS 456
3	L_{eff}	Yes	No	Boundary conditions	3.6.4.2	cl.22.2 of IS 456
4	$p = A_{st}/bd$	No	Yes	Ductility (p = 75 to 80% of $p_{t,\ lim}$)	3.6.4.3	Table 3.1 of Lesson 5 for $p_{t,\ lim}$
5	d	Yes	No	Control of deflection	3.6.4.4	cl.23.2 of IS 456
6	D, b	Yes for D	b	Economy	3.6.4.5	$D = d$ + (40 to 80 mm) b = (0.5 to 0.67)D
7	F_d, M_u	Yes	No	Strength	3.6.4.6	Strength of material

						books
8	d	Yes	No	Limiting depth considering $M_u = M_{u,lim}$	3.6.4.7	Eq. 3.24 of Lesson 5
9	A_{st}	Yes	No	Strength	3.6.4.8	Eq. 3.23 of Lesson 5
10	d, D, A_{st}	No	d, D, L_{eff}	Under-reinforced	3.6.4.9	$D = d + 50$ Eq. 3.23 of Lesson 5
11	A_{st}	No	A_{st}	Strength	3.6.4.10	Eq. 3.23 of Lesson 5

Use of Design Aids

From the solution of the illustrative numerical problems, it is clear that b, d, D and A_{st} are having individual requirements and they are mutually related. Thus, any design problem has several possible sets of these four parameters. After getting one set of values, obtaining the second set, however, involves the same steps as those of the first one. The steps are simple but time consuming and hence, the designer may not have interest to compare between several sets of these parameters. The client, contractor or the architect may request for alternatives also. Thus, there is a need to get several sets of these four parameters as quickly as possible. One way is to write a computer program which also may restrict average designer not having a computer. Bureau of Indian Standard (BIS), New Delhi published SP-16, Design Aids for Reinforced Concrete to IS: 456, Special Publication No. 16, which is very convenient to get several sets of these values quickly.

SP-16 provides both charts (graphs) and tables explaining their use with illustrative examples. On top left or right corner of these charts and tables, the governing parameters are provided for which that chart/table is to be used.

Solution by using Design Aids Charts (SP-16)

The initial dimension of effective depth d of Design Problem 3.1 is modified from 400 mm to 410 mm first to satisfy the deflection and other requirements and then to 450 mm as the final dimension. While using only the charts or tables of SP-16, the final results as obtained for this problem by direct calculation method will not be available. So, we will assume the percentage of steel as 0.75 (0.96) = 0.72 initially.

Effective depth *d*

Chart 22 of SP-16 for f_y = 415 N/mm^2 and f_{ck} = 20 N/mm^2 gives maximum ratio of span to effective depth as 21.5 when the percentage of steel assumed = 0.75 (0.96) = 0.72. Thus, we get effective depth d = 8000/21.5 = 372.09 mm with d = 372.09 mm and effective span L_{eff} = 8000 mm. Total depth D = 372.09 + 50 = 422.09 = 425 mm (say).

Breadth *b* and factored moment M_u

Here also b = 250 mm is assumed and accordingly, Dead load =

0.25 (0.425) (25) = 2.66 kN/m Imposed loads = 7.00 kN/m

Total factored load, F_d = 1.5 (9.66) = 14.50 kN/m

Factored bending moment = (14.5) (8) = 116.00 kN/m

Checking of effective depth d and area of steel A_{st}

Chart 14 of SP-16 is for f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2 and d varying from 300 to 550 mm. For this problem, M_u per metre width of the beam = 464 kNm/m . For the percentage of reinforcement = 0.72, chart 14 gives d = 460 mm and then D = 510 mm. Area of steel reinforcement 0.72 (25) (460)/100 = 828 mm^2.

As in the earlier problem, the increased dead load due to the increased D to 510 mm is checked below:

Revised dead load = 0.25 (0.51) (25) = 3.188 kN/m Imposed loads =

7.000 kN/m

Total factored load F_d = 1.5 (10.188) = 15.282 kN/m Factored

moment M_u = 15.282 (8) = 122.256 kN/m

M_u per metre width of the beam = 122.256/0.25 = 489.02 kNm/m.

Chart 14 of Sp-16 gives the effective depth of the beam d = 472 mm and D = 475 + 50 = 525 mm assuming d = 475 mm.

A_{st} required = (0.72/100) (250) (475) = 855.0 mm^2

Thus, we have b = 250 mm, d = 475 mm, D = 525 mm and A_{st} = 855 mm^2

Solution by using Design Aids Tables (SP-16)

Effective depth d

Tables 1 to 4 of SP-16 present p_t for different values of M_u/bd^2 covering a wide range of f_y and f_{ck}. Table 2 is needed for this problem.

To have more confidence while employing this method, we are starting with the effective depth d as 400 mm as in the direct computational method. The total depth D is (400 + 50) mm = 450 mm. The breadth b of the beam is taken as 250 mm.

Factored load and bending moment

Dead load = 0.25 (0.45) (25) = 2.8125 kN/m

Imposed loads = 7.00 kN/m

Factored load F_d = 1.5 (2.8125 + 7.00) = 14.71875 kN/m Factored

bending moment M_u = 14.71875 (8) = 117.75 kNm

$$\frac{M_u}{b\,d^2} = \frac{117.75\,(10^6)}{250\,(400)(400)} = 2.94375$$

Use of Tables of SP-16

Table 2 of SP-16 shows that $\frac{M_u}{b\,d^2}$ is restricted up to 2.76 when p_t = 0.955, i.e. the limiting condition. So, increasing the effective depth by another 50 mm to have D = 500 m, the total factored moment as calculated in sec. 3.6.4.10 is 121.5 kNm,

Now, $$\frac{M_u}{b\,d^2} = \frac{121.5\,(10^6)}{250\,(450)(450)} = 2.4$$

From Table 2 of SP-16, the corresponding p_t becomes 0.798. Therefore, A_{st} =

0.01 (0.798) (250) (450) = 897.75 mm^2

Comparison of Results of Three Methods

Results of this problem by three methods: (i) direct computation method, (ii) use of charts of SP-16 and (iii) use of tables of SP-16 are summarised for the purpose of comparison. The tabular summary includes the last two values of d and A_{st}. Other parameters (b, f_{ck} and f_y) are remaining constants in all the three methods.

Table 3.5 Comparison of d and A_{st} by three methods

Cycle	Direct computation method		Use of charts of SP-16		Use of tables of SP- 16	
	d (mm)	A_{st} (mm^2)	d (mm)	A_{st} (mm^2)	d (mm)	A_{st} (mm^2)
1	410	966.5168	460	828	400	Not possible
2	450	895.84145 (2-20+2-14 = 935 mm^2)	475	855 (2-20 + 2-12 = 854 mm^2)	450	897.75 (2-20+2-14 = 935 mm^2)

Other Alternatives using Charts and Tables of SP-16

Any alternative solution of d will involve computations of factored loads F_d and bending moment M_u. Thereafter, Eq. 3.23 of Lesson 5 has to be solved to get the value of A_{st} by direct computation method. On the other hand, it is very simple to get the A_{st} with the help of either charts or tables of SP-16 from the value of factored bending moment. Some alternatives are given below in Table 3.6 by the use of tables of SP-16.

In sec. 3.6.7.3, it is observed that an effective depth of 400 mm is not acceptable. Hence, the effective depth is increased up to 450 mm at intervals of 10 mm and the corresponding A_{st} values are presented in Table 3.6. The width b is kept as 250 mm and M 20 and Fe 415 are used for all the alternatives.

Table 3.6 Alternative values of d, D, F_d, M_u, $\frac{M_u}{b\,d^2}$, p_t and A_{st}

Sl. No.	d (mm)	D (mm)	F_d (kN/m)	M_u (kNm)	$\frac{M_u}{b\,d^2}$ (N/mm^2)	p_t (%)	A_{st} (mm^2)
1	410	460	14.825	118.5	2.8197	Not acceptable	Not acceptable
2	420	470	14.9062	119.25	2.7041	0.93	976.5
3	430	480	15.0	120.00	2.596	0.88	946.0
4	440	490	15.09	120.75	2.4948	0.839	922.9
5	450	500	15.1875	121.5	2.4	0.798	897.75

Advantages of using SP-16

The following are the advantages:

(i) Alternative sets of b, d and A_{st} are obtained very quickly.

(ii) The results automatically exclude those possibilities where the steel reinforcement is inadmissible.

It has been mentioned that the reinforcement should be within 75 to 80 per cent of limiting reinforcement to ensure ductile failure. The values of charts and tables are given up to the limiting reinforcement. Hence, the designer should be careful to avoid the reinforcement up to the limiting amount. Moreover, these charts and tables can be used for the design of slabs also. Therefore, the values are also taking care of the minimum reinforcement of slabs. The minimum reinforcement of beams are higher than that of slabs. Accordingly, the designer should also satisfy the requirement of minimum reinforcement for beams while using SP-16.

It is further suggested to use the tables than the charts as the values of the charts may have personal error while reading from the charts. Tabular values have the advantage of numerical, which avoid personal error. Moreover, intermediate values can also be evaluated by linear interpolation.

Introduction to Analysis Type of Problem

This lesson explains the determination of moment of resistance of given singly reinforced rectangular beam sections with the help of illustrative analysis type of problem. The numerical problem is solved by (i) direct computation method, (ii) using charts of SP-16 and (iii) using tables of SP-16. Step by step solutions illustrate the procedure clearly.

Analysis Type of Problems

It may be required to estimate the moment of resistance and hence the service load of a beam already designed earlier with specific dimensions of b, d and D and amount of steel reinforcement A_{st}. The grades of concrete and steel are also known. In such a situation, the designer has to find out first if the beam is under-reinforced or over-reinforced. The following are the steps to be followed for such problems.

$x_{u,\ max}$

The maximum depth of the neutral axis $x_{u,\ max}$ is determined from Table 3.2 of Lesson 5 using the known value of f_y.

x_u

The depth of the neutral axis for the particular beam is determined from Eq. 3.16 of Lesson 5 employing the known values of f_y, f_{ck}, b and A_{st}.

M_u and service imposed loads

The moment of resistance M_u is calculated for the three different cases as follows:

(a) If $x_u < x_{u,\ max}$, the beam is under-reinforced and M_u is obtained from Eq. 3.22 of Lesson 5.

If $x_u = x_{u,\ max}$, the M_u is obtained from Eq. 3.24 of Lesson 5

(b) If $x_u > x_{u,\ max}$, the beam is over-reinforced for which x_u is taken as $x_{u,\ max}$ and then M_u is calculated from Eq. 3.24 of Lesson 5, using $x_u = x_{u,\ max}$.

With the known value of M_u, which is the factored moment, the total factored load can be obtained from the boundary condition of the beam. The total service imposed loads is then determined dividing the total factored load by partial safety factor for loads (= 1.5). The service imposed loads are then obtained by subtracting the dead load of the beam from the total service loads.

Analysis Problems 3.2 and 3.3

Determine the service imposed loads of two simply supported beam of same effective span of 8 m (Figs. 3.7.1 and 2) and same cross-sectional dimensions, but having two different amounts of reinforcement.

Both the beams are made of M 20 and Fe 415.

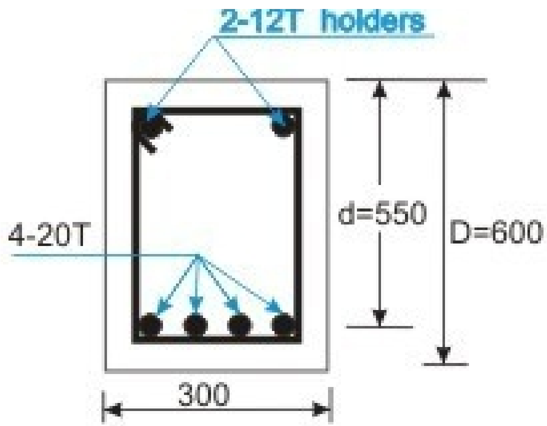


Fig. 3.7.1: Analysis Problem 3.2

Solution by Direct Computation Method - Problem 3.2

Given data: b = 300 mm, d = 550 mm, D = 600 mm, A_{st} = 1256 mm^2 (4-20 T), L_{eff} = 8 m and boundary condition = simply supported (Fig. 3.7.1).

$x_{u,\,max}$

From Table 3.2 of Lesson 5, we get $x_{u,\,max} = 0.479\,d = 263.45$ mm

x_u

$$x_u = \frac{0.87\, f_y A_{st}}{0.36\, b\, f_{ck}}$$

(3.16)

$$= \frac{0.87\ (415)\ (1256)}{0.36\ (300)\ (20)}$$

= 209.94385 mm < $x_{u,\ max}$ = (263.45 mm) Hence, the beam is under-reinforced.

M_u and service imposed loads

For $x_u < x_{u,\ max}$, we have

$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$

(3.22)

= 0.87 (415) (1256) {550 - 0.42(209.94385)}

= 209.4272 kNm

Total factor load F_d $= \frac{8\, M_u}{L_{eff}^2} = \frac{8\ (209.4272)}{8\ (8)}$

= 26.1784 kN/m

Total service load = $\frac{F_d}{1.5} = \frac{26.1784}{1.5}$ = 17.452266 kN/m

Dead load of the beam = 0.3 (0.6) (25) = 4.5 kN/m Hence, service imposed loads = (17.452266 - 4.5) kN/m

= 12.952266 kN/m

Solution by Direct Computation Method - Problem 3.3

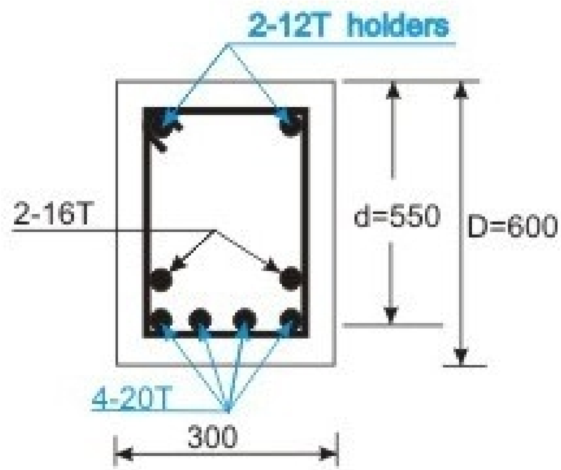


Fig. 3.7.2: Analysis Problem 3.3

Given data: b = 300 mm, d = 550 mm, D = 600 mm, A_{st} = 1658 mm^2 (4-20 T + 2-16 T), L_{eff} = 8 m and boundary conditions = simply supported (Fig. 3.7.2)

$x_{u,\,max}$

From Table 3.2 of Lesson 5, we get $x_{u,\,max} = 0.479\,d = 263.45$ mm

x_u

$$x_u = \frac{0.87\,f_y\,A_{st}}{0.36\,b\,f_{ck}} \tag{3.16}$$

$$= \frac{0.87\,(415)\,(1658)}{0.36\,(300)\,(20)}$$

$= 277.13924$ mm $> x_{u,\,max} = (263.45$ mm$)$

Hence, the beam is over-reinforced.

M_u and service imposed loads

For $x_u > x_{u,max}$, we have

$$M_u = 0.36\,\frac{x_{u,\max}}{d}\,(1 - 0.42\,\frac{x_{u,\max}}{d})\,b\,d^2\,f_{ck} \tag{3.24}$$

$= 0.36\ (0.479)\ \{1 - 0.42(0.479)\}\ (300)\ (550)\ (550)\ (20)$ Nmm

= 250.01356 kNm

If we use Eq. 3.22 using $x_u = x_{u,max}$, for M_u M_u =

$0.87\, f_y\, A_{st}\, (d - 0.42\, x_{u,\, max})$

(3.22)

Then, $(M_u)_{steel} = 0.87\ (415)\ (1658)\ \{550 - 0.42\ (263.45)\}$ Nmm

= 263.00468 kNm > $(M_u)_{concrete}$ (= 250.01356 kNm)

The higher M_u as obtained from steel is not true because the entire amount of steel (1658 mm^2) cannot yield due to over-reinforcing. Prior to that, concrete fails at 250.01356 kNm. However, we can get the same of M_u as obtained from Eq.
3.24 of Lesson 5, if we can find out how much A_{st} is needed to have x_u = 263.45 mm. From Eq. 3.16 of Lesson 5, we can write:

$$(A_{st})_{needed\ for\ x_u = 263.45} = \frac{0.36\, b\, f_{ck}\, x_u}{0.87\, f_y}$$

$$= \frac{0.36(300)(20)(263.45)}{0.87\ (415)}$$

= 1576.1027 mm^2

If we use this value for A_{st} in Eq. 3.22 of Lesson 5, we get

$$(M_u) = 0.87\,(415)\,(1576.1027)\{550 - 0.42\,(263.45)\}$$

= 250.0135 (same as obtained from Eq. 3.24).

From the factored moment M_u = 250.01356 kNm, we have:

$$\text{Total factored load} = F_d = \frac{8\, M_u}{L^2_{eff}} = \frac{8(250.01356)}{8\ (8)} = 31.251695$$

kN/m

$$\text{Total service load} = \frac{31.251695}{1.5} = 20.834463 \text{ kN/m}$$

Now, Dead load of the beam = 0.3 (0.6) (25) = 4.5 kN/m

Hence, service imposed loads = 20.834463 - 4.5 = 16.334463 kN/m

Solution by Design Chart - Problems 3.2 and 3.3

For the two problems with known *b, d, D,* A_{st}, grade of concrete and grade of steel, chart 14 of SP-16 is applicable. From the effective depth *d* and percentage of reinforcement $p_t (= \frac{A_{st}\, 100}{b\, d})$, the chart is used to find M_u per metre width. Multiplying M_u per meter width with *b*, we get M_u for the beam. After that, the service imposed load is calculated using the relation

$$\text{Service imposed load} = \frac{8\, M_u}{(1.5)\, L^2} - \text{Dead load} \qquad (3.27)$$

The results of the two problems are furnished in Table 3.7.

Table 3.7 Results of Problems 3.2 and 3.3 (Chart of SP-16)

Prob-lem	A_{st} (mm^2) p_t (%)	*d*, *b* (mm)	M_u (kNm/m) (Chart 14)	M_u (kNm)	$\frac{8 M_u}{1.5 L^2}$ (kN/m)	Dead load (kN/m)	Service imposed loads (kN/m)	Remarks
3.2	1256 0.7612	550, 300	700	210	17.5	4.5	13	M_u per m of 700 is well within the chart, hence under-reinforced.
3.3	1658 1.205818	550, 300	820	246	20.5	4.5	16	Maximum M_u = 820, so

	1								over-reinforced.

Solution by Design Tables - Problems 3.2 and 3.3

Table 2 of SP-16 presents the value of reinforcement percentage for different combinations of f_y and (M_u/bd^2) for M-20. Here, from the known values of p and f_y, the corresponding values of M_u/bd^2 are determined. These in turn give M_u of the beam. Then the service imposed load can be obtained using Eq. 3.27 as explained in the earlier section (sec. 3.7.6). The results of the two problems are presented in Table 3.8.

Table 3.8 Results of Problems 3.2 and 3.3 (Table of SP-16)

Prob-lem	A_{st} (mm^2) p_t (%)	d, b (mm)	f_y (N/mm^2)	$\frac{M_u}{b\,d^2}$ (N/mm^2) from Table 2	M_u (kNm)	$\frac{8M_u}{1.5L^2}$	Dead load (kN/m)	Service imposed loads (kN/m)	Remarks
3.2	1256 0.7612	550, 300	415	2.3105	209.67784	17.473155	4.5	12.973155	*
3.3	1658 1.2058181	500, 300	415	2.76	250.47	20.8725	4.5	16.3725	**

* Linear interpolated
$$\frac{M_u}{b\,d^2} = 2.30 + \frac{(0.7612 - 0.757)}{(0.765 - 0.757)}(2.32 - 2.30) = 2.3105$$

** p_t = 1.2058181 is not admissible, i.e. over-reinforced. So at p_t = 0.955, M_u/bd^2 =2.76.

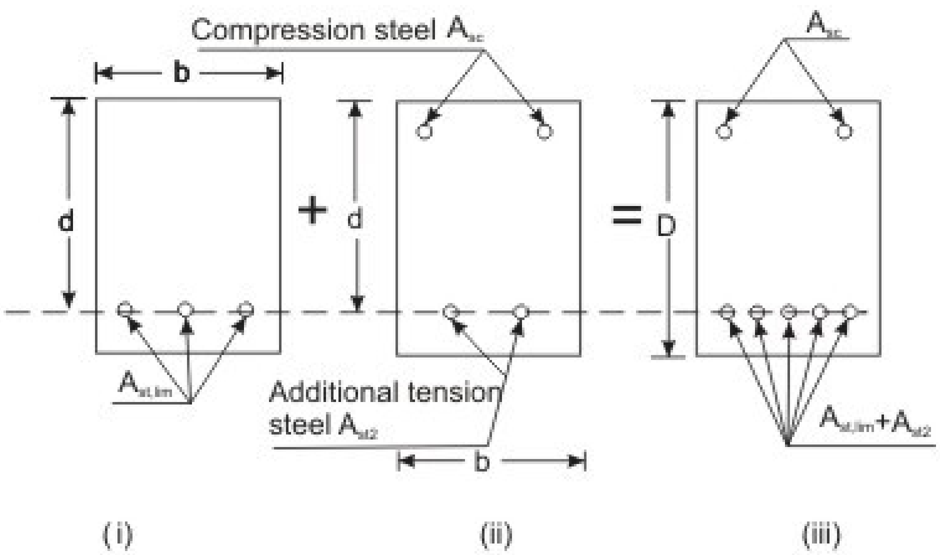
Comparison of Results of Three Methods

The values of M_u and service imposed loads of the under-reinforced and over-reinforced problems (Problems 3.2 and 3.3), computed by three methods, are presented in Table 3.9.

Table 3.9 Comparison of results of Problems 3.2 and 3.3

Problem	M_u (kNm)			Service imposed loads (kN/m)		
	Direct computation	Chart of SP-16	Table of SP-16	Direct computation	Chart of SP-16	Table of SP-16
3.2	209.4272	210	209.67784	12.9522	13	12.973155
3.3	250.01356	246	250.47	16.334463	16	16.3725

Introduction to beam



For(i) » $M_{u1}=M_{u,lim}$

For(ii) » M_{u2} = Due to A_{sc} and A_{st2}

For(iii) » $M_u = M_{u,lim}+M_{u2}$

Fig. 4.8.1: Doubly reinforced beam

Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension. However, these beams have their respective limiting moments of resistance with specified width, depth and grades of concrete and steel. The amount of steel reinforcement needed is known as $A_{st,lim}$. Problem will arise, therefore, if such a section is subjected to bending moment greater than its limiting moment of resistance as a singly reinforced section.

There are two ways to solve the problem. First, we may increase the depth of the beam, which may not be feasible in many situations. In those cases, it is possible to increase both the compressive and tensile forces of the beam by providing steel reinforcement in compression face and additional reinforcement in tension face of the beam without increasing the depth (Fig. 4.8.1). The total compressive force of such beams comprises (i) force due to concrete in compression and (ii) force due to steel in compression. The tensile force also has two components: (i) the first provided by $A_{st,lim}$ which is equal to the compressive force of concrete in compression. The second part is due to the additional steel in tension - its force will be equal to the compressive force of steel in compression. Such reinforced concrete beams having steel reinforcement both on tensile and compressive faces are known as doubly reinforced beams.

Doubly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams. However, other than in doubly reinforced beams compression steel reinforcement is provided when:

(i) some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.

(ii) the ductility requirement has to be followed.

(iii) the reduction of long term deflection is needed.

It may be noted that even in so called singly reinforced beams there would be longitudinal hanger bars in compression zone for locating and fixing stirrups.

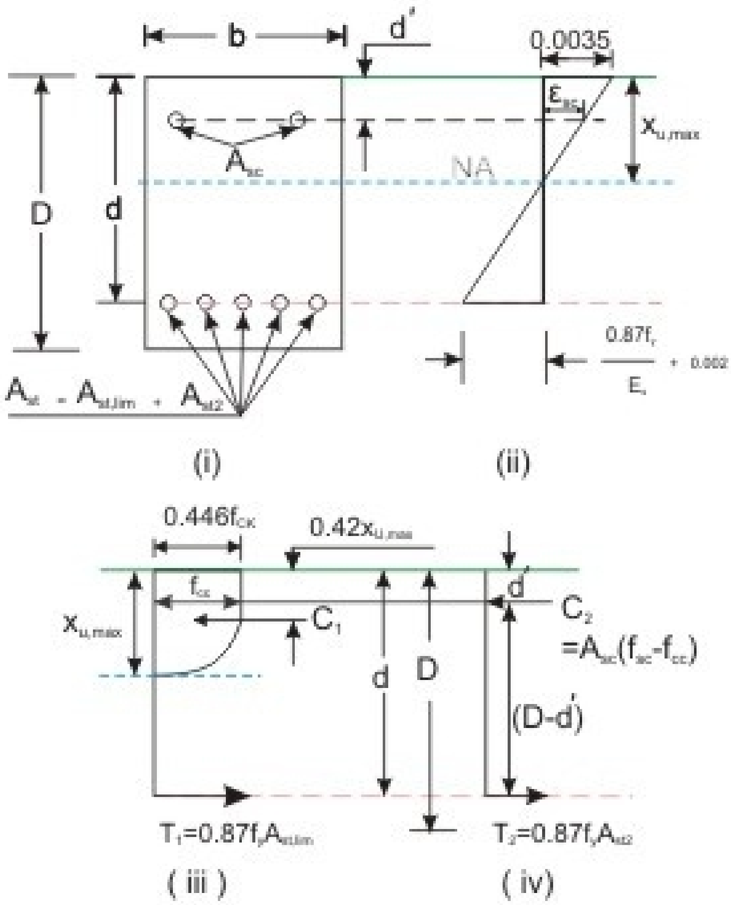
Assumptions

(iv) The assumptions of sec. 3.4.2 of Lesson 4 are also applicable here.

(v) Provision of compression steel ensures ductile failure and hence, the limitations of *x/d* ratios need not be strictly followed here.

(vi) The stress-strain relationship of steel in compression is the same as that in tension. So, the yield stress of steel in compression is 0.87 *fy*.

Basic Principle



(I) Beam cross section
(ii) Strain diagram
(iii) Force diagram of beam of $M_{u,lim}$
(iv) Force diagram of beam of M_{u2}

Fig. 4.8.2: Stress, strain and force diagrams of doubly reinforced beam

As mentioned in sec. 4.8.1, the moment of resistance M_u of the doubly reinforced beam consists of (i) $M_{u,lim}$ of singly reinforced beam and (ii) M_{u2} because of equal and opposite compression and tension forces (C_2 and T_2) due to additional steel reinforcement on compression and tension faces of the beam (Figs. 4.8.1 and 2). Thus, the moment of resistance M_u of a doubly reinforced beam is

$$M_u = M_{u,lim} + M_{u2} \quad (4.1)$$

The $M_{u,lim}$ is as given in Eq. 3.24 of Lesson 5, i.e.,

$$M_{u,lim} = 0.36 \left(\frac{x_{u,\max}}{d}\right)\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) b\, d^2 f_{ck} \quad (4.2)$$

Also, $M_{u,lim}$ can be written from Eq. 3.22 of Lesson 5, using $x_u = x_{u,\,max}$, i.e.,

$$M_{u,\,lim} = 0.87\, A_{st,\,lim}\, f_y\, (d - 0.42\, x_{u,\,max})$$

$$= 0.87\, p_{t,\,lim} \left(1 - 0.42 \frac{x_{u\,,\,\max}}{d}\right) b\, d^2 f_y \quad (4.3)$$

The additional moment M_{u2} can be expressed in two ways (Fig. 4.8.2): considering (i) the compressive force C_2 due to compression steel and (ii) the tensile force T_2 due to additional steel on tension face. In both the equations, the lever arm is $(d - d')$. Thus, we have

$$M_{u2} = A_{sc}\, (f_{sc} - f_{cc})\, (d - d') \quad (4.4)$$

$$M_{u2} = A_{st2}\, (0.87\, f_y)\, (d - d') \quad (4.5)$$

where A_{sc} = area of compression steel reinforcement

f_{sc} = stress in compression steel reinforcement

f_{cc} = compressive stress in concrete at the level of centroid of compression steel reinforcement

A_{st2} = area of additional steel reinforcement

Since the additional compressive force C_2 is equal to the additional tensile force T_2, we have

$$A_{sc}\, (f_{sc} - f_{cc}) = A_{st2}\, (0.87\, f_y) \quad (4.6)$$

Any two of the three equations (Eqs. 4.4 - 4.6) can be employed to determine A_{sc} and A_{st2}.

The total tensile reinforcement A_{st} is then obtained from:

$$A_{st} = A_{st1} + A_{st2} \quad (4.7)$$

where $$A_{st1} = \frac{p_{t,\lim}\, b\, d}{100} = \frac{M_{u,\lim}}{0.87\, f_y (d - 0.42\, x_{u,\max})} \tag{4.8}$$

Determination of f_{sc} and f_{cc}

It is seen that the values of f_{sc} and f_{cc} should be known before calculating A_{sc}. The following procedure may be followed to determine the value of f_{sc} and f_{cc} for the design type of problems (and not for analysing a given section). For the design problem the depth of the neutral axis may be taken as $x_{u,max}$ as shown in Fig. 4.8.2. From Fig. 4.8.2, the strain at the level of compression steel reinforcement ε_{sc} may be written as

$$\varepsilon_{sc} = 0.0035 \left(1 - \frac{d'}{x_{u,\max}}\right) \tag{4.9}$$

The stress in compression steel f_{sc} is corresponding to the strain ε_{sc} of Eq. 4.9 and is determined for (a) mild steel and (b) cold worked bars Fe 415 and 500 as given below:

(a) Mild steel Fe 250

The strain at the design yield stress of 217.39 N/mm^2 ($f_d = 0.87\, f_y$) is 0.0010869 (= 217.39/E_s). The f_{sc} is determined from the idealized stress-strain diagram of mild steel (Fig. 1.2.3 of Lesson 2 or Fig. 23B of IS 456) after computing the value of ε_{sc} from Eq. 4.9 as follows:

(i) If the computed value of $\varepsilon_{sc} \le 0.0010869$, $f_{sc} = \varepsilon_{sc} E_s = 2\,(10^5)\,\varepsilon_{sc}$

(ii) If the computed value of $\varepsilon_{sc} > 0.0010869$, $f_{sc} = 217.39$ N/mm^2.

(b) Cold worked bars Fe 415 and Fe 500

The stress-strain diagram of these bars is given in Fig. 1.2.4 of Lesson 2 and in Fig. 23A of IS 456. It shows that stress is proportional to strain up to a stress of 0.8 f_y. The stress-strain curve for the design purpose is obtained by substituting f_{yd} for f_y in the figure up to 0.8 f_{yd}. Thereafter, from 0.8 f_{yd} to f_{yd}, Table A of SP-16 gives the values of total strains and design stresses for Fe 415 and Fe 500. Table 4.1 presents these values as a ready reference here.

Table 4.1 Values of f_{sc} and ε_{sc}

Stress level	Fe 415		Fe 500	
	Strain ε_{sc}	Stress f_{sc} (N/mm^2)	Strain ε_{sc}	Stress f_{sc} (N/mm^2)
0.80 f_{yd}	0.00144	288.7	0.00174	347.8
0.85 f_{yd}	0.00163	306.7	0.00195	369.6
0.90 f_{yd}	0.00192	324.8	0.00226	391.3
0.95 f_{yd}	0.00241	342.8	0.00277	413.0
0.975 f_{yd}	0.00276	351.8	0.00312	423.9
1.0 f_{yd}	0.00380	360.9	0.00417	434.8

Linear interpolation may be done for intermediate values.

The above procedure has been much simplified for the cold worked bars by presenting the values of f_{sc} of compression steel in doubly reinforced beams for different values of d'/d only taking the practical aspects into consideration. In most of the doubly reinforced beams, d'/d has been found to be between 0.05 and 0.2. Accordingly, values of f_{sc} can be computed from Table 4.1 after determining the value of ε_{sc} from Eq. 4.9 for known values of d'/d as 0.05, 0.10, 0.15 and 0.2. Table F of SP-16 presents these values of f_{sc} for four values of d'/d (0.05, 0.10, 0.15 and 0.2) of Fe 415 and Fe 500. Table 4.2 below, however, includes Fe 250 also whose f_{sc} values are computed as laid down in sec. 4.8.4(a) (i) and (ii) along with those of Fe 415 and Fe 500. This table is very useful and easy to determine the f_{sc} from the given value of d'/d. The table also includes strain values at yield which are explained below:

(i) The strain at yield of Fe 250 =

$$\frac{\text{Design Yield Stress}}{E_s} = \frac{250}{1.15\,(200000)} = 0.0010869$$

Here, there is only elastic component of the strain without any inelastic strain.

(ii) The strain at yield of Fe 415 = $\text{Inelastic Strain} + \dfrac{\text{Design Yield Stress}}{E_s}$

$$= 0.002 + \frac{415}{1.15\,(200000)} = 0.0038043$$

(iii) The strain at yield of Fe 500 = $0.002 + \dfrac{500}{1.15\,(200000)} = 0.0041739$

Table 4.2 Values of f_{sc} for different values of d'/d

f_y (N/mm^2)	d'/d				Strain at yield
	0.05	0.10	0.15	0.20	
250	217.4	217.4	217.4	217.4	0.0010869
415	355	353	342	329	0.0038043
500	412	412	395	370	0.0041739

Minimum and maximum steel

In compression

There is no stipulation in IS 456 regarding the minimum compression steel in doubly reinforced beams. However, hangers and other bars provided up to 0.2% of the whole area of cross section may be necessary for creep and shrinkage of concrete. Accordingly, these bars are not considered as compression reinforcement. From the practical aspects of consideration, therefore, the minimum steel as compression reinforcement should be at least 0.4% of the area of concrete in compression or 0.2% of the whole cross-sectional area of the beam so that the doubly reinforced beam can take care of the extra loads in addition to resisting the effects of creep and shrinkage of concrete.

The maximum compression steel shall not exceed 4 per cent of the whole area of cross-section of the beam as given in cl. 26.5.1.2 of IS 456.

In tension

As stipulated in cl. 26.5.1.1(a) and (b) of IS 456, the minimum amount of tensile reinforcement shall be at least ($0.85\, bd/f_y$) and the maximum area of tension reinforcement shall not exceed ($0.04\, bD$).

It has been discussed in sec. 3.6.2.3 of Lesson 6 that the singly reinforced beams shall have A_{st} normally not exceeding 75 to 80% of $A_{st,lim}$ so that x_u remains less than $x_{u,max}$ with a view to ensuring ductile failure. However, in the case of doubly reinforced beams, the ductile failure is ensured with the presence of compression steel. Thus, the depth of the neutral axis may be taken as $x_{u,\,max}$ if the beam is over-reinforced. Accordingly, the A_{st1} part of tension steel can go up to $A_{st,\,lim}$ and the additional tension steel A_{st2} is provided for the additional moment M_u - $M_{u,\,lim}$. The quantities of A_{st1} and A_{st2} together form the total A_{st}, which shall not exceed $0.04\, bD$.

Types of problems and steps of solution

Similar to the singly reinforced beams, the doubly reinforced beams have two types of problems: (i) design type and (ii) analysis type. The different steps of solutions of these problems are taken up separately.

Design type of problems

In the design type of problems, the given data are b, d, D, grades of concrete and steel. The designer has to determine A_{sc} and A_{st} of the beam from the given factored moment. These problems can be solved by two ways: (i) use of the equations developed for the doubly reinforced beams, named here as direct computation method, (ii) use of charts and tables of SP-16.

(a) Direct computation method

Step 1: To determine $M_{u,\,lim}$ and $A_{st,\,lim}$ from Eqs. 4.2 and 4.8, respectively.

Step 2: To determine M_{u2}, A_{sc}, A_{st2} and A_{st} from Eqs. 4.1, 4.4, 4.6 and 4.7, respectively.

Step 3: To check for minimum and maximum reinforcement in compression and tension as explained in sec. 4.8.5.

Step 4: To select the number and diameter of bars from known values of A_{sc} and A_{st}.

(b) Use of SP table

Tables 45 to 56 present the p_t and p_c of doubly reinforced sections for d'/d = 0.05, 0.10, 0.15 and 0.2 for different f_{ck} and f_y values against M_u/bd^2. The values of p_t and p_c are obtained directly selecting the proper table with known values of M_u/bd^2 and d'/d.

Analysis type of problems

In the analysis type of problems, the data given are b, d, d', D, f_{ck}, f_y, A_{sc} and A_{st} . It is required to determine the moment of resistance M_u of such beams. These problems can be solved: (i) by direct computation method and (ii) by using tables of SP-16.

(a) Direct computation method

Step 1: To check if the beam is under-reinforced or over-reinforced.

First, $\frac{x_{u,\max}}{d}$ is determined assuming it has reached limiting stage using coefficients as given in cl. 38.1, Note of IS 456. The strain of tensile steel ε_{st} is computed from $\varepsilon_{st} = \frac{\varepsilon_c (d - x_{u,\max})}{x_{u,\max}}$ and is checked if ε_{st} has reached the yield strain of steel:

$$\varepsilon_{st\ at\ yield} = \frac{f_y}{1.15\,(E)} + 0.002$$

The beam is under-reinforced or over-reinforced if ε_{st} is less than or more than the yield strain.

Step 2: To determine $M_{u,lim}$ from Eq. 4.2 and $A_{st,lim}$ from the $p_{t,\ lim}$ given in Table 3.1 of Lesson 5.

Step 3: To determine A_{st2} and A_{sc} from Eqs. 4.7 and 4.6, respectively.

Step 4: To determine M_{u2} and M_u from Eqs. 4.4 and 4.1, respectively.

(b) Use of tables of SP-16

As mentioned earlier Tables 45 to 56 are needed for the doubly reinforced beams. First, the needed parameters d'/d, p_t and p_c are calculated. Thereafter, M_u/bd^2 is computed in two stages: first, using d'/d and p_t and then using d'/d and p_c . The lower value of M_u is the moment of resistance of the beam.

Introduction

This lesson illustrates the application of the theory of doubly reinforced beams in solving the two types of problems mentioned in Lesson 8. Both the design and analysis types of problems are solved by (i) direct computation method, and (ii) using tables of SP-16. The step by step solution of the problems will help in understanding the theory of Lesson 8 and its application.

Numerical problems

Problem 4.1

Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: b = 300 mm, D = 700 mm, M 20 concrete, Fe 415 steel (Fig. 4.9.1). Determine f_{sc} from d'/d as given in Table 4.2 of Lesson 8.

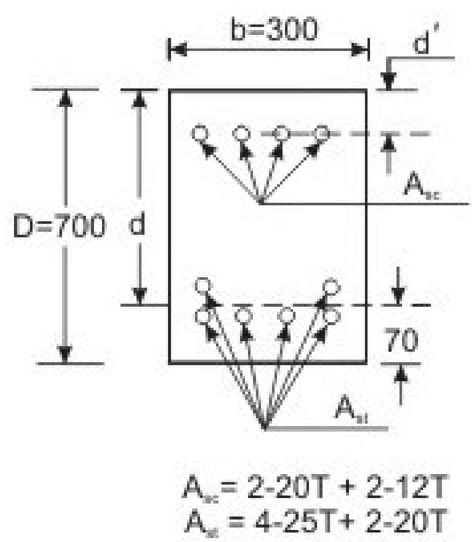


Fig. 4.9.1: Problem 4.1

(a) Solution by direct computation method

Dead load of the beam = 0.3 (0.7) (25) = 5.25 kN/m Imposed loads

(given) = 35.00 kN/m

Total loads = 5.25 + 35.00 = 40.25 kN/m

Factored bending moment = (1.5) $\frac{wl^2}{8} = \frac{(1.5)(40.25)(8)(8)}{8}$ = 482.96 kNm

Assuming d' = 70 mm, d = 700 - 70 = 630 mm

$\frac{x_{u,\max}}{d} = 0.48$ gives $x_{u,\max}$ = 0.48 (630) = 302.4 mm

Step 1: Determination of $M_{u,lim}$ and $A_{st,lim}$

$$M_{u,\lim} = 0.36\left(\frac{x_{u,\max}}{d}\right)\left(1 - 0.42\frac{x_{u,\max}}{d}\right) b\, d^2 f_{ck} \qquad (4.2)$$

$$= 0.36(0.48)\ \{1 - 0.42\ (0.48)\}\ (300)\ (630)^2\ (20)\ (10^{-6})\ \text{kNm}$$

$$= 328.55\ \text{kNm}$$

$$A_{st,\lim} = \frac{M_{u,\lim}}{0.87 f_y\,(d - 0.42\, x_{u,\max})} \qquad (6.8)$$

So, $A_{st1} = \frac{328.55\,(10^6)\ \text{Nmm}}{0.87\ (415)\ \{630 - 0.42\ (0.48)\ 630\}} = 1809.14\ \text{mm}^2$

Step 2: Determination of M_{u2}, A_{sc}, A_{st2} and A_{st}

(Please refer to Eqs. 4.1, 4.4, 4.6 and 4.7 of Lesson 8.)

$$M_{u2} = M_u - M_{u,lim} = 482.96 - 328.55 = 154.41\ \text{kNm}$$

Here, d'/d = 70/630 = 0.11

From Table 4.2 of Lesson 8, by linear interpolation, we get,

$$f_{sc} = 353 - \frac{353 - 342}{5} = 350.8 \text{ N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{154.41(10^6) \text{ Nmm}}{\{350.8 - 0.446(20)\}(630 - 70) \text{ N/mm}} = 806.517 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{806.517\,(350.8 - 8.92)}{(0.87)\,(415)} = 763.694 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1809.14 + 783.621 = 2572.834 \text{ mm}^2$$

Step 3: Check for minimum and maximum tension and compression steel. (vide sec.4.8.5 of Lesson 8)

(i) In compression:

(a) Minimum $A_{sc} = \frac{0.2}{100}(300)(700) = 420 \text{ mm}^2$

(b) Maximum $A_{sc} = \frac{4}{100}(300)(700) = 8400 \text{ mm}^2$

Thus, 420 mm^2 < 806.517 mm^2 < 8400 mm^2 . Hence, o.k.

(i) In tension:

(a) Minimum $A_{st} = \frac{0.85\, b\, d}{f_y}$ $\frac{0.85\,(300)\,(630)}{415} = 387.1 \text{ mm}^2$

(b) Maximum $A_{st} = \frac{4}{100}(300)(700) = 8400 \text{ mm}^2$

Here, 387.1 mm^2 < 2572.834 mm^2 < 8400 mm^2 . Hence, o.k. Step 4: Selection of bar diameter and numbers.

(i) for A_{sc}: Provide 2-20 T + 2-12 T (= 628 + 226 = 854 mm^2)

(ii) for A_{st}: Provide 4-25 T + 2-20 T (= 1963 + 628 = 2591 mm^2)

It may be noted that A_{st} is provided in two layers in order to provide adequate space for concreting around reinforcement. Also the centroid of the tensile bars is at 70 mm from bottom (Fig. 4.9.1).

(b) Solution by use of table of SP-16

For this problem, $\dfrac{M_u}{b\,d^2} = \dfrac{482.96\,(10^6)}{300\,(630)^2} = 4.056$ and $d'/d = \dfrac{70}{630} = 0.11$.

Table 50 of SP-16 gives p_t and p_c for $\dfrac{M_u}{b\,d^2} = 4$ and 4.1 and $d'/d = 0.1$ and 0.15.

The required p_t and p_c are determined by linear interpolation. The values are presented in Table 4.3 to get the final p_t and p_c of this problem.

Table 4.3 Calculation of p_t and p_c

$\dfrac{M_u}{b\,d^2}$		$d'/d = 0.1$	$d'/d = 0.15$	$d'/d = 0.11$
4.0	p_t	1.337	1.360	$1.337 + \dfrac{0.023\,(0.01)}{0.05} = 1.342$
	p_c	0.401	0.437	$0.433 + \dfrac{0.036\,(0.01)}{0.05} = 0.408$
4.1	p_t	1.368	1.392	$1.368 + \dfrac{0.024\,(0.01)}{0.05} = 1.373$
	p_c	0.433	0.472	$0.433 + \dfrac{0.039\,(0.01)}{0.05} = 0.441$
4.056	p_t	Not Applicable (NA)	NA	$1.342 + \dfrac{0.031\,(0.056)}{0.1}$ 1.3594

	p_c	NA	NA	$0.408 + \frac{0.033\ (0.056)}{0.1} = 0.426$

So, $$A_{st} = \frac{1.3594\ (300)\ (630)}{100} = 2569.26 \text{ m}^2$$

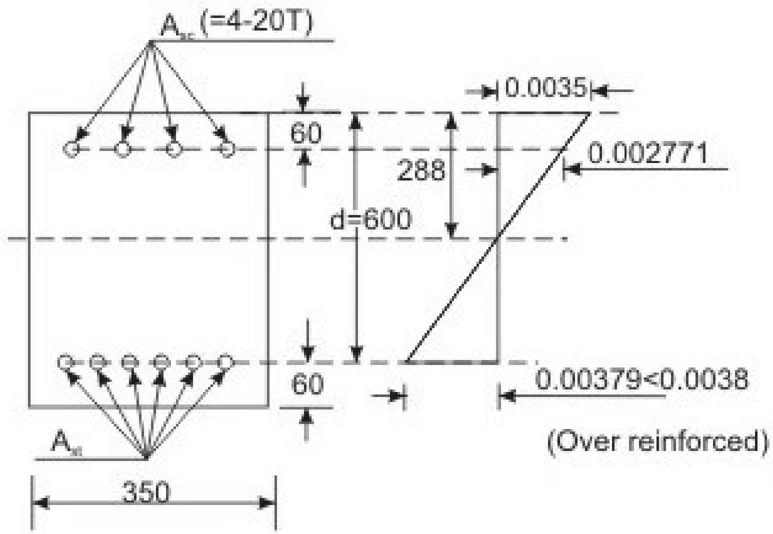
and $$A_{sc} = \frac{0.426\ (300)(630)}{100} = 805.14 \text{ mm}^2$$

These values are close to those obtained by direct computation method where A_{st} = 2572.834 mm^2 and A_{sc} = 806.517 mm^2. Thus, by using table of SP-16 we get the reinforcement very close to that of direct computation method. Hence, provide

(i) for A_{sc}: 2-20 T + 2-12 T (= 628 + 226 = 854 mm^2)

(ii) for A_{st}: 4-25 T + 2-20 T (= 1963 + 628 = 2591 mm^2)

4.9.2.2 Problem 4.2



A_{st} = 6-25T (= 2945 mm^2)
A_{sc} = 4-20T (= 1256 mm^2)

Fig. 4.9.2: Problem 4.2

Determine the ultimate moment capacity of the doubly reinforced beam of b = 350 mm, d' = 60 mm, d = 600 mm, A_{st} = 2945 mm^2 (6-25 T), A_{sc} = 1256 mm^2 (4-20 T), using M 20 and Fe 415 (Fig.4.9.2). Use direct computation method only.

Solution by direct computation method

Step 1: To check if the beam is under-reinforced or over-reinforced.

$$x_{u,\max} = 0.48\ (600) = 288 \text{ mm}$$

$$\varepsilon_{st} = \frac{\varepsilon_c (d - x_{u,\max})}{x_{u,\max}} = \frac{0.0035\ (600 - 288)}{288} = 0.00379$$

Yield strain of Fe 415 = $\frac{f_y}{1.15\,(E_s)} + 0.002 = \frac{415}{(1.15)\,(2)\,(10^5)} + 0.002$

= 0.0038 > 0.00379.

Hence, the beam is over-reinforced.

Step 2: To determine $M_{u,lim}$ and $A_{st,lim}$

(vide Eq. 4.2 of Lesson 8 and Table 3.1 of Lesson 5)

$$M_{u,\mathrm{lim}} = 0.36\,(\frac{x_{u,\max}}{d})\,(1 - 0.42\,\frac{x_{u,\max}}{d})\,b\,d^2 f_{ck}$$

= 0.36 (0.48) {1 - 0.42 (0.48)} (350) $(600)^2$ (20) (10^{-6}) kNm

= 347.67 kNm

From Table 3.1 of Lesson 5, for f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2,

$$A_{st,\mathrm{lim}} = \frac{0.96\ (350)\ (600)}{100} = 2016\ \ \mathrm{mm}^2$$

Step 3: To determine A_{st2} and A_{sc}

(vide Eqs.4.7 and 4.6 of Lesson 8)

$A_{st2} = A_{st} - A_{st,\ lim}$ = 2945 - 2016 = 929 mm^2

The required A_{sc} will have the compression force equal to the tensile force as given by 929 mm^2 of A_{st2}.

So, $$A_{sc} = \frac{A_{st2}\,(0.87\,f_y)}{(f_{sc} - f_{cc})}$$

For f_{sc} let us calculate ε_{sc}: (vide Eq. 4.9 of Lesson 8)

$$\varepsilon_{sc} = \frac{0.0035\ (x_{u,\max} - d')}{x_{u,\max}} = \frac{0.0035\ (288 - 60)}{288} = 0.002771$$

Table 4.1 of Lesson 8 gives:

$$f_{sc} = 351.8 + \frac{(360.9 - 351.8)(0.002771 - 0.002760)}{(0.00380 - 0.00276)} = 351.896 \text{ N/mm}^2$$

So, $A_{sc} = \dfrac{929\,(0.87)(415)}{\{351.89 - 0.446(20)\}} = 977.956 \quad mm^2$

Step 4: To determine M_{u2}, M_u and A_{st}

(Please refer to Eqs. 4.4 and 4.1 of Lesson 8)

$$M_{u2} = A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$= 977.956\ \{351.896 - 0.446\ (20)\}\ (600 - 60)\ (10^{-6})\ \text{kNm}$$

$$= 181.12\ \text{kNm}$$

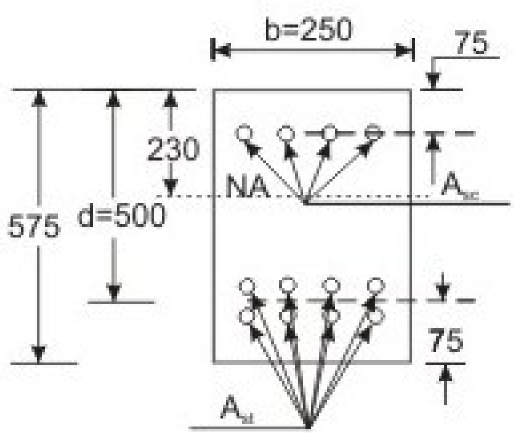
$$M_u = M_{u,lim} + M_{u2} = 347.67 + 181.12 = 528.79\ \text{kNm}$$

Therefore, with $A_{st} = A_{st,\,lim} + A_{st2} = 2016 + 929 = 2945$ mm^2 the required

$A_{sc} = 977.956$ mm^2 (much less than the provided 1256 mm^2). Hence, o.k.

Practice Questions and Problems with Answers

Q.1: Design a doubly reinforced beam (Fig. 4.9.3) to resist $M_u = 375$ kNm when $b = 250$ mm, $d = 500$ mm, $d' = 75$ mm, $f_{ck} = 30$ N/mm^2 and $f_y = 500$ N/mm^2, using (i) direct computation method and (ii) using table of SP- 16.



A_{sc} = 4-16T (= 804 mm^2)
A_{st} = 6-20T + 2-12T (= 2110 mm^2)

Fig. 4.9.3: Problem of Q 1

A.1: (A) Solution by direct computation method:

From the given data

$$M_{u,\,lim} = 0.36\left(\frac{x_{u,\max}}{d}\right)\left(1 - 0.42\,\frac{x_{u,\max}}{d}\right) b\, d^2\, f_{ck}$$

$= 0.36\ (0.46)\ \{1 - 0.42\ (0.46)\}\ (250)\ (500)^2\ (30)\ (10^{-6})$ kNm

$= 250.51$ kNm

Using the value of $p_t = 1.13$ from Table 3.1 of Lesson 5 for $f_{ck} = 30$ N/mm^2 and $f_y = 500$ N/mm^2,

$$A_{st,\,\lim} = \frac{1.13\ (250)\ (500)}{100} = 1412.5\ \text{mm}^2$$

$M_{u2} = 375 - 250.51 = 124.49$ kNm

From Table 4.2 of Lesson 8, for $d'/d = 75/500 = 0.15$ and $f_y = 500$ N/mm^2, we get $f_{sc} = 395$ N/mm^2

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})\,(d - d')} = \frac{124.49\ (10^6)}{\{395 - 0.446\ (30)\}(500 - 75)} = 767.56\ \text{mm}^2$$

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{767.56\ \{395 - 0.446\ (30)\}}{0.87\ (500)} = 673.37\ \text{mm}^2$$

$$A_{st} = A_{st,\lim} + A_{st2} = 1412.5 + 673.37 = 2085.87\ \text{mm}^2$$

Alternatively: (use of Table 4.1 of Lesson 8 to determine f_{sc} from ε_{sc})

$x_{u,\,max} = 0.46\ (500) = 230$ mm

$$\varepsilon_{sc} = \frac{0.0035\ (230 - 75)}{230} = \frac{0.0035\ (155)}{230} = 0.002359$$

From Table 4.1

$$f_{sc} = 391.3 + \frac{(413.0 - 391.3)\,(0.002359 - 0.00226)}{(0.00277 - 0.00226)} = 395.512 \ \text{N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})\,(d - d')} = \frac{124.49\,(10^6)}{\{395.512 - 0.446\,(30)\}(500 - 75)} = 766.53 \ \text{mm}^2$$

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{766.53\ (382.132)}{0.87\ (500)} = 673.369 \ \text{mm}^2$$

$$A_{st} = A_{st,\,\lim} + A_{st2} = 1412.5 + 673.369 = 2085.869 \ \text{mm}^2$$

Check for minimum and maximum A_{st} and A_{sc}

(i) Minimum $A_{st} = \dfrac{0.85\, b\, d}{f_y} = \dfrac{0.85\ (250)\ (500)}{500} = 212.5 \ \text{mm}^2$

(ii) Maximum $A_{st} = 0.04\, b\, D = 0.04\ (250)\ (575) = 5750 \ \text{mm}^2$

(iii) Minimum $A_{st} = \dfrac{0.2\, b\, D}{100} \dfrac{0.2\,(250)\ (575)}{100} = 287.5 \ \text{mm}^2$

(iv) Maximum $A_{st} = 0.04\, b\, D = 0.04\ (250)\ (575) = 5750 \ \text{mm}^2$

Hence, the areas of reinforcement satisfy the requirements.

So, provide (i) 6-20 T + 2-12 T = 1885 + 226 = 2111 mm^2 for A_{st}

(ii) 4-16 T = 804 mm^2 for A_{sc}

(B) Solution by use of table of SP-16

From the given data, we have

$$\frac{M_u}{b\,d^2} = \frac{375\,(10^6)}{250\,(500)^2} = 6.0$$

d'/d = 75/500 = 0.15

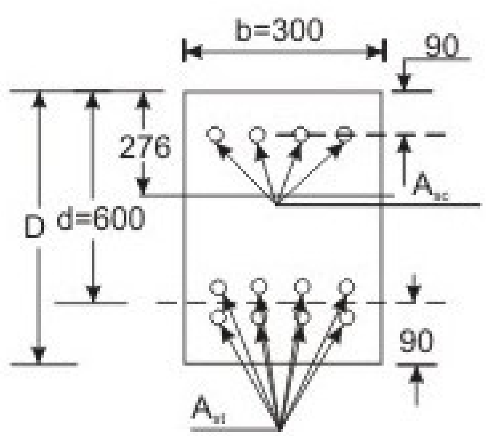
Table 56 of SP-16 gives: p_t = 1.676 and p_c = 0.619

$$\text{So,} \quad A_{st} = \frac{(1.676)(250)(500)}{100} = 2095 \text{ mm}^2$$

$$\text{and} \quad A_{sc} = \frac{(0.619(250)(500)}{100} = 773.75 \text{ mm}^2$$

These values are close to those of (A). Hence, provide 6-20 T + 2-12 T as A_{st} and 4-16 T as A_{sc}.

Q.2: Determine the moment of resistance of the doubly reinforced beam (Fig. 4.9.4) with b = 300 mm, d = 600 mm, d' = 90 mm, f_{ck} = 30 N/mm², f_y = 500 N/mm², A_{sc} = 2236 mm² (2-32 T + 2-20 T), and A_{st} = 4021 mm² (4-32 T + 4-16 T). Use (i) direct computation method and (ii) tables of SP-16.



A_{sc} = 2-32T + 2-20T (= 2236 mm²)
A_{st} = 4-32T + 4-16T (= 4021 mm²)

Fig. 4.9.4: Problem of Q 2

A.2: (i) Solution by direct computation method:

$x_{u,\,max}$ = 0.46 (600) = 276 mm

$$\varepsilon_{st} = \frac{0.0035\ (600 - 276)}{276} = 0.0041086$$

ε_{yield} = 0.00417. So $\varepsilon_{st} < \varepsilon_{yield}$ i.e. the beam is over-reinforced.

For d'/d = 0.15 and f_y = 500 N/mm^2, Table 4.2 of Lesson 8 gives: f_{sc} = 395 N/mm^2 and with f_{ck} = 30 N/mm^2 , Table 3.1 of Lesson 5 gives $p_{t,\ lim}$ = 1.13.

$$A_{st,\lim} = \frac{1.13\,(300)\ (600)}{100} = 2034\ \ \text{mm}^2$$

$$M_{u,lim} = 0.36\ (\frac{x_{u,\max}}{d})\ (1 - 0.42\ \frac{x_{u,\max}}{d})\ b\ d^2\ f_{ck}$$

$$= \quad 0.36\ (0.46)\ \{1 - 0.42\ (0.46)\}\ (300)\ (600)^2\ (30)\ (10^{-6})\ \text{kNm}$$

$$= \quad 432.88\ \text{kNm}$$

A_{st2} = 4021 - 2034 = 1987 mm^2

$$(A_{sc})_{required} = \frac{A_{st2}\,(0.87)\,f_y}{(f_{sc} - f_{cc})} = \frac{1987\ (0.87)\ (500)}{\{395 - 0.446\ (30)\}} = 2264.94 \quad \text{mm}^2 > 2236 \quad \text{mm}^2$$

So, A_{st2} of 1987 mm^2 is not fully used. Let us determine A_{st2} required when A_{sc} = 2236 mm^2.

$$A_{st2} = \frac{A_{sc}\,(f_{sc} - f_{cc})}{0.87\,f_y} = \frac{2236\ \{395 - 0.446\ (30)\}}{(0.87)\ (500)} = 1961.61 \quad \text{mm}^2$$

$A_{st} = A_{st,\ lim} + A_{st2}$ = 2034 + 1961.61 = 3995.61 mm^2 < 4021 mm^2.

Hence, o.k.

With A_{st2} = 1961.61 mm^2, $\quad M_{u2} = A_{st2}\ (0.87\, f_y)\ (d - d')$

$$= \ 1961.61\ (0.87)\ (500)\ (600 - 75)\,(10^{-6})\ \text{kNm} \qquad = 447.98268\ \ \text{kNm}$$

Again, when A_{sc} = 2236 mm^2 (as provided)

$$M_{u2} = A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$= 2236\ \{395 - 0.446\ (30)\}\ (600 - 75)\ (10^{-6})\ \text{kNm} = 447.9837\ \text{kNm}$$

M_u = $M_{u,\,lim}$ + M_{u2} = 432.88 + 447.98 (M_{u2} is taken the lower ofthe two)

= 880.86 kNm

Hence, the moment of resistance of the beam is 880.86 kNm. Alternatively f_{sc} can be determined from Table 4.1 of Lesson 8. Using the following from the above:

$x_{u,\,max}$ = 276 mm $A_{st,lim}$ =

2034 mm^2 $M_{u,lim}$ =

432.88 kNm A_{st2} = 1987

mm^2

To find $(A_{sc})_{required}$

$$\varepsilon_{st} = \frac{0.0035\ (276 - 90)}{276} = 0.00236$$

Table 4.1 of Lesson 8 gives:

$$f_{sc} = 391.3 + \frac{(413 - 391.3)\ (0.00236 - 0.00226)}{(0.00277 - 0.00226)} = 395.55\ \text{N/mm}^2$$

$$(A_{sc})_{required} = \frac{A_{st2}\,(0.87)\,f_y}{(f_{sc} - f_{cc})} = \frac{1987\ (0.87)\ (500)}{\{395.55 - 0.446\ (30)\}}$$

$$= 2261.68\ \text{mm}^2 \quad > 2236\ \text{mm}^2$$

So, it is not o.k.

Let us determine A_{st2} required when A_{sc} = 2236 mm^2.

$$A_{st2} = \frac{A_{sc}\ (f_{sc} - f_{cc}\ )}{0.87 f_y} = \frac{2236\ \{395.55 - 0.446\ (30)\}}{(0.87)\ (500)} = 1964.44 \quad mm^2$$

$A_{st} = A_{st,\ lim} + A_{st2} = 2034 + 1964.44 = 3998.44\ mm^2 < 4021\ mm^2$.

So, o.k.

M_{u2} (when $A_{st2} = 1964.44\ mm^2$) = $A_{st2}\ (0.87\ f_y\ )\ (d - d')$

$= 1964.44\ (0.87)\ (500)\ (600 - 75)\ (10^{-6})$ kNm

$= 448.63$ kNm

For $A_{sc} = 2236\ mm^2$,

$$M_{u2} = A_{sc}\,(f_{sc} - f_{cc})\ (d - d')$$

$= 2236\ \{(395.55 - 0.446\ (30)\}\ (600 - 75)\ (10^{-6})$ kNm

$= 2236\ (382.17)\ (525)\ (10^{-6})$ kNm

$= 448.63$ kNm

Both the M_{u2} values are the same. So,

$M_u = M_{u,lim} + M_{u2} = 432.88 + 448.63$

$= 881.51$ kNm

Here, the $M_u = 881.51$ kNm.

(ii) Solution by using table of SP-16

From the given data:

$$p_t = \frac{4021(100)}{300\ (600)} = 2.234$$

$$p_c = \frac{2236\,(100)}{300\,(600)} = 1.242$$

$d'/d =$ 0.15

Table 56 of SP-16 is used first considering d'/d = 0.15 and p_t = 2.234, and secondly, considering d'/d = 0.15 and p_c = 1.242. The calculated values of p_c and M_u/bd^2 for the first and p_t and M_u/bd^2 for the second cases are presented below separately. Linear interpolation has been done.

(i) When d'/d = 0.15 and p_t = 2.234

$$\frac{M_u}{b\,d^2} = 8.00 + \frac{(8.1-8.0)(2.234-2.218)}{(2.245-2.218)} = 8.06$$

$$p_c = 1.235 + \frac{(1.266-1.235)(0.016)}{(0.027)} = 1.253 > 1.242$$

So, this is not possible.

(ii) When d'/d = 0.15 and p_c = 1.242

$$\frac{M_u}{b\,d^2} = 8.00 + \frac{(8.1-8.0)(1.242-1.235)}{(1.266-1.235)} = 8.022$$

$$p_t = 2.218 + \frac{(2.245-2.218)(1.242-1.235)}{(1.266-1.235)} = 2.224 < 2.234$$

So, M_u = 8.022 (300) $(600)^2$ (10^{-6})= 866.376 kNm.

Hence, o.k.

Solutions

Maximum Marks = 50, Maximum Time = 30 minutes Answer

all questions.

TQ.1: Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: b = 300 mm, D = 700 mm, M 20 concrete, Fe 415 steel (Fig. 4.9.1). Determine f_{sc} from strain ε_{sc} as given in Table 4.1 of Lesson 8.

A.TQ.1: This problem is the same as Problem 4.1 in sec. 4.9.2.1 except that here the f_{sc} is to be calculated using Table 4.1 instead of Table 4.2.

Step 1: Here, the Step 1 will remain the same as that of Problem 4.1.

Step 2: Determination of M_{u2}, A_{sc}, A_{st2} and A_{st}

$$M_{u2} = M_u - M_{u,\lim} = 482.96 - 328.55 = 154.41 \text{ kNm}$$

From strain triangle: (Fig. 4.8.2 of Lesson 8)

$$\varepsilon_{sc} = \frac{0.0035\,(302.4 - 70)}{302.4} = 0.00269$$

$$f_{sc} \text{ (from Table 4.1 of Lesson 8)} = 342.8 + \frac{(351.8 - 342.8)}{(0.00276 - 0.00241)}(0.00269 - 0.00241)$$

$$= 350 \text{ N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{154.41(10^6)}{\{350 - 0.446(20)\}(630 - 70)} = 808.41 \text{ mm}^2$$

N/mm

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{808.41\,(341.08)}{(0.87)\,(415)} = 763.696 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1809.14 + 763.696 = 2572.836 \text{ mm}^2$$

$$A_{sc} = 808.41 \text{ mm}^2$$

Steps 3 & 4 will also remain the same as those of Problem 4.1.

Hence, provide 2-20 T + 2-12 T (854 mm^2) as A_{sc} and 4-25 T + 2-20 T (2591 mm^2) as A_{st} .

TQ.2: Determine the ultimate moment capacity of the doubly reinforced beam of b = 350 mm, d' = 60 mm, d = 600 mm, A_{st} = 2945 mm^2 (6-25 T), A_{sc} = 1256 mm^2 (4-20 T), using M 20 and Fe 415 (Fig. 4.9.2). Use table of SP-16 only.

A.TQ.2: Solution by using table of SP-16

This problem is the same as that of Problem 4.2 of sec. 4.9.2.2, which has been solved by direct computation method. Here, the same is to be solved by using SP-16.

The needed parameters are:

$$d'/d = 60/600 = 0.1$$

$$p_t = \frac{A_{st}(100)}{b\,d} = \frac{2945\,(100)}{350(600)} = 1.402$$

$$p_c = \frac{A_{sc}(100)}{b\,d} = \frac{1256(100)}{350\,(600)} = 0.5981$$

Here, we need to use Table 50 for f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. The table gives values of M_u/bd^2 for (I) d'/d and p_t and (ii) d'/d and p_c. So, we will consider both the possibilities and determine M_u.

(i) Considering Table 50 of SP-16 when d'/d = 0.1 and p_t = 1.402: Interpolating

the values of M_u/bd^2 at p_t = 1.399 and 1.429, we get

$$\left(\frac{M_u}{bd^2}\right)_{pt\,=\,1.402} = 4.2 + \frac{(4.3 - 4.2)\,(1.402 - 1.399)}{(1.429 - 1.399)} = 4.21$$

$$\text{the corresponding } (p_c)_{pt\,=1.402} = 0.466 + \frac{(0.498 - 0.466)\,(1.402 - 1.399)}{(1.429 - 1.399)} = 0.4692$$

But, p_c provided is 0.5981 indicates that extra compression reinforcement has been used.

So, we get

$M_u = 4.21\, b\, d^2 = (4.21)\,(350)\,(600)^2\,(10^{-6}) = 530.46$ kNm when A_{st} = 2945 mm^2 and A_{sc} = 985.32 mm^2, i.e. 270.69 mm^2 (= 1256 - 985.32) of compression steel is extra.

(ii) Considering d'/d = 0.1 and p_c = 0.5981, we get by linear interpolation

$$\left(\frac{M_u}{bd^2}\right)_{pc\,=\,0.5981} = 4.6 + \frac{(4.7 - 4.6)\,(0.5981 - 0.595)}{(0.628 - 0.595)} = 4.61$$

the corresponding p_t is:

$$(p_t)_{pc\,=\,0.5981} = 1.522 + \frac{(1.533 - 1.522)\,(0.5981 - 0.595)}{(0.628 - 0.595)} = 1.5231$$

The provided p_t = 1.402 indicates that the tension steel is insufficient by 254.31 mm^2 as shown below:

Amount of additional A_{st} still required =

$$\frac{(1.5231 - 1.402)(350)(600)}{100} = 254.31 \text{ mm}^2$$

If this additional steel is provided, then the M_u of this beam becomes:

$$M_u = 4.61\ b\,d^2 = 4.61\,(350)\,(600)^2\,(10^{-6})\text{ kNm} = 580.86 \quad \text{kNm}$$

The above two results show that the moment of resistance of this beam is the lower of the two. So, $M_u = 530.46$ kNm. By direct computation the M_u = 528.79 kNm. The two results are in good agreement.

Introduction

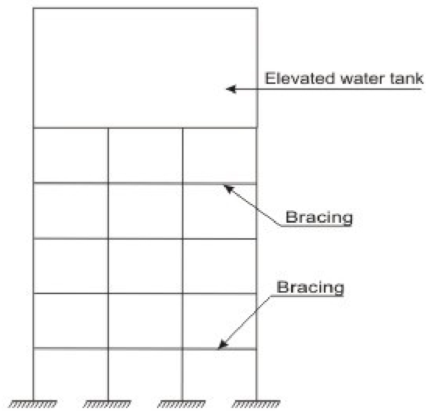


Fig. 5.10.1(a): Bracings of elevated water tank

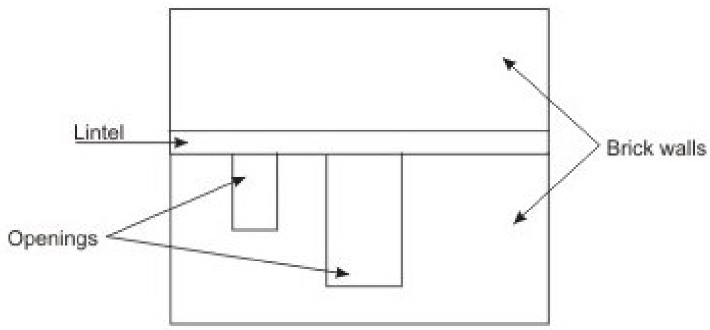


Fig. 5.10.1 (b):Lintels over openings (without effective chajjas)

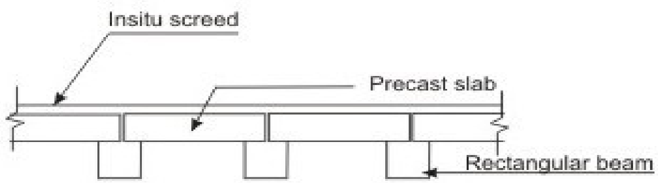


Fig. 5.10.1(c) Precast slab on rectangular beams

Reinforced concrete slabs used in floors, roofs and decks are mostly cast monolithic from the bottom of the beam to the top of the slab. Such rectangular beams having slab on top are different from others having either no slab (bracings of elevated tanks, lintels etc.) or having disconnected slabs as in some pre-cast systems (Figs. 5.10.1 a, b and c). Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e., between the supports of a continuous beam, the slab, up to a certain width greater than the width of the beam, forms the top part of the beam. Such beams having slab on top of the rectangular rib are designated as the flanged beams - either *T* or *L* type depending on whether the slab is on both sides or on one side of the beam (Figs. 5.10.2 a to e). Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression. The continuous beam at support is thus equivalent to a rectangular beam (Figs. 5.10.2 a, c, f and g).

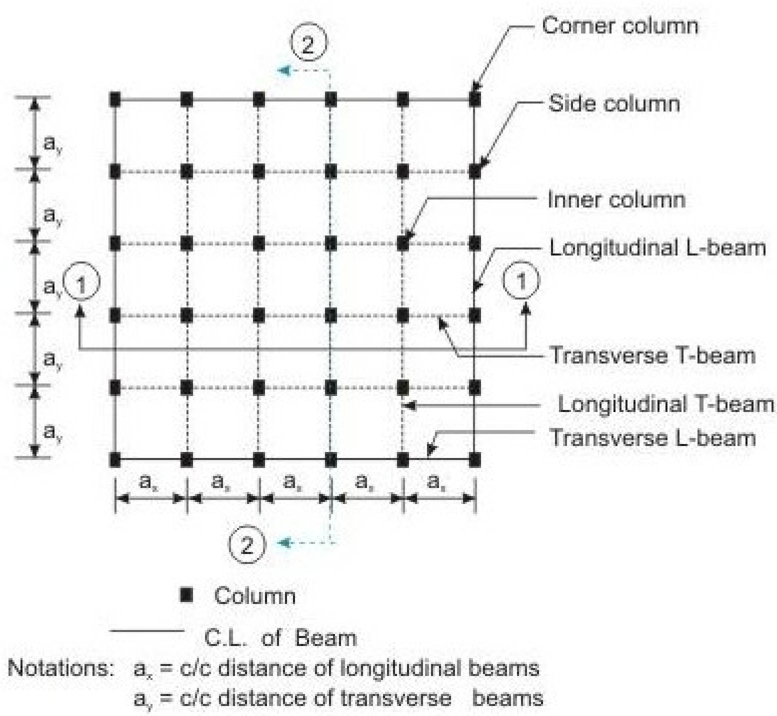
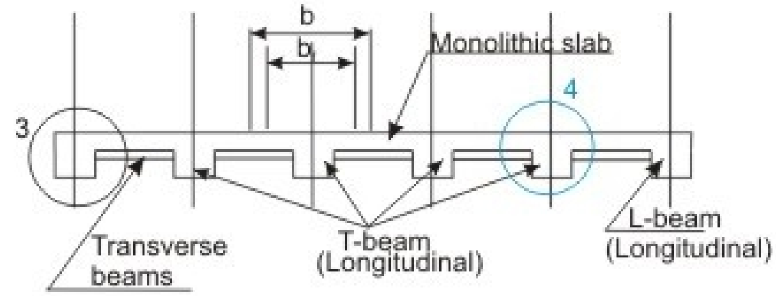


Fig. 5.10.2 (a): Key plan



Notations:
b = Actual width of flange
b_f = Effective width of flange

Fig. 5.10.2 (b): Section 1-1

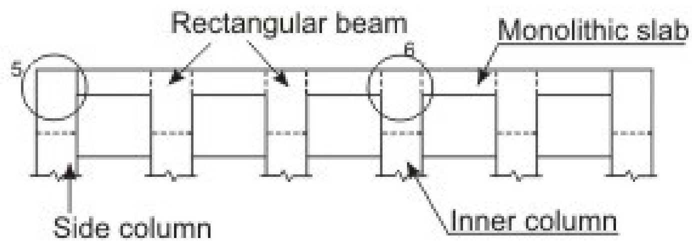
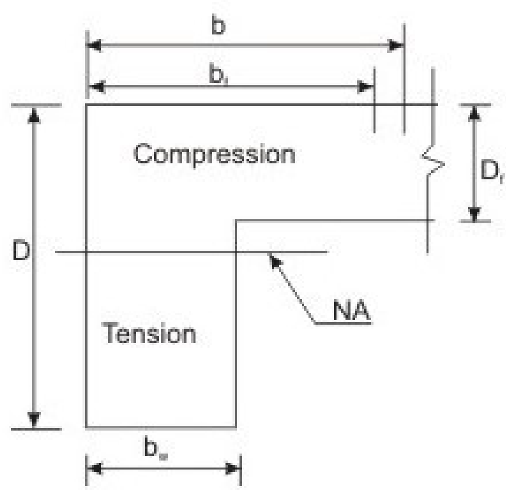
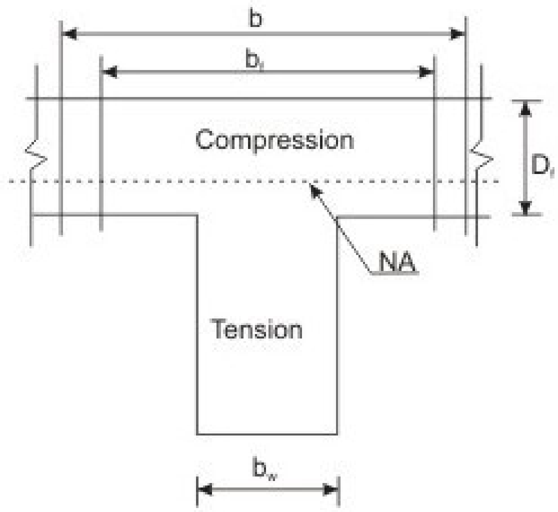


Fig. 5.10.2 (c): Section 2-2



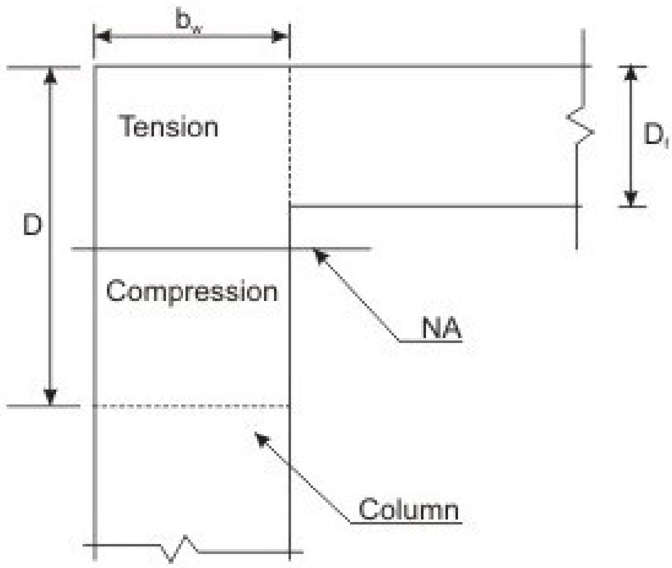
Notations:
b = Actual width of flange
b_f = Effective width of flange
b_w = Width of web
D_f = Depth of flange
NA = Neutral axis

Fig. 5.10.2 (d): Detail at 3 (L-beam)



Notations:
b = Actual width of flange
b_f = Effective width of flange
b_w = Width of web
D_f = Depth of flange
NA = Neutral axis

Fig.5.10.2 (e): Detail at 4 (T-beam)



Notation:
b_f = Effective width of flange
b_w = Width of web
D_f = Depth of flange
NA = Neutral axis

Fig. 5.10.2 (f): Detail at 5 (rectangular beam)

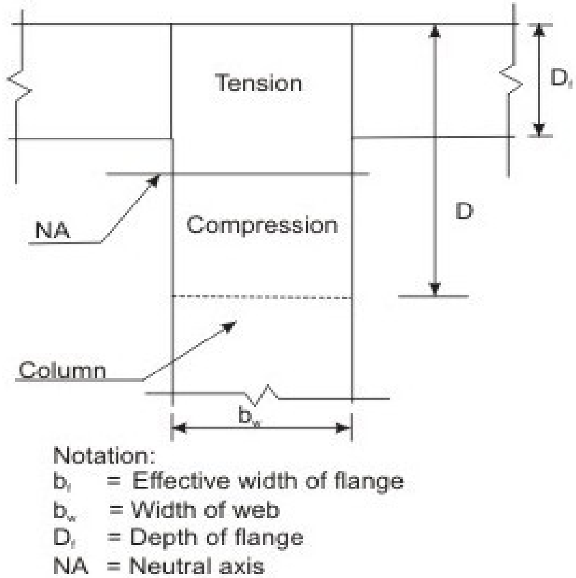


Fig. 5.10.2 (g): Detail at 6 (rectangular beam)

The actual width of the flange is the spacing of the beam, which is the same as the distance between the middle points of the adjacent spans of the slab, as shown in Fig. 5.10.2 b. However, in a flanged beam, a part of the width less than the actual width, is effective to be considered as a part of the beam. This width of the slab is designated as the effective width of the flange.

Effective Width

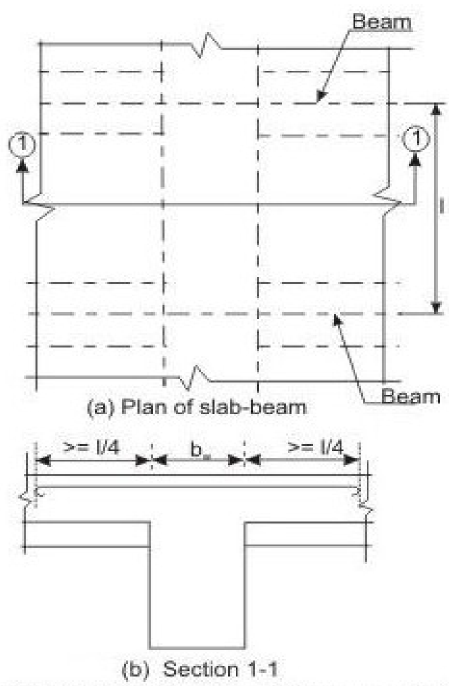


Fig. 5.10.3: Transverse reinforcement of flange of T-beam

IS code requirements

The following requirements (cl. 23.1.1 of IS 456) are to be satisfied to ensure the combined action of the part of the slab and the rib (rectangular part of the beam).

(a) The slab and the rectangular beam shall be cast integrally or they shall be effectively bonded in any other manner.

(b) Slabs must be provided with the transverse reinforcement of at least 60 per cent of the main reinforcement at the mid span of the slab if the main reinforcement of the slab is parallel to the transverse beam (Figs. 5.10.3 a and b).

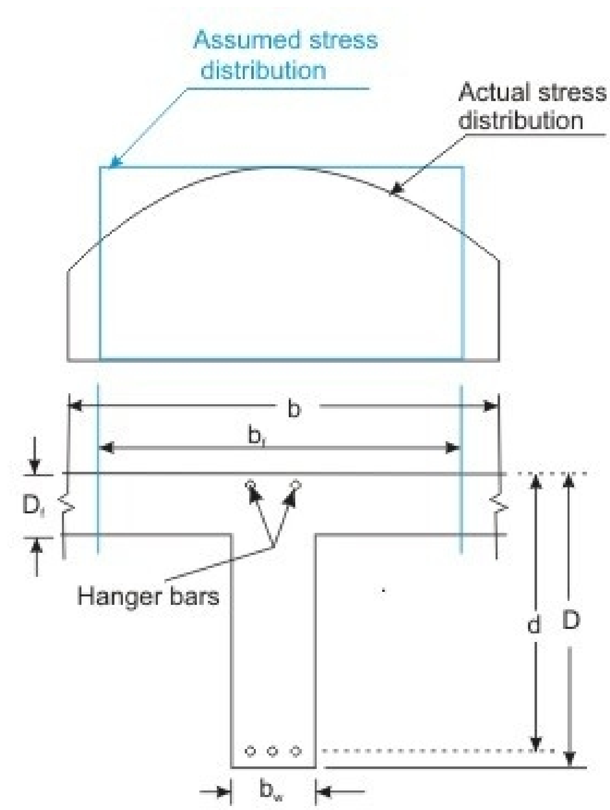


Fig. 5.10.4: Variation of compressive stress

The variation of compressive stress (Fig. 5.10.4) along the actual width of the flange shows that the compressive stress is more in the flange just above the rib than the same at some distance away from it. The nature of variation is complex and, therefore, the concept of effective width has been introduced. The effective width is a convenient hypothetical width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress.

IS code specifications

Clause 23.1.2 of IS 456 specifies the following effective widths of *T* and *L*-beams:

(a) For *T*-beams, the lesser of

(i) $b_f = l_o/6 + b_w + 6\,D_f$

(ii) b_f = Actual width of the flange

(b) For isolated *T*-beams, the lesser of

(i) $$b_f = \frac{l_o}{(l_o/b) + 4} + b_w$$

(ii) b_f = Actual width of the flange

(c) For *L*-beams, the lesser of

(i) $b_f = l_o/12 + b_w + 3\,D_f$

(ii) b_f = Actual width of the flange

(d) For isolated *L*-beams, the lesser of

(i) $$b_f = \frac{0.5\,l_o}{(l_o/b) + 4} + b_w$$

(ii) b_f = Actual width of the flange

where b_f = effective width of the flange,

l_o = distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

b_w = beadth of the web,

D_f = thickness of the flange, and

b = actual width of the flange.

Four Different Cases

The neutral axis of a flanged beam may be either in the flange or in the web depending on the physical dimensions of the effective width of flange b_f, effective width of web b_w, thickness of flange D_f and effective depth of flanged beam d (Fig. 5.10.4). The flanged beam may be considered as a rectangular beam of width b_f and effective depth d if the neutral axis is in the flange as the concrete in tension is ignored. However, if the neutral axis is in the web, the compression is taken by the flange and a part of the web.

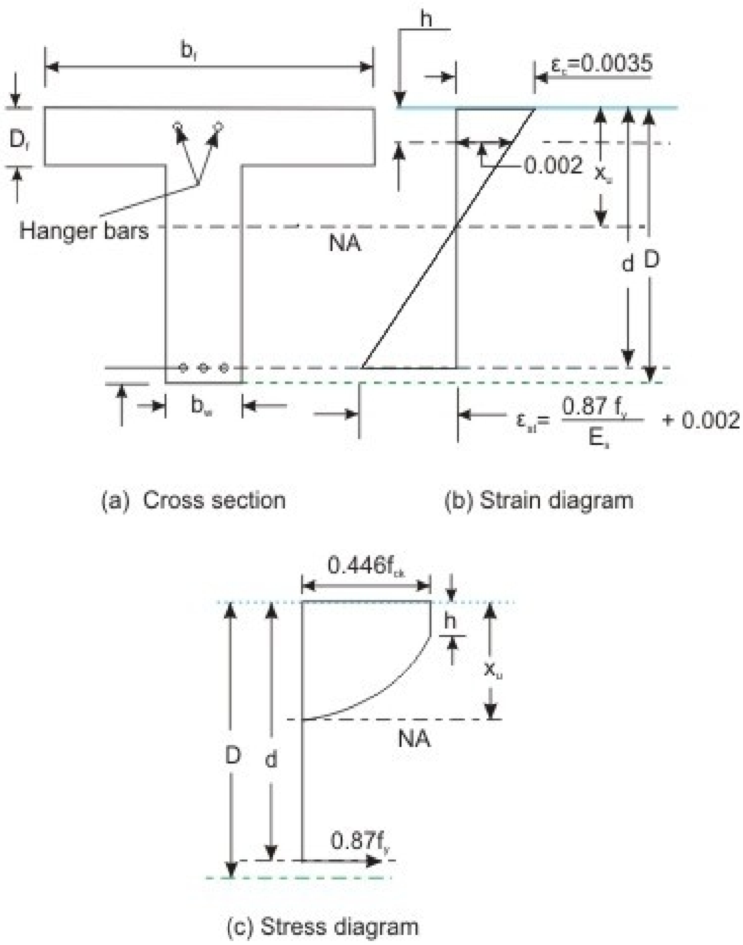


Fig. 5.10.5: A typical T-beam section

All the assumptions made in sec. 3.4.2 of Lesson 4 are also applicable for the flanged beams. As explained in Lesson 4, the compressive stress remains constant between the strains of 0.002 and 0.0035. It is important to find the depth h of the beam where the strain is 0.002 (Fig. 5.10.5 b). If it is located in the web, the whole of flange will be under the constant stress level of 0.446 f_{ck}. The following gives the relation of D_f and d to facilitate the determination of the depth h where the strain will be 0.002.

From the strain diagram of Fig. 5.10.5 b:

$$\frac{0.002}{0.0035} = \frac{x_u - h}{x_u}$$

or
$$\frac{h}{x_u} = \frac{3}{7} = 0.43 \qquad (5.1)$$

when $x_u = x_{u,\max}$, we get

$h = \frac{3}{7} x_{u,\max} = 0.227\,d,\ 0.205\,d$ and $0.197\,d$, for Fe 250, Fe 415 and Fe 500, respectively. In general, we can adopt, say

$$h/d = 0.2 \qquad (5.2)$$

The same relation is obtained below from the values of strains of concrete and steel of Fig. 5.10.5 b.

$$\frac{\varepsilon_{st}}{\varepsilon_c} = \frac{d - x_u}{x_u}$$

or
$$\frac{d}{x_u} = \frac{\varepsilon_{st} + \varepsilon_c}{\varepsilon_c} \qquad (5.3)$$

Dividing Eq. 5.1 by Eq. 5.3

$$\frac{h}{d} = \frac{0.0015}{\varepsilon_{st} + 0.0035} \qquad (5.4)$$

Using $\varepsilon_{st} = (0.87\ f_y/E_s) + 0.002$ in Eq. 5.4, we get h/d = 0.227, 0.205 and 0.197 for Fe 250, Fe 415 and Fe 500 respectively, and we can adopt h/d = 0.2 (as in Eq. 5.2).

Thus, we get the same Eq. 5.2 from Eq. 5.4,

$$h/d = 0.2 \qquad (5.2)$$

It is now clear that the three values of h are around 0.2 d for the three grades of steel. The maximum value of h may be D_f, at the bottom of the flange where the strain will be 0.002, if D_f /d = 0.2. This reveals that the thickness of the flange may be considered small if D_f /d does not exceed 0.2 and in that case, the position of the fibre of 0.002 strain will be in the web and the entire flange will be under a constant compressive stress of 0.446 f_{ck} .

On the other hand, if D_f is > 0.2 d, the position of the fibre of 0.002 strain will be in the flange. In that case, a part of the slab will have the constant stress of 0.446 f_{ck} where the strain will be more than 0.002.

Thus, in the balanced and over-reinforced flanged beams (when $x_u = x_{u\text{ , max}}$), the ratio of D_f /d is important to determine if the rectangular stress block is for the full depth of the flange (when D_f /d does not exceed 0.2) of for a part of the flange (when D_f /d > 0.2). Similarly, for the under-reinforced flanged beams, the ratio of D_f /x_u is considered in place of D_f /d. If D_f /x_u does not exceed 0.43 (see Eq. 5.1), the constant stress block is for the full depth of the flange. If D_f /x_u > 0.43, the constant stress block is for a part of the depth of the flange.

Based on the above discussion, the four cases of flanged beams are as follows:

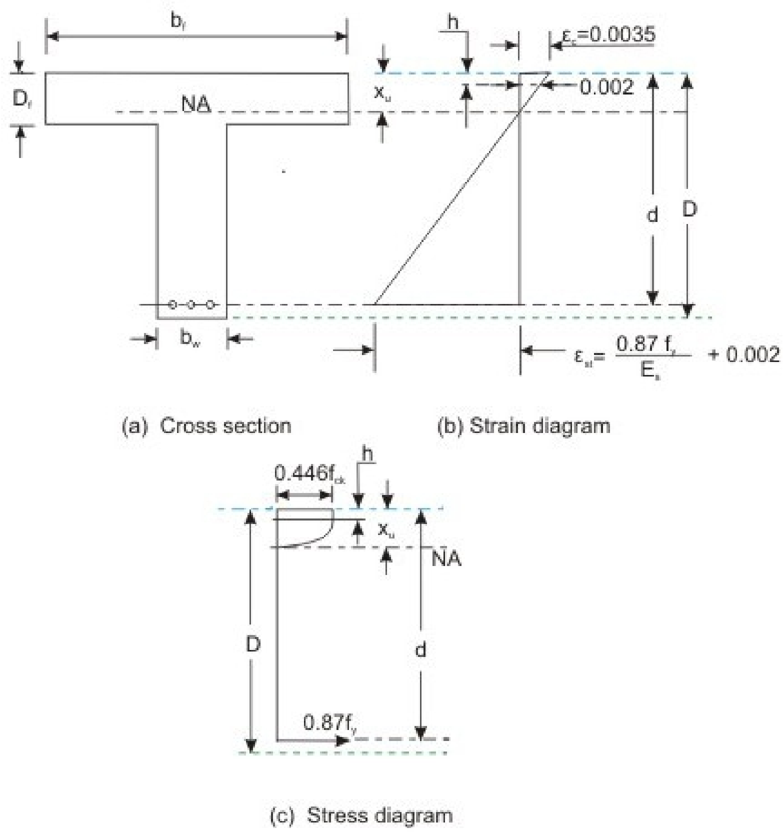


Fig. 5.10.6: T-beam, case (i), when $x_u < D_f$

(i) Neutral axis is in the flange ($x_u < D_f$), (Fig. 5.10.6 a to c)

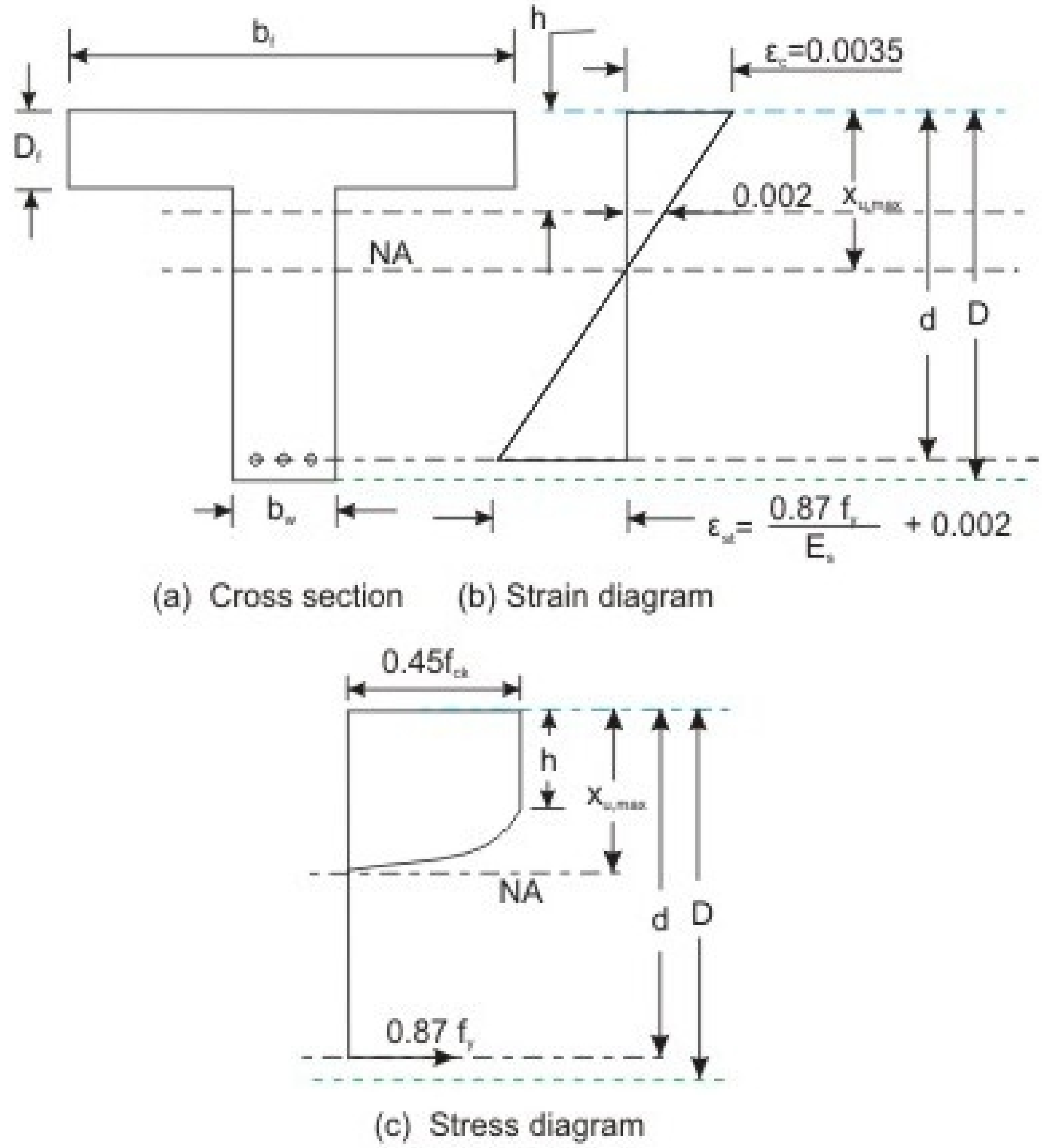
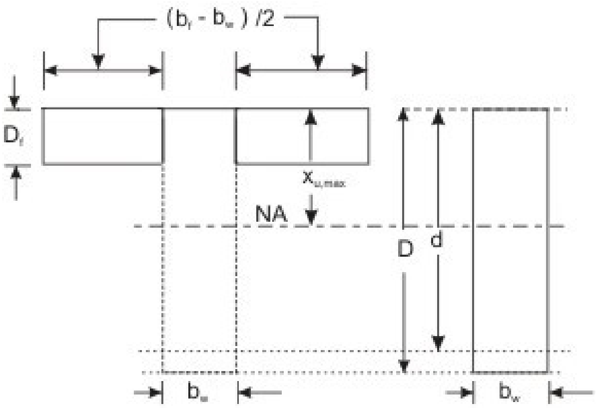


Fig. 5.10.7: T-beam, case (ii a), when D_f/d <= 0.2 and balanced $x_{u,max} > D_f$



(d) Flange (e) Web

Fig. 5.10.7: T-beam, case (ii a), when $D_f/d <= 0.2$ and balanced $x_{u,max} > D_f$

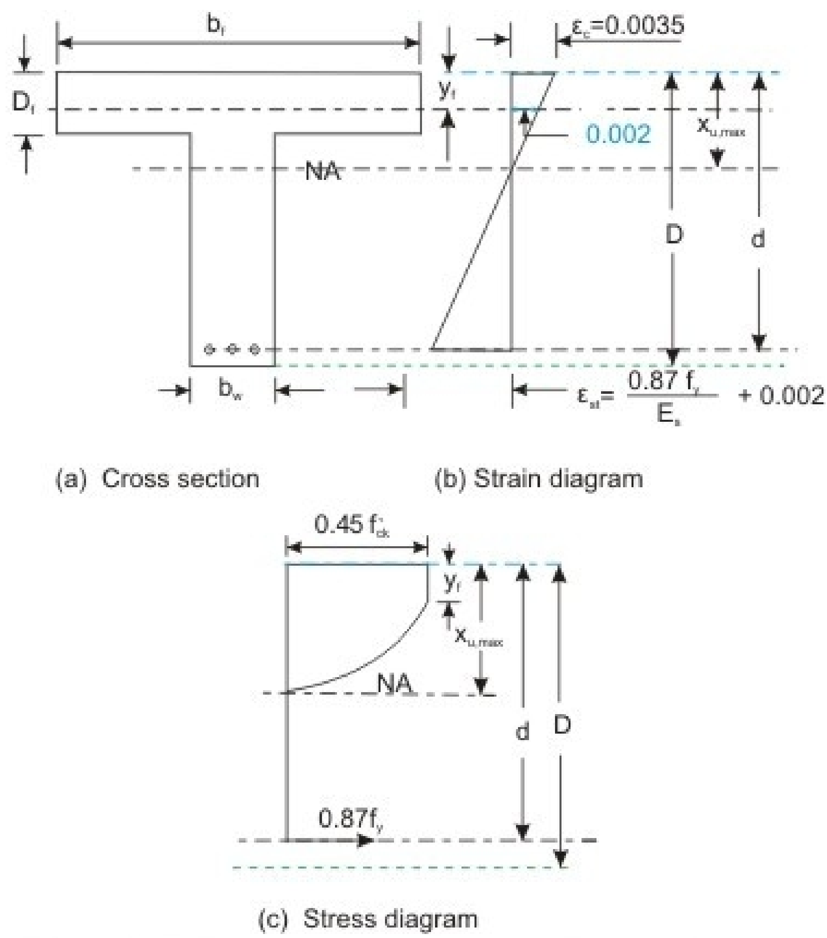


Fig. 5.10.8: T-beam, case (ii b), when $D_f/d > 0.2$ and balanced $x_{u,max} > D_f$

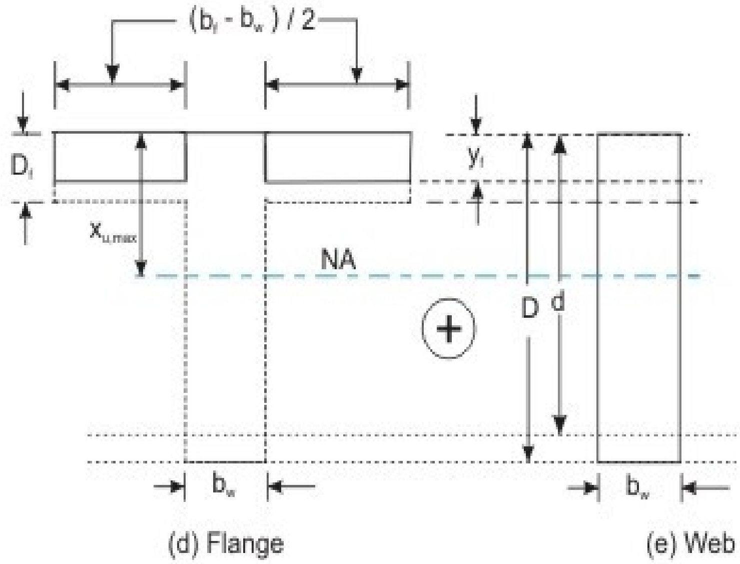
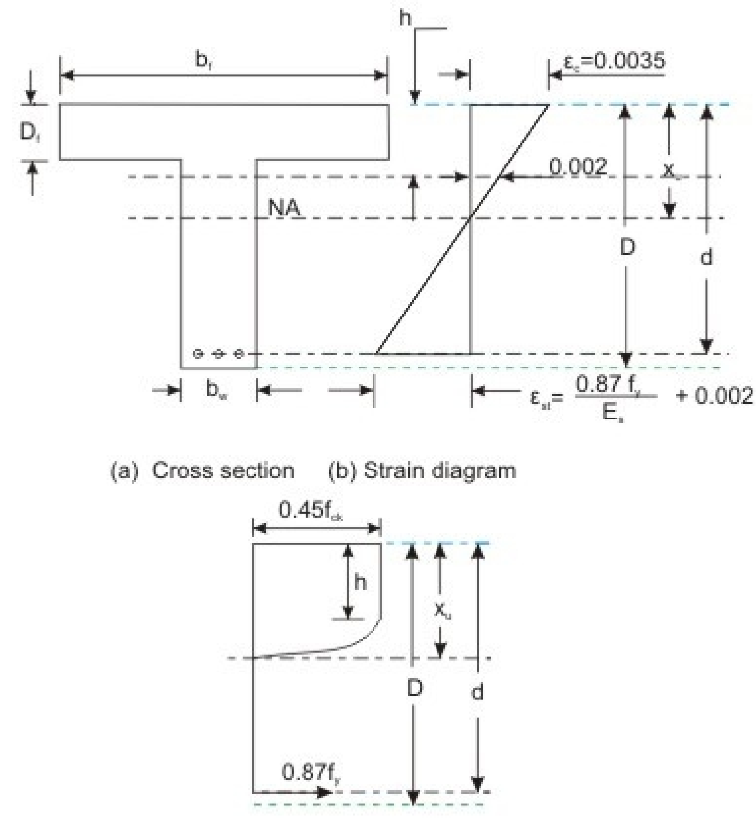


Fig. 5.10.8: T-beam, case (ii b), when $D_f/d > 0.2$ and balanced $x_{u,max} > D_f$

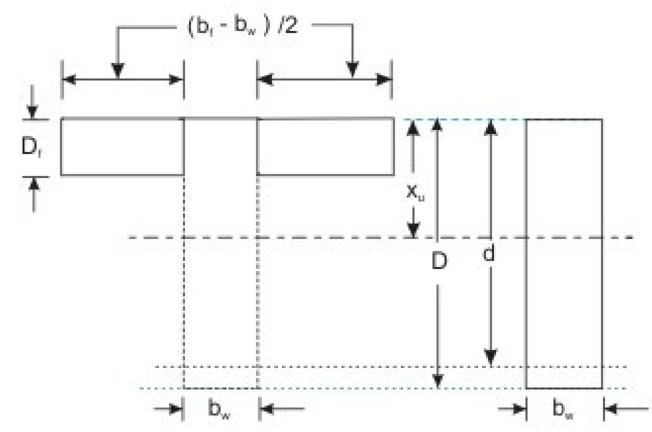
(ii) Neutral axis is in the web and the section is balanced ($x_u = x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a to e)

It has two situations: (a) when D_f/d does not exceed 0.2, the constant stress block is for the entire depth of the flange (Fig. 5.10.7), and
(b) when $D_f/d > 0.2$, the constant stress block is for a part of the depth of flange (Fig. 5.10.8).



(c) Stress diagram

Fig. 5.10.9: T-beam, case (iii a), when $D_f / x_u <= 0.43$ and under-reinforced $x_u > D_f$



(d) Flange (e) Web

Fig. 5.10.9: T-beam, case (iii a), when $D_f / x_u <= 0.43$ and under-reinforced $x_u > D_f$

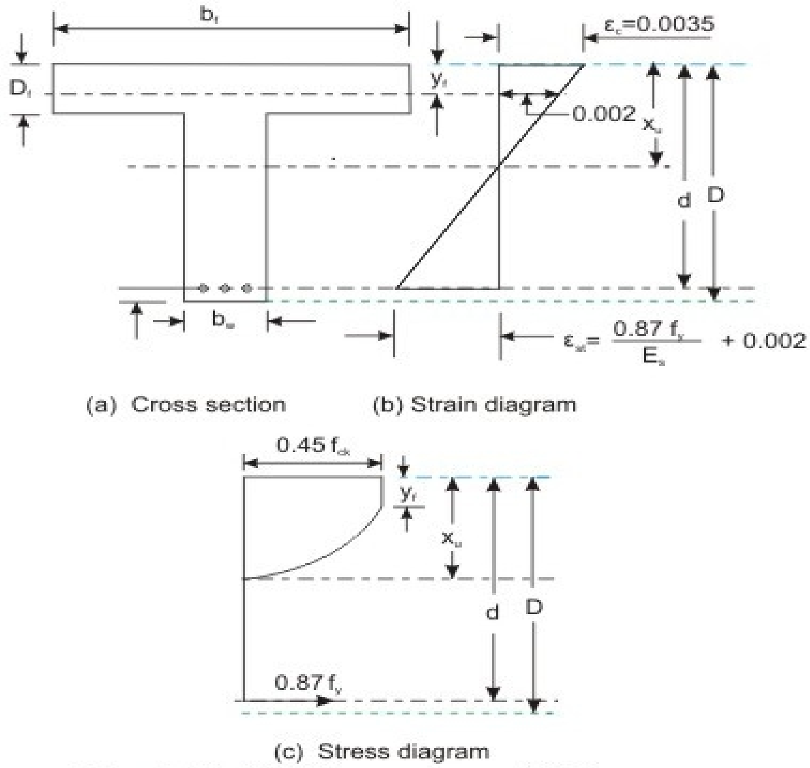


Fig. 5.10.10: T-beam, case (iii b), when $D_f/x_u > 0.43$ and under-reinforced $x_u > D_f$

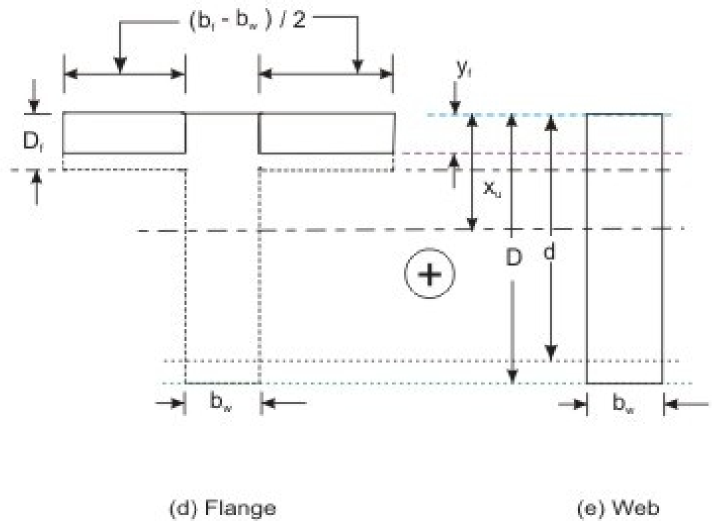


Fig. 5.10.10: T-beam, case (iii b), when $D_f/x_u > 0.43$ and under-reinforced $x_u > D_f$

(iii) Neutral axis is in the web and the section is under-reinforced ($x_{u,max} > x_u > D_f$), (Figs. 5.10.9 and 10 a to e)

This has two situations: (a) when D_f / x_u does not exceed 0.43, the full depth of flange is having the constant stress (Fig. 5.10.9), and (b) when $D_f / x_u > 0.43$, the constant stress is for a part of the depth of flange (Fig. 5.10.10).

(iv) Neutral axis is in the web and the section is over-reinforced ($x_u > x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a to e)

As mentioned earlier, the value of x_u is then taken as $x_{u,max}$ when $x_u > x_{u,max}$. Therefore, this case also will have two situations depending on D_f / d not exceeding 0.2 or > 0.2 as in (ii) above. The governing equations of the four different cases are now taken up.

Governing Equations

The following equations are only for the singly reinforced *T*-beams. Additional terms involving $M_{u,lim}$, M_{u2}, A_{sc} , A_{st1} and A_{st2} are to be included from Eqs. 4.1 to 4.8 of sec. 4.8.3 of Lesson 8 depending on the particular case. Applications of these terms are explained through the solutions of numerical problems of doubly reinforced *T*-beams in Lessons 11 and 12.

Case (i): When the neutral axis is in the flange ($x_u < D_f$), (Figs. 5.10.6 a to c)

Concrete below the neutral axis is in tension and is ignored. The steel reinforcement takes the tensile force (Fig. 5.10.6). Therefore, *T* and *L*-beams are considered as rectangular beams of width b_f and effective depth d. All the equations of singly and doubly reinforced rectangular beams derived in Lessons 4 to 5 and 8 respectively, are also applicable here.

Case (ii): When the neutral axis is in the web and the section is balanced ($x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a to e)

(a) When D_f / d does not exceed 0.2, (Figs. 5.10.7 a to e)

As explained in sec. 5.10.3, the depth of the rectangular portion of the stress block (of constant stress = 0.446 f_{ck}) in this case is greater than D_f (Figs. 5.10.7 a, b and c). The section is split into two parts: (i) rectangular web of width b_w and effective depth d, and (ii) flange of width (b_f - b_w) and depth D_f (Figs. 5.10.7 d and e).

Total compressive force = Compressive force of rectangular beam of width b_w and depth d + Compressive force of rectangular flange of width (b_f - b_w) and depth D_f .

Thus, total compressive force

$$C = 0.36 f_{ck} b_w x_{u,\,max} + 0.45 f_{ck} (b_f - b_w) D_f \tag{5.5}$$

(Assuming the constant stress of concrete in the flange as $0.45 f_{ck}$ in place of $0.446 f_{ck}$, as per G-2.2 of IS 456), and the tensile force

$$T = 0.87 f_y A_{st} \tag{5.6}$$

The lever arm of the rectangular beam (web part) is (d - 0.42 $x_{u,\,max}$) and the same for the flanged part is (d - 0.5 D_f).

So, the total moment = Moment due to rectangular web part + Moment due to rectangular flange part

or $$M_u = 0.36 f_{ck} b_w x_{u,\,max} (d - 0.42 x_{u,\,max}) + 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2)$$

or $$M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f(d - D_f/2) \tag{5.7}$$

Equation 5.7 is given in G-2.2 of IS 456.

(b) When $D_f/d > 0.2$, (Figs. 5.10.8 a to e)

In this case, the depth of rectangular portion of stress block is within the flange (Figs. 5.10.8 a, b and c). It is assumed that this depth of constant stress ($0.45 f_{ck}$) is y_f, where

$$y_f = 0.15 x_{u,\,max} + 0.65 D_f, \text{ but not greater than } D_f \tag{5.8}$$

The above expression of y_f is derived in sec. 5.10.4.5.

As in the previous case (ii a), when D_f/d does not exceed 0.2, equations of C, T and M_u are obtained from Eqs. 5.5, 6 and 7 by changing D_f to y_f. Thus, we have (Figs. 5.10.8 d and e)

$$C = 0.36 f_{ck} b_w x_{u,\,max} + 0.45 f_{ck} (b_f - b_w) y_f \tag{5.9}$$

$$T = 0.87 f_y A_{st} \tag{5.10}$$

The lever arm of the rectangular beam (web part) is (d - 0.42 $x_{u,\,max}$) and the same for the flange part is (d - 0.5 y_f). Accordingly, the expression of M_u is as follows:

$$M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f(d - y_f/2) \quad (5.11)$$

Case (iii): When the neutral axis is in the web and the section is under-reinforced ($x_u > D_f$), (Figs. 5.10.9 and 10 a to e)

(a) When D_f / x_u does not exceed 0.43, (Figs. 5.10.9 a to e)

Since D_f does not exceed 0.43 x_u and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the entire flange will be under a constant stress of 0.45 f_{ck} (Figs. 5.10.9 a, b and c). The equations of C, T and M_u can be written in the same manner as in sec. 5.10.4.2, case (ii a). The final forms of the equations are obtained from Eqs. 5.5, 6 and 7 by replacing $x_{u,\,max}$ by x_u. Thus, we have (Figs. 5.10.9 d and e)

$$C = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\,(b_f - b_w)\, D_f \quad (5.12)$$

$$T = 0.87\, f_y\, A_{st} \quad (5.13)$$

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, D_f\,(d - D_f/2) \quad (5.14)$$

(b) When $D_f / x_u > 0.43$, (Figs. 5.10.10 a to e)

Since $D_f > 0.43\, x_u$ and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the part of the flange having the constant stress of 0.45 f_{ck} is assumed as y_f (Fig. 5.10.10 a, b and c). The expressions of y_f, C, T and M_u can be written from Eqs. 5.8, 9, 10 and 11 of sec. 5.10.4.2, case (ii b), by replacing $x_{u,max}$ by x_u. Thus, we have (Fig. 5.10.10 d and e)

$$y_f = 0.15\, x_u + 0.65\, D_f, \text{ but not greater than } D_f \quad (5.15)$$

$$C = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\,(b_f - b_w)\, y_f \quad (5.16)$$

$$T = 0.87\, f_y\, A_{st} \quad (5.17)$$

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f\,(d - y_f/2) \quad (5.18)$$

Case (iv): When the neutral axis is in the web and the section is over-reinforced (x_u > D_f), (Figs. 5.10.7 and 8 a to e)

For the over-reinforced beam, the depth of neutral axis x_u is more than $x_{u,\ max}$ as in rectangular beams. However, x_u is restricted up to $x_{u,max}$. Therefore, the corresponding expressions of C, T and M_u for the two situations (a) when D_f / d does not exceed 0.2 and (b) when $D_f / d >$ 0.2 are written from Eqs. 5.5 to 5.7 and 5.9 to 5.11, respectively of sec. 5.10.4.2 (Figs. 5.10.7 and 8). The expression of y_f for (b) is the same as that of Eq. 5.8.

(a) When D_f /d does not exceed 0.2 (Figs. 5.10.7 a to e) The

equations are:

$$C = 0.36 f_{ck}\ b_w\ x_{u,\ max} + 0.45\ f_{ck} (b_f - b_w) D_f \qquad (5.5)$$

$$T = 0.87\ f_y\ A_{st} \qquad (5.6)$$

/2)

$$M_u = 0.36(x_{u,\ max} /d)\{1 - 0.42(x_{u,\ max}/d)\} f_{ck}\, b_w\, d^2 + 0.45 f_{ck}(b_f - b_w)\, D_f(d - \ D_f$$

(5.7)

(b) When $D_f /d > 0.2$ (Figs. 5.10.8 a to e)

$y_f = 0.15\ x_{u,\ max} + 0.65\ D_f$, but not greater than D_f

(5.8)

$$C = 0.36 f_{ck}\ b_w\ x_{u,\ max} + 0.45\ f_{ck} (b_f - b_w)\ y_f \qquad (5.9)$$

(5.10) $T = 0.87 f_y\ A_{st}$

/2)

(5.11) $M_u = 0.36(x_{u,\ max} /d)\{1 - 0.42(x_{u,\ max}/d)\} f_{ck}\, b_w\, d^2 + 0.45 f_{ck}(b_f - b_w)\, y_f(d - y_f$

It is clear from the above that the over-reinforced beam will not have additional moment of resistance beyond that of the balanced one. Moreover, it will prevent steel failure. It is, therefore, recommended either to re-design or to go for doubly reinforced flanged beam than designing over-reinforced flanged beam.

Derivation of the equation to determine y_f, Eq. 5.8, Fig. 5.10.11

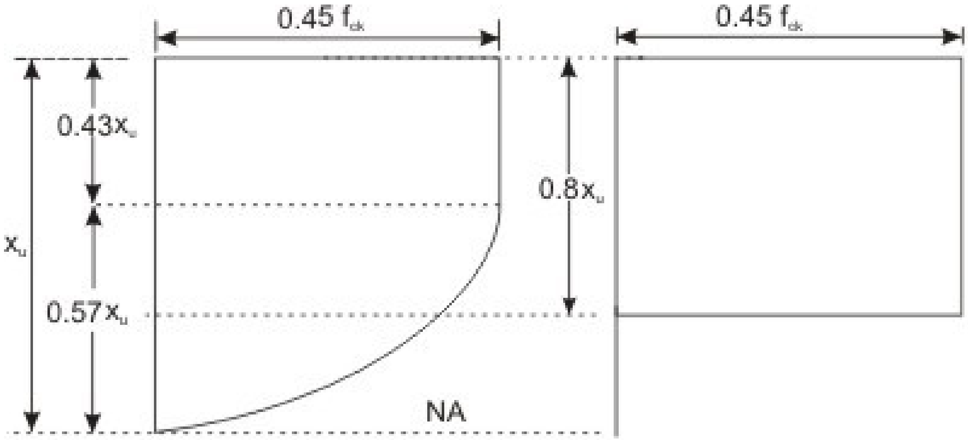


Fig. 5.10.11: (a) IS 456 stress block (b) Whitney's stress block

Whitney's stress block has been considered to derive Eq. 5.8. Figure 5.10.11 shows the two stress blocks of IS code and of Whitney.

y_f = Depth of constant portion of the stress block when $D_f/d > 0.2$. As y_f is a function of x_u and D_f and let us assume

$$y_f = A\,x_u + B\,D_f \tag{5.19}$$

where A and B are to be determined from the following two conditions:

$$(i)\quad y_f = 0.43\,x_u, \text{ when } D_f = 0.43\,x_u \tag{5.20}$$

$$(ii)\quad y_f = 0.8\,x_u, \quad \text{when } D_f = x_u \tag{5.21}$$

Using the conditions of Eqs. 5.20 and 21 in Eq. 5.19, we get $A = 0.15$ and $B = 0.65$. Thus, we have

$$y_f = 0.15\,x_u + 0.65\,D_f \tag{5.8}$$

Practice Questions and Problems with Answers

Q.1: Why do we consider most of the beams as *T* or *L*-beams between the supports and rectangular beams over the support of continuous span?

A.1: Sec. 5.10.1, first paragraph.

Q.2: Draw cross-section of a beam with top slab and show the actual width and effective width of the *T*-beam.

A.2: Fig. 5.10.2 b.

Q.3: State the requirements with figures as per IS 456 which ensure the combined action of the part of the slab and the rib of flanged beams.

A.3: Sec. 5.10.2.1(a) and (b), Figure 5.10.3 (a and b).

Q.4: Define "effective width" of flanged beams.

A.4: Effective width is an imaginary width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress (Fig. 5.10.4 of text).

Q.5: Write the expressions of effective widths of *T* and *L*-beams and isolated beams.

A.5: Sec. 5.10.2.2.

Q.6: Name the four different cases of flanged beams.

A.6: The four different cases are:

(i) When the neutral axis is in the flange ($x_u < D_f$) (discussed in sec. 5.10.4.1).

(ii) When the neutral axis is in the web and the section is balanced ($x_{u,max} > D_f$). It has two situations: (a) when D_f /d does not exceed 0.2 and (b) when $D_f /d > 0.2$ (discussed in sec. 5.10.4.2).

(iii) When the neutral axis is in the web and the section is under-reinforced ($x_{u,max} > x_u > D_f$). It has two situations: (a) when D_f /x_u does not exceed 0.43 and (b) when $D_f /x_u > 0.43$ (discussed in sec. 5.10.4.3).

(iv) When the neutral axis is in the web and the section is over-reinforced ($x_u > x_{u,max} > D_f$). It has two situations: (a) when D_f /d does not exceed 0.2 and (b) when $D_f /d > 0.2$ (discussed in sec. 5.10.4.4).

Q.7: (a) Derive the following equation:

$$y_f = 0.15\, x_{u,max} + 0.65\, D_f$$

(b) State when this equation is to be used.

(c) What is the limiting value of y_f ?

A.7: (a) For derivation of the equation, see sec. 5.10.4.5.

(b) This equation gives the depth of flange over which the stress is constant at 0.45 f_{ck} (i.e. strain is more than 0.002) when the neutral axis is in web. This occurs when D_f /d > 0.2 for balanced beam and when D_f /x_u > 0.43 for under-reinforced beams.

(c) Limiting value of y_f is D_f.

Analysis Type of Problems

The dimensions of the beam b_f, b_w, D_f, d, D, grades of concrete and steel and the amount of steel A_{st} are given. It is required to determine the moment of resistance of the beam.

Step 1: To determine the depth of the neutral axis x_u

The depth of the neutral axis is determined from the equation of equilibrium $C = T$. However, the expression of C depends on the location of neutral axis, D_f /d and D_f /x_u parameters. Therefore, it is required to assume first that the x_u is in the flange. If this is not the case, the next step is to assume x_u in the web and the computed value of x_u will indicate if the beam is under- reinforced, balanced or over-reinforced.

Other steps:

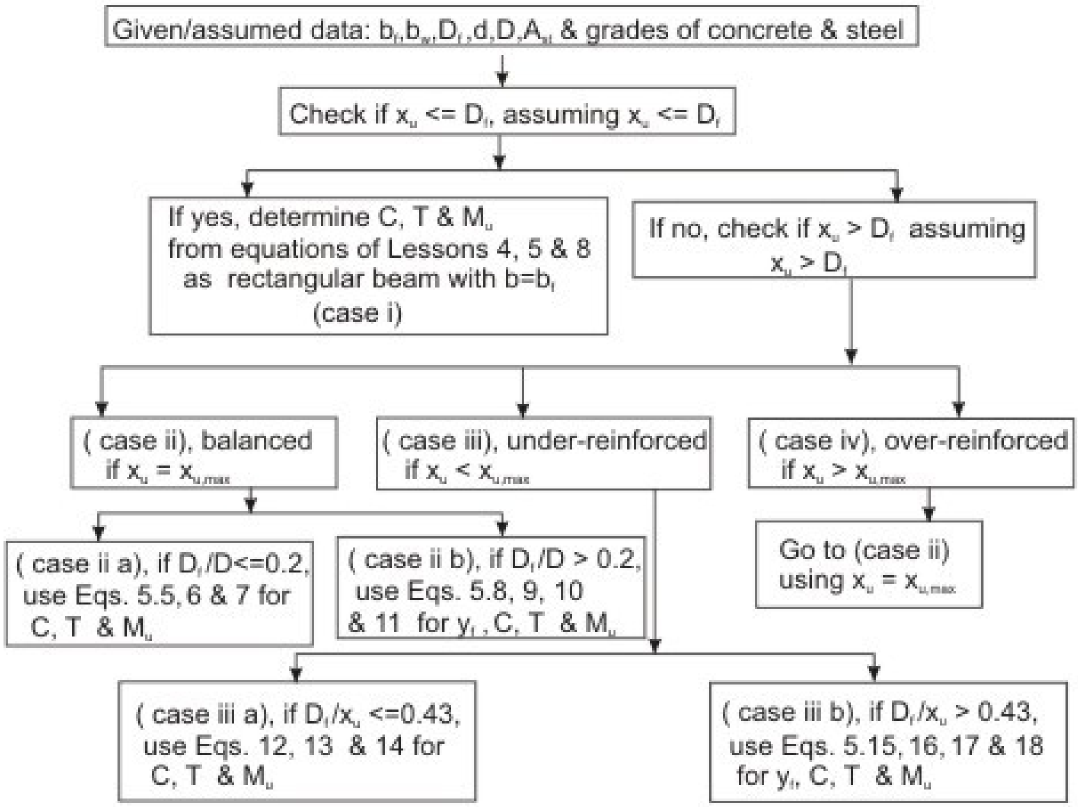


Fig. 5.11.1: Steps of solution of analysis type of problems

After knowing if the section is under-reinforced, balanced or over- reinforced, the respective parameter D_f/d or D_f/x_u is computed for the under- reinforced, balanced or over-reinforced beam. The respective expressions of C, T and M_u, as established in Lesson 10, are then employed to determine their values. Figure 5.11.1 illustrates the steps to be followed.

5.11.1 Numerical Problems (Analysis Type)

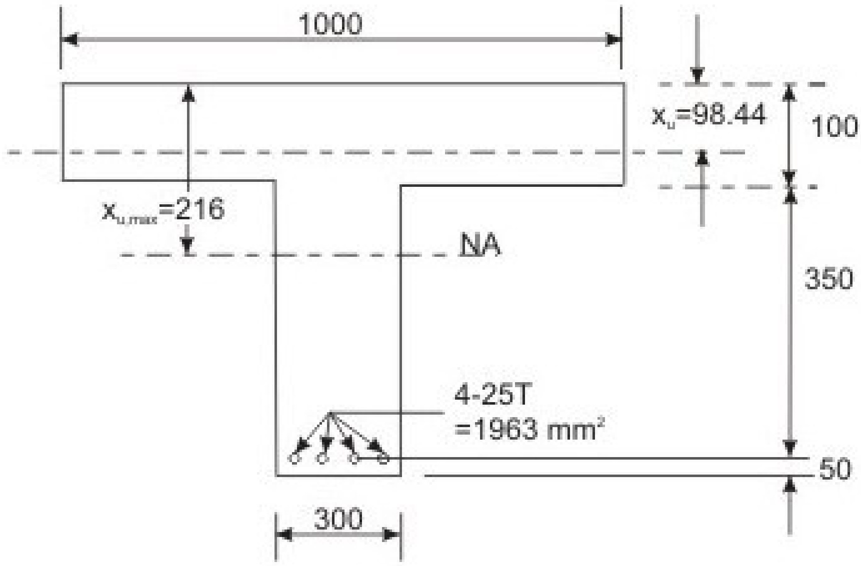


Fig. 5.11.2: Example 1, case (i)

Ex.1: Determine the moment of resistance of the *T*-beam of Fig. 5.11.2. Given data: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm, d = 450 mm and A_{st} = 1963 mm^2 (4- 25 T). Use M 20 and Fe 415.

Step 1: To determine the depth of the neutral axis x_u

Assuming x_u in the flange and equating total compressive and tensile forces from the expressions of *C* and *T* (Eq. 3.16 of Lesson 5) as the *T*-beam can be treated as rectangular beam of width b_f and effective depth d, we get:

$$x_u = \frac{0.87 f_y \; A_{st}}{0.36\, b_f \; f_{ck}} = \frac{0.87\,(415)(1963)}{0.36\,(1000)(20)} = 98.44 \quad \text{mm} < 100 \text{ mm}$$

So, the assumption of x_u in the flange is correct.

$x_{u,\,max}$ for the balanced rectangular beam = 0.48 d = 0.48 (450) = 216 mm.

It is under-reinforced since $x_u < x_{u,max}$.

Step 2: To determine *C*, *T* and M_u

From Eqs. 3.9 (using $b = b_f$) and 3.14 of Lesson 4 for *C* and *T* and Eq. 3.23 of Lesson 5 for M_u, we have:

$C = 0.36\ b_f\ x_u\ f_{ck}$ (3.9)

$= 0.36\ (1000)\ (98.44)\ (20) = 708.77$ kN

$T = 0.87\ f_y\ A_{st}$

(3.14)

$= 0.87\ (415)\ (1963) = 708.74$ kN

$$M_u = 0.87 f_y A_{st}\, d\, (1 - \frac{A_{st} f_y}{f_{ck} b_f d}) \quad (3.23)$$

$$= 0.87\ (415)\ (1963)\ (450)\ \{1 - \frac{(1963)\ (415)}{(20)\ (1000)\ (450)}\} = 290.06 \text{ kNm}$$

This problem belongs to the case (i) and is explained in sec. 5.10.4.1 of Lesson 10.

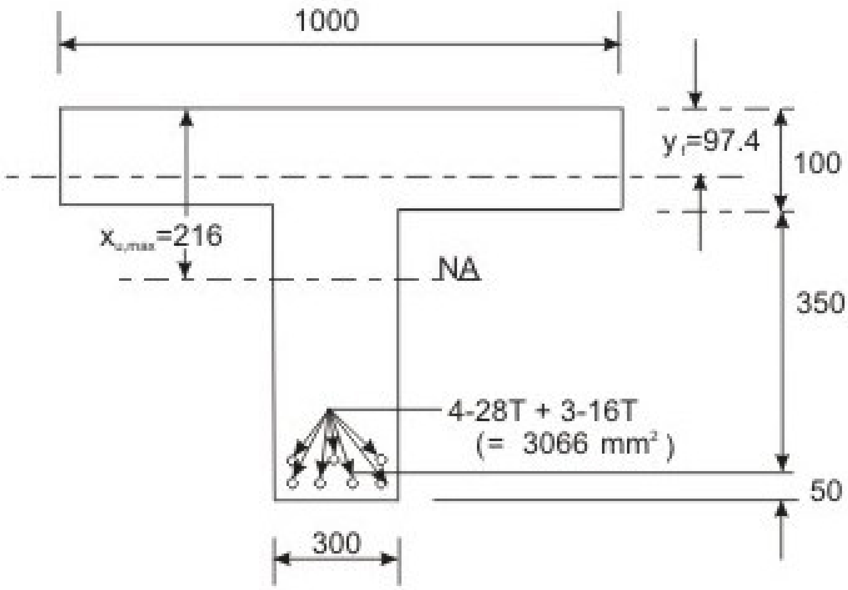


Fig. 5.11.3: Example 2, case (ii b)

Ex.2: Determine $A_{st,lim}$ and $M_{u,lim}$ of the flanged beam of Fig. 5.11.3. Given data are: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine D_f/d ratio

For the limiting case $x_u = x_{u,max} = 0.48\ (450) = 216$ mm $> D_f$. The ratio D_f/d is computed.

$D_f/d = 100/450 = 0.222 > 0.2$

Hence, it is a problem of case (ii b) and discussed in sec. 5.10.4.2 b of Lesson 10.

Step 2: Computations of y_f , C and T

First, we have to compute y_f from Eq.5.8 of Lesson 10 and then employ Eqs. 5.9, 10 and 11 of Lesson 10 to determine C, T and Mu, respectively.

$y_f = 0.15\, x_{u,max} + 0.65\, D_f = 0.15\,(216) + 0.65\,(100) = 97.4$ mm. (from Eq. 5.8)

$$C = 0.36\, f_{ck}\, b_w\, x_{u,max} + 0.45\, f_{ck}\, (b_f - b_w)\, y_f \qquad (5.9)$$

$$= 0.36\,(20)\,(300)\,(216) + 0.45\,(20)\,(1000 - 300)\,(97.4) = 1{,}080.18 \text{ kN.}$$

$$T = 0.87\, f_y\, A_{st} = 0.87\,(415)\, A_{st} \qquad (5.10)$$

Equating C and T, we have

$$A_{st} = \frac{(1080.18)\,(1000)\,\text{N}}{0.87\,(415)\,\text{N/mm}^2} = 2{,}991.77 \text{ mm}^2$$

Provide 4-28 T (2463 mm^2) + 3-16 T (603 mm^2) = 3,066 mm^2

Step 3: Computation of M_u

$$M_{u,\,lim} = 0.36\left(\frac{x_{u,max}}{d}\right)\left\{1 - 0.42\left(\frac{x_{u,max}}{d}\right)\right\} f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\,(b_f - b_w)\, y_f\, (d - y_f/2) \qquad (5.11)$$

$= 0.36\,(0.48)\,\{1 - 0.42\,(0.48)\}\,(20)\,(300)\,(450)^2$

$+ 0.45\,(20)\,(1000 - 300)\,(97.4)\,(450 - 97.4/2) = 413.87$ kNm

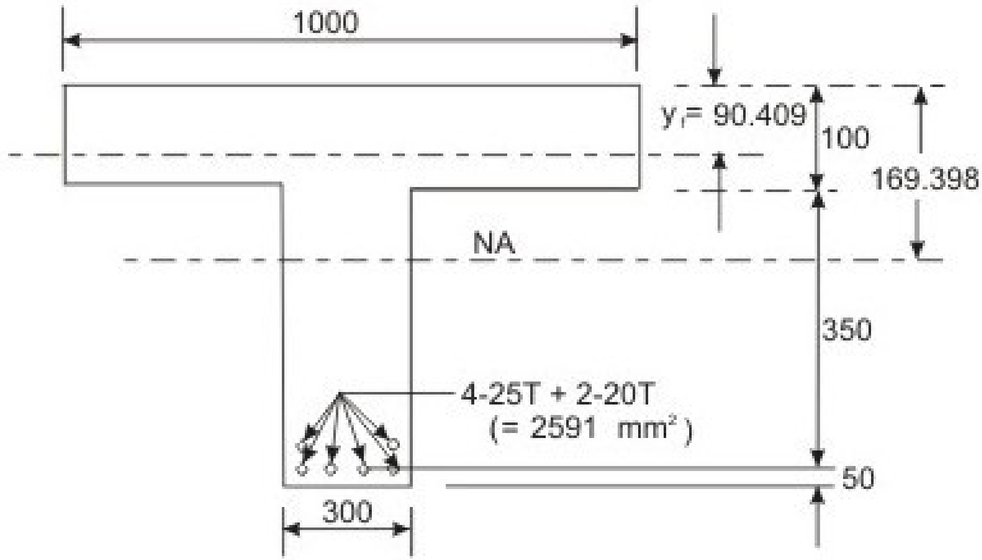


Fig. 5.11.4: Example 3, case (iii b)

Ex.3: Determine the moment of resistance of the beam of Fig. 5.11.4 when A_{st} = 2,591 mm^2 (4- 25 T and 2- 20 T). Other parameters are the same as those of Ex.1: b_f = 1,000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u to be in the flange and the beam is under-reinforced, we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87\,(415)\,(2591)}{0.36\,(1000)\,(20)} = 129.93 \text{ mm} \quad > 100 \text{ mm}$$

Since $x_u > D_f$, the neutral axis is in web. Here, D_f/d = 100/450 = 0.222 > 0.2. So, we have to substitute the term y_f from Eq. 5.15 of Lesson 10, assuming $D_f / x_u > 0.43$ in the equation of $C = T$ from Eqs. 5.16 and 17 of sec. 5.10.4.3 b of Lesson 10. Accordingly, we get:

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

or $0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\,x_u + 0.65\,(100)\}$

$= 0.87\,(415)\,(2591)$

or x_u = 169.398 mm < 216 mm ($x_{u,max} = 0.48\, x_u$ = 216 mm)

So, the section is under-reinforced.

Step 2: To determine M_u

$D_f / x_u = 100/169.398 = 0.590 > 0.43$

This is the problem of case (iii b) of sec. 5.10.4.3 b. The corresponding equations are Eq. 5.15 of Lesson 10 for y_f and Eqs. 5.16 to 18 of Lesson 10 for C, T and M_u, respectively. From Eq. 5.15 of Lesson 10, we have:

$y_f = 0.15\, x_u + 0.65\, D_f = 0.15\,(169.398) + 0.65\,(100) = 90.409$ mm

From Eq. 5.18 of Lesson 10, we have

$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f\, (d - y_f/2)$ or

$M_u = 0.36\,(169.398/450)\,\{1 - 0.42\,(169.398/450)\}\,(20)\,(300)\,(450)\,(450)$

$+ 0.45\,(20)\,(1000 - 300)\,(90.409)\,(450 - 90.409/2)$

$= 138.62 + 230.56 = 369.18$ kNm.

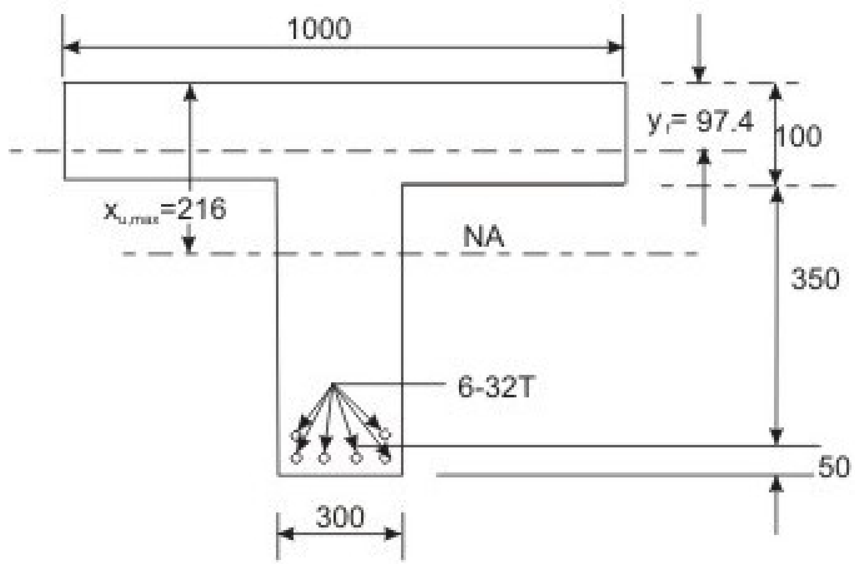


Fig. 5.11.5: Example 4, case (iv b)

Ex.4: Determine the moment of resistance of the flanged beam of Fig. 5.11.5 with A_{st} = 4,825 mm^2 (6- 32 T). Other parameters and data are the same as those of Ex.1: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u in the flange of under-reinforced rectangular beam we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87\,(415)\,(4825)}{0.36(1000)(20)} = 241.95 \text{ mm} > D_f$$

Here, D_f/d = 100/450 = 0.222 > 0.2. So, we have to determine y_f from Eq. 5.15 and equating C and T from Eqs. 5.16 and 17 of Lesson 10.

$$y_f = 0.15\, x_u + 0.65\, D_f \qquad (5.15)$$

$$0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, y_f = 0.87\, f_y\, A_{st} \qquad (5.16 \text{ and } 5.17)$$

or $\quad 0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\, x_u + 0.65\,(100)\}$

$\quad = 0.87\,(415)\,(4825)$

or $\quad 2160\, x_u + 945\, x_u = -\ 409500 + 1742066$ or x_u

$= 1332566/3105 = 429.17$ mm

$x_{u,max} = 0.48\,(450) = 216$ mm

Since $x_u > x_{u,max}$, the beam is over-reinforced. Accordingly.

$x_u = x_{u,\,max} = 216$ mm.

Step 2: To determine M_u

This problem belongs to case (iv b), explained in sec.5.10.4.4 b of Lesson 10. So, we can determine Mu from Eq. 5.11 of Lesson 10.

$$M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f\, (d - y_f/2) \qquad (5.11)$$

where $y_f = 0.15\, x_{u,\,max} + 0.65\, D_f = 97.4$ mm $\qquad$ (5.8)

From Eq. 5.11, employing the value of y_f = 97.4 mm, we get:

$$M_u = 0.36\ (0.48)\ \{1 - 0.42\ (0.48)\}\ (20)\ (300)\ (450)\ (450)$$

$$+ 0.45 (20) (1000 - 300) (97.4) (450 - 97.4/2)$$

$$= 167.63 + 246.24 = 413.87 \text{ kNm}$$

It is seen that this over-reinforced beam has the same M_u as that of the balanced beam of Example 2.

Summary of Results of Examples 1-4

The results of four problems (Exs. 1-4) are given in Table 5.1 below. All the examples are having the common data except A_{st}.

Table 5.1 Results of Examples 1-4 (Figs. 5.11.2 – 5.11.5)

Ex. No.	A_{st} (mm^2)	Case	Section No.	M_u (kNm)	Remarks
1	1,963	(i)	5.10.4.1	290.06	x_u = 98.44 mm < $x_{u,\ max}$ (= 216 mm), x_u < D_f (= 100 mm), Under-reinforced, (NA in the flange).
2	3,066	(ii b)	5.10.4.2 (b)	413.87	$x_u = x_{u,\ max}$ = 216 mm, D_f/d = 0.222 > 0.2, Balanced, (NA in web).
3	2,591	(iii b)	5.10.4.3 (b)	369.18	x_u = 169.398 mm < $x_{u,\ max}$(= 216 mm), D_f/x_u = 0.59 > 0.43, Under-reinforced, (NA in the web).
4	4,825	(iv b)	5.10.4.4 (b)	413.87	x_u = 241.95 mm > $x_{u,\ max}$ (= 216 mm), D_f/d = 0.222 > 0.2, Over-reinforced, (NA in web).

It is clear from the above table (Table 5.1), that Ex.4 is an over-reinforced flanged beam. The moment of resistance of this beam is the same as that of balanced beam of Ex.2. Additional reinforcement of 1,759 mm^2 (= 4,825 mm^2 – 3,066 mm^2) does not improve the M_u of the over-reinforced beam. It rather prevents the beam from tension failure. That is why over-reinforced beams are to be avoided. However, if the M_u has to be increased beyond 413.87 kNm, the flanged beam may be doubly reinforced.

Use of SP-16 for the Analysis Type of Problems

Using the two governing parameters (b_f /b_w) and (D_f /d), the $M_{u,lim}$ of balanced flanged beams can be determined from Tables 57-59 of SP-16 for the three grades of steel (250, 415 and 500). The value of the moment coefficient $M_{u,lim} /b_w d^2 f_{ck}$ of Ex.2, as obtained from SP-16, is presented in Table 5.2 making linear interpolation for both the parameters, wherever needed. $M_{u,lim}$ is then calculated from the moment coefficient.

Table 5.2 $M_{u,lim}$ of Example 2 using Table 58 of SP-16 Parameters: (i)

b_f /b_w = 1000/300 = 3.33

(ii) D_f /d = 100/450 = 0.222

($M_{u,lim} /b_w d^2 f_{ck}$) in N/mm^2			
D_f /d	b_f /b_w		
	3	4	3.33
0.22	0.309	0.395	
0.23	0.314	0.402	
0.222	0.31*	0.3964*	0.339*

* by linear interpolation

So, from Table 5.2, $$\frac{M_{u,\text{lim}}}{b_w d^2 f_{ck}} = 0.339$$

$M_{u,lim}$ = 0.339 $b_w d^2 f_{ck}$ = 0.339 (300) (450) (450) (20) 10^{-6} = 411.88 kNm

$M_{u,lim}$ as obtained from SP-16 is close to the earlier computed value of $M_{u,lim}$ = 413.87 kNm (see Table 5.1).

Practice Questions and Problems with Answers

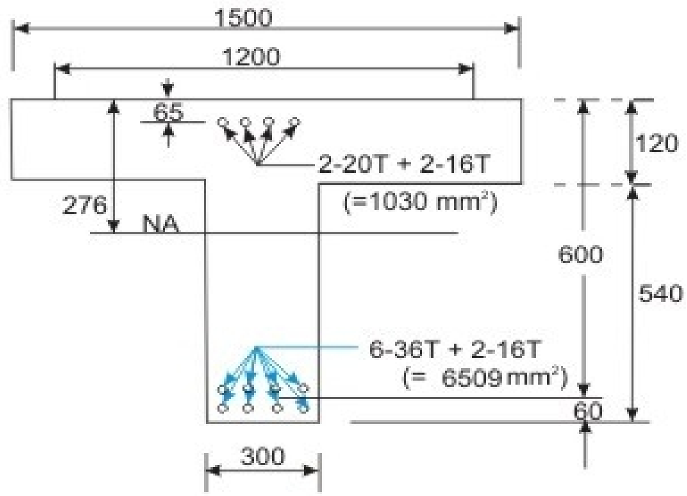

1500
1200
65
2-20T + 2-16T
(=1030 mm²)
120
276
NA
600
540
6-36T + 2-16T
(= 6509 mm²)
60
300

Fig. 5.11.6: Q. 1

Q.1: Determine the moment of resistance of the simply supported doubly reinforced flanged beam (isolated) of span 9 m as shown in Fig. 5.11.6. Assume M 30 concrete and Fe 500 steel.

A.1: Solution of Q.1:

Effective width $b_f = \dfrac{l_o}{(l_o/b)+4} + b_w = \dfrac{9000}{(9000/1500)+4} + 300 = 1200$ mm

Step 1: To determine the depth of the neutral axis

Assuming neutral axis to be in the flange and writing the equation $C = T$, we have:

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_f\, x_u + (f_{sc}\, A_{sc} - f_{cc}\, A_{sc})$$

Here, $d'/d = 65/600 = 0.108 = 0.1$ (say). We, therefore, have $f_{sc} = 353$ N/mm^2 .

From the above equation, we have:

$$x_u = \frac{0.87\,(500)\,(6509) - \{(353)\,(1030) - 0.446\,(30)\,(1030)\}}{0.36\,(30)\,(1200)} = 191.48 \text{ mm} > 120 \text{ mm}$$

So, the neutral axis is in web.

$D_f/d = 120/600 = 0.2$

Assuming $D_f/x_u < 0.43$, and Equating $C = T$

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\, (b_f - b_w)\, D_f + (f_{sc} - f_{cc})\, A_{sc}$$

$$x_u = \frac{0.87\,(500)\,(6509) - 1030\{353 - 0.446\,(30)\} - 0.446\,(30)\,(1200 - 300)\,(120)}{0.36\,(30)\,(300)}$$

$= 319.92 > 276$ mm ($x_{u,max} = 276$ mm)

So, $x_u = x_{u,max} = 276$ mm (over-reinforced beam).

$D_f/x_u = 120/276 = 0.4347 > 0.43$

Let us assume $D_f/x_u > 0.43$. Now, equating $C = T$ with y_f as the depth of flange having constant stress of $0.446\, f_{ck}$. So, we have:

$y_f = 0.15\, x_u + 0.65\, D_f = 0.15\, x_u + 78$

$$0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\, (b_f - b_w)\, y_f + A_{sc}\, (f_{sc} - f_{cc}) = 0.87\, f_y\, A_{st}$$

$0.36\ (30)\ (300)\ x_u + 0.446\ (30)\ (900)\ (0.15\ x_u + 78)$

$= 0.87\ (500)\ (6509) - 1030\ \{353 - 0.446\ (30)\}$

or x_u = 305.63 mm > $x_{u,max}$. ($x_{u,max}$ = 276 mm)

The beam is over-reinforced. Hence, $x_u = x_{u,max}$ = 276 mm. This is a problem of case (iv), and we, therefore, consider the case (ii) to find out the moment of resistance in two parts: first for the balanced singly reinforced beam and then for the additional moment due to compression steel.

Step 2: Determination of $x_{u,lim}$ for singly reinforced flanged beam

Here, D_f/d = 120/600 = 0.2, so y_f is not needed. This is a problem of case (ii a) of sec. 5.10.4.2 of Lesson 10. Employing Eq. 5.7 of Lesson 10, we have:

$$M_{u,lim} = 0.36\ (x_{u,max}/d)\ \{1 - 0.42\ (x_{u,max}/d)\}\ f_{ck}\ b_w\ d^2$$

$$+ 0.45\ f_{ck}\ (b_f - b_w)\ D_f\ (d - D_f/2)$$

$$= 0.36(0.46)\ \{1 - 0.42(0.46)\}\ (30)\ (300)\ (600)\ (600) + 0.45(30)\ (900)\ (120)\ (540)$$

$$= 1{,}220.20 \text{ kNm}$$

$$A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y d\ \{1 - 0.42\ (x_{u,max}/d)\}}$$

$$= \frac{(1220.20)\ (10^6)}{(0.87)\ (500)\ (600)\ (0.8068)} = 5{,}794.6152 \text{ mm}^2$$

Step 3: Determination of M_{u2}

Total A_{st} = 6,509 mm^2, $A_{st,lim}$ = 5,794.62 mm^2

A_{st2} = 714.38 mm^2 and A_{sc} = 1,030 mm^2

It is important to find out how much of the total A_{sc} and A_{st2} are required effectively. From the equilibrium of C and T forces due to additional steel (compressive and tensile), we have:

$(A_{st2})\ (0.87)\ (f_y) = (A_{sc})\ (f_{sc})$ If

we assume A_{sc} = 1,030 mm^2

$$A_{st2} = \frac{1030(353)}{0.87(500)} = 835.84 \text{ mm}^2 > 714.38 \text{ mm}^2,$$ (714.38 mm^2 is the total A_{st2} provided). So, this is not possible.

Now, using A_{st2} = 714.38 mm^2 , we get A_{sc} from the above equation.

$$A_{sc} = \frac{(714.38)(0.87)(500)}{353} = 880.326 < 1{,}030 \text{ mm}^2,$$ (1,030 mm^2 is the total A_{sc} provided).

$$M_{u2} = A_{sc} f_{sc}(d - d') = (880.326)(353)(600 - 60) = 167.807 \text{ kNm}$$

Total moment of resistance = $M_{u,lim} + M_{u2}$ = 1,220.20 + 167.81 = 1,388.01 kNm

Total A_{st} required = $A_{st,lim} + A_{st2}$ = 5,794.62 + 714.38 = 6,509.00 mm^2 , (provided A_{st} = 6,509 mm^2)

A_{sc} required = 880.326 mm^2 (provided 1,030 mm^2).

Solutions

Maximum Marks = 50, Maximum Time = 30 minutes Answer all

questions.

TQ.1: Determine $M_{u,lim}$ of the flanged beam of Ex. 2 (Fig. 5.11.3) with the help of SP-16 using (a) M 20 and Fe 250, (b) M 20 and Fe 500 and (c) compare the results with the $M_{u,lim}$ of Ex. 2 from Table 5.2 when grades of concrete and steel are M 20 and Fe 415, respectively. Other data are: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm.

(10 X 3 = 30 marks)

A.TQ.1: From the results of Ex. 2 of sec. 5.11.5 (Table 5.2), we have:

Parameters: (i) b_f / b_w = 1000/300 = 3.33 (ii) D_f/d = 100/450 = 0.222

For part (a): When Fe 250 is used, the corresponding table is Table 57 of SP-16. The computations are presented in Table 5.3 below:

Table 5.3 ($M_{u,lim} /b_w d^2 f_{ck}$) in N/mm^2 Of TQ.1 (PART a for M 20 and Fe 250)

($M_{u,lim} /b_w d^2 f_{ck}$) in N/mm^2			
D_f /d	b_f /b_w		
	3	4	3.33
0.22	0.324	0.411	
0.23	0.330	0.421	
0.222	0.3252*	0.413*	0.354174*

- by linear interpolation

$M_{u,lim} /b_w d^2 f_{ck}$ = 0.354174 = 0.354 (say)

So, $M_{u,lim}$ = (0.354) (300) (450) (450) (20) N mm = 430.11 kNm

For part (b): When Fe 500 is used, the corresponding table is Table 59 of SP-16. The computations are presented in Table 5.4 below:

Table 5.4 ($M_{u,lim} /b_w d^2 f_{ck}$) in N/mm^2 Of TQ.1 (PART b for M 20 and Fe 500)

($M_{u,lim} /b_w d^2 f_{ck}$) in N/mm^2			
D_f /d	b_f /b_w		
	3	4	3.33
0.22	0.302	0.386	
0.23	0.306	0.393	
0.222	0.3028*	0.3874*	0.330718*

* by linear interpolation

$M_{u,lim} /b_w d^2 f_{ck}$ = 0.330718 = 0.3307 (say)

So, $M_{u,lim}$ = (0.3307) (300) (450) (450) (20) mm = 401.8 kNm

For part (c): Comparison of results of this problem with that of Table 5.2 (M 20 and Fe 415) is given below in Table 5.5.

Table 5.5 Comparison of results of $M_{u,lim}$

Sl. No.	Grade of Steel	$M_{u,lim}$ (kNm)
1	Fe 250	430.11
2	Fe 415	411.88
3	Fe 500	401.80

It is seen that $M_{u,lim}$ of the beam decreases with higher grade of steel for a particular grade of concrete.

TQ.2: With the aid of SP-16, determine separately the limiting moments of resistance and the limiting areas of steel of the simply supported isolated, singly reinforced and balanced flanged beam of Q.1 as shown in Fig. 5.11.6 if the span = 9 m. Use M 30 concrete and three grades of steel, Fe 250, Fe 415 and Fe 500, respectively. Compare the results obtained above with that of Q.1 of sec. 5.11.6, when balanced.

(15 + 5 = 20 marks)

A.TQ.2: From the results of Q.1 sec. 5.11.6, we have:

Parameters: (i) b_f / b_w = 1200/300 = 4.0 (ii) D_f/d = 120/600 = 0.2

For Fe 250, Fe 415 and Fe 500, corresponding tables are Table 57, 58 and 59, respectively of SP-16. The computations are done accordingly. After computing the limiting moments of resistance, the limiting areas of steel are determined as explained below. Finally, the results are presented in Table 5.6 below:

$$A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y \, d \, \{1 - 0.42 \, (x_{u,max} / d)\}}$$

Table 5.6 Values of $M_{u,lim}$ in N/mm^2 Of TQ.2

Grade of Fe / Q.1 of sec. 5.11.6	($M_{u,lim}/b_w \; d^2 \; f_{ck}$) (N/mm^2)	$M_{u,lim}$ (kNm)	$A_{st,lim}$ (mm^2)
Fe 250	0.39	1, 263.60	12,455.3 2
Fe 415	0.379	1, 227.96	7,099.78
Fe 500	0.372	1, 205.28	5,723.76
Q.1 of sec. 5.11.6 (Fe 415)		1, 220.20	5,794.62

The maximum area of steel allowed is .04 $b\,D$ = (.04) (300) (660) = 7,920 mm^2 . Hence, Fe 250 is not possible in this case.

Introduction to flange beams

Lesson 10 illustrates the governing equations of flanged beams and Lesson 11 explains their applications for the solution of analysis type of numerical problems. It is now necessary to apply them for the solution of design type, the second type of the numerical problems. This lesson mentions the different steps of the solution and solves several numerical examples to explain their step-by-step solutions.

Design Type of Problems

We need to assume some preliminary dimensions of width and depth of flanged beams, spacing of the beams and span for performing the structural analysis before the design. Thus, the assumed data known for the design are: D_f, b_w, D, effective span, effective depth, grades of concrete and steel and imposed loads.

There are four equations: (i) expressions of compressive force C, (ii) expression of the tension force T, (iii) $C = T$ and (iv) expression of M_u in terms of C or T and the lever arm $\{M = (C \text{ or } T)$ (lever arm)$\}$. However, the relative dimensions of D_f, D and x_u and the amount of steel (under-reinforced, balanced or over-reinforced) influence the expressions. Accordingly, the respective equations are to be employed assuming a particular situation and, if necessary, they need to be changed if the assumed parameters are found to be not satisfactory. The steps of the design problems are as given below.

Step 1: To determine the factored bending moment M_u

Step 2: To determine the $M_{u,lim}$ of the given or the assumed section

The beam shall be designed as under-reinforced, balanced or doubly reinforced if the value of M_u is less than, equal to or more than $M_{u,lim}$. The design of over-reinforced beam is to be avoided as it does not increase the bending moment carrying capacity beyond $M_{u,lim}$ either by increasing the depth or designing a doubly reinforced beam.

Step 3: To determine x_u, the distance of the neutral axis, from the expression of M_u

Here, it is necessary to assume first that x_u is in the flange. Later on, it may be necessary to calculate x_u if the value is found to be more than D_f. This is to be done assuming first that $D_f/x_u < 0.43$ and then $D_f/x_u > 0.43$ separately.

Step 4: To determine the area(s) of steel

For doubly reinforced beams $A_{st} = A_{st,lim} + A_{st2}$ and A_{sc} are to be obtained, while only A_{st} is required to be computed for under-reinforced and balanced beams. These are calculated employing $C = T$ (for A_{st} and $A_{st,\,lim}$) and the expression of M_{u2} to calculate A_{st2} and A_{sc}.

Step 5: It may be necessary to check the x_u and A_{st} once again after Step 4

It is difficult to prescribe all the relevant steps of design problems. Decisions are to be taken judiciously depending on the type of problem. For the design of a balanced beam, it is necessary to determine the effective depth in Step 3 employing the expression of bending moment M_u. For such beams and for under-reinforced beams, it may be necessary to estimate the A_{st} approximately immediately after Step 2. This value of A_{st} will facilitate to determine x_u.

5.12.1 Numerical Problems

Four numerical examples are solved below explaining the steps involved in the design problems.

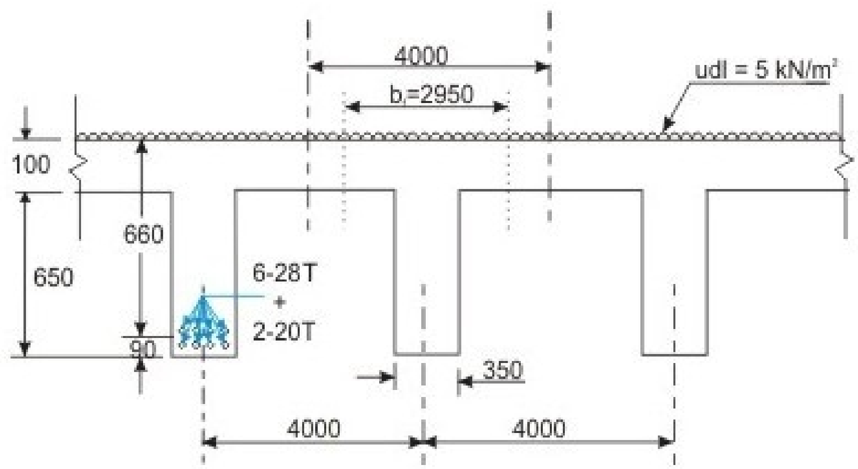


Fig. 5.12.1: Example 5

Ex.5: Design the simply supported flanged beam of Fig. 5.12.1, given the following: D_f = 100 mm, D = 750 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m, cover = 90 mm, d = 660 mm and imposed loads = 5 kN/m^2. Fe 415 and M 20 are used.

Solution:

Step 1: Computation of factored bending moment

Weight of slab per m^2 = (0.1) (1) (1) (25) = 2.5 kN/m^2 So, Weight of slab per m = (4) (2.5) = 10.00 kN/m

Dead loads of web part of the beam = (0.35) (0.65) (1) (25) = 5.6875 kN/m

Imposed loads = (4) (5) = 20 kN/m

Total loads = 30 + 5.6875 = 35.6875 kN/m

$$\text{Factored Bending moment} = (1.5)\frac{(35.6875)\,(12)(12)}{8} = 963.5625 \text{ kNm}$$

Step 2: Computation of $x_{u,lim}$

Effective width of flange =(l_o/6) + b_w+ 6 D_f = (12000/6) + 350 + 600 = 2,950 mm.

$x_{u,max}$ = 0.48 d = 0.48 (660) = 316.80 mm. This shows that the neutral axis is in the web of this beam.

D_f/d = 100/660 = 0.1515 < 0.2, and

D_f/x_u = 100/316.8 = 0.316 < 0.43

The expression of $M_{u,lim}$ is obtained from Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2) and is as follows:

$$M_{u,lim} = 0.36(x_{u,max}/d)\{1 - 0.42\,(x_{u,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\,(b_f - b_w)\, D_f\,(d - D_f/2)$$

$$= 0.36(0.48)\ \{1 - 0.42(0.48)\}\ (20)\ (350)\ (650)\ (650)$$

$$+ 0.45\ (20)\ (2950 - 350)\ (100)\ (660 - 50) = 1{,}835.43 \text{ kNm}$$

The design moment M_u = 963.5625 kNm is less than $M_{u,lim}$. Hence, one under- reinforced beam can be designed.

Step 3: Determination of x_u

Since the design moment M_u is almost 50% of $M_{u,lim}$, let us assume the neutral axis to be in the flange. The area of steel is to be calculated from the moment equation (Eq. 3.23 of Lesson 5), when steel is ensured to reach the design stress f_d = 0.87 (415) = 361.05 N/mm^2. It is worth mentioning that the term b of Eq. 3.23 of Lesson 5 is here b_f as the T- beam is treated as a rectangular beam when the neutral axis is in the flange.

$$M_u = 0.87\, f_y\, A_{st}\, d \left\{ 1 - \frac{A_{st}\, f_y}{f_{ck}\, b\, d} \right\} \tag{3.23}$$

Here, all but A_{st} are known. However, this will give a quadratic equation of A_{st} and the lower one of the two values will be provided in the beam. The above equation gives:

$$A_{st}^2 - 93831.3253\, A_{st} + 379416711.3 = 0$$

which gives the lower value of A_{st} as:

A_{st} = 4,234.722097 mm^2. The reason of selecting the lower value of A_{st}

is explained in sec 3.6.4.8 of Lesson 6 in the solution of Design Problem 3.1. Then, employing Eq. 3.16 of

Lesson 5, we get

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, b\, f_{ck}} \tag{3.16}$$

or x_u = 71.98 mm.

Again, employing Eq. 3.24 of Lesson 5, we can determine x_u first and then A_{st} from Eq. 3.16 or 17 of Lesson 5, as explained in the next step. Eq. 3.24 of Lesson

5 gives:

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_f\, d^2$$

$$= 0.36\,(x_u)\,\{1 - 0.42\,(x_u/d)\}\, f_{ck}\, b_f\, d$$

$$963.5625\,(10^6) = 0.36\,(x_u)\,\{1 - 0.42\,(x_u/660)\}\,(20)\,(2950)\,(660)$$

or x_u = 72.03 mm.

The two values of x_u are the same. It is thus seen that, the value of x_u can be determined either first finding the value of A_{st}, from Eq. 3.23 of Lesson 5 or directly from Eq. 3.24 of Lesson 5 first and then the value of A_{st} can be determined.

Step 4: Determination of A_{st}

Equating $C = T$, we have from Eq. 3.17 of Lesson 5:

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f d}$$

$$A_{st} = \frac{0.36 f_{ck} b_f x_u}{0.87 f_y} = \frac{0.36\,(20)\,(2950)\,(72.03)}{0.87\,(415)} = 4{,}237.41 \text{ mm}^2$$

Minimum A_{st} = $(0.85/f_y)\ b_w\ d$ = (0.85/415) (350) (660) = 473.13 mm^2 Maximum A_{st} = 0.04 b_w D

= (0.04) (350) (660) = 9,240 mm^2

Hence, A_{st} = 4,237.41 mm^2 is o.k.

Provide 6 - 28 T (= 3694 mm^2) + 2-20 T (= 628 mm^2) to have total A_{st} = 4,322 mm^2.

Ex.6: Design a beam in place of the beam of Ex.5 (Fig. 5.12.1) if the imposed loads are increased to 12 kN/m^2. Other data are: D_f = 100 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m simply supported and cover = 90 mm. Use Fe 415 and M 20.

Solution: As in Ex.5, b_f = 2,950 mm.

Step 1: Computation of factored bending moment Weight of

slab/m^2 = 2.5 kN/m^2 (as in Ex.1) Imposed loads = 12.0

kN/m^2 (given)

Total loads = 14.5 kN/m^2

Total weight of slab and imposed loads = 14.5 (4) = 58.0 kN/m Dead loads of the

beam = 0.65 (0.35) (25) = 5.6875 kN/m Total loads = 63.6875 kN/m

$$(M_u)_{factored} = \frac{1.5\ (63.6875)\ (12)\ (12)}{8} = 1{,}719.5625 \text{ kNm}$$

Step 2: Determination of $M_{u,lim}$

$M_{u,lim}$ of the beam of Ex.5 = 1,835.43 kNm. The factored moment of this problem (1,719.5625 kNm) is close to the value of $M_{u,lim}$ of the section.

Step 3: Determination of d

Assuming $D_f /d < 0.2$, we have from Eq. 5.7 of Lesson 10,

$$M_u = 0.36(x_{u,max} /d)\{1 - 0.42\ (x_{u,max} /d)\}\ f_{ck}\ b_w\ d^2 + 0.45\ f_{ck}\ (b_f - b_w)\ D_f\ (d - D_f /2)$$

$$1719.5625\ (10^6) = 0.36(0.48)\ \{1 - 0.42(0.48)\}\ (20)\ (350)\ d^2$$

$$+\ 0.45\ (2600)\ (20)\ (100)\ (d - 50)$$

Solving the above equation, we get d = 624.09 mm, giving total depth = 624.09 + 90 = 715 mm (say).

Since the dead load of the beam is reduced due to decreasing the depth of the beam, the revised loads are calculated below:

Loads from the slab = 58.0 kN/m

Dead loads (revised) = 0.615 (0.35) (25) = 5.38125 kN/m Total loads =

63.38125 kN/m

$$(M_u)_{factored} = \frac{1.5\ (63.38125)\ (12)\ (12)}{8} = 1{,}711.29 \text{ kNm}$$

Approximate value of A_{st}:

$$A_{st} = \frac{M_u}{0.87 f_y\ (d - \frac{D_f}{2})} = \frac{1711.29\ (10^6)}{0.87\ (415)\ (625 - 50)} = 8{,}243.06 \text{ mm}^2$$

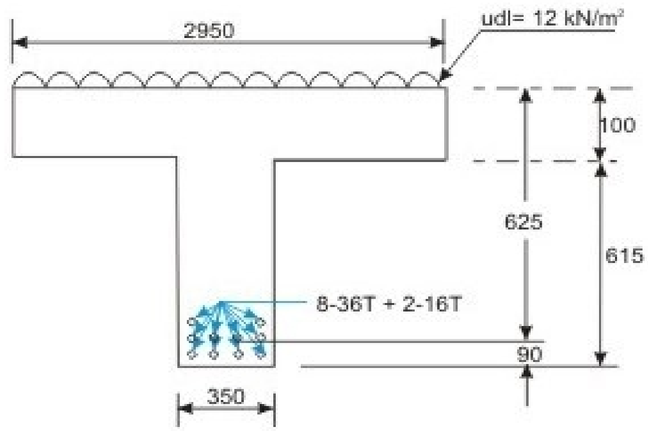


Fig. 5.12.2: Example 6

Step 4: Determination of A_{st} (Fig. 5.12.2)

x_u = $x_{u,max}$ = 0.48 (625) = 300 mm Equating T and C

(Eq. 5.5 of Lesson 10), we have:

$$0.87 f_y A_{st} = 0.36 x_{u,max} b_w f_{ck} + 0.45 f_{ck} (b_f - b_w) D_f$$

or $$A_{st} = \frac{0.36 (300) (350) (20) + 0.45 (20) (2600) (100)}{0.87 (415)} = 8{,}574.98 \text{ mm}^2$$

Maximum $A_{st} = 0.04\, b\, D = 0.04\, (350)\, (715) = 10{,}010.00$ mm^2

Minimum $A_{st} = (0.85/f_y)\, b_w\, d = (0.85/415)\, (350)\, (625) = 448.05$ mm^2 Hence, A_{st} =

8,574.98 mm^2 is o.k.

So, provide 8-36 T + 2-18 T = 8143 + 508 = 8,651 mm^2

Step 5: Determination of x_u

Using A_{st} = 8,651 mm^2 in the expression of $T = C$ (Eq. 5.5 of Lesson 10), we have:

$$0.87 f_y A_{st} = 0.36 x_u b_w f_{ck} + 0.45 f_{ck} (b_f - b_w) D_f$$

or $$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_f - b_w)}{0.36 b_w f_{ck}}$$

$$= \frac{0.87\,(415)\,(8651) - 0.45\,(20)\,(2600)\,(100)}{0.36\,(350)\,(20)} = 310.89 > x_{u,\max} \;(= 300 \text{ mm})$$

So, A_{st} provided is reduced to 8-36 + 2-16 = 8143 + 402 = 8,545 mm^2. Accordingly,

$$x_u = \frac{0.87\,(415)\,(8545) - 0.45\,(20)\,(2600)\,(100)}{0.36\,(350)\,(20)} = 295.703 \text{ mm} < x_{u,\max} \;(= 300 \text{ mm})$$

Step 6: Checking of M_u

D_f/d = 100/625 = 0.16 < 0.2

D_f/x_u = 100/215.7 = 0.33 < 0.43. Hence, it is a problem of case (iii a) and M_u can be obtained from Eq. 5.14 of Lesson 10.

So, $M_u = 0.36(x_u/d)\,\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_f\, d^2 + 0.45\, f_{ck}\,(b_f - b_w)\,(D_f)\,(d - D_f/2)$

= 0.36 (295.703/625) {1 - 0.42 (295.703/625)} (20) (350) (625) (625)

+ 0.45 (20) (2600) (100) (625 - 50)

= 1,718.68 kNm > $(M_u)_{\text{design}}$ (= 1,711.29 kNm)

Hence, the design is o.k.

Ex.7: Determine the tensile reinforcement A_{st} of the flanged beam of Ex.5 (Fig. 5.12.1) when the imposed loads = 12 kN/m^2. All other parameters are the same as those of Ex.5: D_f = 100 mm, D = 750 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m, simply supported, cover = 90 mm and d = 660 mm. Use Fe 415 and M 20.

Solution:

Step 1: Computation of factored bending moment M_u Dead

loads of the slab (see Ex.5) = 2.5 kN/m^2 Imposed loads

= 12.0 kN/m^2

Total loads = 14.5 kN/m^2 Loads/m = 14.5 (4)

= 58.0 kN/m

Dead loads of beam = 0.65 (0.35) (25) = 5.6875 kN/m

Total loads = 63.6875 kN/m

Factored M_u = (1.5) (63.6875) (12) (12)/8 = 1,719.5625 kNm.

Step 2: Determination of $M_{u,lim}$

From Ex.5, the $M_{u,lim}$ of this beam = 1,835.43 kNm. Hence, this beam shall be designed as under-reinforced.

Step 3: Determination of x_u

Assuming x_u to be in the flange, we have from Eq. 3.24 of Lesson 5 and considering $b = b_f$,

$$M_u = 0.36 x_u \{1 - 0.42(x_u/d)\} f_{ck}\, b_f\, d$$

$$1719.5625\,(10^6) = 0.36\, x_u\, \{1 - 0.42\,(x_u/660)\}\,(20)\,(2950)\,(550)$$

Solving, we get x_u = 134.1 > 100 mm

So, let us assume that the neutral axis is in the web and D_f/x_u < 0.43, from Eq. 5.14 of Lesson 10 (case iii a of sec. 5.10.4.3), we have:

$$M_u = 0.36(x_u/d)\,\{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\,(b_f - b_w)\,(D_f)\,(d - D_f/2)$$

$$= 0.36\, x_u\, \{1 - 0.42\,(x_u/660)\}\,(20)\,(350)\,(660)$$

$$+ 0.45\,(20)\,(2600)\,(100)\,(660 - 50)$$

Substituting the value of M_u = 1,719.5625 kNm in the above equation and simplifying,

$$x_u^2 - 1571.43\, x_u + 276042 = 0$$

Solving, we have x_u = 201.5 mm

$$D_f/x_u = 100/201.5 = 0.496 > 0.43.$$

So, we have to use Eq. 5.15 and 5.18 of Lesson 10 for y_f and M_u (case iii b of sec. 5.10.4.3). Thus, we have:

$$M_u = 0.36\, x_u\, \{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d + 0.45\, f_{ck}\,(b_f - b_w)\, y_f\,(d - y_f/2) \text{ where,}$$

$$y_f = (0.15\, x_u + 0.65\, D_f)$$

So, $M_u = 0.36\, x_u\, \{1 - 0.42\,(x_u/660)\}\,(20)\,(350)\,(660)$

$+ 0.45 (20) (2600) (0.15 x_u + 65) (660 - 0.075 x_u - 32.5)$

or $1719.5625 (10^6) = 3.75165 (10^6) x_u - 795.15 x_u^2 + 954.4275 (10^6)$

Solving, we get $x_u = 213.63$ mm.

$D_f / x_u = 100/213.63 = 0.468 > 0.43$.
Hence, o.k.

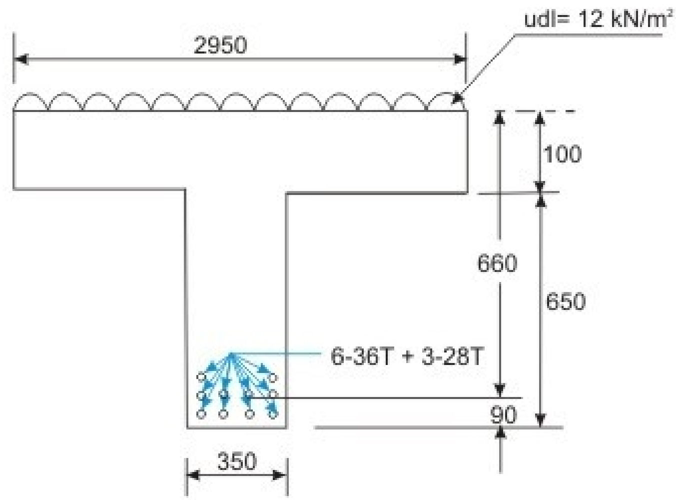


Fig. 5.12.3: Example 7

Step 4: Determination of A_{st}

Equating $C = T$ from Eqs. 5.16 and 5.17 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

$$0.87 f_y A_{st} = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f$$

where, $y_f = 0.15 x_u + 0.65 D_f$

Here, using $x_u = 213.63$ mm, $D_f = 100$ mm, we get

$y_f = 0.15 (213.63) + 0.65 (100) = 97.04$ mm

So, $$A_{st} = \frac{0.36 (20) (350) (213.63) + 0.45 (20) (2600)(97.04)}{0.87 (415)} = 7{,}780.32 \text{ mm}^2$$

Minimum $A_{st} = (0.85/f_y) (b_w) (d) = 0.85 (350) (660)/(415) = 473.13$ mm^2 Maximum $A_{st} = 0.04 b_w D = 0.04 (350) (750) = 10{,}500$ mm^2

Hence, $A_{st} = 7{,}780.32$ mm^2 is o.k.

Provide 6-36 T + 3-28 T (6107 + 1847 = 7,954 mm^2). Please refer to Fig. 5.12.3.

Step 5: Checking of x_u and M_u using A_{st} = 7,954 mm^2

From $T = C$ (Eqs. 5.16 and 5.17 of Lesson 10), we have

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$$

where, $y_f = 0.15\, x_u + 0.65\, D_f$

or $0.87\ (415)\ (7954) = 0.36\ (20)\ (350)\ x_u + 0.45\ (20)\ (2600)\ (0.15\ x_u + 0.65\ D_f)$

or $x_u = 224.01$ mm

$D_f / x_u = 100/224.01 = 0.446 > 0.43$. Accordingly, employing Eq. 5.18 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

So, $M_u = 0.36\, x_u\, \{1 - 0.42(\, x_u/d)\}\, f_{ck}\, b_w\, d + 0.45\ f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2)$

$= 0.36\ (224.01)\{1 - 0.42\ (224.01/660)\}\ (20)\ (350)\ (660)$

$+ 0.45\ (20)\ (2600)\ \{(0.15)\ 224.01 + 65\}\ \{(660) - 0.15\ (112) - 32.5\}$

$= 1{,}779.439$ kNm $> 1{,}719.5625$ kNm

Hence, o.k.

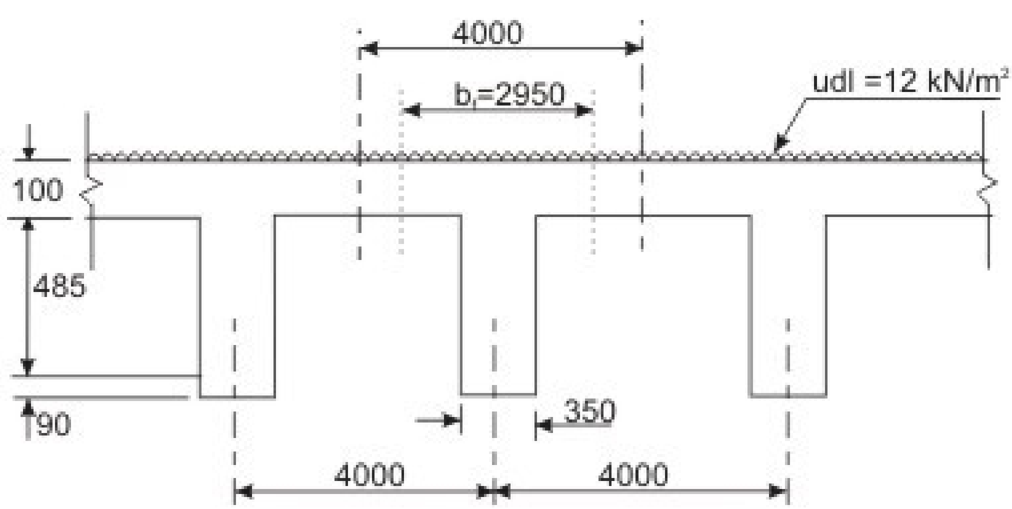


Fig. 5.12.4: Example 8

Ex.8: Design the flanged beam of Fig. 5.12.4, given in following: D_f = 100 mm, D = 675 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m simply supported, cover = 90 mm, d = 585 mm and imposed loads = 12 kN/m^2. Use Fe 415 and M 20.

Step 1: Computation of factored bending moment, M_u

Weight of slab/m^2 = (0.1) (25) = 2.5 kN/m^2 Imposed

loads = 12.0 kN/m^2

Total loads = 14.5 kN/m^2

Total weight of slab + imposed loads/m = 14.5 (4) = 58 kN/m Dead loads of

beam = 0.575 (0.35) (25) = 5.032 kN/m

Total loads = 63.032 kN/m

Factored M_u = (1.5) (63.032) (12) (12)/8 = 1,701.87 kNm

Step 2: Determination of $M_{u,lim}$

Assuming the neutral axis to be in the web, $D_f / x_u < 0.43$ and D_f / d = 100 / 585 = 0.17 < 0.2, we consider the case (ii a) of sec. 5.10.4.2 of Lesson 10 to get the following:

$$M_{u,lim} = 0.36\,(x_{u,max}/d)\,\{1 - 0.42\,(x_{u,max}/d)\}\,f_{ck}\,b_w\,d^2$$

$$+ 0.45\,f_{ck}\,(b_f - b_w)\,D_f\,(d - D_f/2)$$

$$= 0.36(0.48)\,\{1 - 0.42\,(0.48)\}\,(20)\,(350)\,(585)\,(585)$$

$$+ 0.45(20)\,(2600)\,(100)\,(585 - 50) = 1{,}582.4 \text{ kNm}$$

Since, factored $M_u > M_{u,lim}$, the beam is designed as doubly reinforced.

$$M_{u2} = M_u - M_{u,lim} = 1701.87 - 1582.4 = 119.47 \text{ kNm}$$

Step 3: Determination of area of steel

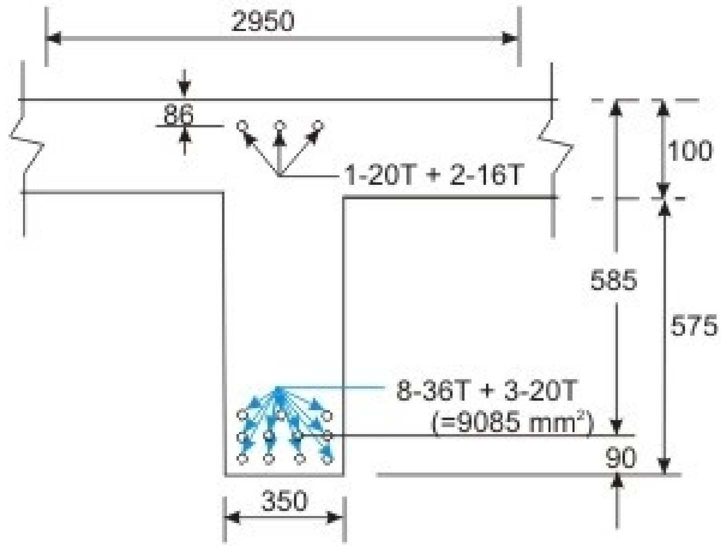


Fig. 5.12.5: Example 8

$A_{st,lim}$ is obtained equating $T = C$ (Eqs. 5.5 and 6 of Lesson 10).

$$0.87\, f_y\, (A_{st,lim}) = 0.36\, b_w\, (x_{u,max}/d)\, d\, f_{ck} + 0.45\, f_{ck}\, (b_f - b_w)\, D_f$$

or $$A_{st,\lim} = \frac{0.36\, b_w\, (x_{u,\max}/d)\, d\, f_{ck} + 0.45\, f_{ck}\, (b_f - b_w)\, D_f}{0.87\, f_y}$$

$$= \frac{0.36\,(350)\,(0.48)\,(585)\,(20) + 0.45\,(20)\,(2600)\,(100)}{0.87\,(415)} = 8{,}440.98 \text{ mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})\,(d - d')}$$ (Eq. 4.4 of Lesson 8).

where f_{sc} = 353 N/mm^2 for d'/d = 0.1

f_{cc} = 0.446 f_{ck} = 0.446 (20) = 8.92 N/mm^2

M_{u2} = 119.47 (10^6) Nmm

d' = 58.5 mm

d = 585 mm

Using the above values in the expression of A_{sc} (Eq. 4.4 of Lesson 8), we get

A_{sc} = 659.63 mm^2

$$A_{st\,2} = \frac{A_{sc}\,(f_{sc} - f_{cc})}{0.87\,f_y}$$ (Eqs. 4.4 and 4.5 of Lesson 8).

Substituting the values of A_{sc}, f_{sc}, f_{cc} and f_y we get

A_{st2} = 628.48 mm^2

Total $A_{st} = A_{st,lim} + A_{st2}$ = 8,440.98 + 628.48 = 9,069.46 mm^2

Maximum $A_{st} = 0.04\ b_w\ D$ = 0.04 (350) (675) = 9,450 mm^2

and minimum $A_{st} = (0.85/f_y)\ b_w\ d$ = (0.85/415) (350) (585) = 419.37 mm^2 Hence, A_{st} = 9,069.46 mm^2 is o.k.

Provide 8-36 T + 3-20 T = 8143 + 942 = 9,085 mm^2 for A_{st} and 1-20 + 2-16 = 314 + 402 = 716 mm^2 for A_{sc} (Fig. 5.12.5).

Step 4: To check for x_u and M_u (Fig. 5.12.5)

Assuming x_u in the web and $D_f/x_u < 0.43$ and using $T = C$ (case ii a of sec. 5.10.4.2 of Lesson 10 with additional compression force due to compression steel), we have:

$0.87\ f_y\ A_{st} = 0.36\ b_w\ x_u\ f_{ck} + 0.45\ (b_f - b_w)\ f_{ck}\ D_f + A_{sc}\ (f_{sc} - f_{cc})$ or $0.87\ (415)\ (9085) = 0.36\ (350)\ x_u\ (20) + 0.45\ (2600)\ (20)\ (100)$

$+ 716\ \{353 - 0.45\ (20)\}$

This gives x_u = 275.33 mm.

$x_{u,max}$ = 0.48 (d) = 0.48 (585) = 280.8 mm.

So, $x_u < x_{u,max}$, D_f/x_u = 100/275.33 = 0.363 < 0.43 and D_f/d = 100/585 = 0.17 < 0.2.

The assumptions, therefore, are correct. So, M_u can be obtained from Eq. 5.14 of sec. 5.10.4.3 of Lesson 10 with additional moment due to compression steel, as given below:

So, $M_u = 0.36\, b_w\, x_u\, f_{ck}\, (d - 0.42\, x_u) + 0.45\, (b_f - b_w)\, f_{ck}\, D_f\, (d - D_f/2)$

$+ A_{sc}\, (f_{sc} - f_{cc})\, (d - d')$

$= 0.36\,(350)\,(275.33)\,(20)\,\{585 - 0.42\,(275.33)\}$

$+ 0.45\,(2600)\,(20)\,(100)\,(585 - 50) + 716\,(344)\,(585 - 58.5)$

$= 325.66 + 1251.9 + 129.67 = 1{,}707.23$ kNm

Factored moment = 1,701.87 kNm < 1,707.23 kNm. Hence, o.k.

5.12.2 Practice Questions and Problems with Answers

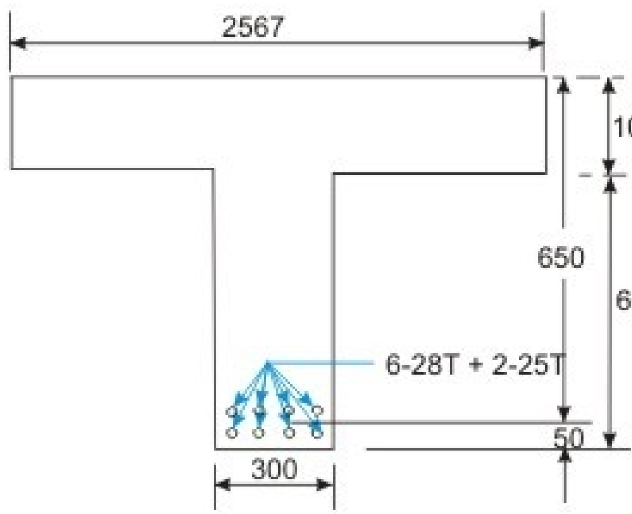


Fig. 5.12.6: Q. 1

Q.1: Determine the steel reinforcement of a simply supported flanged beam (Fig. 5.12.6) of D_f = 100 mm, D = 700 mm, cover = 50 mm, d = 650 mm, b_w = 300 mm, spacing of the beams = 4,000 mm c/c, effective span = 10 m and imposed loads = 10 kN/m^2. Use M 20 and Fe 415.

A.1: Solution:

Step 1: Computation of $(M_u)_{\text{factored}}$

Weight of slab = (0.1) (25) = 2.5 kN/m^2
Imposed loads = 10.0 kN/m^2

12.5 kN/m^2

Total loads per m = (12.5) (4) = 50 kN/m

Dead loads of beam = (0.3) (0.6) (25) = 4.50 kN/m

Total loads = 54.50 kN/m

Factored M_u = (1.5) (54.50) (10) (10)/8 = 1,021.87 kNm

Step 2: Determination of $M_{u,lim}$

Effective width of the flange $b_f = l_o/6 + b_w + 6\,D_f$ = (10,000/6) + 300 + 600 = 2,567 mm.
$x_{u,max} = 0.48\,d = 0.48\,(650) = 312$ mm

Hence, the balanced neutral axis is in the web of the beam.

$D_f/d = 100/650 = 0.154 < 0.2$

$D_f/x_u = 100/312 = 0.32 < 0.43$

So, the full depth of flange is having a stress of 0.446 f_{ck}. From Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2), we have,

$$M_{u,lim} = 0.36\,(x_{u,max}/d)\,\{1 - 0.42\,(x_{u,max}/d)\}\,f_{ck}\,b_w\,d^2$$

$$+ 0.45\,f_{ck}\,(b_f - b_w)\,D_f\,(d - D_f/2)$$

$$= 0.36(0.48)\,\{1 - 0.42(0.48)\}\,(20)\,(300)\,(650)\,(650)$$

$$+ 0.45(20)\,(2267)\,(100)\,(650 - 50)$$

$$= 1573.92 \text{ kNm} > M_u\ (= 1021.87 \text{ kNm})$$

So, the beam will be under-reinforced one.

Step 3: Determination of x_u

Assuming x_u is in the flange, we have from Eq. 3.24 of Lesson 5 (rectangular beam when $b = b_f$).

$$M_u = 0.36\,(x_u/d)\,\{1 - 0.42\,(x_u/d)\}\,f_{ck}\,b_f\,d^2$$

$$= 0.36\,x_u\,\{1 - 0.42\,(x_u/d)\}\,f_{ck}\,b_f\,d$$

$$1021.87\,(10^6) = 0.36\,x_u\,\{1 - 0.42(x_u/650)\}\,(20)\,(2567)\,(650)$$

x_u = 89.55 mm^2 < 100 mm (Hence, the neutral axis is in the flange.)

Step 4: Determination of A_{st}

Equating $C = T$, we have from Eq. 3.17 of Lesson 5:

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f d}$$

or $$A_{st} = \frac{0.36 f_{ck} b_f x_u}{0.87 f_y} = \frac{0.36\,(20)\,(2567)\,(89.55)}{0.87\,(415)} = 4{,}584.12 \text{ mm}^2$$

Minimum $$A_{st} = (0.85/f_y)\,(b_w)\,d = \frac{0.85\,(300)\,(650)}{415} = 399.39 \text{ mm}^2$$

Maximum $A_{st} = 0.04\ b_w\ D = (0.04)\ (300)\ (700) = 8{,}400\ \ mm^2$

So, $A_{st} = 4{,}584.12\ mm^2$ is o.k.

Provide 6-28 T + 2-25 T = 3694 + 981 = 4,675 mm^2 (Fig. 5.12.6).

Solutions

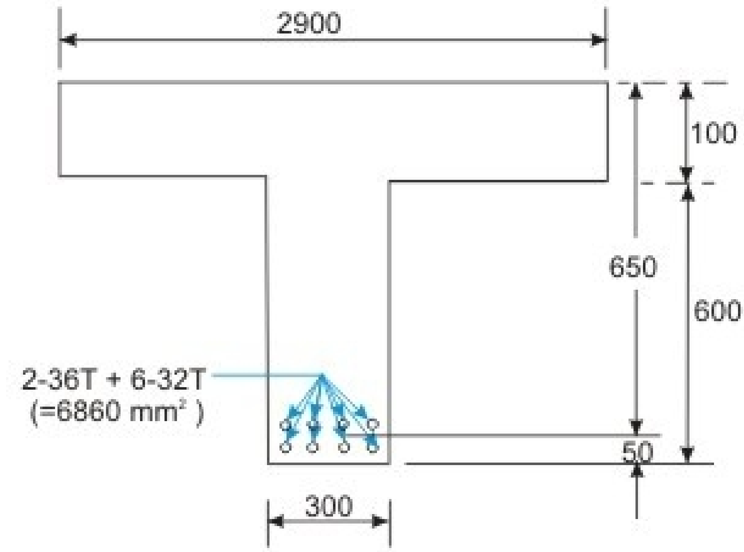


Fig. 5.12.7: TQ. 1

TQ.1: Determine the steel reinforcement A_{st} of the simply supported flanged beam of Q.1 (Fig. 5.12.6) having D_f = 100 mm, D = 700 mm, cover = 50 mm, d = 650 mm, b_w = 300 mm, spacing of the beams = 4,000 mm c/c, effective span = 12 m and imposed loads = 10 kN/m^2. Use M 20 and Fe 415.

A.TQ.1: Solution:

Step 1: Computation of $(M_u)_{\text{factored}}$

Total loads from Q.1 of sec. 5.12.4 = 54.50 kN/m Factored M_u = (1.5)

(54.50) (12) (12)/8 = 1,471.5 kNm

Step 2: Determination of $M_{u,lim}$

Effective width of flange = $l_o/6 + b_w + 6\ D_f$

= (12000/6) + 300 + 600 = 2,900 mm (Fig. 5.12.7)

$x_{u,max} = 0.48\ d = 0.48\ (650) = 312$ mm

Hence, the balanced neutral axis is in the web.

$D_f/d = 100/650 = 0.154 < 0.2$

$D_f/x_u = 100/312 = 0.32 < 0.43$

So, the full depth of flange is having constant stress of 0.446 f_{ck}. From Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2), we have

$$M_{u,lim} = 0.36\ (x_{u,max}/d)\ \{1 - 0.42\ (x_{u,max}/d)\}\ f_{ck}\ b_w\ d^2$$

$$+ 0.45\ f_{ck}\ (b_f - b_w)\ D_f\ (d - D_f/2)$$

= 0.36(0.48) {1 – 0.42(0.48)} (20) (300) (650) (650)

+ 0.45(20) (2600) (100) (650 – 50) = 1,753.74 kNm > 1,471.5 kNm

So, the beam will be under-reinforced.

Step 3: Determination of x_u

Assuming x_u to be in the flange, we have from Eq. 3.24 of Lesson 5 (singly reinforced rectangular beam when $b = b_f$):

$$M_u = 0.36\ x_u\ \{1 - 0.42\ (x_u/d)\}\ f_{ck}\ b_f\ d$$

or $1471.5 (10^6) = 0.36 (x_u) \{1 - 0.42 (x_u /650)\} (20) (2900) (650)$

or $x_u^2 - 1547.49 x_u + 167.81 (10^3) = 0$

Solving, we have x_u = 117.34 mm > 100 mm So, neutral axis

is in the web.

Assuming $D_f/x_u < 0.43$, we have from Eq. 5.14 of Lesson 10 (case iii a of sec. 5.10.4.3),

$$M_u = 0.36\, x_u \{1 - 0.42\, (x_u/d)\}\, f_{ck}\, b_w\, d + 0.45\, f_{ck}\, (b_f - b_w)\, D_f\, (d - D_f/2)$$

$$= 0.36\, x_u \{1 - 0.42\, (x_u/650)\}\, (20)\, (300)\, (650)$$

$$+ 0.45(20)\, (2600)\, (100)\, (650 - 50)$$

or $\quad x_u^2 - 1547.62\, x_u + 74404.7 = 0$

Solving, we have x_u = 49.67 < 100 mm

However, in the above when it is assumed that the neutral axis is in the flange x_u is found to be 117.34 mm and in the second trial when x_u is assumed in the web x_u is seen to be 49.67 mm. This indicates that the full depth of the flange will not have the strain of 0.002, neutral axis is in the web and D_f/x_u is more than 0.43. So, we have to use Eq. 5.18 of Lesson 10, with the introduction of y_f from Eq. 5.15 of Lesson 10.

Assuming $D_f/x_u > 0.43$, from Eqs. 5.15 and 5.18 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

$$M_u = 0.36\, x_u \{1 - 0.42\, (x_u/d)\}\, f_{ck}\, b_w\, d + 0.45\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2) \text{ where,}$$

$$y_f = (0.15\, x_u + 0.65\, D_f)$$

So, $\quad M_u = 0.36\, x_u \{1 - 0.42\, (x_u/650)\}\, (20)\, (300)\, (650)$

$$+ 0.45(20)\, (2600)\, (0.15\, x_u + 0.65)\, (650 - 0.075\, x_u - 0.325\, x_u)$$

or, $\quad 1471.5\, (10^6) = -\, 1170.45\, x_u^2 + 3.45735\, x_u + 939.2175\, (10^6)$

Solving, we get x_u = 162.9454 mm. This shows that the assumption of $D_f/x_u > 0.43$ is correct as D_f/x_u = 100 / 162.9454 = 0.614.

Step 4: Determination of A_{st}

Equating $C = T$ from Eqs. 5.16 and 5.17 of Lesson 10 (case iii b of sec. 5.10.4.3), we have

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$$

or $$A_{st} = \frac{0.36\,(20)\,(30)\,(162.9454) + 0.45\,(20)\,(2600)\{0.15\,(162.9454) + 65\}}{0.87\,(415)}$$

$$= 974.829 + 5{,}796.81 = 6{,}771.639 \text{ mm}^2$$

Minimum $A_{st} = (0.85/f_y)\,(b_w)\,(d) = 0.85\,(300)\,(650)/415 = 399.39 \text{ mm}^2$

Maximum $A_{st} = 0.04\,(b_w)\,(D) = 0.04\,(300)\,(700) = 8{,}400 \text{ mm}^2$

So, $A_{st} = 6{,}771.639$ is o.k.

Provide 2-36 T + 6-32 T = 2035 + 4825 = 6,860 mm^2 > 6,771.639 mm^2 (Fig. 5.12.7).

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – II – LIMIT STATE OF COLLAPSE - SHEAR – SCIA1203

LIMIT STATE OF COLLAPSE SHEAR AND BOND

Limit State of Collapse

This limit state is also called as strength limit state as it corresponds to the maximum load carrying capacity i.e., the safety requirements of the structure. The limit state of collapse is assessed from collapse of the whole or part of the structure. As per this limit state, the resistance to bending, shear, torsion and axial loads at every section shall not be less than that produced by the most unfavorable combination of loads on that structure.

INTRODUCTION

Limit state of collapse shear and bond :

A beam loaded with transverse loads is subjected to shear force and bending moment. The shear force at any section is equal to the rate of change of bending moment. The shear force results into shear stresses across the cross-section is given by the following equations.

$$q=V(A.\bar{y})/I.b$$

where, q = Shear stress
I = Moment of inertia of the beam section
b = Width of section
V = Shear force at the section
$(A.\bar{y})$ = First moment of the area above the section about neutral axis
On the basis of above equation the shear stress distribution across a rectangular cross-section is shown in Fig. 5.1(b).

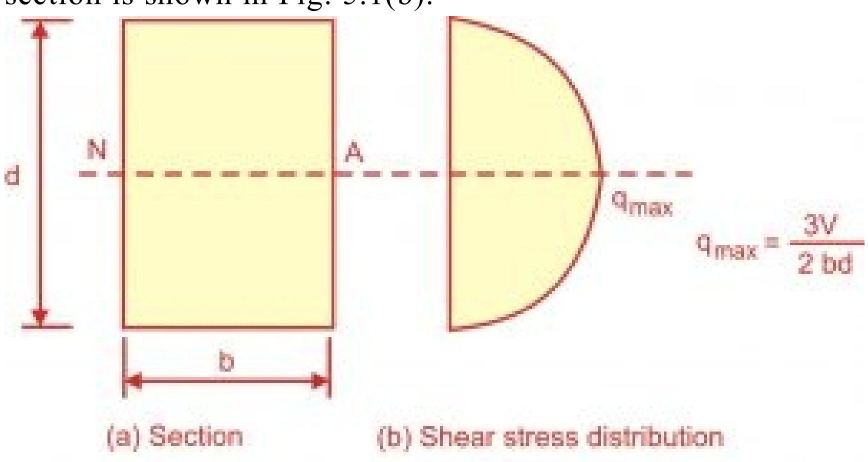


Fig. 5.1. Shear stress distribution in rectangular section.

LIMIT STATE OF COLLAPSE SHEAR AND BOND
It is parabolic with zero at top and bottom and the maximum shear stress, occurs at neutral axis is equal to

3V/2bd

Types of shear reinforcement

Types of shear reinforcement : Vertical stirrups, Bent up bars along with stirrups, Inclined stirrups, contribution of bent up bars

TYPES OF SHEAR REINFORCEMENT

The following three types of shear reinforcement are used : Vertical stirrups.

Bent up bars along with stirrups.

Inclined stirrups.

Vertical Stirrups

These are the steel bars vertically placed around the tensile reinforcement at suitable spacing along the length of the beam. Their diameter varies from 6 mm to 16 mm. The free ends of the stirrups are anchored in the compression zone of the beam to the anchor bars (hanger bar) or the compressive reinforcement. Depending upon the magnitude of the shear force to be resisted the vertical stirrups may be one legged, two legged, four legged and so on as shown in Fig. 5.5.

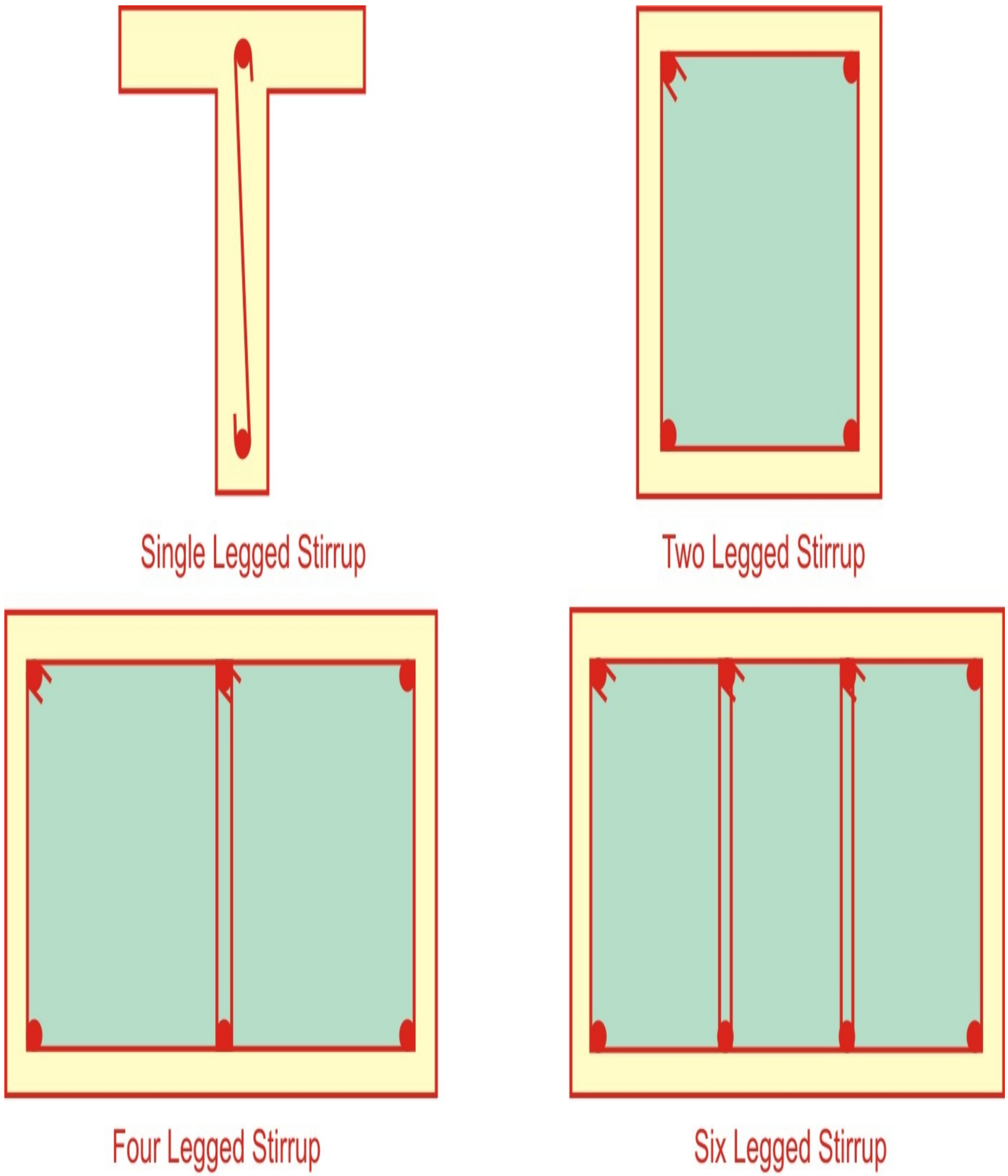


Fig. 5.5. Vertical stirrups

It is desirable to use closely spaced stirrups for better prevention of the diagonal cracks. The spacing of stirrups near the supports is less as compared to spacing near the mid-span since shear force is maximum at the supports.

Bent up Bars along with Vertical Stirrups

Some of the longitudinal bars in a beam can be bent up near the supports where they are not required to resist bending moment (Bending Moment is very less near the supports).

These bent up bars resist diagonal tension. Equal number of bars are to be bent on both sides to maintain symmetry. The bars can be bent up at more than one point uniformly along the length of the beam. These bars are usually bent at 45º as shown in Fig. 5.6.

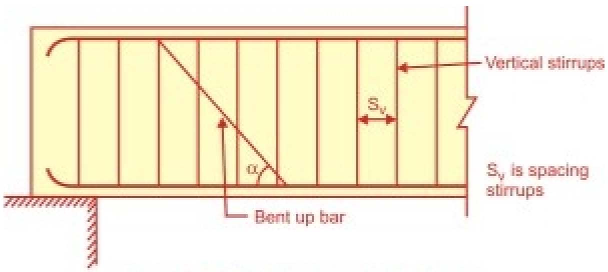


Fig. 5.6. Bent up bars alongwith stirrups.

Bent up Bars along with Vertical Stirrups

This system is used for heavier shear forces. The total shear resistance of the beam is calculated by adding the contribution of bent up bars and vertical stirrups. The contribution of bent up bars is not greater than half of the total shear reinforcement.

Inclined Stirrups

Inclined stirrups are also provided generally at 45º for resisting diagonal tension as shown in Fig. 5.7. They are provided throughout the length of the beam.

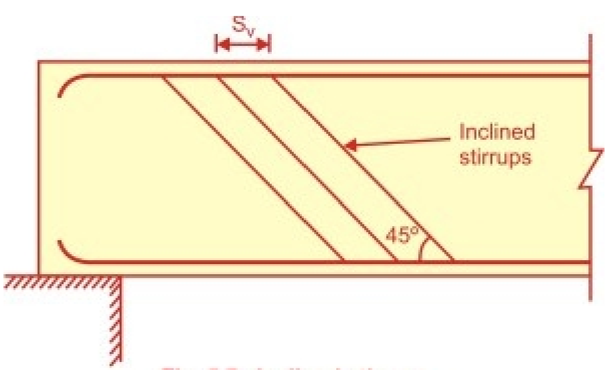


Fig. 5.7. Inclined stirrups.

Inclined Stirrups

Effects of Shear Diagonal Tension

Effects of Shear Diagonal Tension, maximum bending tensile stress, tensile stress (σ) as well as shear stress (τ),crack pattern for a simply supported beam, The maximum bending moment in this beam will be at midspan and the maximum shear force, at the supports.

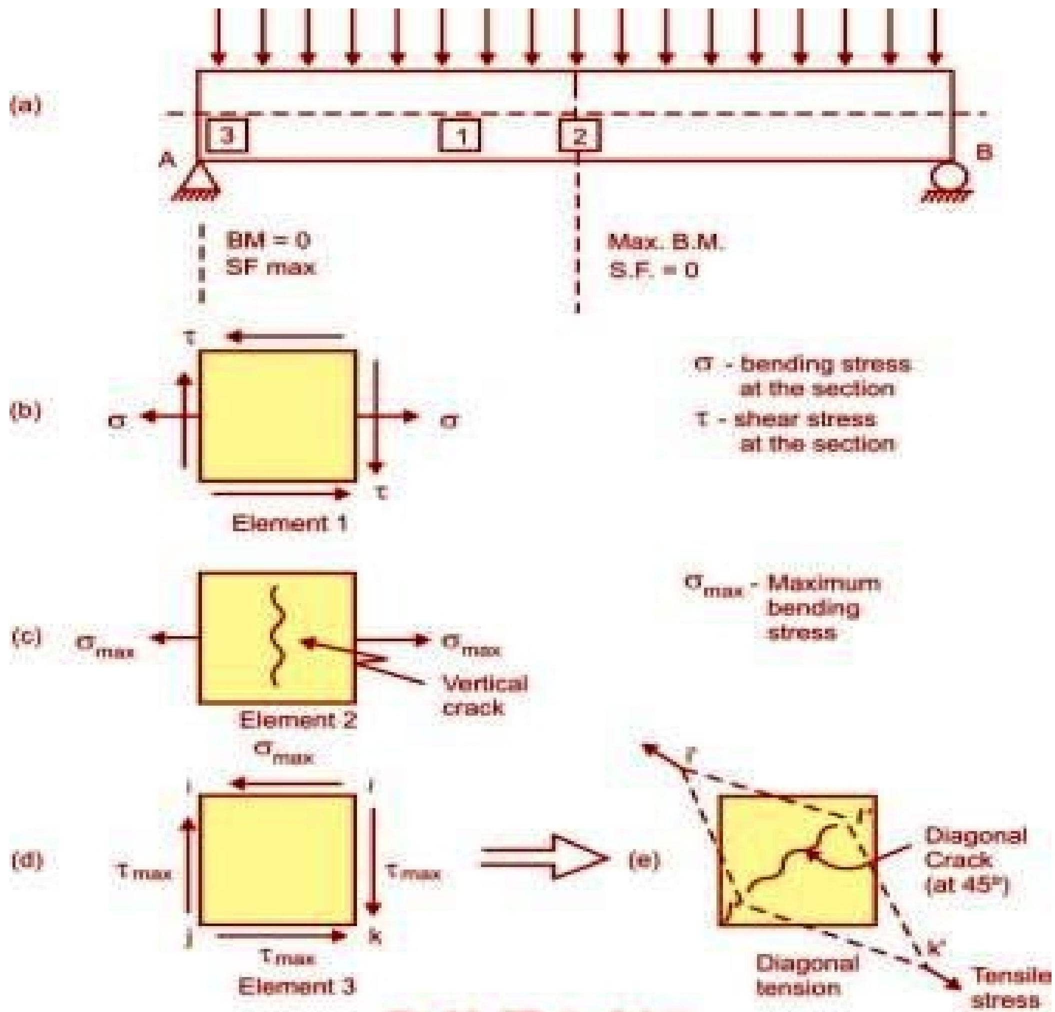
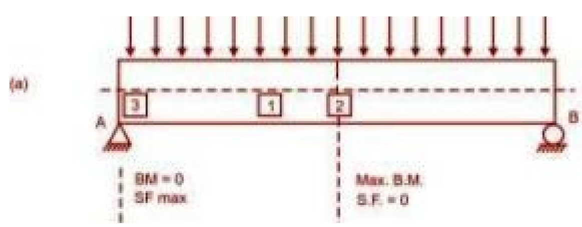


Fig. 5.3. Effects of shear.

Effects of Shear Diagonal Tension, maximum bending tensile stress, tensile stress (σ) as well as shear stress (τ),

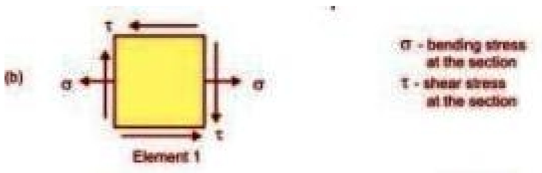
EFFECTS OF SHEAR DIAGONAL TENSION

Consider a beam AB subjected to transvers loads as shown in Fig. 5.3(a).



beam AB subjected to transvers loads

The maximum bending moment in this beam will be at midspan and the maximum shear force, at the supports. The beam is subjected to bending and shear stresses across the cross-section. Let us consider a small element (1) from the tensile zone of the beam. It is subjected to bending tensile stress (σ) as well as shear stress (τ) as shown in Fig. 5.3. (b).



tensile stress (σ) as well as shear stress (τ)

At the midspan, the bending moment is maximum and the shear force is zero. So the element 2 is subjected to maximum bending tensile stress only. This tensile stress tries to pull apart the section as shown in Fig. 5.3(c) and the crack developed is vertical.

maximum bending tensile stress

At the support, the bending moment is zero and the shear force is maximum. Thus the element 3 is subjected to maximum shear stress and no bending stresses as shown in Fig. 5.3 (d).

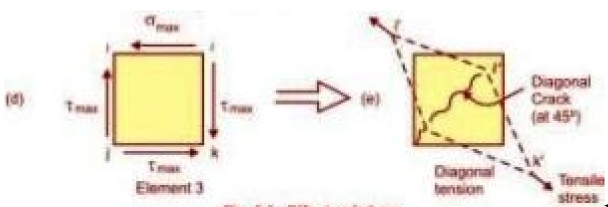


Fig. 5.3. Effects of shear.

maximum shear stress and no bending stresses

Due to this stress condition the diagonal ik of the element is subjected to tensile stresses as shown in Fig. 5.3 (e). As the concrete is very weak in tension it split along the diagonal (at 45º) and develops crack as shown.

This tension which is caused in the tensile zone of the beam due to shear, at or near the supports is called as diagonal tension which is caused by shear, cannot be resisted by concrete alone. So shear reinforcement is provided in the R.C.C. beams to take up diagonal tension and prevent cracking of beam.

The crack pattern for a simply supported beam is shown in Fig. 5.4.

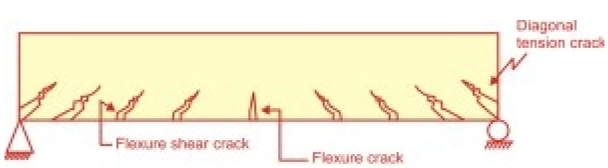


Fig. 5.4. Crack pattern in simply supported beam.

crack pattern for a simply supported beam

1. At or near the midspan, the crack will be vertical (flexure cracks due to bending alone).
2. At or near the supports the cracks are inclined at 45º (shear or diagonal tension cracks).
3. In between the supports and mid-span the cracks inclination vary from 45º to 90º gradually (flexure-shear cracks).

Shear Stresses in R.C.C. beams
Shear Stresses in R.C.C. beam (Reinforced cement concrete beam), Stress Based Approach (Elastic Theory), IS Code Approach

SHEAR STRESSES IN R.C.C. BEAMS

Stress Based Approach (Elastic Theory)

R. C.C. is a composite materials so the exact shear distribution as per elastic theory is very complex. It is shown in Fig. 5.2(b)

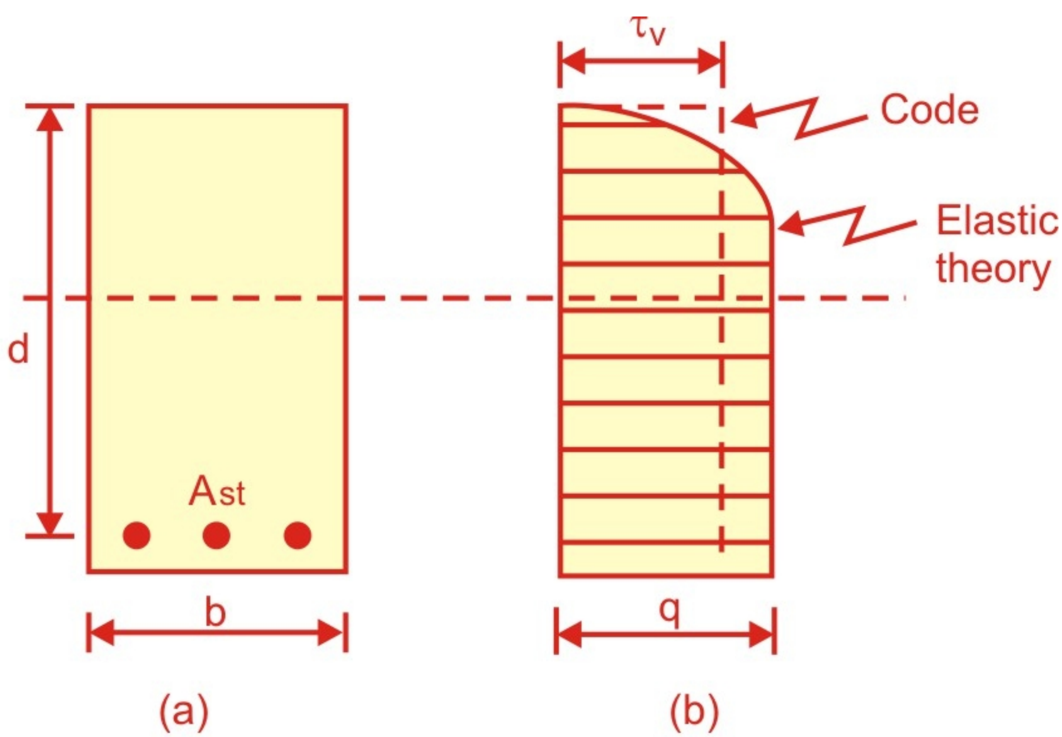


Fig. 5.2. Shear-stress distribution in RCC beam.

by the hatched portion of the curve.It is parabolic in the compression zone with zero at the top and maximum at the neutral axis. The value of shear-stress is constant in the tensile zone and is equal to the maximum shear-stress (q) because the concrete, below the neutral axis (tensile zone) is assumed to be cracked and neglected. The maximum value of shear stress (q) as per elastic theory is given by

$$q=Vbjd$$

where V = shear force at the section b and d = dimensions of the section j = Lever arm depth factor

IS Code Approach

As per IS code 456:2000 the stress based approach does not represent the true behaviour of the R.C.C. beam in shear. Hence, the equation for shear stress i.e.,

$$q = V/bjd$$

has been simplified. IS code recommends the use of nominal shear stress

$$(\tau y)$$

for R.C.C. beams. The nominal shear stress

$$(\tau v)$$

or average shear stress distribution is shown in Fig. 5.2(b) and is given by

$\tau v = V/bd$

Introduction

This lesson explains the three failure modes due to shear force in beams and defines different shear stresses needed to design the beams for shear. The critical sections for shear and the minimum shear reinforcement to be provided in beams are mentioned as per IS 456. The design of shear reinforcement has been illustrated in Lesson 14 through several numerical problems including the curtailment of tension reinforcement in flexural members.

Failure Modes due to Shear

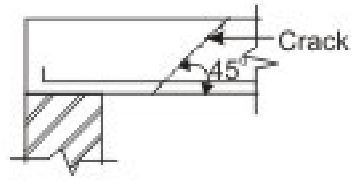


Fig.6.13.1(a): Web shear progresses along dotted dotted lines

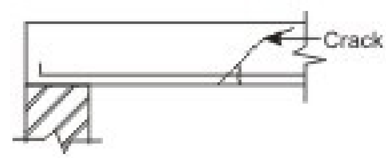


Fig.6.13.1(b): Flexural tension (steel yields)

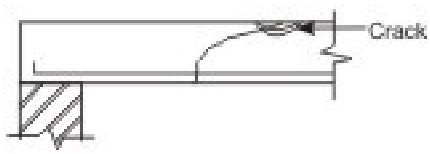


Fig.6.13.1(c): Flexural compression
(concrete crushes in compression)

Fig.6.13.1: Failure modes

Bending in reinforced concrete beams is usually accompanied by shear, the exact analysis of which is very complex. However, experimental studies confirmed the following three different modes of failure due to possible combinations of shear force and bending moment at a given section (Figs. 6.13.1a to c):

(i) Web shear (Fig. 6.13.1a)
(ii) Flexural tension shear (Fig. 6.13.1b)
(iii) Flexural compression shear (Fig. 6.13.1c)

Web shear causes cracks which progress along the dotted line shown in Fig. 6.13.1a. Steel yields in flexural tension shear as shown in Fig. 6.13.1b, while concrete crushes in compression due to flexural compression shear as shown in Fig. 6.13.1c. An in-depth presentation of the three types of failure modes is beyond the scope here. Only the salient points needed for the routine design of beams in shear are presented here.

Shear Stress

The distribution of shear stress in reinforced concrete rectangular, *T* and *L*-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):

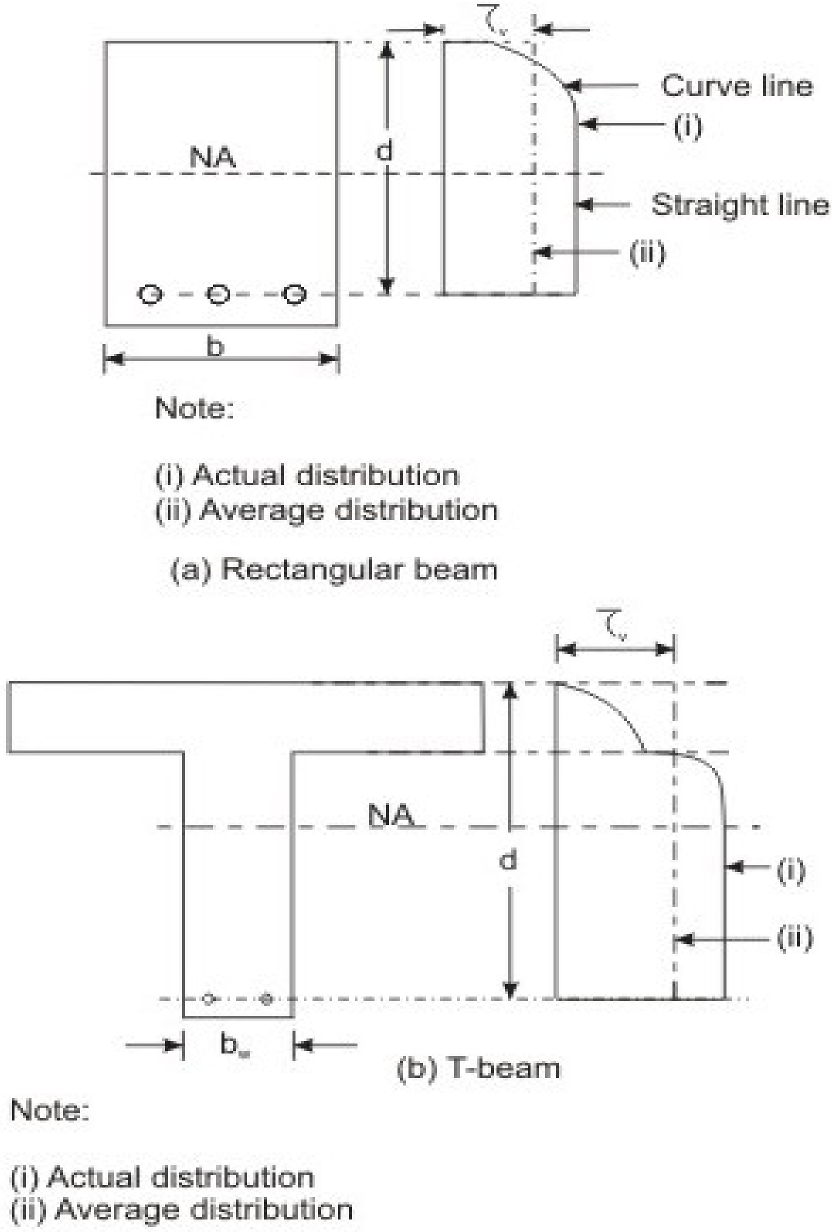


Fig. 6.13.2: Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs. 6.13.2a and b):

$$v \quad \frac{V_u}{b\,d} \qquad (6.1)$$

where V_u = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

d = effective depth.

(ii) In beams of varying depth:

$$v \quad \frac{V_u \quad \frac{M_u}{d} \tan}{b\,d} \qquad (6.2)$$

where τ_v, V_u, b or b_w and d are the same as in (i),

M_u = bending moment at the section, and

β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment M_u decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment M_u increases numerically in the same direction as the effective depth increases.

6.13.1 Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value τ_{cmax} depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c given in Table 6.1.

Table 6.1 Design shear strength of concrete, τ_c in N/mm^2

(100 A_s /b d)	Grade of concrete				
	M 20	M 25	M 30	M 35	M40 and above
$\leq$ 0.15	0.28	0.29	0.29	0.29	0.30
0.25	0.36	0.36	0.37	0.37	0.38
0.50	0.48	0.49	0.50	0.50	0.51
0.75	0.56	0.57	0.59	0.59	0.60
1.00	0.62	0.64	0.66	0.67	0.68
1.25	0.67	0.70	0.71	0.73	0.74
1.50	0.72	0.74	0.76	0.78	0.79
1.75	0.75	0.78	0.80	0.82	0.84
2.00	0.79	0.82	0.84	0.86	0.88
2.25	0.81	0.85	0.88	0.90	0.92
2.50	0.82	0.88	0.91	0.93	0.95
2.75	0.82	0.90	0.94	0.96	0.98
$\geq$ 3.00	0.82	0.92	0.96	0.99	1.01

In Table 6.1, A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section considered except at support where the full area of tension reinforcement may be used provided the detailing is as per IS 456, cls. 26.2.2 and 26.2.3.

Maximum shear stress τ_{cmax} with shear reinforcement (cls. 40.2.3, 40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress of reinforced concrete in beams τ_{cmax} as given below in Table 6.2. Under no circumstances, the nominal shear stress in beams τ_v shall exceed τ_{cmax} given in Table 6.2 for different grades of concrete.

Table 6.2 Maximum shear stress, τ_{cmax} in N/mm^2

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
τ_{cmax}, N/mm^2	2.8	3.1	3.5	3.7	4.0

Critical Section for Shear

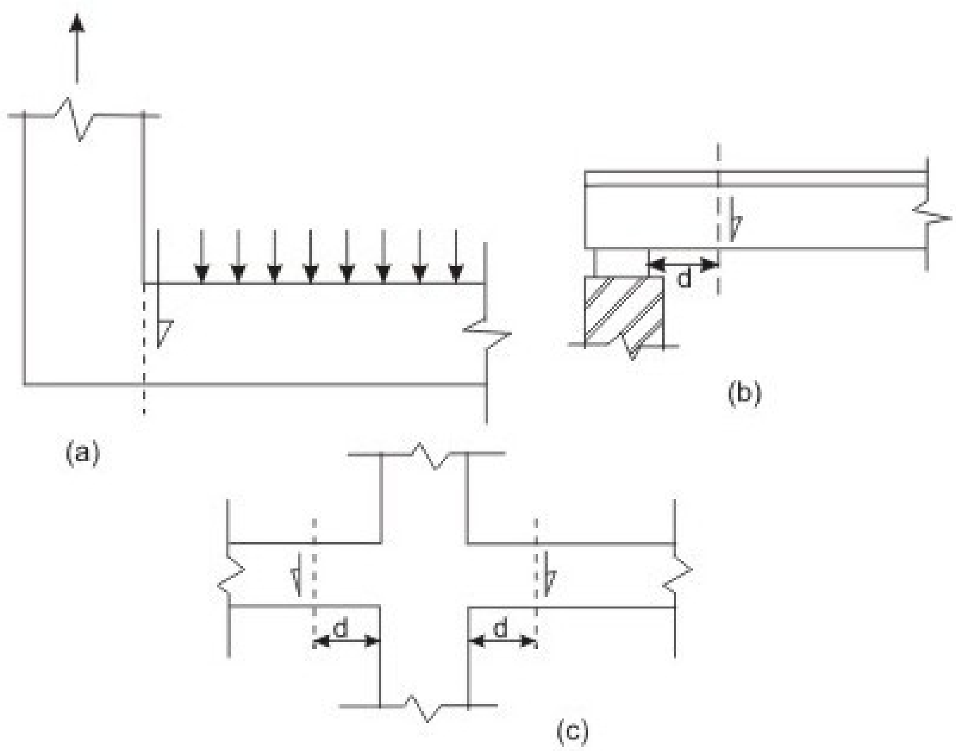


Fig. 6.13.3: Support conditions for locating factored shear force

Clauses 22.6.2 and 22.6.2.1 stipulate the critical section for shear and are as follows:

For beams generally subjected to uniformly distributed loads or where the principal load is located further than $2d$ from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in Figs. 6.13.3 a, b and c and are mentioned below.

(i) When the reaction in the direction of the applied shear introduces tension (Fig. 6.13.3a) into the end region of the member, the shear force is to be computed at the face of the support of the member at that section.

(ii) When the reaction in the direction of the applied shear introduces compression into the end region of the member (Figs. 6.13.3b and c), the shear force computed at a distance d from the face of the support is to be used for the design of sections located at a distance less than d from the face of the support. The enhanced shear strength of sections close to supports, however, may be considered as discussed in the following section.

Enhanced Shear Strength of Sections Close to Supports (cl. 40.5 of IS 456)

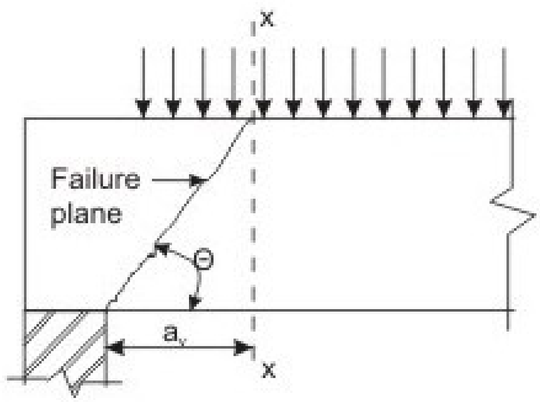


Fig. 6.13.4: Shear failure near support

Figure 6.13.4 shows the shear failure of simply supported and cantilever beams without shear reinforcement. The failure plane is normally inclined at an angle of 30° to the horizontal. However, in some situations the angle of failure is more steep either due to the location of the failure section closed to a support or for some other reasons. Under these situations, the shear force required to produce failure is increased.

Such enhancement of shear strength near a support is taken into account by increasing the design shear strength of concrete to ($2d\tau_c/a_v$) provided that the design shear stress at the face of the support remains less than the value of τ_{cmax} given in Table 6.2 (Table 20 of IS 456). In the above expression of the enhanced shear strength

d = effective depth of the beam,

τ_c = design shear strength of concrete before the enhancement as given in Table 6.1

(Table 19 of IS 456),

a_v = horizontal distance of the section from the face of the support (Fig. 6.13.4).

Similar enhancement of shear strength is also to be considered for sections closed to point loads. It is evident from the expression ($2d\tau_c/a_v$) that when a_v is equal to $2d$, the enhanced shear strength does not come into picture. Further, to increase the effectivity, the tension reinforcement is recommended to be extended on each side of the point where it is intersected by a possible failure plane for a distance at least equal to the effective depth, or to be provided with an equivalent anchorage.

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table 6.1 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{b\, s_v} \quad \frac{0.4}{0.87 f_y} \qquad (6.3)$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam b_w, and

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2.

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.

The minimum shear reinforcement is provided for the following:

(i) Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

(ii) Brittle shear failure is arrested which would have occurred without shear reinforcement.

(iii) Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.

(iv) To hold the reinforcement in place when concrete is poured.

Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 *d* for vertical stirrups and *d* for inclined stirrups at 45°, where *d* is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

Design of Shear Reinforcement (cl. 40.4 of IS 456)

When τ_v is more than τ_c given in Table 6.1, shear reinforcement shall be provided in any of the three following forms:

(a) Vertical stirrups,
(b) Bent-up bars along with stirrups, and
(c) Inclined stirrups.

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement.

The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to

$$V_{us} = V_u - \tau_c b d \tag{6.4}$$

where b is the breadth of rectangular beams or b_w in the case of flanged beams.

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) Vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} \tag{6.5}$$

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} (\sin \alpha + \cos \alpha) \tag{6.6}$$

(c) For single bar or single group of parallel bars, all bent-up at the same cross- section:

$$V_{us} \quad 0.87 \quad f_y A_{sv} s_v \sin \qquad (6.7)$$

where A_{sv} = total cross-sectional area of stirrup legs or bent-up bars within a distance s_v,

s_v = spacing of stirrups or bent-up bars along the length of the member,

τ_v = nominal shear stress,

τ_c = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web b_w,

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm^2,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45^o, and

d = effective depth.

The following two points are to be noted:

(i) The total shear resistance shall be computed as the sum of the resistance for the various types separately where more than one type of shear reinforcement is used.

(ii) The area of stirrups shall not be less than the minimum specified in cl. 26.5.1.6.

6.13.2 Shear Reinforcement for Sections Close to Supports

As stipulated in cl. 40.5.2 of IS 456, the total area of the required shear reinforcement A_s is obtained from:

$$A_s = a_v\, b\, (\tau_v - 2d\, \tau_c /a_v)/0.87\, f_y$$

and $\geq 0.4\, a_v\, b/0.87\, f_y$

(6.8)

For flanged beams, b will be replaced by b_w, the breadth of the web of flanged beams.

This reinforcement should be provided within the middle three quarters of a_v, where a_v is less than d, horizontal shear reinforcement will be effective than vertical.

Alternatively, one simplified method has been recommended in cl. 40.5.3 of IS 456 and the same is given below.

The following method is for beams carrying generally uniform load or where the principal load is located further than $2d$ from the face of support. The shear stress is calculated at a section a distance d from the face of support. The value of τ_c is calculated in accordance with Table 6.1 and appropriate shear reinforcement is provided at sections closer to the support. No further check for shear at such sections is required.

Curtailment of Tension Reinforcement in Flexural Members (cl. 26.2.3.2 of IS 456)

Curtailment of tension reinforcement is done to provide the required reduced area of steel with the reduction of the bending moment. However, shear force increases with the reduction of bending moment. Therefore, it is necessary to satisfy any one of following three conditions while terminating the flexural reinforcement in tension zone:

(i) The shear stress τ_v at the cut-off point should not exceed two-thirds of the permitted value which includes the shear strength of the web reinforcement. Accordingly,

$$\tau_v \le (2/3)\,(\tau_c + V_{us}/b\,d) \text{ or}$$

$$V_{us} \ge (1.5\,\tau_v - \tau_c)\,b\,d$$

(6.9)

(ii) For each of the terminated bars, additional stirrup area should be provided over a distance of three-fourth of effective depth from the cut-off point. The additional stirrup area shall not be less than $0.4\,b\,s/f_y$, where b is the breadth of rectangular beams and is replaced by b_w, the breadth of the web for flanged beams, s = spacing of additional stirrups and f_y is the characteristic strength of stirrup reinforcement in N/mm^2. The value of s shall not exceed $d/(8\,\beta_b)$, where β_b is the ratio of area of bars cut-off to the total area of bars at that section, and d is the effective depth.

(iii) For bars of diameters 36 mm and smaller, the continuing bars provide double the area required for flexure at the cut-off point. The shear stress should not exceed three-fourths that permitted. Accordingly,

$\tau_v \le (3/4)\ (\tau_c + V_{us}/b\ d)$ or

$V_{us} \ge (1.33\ \tau_v - \tau_c)\ b\ d$

(6.10)

In the above expression b is the breadth of the rectangular beams which will be b_w in the case of flanged beams.

Placement of Stirrups

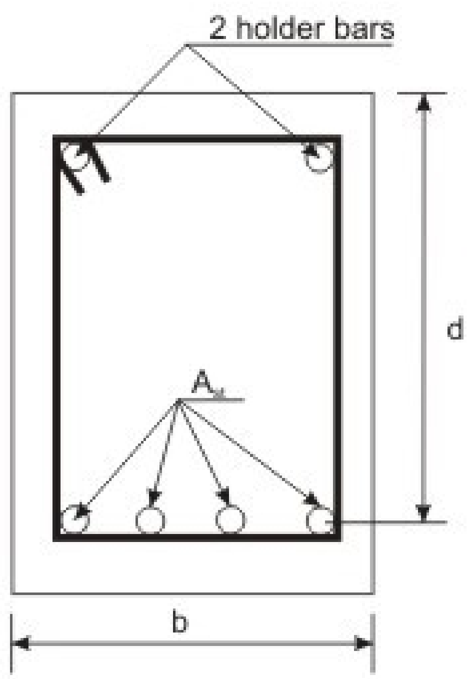


Fig. 6.13.5: Placement of stirrups

The stirrups in beams shall be taken around the outer-most tension and compression bars. In T and L-beams, the stirrups will pass around longitudinal bars located close to the outer face of the flange. In the rectangular beams, two holder bars of diameter 10 or 12 mm are provided if there is no particular need for compression reinforcement (Fig. 6.13.5).

Practice Questions and Problems with Answers

Q.1: Name and explain the three different failure modes of reinforced concrete beams under the combined effects of bending moment and shear force.

A.1: Sec. 6.13.2

Q.2: Define nominal shear stress τ_v of rectangular and *T*-beams of (i) uniform depth and (ii) varying depth subjected to bending moment and shear force.

A.2: Sec. 6.13.3

Q.3: What is meant by "Design shear strength of concrete τ_c" ?

A.3: Sec. 6.13.4

Q.4: On what parameters τ_c of beams without shear reinforcement depends ? How do you get τ_c for different grades of concrete ?

A.4: τ_c depends on (i) grade of concrete and (ii) percentage of tensile steel in the beam.

Table 19 of cl. 40.2.1 of IS 456 gives the values of τ_c and the same table is presented in Table 6.1 of sec. 6.13.4.1 of this lesson.

Q.5: How do you know the maximum shear stress of concrete beams τ_{cmax} with shear reinforcement ?

A.5: Sec. 6.13.4.2

Q.6: How do you determine the critical sections for shear in a beam ?

A.6: Sec. 6.13.5

Q.7: When and why do we consider enhanced shear strength of concrete ?

A.7: Sec. 6.13.6

Q.8: How do we determine the minimum shear reinforcement in rectangular and *T*-beams ? Why do we provide the minimum shear reinforcement ?

A.8: Sec. 6.13.7

Q.9: What are the three different ways to provide shear reinforcement ? Explain the method of design of each of them.

A.9: Sec. 6.13.8

Q.10: How do we design the shear reinforcement close to the support of a beam ?

A.10: Sec. 6.13.9

Q.11: State the conditions to be satisfied for the curtailment of tension reinforcement when designing the shear reinforcement.

A.11: Sec. 6.13.10

Q.12: How do we place the vertical stirrups in a beam ?

A.12: Sec. 6.13.11

Solutions

TQ.1: Define nominal shear stress τ_v of rectangular and *T*-beams of (i) uniform depth and (ii) varying depth subjected to bending moment and shear force. (5 marks)

A.TQ.1: Sec. 6.13.3

TQ.2: How do you determine the critical sections for shear in a beam ? (5 marks)

A.TQ.2: Sec. 6.13.5

TQ.3: When and why do we consider enhanced shear strength of concrete? (5 marks)

A.TQ.3: Sec. 6.13.6

TQ.4: How do we determine the minimum shear reinforcement in rectangular and *T*-beams? Why do we provide the minimum shear reinforcement ? (5 marks)

A.TQ.4: Sec. 6.13.7

TQ.5: What are the three different ways to provide shear reinforcement ? Explain the method of design of each of them. (5 marks)

A.TQ.5: Sec. 6.13.8

TQ.6: How do we design the shear reinforcement close to the support of a beam? (5 marks)

A.TQ.6: Sec. 6.13.9

TQ.7: State the conditions to be satisfied for the curtailment of tension reinforcement when designing the shear reinforcement. (5 marks)

A.TQ.7: Sec. 6.13.10

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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – III – SLABS AND CONTINUOUS BEAMS – SCIA1203

Introduction

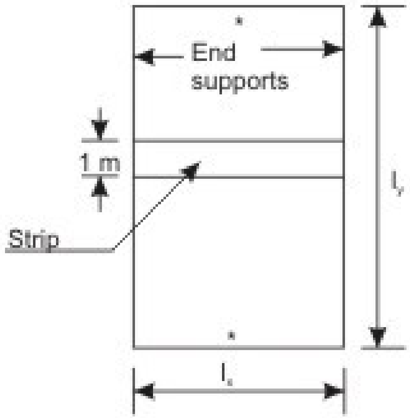

Fig. 8.18.1(a): One span

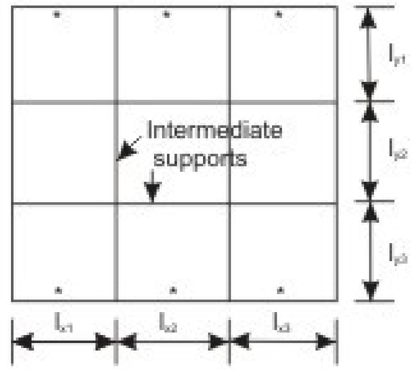

Fig. 8.18.1(c): Continuous in both directions

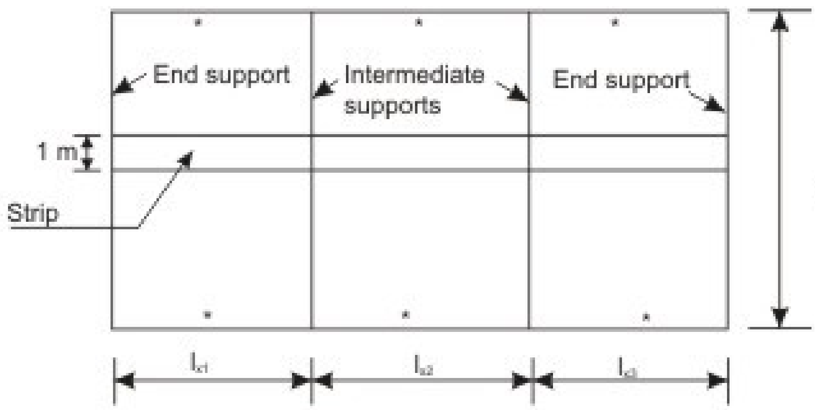

Fig. 8.18.1(b): Continuous in one direction

Notes:
1. One-way slab if $l_y > 2l_x$
2. * indicates that no support is needed if $l_y > 2l_x$ and is needed if $l_y <= 2l_x$
3. End supports may be simply supported or clamped

Fig. 8.18.1: Horizontal slabs

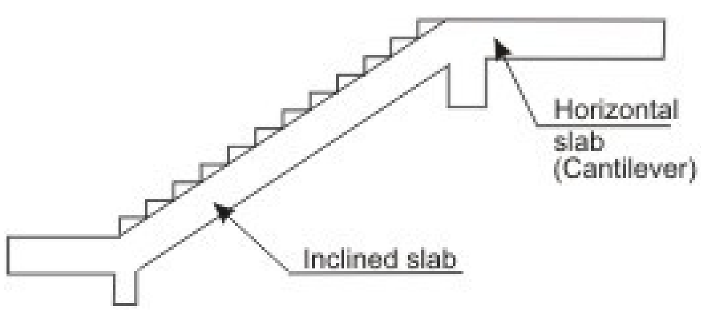

Fig. 8.18.2: Stair case

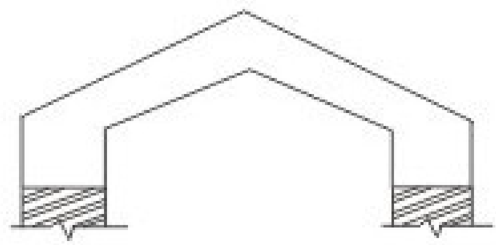

Fig. 8.18.3: Inclined roof

Slabs, used in floors and roofs of buildings mostly integrated with the supporting beams, carry the distributed loads primarily by bending. It has been mentioned in sec. 5.10.1 of Lesson 10 that a part of the integrated slab is considered as flange of *T*- or *L*-beams because of monolithic construction. However, the remaining part of the slab needs design considerations. These slabs are either single span or continuous having different support conditions like fixed, hinged or free along the edges (Figs.8.18.1a,b and c). Though normally these slabs are horizontal, inclined slabs are also used in ramps, stair cases and inclined roofs (Figs.8.18.2 and 3). While square or rectangular plan forms are normally used, triangular, circular and other plan forms are also needed for different functional requirements. This lesson takes up horizontal and rectangular /square slabs of buildings supported by beams in one or both directions and subjected to uniformly distributed vertical loadings.

The other types of slabs, not taken up in this module, are given below. All these slabs have additional requirements depending on the nature and magnitude of loadings in respective cases.

horizontal or inclined bridge and fly over deck slabs carrying heavy concentrated loads,

horizontal slabs of different plan forms like triangular, polygonal or circular,

flat slabs having no beams and supported by columns only,

inverted slabs in footings with or without beams,

slabs with large voids or openings,

grid floor and ribbed slabs.

One-way and Two-way Slabs

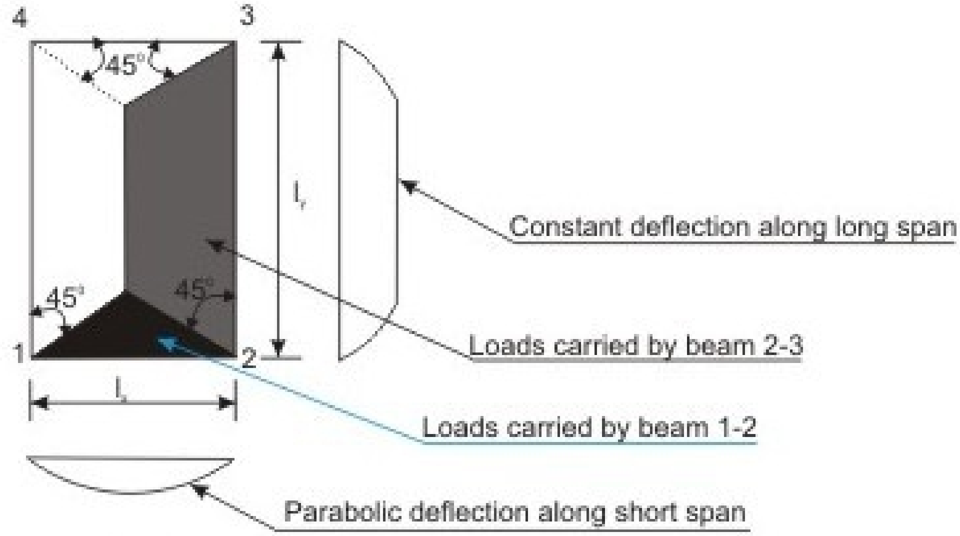

Fig. 8.18.4(a): One-way slab (l_y/l_x > 2) Gtg

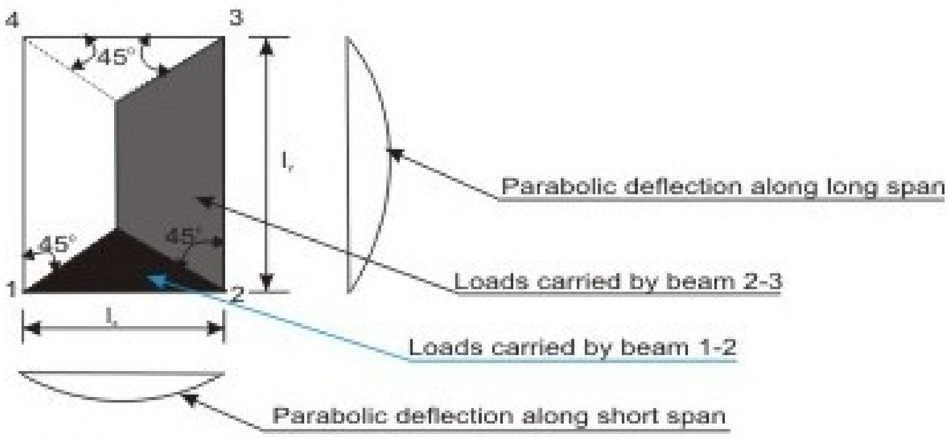

Fig. 8.18.4(b): Two-way slab (l_y/l_x <= 2)

Fig. 8.18.4: Sharing of loads

Figures 8.18.4a and b explain the share of loads on beams supporting solid slabs along four edges when vertical loads are uniformly distributed. It is evident from the figures that the share of loads on beams in two perpendicular directions depends upon the aspect ratio l_y /l_x of the slab, l_x being the shorter span. For large values of l_y, the triangular area is much less than the trapezoidal area (Fig.8.18.4a). Hence, the share of loads on beams along shorter span will gradually reduce with increasing ratio of l_y /l_x. In such cases, it may be said that the loads are primarily taken by beams along longer span. The deflection profiles of the slab along both directions are also shown in the figure. The deflection profile is found to be constant along the longer span except near the edges for the slab panel of Fig.8.18.4a. These slabs are designated as one-way slabs as

they span in one direction (shorter one) only for a large part of the slab when l_y
$/l_x > 2$.

On the other hand, for square slabs of $l_y /l_x = 1$ and rectangular slabs of l_y /l_x up to 2, the deflection profiles in the two directions are parabolic (Fig.8.18.4b). Thus, they are spanning in two directions and these slabs with l_y /l_x up to 2 are designated as two-way slabs, when supported on all edges.

It would be noted that an entirely one-way slab would need lack of support on short edges. Also, even for $l_y /l_x < 2$, absence of supports in two parallel edges will render the slab one-way. In Fig. 8.18.4b, the separating line at 45 degree is tentative serving purpose of design. Actually, this angle is a function of l_y /l_x .

This lesson discusses the analysis and design aspects of one-way slabs.
The two-way slabs are taken up in the next lesson.

Design Shear Strength of Concrete in Slabs

Experimental tests confirmed that the shear strength of solid slabs up to a depth of 300 mm is comparatively more than those of depth greater than 300 mm. Accordingly, cl.40.2.1.1 of IS 456 stipulates the values of a factor *k* to be
multiplied with τ_c given in Table 19 of IS 456 for different overall depths of slab.
Table 8.1 presents the values of *k* as a ready reference below: Table 8.1 Values of

the multiplying factor *k*

Overall depth of slab (mm)	300 or more	275	250	225	200	175	150 or less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

Thin slabs, therefore, have more shear strength than that of thicker slabs. It is the normal practice to choose the depth of the slabs so that the concrete can resist the shear without any stirrups for slab subjected to uniformly distributed loads. However, for deck slabs, culverts, bridges and fly over, shear reinforcement should be provided as the loads are heavily concentrated in those slabs. Though, the selection of depth should be made for normal floor and roof slabs to avoid stirrups, it is essential that the depth is checked for the shear for these slabs taking due consideration of enhanced shear strength as discussed above depending on the overall depth of the slabs.

Structural Analysis

As explained in sec. 8.18.2, one-way slabs subjected to mostly uniformly distributed vertical loads carry them primarily by bending in the shorter direction. Therefore, for the design, it is important to analyse the slab to find out the bending moment (both positive and negative) depending upon the supports. Moreover, the shear forces are also to be computed for such slabs. These internal bending moments and shear forces can be determined using elastic method of analysis considering the slab as beam of unit width i.e. one metre (Fig.8.18.1a). However, these values may also be determined with the help of the coefficients given in Tables 12 and 13 of IS 456 in cl.22.5.1. It is worth mentioning that these coefficients are applicable if the slab is of uniform cross- section and subjected to substantially uniformly distributed loads over three or more spans and the spans do not differ by more than fifteen per cent of the longer span. It is also important to note that the average of the two values of the negative moment at the support should be considered for unequal spans or if the spans are not equally loaded. Further, the redistribution of moments shall not be permitted to the values of moments obtained by employing the coefficients of bending moments as given in IS 456.

For slabs built into a masonry wall developing only partial restraint, the negative moment at the face of the support should be taken as $Wl/24$, where W is the total design loads on unit width and l is the effective span. The shear coefficients, given in Table 13 of IS 456, in such a situation, may be increased by 0.05 at the end support as per cl.22.5.2 of IS 456.

Design Considerations

The primary design considerations of both one and two-way slabs are strength and deflection. The depth of the slab and areas of steel reinforcement are to be determined from these two aspects. The detailed procedure of design of one-way slab is taken up in the next section. However, the following aspects are to be decided first.

(iv) Effective span (cl.22.2 of IS 456)

The effective span of a slab depends on the boundary condition. Table 8.2 gives the guidelines stipulated in cl.22.2 of IS 456 to determine the effective span of a slab.

Table 8.2 Effective span of slab (cl.22.2 of IS 456)

Sl.No.	Support condition	Effective span
1	Simply supported not built integrally with its supports	Lesser of (i) clear span + effective depth of slab, and (ii) centre to centre of supports

2	Continuous when the width of the support is < 1/12th of clear span	Do
3	Continuous when the width of the support is > lesser of 1/12th of clear span or 600 mm (iii) for end span with one end fixed and the other end continuous or for intermediate spans, (iv)for end span with one end free and the other end continuous, (v) spans with roller or rocker bearings.	(iii) Clear span between the supports (iv) Lesser of (a) clear span + half the effective depth of slab, and (b) clear span + half the width of the discontinuous support (v) The distance between the centres of bearings
4	Cantilever slab at the end of a continuous slab	Length up to the centre of support
5	Cantilever span	Length up to the face of the support + half the effective depth
6	Frames	Centre to centre distance

(v) Effective span to effective depth ratio (cls.23.2.1a-e of IS 456)

The deflection of the slab can be kept under control if the ratios of effective span to effective depth of one-way slabs are taken up from the provisions in cl.23.2.1a-e of IS 456. These stipulations are for the beams and are also applicable for one-way slabs as they are designed considering them as beam of unit width. These provisions are explained in sec.3.6.2.2 of Lesson6.

(vi) Nominal cover (cl.26.4 of IS 456)

The nominal cover to be provided depends upon durability and fire resistance requirements. Table 16 and 16A of IS 456 provide the respective values. Appropriate value of the nominal cover is to be provided from these tables for the particular requirement of the structure.

(vii) Minimum reinforcement (cl.26.5.2.1 of IS 456)

Both for one and two-way slabs, the amount of minimum reinforcement in either direction shall not be less than 0.15 and 0.12 per cents of the total cross- sectional area for mild steel (Fe 250) and high strength deformed bars (Fe 415 and Fe 500)/welded wire fabric, respectively.

(viii) Maximum diameter of reinforcing bars (cl.26.5.2.2)

The maximum diameter of reinforcing bars of one and two-way slabs shall not exceed one-eighth of the total depth of the slab.

(ix) Maximum distance between bars (cl.26.3.3 of IS 456)

The maximum horizontal distance between parallel main reinforcing bars shall be the lesser of (i) three times the effective depth, or (ii) 300 mm. However, the same for secondary/distribution bars for temperature, shrinkage etc. shall be the lesser of (i) five times the effective depth, or (ii) 450 mm.

Design of One-way Slabs

The procedure of the design of one-way slab is the same as that of beams. However, the amounts of reinforcing bars are for one metre width of the slab as to be determined from either the governing design moments (positive or negative) or from the requirement of minimum reinforcement. The different steps of the design are explained below.

Step 1: Selection of preliminary depth of slab

The depth of the slab shall be assumed from the span to effective depth ratios as given in section 3.6.2.2 of Lesson 6 and mentioned here in sec.8.18.5b.

Step 2: Design loads, bending moments and shear forces

The total factored (design) loads are to be determined adding the estimated dead load of the slab, load of the floor finish, given or assumed live loads etc. after multiplying each of them with the respective partial safety factors. Thereafter, the design positive and negative bending moments and shear forces are to be determined using the respective coefficients given in Tables 12 and 13 of IS 456 and explained in sec.8.18.4 earlier.

Step 3: Determination/checking of the effective and total depths of slabs

The effective depth of the slab shall be determined employing Eq.3.25 of sec.3.5.6 of Lesson 5 and is given below as a ready reference here,

$$M_{u,lim} = R_{,lim}\, bd^2 \qquad .... \qquad (3.25)$$

where the values of $R_{,lim}$ for three different grades of concrete and three different grades of steel are given in Table 3.3 of Lesson 5 (sec.3.5.6). The value of b shall be taken as one metre.

The total depth of the slab shall then be determined adding appropriate nominal cover (Table 16 and 16A of cl.26.4 of IS 456) and half of the diameter of the larger bar if the bars are of different sizes. Normally, the computed depth of the slab comes out to be much less than the assumed depth in Step 1. However, final selection of the depth shall be done after checking the depth for shear force.

Step 4: Depth of the slab for shear force

Theoretically, the depth of the slab can be checked for shear force if the design shear strength of concrete is known. Since this depends upon the percentage of tensile reinforcement, the design shear strength shall be assumed considering the lowest percentage of steel. The value of τ_c shall be modified after knowing the multiplying factor *k* from the depth tentatively selected for the slab in Step 3. If necessary, the depth of the slab shall be modified.

Step 5: Determination of areas of steel

Area of steel reinforcement along the direction of one-way slab should be determined employing Eq.3.23 of sec.3.5.5 of Lesson 5 and given below as a ready reference.

$$M_u = 0.87 f_y A_{st} d \{1 - (A_{st})(f_y)/(f_{ck})(bd)\} \quad .... \quad (3.23)$$

The above equation is applicable as the slab in most of the cases is under- reinforced due to the selection of depth larger than the computed value in Step 3. The area of steel so determined should be checked whether it is at least the minimum area of steel as mentioned in cl.26.5.2.1 of IS 456 and explained in sec.8.18.5d.

Alternatively, tables and charts of SP-16 may be used to determine the depth of the slab and the corresponding area of steel. Tables 5 to 44 of SP-16 covering a wide range of grades of concrete and Chart 90 shall be used for determining the depth and reinforcement of slabs. Tables of SP-16 take into consideration of maximum diameter of bars not exceeding one-eighth the depth of the slab. Zeros at the top right hand corner of these tables indicate the region where the percentage of reinforcement would exceed $p_{t,lim}$. Similarly, zeros at the lower left and corner indicate the region where the reinforcement is less than the minimum stipulated in the code. Therefore, no separate checking is needed for the allowable maximum diameter of the bars or the computed area of steel exceeding the minimum area of steel while using tables and charts ofSP-16.

The amount of steel reinforcement along the large span shall be the minimum amount of steel as per cl.26.5.2.1 of IS 456 and mentioned in sec.8.18.5d earlier.

Step 6: Selection of diameters and spacings of reinforcing bars (cls.26.5.2.2 and 26.3.3 of IS 456)

The diameter and spacing of bars are to be determined as per cls.26.5.2.2 and 26.3.3 of IS 456. As mentioned in Step 5, this step may be avoided when using the tables and charts of SP-16.

Detailing of Reinforcement

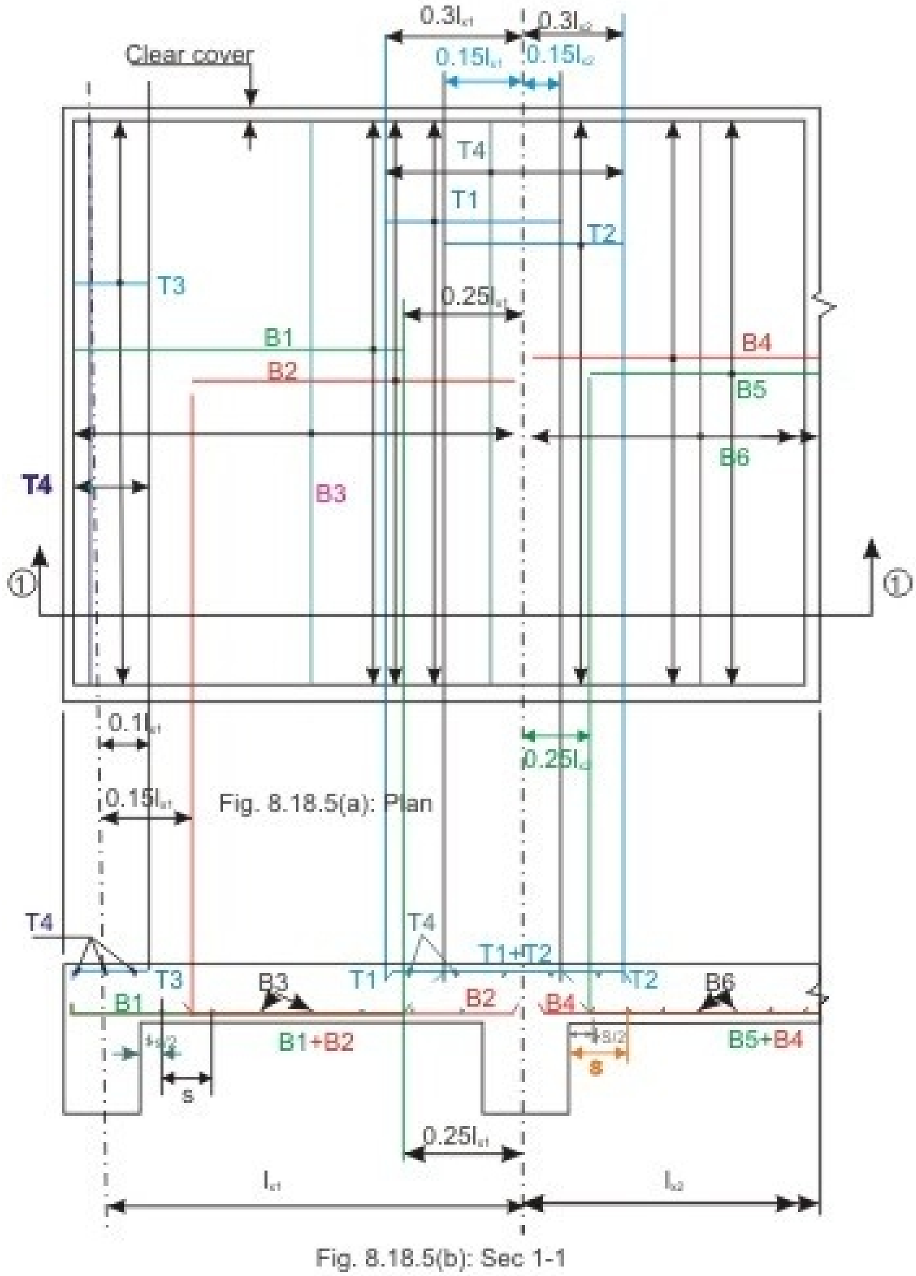

Fig. 8.18.5: Reinforcement of one-way slab

Figures 8.18.5a and b present the plan and section 1-1 of one-way continuous slab showing the different reinforcing bars in the discontinuous and continuous ends (DEP and CEP, respectively) of end panel and continuous end of adjacent panel (CAP). The end panel has three bottom bars B1, B2 and B3 and four top bars T1, T2, T3 and T4. Only three bottom bars B4, B5 and B6 are shown in the adjacent panel. Table 8.3 presents these bars mentioning the respective zone of their placement (DEP/CEP/CAP), direction of the bars (along x or y) and the resisting moment for which they shall be designed or if to be provided on the basis of minimum reinforcement. These bars are explained below for the three types of ends of the two panels.

Table 8.3 Steel bars of one-way slab (Figs.8.18.5a and b)

Sl.No.	Bars	Panel	Along	Resisting moment
1	B1, B2	DEP	x	+ 0.5 M_x for each,
2	B3	DEP	y	Minimum steel
3	B4, B5	CAP	x	+ 0.5 M_x for each,
4	B6	CAP	y	Minimum steel
5	T1, T2	CEP	x	- 0.5 M_x for each,
6	T3	DEP	x	+ 0.5 M_x
7	T4	DEP	y	Minimum steel

Notes: (i) DEP = Discontinuous End Panel
(v) CEP = Continuous End Panel
(vi) CAP = Continuous Adjacent Panel

(d) Discontinuous End Panel (DEP)

- Bottom steel bars B1 and B2 are alternately placed such that B1 bars are curtailed at a distance of 0.25 lx_1 from the adjacent support and B2 bars are started from a distance of 0.15lx_1 from the end support. Thus, both B1 and B2 bars are present in the middle zone covering 0.6lx_1, each of which is designed to resist positive moment 0.5M_x. These bars are along the direction of x and are present from one end to the other end of l_y.

- Bottom steel bars B3 are along the direction of y and cover the entire span lx_1 having the minimum area of steel. The first bar shall be placed at a distance not exceeding $s/2$ from the left discontinuous support, where s is the spacing of these bars in y direction.

Top bars T3 are along the direction of x for resisting the negative moment which is numerically equal to fifty per cent of positive M_x. These bars are continuous up to a distance of $0.1lx_1$ from the centre of support at the discontinuous end.

- Top bars T4 are along the direction of y and provided up to a distance of $0.1lx_1$ from the centre of support at discontinuous end. These are to satisfy the requirement of minimum steel.

(e) Continuous End Panel (CEP)

- Top bars T1 and T2 are along the direction of x and cover the entire l_y. They are designed for the maximum negative moment M_x and each has a capacity of $-0.5M_x$. Top bars T1 are continued up to a distance of $0.3lx_1$, while T2 bars are only up to a distance of $0.15lx_1$.

- Top bars T4 are along y and provided up to a distance of $0.3lx_1$ from the support. They are on the basis of minimum steel requirement.

(f) Continuous Adjacent Panel (CAP)

- Bottom bars B4 and B5 are similar to B1 and B2 bars of (i) above.

- Bottom bars B6 are similar to B3 bars of (i) above.

Detailing is an art and hence structural requirement can be satisfied by more than one mode of detailing each valid and acceptable.

Numerical Problems

(a) Problem 8.1

Design the one-way continuous slab of Fig.8.18.6 subjected to uniformly distributed imposed loads of 5 kN/m^2 using M 20 and Fe 415. The load of floor finish is 1 kN/m^2. The span dimensions shown in the figure are effective spans. The width of beams at the support = 300 mm.

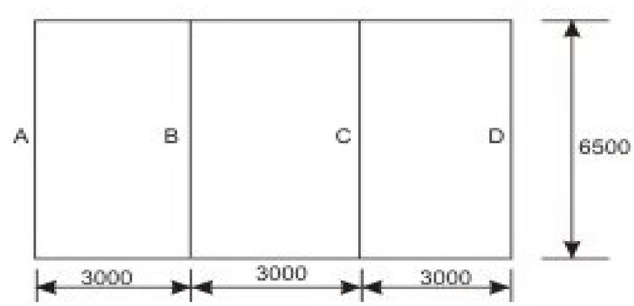

Fig. 8.18.6: Problem 8.1

Solution of Problem 8.1

Step 1: Selection of preliminary depth of slab

The basic value of span to effective depth ratio for the slab having simple support at the end and continuous at the intermediate is (20+26)/2 = 23 (cl.23.2.1 of IS 456).

Modification factor with assumed p = 0.5 and f_s = 240 N/mm^2 is obtained as 1.18 from Fig.4 of IS 456.

Therefore, the minimum effective depth = 3000/23(1.18) = 110.54 mm. Let us take the effective depth d = 115 mm and with 25 mm cover, the total depth D = 140 mm.

Step 2: Design loads, bending moment and shear force

Dead loads of slab of 1 m width = 0.14(25) = 3.5 kN/m

Dead load of floor finish =1.0 kN/m

Factored dead load = 1.5(4.5) = 6.75 kN/m

Factored live load = 1.5(5.0) = 7.50 kN/m

Total factored load = 14.25 kN/m

Maximum moments and shear are determined from the coefficients given in Tables 12 and 13 of IS 456.

Maximum positive moment = 14.25(3)(3)/12 = 10.6875 kNm/m

Maximum negative moment = 14.25(3)(3)/10 = 12.825 kNm/m

Maximum shear V_u = 14.25(3)(0.4) = 17.1 kN

Step 3: Determination of effective and total depths of slab

From Eq.3.25 of sec. 3.5.6 of Lesson 5, we have

$M_{u,lim} = R_{,lim}\, bd^2$ where $R_{,lim}$ is 2.76 N/mm^2 from Table 3.3 of sec. 3.5.6 of Lesson 5. So, d = {12.825(10^6)/(2.76)(1000)}$^{0.5}$ = 68.17 mm

Since, the computed depth is much less than that determined in Step 1, let us keep D = 140 mm and d = 115 mm.

Step 4: Depth of slab for shear force

Table 19 of IS 456 gives τ_c = 0.28 N/mm^2 for the lowest percentage of steel in the slab. Further for the total depth of 140 mm, let us use the coefficient k of cl. 40.2.1.1 of IS 456 as 1.3 to get $\tau_c = k\,\tau_c$ = 1.3(0.28) = 0.364 N/mm^2.

Table 20 of IS 456 gives $\tau_{c\,\max}$ = 2.8 N/mm^2. For this problem $\tau_v = V_u/bd$ = 17.1/115 = 0.148 N/mm^2. Since, $\tau_v < \tau_c < \tau_{c\max}$, the effective depth d = 115 mm is acceptable.

Step 5: Determination of areas of steel

From Eq.3.23 of sec. 3.5.5 of Lesson 5, we have

$$M_u = 0.87\, f_y\, A_{st}\, d\, \{1 - (A_{st})(f_y)/(f_{ck})(bd)\}$$

(d) For the maximum negative bending moment

$$12825000 = 0.87(415)(A_{st})(115)\{1 - (A_{st})(415)/(1000)(115)(20)\}$$

or $A_{st}^2 - 5542.16\, A_{st} + 1711871.646 = 0$

Solving the quadratic equation, we have the negative A_{st} = 328.34 mm^2

(e) For the maximum positive bending moment

$$10687500 = 0.87(415)\, A_{st}(115)\ \{1 - (A_{st})(415)/(1000)(115)(20)\}$$

or $A_{st}^2 - 5542.16\, A_{st} + 1426559.705 = 0$

Solving the quadratic equation, we have the positive A_{st} = 270.615 mm^2

Alternative approach: Use of Table 2 of SP-16

(iv) For negative bending moment

M_u/bd^2 = 0.9697

Table 2 of SP-16 gives: p_s = 0.2859 (by linear interpolation). So, the area of negative steel = 0.2859(1000)(115)/100 = 328.785 mm^2.

(v) For positive bending moment

$$M_u/bd^2 = 0.8081$$

Table 2 of SP-16 gives: p_s = 0.23543 (by linear interpolation). So, the area of positive steel = 0.23543(1000)(115)/100 = 270.7445 mm^2.

These areas of steel are comparable with those obtained by direct computation using Eq.3.23.

Distribution steel bars along longer span l_y

Distribution steel area = Minimum steel area = 0.12(1000)(140)/100 = 168 mm^2. Since, both positive and negative areas of steel are higher than the minimum area, we provide:

- For negative steel: 10 mm diameter bars @ 230 mm c/c for which A_{st} = 341 mm^2 giving p_s = 0.2965.

- For positive steel: 8 mm diameter bars @ 180 mm c/c for which A_{st} = 279 mm^2 giving p_s = 0.2426
- For distribution steel: Provide 8 mm diameter bars @ 250 mm c/c for which A_{st} (minimum) = 201 mm^2.

Step 6: Selection of diameter and spacing of reinforcing bars

The diameter and spacing already selected in step 5 for main and distribution bars are checked below:

For main bars (cl. 26.3.3.b.1 of IS 456), the maximum spacing is the lesser of 3*d* and 300 mm i.e., 300 mm. For distribution bars (cl. 26.3.3.b.2 of IS 456), the maximum spacing is the lesser of 5*d* or 450 mm i.e., 450 mm. Provided spacings, therefore, satisfy the requirements.

Maximum diameter of the bars (cl. 26.5.2.2 of IS 456) shall not exceed 140/8 = 17 mm is also satisfied with the bar diameters selected here.

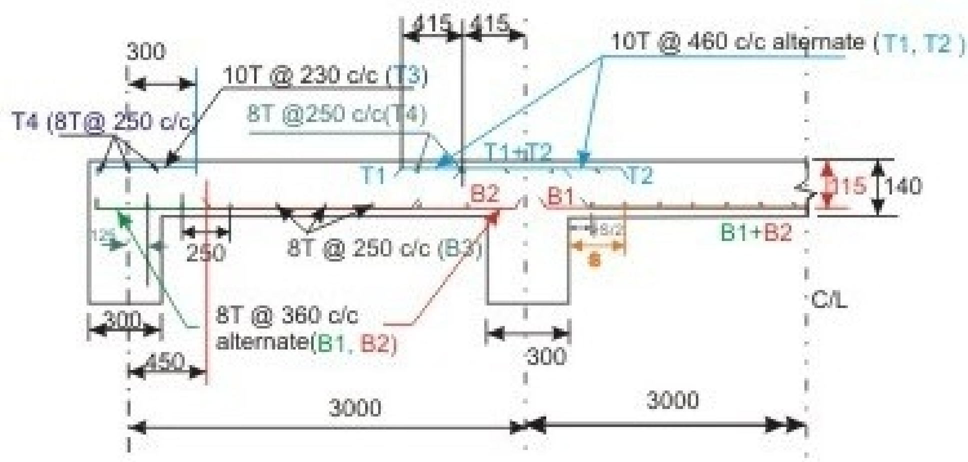

Fig. 8.18.7: Problem 8.1

Figure 8.18.7 presents the detailing of the reinforcement bars. The abbreviation B1 to B3 and T1 to T4 are the bottom and top bars, respectively which are shown in Fig.8.18.5 for a typical one-way slab.

The above design and detailing assume absence of support along short edges. When supports along short edges exist and there is eventual clamping top reinforcement would be necessary at shorter supports also.

Practice Questions and Problems with Answers

Q.1: State the names of different types of slabs used in construction.

A.1: See sec. 8.18.1.

Q.2: (a) State the limit of the aspect ratio of l_y/l_x of one- and two-way slabs.
(b) Explain the share of loads by the supporting beams in one- and two- way slabs.

A.2: (a) The aspect ratio l_y/l_x (l_x is the shorter one) is from 1 to 2 for two-way slabs and beyond 2 for one-way slabs.

(b) See sec. 8.18.2.

Q.3: How to determine the design shear strength of concrete in slabs of different depths having the same percentage of reinforcement?

A.3: See sec. 8.18.3.

Q.4: State span to depth ratios of one-way slabs for different support conditions to be considered for the control of deflection.

A.4: See sec. 8.18.5b.

Q.5: State the minimum amounts of reinforcing bars to be provided in slabs?

A.5: See sec. 8.18.5d.

Q.6: State the maximum diameter of a bar to be used in slabs.

A.6: See sec. 8.18.5e.

Q.7: How do we determine the effective depth of a slab for a given factored moment?

A.7: See sec. 8.18.6, Step 3, Eq.3.25.

Q.8: How do we determine the area of steel to be provided for a given factored moment?

A.8: See sec. 8.18.6, Step 5, Eq.3.23.

Q.9: How do we determine the amount of steel in the longer span direction?

A.9: Minimum amount of steel shall be provided for temperature, shrinkage etc. as per cl. 26.5.2.1 of IS 456. These are known as distribution bars.

Q.10: Design the cantilever panel of the one-way slab shown in Fig.8.18.8 subjected to uniformly distributed imposed loads 5 kN/m^2 using M 20 and Fe 415. The load of floor finish is 0.75 kN/m^2. The span dimensions shown in the figure are effective spans. The width of the support is 300 mm.

A.10:

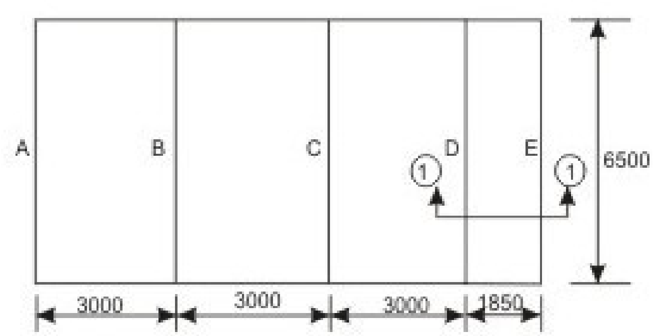

Fig. 8.18.8: Problem Q-10

Step 1: Selection of preliminary depth of slab

Basic value of span to depth ratio (cl. 23.2.1 of IS 456) = 7 Modification factor = 1.18 (see Problem 8.1)

Minimum effective depth = 1850/7(1.18) = 223.97 mm Assume d = 225 mm and D = 250 mm.

Step 2: Design loads, bending moment and shear force Factored dead loads = (1.5)(0.25)(25) = 9.375 kN/m Factored load of floor finish = (1.5)(0.75) = 1.125 kN/m Factored live loads = (1.5)(5) = 7.5 kN/m

Total factored loads = 18.0 kN/m

Maximum negative moment = 18.0(1.85)(1.85)(0.5) = 30.8025 kNm/m Maximum shear force = 18.0(1.85) = 33.3 kN/m

Step 3: Determination of effective and total depths of slab

From Eq.3.25, Step 3 of sec. 8.18.6, we have

d = $\{30.8025(10^6)/2.76(10^3)\}^{0.5}$ = 105.64 mm, considering the value of R = 2.76 N/mm^2 from Table 3.3 of sec. 3.5.5 of Lesson 5. This depth is less than assumed depth of slab in Step 1. Hence, assume d = 225 mm and D = 250 mm.

Step 4: Depth of slab for shear force

Using the value of k = 1.1 (cl. 40.2.1.1 of IS 456) for the slab of 250 mm depth, we have τ_c (from Table 19 of IS 456) = 1.1(0.28) = 0.308 N/mm^2. Table 20 of IS 456 gives $\tau_{c\max}$ = 2.8 N/mm^2. Here, $\tau_v = V_u / bd$ = 33.3/225 = 0.148 N/mm^2.

The depth of the slab is safe in shear as $\tau_v < \tau_c < \tau_{c\max}$.

Step 5: Determination of areas of steel (using table of SP-16)

Table 44 gives 10 mm diameter bars @ 200 mm c/c can resist 31.43 kNm/m > 30.8025 kNm/m. Fifty per cent of the bars should be curtailed at a

distance of larger of L_d or $0.5l_x$. Table 65 of SP-16 gives L_d of 10 mm bars =
470 mm and $0.5l_x$ = 0.5(1850) = 925 mm from the face of the column. The curtailment distance from the centre line of beam = 925 + 150 = 1075, say 1100 mm.

The above, however, is not admissible as the spacing of bars after the curtailment exceeds 300 mm. So, we provide 10 mm @ 300 c/c and 8 mm @ 300 c/c. The moment of resistance of this set is 34.3 kNm/m > 30.8025 kNm/m (see Table 44 of SP-16).

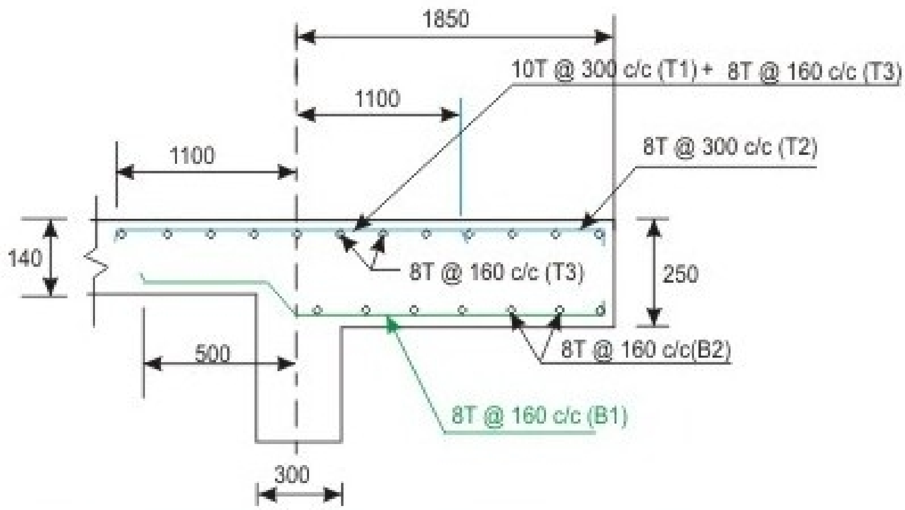

Fig. 8.18.9: Detailing of bars of Problem Q-10 (sec. 1-1 of Fig. 8.18.8)

Figure 8.18.9 presents the detailing of reinforcing bars of this problem.

Solutions

TQ.1: (a) State the limit of the aspect ratio of l_y/l_x of one- and two-way slabs.
(b) Explain the share of loads by the supporting beams in one- and two- way slabs.
(10 marks)

A.TQ.1: (a) The aspect ratio l_y/l_x (l_x is the shorter one) is from 1 to 2 for two-way slabs and beyond 2 for one-way slabs.

(b) See sec. 8.18.2.

TQ.2: How to determine the design shear strength of concrete in slabs of different depths having the same percentage of reinforcement? (10 marks)

A.TQ.2: See sec. 8.18.3.

TQ3: Determine the areas of steel, bar diameters and spacings in the two directions of a simply supported slab of effective spans 3.5 m x 8 m (Figs.8.18.10a and b) subjected to live loads of 4 kN/m^2 and the load of floor finish is 1 kN/m^2. Use M 20 and Fe 415. Draw the diagram showing the detailing of reinforcement. (30 marks)

A.TQ.3:

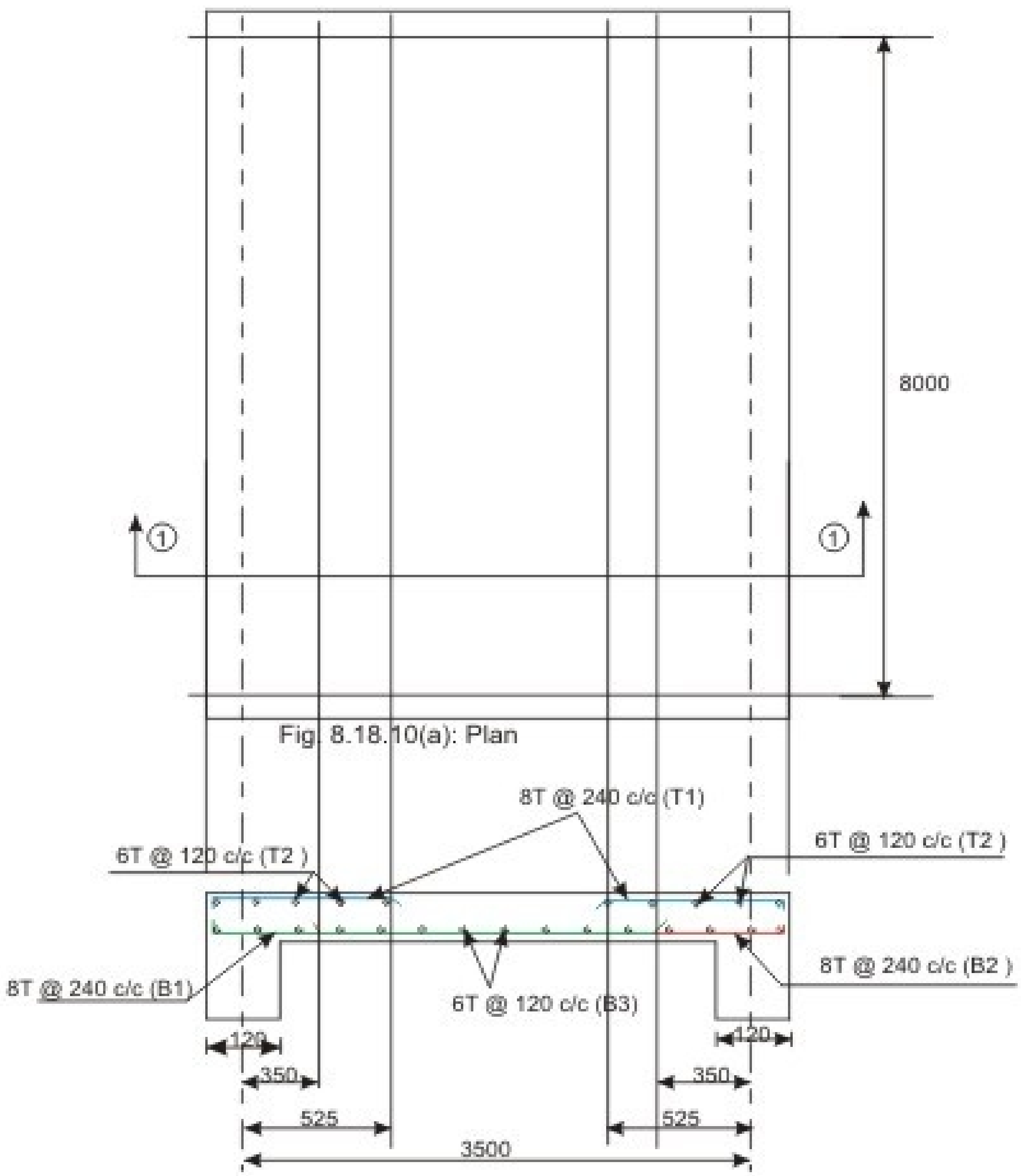

Fig. 8.18.10(b): Section 1 1

One-way slab

Fig. 8.18.10: Detailing of bars of Problem TQ. 1

This is one-way slab as $l_y/l_x = 8/3.5 = 2.285 > 2$. The calculations are shown in different steps below:

Step 1: Selection of preliminary depth of slab

Clause 23.2.1 stipulates the basic value of span to effective depth ratio of 20. Using the modification factor of 1.18 from Fig.4 of IS 456, with p = 0.5 per cent and f_s = 240 N/mm^2, we have the span to effective depth ratio = 20(1.18) = 23.6.

So, the minimum effective depth of slab = 3500/23.6 = 148.305 mm. Let us take d = 150 mm and D = 175 mm.

Step 2: Design loads, bending moment and shear force

Factored dead loads of slab = (1.5)(0.175)(25) = 6.5625 kN/m Factored load of floor finish =

(1.5)(1) = 1.5 kN/m

Factored live load = (1.5)(4) = 6.0 kN/m Total factored

load = 14.0625 kN/m

Maximum positive bending moment = 14.0625(3.5)(3.5)/8 = 21.533 kNm/m

Maximum shear force = 14.0625(3.5)(0.5) = 24.61 kN/m

Step 3: Determination/checking of the effective and total depths of slab

Using Eq.3.25 as explained in Step 3 of sec. 8.18.6, we have

$d = \{21.533(10^6)/(2.76)(10^3)\}^{0.5} = 88.33$ mm < 150 mm, as assumed in Step 1. So, let us keep d = 150 mm and D = 175 mm.

Step 4: Depth of the slab for shear force

With the multiplying factor k = 1.25 for the depth as 175 mm (vide Table 8.1 of this lesson) and τ_c = 0.28 N/mm^2 from Table 19 of IS 456, we have τ_c = 1.25(0.28) = 0.35 N/mm^2.

Table 20 of IS 456 gives $\tau_{c\,max}$ = 2.8 N/mm^2. For this problem:

$\tau_v = 24.61(1000)/(1000)(150) = 0.1641$ N/mm^2.

Thus, the effective depth of slab as 150 mm is safe as $\tau_v < \tau_c < \tau_{c\,max}$.

Step 5: Determination of areas of steel

Table 41 of SP-16 gives 8 mm diameter bars @ 120 mm c/c have 22.26 kNm/m > 21.533 kNm/m. Hence, provide 8 mm T @ 120 mm c/c as main positive steel bars along the short span of 3.5 mm.

The minimum amount of reinforcement (cl. 26.5.2.1 of IS 456) = 0.12(175)(1000)/100 = 210 mm^2. Provide 6 mm diameter bars @ 120 mm c/c (236 mm^2) along the large span of 8m.

Figure 8.18.10b shows the detailing of reinforcing bars.

Introduction

Lesson 18 explains the various types of slabs with different support conditions, plan forms, horizontal/inclined etc. Moreover, sec. 8.18.2 of Lesson 18 illustrates the sharing of uniformly distributed loads to the supporting beams of both one and two-way slabs including the profiles of deflection (Figs.8.18.4a and b). It is, thus, understood that two-way slabs span in both directions having the aspect ratio of l_y/l_x up to 2, considering l_x as the shorter span. This lesson presents the different aspects of analysis and design of two-way slabs. Many of the stipulations of IS 456 are the same as those of one-way slabs. While mentioning the common stipulations with their respective section in Lesson 18, this lesson presents other relevant requirements regarding the analysis, design and detailing of two-way slabs. Numerical problems are also solved to illustrate the applications of the theory in the design of two-way slabs.

Two-way Slabs

Two-way slabs subjected mostly to uniformly distributed loads resist them primarily by bending about both the axis. However, as in the one-way slab, the depth of the two-way slabs should also be checked for the shear stresses to avoid any reinforcement for shear. Moreover, these slabs should have sufficient depth for the control deflection. Thus, strength and deflection are the requirements of design of two-way slabs.

Design Shear Strength of Concrete

Design shear strength of concrete in two-way slabs is to be determined incorporating the multiplying factor *k* from Table 8.1 of Lesson 18 in the same manner as discussed in sec. 8.18.3 of Lesson 18.

Structural Analysis

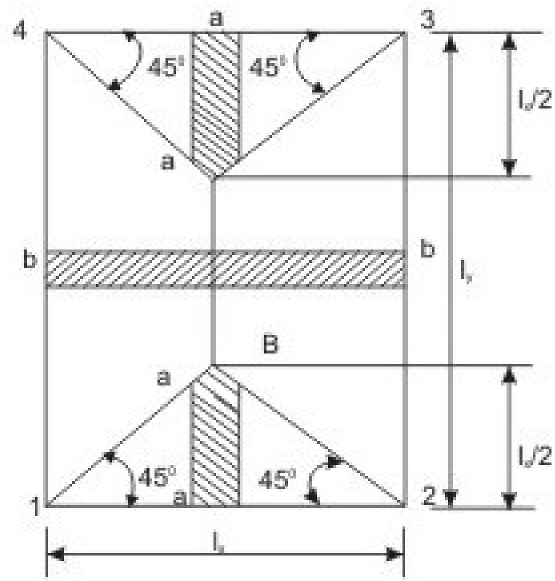

Fig. 8.19.1: Strips for shear

Computation of shear force

Shear forces are computed following the procedure stated below with reference to Fig.8.19.1.

The two-way slab of Fig. 8.19.1 is divided into two trapezoidal and two triangular zones by drawing lines from each corner at an angle of 45°. The loads of triangular segment A will be transferred to beam 1-2 and the same of trapezoidal segment B will be beam 2-3. The shear forces per unit width of the strips aa and bb are highest at the ends of strips. Moreover, the length of half the strip bb is equal to the length of the strip aa. Thus, the shear forces in both strips are equal and we can write,

$$V_u = W\,(l_x/2) \tag{8.1}$$

where W = intensity of the uniformly distributed loads.

The nominal shear stress acting on the slab is then determined from

$$\tau_v = V_u / bd \tag{8.2}$$

Computation of bending moments

Two-way slabs spanning in two directions at right angles and carrying uniformly distributed loads may be analysed using any acceptable theory. Pigeoud's or Wester-guard's theories are the suggested elastic methods and Johansen's yield line theory is the most commonly used in the limit state of collapse method and suggested by IS 456 in the note of cl. 24.4. Alternatively, Annex D of IS 456 can be employed to determine the bending moments in the two directions for two types of slabs: (i) restrained slabs, and (ii) simply supported slabs. The two methods are explained below:

(i) Restrained slabs

Restrained slabs are those whose corners are prevented from lifting due to effects of torsional moments. These torsional moments, however, are not computed as the amounts of reinforcement are determined from the computed areas of steel due to positive bending moments depending upon the intensity of torsional moments of different corners. This aspect has been explained in Step 7 of sec. 8.19.6. Thus, it is essential to determine the positive and negative bending moments in the two directions of restrained slabs depending on the various types of panels and the aspect ratio l_y/l_x.

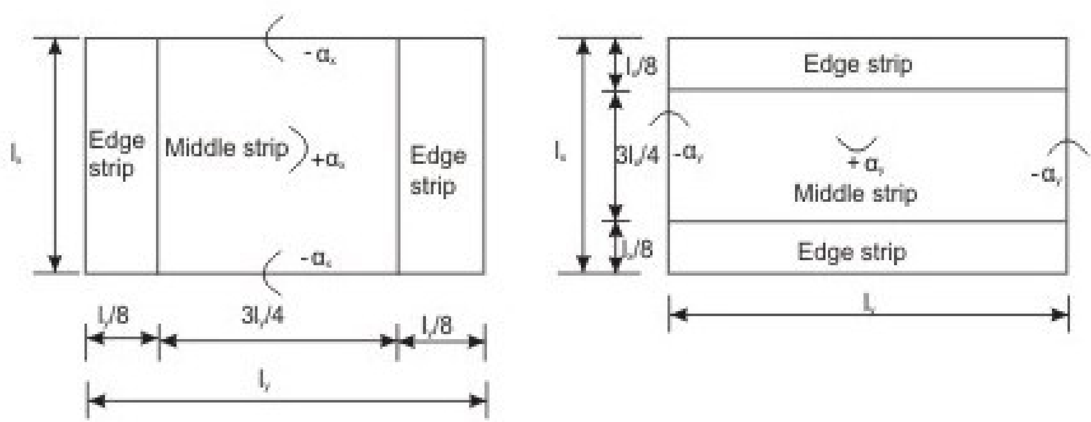

Fig. 8.19.2(a): For span l_x **Fig. 8.19.2(b)**: For span l_y

Fig. 8.19.2: Middle and edge strips

Restrained slabs are considered as divided into two types of strips in each direction: (i) one middle strip of width equal to three-quarters of the respective length of span in either directions, and (ii) two edge strips, each of width equal to one-eighth of the respective length of span in either directions. Figures 8.19.2a and b present the two types of strips for spans l_x and l_y separately.

The maximum positive and negative moments per unit width in a slab are determined from

$$M_x = \alpha_x w l_x^2 \tag{8.3}$$

$$M_y = \alpha_x w l_y^2 \tag{8.4}$$

where α_x and α_y are coefficients given in Table 26 of IS 456, Annex D, cl. D-

1.1. Total design load per unit area is *w* and lengths of shorter and longer spans are represented by l_x and l_y, respectively. The values of α_x and α_y, given in Table 26 of IS 456, are for nine types of panels having eight aspect ratios of l_y/l_x from one to two at an interval of 0.1. The above maximum bending moments are applicable only to the middle strips and no redistribution shall be made.

Tension reinforcing bars for the positive and negative maximum moments are to be provided in the respective middle strips in each direction. Figure 8.19.2 shows the positive and negative coefficients α_x and α_y.

The edge strips will have reinforcing bars parallel to that edge following the minimum amount as stipulated in IS 456.

The detailing of all the reinforcing bars for the respective moments and for the minimum amounts as well as torsional requirements are discussed in sec. 8.19.7(i).

(ii) Simply supported slabs

The maximum moments per unit width of simply supported slabs, not having adequate provision to resist torsion at corners and to prevent the corners from lifting, are determined from Eqs.8.3 and 8.4, where α_x and α_y are the respective coefficients of moments as given in Table 27 of IS 456, cl. D-2. The notations M_x, M_y, w, l_x and l_y are the same as mentioned below Eqs.8.3 and 8.4 in (i) above.

The detailing of reinforcing bars for the respective moments is explained in sec. 8.19.7(ii).

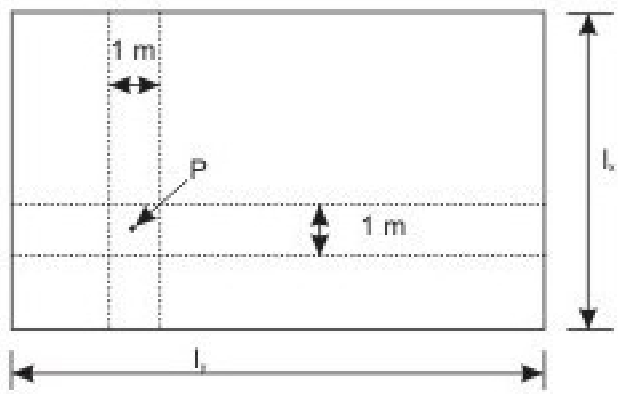

Fig. 8.19.3: Interconnected two strips containing P

The coefficients α_x and α_y of simply supported two-way slabs are derived from the Grashoff-Rankine formula which is based on the consideration of the same deflection at any point P (Fig.8.19.3) of two perpendicular interconnected strips containing the common point P of the two-way slab subjected to uniformly distributed loads.

Design Considerations

The design considerations mentioned in sec. 8.18.5 of Lesson 18 in (a), (c), (d), (e) and (f) are applicable for the two-way slabs also. However, the effective span to effective depth ratio is different from those of one-way slabs. Accordingly, this item for the two-way slabs is explained below.

Effective span to effective depth ratio (cl. 24.1 of IS 456)

The following are the relevant provisions given in Notes 1 and 2 of cl. 24.1.

- The shorter of the two spans should be used to determine the span to effective depth ratio.
- For spans up to 3.5 m and with mild steel reinforcement, the span to overall depth ratios satisfying the limits of vertical deflection for loads up to 3 kN/m^2 are as follows:

Simply supported slabs	35
Continuous slabs	40

- The same ratios should be multiplied by 0.8 when high strength deformed bars (Fe 415) are used in the slabs.

Design of Two-way Slabs

The procedure of the design of two-way slabs will have all the six steps mentioned in sec. 8.18.6 for the design of one-way slabs except that the bending moments and shear forces are determined by different methods for the two types of slab.

While the bending moments and shear forces are computed from the coefficients given in Tables 12 and 13 (cl. 22.5) of IS 456 for the one-way slabs, the same are obtained from Tables 26 or 27 for the bending moment in the two types of two-way slabs and the shear forces are computed from Eq.8.1 for the two-way slabs.

Further, the restrained two-way slabs need adequate torsional reinforcing bars at the corners to prevent them from lifting. There are three types of corners having three different requirements. Accordingly, the determination of torsional reinforcement is discussed in Step 7, as all the other six steps are common for the one and two-way slabs.

Step 7: Determination of torsional reinforcement

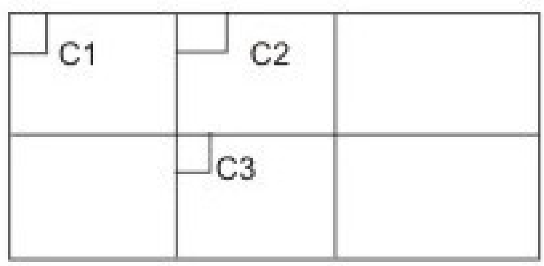

Fig. 8.19.4: Three types of corners

Three types of corners, C1, C2 and C3, shown in Fig.8.19.4, have three different requirements of torsion steel as mentioned below.

(a) At corner C1 where the slab is discontinuous on both sides, the torsion reinforcement shall consist of top and bottom bars each with layers of bar placed parallel to the sides of the slab and extending a minimum distance of one- fifth of the shorter span from the edges. The amount of reinforcement in each of the four layers shall be 75 per cent of the area required for the maximum mid- span moment in the slab. This provision is given in cl. D-1.8 of IS 456.

(b) At corner C2 contained by edges over one of which is continuous, the torsional reinforcement shall be half of the amount of (a) above. This provision is given in cl. D-1.9 of IS 456.

(c) At corner C3 contained by edges over both of which the slab is continuous, torsional reinforcing bars need not be provided, as stipulated in cl. D-1.10 of IS 456.

Detailing of Reinforcement

As mentioned in sec. 8.19.6, Step 5 of sec. 8.18.6 explains the two methods of determining the required areas of steel required for the maximum positive and negative moments. The two methods are (i) employing Eq.3.23 as given in Step 5 of sec. 8.18.6 or (ii) using tables and charts of SP-16. Thereafter, Step 7 of sec. 8.19.6 explains the method of determining the areas steel for corners of restrained slab depending on the type of corner. The detailing of torsional reinforcing bars is explained in Step 7 of sec. 8.19.6. In the following, the detailings of reinforcing bars for (i) restrained slabs and (ii) simply supported slabs are discussed separately for the bars either for the maximum positive or negative bending moments or to satisfy the requirement of minimum amount of steel.

(i) Restrained slabs

The maximum positive and negative moments per unit width of the slab calculated by employing Eqs.8.3 and 8.4 as explained in sec. 8.19.4.2(i) are applicable only to the respective middle strips (Fig.8.19.2). There shall be no redistribution of these moments. The reinforcing bars so calculated from the maximum moments are to be placed satisfying the following stipulations of IS 456.

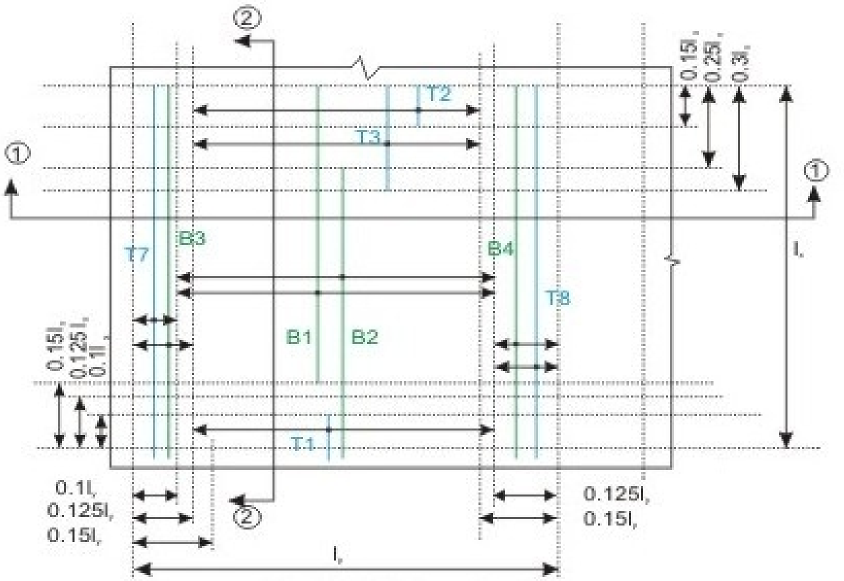

Fig. 8.19.5(a): Bars along l_x only

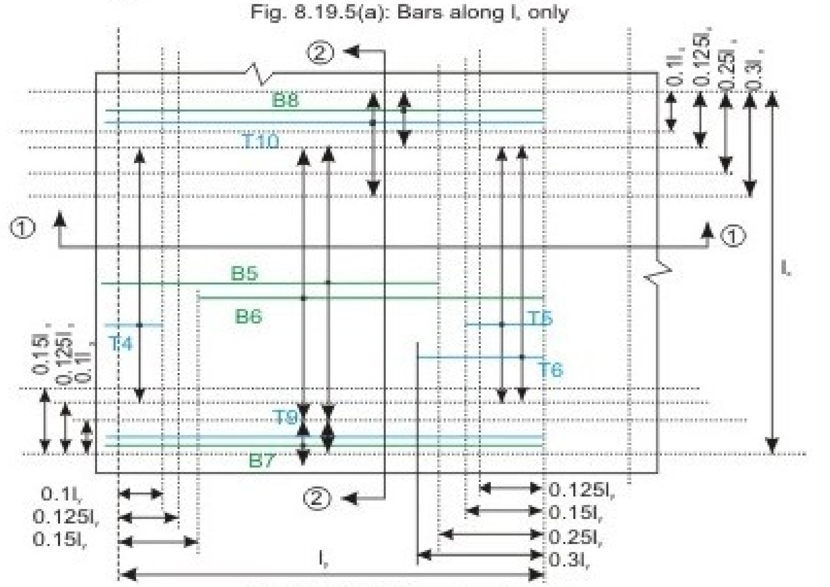

Fig. 8.19.5(b): Bars along l_y only

Fig. 8.19.5: Reinforcement of two-way slab, $l_x < l_y$ (except torsion reinforcement)

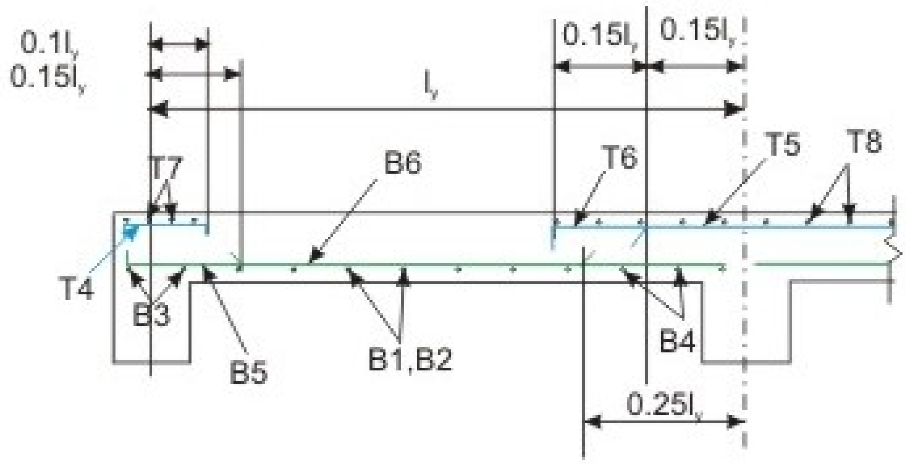

Fig. 8.19.5(c): Sec 1 1

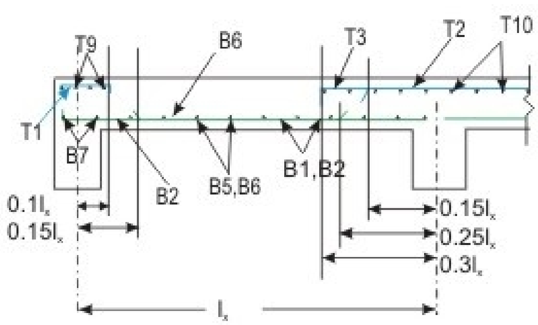

Fig 8.19.5(d): Sec 2 2

Fig 8.19.5:Reinforcement of Two-way slab, $l_x < l_y$ (except torsional reinforcement)

- Bottom tension reinforcement bars of mid-span in the middle strip shall extent in the lower part of the slab to within 0.25*l* of a continuous edge, or 0.15*l* of a discontinuous edge (cl. D-1.4 of IS 456). Bars marked as B1, B2, B5 and B6 in Figs.8.19.5 a and b are these bars.

- Top tension reinforcement bars over the continuous edges of middle strip shall extend in the upper part of the slab for a distance of 0.15*l* from the support, and at least fifty per cent of these bars shall extend a distance of 0.3*l* (cl. D-1.5 of IS 456). Bars marked as T2, T3, T5 and T6 in Figs.8.19.5 a and b are these bars.

- To resist the negative moment at a discontinuous edge depending on the degree of fixity at the edge of the slab, top tension reinforcement bars equal to fifty per cent of that provided at mid-span shall extend 0.1*l* into the span (cl. D-1.6 of IS 456). Bars marked as T1 and T4 in Figs.8.19.5 a and b are these bars.

- The edge strip of each panel shall have reinforcing bars parallel to that edge satisfying the requirement of minimum amount as specified in sec. 8.18.15d of Lesson 18 (cl. 26.5.2.1 of IS 456) and the requirements for torsion, explained in Step 7 of sec. 8.19.6 (cls. D-1.7 to D-1.10 of IS 456). The bottom and top bars of the edge strips are explained below.

- Bottom bars B3 and B4 (Fig.8.19.5 a) are parallel to the edge along l_x for the edge strip for span l_y, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).

- Bottom bars B7 and B8 (Fig.8.19.5 b) are parallel to the edge along l_y for the edge strip for span l_x, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).

- Top bars T7 and T8 (Fig.8.19.5 a) are parallel to the edge along l_x for the edge strip for span l_y, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).

- Top bars T9 and T10 (Fig.8.19.5 b) are parallel to the edge along l_y for the edge strip for span l_x, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).

The detailing of torsion bars at corners C1 and C2 is explained in Fig.8.19.7 of Problem 8.2 in sec. 8.19.8.

The above explanation reveals that there are eighteen bars altogether comprising eight bottom bars (B1 to B8) and ten top bars (T1 to T10). Tables 8.4 and 8.5 present them separately for the bottom and top bars, respectively, mentioning the respective zone of their placement (MS/LDES/ACES/BDES to designate Middle Strip/Left Discontinuous Edge Strip/Adjacent Continuous Edge Strip/Bottom Discontinuous Edge Strip), direction of the bars (along *x* or *y*), the resisting moment for which they shall be determined or if to be provided on the basis of minimum reinforcement clause number of IS 456 and Fig. No. For easy understanding, plan views in (a) and (b) of Fig.8.19.5 show all the bars separately along *x* and *y* directions, respectively. Two sections (1-1 and 2-2), however, present the bars shown in the two plans. Torsional reinforcements are not included in Tables 8.4 and 8.5 and Figs.8.19.5 a and b.

Table 8.4 Details of eight bottom bars

Sl.No.	Bars	Into	Along	Resisting Moment	Cl.No. of IS 456	Fig.No.
1	B1, B2	MS	x	Max. + M_x	D-1.3,1.4	8.19.5a, c, d
2	B3	LDES	x	Min. Steel	D-1.7	8.19.5 a, c

3	B4	ACES	x	Min. Steel	D-1.7	8.19.5a, c
4	B5, B6	MS	y	Max. + M_y	D-1.3,1.4	8.19.5b, c, d
5	B7	BDES	y	Min. Steel	D-1.7	8.19.5b, d
6	B8	ACES	y	Min . Ste el	D-1.7	8.19.5b, d

Notes: (i) MS = Middle Strip
(ii) LDES = Left Discontinuous Edge Strip
(iii) ACES = Adjacent Continuous Edge Strip
(iv) BDES = Bottom Discontinuous Edge Strip

Table 8.5 Details of eight top bars

Sl.No.	Bars	Into	Along	Resisting Moment	Cl.No. of IS 456	Fig. No.
1	T1	BDES	x	+ 0.5 M_x	D-1.6	8.19.5 a, d
2	T2, T3	ACES	x	- 0.5 M_x for each	D-1.5	8.19.5 a, d
3	T4	LDES	y	+ 0.5 M_y	D-1.6	8.19.5 b, c
4	T5, T6	ACES	y	-0.5 M_y for	D-1.5	8.19.5 b, c

				each		
5	T7	LDES	*x*	Min. Steel	D-1.7	8.19.5 a, c
6	T8	ACES	*x*	Min. Steel	D-1.7	8.19.5 a, c
7	T9	LDES	*y*	Min. Steel	D-1.7	8.19.5 b, d
8	T10	ACES	*y*	Min. Steel	D-1.7	8.19.5 b, d

Notes: (i) MS = Middle Strip
(ii) LDES = Left Discontinuous Edge Strip
(iii) ACES = Adjacent Continuous Edge Strip
(iv) BDES = Bottom Discontinuous Edge Strip

(ii) Simply supported slabs

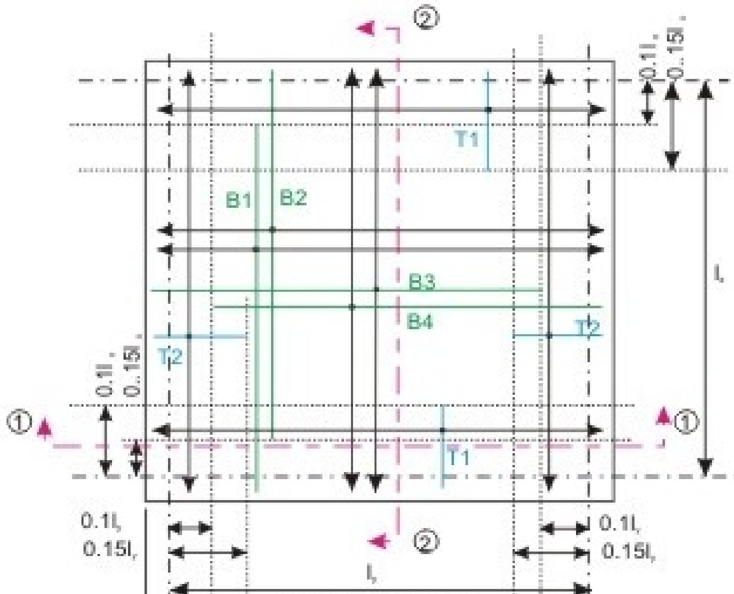

Fig. 8.19.6(a): Plan

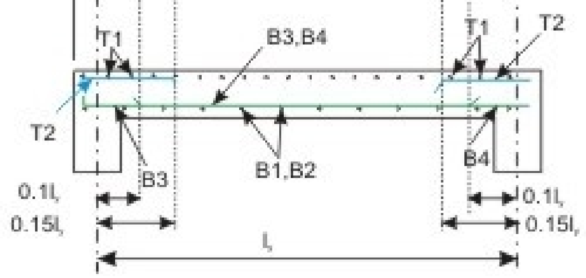

Fig. 8.19.6(b): Section 1-1

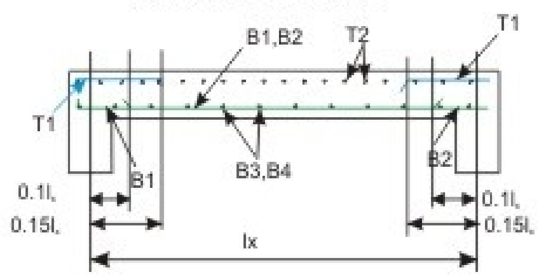

Fig. 8.19.6(c): Section 2-2

Fig. 8.19.6: Simply supported two-way slab, corners not held down

Figures 8.19.6 a, b and c present the detailing of reinforcing bars of simply supported slabs not having adequate provision to resist torsion at corners and to prevent corners from lifting. Clause D-2.1 stipulates that fifty per cent of the tension reinforcement provided at mid-span should extend to the supports. The remaining fifty per cent should extend to within $0.1l_x$ or $0.1l_y$ of the support, as appropriate.

Numerical Problems

Problem 8.2

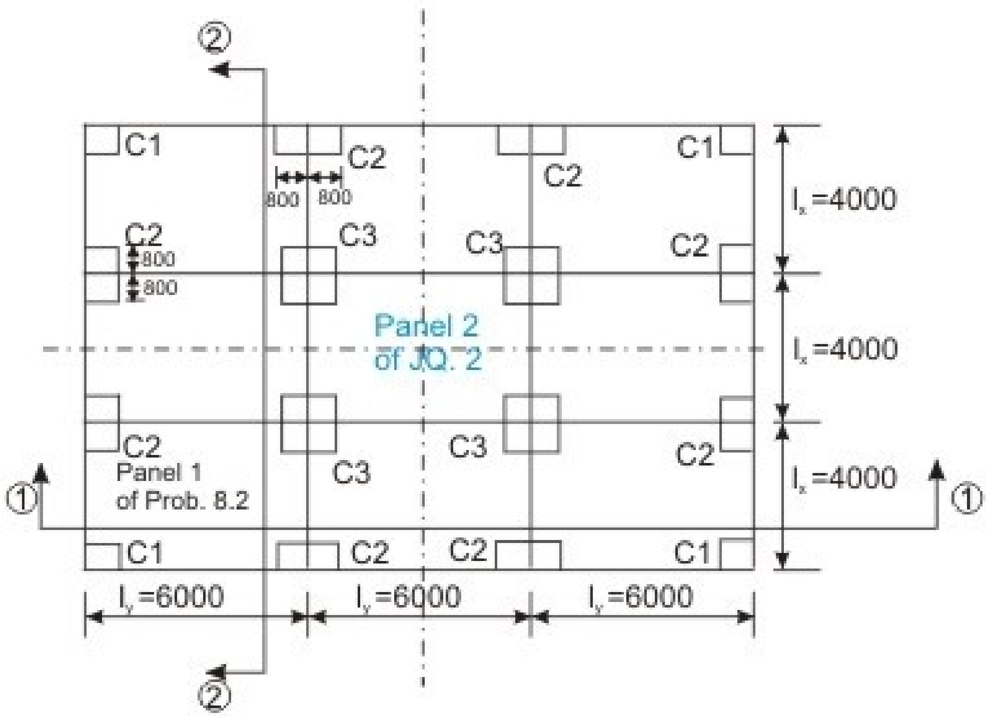

Fig. 8.19.7: Problem 8.2 (panel 1) and TQ 2 (panel 2)

Design the slab panel 1 of Fig.8.19.7 subjected to factored live load of 8 kN/m^2 in addition to its dead load using M 20 and Fe 415. The load of floor finish is 1 kN/m^2. The spans shown in figure are effective spans. The corners of the slab are prevented from lifting.

Solution of Problem 8.2

Step 1: Selection of preliminary depth of slab

The span to depth ratio with Fe 415 is taken from cl. 24.1, Note 2 of IS 456 as 0.8 (35 + 40) / 2 = 30. This gives the minimum effective depth d = 4000/30 = 133.33 mm, say 135 mm. The total depth D is thus 160 mm.

Step 2: Design loads, bending moments and shear forces

Dead load of slab (1 m width) = 0.16(25) = 4.0 kN/m^2

Dead load of floor finish (given) = 1.0 kN/m^2 Factored dead load

= 1.5(5) = 7.5 kN/m^2 Factored live load (given) = 8.0 kN/m^2

Total factored load = 15.5 kN/m^2

The coefficients of bending moments and the bending moments M_x and M_y per unit width (positive and negative) are determined as per cl. D-1.1 and Table 26 of IS 456 for the case 4, "Two adjacent edges discontinuous" and presented in Table 8.6. The l_y / l_x for this problem is 6/4 = 1.5.

Table 8.6 Maximum bending moments of Problem 8.2

For	Short span		Long span	
	α_x	M_x (kNm/m)	α_y	M_y (kNm/m)
Negative moment at continuous edge	0.075	18.6	0.047	11.66
Positive moment at mid-span	0.056	13.89	0.035	8.68

Maximum shear force in either direction is determined from Eq.8.1 (Fig.8.19.1) as

$V_u = w(l_x/2) = 15.5\ (4/2) = 31$ kN/m

Step 3: Determination/checking of the effective depth and total depth of slab

Using the higher value of the maximum bending moments in x and y directions from Table 8.6, we get from Eq.3.25 of Lesson 5 (sec. 3.5.5):

$$M_{u,lim} = R_{,lim}\, bd^2$$

or $\quad d = [(18.6)(10^6)/\{2.76(10^3)\}]^{1/2} = 82.09$ mm,

where 2.76 N/mm^2 is the value of $R_{,lim}$ taken from Table 3.3 of Lesson 5 (sec. 3.5.5). Since, this effective depth is less than 135 mm assumed in Step 1, we retain d = 135 mm and D = 160 mm.

Step 4: Depth of slab for shear force

Table 19 of IS 456 gives the value of τ_c = 0.28 N/mm^2 when the lowest percentage of steel is provided in the slab. However, this value needs to be modified by multiplying with *k* of cl. 40.2.1.1 of IS 456. The value of *k* for the total depth of slab as 160 mm is 1.28. So, the value of τ_c is 1.28(0.28) = 0.3584 N/mm^2.

Table 20 of IS 456 gives $\tau_{c\,max}$ = 2.8 N/mm^2. The computed shear stress

$\tau_v = V_u/bd$ = 31/135 = 0.229 N/mm^2.

Since, $\tau_v < \tau_c < \tau_{cmax}$, the effective depth of the slab as 135 mm and the total depth as 160 mm are safe.

Step 5: Determination of areas of steel

The respective areas of steel in middle and edge strips are to be determined employing Eq.3.23 of Step 5 of sec. 8.18.6 of Lesson 18. However, in Problem 8.1 of Lesson 18, it has been shown that the areas of steel computed from Eq.3.23 and those obtained from the tables of SP-16 are in good agreement. Accordingly, the areas of steel for this problem are computed from the respective Tables 40 and 41 of SP-16 and presented in Table 8.7. Table 40 of SP-16 is for the effective depth of 150 mm, while Table 41 of SP-16 is for the effective depth of 175 mm. The following results are, therefore, interpolated values obtained from the two tables of SP-16.

Table 8.7 Reinforcing bars of Problem 8.2

Particulars	Short span l_x			Long span l_y		
	Table No.	M_x (kNm/m)	Dia. & spacing	Table No.	M_y (kNm/m)	Dia. & spacing
Top steel for negative moment	40,41	18.68 > 18.6	10 mm @ 200 mm c/c	40,41	12.314 > 11.66	8 mm @ 200 mm c/c
Bottom steel for positive	40,41	14.3	8 mm @ 170 mm c/c	40,41	9.20 >	8 mm @ 250 mm

moment		88 > 13.8 9			8.68	c/c

The minimum steel is determined from the stipulation of cl. 26.5.2.1 of IS 456 and is
A_s = (0.12/100)(1000)(160) = 192 mm^2

and 8 mm bars @ 250 mm c/c (= 201 mm^2) is acceptable. It is worth mentioning that the areas of steel as shown in Table 8.7 are more than the minimum amount of steel.

Step 6: Selection of diameters and spacings of reinforcing bars

The advantages of using the tables of SP-16 are that the obtained values satisfy the requirements of diameters of bars and spacings. However, they are checked as ready reference here. Needless to mention that this step may be omitted in such a situation.

Maximum diameter allowed, as given in cl. 26.5.2.2 of IS 456, is 160/8 = 20 mm, which is more that the diameters used here.

The maximum spacing of main bars, as given in cl. 26.3.3(1) of IS 456, is the lesser of 3(135) and 300 mm. This is also satisfied for all the bars.

The maximum spacing of minimum steel (distribution bars) is the lesser of 5(135) and 450 mm. This is also satisfied.

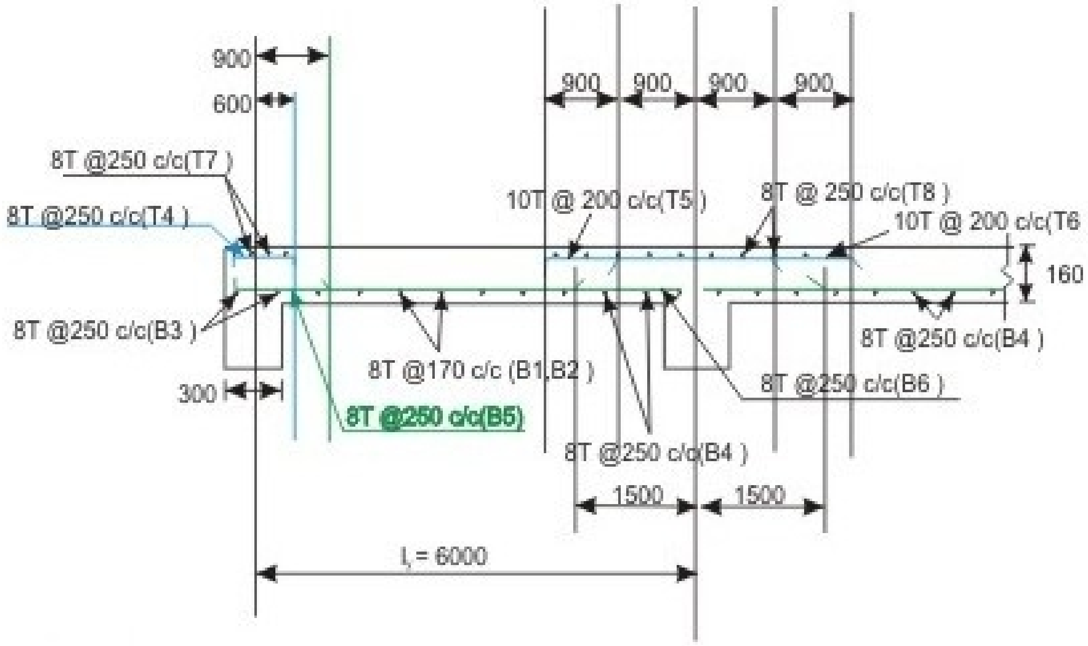

Fig. 8.19.8: Problem 8.2, Sec 1-1 of Panel 1 of Fig. 8.19.7

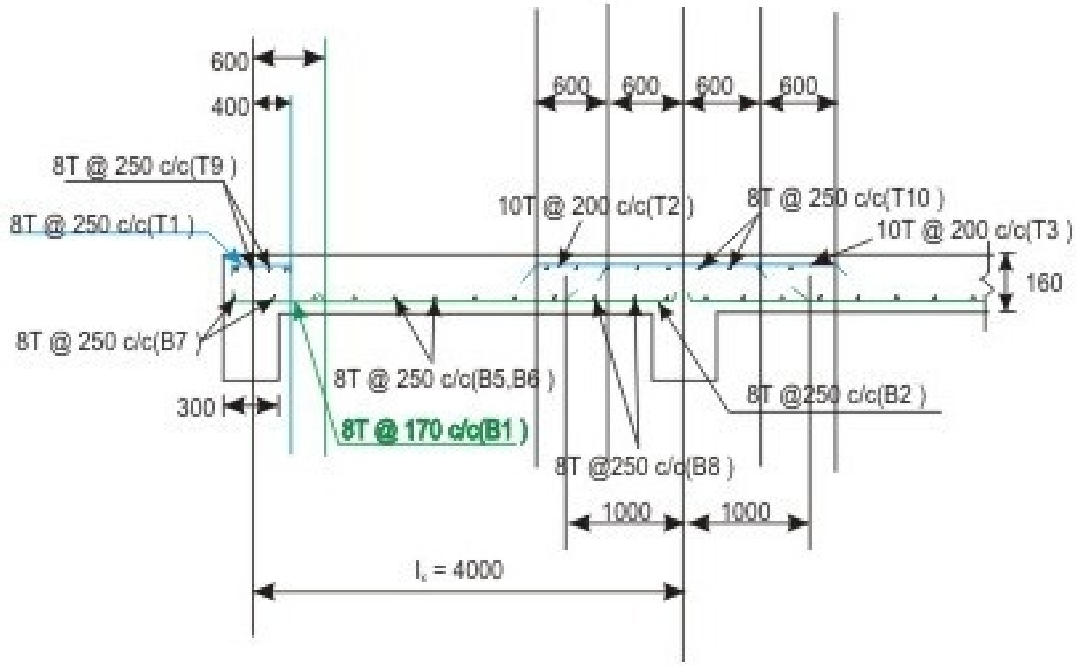

Fig. 8.19.9: Problem 8.2, Sec 2-2 of Panel 1 of Fig. 8.19.7

Figures 8.19.8 and 9 present the detailing of reinforcing bars.

Step 7: Determination of torsional reinforcement

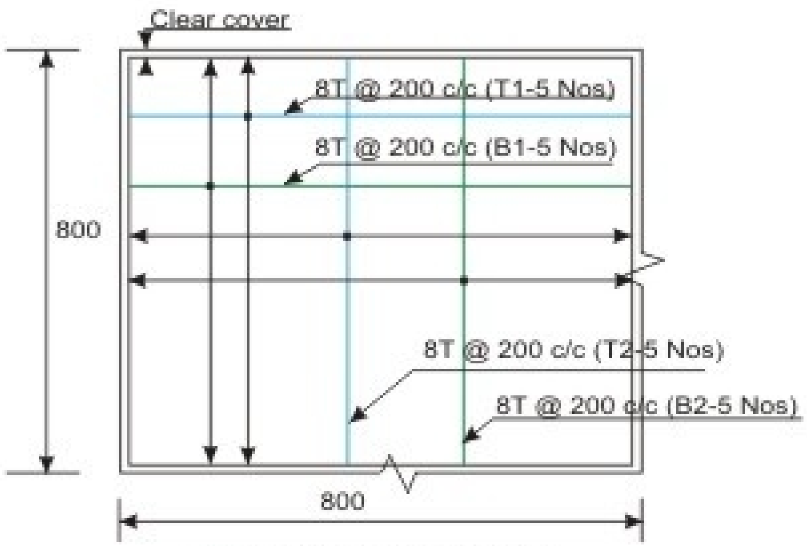

Fig. 8.19.10(a): Corner C1

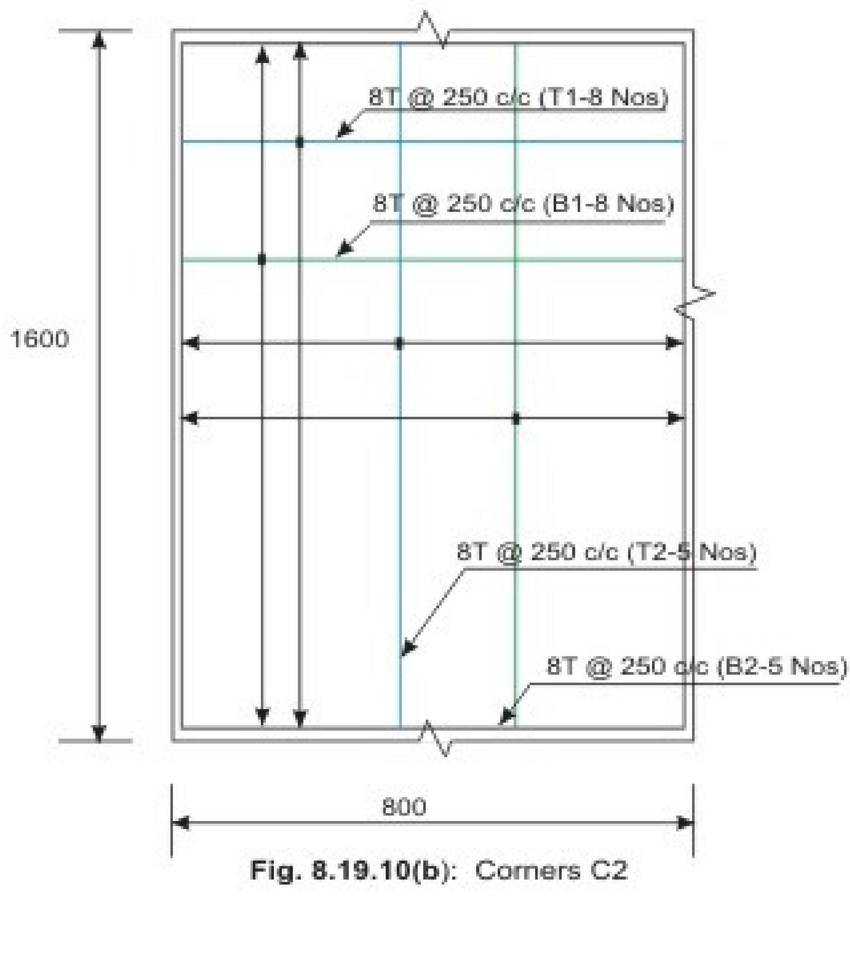

Fig. 8.19.10(b): Corners C2

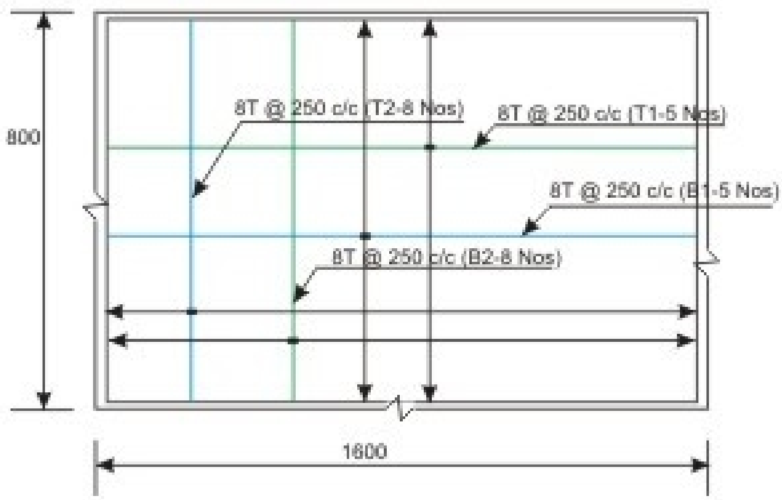

Fig. 8.19.10(c): Corners C2

Fig. 8.19.10: Torsion reinforcement bars of Problem 8.2

Torsional reinforcing bars are determined for the three different types of corners as explained in sec. 8.19.6 (Fig.8.19.4). The length of torsional strip is 4000/5 = 800 mm and the bars are to be provided in four layers. Each layer will have 0.75 times the steel used for the maximum positive moment. The C1 type of corners will have the full amount of torsional steel while C2 type of corners will have half of the amount provided in C1 type. The C3 type of corners do not need any torsional steel. The results are presented in Table 8.8 and Figs.8.19.10 a, b and c.

Table 8.8 Torsional reinforcement bars of Problem 8.2

Type	Dimensions along		Bar diameter & spacing	No. of bars along		Cl. no. of IS 456
	x (mm)	*y* (mm)		*x*	*y*	
C1	800	800	8 mm @ 200 mm c/c	5	5	D-1.8
C2	800	1600	8 mm @ 250 mm c/c	5	8	D-1.9
C2	1600	800	8 mm @ 250 mm c/c	8	5	D-1.9

Practice Questions and Problems with Answers

Q.1: How do you determine the shear force of a two-way slab subjected to uniformly distributed loads?

A.1: See sec. 8.19.4.1.

Q.2: Name the two types of two-way slabs.

A.2: The two types of two-way slabs are: (i) restrained slabs and (ii) simply supported slabs.

Q.3: What is the difference in the design of the two types of slabs of Q.2?

A.3: The restrained slabs are those whose corners are prevented from lifting and accordingly, there are torsional reinforcing bars in the two types of corners. The simply supported slabs do not have adequate provision to resist torsion at corners and to prevent the corners from lifting. So, torsional reinforcing bars are not provided in these slabs.

Q.4: State span to depth ratios of two-way slabs for different support conditions to be considered for the control of deflection.

A.4: See sec. 8.19.5.

Q.5: Explain the provisions of torsional reinforcing bars in restrained type of two- way slabs.

A.5: Step 7 of sec. 8.19.6.

Q.6:

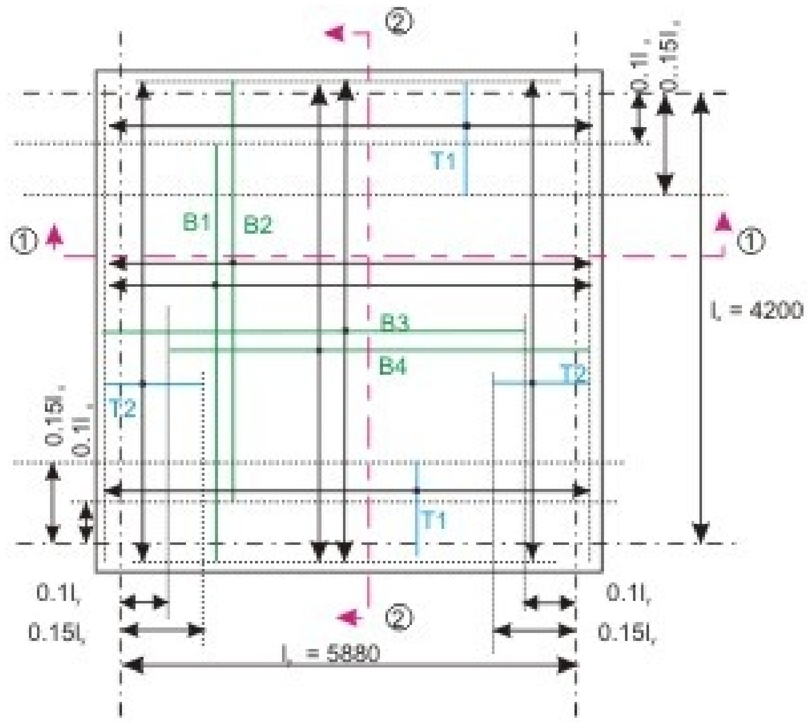

Fig. 8.19.11: Problem Q. 6

Design a two-way simply supported slab of Fig.8.19.11, not having adequate provision to resist torsion at corners and to prevent the corners from lifting. The factored live load is 6 kN/m^2 and the load of the floor finish in 1 kN/m^2. The spans shown in the figure are effective spans. Use M 20 and Fe 415. The width of the support is 300 mm.

A.6:

Step 1: Selection of preliminary depth of slab

As per cl.24.1, Note 2, the span to effective depth ratio = 0.8(35) = 28. The minimum effective depth = d = 4200/28 = 150 mm and, therefore, D = 175 mm.

Step 2: Design loads, bending moments and shear forces Factored dead

load of the slab = 1.5(0.175)(25) = 6.5625 kN/m^2 Factored load of

floor finish = 1.5(1) = 1.5 kN/m^2

Factored live loads = 6.0 kN/m^2

Total factored loads = 14.0625 kN/m^2

For this slab l_y/l_x = 5880/4200 = 1.4, Table 27 of IS 456 gives α_x = 0.099 and α_y = 0.051.

$$+M_x = \alpha_x \; w \, l_x^2 = (0.099)(14.0625)(4.2)(4.2) = 24.558 \text{ kNm/m}$$

$$+M_y = \alpha_y \; w \, l_x^2 = (0.051)(14.0625)(4.2)(4.2) = 12.651 \text{ kNm/m}$$

$$V_u = 0.5 \, w \, l_x = 0.5(14.0625)(4.2) = 29.531 \text{ kN/m}$$

Step 3: Determination/checking of the effective depth and total depth of slab

$$d = \{24.558(10^3)/2.76\}^{0.5} = 94.328 \text{ mm} < 150 \text{ mm}$$

So, we keep d = 150 mm and D = 175 mm.

Step 4: Depth of slab for shear forces

Table 19 of IS 456 gives τ_c = 0.28 N/mm². Clause 40.2.1.1 of IS 456 gives k = 1.25 for D = 175 mm. So, τ_c = (1.25)(0.28) = 0.35 N/mm².

Table 20 of IS 456 gives $\tau_{c\,max}$ = 2.8 N/mm². For this problem $\tau_v = V_u/bd$ = 29.531/150 = 0.1968 N/mm². Since $\tau_v < \tau_c < \tau_{cmax}$, the depth is safe.

Step 5: Determination of areas of steel

The positive steel in the two directions and the minimum steel are furnished below in Table 8.9. These are the results obtained from the use of Table 41 of SP-16.

Table 8.9 Reinforcing bars of Problem Q.6

Particulars	Short span l_x			Long span l_y		
	SP Table No.	M_x (kNm/m)	Dia. & spacing	SP Table No.	M_y (kNm/m)	Dia. & spacing
Positive steel	41	26.40 > 24.55	8 mm @ 100 mm c/c (503 mm²)	41	12.93 > 12.651	6 mm @ 120 mm c/c (236 mm²)

		8				
Minimum steel = 0.12(1750)	96	Min. steel	6 mm @ 120 mm c/c (236	96	Min. steel	6 mm @ 120 mm c/c (236

= 210 mm^2			mm^2) > 210 mm^2			mm^2) > 210 mm^2

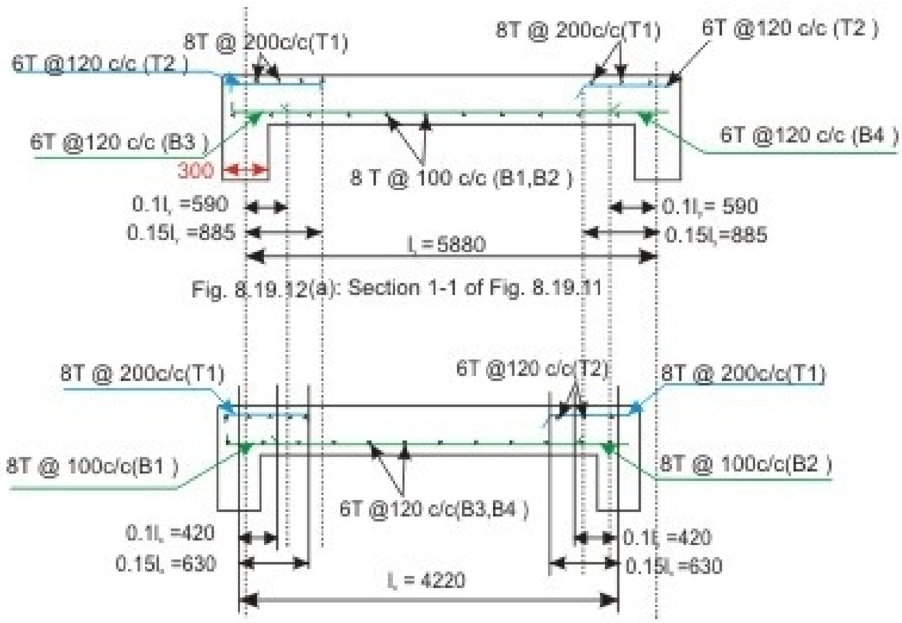

Fig. 8.19.12(b): Section 2-2 of Fig. 8.19.11

Fig. 8.19.12: Detailing of bars of Problem Q. 6, corners not held down

Figures 8.19.12a and b show the detailing of reinforcing bars.

Solutions

TQ.1: Explain the provisions of torsional reinforcing bars in restrained type of two-way slabs. (20 marks)

A.TQ.1: Step 7 of sec. 8.19.6.

TQ.2: Design the interior panel (Panel 2) of Problem 8.2 (Fig.8.19.7). Other data are the same as those of Problem 8.2. (30 marks)

A.TQ.2: Let us keep the effective and total depths of the slab as 135 mm and 160 mm, respectively (see Problem 8.2). The total factored load = 15.5 kN/m^2 (see Problem 8.2). The coefficients of bending moments and the bending moments M_x and M_y (positive and negative) per unit width are determined as per cl. D-1.1 and Table 26 of IS 456 for the case 1 (interior panel) and presented in Table 8.10. The l_y/l_x for this problem is 1.5.

Table 8.10 Bending moments of Problem TQ.2

For	Short span		Long span	
	α_x	M_x (kNm/m)	α_y	M_y (kNm/m)
Negative moment at continuous edge	0.053	13.144	0.032	7.936
Positive moment at mid-span	0.041	10.168	0.024	5.952

The maximum shear force in either direction is the same as that of Problem 8.2 = 31 kN/m.

Since the bending moments are much less than those of Problem 8.2, the effective depth of 135 m and total depth of 160 mm are safe.

In Step 4 of solution of Problem 8.2, this depth has been found to be safe in shear. So, the depths 135 mm and 160 mm are safe.

Step 5: Determination of areas of steel

The areas of steel using the table of SP-16 are presented in Table 8.11. The maximum diameter and spacing of bars are not needed to check separately as the results obtained from tables of SP-16 already take into consideration these aspects.

Table 8.11 Reinforcing bars of Problem TQ.2

Particulars	Short span l_x			Long span l_y		
	Table No.	M_x (kNm/m)	Dia. & spacing	Table No.	M_y (kNm/m)	Dia. & spacing
Top steel for negative moment	40,41	13.618 > 13.144	8 T @ 180 mm c/c	40	9.2 > 7.936	8 T @ * 250 mm c/c
Bottom steel for positive moment	40,41	10.758	8 T @ 230 mm c/c	40	9.2 > 5.95	8 T @ * 250 mm

		> 10.168			2	c/c

* Note: The areas of bars are selected to satisfy the minimum amount of steel.

The minimum steel will be the same as that of Problem 8.2 i.e., 8 mm diameter @ 250 mm c/c.

Since this is an internal panel, torsional reinforcing bars are not needed at any of the four corners.

The detailing of the bars can be drawn following the same of Problem 8.2 shown in Figs.8.19.8 and 9.

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DEPARTMENT OF CIVIL ENGINEERING

UNIT – IV – LIMIT STATE OF COLLAPSE - COMPRESSION – SCIA1203

Introduction

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. Figures 10.21.1a to c show their examples: pedestal, column, wall and strut. While pedestal, column and wall carry the loads along its length l in vertical direction, the strut in truss carries loads in any direction. The letters l, b and D represent the unsupported vertical length, horizontal lest lateral dimension, width and the horizontal longer lateral dimension, depth. These compression members may be made of bricks or reinforced concrete. Herein, reinforced concrete compression members are only discussed.

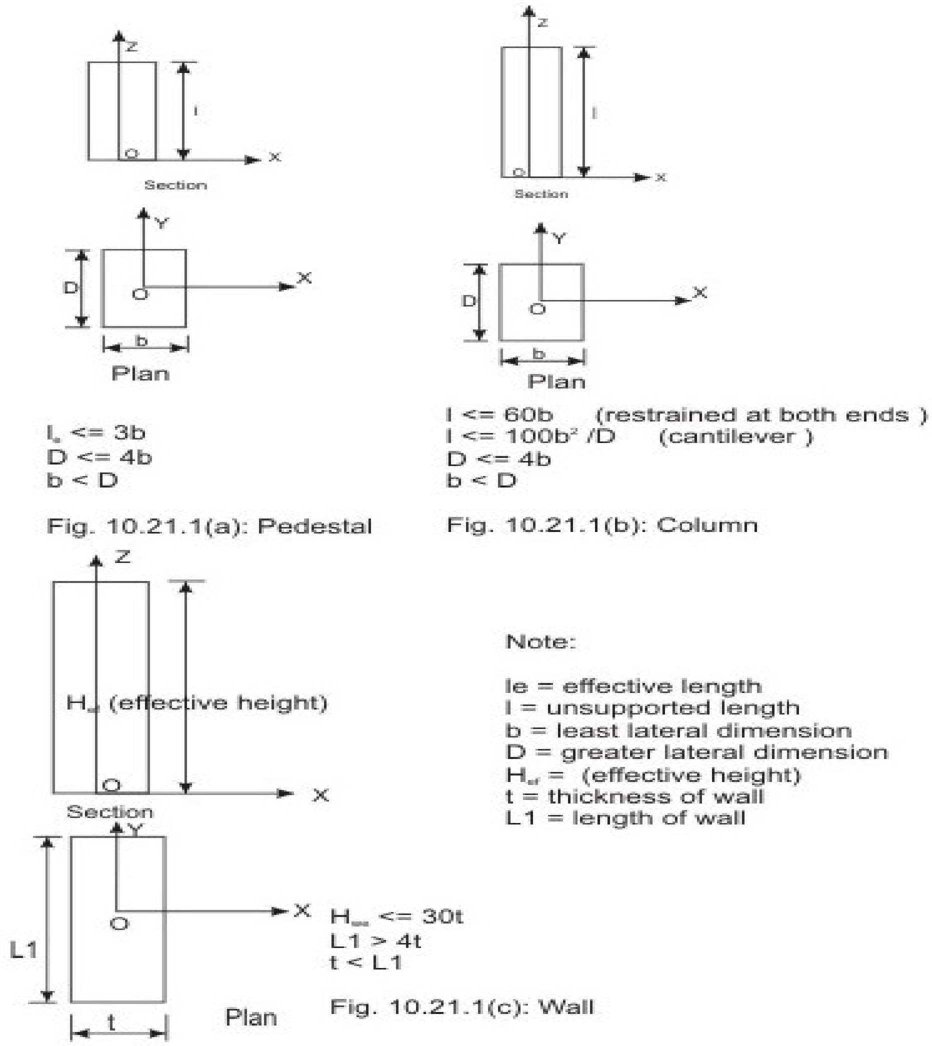


Fig. 10.21.1: Pedestal, column and wall

This module is intended to explain the definition of some common terminologies and to illustrate the design of compression members and other related issues. This lesson, however, explain the definitions and classifications of columns depending on different aspects. Further, the recommendations of IS 456 to be followed in the design are discussed regarding the longitudinal and lateral reinforcing bars. The assumptions made in the design of compression member by limit sate of collapse are illustrated.

Definitions

(a) Effective length: The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length l_e of that compression member in that plane. The effective length is different from the unsupported length l of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is

$$l_e = k\,l \tag{10.1}$$

where k is the ratio of effective to the unsupported lengths. Clause 25.2 of IS 456 stipulates the effective lengths of compression members (vide Annex E of IS 456). This parameter is needed in classifying and designing the compression members.

(b) Pedestal: Pedestal is a vertical compression member whose effective length l_e does not exceed three times of its least horizontal dimension b (cl. 26.5.3.1h, Note). The other horizontal dimension D shall not exceed four times of b (Fig.10.21.1a).

(c) Column: Column is a vertical compression member whose unsupported length l shall not exceed sixty times of b (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed $100b^2/D$, where D is the larger lateral dimension which is also restricted up to four times of b (vide cl. 25.3 of IS 456 andFig.10.21.1b).

(d) Wall: Wall is a vertical compression member whose effective height H_{we} to thickness t (least lateral dimension) shall not exceed 30 (cl. 32.2.3 of IS 456). The larger horizontal dimension i.e., the length of the wall L is more than 4t (Fig.10.21.1c).

Classification of Columns Based on Types of Reinforcement

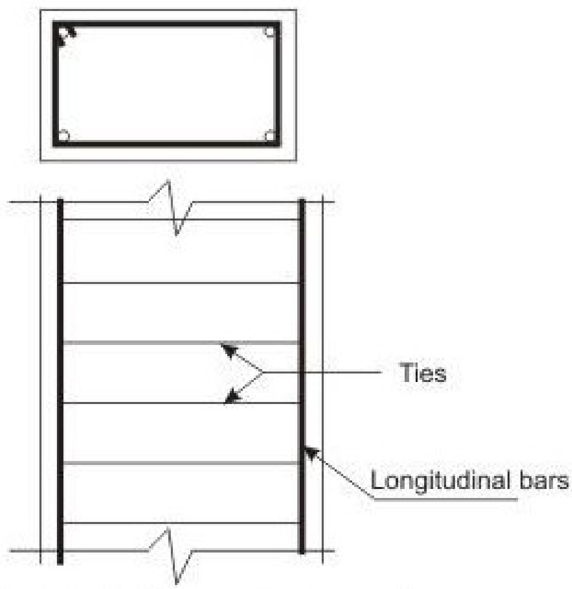


Fig. 10.21.2(a): Tied column

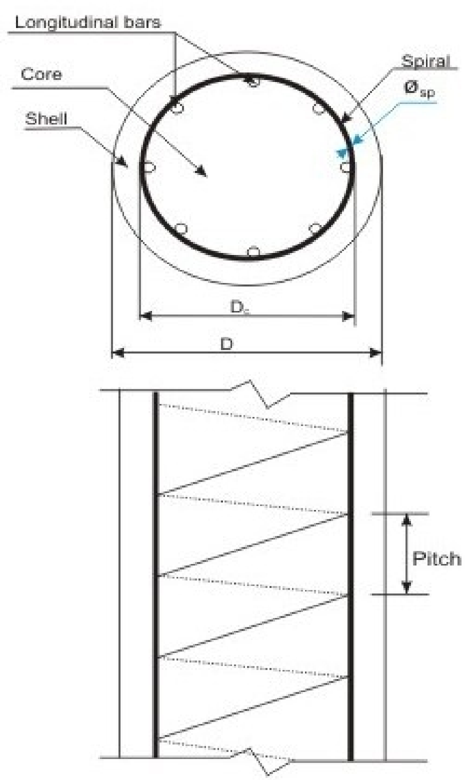


Fig. 10.21.2b: Column with helical reinforcement

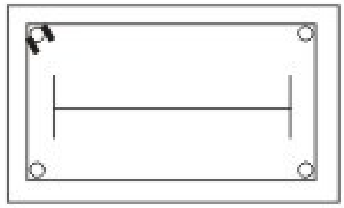


Fig. 10.21.2(c): Composite column (steel section)

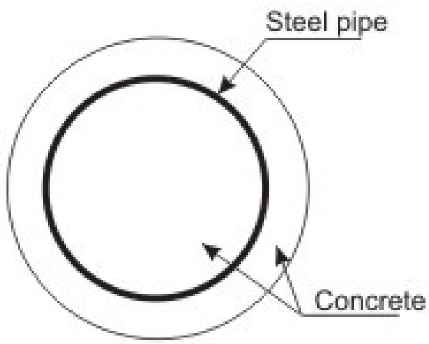


Fig. 10.21.2(d): Composite column (steel pipe)

Fig. 10.21.2: Tied, helically bound & composite columns

Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:

(i) Tied columns: The main longitudinal reinforcement bars are enclosed within closely spaced lateral ties (Fig.10.21.2a).

(ii) Columns with helical reinforcement: The main longitudinal reinforcement bars are enclosed within closely spaced and continuously wound spiral reinforcement. Circular and octagonal columns are mostly of this type (Fig.10.21.2b).

(iii) Composite columns: The main longitudinal reinforcement of the composite columns consists of structural steel sections or pipes with or without longitudinal bars (Fig.20.21.2c and d).

Out of the three types of columns, the tied columns are mostly common with different shapes of the cross-sections viz. square, rectangular, T-, L-, cross etc. Helically bound columns are also used for circular or octagonal shapes of cross-sections. Architects prefer circular columns in some specific situations for the functional requirement. This module, accordingly takes up these two types (tied and helically bound) of reinforced concrete columns.

Classification of Columns Based on Loadings

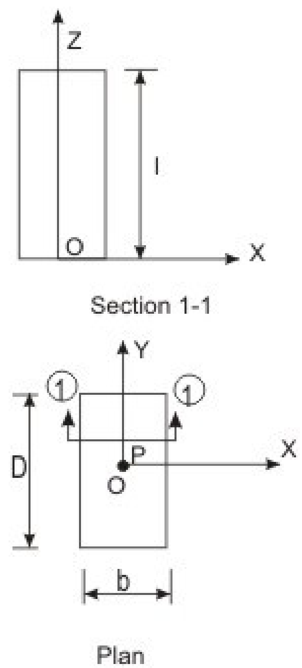


Fig. 10.21.3(a): Axial loading (concentric)

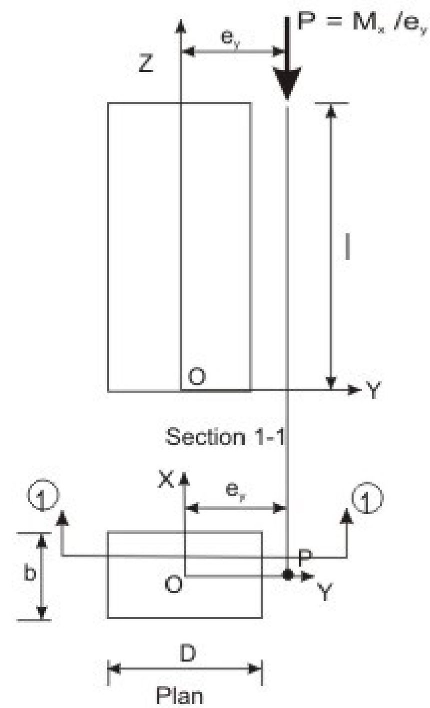


Fig. 10.21.3(b): Axial loading with uniaxial bending

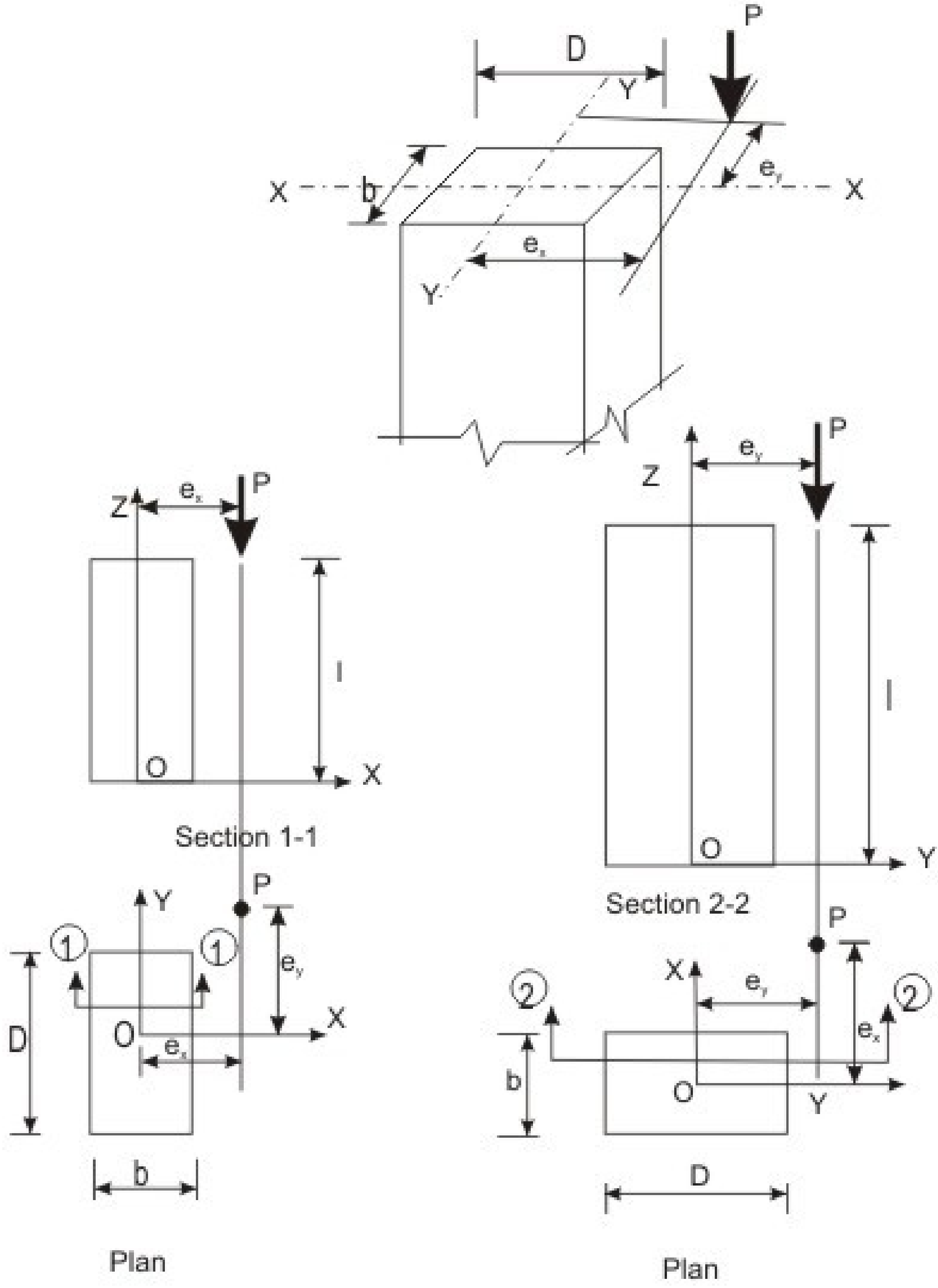


Fig. 10.21.3(c): Axial loading with biaxial bending

Fig. 10.21.3: Concentric and eccentric loadings on columns

Columns are classified into the three following types based on the loadings:

(i) Columns subjected to axial loads only (concentric), as shown in Fig.20.21.3a.

(ii) Columns subjected to combined axial load and uniaxial bending, as shown in Fig.10.21.3b.

(iii) Columns subjected to combined axial load and bi-axial bending, as shown in Fig.10.21.3c.

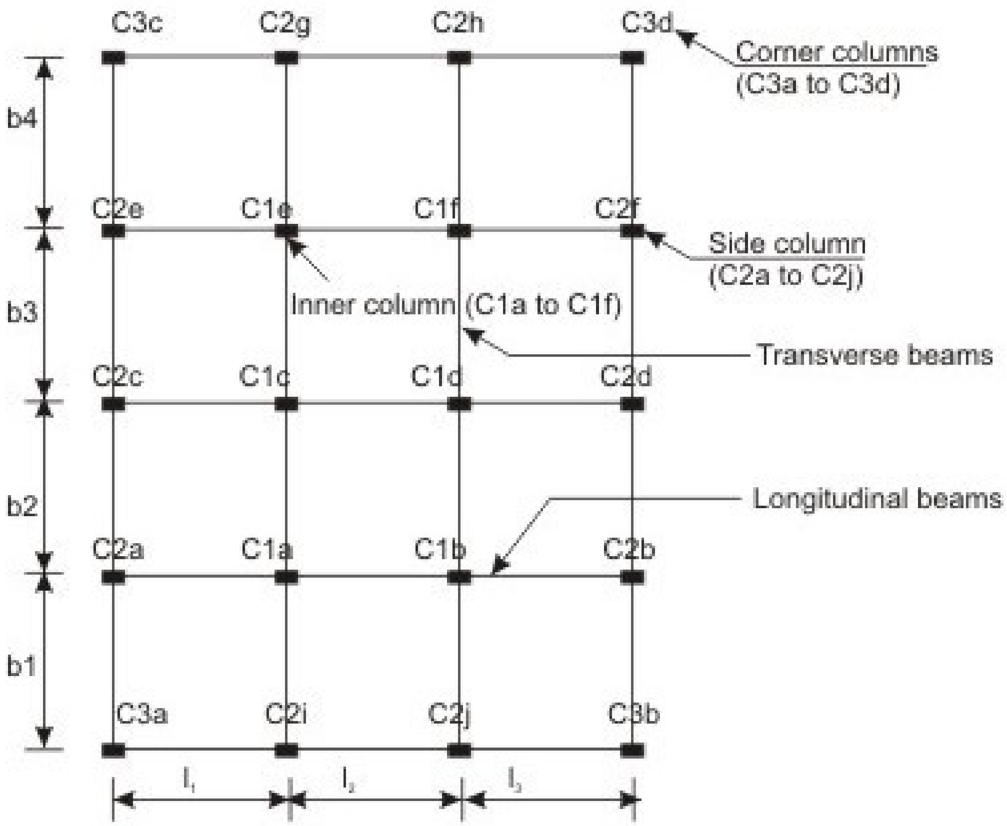


Fig. 10.21.4: Grid of beams and columns

Figure 10.21.4 shows the plan view of a reinforced concrete rigid frame having columns and inter-connecting beams in longitudinal and transverse directions. From the knowledge of structural analysis it is well known that the bending moments on the left and right of columns for every longitudinal beam will be comparable as the beam is continuous. Similarly, the bending moments at the two sides of columns for every continuous transverse beam are also comparable (neglecting small amounts due to differences of l_1, l_2, l_3 and b_1, b_2, b_3, b_4). Therefore, all internal columns (C1a to C1f) will be designed for axial force only. The side columns (C2a to C2j) will have axial forces with uniaxial bending moment, while the four corner columns (C3a to C3d) shall have axial forces with bi-axial bending moments. Thus, all internal columns (C1a to C1f), side columns

(C2a to C2j) and corner columns (C3a to C3d) are the columns of type (i), (ii) and (iii), respectively.

It is worth mentioning that pure axial forces in the inside columns is a rare case. Due to rigid frame action, lateral loadings and practical aspects of construction, there will be bending moments and horizontal shear in all the inside columns also. Similarly, side columns and corner columns will have the column shear along with the axial force and bending moments in one or both directions, respectively. The effects of shear are usually neglected as the magnitude is very small. Moreover, the presence of longitudinal and transverse reinforcement is sufficient to resist the effect of column shear of comparatively low magnitude. The effect of some minimum bending moment, however, should be taken into account in the design even if the column is axially loaded. Accordingly, cls. 39.2 and 25.4 of IS 456 prescribes the minimum eccentricity for the design of all columns. In case the actual eccentricity is more than the minimum, that should be considered in the design.

Classification of Columns Based on Slenderness Ratios

Columns are classified into the following two types based on the slenderness ratios:

(i) Short columns

(ii) Slender or long columns

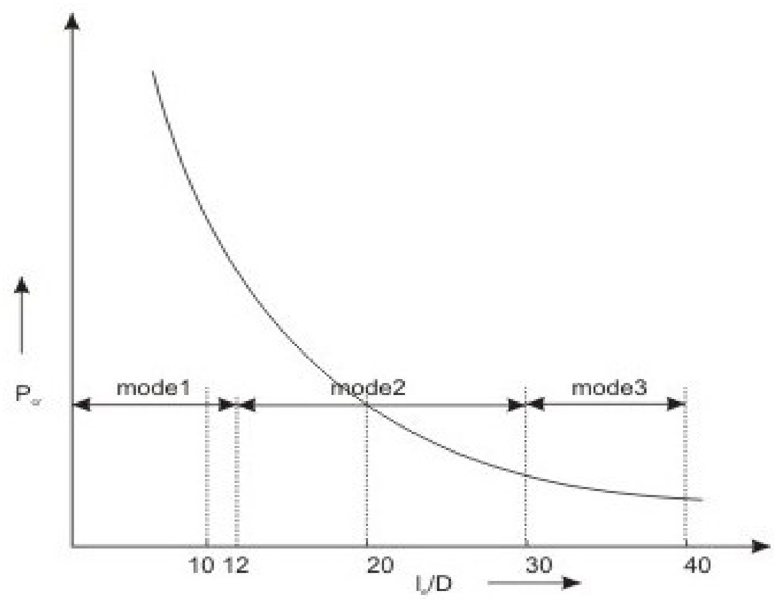


Fig. 10.21.5: Modes of failure of columns

Figure 10.21.5 presents the three modes of failure of columns with different slenderness ratios when loaded axially. In the mode 1, column does not undergo any lateral deformation and collapses due to material failure. This is known as compression failure. Due to the combined effects of axial load and moment a short column may have material failure of mode 2. On the other hand, a slender column subjected to axial load only undergoes deflection due to beam- column effect and may have material failure under the combined action of direct load and bending moment. Such failure is called combined compression and bending failure of mode 2. Mode 3 failure is by elastic instability of very long column even under small load much before the material reaches the yield stresses. This type of failure is known as elastic buckling.

The slenderness ratio of steel column is the ratio of its effective length l_e to its least radius of gyration r. In case of reinforced concrete column, however, IS 456 stipulates the slenderness ratio as the ratio of its effective length l_e to its least lateral dimension. As mentioned earlier in sec. 10.21.2(a), the effective length l_e is different from the unsupported length, the rectangular reinforced concrete column of cross-sectional dimensions b and D shall have two effective lengths in the two directions of b and D. Accordingly, the column may have the possibility of buckling depending on the two values of slenderness ratios as given below:

Slenderness ratio about the major axis $= l_{ex}/D$

Slenderness ratio about the minor axis $= l_{ey}/b$

Based on the discussion above, cl. 25.1.2 of IS 456 stipulates the following:

A compression member may be considered as short when both the slenderness ratios l_{ex}/D and l_{ey}/b are less than 12 where l_{ex} = effective length in respect of the major axis, D = depth in respect of the major axis, l_{ey} = effective length in respect of the minor axis, and b = width of the member. It shall otherwise be considered as a slender compression member.

Further, it is essential to avoid the mode 3 type of failure of columns so that all columns should have material failure (modes 1 and 2) only. Accordingly, cl. 25.3.1 of IS 456 stipulates the maximum unsupported length between two restraints of a column to sixty times its least lateral dimension. For cantilever columns, when one end of the column is unrestrained, the unsupported length is restricted to $100b^2/D$ where b and D are as defined earlier.

Braced and unbraced columns

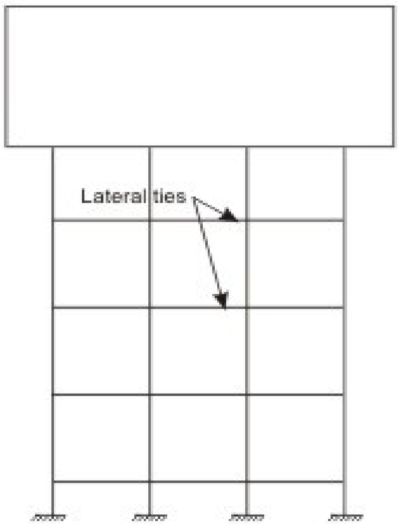


Fig. 10.21.6(a): water tank

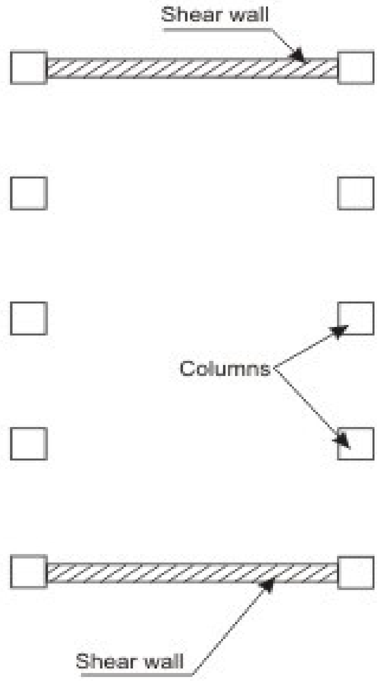


Fig. 10.21.6(b): Tall building frame

Fig. 10.21.6: Bracing of columns

It is desirable that the columns do not have to resist any horizontal loads due to wind or earthquake. This can be achieved by bracing the columns as in the case of columns of a water tank or tall buildings (Figs.10.21.6a and b). Lateral tie members for the columns of water tank or shear walls for the columns of tall buildings resist the horizontal forces and these columns are called braced columns. Unbraced columns are supposed to resist the horizontal loads also. The bracings can be in one or more directions depending on the directions of the lateral loads. It is worth mentioning that the effect of bracing has been taken into account by the IS code in determining the effective lengths of columns (vide Annex E of IS 456).

Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. Clause 26.5.3.1 stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

(e) The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.

(f) The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.

(g) Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.

(h) The diameter of the longitudinal bars should be at least 12 mm.

(i) Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.

(j) The bars shall be spaced not exceeding 300 mm along the periphery of the column.

(k) The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

Transverse Reinforcement

Transverse reinforcing bars are provided in forms of circular rings, polygonal links (lateral ties) with internal angles not exceeding 135^{o} or helical reinforcement. The transverse reinforcing bars are provided to ensure that every longitudinal bar nearest to the compression face has effective lateral support against buckling. Clause 26.5.3.2 stipulates the guidelines of the arrangement of transverse reinforcement. The salient points are:

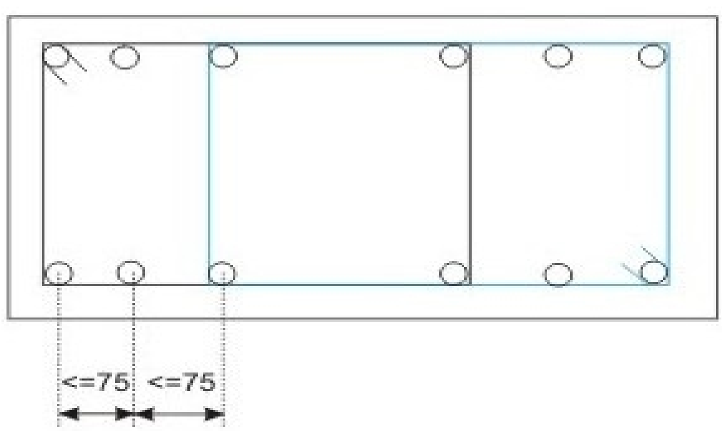


Fig. 10.21.7: Lateral tie (Scheme 1)

(l) Transverse reinforcement shall only go round corner and alternate bars if the longitudinal bars are not spaced more than 75 mm on either side (Fig.10.21.7).

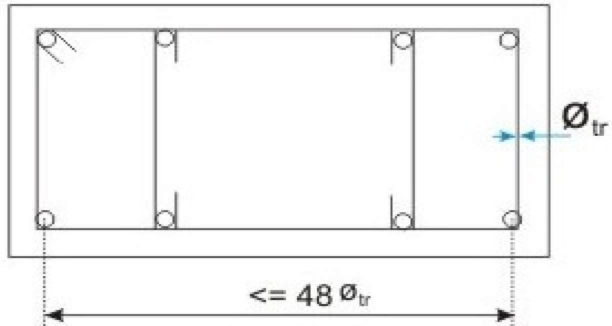


Fig. 10.21.8: Lateral tie (Scheme 2)

(m) Longitudinal bars spaced at a maximum distance of 48 times the diameter of the tie shall be tied by single tie and additional open ties for in between longitudinal bars (Fig.10.21.8).

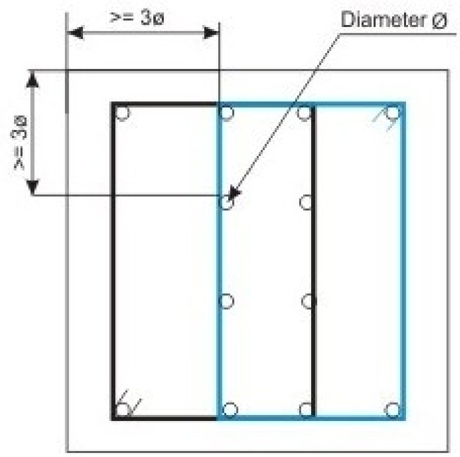


Fig. 10.21.9: Lateral tie (Scheme 3)

(n) For longitudinal bars placed in more than one row (Fig.10.21.9): (i) transverse reinforcement is provided for the outer-most row in accordance with (a) above, and (ii) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.

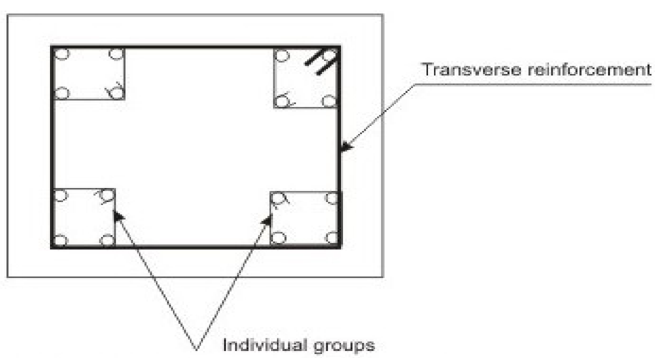


Fig. 10.21.10: Lateral tie (Scheme 4)

(d) For longitudinal bars arranged in a group such that they are not in contact and each group is adequately tied as per (a), (b) or (c) above, as appropriate, the transverse reinforcement for the compression member as a whole may be provided assuming that each group is a single longitudinal bar for determining the pitch and diameter of the transverse reinforcement as given in sec.10.21.9. The diameter of such transverse reinforcement should not, however, exceed 20 mm (Fig.10.21.10).

Pitch and Diameter of Lateral Ties

(o) Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:

(i) the least lateral dimension of the compression members;

(ii) sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and

(iii) 300 mm.

(p) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

Helical Reinforcement

(q) Pitch: Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and a half extra turns of the spiral bar. The pitch of helical reinforcement shall be determined as given in sec.10.21.9 for all cases except where an increased load on the column is allowed for on the strength of the helical reinforcement. In such cases only, the maximum pitch shall be the lesser of 75 mm and one-sixth of the core diameter of the column, and the minimum pitch shall be the lesser of 25 mm and three times the diameter of the steel bar forming the helix.

(r) Diameter: The diameter of the helical reinforcement shall be as mentioned in sec.10.21.9b.

Assumptions in the Design of Compression Members by Limit State of Collapse

It is thus seen that reinforced concrete columns have different classifications depending on the types of reinforcement, loadings and slenderness ratios. Detailed designs of all the different classes are beyond the scope here. Tied and helically reinforced short and slender columns subjected to axial loadings with or without the combined effects of uniaxial or biaxial bending will be taken up. However, the basic assumptions of the design of any of the columns under different classifications are the same. The assumptions (i) to (v) given in sec.3.4.2 of Lesson 4 for the design of flexural members are also applicable here. Furthermore, the following are the additional assumptions for the design of compression members (cl. 39.1 of IS 456).

(i) The maximum compressive strain in concrete in axial compression is taken as 0.002.

(ii) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

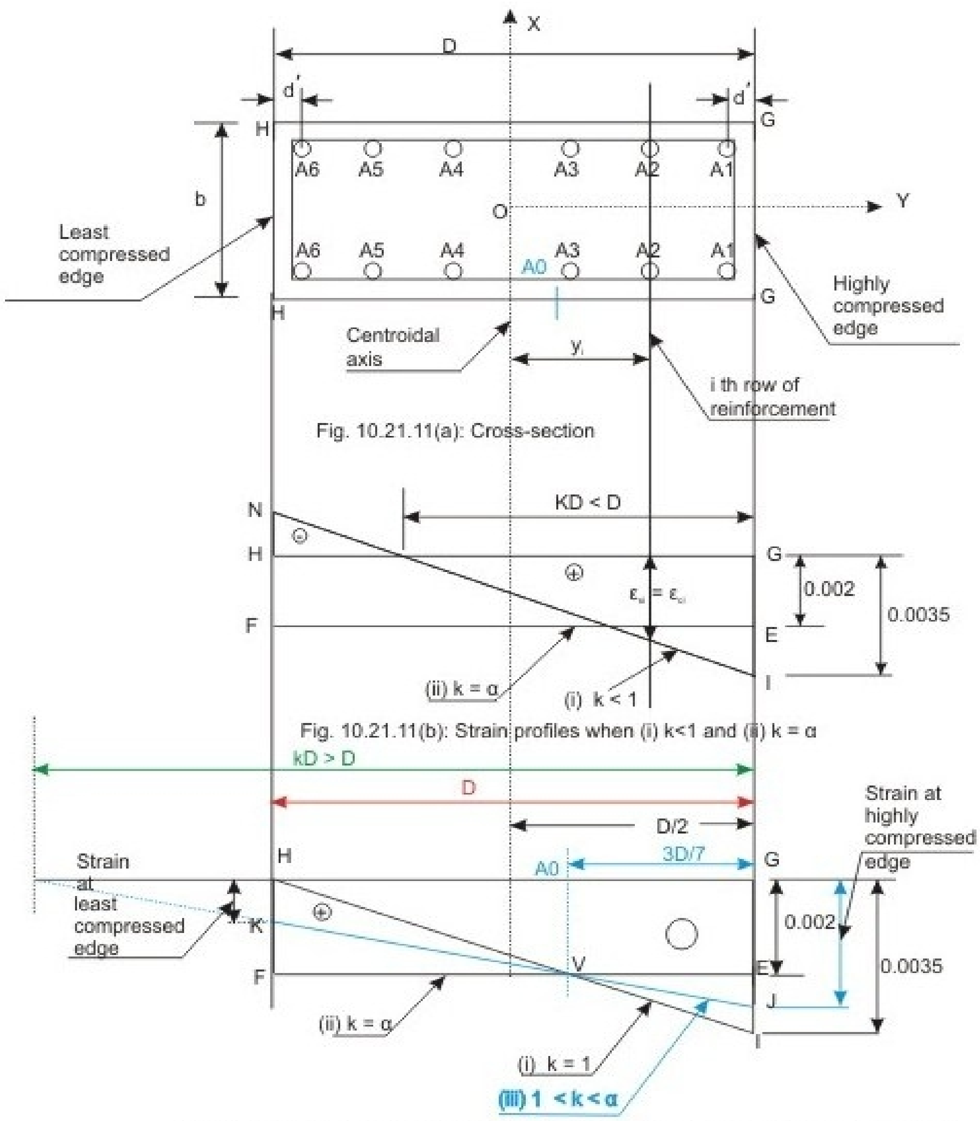


Fig. 10.21.11: Strain profiles for different positions of neutral axis

The assumptions (i) to (v) of section 3.4.2 of Lesson 4 and (i) and (ii) mentioned above are discussed below with reference to Fig.10.21.11a to c presenting the cross-section and strain diagrams for different location of the neutral axis.

The discussion made in sec. 3.4.2 of Lesson 4 regarding the assumptions (i), (iii), (iv) and (v) are applicable here also. Assumption (ii) of sec.3.4.2 is also applicable here when kD, the depth of neutral axis from the highly compressed right edge is within the section i.e., $k < 1$. The corresponding strain profile IN in Fig.10.21.11b is for particular value of P and M such that the maximum compressive strain is 0.0035 at the highly compressed right edge and tensile strain develops at the opposite edge. This strain profile is very much similar to that of a beam in flexure of Lesson 4.

The additional assumption (i) of this section refers to column subjected axial load P only resulting compressive strain of maximum (constant) value of
0.002 and for which the strain profile is EF in Fig.10.21.11b. The neutral axis is at infinity (outside the section).

Extending the assumption of the strain profile IN (Fig.10.21.11b), we can draw another strain profile IH (Fig.10.21.11c) having maximum compressive strain of 0.0035 at the right edge and zero strain at the left edge. This strain profile 1H along with EF are drawn in Fig.10.21.11c to intersect at V. From the two similar triangles EVI and GHI, we have

EV/GH = 0.0015/0.0035 = 3/7, which gives

$$EV = 3D/7 \tag{10.2}$$

The point V, where the two profiles intersect is assumed to act as a fulcrum for the strain profiles when the neutral axis lies outside the section. Another strain profile JK drawn on this figure passing through the fulcrum V and whose neutral axis is outside the section. The maximum compressive strain GJ of this profile is related to the minimum compressive strain HK as explained below.

GJ = GI – IJ = GI – 0.75 HK, as we can write IJ in term of HK from two similar triangles JVI and HVK:

IJ/HK = VE/VF = 0.75.

The value of the maximum compressive strain GJ for the profile JK is, therefore, 0.0035 minus 0.75 times the strain HK on the least compressed edge. This is the assumption (ii) of this section (cl. 39.1b of IS 456).

Minimum Eccentricity

Section 10.21.4 illustrates that in practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity as stipulated in cl. 25.4 of IS 456 and given below (Fig.10.21.3c)

$e_{x\ min} \geq$ greater of)l/500 + D/30) or 20 mm

(10.3)

$e_{y\ min} \geq$ greater of)l/500 + b/30) or 20 mm where l, D and b are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

Further Assumptions Regarding the Strengths of Concrete and Steel

All the assumptions required for the derivation of the governing equations are given in sec.10.21.11 of Lesson 21. The stress-strain diagrams of mild steel (Fe 250) and cold worked deformed bars (Fe 415 and Fe 500) are given in Figs.1.2.3 and 4, respectively of Lesson 2. The stress block of compressive part of concrete is given in Fig.3.4.1.9 of Lesson 4, which is used in the design of beam by limit state of collapse. The maximum design strength of concrete is shown as constant at 0.446 f_{ck} when the strain ranges from 0.002 to 0.0035. The maximum design stress of steel is 0.87 f_y.

Sections 10.21.4 and 12 of Lesson 21 explain that all columns including the short axially loaded columns shall be designed with a minimum eccentricity (cls. 25.4 and 39.2 of IS 456). Moreover, the design strengths of concrete and steel are further reduced to 0.4 f_{ck} and 0.67 f_y, respectively, to take care of the minimum eccentricity of 0.05 times the lateral dimension, as stipulated in cl.39.3 of IS 456. It is noticed that there is not attempt at strain compatibility. Also the phenomenon of creep has not been directly considered.

$$e_{x\,min} \geq \text{greater of } (l/500 + D/30) \text{ or } 20 \text{ mm}$$

(10.3)

$$e_{y\,min} \geq \text{greater of } (l/500 + b/30) \text{ or } 20 \text{ mm}$$

The maximum values of l_{ex}/D and l_{ey}/b should not exceed 12 in a short column as per cl.25.1.2 of IS 456. For a short column, when the unsupported length $l = l_{ex}$ (for the purpose of illustration), we can assume l = 12 D (or 12b when b is considered). Thus, we can write the minimum eccentricity = $12D/500 + D/30$ = 0.057D, which has been taken as 0.05D or 0.05b as the maximum amount of eccentricity of a short column.

It is, therefore, necessary to keep provision so that the short columns can resist the accidental moments due to the allowable minimum eccentricity by lowering the design strength of concrete by ten per cent from the value of $0.446f_{ck}$, used for the design of flexural members. Thus, we have the design strength of concrete in the design of short column as $(0.9)(0.446f_{ck}) = 0.4014f_{ck}$, say 0.40 f_{ck}. The reduction of the design strength of steel is explained below.

For mild steel (Fe 250), the design strength at which the strain is 0.002 is $f_y/1.15 = 0.87f_y$. However, the design strengths of cold worked deformed bars (Fe 415 and Fe 500) are obtained from Fig.1.2.4 of Lesson 2 or Fig.23A of IS 456. Table A of SP-16 presents the stresses and corresponding strains of Fe 415 and Fe 500. Use of Table A of SP-16 is desirable as it avoids error while reading from figures (Fig.1.2.4 or Fig.23A, as mentioned above). From Table A of SP-16, the corresponding design strengths are obtained by making linear interpolation. These values of design strengths for which the strain is 0.002 are as follows:

(i) Fe 415: $\{0.9f_{yd} + 0.05f_{yd}(0.002 - 0.00192)/(0.00241 - 0.00192)\} = 0.908f_{yd} = 0.789f_y$

(ii) Fe 500: $\{0.85f_{yd} + 0.05f_{yd}(0.002 - 0.00195)/(0.00226 - 0.00195)\} = 0.859f_{yd} = 0.746f_y$

A further reduction in each of three values is made to take care of the minimum eccentricity as explained for the design strength of concrete. Thus, the acceptable design strength of steel for the three grades after reducing 10 per cent from the above mentioned values are $0.783f_y$, $0.710f_y$ and $0.671f_y$ for Fe 250, Fe 415 and Fe 500, respectively. Accordingly, cl. 39.3 of IS 456 stipulates $0.67f_y$ as the design strength for all grades of steel while designing the short columns. Therefore, the assumed design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, for the design of short axially loaded columns.

Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area A_c and longitudinal steel of areas A_{sc} effectively held by lateral ties at intervals (Fig.10.21.2a of Lesson 21). Assuming the design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, as explained in sec. 10.22.2, we can write

$$P_u = 0.4f_{ck}\,A_c + 0.67f_y\,A_{sc}$$

(10.4)

where P_u = factored axial load on the member,

f_{ck} = characteristic compressive strength of the concrete,

A_c = area of concrete,

f_y = characteristic strength of the compression reinforcement, and

A_{sc} = area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns Ac and A_{sc} to be determined from one equation. The equation is recast in terms of A_g, the gross area of concrete and p, the percentage of compression reinforcement employing

$$A_{sc} = pA_g/100$$

(10.5)

$$A_c = A_g(1 - p/100)$$

(10.6)

Accordingly, we can write

$$P_u/A_g = 0.4f_{ck} + (p/100)\,(0.67f_y - 0.4f_{ck})$$

(10.7)

Equation 10.7 can be used for direct computation of A_g when P_u, f_{ck} and f_y are known by assuming p ranging from 0.8 to 4 as the minimum and maximum percentages of longitudinal reinforcement. Equation 10.4 also can be employed to determine A_g and p in a similar manner by assuming p. This method has been illustrated with numerical examples and is designated as Direct Computation Method.

On the other hand, SP-16 presents design charts based on Eq.10.7. Each chart of charts 24 to 26 of SP-16 has lower and upper sections. In the lower section, P_u/A_g is plotted against the reinforcement percentage $p(= 100A_s/A_g)$ for different grades of concrete and for a particular grade of steel. Thus, charts 24 to 26 cover the three grades of steel with a wide range of grades of concrete. When the areas of cross-section of the columns are known from the computed value of P_u/A_g, the percentage of reinforcement can be obtained directly from the lower section of the chart. The upper section of the chart is a plot of P_u/A_g versus P_u for different values of A_g. For a known value of P_u, a horizontal line can be drawn in the upper section to have several possible A_g values and the corresponding P_u/A_g values. Proceeding vertically down for any of the selected P_u/A_g value, the corresponding percentage of reinforcement can be obtained. Thus, the combined use of upper and lower sections of the chart would give several possible sizes of the member and the corresponding A_{sc} without performing any calculation. It is worth mentioning that there may be some parallax error while using the charts. However, use of chart is very helpful while deciding the sizes of columns at the preliminary design stage with several possible alternatives.

Another advantage of the chart is that, the amount of compression reinforcement obtained from the chart are always within the minimum and maximum percentages i.e., from 0.8 to 4 per cent. Hence, it is not needed to examine if the computed area of steel reinforcement is within the allowable range as is needed while using Direct Computation Method. This method is termed as SP-16 method while illustrating numerical examples.

Governing Equation of Short Axially Loaded Columns with Helical Ties

Columns with helical reinforcement take more load than that of tied columns due to additional strength of spirals in contributing to the strength of columns. Accordingly, cl. 39.4 recommends a multiplying factor of 1.05 regarding the strength of such columns. The code further recommends that the ratio of volume of helical reinforcement to the volume of core shall not be less than 0.36 $(A_g/A_c - 1)\,(f_{ck}/f_y)$, in order to apply the additional strength factor of 1.05 (cl. 39.4.1). Accordingly, the governing equation of the spiral columns may be written as

$$P_u = 1.05(0.4\,f_{ck}\,A_c + 0.67\,f_y\,A_{sc})$$

(10.8)

All the terms have been explained in sec.10.22.3.

Earlier observations of several investigators reveal that the effect of containing holds good in the elastic stage only and it gets lost when spirals reach the yield point. Again, spirals become fully effective after spalling off the concrete cover over the spirals due to excessive deformation. Accordingly, the above two points should be considered in the design of such columns. The first point is regarding the enhanced load carrying capacity taken into account by the multiplying factor of 1.05. The second point is maintaining specified ratio of volume of helical reinforcement to the volume of core, as specified in cl.39.4.1 and mentioned earlier.

The second point, in fact, determines the pitch p of the helical reinforcement, as explained below with reference to Fig.10.21.2b of Lesson 21.

Volume of helical reinforcement in one loop = $\pi (D_c - \phi_{sp})\, a_{sp}$ (10.9)

Volume of core = $(\pi / 4)\, D_c^2\, p$ (10.10)

where D_c = diameter of the core (Fig.10.21.2b)

ϕ_{sp} = diameter of the spiral reinforcement (Fig.10.21.2b)

a_{sp} = area of cross-section of spiral reinforcement

p = pitch of spiral reinforcement (Fig.10.21.2b) To satisfy the condition of cl.39.4.1 of IS 456, we have

$$\{\pi(D_c - \phi_{sp})\, a_{sp}\}/\{(\pi/4)\, D_c^2\, p\} \geq 0.36(A_g / A_c - 1)\,(f_{ck} / f_y)$$

which finally gives

$$p \leq 11.1(D_c - \phi_{sp})\, a_{sp}\, f/(D^2 - D^2)\, f$$

y c ck

(10.11)

Thus, Eqs.10.8 and 11 are the governing equations to determine the diameter of column, pitch of spiral and area of longitudinal reinforcement. It is worth mentioning that the pitch p of the spiral reinforcement, if determined from Eq.10.11, automatically satisfies the stipulation of cl.39.4.1 of IS 456. However, the pitch and diameter of the spiral reinforcement should also satisfy cl. 26.5.3.2 of IS 456:2000.

Illustrative Examples

Problem 1:

Design the reinforcement in a column of size 400 mm x 600 mm subjected to an axial load of 2000 kN under service dead load and live load. The column has an unsupported length of 4.0 m and effectively held in position and restrained against rotation in both ends. Use M 25 concrete and Fe 415 steel.

Solution 1:

Step 1: To check if the column is short or slender

Given l = 4000 mm, b = 400 mm and D = 600 mm. Table 28 of IS 456 = $l_{ex} = l_{ey} = 0.65(l)$ = 2600 mm. So, we have

l_{ex}/D = 2600/600 = 4.33 < 12

l_{ey}/b = 2600/400 = 6.5 < 12

Hence, it is a short column.

Step 2: Minimum eccentricity

$e_{x\,min}$ = Greater of (l_{ex}/500 + D/30) and 20 mm = 25.2 mm $e_{y\,min}$ = Greater of (l_{ey}/500 + b/30) and 20 mm = 20 mm 0.05 D = 0.05(600) = 30 mm > 25.2 mm (= $e_{x\,min}$)0.05 b = 0.05(400) = 20 mm = 20 mm (= $e_{y\,min}$)

Hence, the equation given in cl.39.3 of IS 456 (Eq.10.4) is applicable for the design here.

Step 3: Area of steel

Fro Eq.10.4, we have

$P_u = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}$ .... (10.4) 3000(10^3)

$= 0.4(25)\{(400)(600) - A_{sc}\} + 0.67(415)\, A_{sc}$

which gives,

A_{sc} = 2238.39 mm^2

Provide 6-20 mm diameter and 2-16 mm diameter rods giving 2287 mm^2 (> 2238.39 mm^2) and p = 0.953 per cent, which is more than minimum percentage of 0.8 and less than maximum percentage of 4.0. Hence, o.k.

Step 4: Lateral ties

The diameter of transverse reinforcement (lateral ties) is determined from cl.26.5.3.2 C-2 of IS 456 as not less than (i) ϕ /4 and (ii) 6 mm. Here, ϕ = largest bar diameter used as longitudinal reinforcement = 20 mm. So, the diameter of bars used as lateral ties = 6 mm.

The pitch of lateral ties, as per cl.26.5.3.2 C-1 of IS 456, should be not more than the least of

(i) the least lateral dimension of the column = 400 mm

(ii) sixteen times the smallest diameter of longitudinal reinforcement bar to be tied = 16(16) = 256 mm

(iii) 300 mm

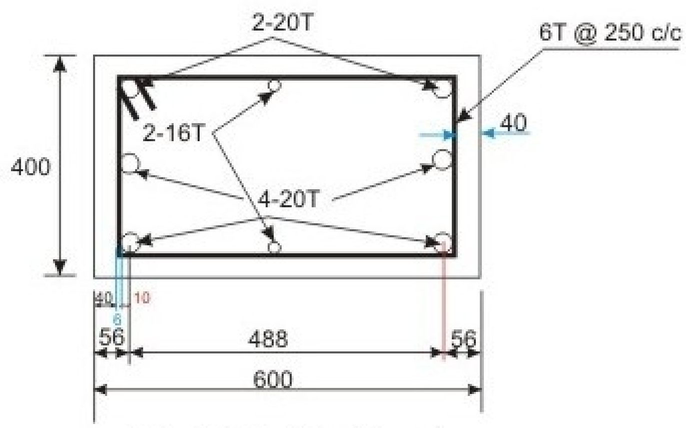


Fig. 10.22.1: Problem 1

Let us use p = pitch of lateral ties = 250 mm. The arrangement of longitudinal and transverse reinforcement of the column is shown in Fig. 10.22.1.

Problem 2:

Design the column of Problem 1 employing the chart of SP-16.

Solution 2:

Steps 1 and 2 are the same as those of Problem 1.

Step 3: Area of steel

$P_u/A_g = 3000(10^3)/(600)(400) = 12.5\ \text{N/mm}^2$

From the lower section of Chart 25 of SP-16, we get p = 0.95% when P_u/A_g = 12.5 N/mm^2 and concrete grade is M 25. This gives A_{sc} = 0.95(400)(600)/100 = 2288 mm^2. The results of both the problems are in good agreement. Marginally higher value of A_{sc} while using the chart is due to parallax error while reading the value from the chart. Here also, 6-20 mm diameter bars + 2-16 mm diameter bars (A_{sc} provided = 2287 mm^2) is o.k., though it is 1 mm^2 less.

Step 4 is the same as that of Problem 1. Figure 10.22.1, thus, is also the figure showing the reinforcing bars (longitudinal and transverse reinforcement) of this problem (same column as that of Problem 1).

Problem 3:

Design a circular column of 400 mm diameter with helical reinforcement subjected to an axial load of 1500 kN under service load and live load. The column has an unsupported length of 3 m effectively held in position at both ends but not restrained against rotation. Use M 25 concrete and Fe 415 steel.

Solution 3:

Step 1: To check the slenderness ratio

Given data are: unsupported length l = 3000 mm, D = 400 mm. Table 28 of Annex E of IS 456 gives effective length $l_e = l$ = 3000 mm. Therefore, l_e/D = 7.5
< 12 confirms that it is a short column.

Step 2: Minimum eccentricity

e_{min} = Greater of (l/500 + D/30) or 20 mm = 20 mm 0.05 D = 0.05(400)

= 20 mm

As per cl.39.3 of IS 456, e_{min} should not exceed 0.05D to employ the equation given in that clause for the design. Here, both the eccentricities are the same. So, we can use the equation given in that clause of IS 456 i.e., Eq.10.8 for the design.

Step 3: Area of steel

From Eq.10.8, we have

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \quad \dots \ (10.8)$$

$$A_c = A_g - A_{sc} = 125714.29 - A_{sc}$$

Substituting the values of P_u, f_{ck}, A_g and f_y in Eq.10.8,

$$1.5(1500)(10^3) = 1.05\{0.4(25)(125714.29 - A_{sc}) + 0.67(415)\ A_{sc}\}$$

we get the value of A_{sc} = 3304.29 mm^2. Provide 11 nos. of 20 mm diameter bars (= 3455 mm^2) as longitudinal reinforcement giving p = 2.75%. This p is between
0.8 (minimum) and 4 (maximum) per cents. Hence o.k.

Step 4: Lateral ties

It has been mentioned in sec.10.22.4 that the pitch p of the helix determined from Eq.10.11 automatically takes care of the cl.39.4.1 of IS 456. Therefore, the pitch is calculated from Eq.10.11 selecting the diameter of helical reinforcement from cl.26.5.3.2 d-2 of IS 456. However, automatic satisfaction of cl.39.4.1 of IS 456 is also checked here for confirmation.

Diameter of helical reinforcement (cl.26.5.3.2 d-2) shall be not less than greater of (i) one-fourth of the diameter of largest longitudinal bar, and (ii) 6 mm.

Therefore, with 20 mm diameter bars as longitudinal reinforcement, the diameter of helical reinforcement = 6 mm.

From Eq.10.11, we have

$$\text{Pitch of helix } p \le 11.1(D_c - \phi_{sp})\, a_{sp} f_y/(D^2 - D_c^2)\, f_{ck} \qquad \text{… (10.11)}$$

where $D_c = 400 - 40 - 40 = 320$ mm, $\phi_{sp} = 6$ mm, $a_{sp} = 28$ mm^2, $f_y = 415$ N/mm^2, $D = 400$ mm and $f_{ck} = 25$ N/mm^2.

So, $p \le 11.1(320 - 6)\,(28)\,(415)/(400^2 - 320^2)\,(25) \le 28.125$ mm

As per cl.26.5.3.2 d-1, the maximum pitch is the lesser of 75 mm and 320/6 = 53.34 mm and the minimum pitch is lesser of 25 mm and 3(6) = 18 mm. We adopt pitch = 25 mm which is within the range of 18 mm and 53.34 mm. So, provide 6 mm bars @ 25 mm pitch forming the helix.

Checking of cl. 39.4.1 of IS 456

The values of helical reinforcement and core in one loop are obtained from Eqs.10.8 and 9, respectively. Substituting the values of D_c, ϕ_{sp}, a_{sp} and pitch p in the above two equations, we have

Volume of helical reinforcement in one loop = 27632 mm^3 and Volume of core in one loop = 2011428.571 mm^3.

Their ratio = 27632/2011428.571 = 0.0137375 $0.36(A_g/A_c - 1)(f_{ck}/f_y) =$ 0.012198795

It is, thus, seen that the above ratio (0.0137375) is not less than $0.36(A_g/A_c - 1)(f_{ck}/f_y)$.

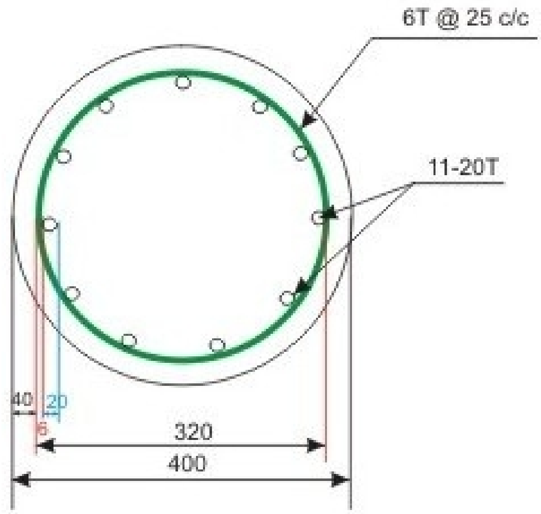


Fig. 10.22.2: Problem 3

Hence, the circular column of diameter 400 mm has eleven longitudinal bars of 20 mm diameter and 6 mm diameter helix with pitch p = 25 mm. The reinforcing bars are shown in Fig.10.22.2.

Design a short rectangular tied column of b = 300 mm having the maximum amount of longitudinal reinforcement employing the equation given in cl.39.3 of IS 456, to carry an axial load of 1200 kN under service dead load and live load using M 25 and Fe 415. The column is effectively held in position at both ends and restrained against rotation at one end. Determine the unsupported length of the column.

Step 1: Dimension *D* and area of steel A_{sc}

Substituting the values of P_u = 1.5(1200) = 1800 kN and A_{sc} = 0.04(300)D in Eq.10.4, we have

$1800(10^3) = 0.4(25)(300D)(1 - 0.04) + 0.67(415)(0.04)(300D)$

we get$D$ = 496.60 mm. Use 300 mm x 500 mm column.

A_{sc} = 0.04(300)(500) = 6000 mm^2, provide 4-36 mm diameter + 4-25 mm diameter bars to give 4071 + 1963 = 6034 mm^2 > 6000 mm^2.

Step 2: Lateral ties

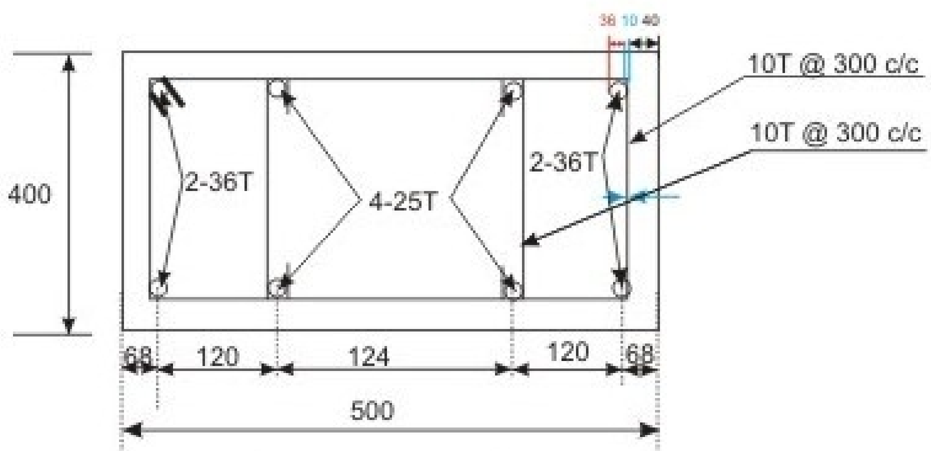


Fig. 10.22.3: Q.5

Diameter of lateral ties shall not be less than the larger of (i) 36/4 = 9 mm and (ii) 6 mm. Use 10 mm diameter bars as lateral ties.

Pitch of the lateral ties *p* shall not be more than the least of (i) 300 mm, (ii) 16(25) = 400 mm and (iii) 300 mm.

So, provide 10 mm diameter bars @ 300 mm c/c. The reinforcement bars are shown in Fig.10.22.3.

The centre to centre distance between two corner longitudinal bas along 500 mm direction is 500 – 2(4) + 10 + 18) = 364 mm which is less than 48 (diameter of lateral tie). Hence, the arrangement is satisfying Fig.9 of cl. 26.5.3.2 b-2 of IS 456.

Step 3: Unsupported length

As per the stipulation in cl. 25.1.2 of IS 456, the column shall be considered as short if l_{ex} = 12(D) = 6000 mm and l_{ey} = 12(300) = 3600 mm. For the type of support conditions given in the problem, Table 28 of IS 456 gives unsupported length is the least of (i) l = l_{ex}/0.8 = 6000/0.8 = 7500 mm and (ii) l_{ey}/0.8 = 3600/0.8 = 4500 mm. Hence, the unsupported length of the column is
4.5 m if the minimum eccentricity clause (cl. 39.3) is satisfied, which is checked in the next step.

Step 4: Check for minimum eccentricity

According to cl. 39.4 of IS 456, the minimum eccentricity of 0.05b or 0.05D shall not exceed as given in cl. 25.4 of IS 456. Thus, we have

(i) 0.05(500) = l/500 + 500/30 giving l = 4165 mm (ii)

0.05(300) = l/500 + 300/10 giving l = 2500 mm

Therefore, the column shall have the unsupported length of 2.5 m.

(a) Suggest five alternative dimensions of square short column with the minimum longitudinal reinforcement to carry a total factored axial load of 3000 kN using concrete of grades 20, 25, 30, 35 and 40 and Fe 415. Determine the respective maximum unsupported length of the column if it is effectively held in position at both ends but not restrained against rotation. Compare the given factored load of the column with that obtained by direct computation for all five alternative columns.

(b) For each of the five alternative sets of dimensions obtained in (a), determine the maximum factored axial load if the column is having maximum longitudinal reinforcement (i) employing SP-16 and (ii) by direct computation.

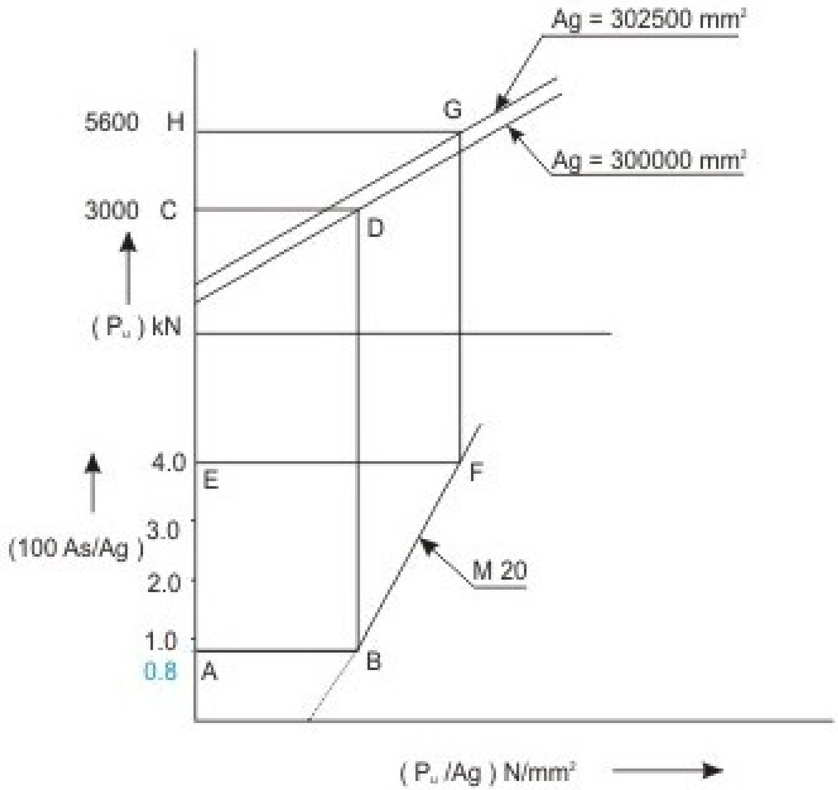


Fig. 10.22.4: Q.6, Chart 25 of SP-16 (not to scale)

Solution of Part (a):

Step 1: Determination of A_g and column dimensions *b* (= *D*)

Chart 25 of SP-16 gives all the dimensions of five cases. The two input data are P_u = 3000 kN and 100 A_s/A_g = 0.8. In the lower section of Chart 25, one horizontal line AB is drawn starting from A where *p* = 0.8 (Fig.10.22.4) to meet the lines for M 20, 25, 30, 35 and 40 respectively. In Fig.10.22.4, B is the meeting point for M 20 concrete. Separate vertical lines are drawn from these points of intersection to meet another horizontal line CD from the point C where P_u = 3000 kN in the upper section of the figure. The point *D* is the intersecting point. *D* happens to be on line when A_g = 3000 cm^2. Otherwise, it may be in between two liens with different values of A_g. For M 20, A_g = 3000 cm^2. However, in case the point is in between two lines with different values of A_g, the particular A_g has to be computed by linear interpolation. Thus, all five values of A_g are obtained.

The dimension $b = D = \sqrt{300000}$ = 550 mm. Other four values are obtained similarly. Table 10.1 presents the values of A_g and *D* along with other results as explained below.

Step 2: Unsupported length of each column

The unsupported length l is determined from two considerations:

(i) Clause 25.1.2 of IS 456 mentions that the maximum effective length l_{ex} is 12 times b or D (as $b = D$ here for a square column). The unsupported length is related to the effective length depending on the type of support. In this problem Table 28 of IS 456 stipulates $l = l_{ex}$. Therefore, maximum value of $l = 12D$.

(ii) The minimum eccentricity of cl. 39.3 should be more than the same as given in cl. 25.4. Assuming them to be equal, we get $l/500 + D/30 = D/20$, which gives $l = 8.33D$. For the column using M 20 and Fe 415, the unsupported length = 8.33(550) = 4581 mm. All unsupported lengths are presented in Table 10.1 using the equation

$$l = 8.33\,D \tag{1}$$

Step 3: Area of longitudinal steel

Step 1 shows that the area provided for the first case is 550 mm x 550 mm = 302500 mm^2, slightly higher than the required area of 300000 mm^2 for the practical aspects of construction. However, the minimum percentage of the longitudinal steel is to the calculated as 0.8 per cent of area required and not area provided (vide cl. 26.5.3.1 b of IS 456). Hence, for this case A_{sc} = 0.8(300000)/100 = 2400 mm^2. Provide 4-25 mm diameter + 4-12 mm diameter bars (area = 1963 + 452 = 2415 mm^2). Table 10.1 presents this and other areas of longitudinal steel obtained in a similar manner.

Step 4: Factored load by direct computation

Equation 10.4 is employed to calculate the factored load by determining A_c from A_g and A_{st}. With a view to comparing the factored loads, we will use the values of A_g as obtained from the chart and not the A_g actually provided. From Eq.10.4, we have

$$P_u \text{ from direct computation} = 0.4(f_{ck})(0.992\,A_g) + 0.67(f_y)(0.008)A_g \text{ or } P_u = A_g(0.3968\,f_{ck} + 0.00536\,f_y) \tag{2}$$

For the first case when A_g = 300000 mm^2, f_{ck} = 20 N/mm^2, and f_y = 415 N/mm^2, Eq.(2) gives P_u = 3048.12 kN. This value and other values of factored loads obtained from the direct computation are presented in Table 10.1.

Table 10.1 Results of Q.6a (Minimum Longitudinal Steel), given factored P_u = 3000 kN

Concrete grade	Gross area of concrete (A_g)		$b = D$ (cm)	Area of steel (A_{sc})			P_u by direct computation	l (m)
	Required (cm^2)	Provided (cm^2)		Required (cm^2)	Provided (cm^2)	Bars		
M 20	3000	3025	55	24	24.15	4-25 + 4-12	3048.12	4.581
M 25	2500	2500	50	20	20.60	4-20 + 4-16	3036.10	4.165
M 30	2200	2209	47	17.60	17.85	2.25 + 4-16	3108.25	3.915
M 35	1800	1806	42.5	14.40	14.57	2-28 + 2-12	2900.23	3.540
M 40	1600	1600	40	12.80	13.06	2-20 + 6-12	2895.42	3.332

Solution of Part (b):

Step 1: Determination of P_u

Due to the known dimensions of the column section, the A_g is now known. With known A_g and reinforcement percentage $100A_s/A_g$ as 4 per cent, the factored P_u shall be determined. For the first case, when $b = D = 550$ mm, $A_g = 302500$ mm^2. In Chart 25, we draw a horizontal line EF from E, where $100A_s/A_g = 4$ in the lower section of the chart (see Fig.10.22.4) to meet the M 20 line at F. Proceeding vertically upward, the line FG intersects the line $A_g = 302500$ at G. A horizontal line towards left from G, say GH, meets the load axis at H where $P_u = 5600$ kN. Similarly, P_u for other sets are determined and these are presented in Table 10.2, except for the last case when M 40 is used, as this chart has ended at $p = 3.8$ per cent.

Step 2: Area of longitudinal steel

The maximum area of steel, 4 per cent of gross area of column = 0.04(550)(550) = 12100 mm^2. Provide 12-36 mm diameter bars to have the actual area of steel = 12214 mm^2 > 12100 mm^2, as presented in Table 10.2.

Step 3: Factored P_u from direct computation

From Eq,10.4, as in Step 4 of the solution of Part (a) of this question, we have

$$P_u = 0.4\, f_{ck}\, (A_g - A_{sc}) + 0.67\, f_y\, A_{sc}$$

(3)

Substituting the values of A_g and A_{sc} actually provided, we get the maximum P_u of the same column when the longitudinal steel is the maximum. For the first case when A_g = 302500 mm^2, A_{sc} = 12214 mm^2, f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2, we get P_u = 5718.4 kN. This value along with other four values are presented in Table 10.2.

Remarks:

Tables 10.1 and 10.2 reveal that two sets of results obtained from charts of SP-16 and by direct computation methods are in good agreement. However, values obtained from the chart are marginally different from those obtained by direct computation both on the higher and lower sides. These differences are mainly due to personal error (parallax error) while reading the values with eye estimation from the chart.

Table 10.2 Results of Q.6(b) (Maximum Longitudinal Steel) given the respective A_g

Concrete grade	$b = D$ (cm)	Gross concrete area (A_s) (cm^2)	Area of steel (A_{sc})			P_u = Factored load	
			Required (cm^2)	Provided (cm^2)	Bars (No.)	SP-chart (kN)	Direct Computation (kN)
M 20	55	3025	121	122.14	12-36	5600	5718.4
M 25	50	2500	100	101.06	8-36 + 4-25	5200	5208.9
M 30	47	2209	88.36	88.97	8-32 +	5000	5017.8

					4-28		
M 35	42.5	1806.25	72.25	73.69	12-28	4500	4474.5
M 40	40	1600	64	64.46	8-28 + 4-32	Not available	4249.2

Design a short, helically reinforced circular column with minimum amount of longitudinal steel to carry a total factored axial load of 3000 kN with the same support condition as that of Q.6, using M 25 and Fe 415. Determine its unsupported length. Compare the results of the dimension and area of longitudinal steel with those of Q.6(a) when M 25 and Fe 415 are used.

Step 1: Diameter of helically reinforced circular column

As per cl. 39.4 of IS 456, applicable for short spiral column, we get from Eq.10.8

$$P_u = 1.05(0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}) \qquad .... \ (10.8)$$

Given data are: P_u = 3000 kN, A_c = /4 (D^2)(0.992), A_{sc} = 0.008(/4) D^2, f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2. So, we have

$$3000(10^3) = 1.05(12.1444)(\quad /4)D^2$$

giving D = 547.2 mm and A_g = 235264.2252 mm^2. Provide diameter of 550 mm.

Step 2: Area of longitudinal steel

Providing 550 mm diameter, the required A_g has been exceeded. Clause 26.5.3.1b stipulates that the minimum amount of longitudinal bar shall be determined on the basis of area required and not area provided for any column. Accordingly, the area of longitudinal steel = 0.008 A_g = 0.008(235264.2252) = 1882.12 mm^2. Provide 6-20 mm diameter bars (area = 1885 mm^2) as longitudinal steel, satisfying the minimum number of six bar (cl. 26.5.3.1c of IS 456).

Step 3: Helical reinforcement

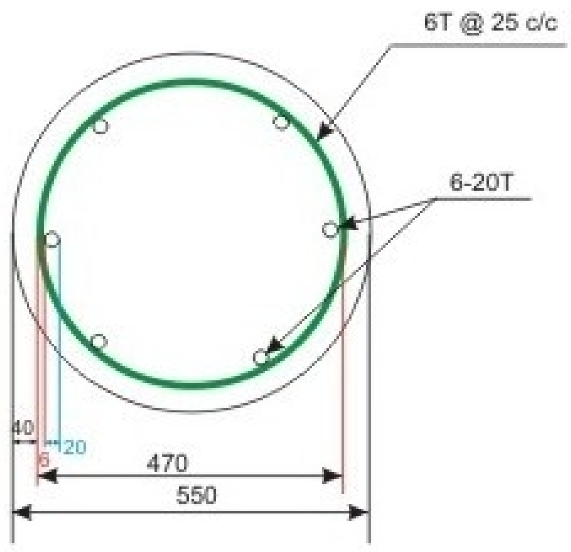


Fig. 10.22.5: Q.7

Minimum diameter of helical reinforcement is greater of (i) 20/4 or (ii) 6 mm. So, provide 6 mm diameter bars for the helical reinforcement (cl. 26.5.3.2d- 2 of IS 456). The pitch of the helix p is determined from Eq.10.11 as follows:

$$p \le 11.1(D_c - \phi_{sp})\, a_{sp}\, f_y/(D^2 - D_c^2)\, f_{ck} \qquad .... \ (10.11)$$

Using $D_c = 550 - 40 - 40 = 470$ mm, $\phi_{sp} = 6$ mm, $a_{sp} = 28$ mm^2, $D = 550$ mm,

$f_{ck} = 25$ N/mm^2 and $f_y = 415$ N/mm^2, we get

$$p \le 11.1(470 - 6)(28)(415)/(550^2 - 470^2)(25) \le 29.34 \text{ mm}$$

Provide 6 mm diameter bar @ 25 mm c/c as helix. The reinforcement bars are shown in Fig. 10.22.5. Though use of Eq.10.11 automatically checks the stipulation of cl. 39.4.1 of IS 456, the same is checked as a ready reference in Step 4 below.

Step 4: Checking of cl. 39.4.1 of IS 456

Volume of helix in one loop = $\pi(D_c - \phi_{sp})\, a_{sp}$ …. (10.9)

Volume of core = $(\pi/4)\ D_c^2\ (p)$ …. (10.10)

The ratio of Eq.10.9 and Eq.10.10 = $4(D_c - \phi_{sp})\, a_{sp}/D_c^2\, p$

$$= 4(470 - 6)(28)/(470)(470)(25) = 0.009410230874$$

This ratio should not be less than $0.36(A_g/A_c - 1)(f_{ck}/f_y)$

$$= 0.36\{(D^2/D_c^2) - 1)\}\ (f_{ck}/f_y) = 0.008011039177$$

Hence, the stipulation of cl. 39.4.1 is satisfied.

Step 5: Unsupported length

The unsupported length shall be the minimum of the two obtained from (i) short column requirement given in cl. 25.1.2 of IS 456 and (ii) minimum eccentricity requirement given in cls. 25.4 and 39.3 of IS 456. The two values are calculated separately:

(i) $l = l_e = 12D = 12(550) = 6600$ mm

(ii) $l/500 + D/30 = 0.05\,D$ gives $l = 4583.3$ mm So, the

unsupported length of this column = 4.58 m. **Step 6:**

Comparison of results

Table 10.3 presents the results of required and actual gross areas of concrete and area of steel bars, dimensions of column and number and diameter of longitudinal reinforcement of the helically reinforced circular and the square columns of Q.6(a) when M 20 and Fe 415 are used for the purpose of comparison.

Table 10.3 Comparison of results of circular and square columns with minimum longitudinal steel (P_u = 3000 kN, M 25, Fe 415)

Column shape and Q.No.	Gross concrete area			Area of steel		
	Required (cm^2)	Provided (cm^2)	Dimension *D* (cm)	Required (cm^2)	Provided (cm^2)	Bar dia. and No. (mm,No.)
Circular (Q.7)	2352.64	2376.78	55	18.82	18.85	6-20
Square (Q.6(a)	2500	2500	50	20	20.6	4-20 + 4-16

Behaviour of Short Columns under Axial Load and Uniaxial Moment

Normally, the side columns of a grid of beams and columns are subjected to axial load *P* and uniaxial moment M_x causing bending about the major axis xx, hereafter will be written as *M*. The moment *M* can be made equivalent to the axial load *P* acting at an eccentricity of *e* (= *M/P*). Let us consider a symmetrically reinforced short rectangular column subjected to axial load P_u at an eccentricity of *e* to have M_u causing failure of the column.

Figure 10.21.11b of Lesson 21 presents two strain profiles IN and EF. For the strain profile IN, the depth of the neutral axis *kD* is less than *D*, i.e., neutral axis is within the section resulting the maximum compressive strain of 0.0035 on the right edge and tensile strains on the left of the neutral axis forming cracks. This column is in a state of collapse for the axial force P_u and moment M_u for which IN is the strain profile. Reducing the eccentricity of the load P_u to zero, we get the other strain profile EF resulting in the constant compressive strain of 0.002, which also is another collapse load. This axial load P_u is different from the other one, i.e., a pair of P_u and M_u, for which IN is the profile. For the strain profile EF, the neutral axis is at infinity (*k =).*

Figure 10.21.11c of Lesson 21 presents the strain profile EF with two more strain profiles IH and JK intersecting at the fulcrum point V. The strain profile IH has the neutral axis depth $kD = D$, while other strain profile JK has $kD >$ *D*. The load and its eccentricity for the strain profile IH are such that the maximum compressive strain reaches 0.0035 at the right edge causing collapse of the column, though the strains throughout the depth is compressive and zero at the left edge. The strain profile JK has the maximum compressive strain at the right edge between 0.002 and 0.0035 and the minimum compressive strain at the left edge. This strain profile JK also causes collapse of the column since the maximum compressive strain at the right edge is a limiting strain satisfying assumption (ii) of sec. 10.21.10 of Lesson 21.

The four strain profiles, IN, EF, JK and IH of Figs.10.21.11b and c, separately cause collapse of the same column when subjected to four different pairs of P_u and M_u. This shows that the column may collapse either due to a uniform constant strain throughout (= 0.002 by EF) or due to the maximum compressive strain at the right edge satisfying assumption (ii) of sec.10.21.10 of irrespective of the strain at the left edge (zero for IH and tensile for IN). The positions of the neutral axis and the eccentricities of the load are widely varying as follows:

(i) For the strain profile EF, *kD* is infinity and the eccentricity of the load is zero.

(ii) For the strain profile JK, *kD* is outside the section (*D* < *kD* <), with appropriate eccentricity having compressive strain in the section.

(iii) For the strain profile IH, *kD* is just at the left edge of the section (*kD* = *D*), with appropriate eccentricity having zero and 0.0035 compressive strains at the left and right edges, respectively.

(iv) For the strain profile IN, *kD* is within the section (*kD* < *D*), with appropriate eccentricity having tensile strains on the left of the neutral axis and 0.0035 compressive strain at the right edge.

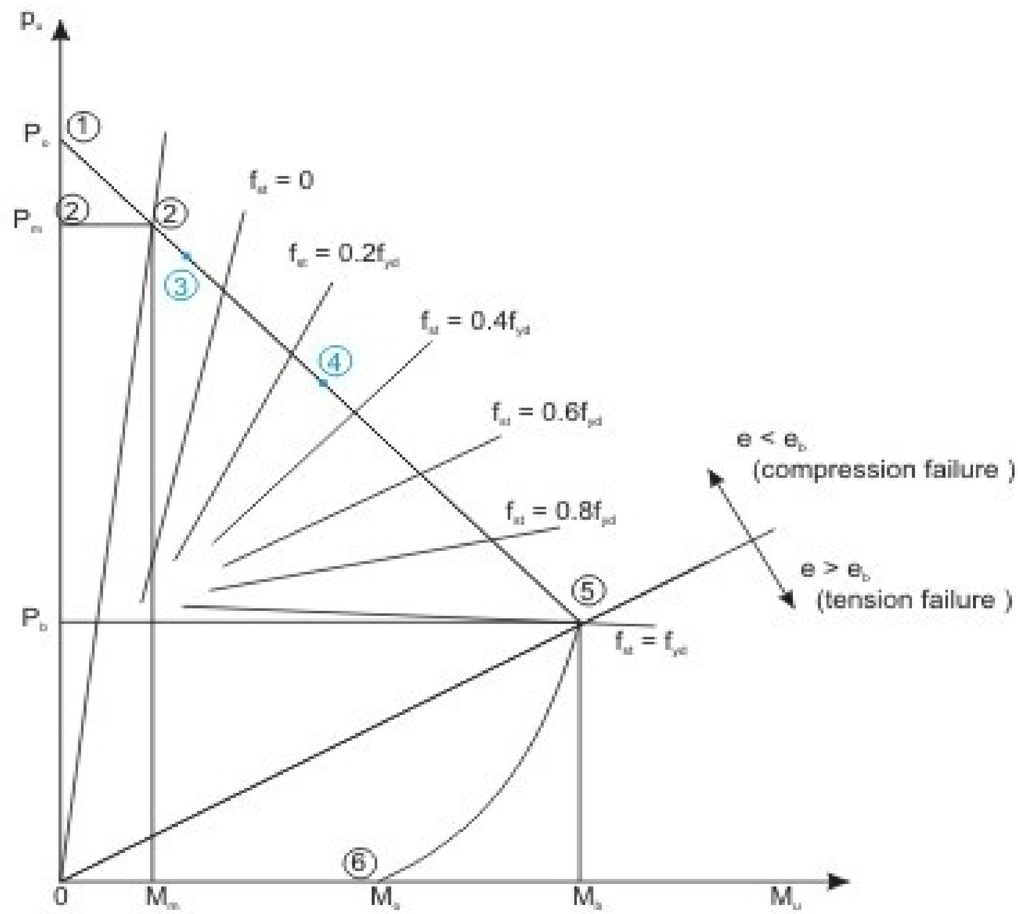


Fig. 10.23.1: Typical interaction diagram

It is evident that gradual increase of the eccentricity of the load P_u from zero is changing the strain profiles from EF to JK, IH and then to IN. Therefore, we can accept that if we increase the eccentricity of the load to infinity, there will be only M_u acting on the column. Designating by P_o as the load that causes collapse of the column when acting alone and M_o as the moment that also causes collapse when acting alone, we mark them in Fig.10.23.1 in the vertical and horizontal axes. These two points are the extreme points on the plot of P_u versus M_u, any point on which is a pair of P_u and M_u (of different magnitudes) that will cause collapse of the same column having the neutral axis either outside or within the column.

The plot of P_u versus M_u of Fig.10.23.1 is designated as interaction diagram since any point on the diagram gives a pair of values of P_u and M_u causing collapse of the same column in an interactive manner. Following the same logic, several alternative column sections with appropriate longitudinal steel bars are also possible for a particular pair of P_u and M_u. Accordingly for the purpose of designing the column, it is essential to understand the different modes of failure of columns, as given in the next section.

Modes of Failure of Columns

The two distinct categories of the location of neutral axis, mentioned in the last section, clearly indicate the two types of failure modes: (i) compression failure, when the neutral axis is outside the section, causing compression throughout the section, and (ii) tension failure, when the neutral axis is within the section developing tensile strain on the left of the neutral axis. Before taking up these two failure modes, let us discuss about the third mode of failure i.e., the balanced failure.

(A) Balanced failure

Under this mode of failure, yielding of outer most row of longitudinal steel near the left edge occurs simultaneously with the attainment of maximum compressive strain of 0.0035 in concrete at the right edge of the column. As a result, yielding of longitudinal steel at the outermost row near the left edge and crushing of concrete at the right edge occur simultaneously. The different yielding strains of steel are determined from the following:

(i) For mild steel (Fe 250): $\varepsilon_y = 0.87f_y/E_s$

(10.12)

(ii) For cold worked deformed bars: $\varepsilon_y = 0.87f_y/E_s + 0.002$

(10.13)

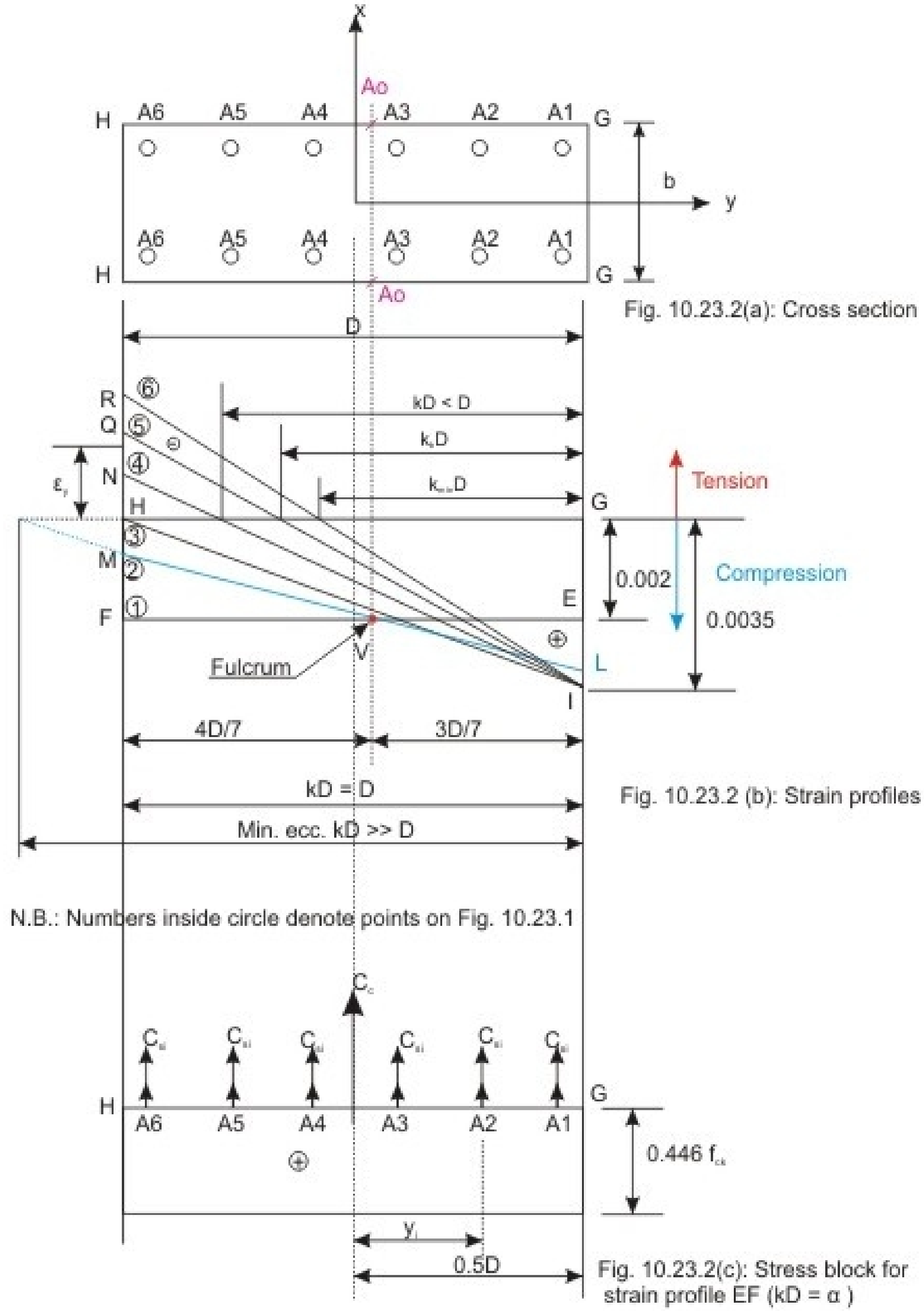


Fig. 10.23.2: Strain profiles and stress block for the strain profile EF

The corresponding numerical values are 0.00109, 0.0038 and 0.00417 for Fe 250, Fe 415 and Fe 500, respectively. Such a strain profile is known a balanced strain profile which is shown by the strain profile IQ in Fig.10.23.2b with a number

5. This number is shown in Fig.10.23.1 lying on the interaction diagram causing

collapse of the column. The depth of the neutral axis is designated as k_bD and shown in Fig.10.23.2b. The balanced strain profile IQ in Fig.10.23.2b also shows the strain ε_y whose numerical value would change depending on the grade of

steel as mentioned earlier. It is also important to observe that this balanced profile IQ does not pass trough the fulcrum point V in Fig.10.23.2b, while other profiles 1, 2 and 3 i.e., EF, LM and IN pass through the fulcrum point V as none of them produce tensile strain any where in the section of the column. The neutral axis depth for the balanced strain profile IQ is less than D, while the same for the other three are either equal to or more than D.

To have the balanced strain profile IQ causing balanced failure of the column, the required load and moment are designated as P_b and M_b , respectively and shown in Fig.10.23.1 as the coordinates of point 5. The corresponding eccentricity of the load P_b is defined by the notation e_b (= M_b/P_b). The four parameters of the balanced failure are, therefore, P_b, M_b, e_b and k_b (the coefficient of the neutral axis depth k_bD).

(B) Compression failure

Compression failure of the column occurs when the eccentricity of the load P_u is less than that of balanced eccentricity ($e < e_b$) and the depth of the neutral axis is more than that of balanced failure. It is evident from Fig.10.23.2b that these strain profiles may develop tensile strain on the left of the neutral axis till $kD = D$. All these strain profiles having $1 > k > k_b$ will not pass through the fulcrum point V. Neither the tensile strain of the outermost row of steel on the left of the neutral axis reaches ε_y .

On the other hand, all strain profiles having kD greater than D pass through the fulcrum point V and cause compression failure (Fig.10.23.2b). The loads causing compression failure are higher than the balanced load P_b having the respective eccentricities less than that of the load of balanced failure. The extreme strain profile is EF marked by 1 in Fig.10.23.2b. Some of these points causing compression failure are shown in Fig.10.23.1 as 1, 2, 3 and 4 having $k > k_b$, either within or outside the section.

Three such strain profiles are of interest and need further elaboration. One of them is the strain profile IH (Fig.10.23.2b) marked by point 3 (Fig.10.23.1) for which $kD = D$. This strain profile develops compressive strain in the section with zero strain at the left edge and 0.0035 in the right edge as explained in sec.
10.23.2. Denoting the depth of the neutral axis by D and eccentricity of the load for this profile by e_D, we observe that the other strain profiles LM and EF (Fig.10.23.2b), marked by 2 and 1 in Fig.10.23.1, have the respective $kD > D$ and $e < e_D$.

The second strain profile is EF (Fig.10.23.2b) marked by point 1 in Fig.10.23.1 is for the maximum capacity of the column to carry the axial load P_o when eccentricity is zero and for which moment is zero and the neutral axis is at infinity. This strain profile has also been discussed earlier in sec.10.23.2.

The third important strain profile LM, shown in Fig.10.23.2b and by point 2 in Figs.10.23.1 and 2, is also due to another pair of collapse P_m and M_m, having the capacity to accommodate the minimum eccentricity of the load, which hardly can be avoided in practical construction or for other reasons. The load P_m , as seen from Fig.10.23.1, is less than P_o and the column can carry P_m and M_m in an interactive mode to cause collapse. Hence, a column having the capacity to carry the truly concentric load P_o (when $M = 0$) shall not be allowed in the design. Instead, its maximum load shall be restricted up to P_m ($< P_o$) along with M_m (due to minimum eccentricity). Accordingly, the actual interaction diagram to be used for the purpose of the design shall terminate with a horizontal line 22' at point 2 of Fig.10.23.1. Point 2 on the interaction diagram has the capacity of P_m with M_m having eccentricity of e_m (= M_m/P_m) and the depth of the neutral axis is >> D (Fig.10.23.2b).

It is thus seen that from points 1 to 5 (i.e., from compression failure to balanced failure) of the interaction diagram of Fig.10.23.1, the loads are gradually decreasing and the moments are correspondingly increasing. The eccentricities of the successive loads are also increasing and the depths of neutral axis are decreasing from infinity to finite but outside and then within the section up to k_bD at balanced failure (point 5). Moreover, this region of compression failure can be subdivided into two zones: (i) zone from point 1 to point 2, where the eccentricity of the load is less than the minimum eccentricity that should be considered in the actual design as specified in IS 456, and (ii) zone from point 2 to point 5, where the eccentricity of the load is equal to or more than the minimum that is specified in IS 456. It has been mentioned also that the first zone from point 1 to point 2 should be avoided in the design of column.

(C) Tension failure

Tension failure occurs when the eccentricity of the load is greater than the balanced eccentricity e_b. The depth of the neutral axis is less than that of the balanced failure. The longitudinal steel in the outermost row on the left of the neutral axis yields first. Gradually, with the increase of tensile strain, longitudinal steel of inner rows, if provided, starts yielding till the compressive strain reaches 0.0035 at the right edge. The line IR of Fig.10.23.2b represents such a profile for which some of the inner rows of steel bars have yielded and compressive strain has reached 0.0035 at the right edge. The depth of the neutral axis is designated by ($k_{min}D$).

It is interesting to note that in this region of the interaction diagram (from 5 to 6 in Fig.10.23.1), both the load and the moment are found to decrease till point 6 when the column fails due to M_o acting alone. This important behaviour is explained below starting from the failure of the column due to M_o alone at point 6 of Fig.10.23.1.

At point 6, let us consider that the column is loaded in simple bending to the point (when $M = M_o$) at which yielding of the tension steel begins. Addition of some axial compressive load P at this stage will reduce the previous tensile stress of steel to a value less than its yield strength. As a result, it can carry additional moment. This increase of moment carrying capacity with the increase of load shall continue till the combined stress in steel due to additional axial load and increased moment reaches the yield strength.

Interaction Diagram

It is now understood that a reinforced concrete column with specified amount of longitudinal steel has different carrying capacities of a pair of P_u and M_u before its collapse depending on the eccentricity of the load. Figure 10.23.1 represents one such interaction diagram giving the carrying capacities ranging from P_o with zero eccentricity on the vertical axis to M_o (pure bending) on the horizontal axis. The vertical axis corresponds to load with zero eccentricity while the horizontal axis represents infinite value of eccentricity. A radial line joining the origin O of Fig.10.23.1 to point 2 represents the load having the minimum eccentricity. In fact, any radial line represents a particular eccentricity of the load. Any point on the interaction diagram gives a unique pair of P_u and M_u that causes the state of incipient failure. The interaction diagram has three distinct zones of failure: (i) from point 1 to just before point 5 is the zone of compression failure, (ii) point 5 is the balanced failure and (iii) from point 5 to point 6 is the zone of tension failure. In the compression failure zone, small eccentricities produce failure of concrete in compression, while large eccentricities cause failure triggered by yielding of tension steel. In between, point 5 is the critical point at which both the failures of concrete in compression and steel in yielding occur simultaneously.

The interaction diagram further reveals that as the axial force P_u becomes larger the section can carry smaller M_u before failing in the compression zone. The reverse is the case in the tension zone, where the moment carrying capacity M_u increases with the increase of axial load P_u. In the compression failure zone, the failure occurs due to over straining of concrete. The large axial force produces high compressive strain of concrete keeping smaller margin available for additional compressive strain line to bending. On the other hand, in the tension failure zone, yielding of steel initiates failure. This tensile yield stress reduces with the additional compressive stress due to additional axial load. As a result, further moment can be applied till the combined stress of steel due to axial force and increased moment reaches the yield strength.

Therefore, the design of a column with given P_u and M_u should be done following the three steps, as given below:

(i) Selection of a trial section with assumed longitudinal steel,

(ii) Construction of the interaction diagram of the selected trial column section by successive choices of the neutral axis depth from infinity (pure axial load) to a very small value (to be found by trial to get $P = 0$ for pure bending),

(iii) Checking of the given P_u and M_u, if they are within the diagram.

We will discuss later whether the above procedure should be followed or not. Let us first understand the corresponding compressive stress blocks of concrete for the two distinct cases of the depth of the neutral axis: (i) outside the cross-section and (ii) within the cross-section in the following sections.

Compressive Stress Block of Concrete when the Neutral Axis Lies Outside the Section

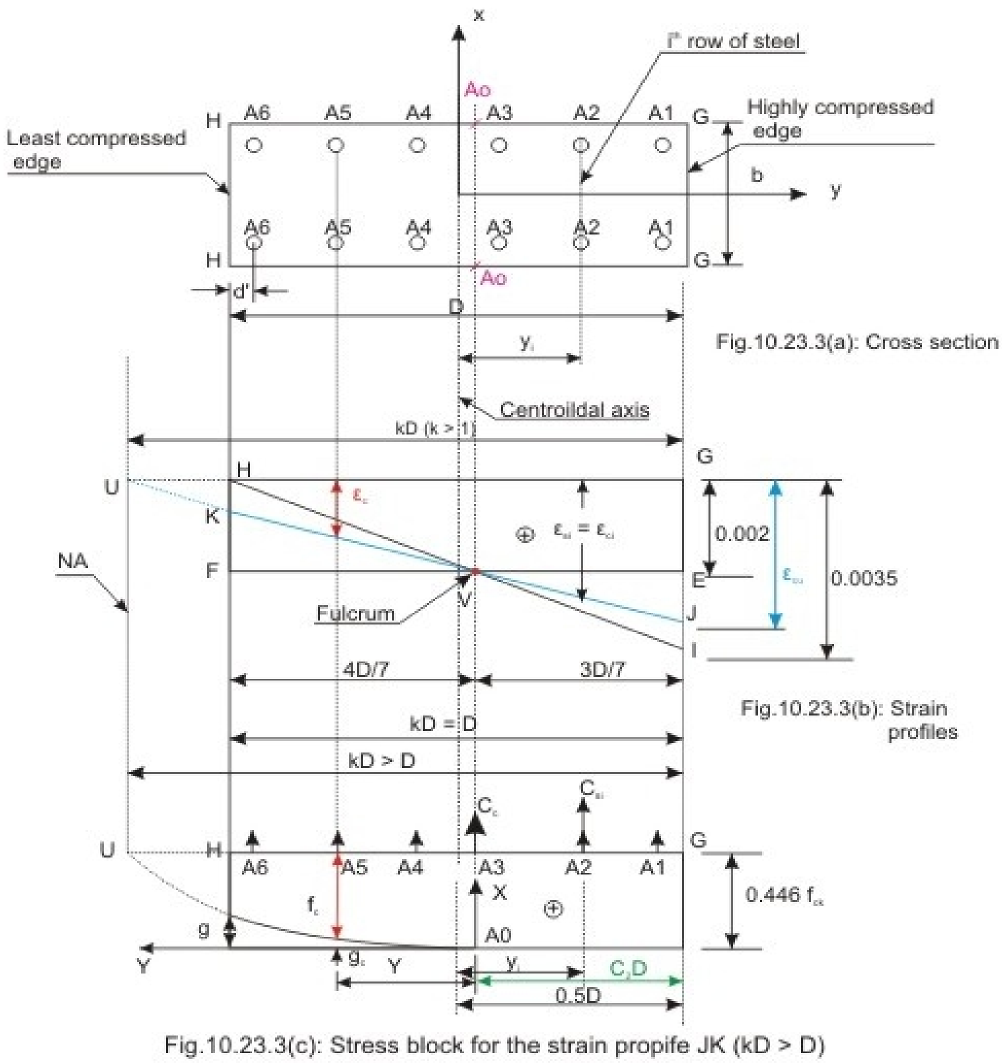


Fig.10.23.3: Cross section of column, strain profiles and stress block for the strain profile JK (kD > D)

Figure 10.23.3c presents the stress block for a typical strain profile JK having neutral axis depth kD outside the section ($k > 1$). The strain profile JK in Fig.10.23.3b shows that up to a distance of $3D/7$ from the right edge (point AO),

the compressive strain is $\geq$ 0.002 and, therefore, the compressive stress shall remain constant at $0.446f_{ck}$. The remaining part of the column section of length $4D/7$, i.e., up to the left edge, has reducing compressive strains (but not zero). The stress block is, therefore, parabolic from AO to H which becomes zero at U (outside the section). The area of the compressive stress block shall be obtained subtracting the parabolic area between AO to H from the rectangular area between G and H. To establish the expression of this area, it is essential to know the equation of the parabola between AO and U, whose origin is at AO. The positive coordinates of X and Y are measured from the point AO upwards and to the left, respectively. Let us assume that the general equation of the parabola as

$X = aY^2 + bY + c$

(10.14)

The values of a, b and c are obtained as follows:

(i) At $Y = 0$, $X = 0$, at the origin: gives $c = 0$

(ii) At $Y = 0$, $dX/dY = 0$, at the origin: gives $b = 0$

(iii) At $Y = (kD - 3D/7)$, i.e., at point U, $X = 0.446f_{ck}$: gives $a = 0.446f_{ck}/D^2(k - 3/7)^2$.

Therefore, the equation of the parabola is:

$X = \{0.446f_{ck}/D^2(k - 3/7)^2\}Y^2$ (10.15)

The value of X at the point H (left edge of the column), g is now determined from Eq.10.15 when $Y = 4D/7$, which gives

$g = 0.446\,f_{ck}\,\{4/(7k - 3)\}^2$ (10.16)

Hence, the area of the compressive stress block = $0.446\,f_{ck}\,D\,[1 - (4/21)\{4/(7k - 3)\}^2]$
$= C_1\,f_{ck}\,D$

(10.17)

where $C_1 = 0.446[1 - (4/21)\{4/(7k - 3)\}^2]$ (10.18)

Equation 10.17 is useful to determine the area of the stress block for any value of $k > 1$ (neutral axis outside the section) by substituting the value of C_1 from Eq.10.18. The symbol C_1 is designated as the coefficient for the area of the stress block.

The position of the centroid of the compressive stress block is obtained by dividing the moment of the stress block about the right edge by the area of the stress block. The moment of the stress block is obtained by subtracting the moment of the parabolic part between AO and H about the right edge from the moment of the rectangular stress block of full depth D about the right edge. The expression of the moment of the stress block about the right edge is:

$0.446\,f_{ck}\,D(D/2) - (1/3)(4D/7)\,0.446\,f_{ck}\,\{4/(7k - 3)\}^2\,\{3D/7 + (3/4)(4D/7)\}$

$= 0.446\, f_{ck}\, D^2\, [(1/2) - (8/49)\{4/(7k - 3)\}^2]$ (10.19)

Dividing Eq.10.19 by Eq.10.17, we get the distance of the centroid from the right edge is:

$D[(1/2) - (8/49)\{4/(7k - 3)\}^2]/[1 - (4/21)\{4/(7k - 3)\}^2]$ (10.20)

$= C_2\, D$

(10.21)

where C_2 is the coefficient for the distance of the centroid of the compressive stress block of concrete measured from the right edge and is:

$C_2 = [(1/2) - (8/49)\{4/(7k - 3)\}^2]/[1 - (4/21)\{4/(7k - 3)\}^2]$ (10.22)

Table 10.4 presents the values of C_1 and C_2 for different values of k greater than 1, as given in Table H of SP-16. For a specific depth of the neutral axis, k is known. Using the corresponding values of C_1 and C_2 from Table 10.4, area of the stress block of concrete and the distance of centroid from the right edge are determined from Eqs.10.17 and 10.21, respectively.

Table 10.4 Stress block parameters C_1 and C_2 when the neutral axis is outside the section

K	C_1	C_2
1.00	0.361	0.416
1.05	0.374	0.432
1.10	0.384	0.443
1.20	0.399	0.458
1.30	0.409	0.468
1.40	0.417	0.475
1.50	0.422	0.480
2.00	0.435	0.491
2.50	0.440	0.495
3.00	0.442	0.497
4.00	0.444	0.499

It is worth mentioning that the area of the stress block is $0.446 f_{ck} D$ and the distance of the centroid from the right edge is $0.5D$, when k is infinite. Values of C_1 and C_2 at $k = 4$ are very close to those when $k = \infty$. In fact, for the practical interaction diagrams, it is generally adequate to consider values of k up to about 1.2.

Determination of Compressive Stress Anywhere in the Section when the Neutral Axis Lies outside the Section

The compressive stress of concrete at any point between G and AO of Fig.10.23.3c is constant at $0.446f_{ck}$ as the strain in this zone is equal to or greater than 0.002. So, we can write

$$f_c = 0.446f_{ck} \text{ if } 0.002 \le \varepsilon_c \le 0.0035$$

(10.23)

However, compressive stress of concrete between AO and H is to be determined using the equation of parabola. Let us determine the concrete stress f_c at a distance of Y from the origin AO. From Fig.10.23.3c, we have

$$f_c = 0.446\, f_{ck} - g_c$$

(10.24)

where g_c is as shown in Fig.10.23.3c and obtained from Eq.10.15. Thus, we get

$$f_c = 0.446\, f_{ck} - \{0.446\, f_{ck}/D^2(k - 3/7)^2\}Y^2 \text{ or } f_c$$

$$= 0.446\, f_{ck}\,\{1 - Y^2/(kD - 3D/7)^2\}$$

(10.25)

Designating the strain of concrete at this point by have from similar ε_c (Fig.10.23.3b), we triangles

$$\varepsilon_c/0.002 = 1 - Y/(kD - 3D/7)$$

which gives

$Y = \{1 - (/0.002)\}(kD - 3D/7)$ (10.26)

Substituting the value of Y from Eq.10.26 in Eq.10.25, we have

$$f_c = 0.446\, f_{ck}\,[2(\varepsilon_c/0.002) - (\varepsilon_c/0.002)^2], \text{ if } 0 \le \varepsilon_c < 0.002$$

(10.27)

Compressive Stress Block of Concrete when the Neutral Axis is within the Section

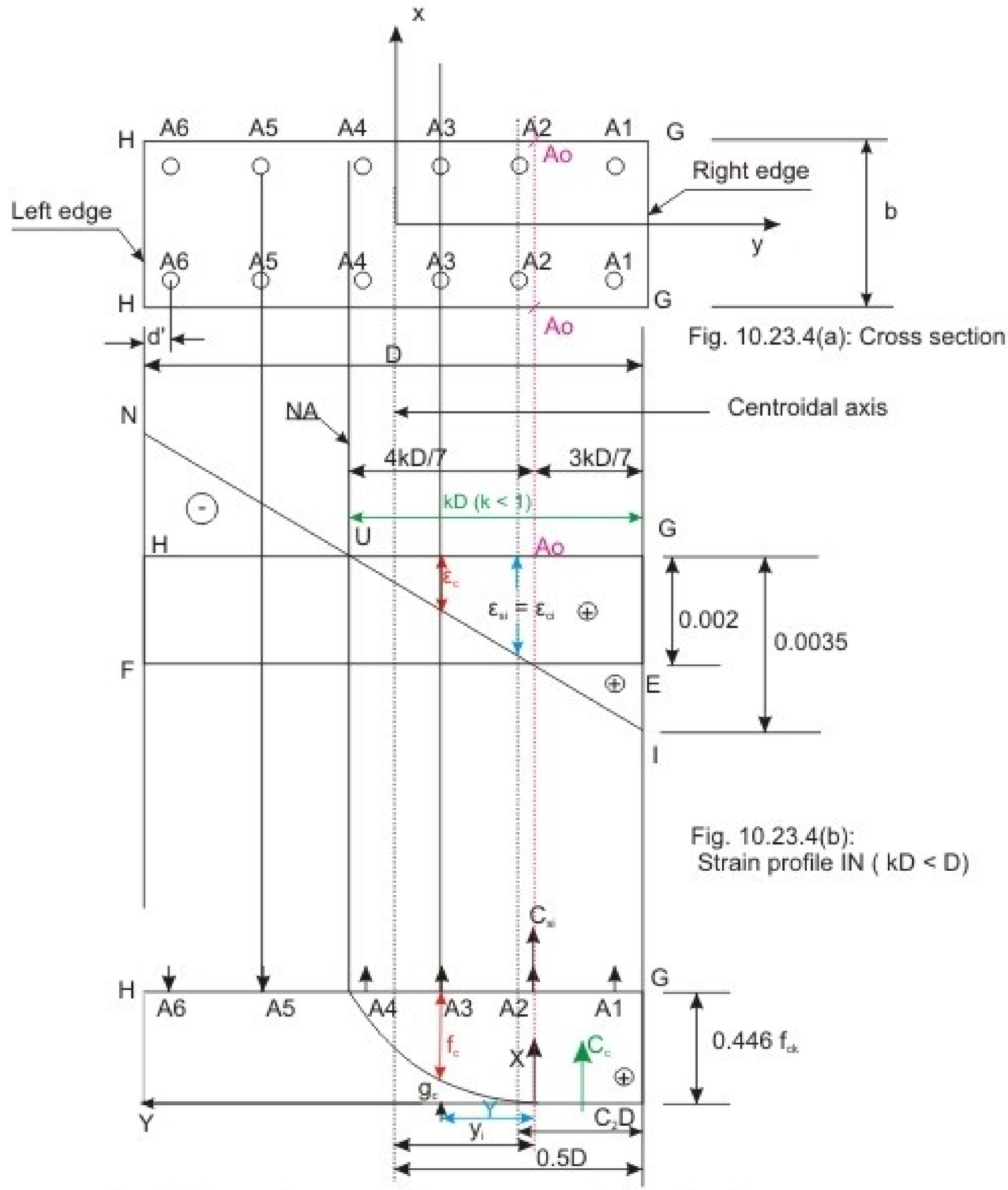


Fig. 10.23.4(c): Stress block for the strain profile IN (kD < D)

Fig. 10.23.4: Cross section, strain profile IN and stress block for strain profile IN

Figure 10.23.4c presents the stress block for a typical strain profile IN having neutral axis depth = kD within the section ($k < 1$). The strain profile IN in Fig.10.23.4b shows that from a to AO, i.e., up to a distance of $3kD/7$ from the

right edge, the compressive strain is $\geq$ 0.002 and, therefore, the compressive stress shall remain constant at $0.446f_{ck}$. From AO to U, i.e., for a distance of $4kD/7$, the strain is reducing from 0.002 to zero and the stress in this zone is parabolic as shown in Fig.10.23.4c. The area of the stress block shall be obtained subtracting the parabolic area between AO and U from the total rectangular area between G and U. As in the case when the neutral axis is outside the section (sec.10.23.5), we have to establish the equation of the parabola with AO as the origin and the positive coordinates X and Y are measured from the point AO upwards for X and from the point AO to the left for Y, as shown in Fig.10.23.4c. Proceeding in the same manner as in sec.10.23.5 and assuming the same equation of the parabola as in Eq.10.14, the values of a, b and c are obtained as:

(iv) At $Y = 0$, $X = 0$, at the origin: gives $c = 0$

(v) At $Y = 0$, $dX/dY = 0$, at the origin: gives $b = 0$

(iii) At U, (i.e., at $Y = 4kD/7$), $X = 0.446\, f_{ck}$: gives $a = 0.446\, f_{ck}/(4kD/7)^2$.

Therefore, the equation of the parabola OR is:

$$X = \{0.446\, f_{ck}/(4kD/7)^2\}Y^2$$

(10.28)

The area of the stress block = $0.446\, f_{ck}\, kD - (1/3)\, 0.446\, f_{ck}\, (4kD/7) = 0.36\, f_{ck}\, kD$, the same as obtained earlier in Eq.3.9 of Lesson 4 for flexural members. Similarly, the distance of the centroid can be obtained by dividing the moment of area of stress block about the right edge by the area of the stress block. The result is the same as in Eq.3.12 for the flexural members. Therefore, we have

Area of the stress block = $0.36\, f_{ck}\, kD$
(10.29)

The distance of the centroid of the stress block from the right edge = $0.42kD$ (10.30)

Thus, the values of C_1 and C_2 of Eqs.10.17 and 10.21, respectively, are 0.36 and 0.42 when the neutral axis is within the section. It is to be noted that the coefficients C_1 and C_2 are multiplied by Df_{ck} and D, respectively when the neutral axis is outside the section. However, they are to be multiplied here, when the neutral axis is within the section, by kDf_{ck} and kD, respectively.

It is further to note that though the expressions of the area of stress block and the distance of the centroid of the stress block from the right edge are the same as those for the flexural members, the important restriction of the maximum depth of the neutral axis x_{umax} in the flexural members is not applicable in case of column. By this restriction, the compression failure of the flexural members is avoided. In case of columns, compression failure is one of the three modes of failure.

Determination of Compressive Stress Anywhere in the Compressive Zone when the Neutral Axis is within the Section

The compressive stress at any point between G and AO of Fig.10.23.4c is constant at $0.446f_{ck}$ as the strain in this zone is equal to or greater than 0.002. So, we can write

$f_c = 0.446\, f_{ck}$ if $0.002 \le \varepsilon_c \le 0.0035$ ….

(10.23)

However, the compressive stress between AO and U is to be determined from the equation of the parabola. Let us determine the compressive stress f_{ci} at a distance of Y from the origin AO. From Fig.10.23.4c, we have

$f_c = 0.446\, f_{ck} - g_c$

(10.31)

where g_c as shown in Fig.10.23.4c, is obtained from Eq.10.28. Thus, we get,

$f_c = \{0.446\, f_{ck} – 0.446\, f_{ck}\, (4kD/7)^2\}Y^2$ (10.32)

Designating the strain of concrete at this point by ε_c (Fig.10.23.4b), we have from similar triangles

$\varepsilon_c /0.002 = 1 – Y/(4kD/7)$, which gives

$Y = \{1 - \varepsilon_c /0.002\}(4kD/7)$ (10.33)

Substituting the value of Y from Eq.10.33 in Eq.10.32, we get the same equation, Eq.10.27 of sec.10.23.6, when the neutral axis is outside the section. Therefore,

$f_c = 0.446\, f_{ck}\, [2(\varepsilon_c /0.002) – (\varepsilon_c /0.002)^2]$ …. (10.26)

From the point U to the left edge H of the cross-section of the column, the compressive stress is zero. Thus, we have

$f_c = 0$ if $\varepsilon_c \le 0$

$f_c = 0.446 f_{ck}$ if $\varepsilon_c \geq 0.002$

$f_c = 0.446 f_{ck}\{2(\varepsilon_c/0.002) - (\varepsilon_c/0.002)^2\}$ if $0 \leq \varepsilon_c < 0.002$ (10.34)

Tensile and Compressive Stresses of Longitudinal Steel

Stresses are compressive in all the six rows (A1 to A6 of Figs.10.23.3a and c) of longitudinal steel provided in the column when the neutral axis depth $kD \geq D$. However, they are tensile on the left side of the neutral axis and compressive on the right side of the neutral axis (Figs.10.23.4a and c) when $kD < D$. These compressive or tensile stresses of longitudinal steel shall be calculated from the strain ε_{si} at that position of the steel which is obtained from the strain profile considered for the purpose.

It should be remembered that the linear strain profiles are based on the assumption that plane sections remain plane. Moreover, at the location of steel in a particular row, the strain of steel ε_{si} shall be the same as that in the adjacent concrete ε_{ci}. Thus, the strain of longitudinal steel can be calculated from the particular strain profile if the neutral axis is within or outside the cross-section of the column.

The corresponding stresses f_{si} of longitudinal steel are determined from the strain ε_{si} (which is the same as that of ε_{ci} in the adjacent concrete) from the respective stress-strain diagrams of mild steel (Fig.1.2.3 of Lesson 2) and High Yield Strength Deformed bars (Fig.1.2.4 of Lesson2). The values are summarized in Table 10.5 below as presented in Table A of SP-16.

Table 10.5 Values of compressive or tensile f_{si} from known values of ε_{si} of longitudinal steel (Fe 250, Fe 415 and Fe 500)

Fe 250		Fe 415		Fe 500	
Strain ε_{si}	Stress (N/mm^2) f_{si}	Strain ε_{si}	Stress (N/mm^2) f_{si}	Strain ε_{si}	Stress (N/mm^2) f_{si}
< 0.00109	ε_{si} (E_s)	< 0.00144	ε_{si} (E_s)	< 0.00174	ε_{si} (E_s)
≥ 0.00109	217.5 (= 0.87 f_y)	0.00144	288.7	0.00174	347.8
		0.00163	306.7	0.00195	369.6
		0.00192	324.8	0.00226	391.3

	0.00241	342.8	0.00277	413.0
	0.00276	351.8	0.00312	423.9
	0.00380	360.9	0.00417	434.8

Notes: 1. Linear interpolation shall be done for intermediate values.

2. Strain at initial yield = f_y/E_s
3. Strain at final yield = f_y/E_s + 0.002

Governing Equations

A column subjected to P_u and M_u (= $P_u e$) shall satisfy the two equations of equilibrium, viz., $\sum V$ = 0 and $\sum M$ = 0, taking moment of vertical forces about the centroidal axis of the column. The two governing equation are, therefore,

$$P_u = C_c + C_s$$

(10.35)

$M_u = C_c$ (appropriate lever arm) + C_s (appropriate lever arm) (10.36)

where C_c = Force due to concrete in compression

C_s = Force due to steel either in compression when $kD \geq D$ or in tension and compression when $kD < D$

However, two points are to be remembered while expanding the equation $\sum V$ = 0. The first is that while computing the force of steel in compression, the force of concrete that is not available at the location of longitudinal steel has to be subtracted. The second point is that the total force of steel shall consist of the summation of forces in every row of steel having different stresses depending on the respective distances from the centroidal axis. These two points are also to be considered while expanding the other equation $\sum M$ = 0. Moreover, negative sign should be used for the tensile force of steel on the left of the neutral axis when $kD < D$.

It is now possible to draw the interaction diagram of a trial section for the given values of P_u and M_u following the three steps mentioned in sec.10.23.4. However, such an attempt should be avoided for the reason explained below.

It has been mentioned in sec.10.23.2 that any point on the interaction diagram gives a pair of values of P_u and M_u causing collapse. On the other hand, it is also true that for the given P_u and M_u, several sections are possible. Drawing of interaction diagrams for all the trial sections is time consuming. Therefore, it is necessary to recast the interaction diagram selecting appropriate non- dimensional parameters instead of P_u versus M_u as has been explained in this lesson. Non-dimensional interaction diagram has the advantage of selecting alternative sections quickly for a given pair of P_u and M_u. It is worth mentioning that all the aspects of the behaviour of column and the modes of failure shall remain valid in constructing the more versatile non-dimensional interaction diagram.

Design Parameters

The following are the four major design parameters to be determined for any column so that it has sufficient pairs of strengths (P_u and M_u) to resist all critical pairs obtained from the analysis:

(i) dimensions *b* and *D* of the rectangular cross-section,

(ii) longitudinal steel reinforcing bars - percentage *p*, nature of distribution (equally on two or four sides) and *d'/D*,

(iii) grades of concrete and steel, and

(iv) transverse reinforcement.

The roles and importance of each of the above four parameters are elaborated below:

(i) Dimensions b and D of the rectangular cross-section

The strength of column depends on the two dimensions *b* and *D*. However, preliminary dimensions of *b* and *D* are already assumed for the analysis of structure, which are usually indeterminate statically. In the subsequent redesign, these dimensions may be revised, if needed, inviting re- analysis with the revised dimensions.

(ii) Longitudinal steel reinforcing bars

It is a very important consideration to utilise the total area of steel bars effectively. The total area of steel, expressed in percentage *p* ranges from the minimum 0.8 to the maximum 4 per cent of the gross area of the cross-section. The bars may be distributed either equally on two sides or on all four sides judiciously having two or multiple rows of steel bars. The strain profiles of Fig.10.23.2 reveals that the rows of bars may be all in compression or both compression and tension depending on the location of the neutral axis. Accordingly, the total strength of the longitudinal bars is determined by adding all the individual strengths of bars of different rows. The effective cover *d'*, though depends on the nominal cover, has to be determined from practical considerations of housing all the steel bars.

(iii) Grades of concrete and steel

The dimensions *b* and *D* of the cross-section and the amount of longitudinal steel bars depend on the grades of concrete and steel.

(iv) Transverse reinforcement

The transverse reinforcement, provided in form of lateral ties or spirals, are important for the following advantages in

(a) preventing premature / local buckling of the longitudinal bars,

(b) improving ductility and strength by the effect of confinement of the core concrete,

(c) holding the longitudinal bars in position during construction, and

(d) providing resistance against shear and torsion, if present.

However, the transverse reinforcement does not have a major contribution in influencing the capacities of the column. Moreover, the design of transverse reinforcement involves selection of bar diameter and spacing following the stipulations in the design code. The bar diameter of the transverse reinforcement also depends on the bar diameter of longitudinal steel. Accordingly, the transverse reinforcement is designed after finalizing other parameters mentioned above.

It is, therefore, clear that the design of columns mainly involves the determination of percentage of longitudinal reinforcement *p*, either assuming or knowing the dimensions *b* and *D*, grades of concrete and steel, distribution of longitudinal bars in two or multiple rows and *d'/D* ratio from the analysis or elsewhere. Needless to mention that any designed column should be able to resist several critical pairs of P_u and M_u obtained from the analysis of the structure. It is also a fact that several trials may be needed to arrive at the final selection revising any or all the assumed parameters. Accordingly, the design charts are prepared to give the results for the unknown parameter quickly avoiding lengthy calculations after selecting appropriate non-dimensional parameters.

Based on the above considerations and making the design simple, quick and fairly accurate, the following are the two non-dimensional parameters:

For axial load: $P_u/f_{ck}bD$

For moment: $M_u/f_{ck}bD^2$

The characteristic strength of concrete f_{ck} has been associated with the non-dimensional parameters as the grade of concrete does not improve the strength of the column significantly. The design charts prepared by SP-16 are assuming the constant value of f_{ck} for M 20 to avoid different sets of design charts for different grades of concrete. However, separate design charts are presented in SP-16 for three grades of steel (Fe 250, Fe 415 and Fe 500), four values of d'/D (0.05, 0.1, 0.15 and 0.2) and two types of distribution of longitudinal steel (distributed equally on two and four sides). Accordingly there are twenty-four design charts for the design of rectangular columns. Twelve separate design charts are also presented in SP-16 for circular sections covering the above mentioned three grades of steel and for values of d'/D ratio.

However, the unknown parameter p, the percentage of longitudinal reinforcement has been modified to p/f_{ck} in all the design charts of SP-16, so that for grades other than M 20, the more accurate value of p can be obtained by multiplying the p/f_{ck} with the actual grade of concrete used in the design of that column.

However, this lesson explains that it is also possible to prepare design chart taking into consideration the actual grade of concrete. As mentioned earlier, the design charts are prepared getting the pairs of values of P_u and M_u in non- dimensional form from the equations of equilibrium for different locations of the neutral axis. We now take up the respective non-dimensional equations for four different cases as follows:

(a) When the neutral axis is at infinity, i.e., $kD = \infty$, pure axial load is applied on the column.

(b) When the neutral axis is outside the cross-section of the column, i.e., $\infty > kD \geq D$.

(c) When the neutral axis is within the cross-section of the column, i.e., $kD < D$.

(d) When the column behaves like a steel beam.

Non-dimensional Equation of Equilibrium when $k = \infty$, (Pure Axial Load)

Figures 10.23.2b and c of Lesson 23 present the strain profile EF and the corresponding stress block for this case. As the load is purely axial, we need to express the terms C_c and C_s of Eq.10.35 of sec.10.23.10 of Lesson 23. The total compressive force due to concrete of constant stress of 0.446 f_{ck} is:

$$C_c = 0.446 f_{ck}\, b\, D$$

(10.37)

However, proper deduction shall be made for the compressive force of concrete not available due to the replacement by steel bars while computing C_s.

The force of longitudinal steel bars in compression is now calculated. The steel bars of area $pbD/100$ are subjected to the constant stress of f_{sc} when the strain is 0.002. Subtracting the compressive force of concrete of the same area $pbD/100$, we have,

$$C_s = (pbD/100)\,(f_{sc} - 0.446 f_{ck}) \quad (10.38)$$

Thus, we have from Eq.10.35 of sec.10.23.10 of Lesson 23 after substituting the expressions of C_c and C_s from Eqs.10.37 and 10.38,

$$P_u = 0.446 f_{ck}\, b\, D + (pbD/100)\,(f_{sc} - 0.446 f_{ck}) \quad (10.39)$$

Dividing both sides of Eq.10.39 by $f_{ck}\, bD$, we have

$$(P_u/f_{ck}\, bD) = 0.446 + (p/100 f_{ck})\,(f_{sc} - 0.446 f_{ck}) \quad (10.40)$$

Thus, Eq.10.40 is the only governing equation for this case to be considered.

Non-dimensional Equations of Equilibrium when Neutral Axis is Outside the Section ($\infty > kD \geq D$)

Figures 10.23.3b and c of Lesson 23 present the strain profile JK and the corresponding stress block for this case. The expressions of C_c, C_s and appropriate lever arms are determined to write the two equations of equilibrium (Eqs.10.35 and 36) of Lesson 23. While computing C_c, the area of parabolic stress block is determined employing the coefficient C_1 from Table 10.4 of Lesson 23. Similarly, the coefficient C_2, needed to write the moment equation, is obtained from Table 10.4 of Lesson 23. The forces and the corresponding lever arms of longitudinal steel bars are to be considered separately and added for each of the n rows of the longitudinal bars. Thus, we have the first equationas,

$$P_u = C_1 f_{ck} bD + \sum_{i=1}^{n} (p_i bD / 100)(f_{si} - f_{ci}) \quad (10.41)$$

where C_1 = coefficient for the area of stress block to be taken from Table 10.4 of Lesson 23,

p_i = A_{si}/bD where A_{si} is the area of reinforcement in the i^th^ row,

f_{si} = stress in the i^th^ row of reinforcement, taken positive for compression and negative for tension,

f_{ci} = stress in concrete at the level of the i^th^ row of reinforcement, and

n = number of rows of reinforcement.

Here also, the deduction of the compressive force of concrete has been made for the concrete replaced by the longitudinal steel bars.

Dividing both sides of Eq.10.41 by $f_{ck}bD$, we have

$$(P_u/f_{ck}bD) = C_1 + \sum_{i=1}^{n} (p_i/100 f_{ck})(f_{si} - f_{ci}) \quad (10.42)$$

Similarly, the moment equation (Eq.10.36) becomes,

$$M_u = C_1 f_{ck}bD(D/2 - C_2D) + \sum_{i=1}^{n} (p_i bD / 100)(f_{si} - f_{ci})\, y_i \quad (10.43)$$

where C_2 = coefficient for the distance of the centroid of the compressive stress block of concrete measured from the highly compressed right edge and is taken from Table 10.4 of Lesson 23, and

y_i = the distance from the centroid of the section to the i^th^ row of reinforcement, positive towards the highly compressed right edge and negative towards the least compressed left edge.

Dividing both sides of Eq.10.43 by $f_{ck}bD^2$, we have

$$(M_u/f_{ck}bD^2) = C_1(0.5 - C_2) + \sum_{i=1}^{n} (p_i /100 f_{ck})(f_{si} - f_{ci})(y_i/D) \quad (10.44)$$

Equations 10.42 and 10.44 are the two non-dimensional equations of equilibrium in this case when $\infty < kD \leq D$.

Non-dimensional Equations of Equilibrium when the Neutral Axis is within the Section (kD < D)

The strain profile IN and the corresponding stress block of concrete are presented in Figs.10.23.4b and c for this case. Following the same procedure of computing C_c, C_s and the respective lever arms, we have the first equation as

$$P_u = 0.36 f_{ck} kbD + \sum_{i=1}^{n} (p_i bD / 100)(f_{si} - f_{ci}) \qquad (10.45)$$

Dividing both sides of Eq.10.45 by $f_{ck}bD$, we have

$$P_u/f_{ck}bD = 0.36 k + \sum_{i=1}^{n} (p_i / 100 f_{ck})(f_{si} - f_{ci}) \qquad (10.46)$$

and the moment equation (Eq.10.36) as

$$M_u = 0.36 f_{ck} kbD(0.5 - 0.42 k) D + \sum_{i=1}^{n} (p_i bD / 100)(f_{si} - f_{ci})(y_i/D) \qquad (10.47)$$

Dividing both sides of Eq.10.47 by $f_{ck}bD^2$, we have

$$(M_u/f_{ck}bD^2) = 0.36 k(0.5 - 0.42 k) + \sum_{i=1}^{n} (p_i / 100 f_{ck})(f_{si} - f_{ci})(y_i/D) \qquad (10.48)$$

where k = Depth of the neutral axis/Depth of column, mentioned earlier in sec.10.21.10 and Fig.10.21.11 of Lesson 21.

Equations 10.46 and 10.48 are the two non-dimensional equations of equilibrium in this case.

Non-dimensional Equation of Equilibrium when the Column Behaves as a Steel Beam

This is a specific situation when the column is subjected to pure moment $M_u = M_o$ only (Point 6 of the interaction diagram in Fig.10.23.1 of Lesson 23).

Since the column has symmetrical longitudinal steel on both sides of the centroidal axis of the column, the column will resist the pure moment by yielding of both tensile and compressive steel bars (i.e., $f_{si} = 0.87 f_y = f_{yd}$). Thus, we have only one equation (Eq.10.36 of Lesson 23), which becomes

$$M_u = \sum_{i=1}^{n} (p_i bD / 100)(0.87 f_y)(y_i/D) \qquad (10.49)$$

Dividing both sides of Eq.10.49 by $f_{ck}\, bD^2$, we have

$$(M_u/f_{ck}bD^2) = \sum_{i=1}^{n} (p_i /100 f_{ck})(0.87 f_y)(y_i/D) \qquad (10.50)$$

Equation 10.50 is the equation of equilibrium in this case.

Preparation of Design Charts

Design charts are prepared employing the equations of four different cases as given in secs.10.24.3 to 6. The advantage of employing the equations is that the actual grade of concrete can be taken into account, though it may not be worthwhile to follow this accurately. However, preparation of interaction diagram will help in understanding the behaviour of column with the change of neutral axis depth for the four cases mentioned in sec.10.24.2. The step by step procedure of preparing the design charts is explained below. It is worth mentioning that the values of $(P_u/f_{ck}bD)$ and $(M_u/f_{ck}bD^2)$ are determined considering different locations of the neutral axis for the four cases mentioned in sec.10.24.2.

Step 1: When the neutral axis is at infinity

The governing equation is Eq.10.40. The strain profile EF and the corresponding stress block are in Fig.10.23.2b and c of Lesson 23, respectively.

Step 2: When the column is subjected to axial load considering minimum eccentricity

Lesson 22 presents the design of short columns subjected to axial load only considering minimum eccentricity as stipulated in cl.29.3 of IS 456, employing Eq.10.4, which is as follows:

$P_u = 0.4 f_{ck}\, b\, D + (pbD/100)(0.67 f_y - 0.4 f_{ck})$ .... (10.4) Dividing both sides of Eq.10.4 by $f_{ck}\, bD$, we have

$$(P_u/f_{ck}\,bD) = 0.4 + (p/100\,f_{ck})\,(0.67\,f_y - 0.4\,f_{ck}) \quad (10.51)$$

The P_u obtained from Eq.10.51 can also resist M_u as per cl.39.3 of IS 456. From the stipulation of cl. 39.3 of IS 456 and considering the maximum value of the minimum eccentricity as 0.05D, we have

$$M_u = (P_u)\,(0.05)D = 0.02\,f_{ck}\,bD^2 + (0.05\,pbD^2/100)\,(0.67\,f_y - 0.4\,f_{ck})$$

Dividing both sides of the above equation by $f_{ck}bD^2$, we have

$$(M_u/f_{ck}\,bD^2) = 0.02 + (0.05p/100\,f_{ck})\,(0.67\,f_y - 0.4\,f_{ck}) \quad (10.52)$$

Equations 10.51 and 10.52 are the two equations to be considered in this case.

Step 3: When the neutral axis is outside the section

Figures 10.23.3b and c of Lesson 23 present one strain profile JK and the corresponding stress block, respectively, out of a large number of values of k from 1 to infinity, only values up to about 1.2 are good enough to consider, as explained in sec.10.23.5 of Lesson 23. Accordingly, we shall consider only one point, where k = 1.1, in this case. With the help of Eqs.10.42 and 10.44, Table 10.4 for the values of C_1 and C_2, Table 10.5 for the values of f_{si} and Eq.10.23 or Eq.10.27 for the values of f_{ci}, the non-dimensional parameters $P_u/f_{ck}\,bD$ and $M_u/f_{ck}\,bD^2$ are determined.

Step 4: When the neutral axis is within the section

One representative strain profile IU and the corresponding stress block are presented in Fig.10.23.4b and c, respectively, of Lesson 23. The following six points of the interaction diagram are considered satisfactory for preparing the design charts:

(a) Where the tensile stress of longitudinal steel is zero i.e., $kD = D - d'$,

(b) Where the tensile stress of longitudinal steel is $0.4f_yd = 0.4(0.87f_y)$,

(c) Where the tensile stress of longitudinal steel is) $0.8f_yd = 0.8(0.87\,f_y)$,

(d) Where the tensile stress of longitudinal steel is $f_yd = 0.87f_y$ and strain $= 0.87f_y/E_s$, i.e., the initial yield point,

(e) Where the tensile stress of longitudinal steel is $f_yd = 0.87f_y$ and strain $= 0.87f_y/E_s + 0.002$, i.e., the final yield point,

(f) When the depth of the neutral axis is $0.25D$.

For all six points, the respective strain profile and the corresponding stress blocks can be drawn. Therefore, values of ($P_u/f_{ck}\ bD$) and ($M_u/f_{ck}\ bD^2$) are determined from Eqs.10.46 and 10.48, using Table 10.5 for f_{sc} and Eq.10.34 for f_{ci}.

Step 5: When the column behaves like a steel beam

As explained in sec.10.24.6, Eq.10.50 is used to compute $M_u/f_{ck}\ bD^2$ in this case.

Step 6: Preparation of design chart

The ten pairs of ($P_u/f_{ck}\ bD$) and ($M_u/f_{ck}\ bD^2$) (one set each in steps 1, 2, 3 and 5 and six sets in step 4) can be plotted to prepare the desired design chart.

One illustrative example is taken up in the next section.

Illustrative Example

Problem 1:

Prepare a design chart for a rectangular column with 3 per cent longitudinal steel distributed equally on two faces using M 25 and Fe 415, and considering $d'/D = 0.15$.

Solution 1:

The solution of this problem is explained in six steps of the earlier section.

Step 1: When the neutral axis is at infinity

Figures 10.23.2b and c present the strain profile EF and the corresponding stress block, respectively. Using the values of $p = 3$ per cent, f_{ck} = 25 N/mm^2 and determining the value of $f_{sc} = 327.7388$ N/mm^2 (using linear interpolation from the values of Table 10.5 of Lesson 23), we get the value of ($P_u/f_{ck}\ bD$) from Eq.10.40 as

$$(P_u/f_{ck}\ bD) = 0.8259.$$

Step 2: When the column is subjected to axial load considering minimum eccentricity

Using the value of $p = 3$ per cent, $f_{ck} = 25$ N/mm^2 and $f_y = 415$ N/mm^2 in Eqs.10.51 and 10.52 of sec.10.24.6, we have

$(P_u/f_{ck}\, bD) = 0.7217$ $(M_u/f_{ck}$

$bD^2) = 0.0361$

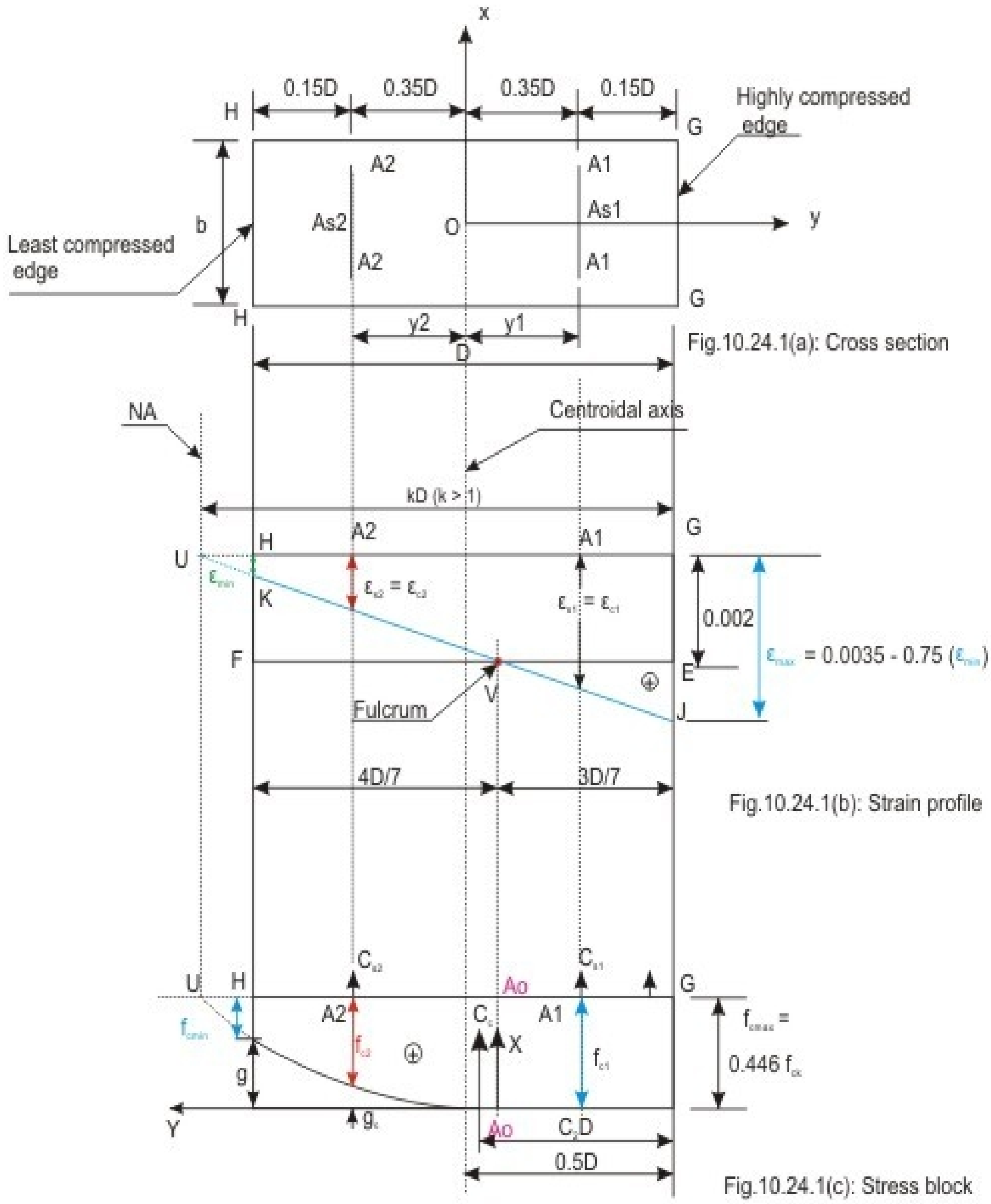


Fig.10.24.1: Problem 1 and Q. 3 (step 3, k=1.1)

Step 3: When the neutral axis depth = 1.1 *D*

Figures 10.24.1a, b and c show the section of the column, strain profile JK and the corresponding stress block, respectively, for this case. We use Eqs.10.42 and 10.44 for determining the value of $(P_u/f_{ck}\, bD)$

and $(M_u/f_{ck}\, bD^2)$ for

this case using $k = 1.1$, $f_{ck} = 25$ N/mm^2, $p_1 = p_2 = 1.5$, $y_1/D = 0.35$ and $y_2/D = -0.35$. Values of C_1, C_2, f_{s1} and f_{s2}, f_{c1} and f_{c2} are obtained from equations mentioned in Step 3 of sec.10.24.6. The values of all the quantities are presented in Table 10.6A, mentioning the source equation no., table no. etc. to get the two non-dimensional parameters as given below:

$(P_u/f_{ck}\ bD) = 0.67405$ $(M_u/f_{ck}\ bD^2) = 0.06370$

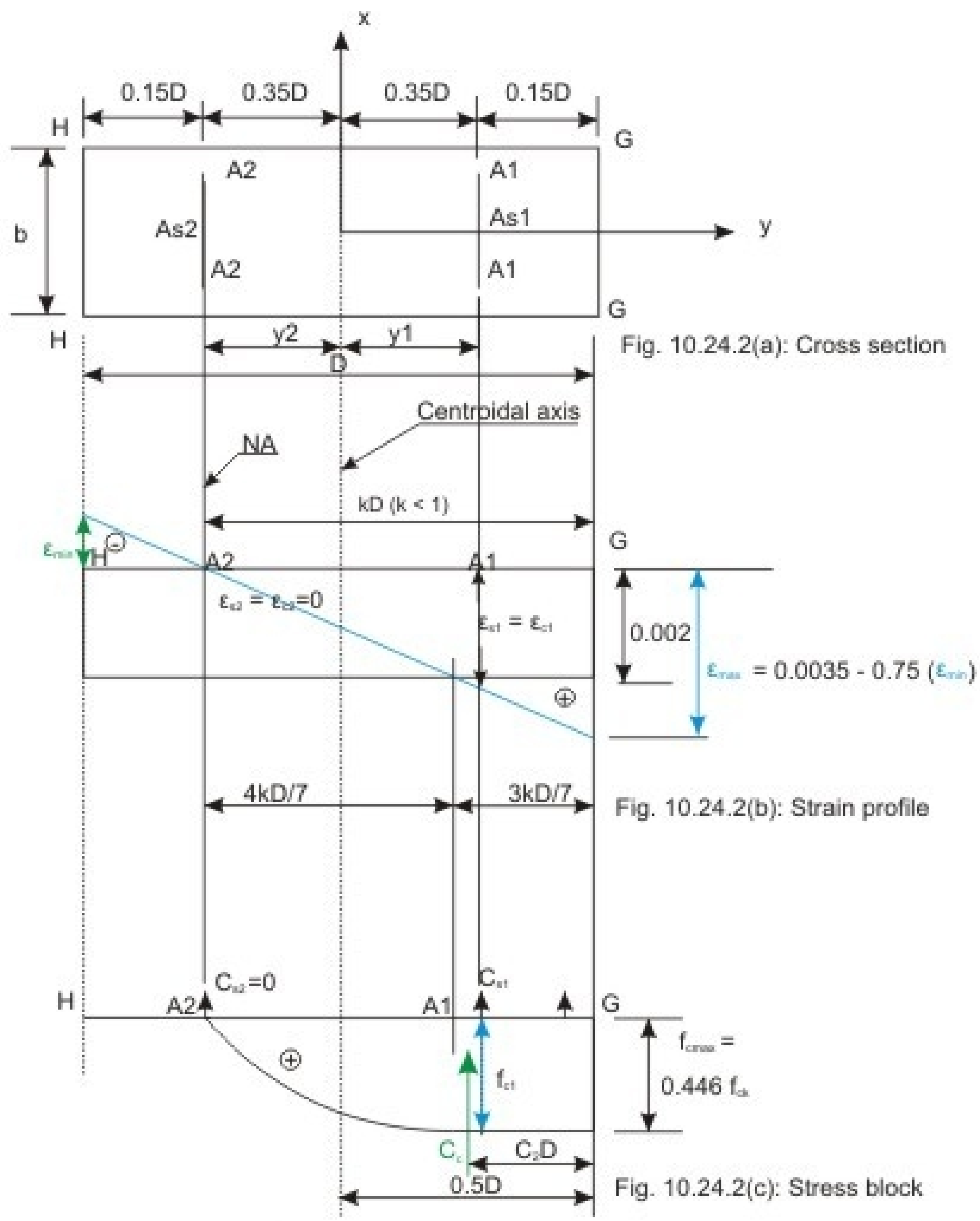


Fig. 10.24.2: Problem 1 and Q. 3 (step 4, fs2 = 0)

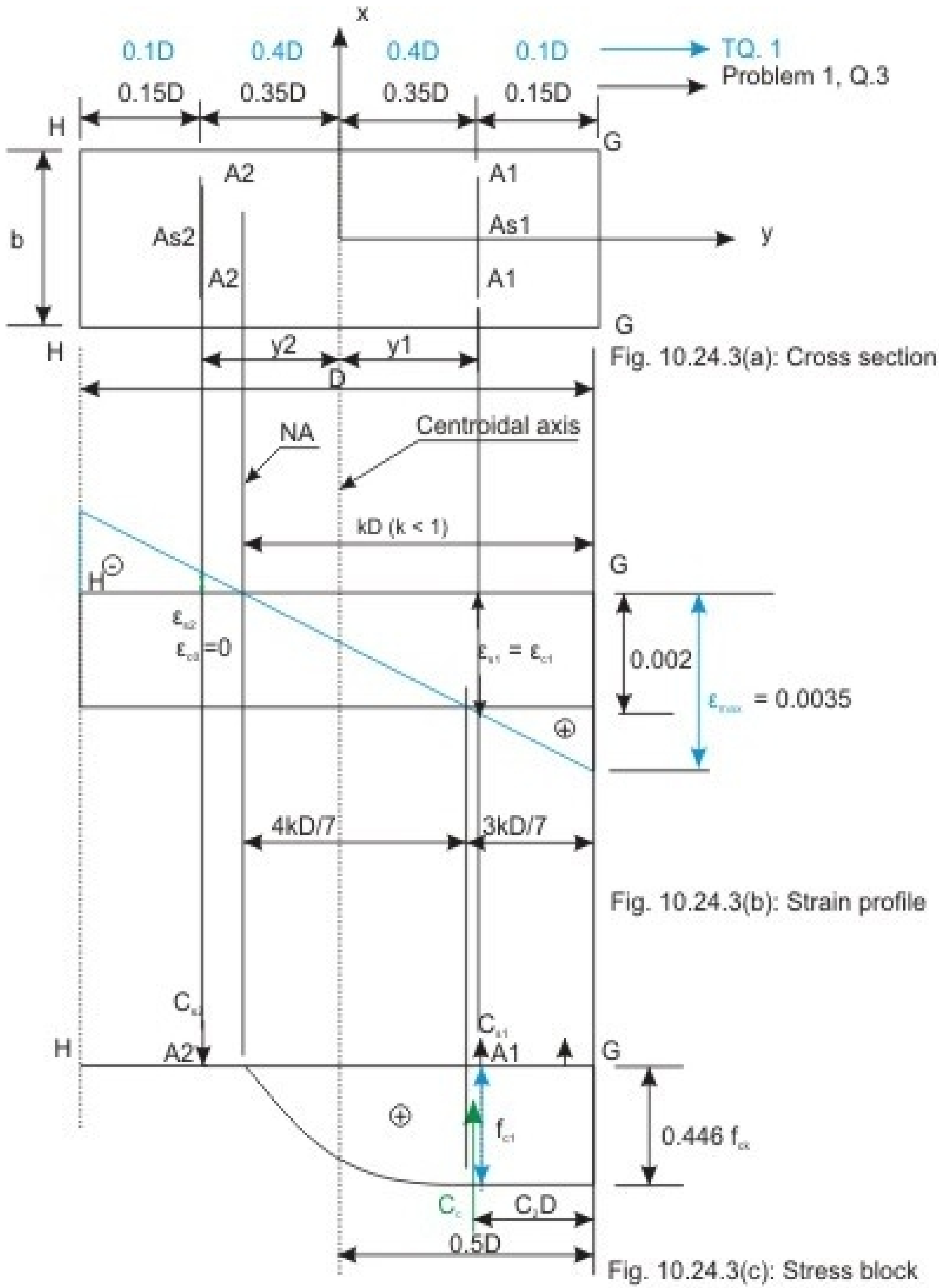


Fig. 10.24.3: Problem 1, Q.3 and TQ. 1 (step 4, fs2 = $-0.4f_{yd}$)

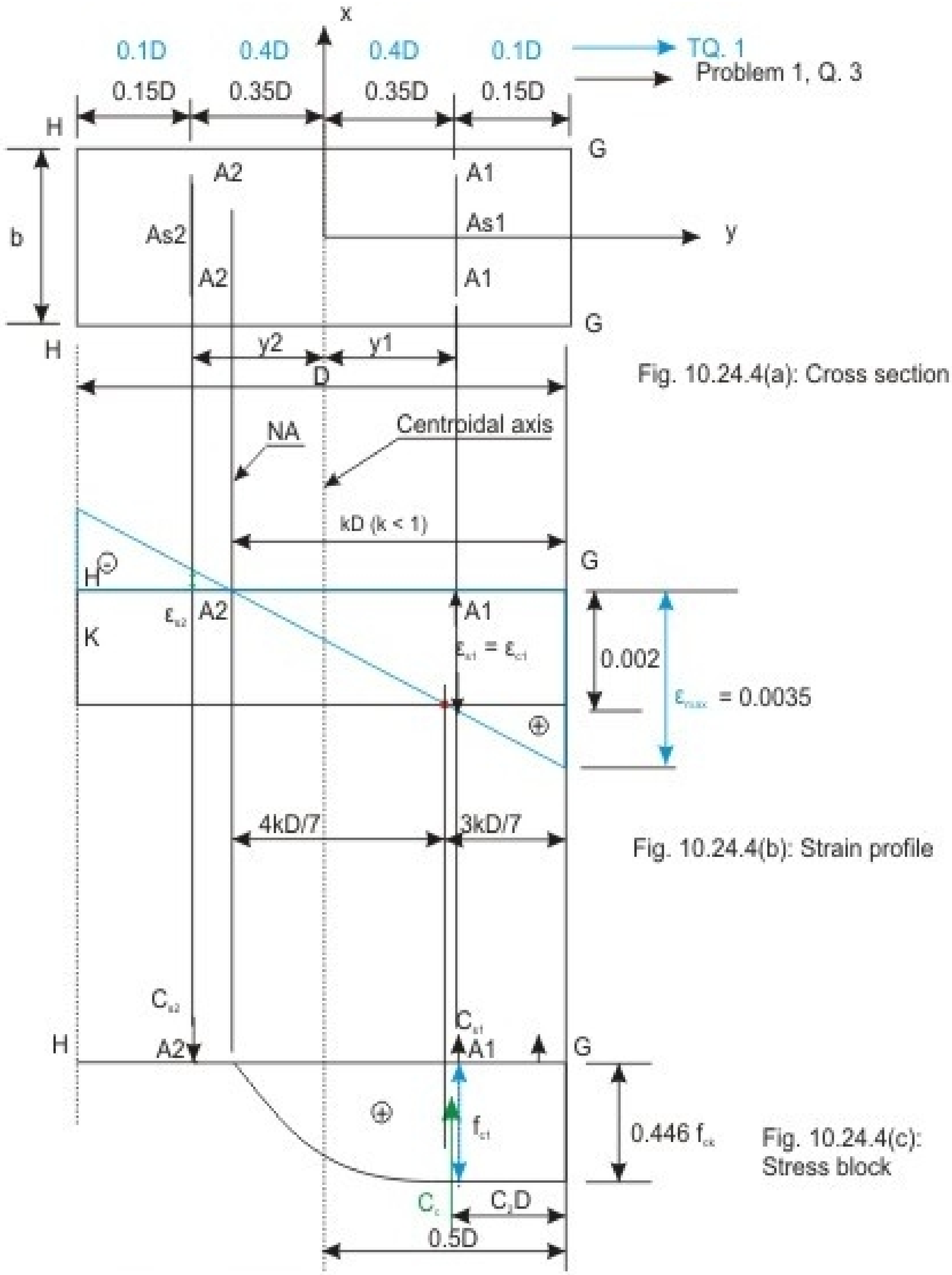


Fig. 10.24.4.: Problem 1, Q.3 and TQ. 1 (step 4, fs2 = - 0.8f_{yd})

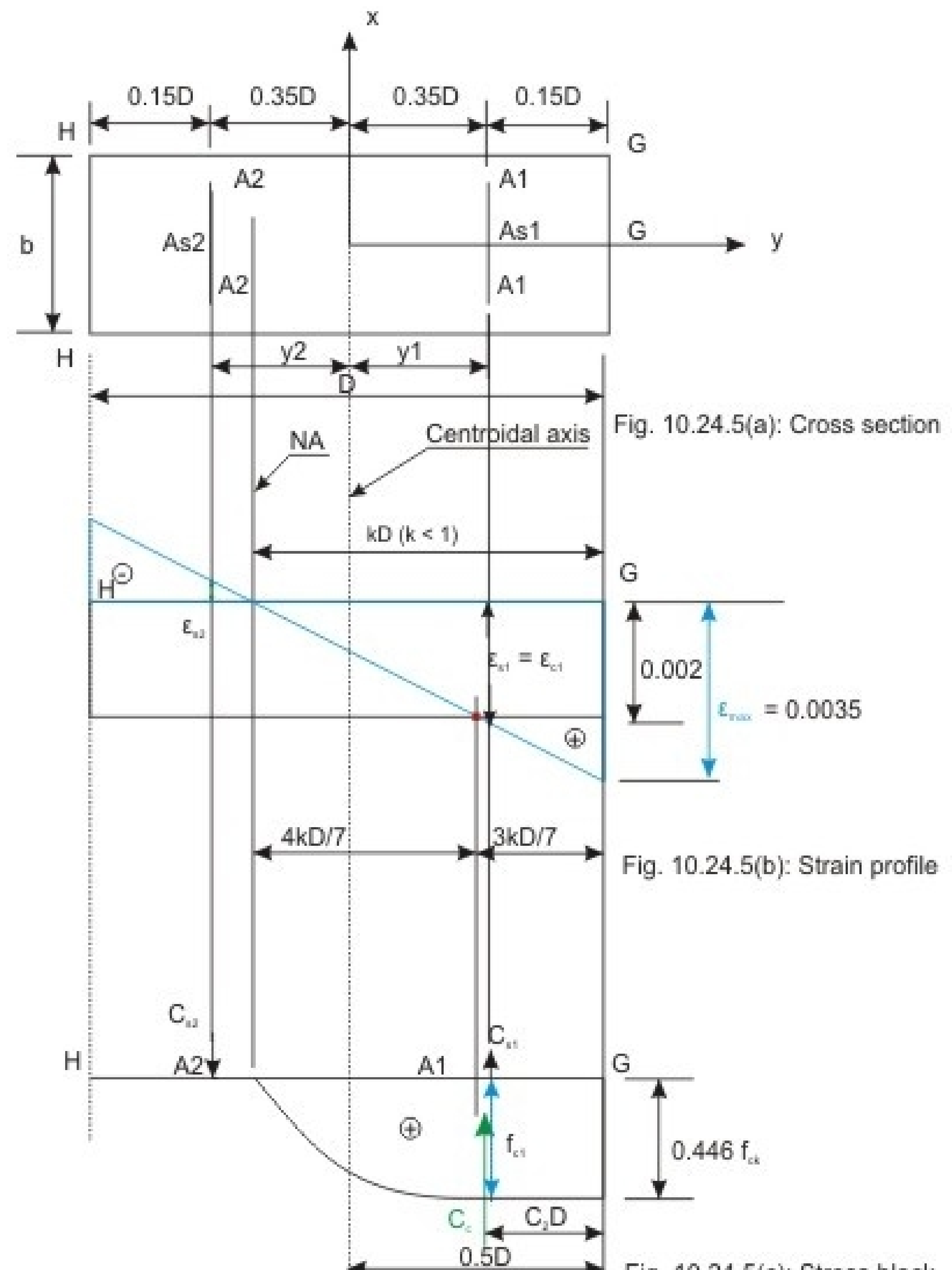


Fig. 10.24.5: Problem 1 and Q.3 (step 4, f_{s2} = - f_{yd} , initial yield)

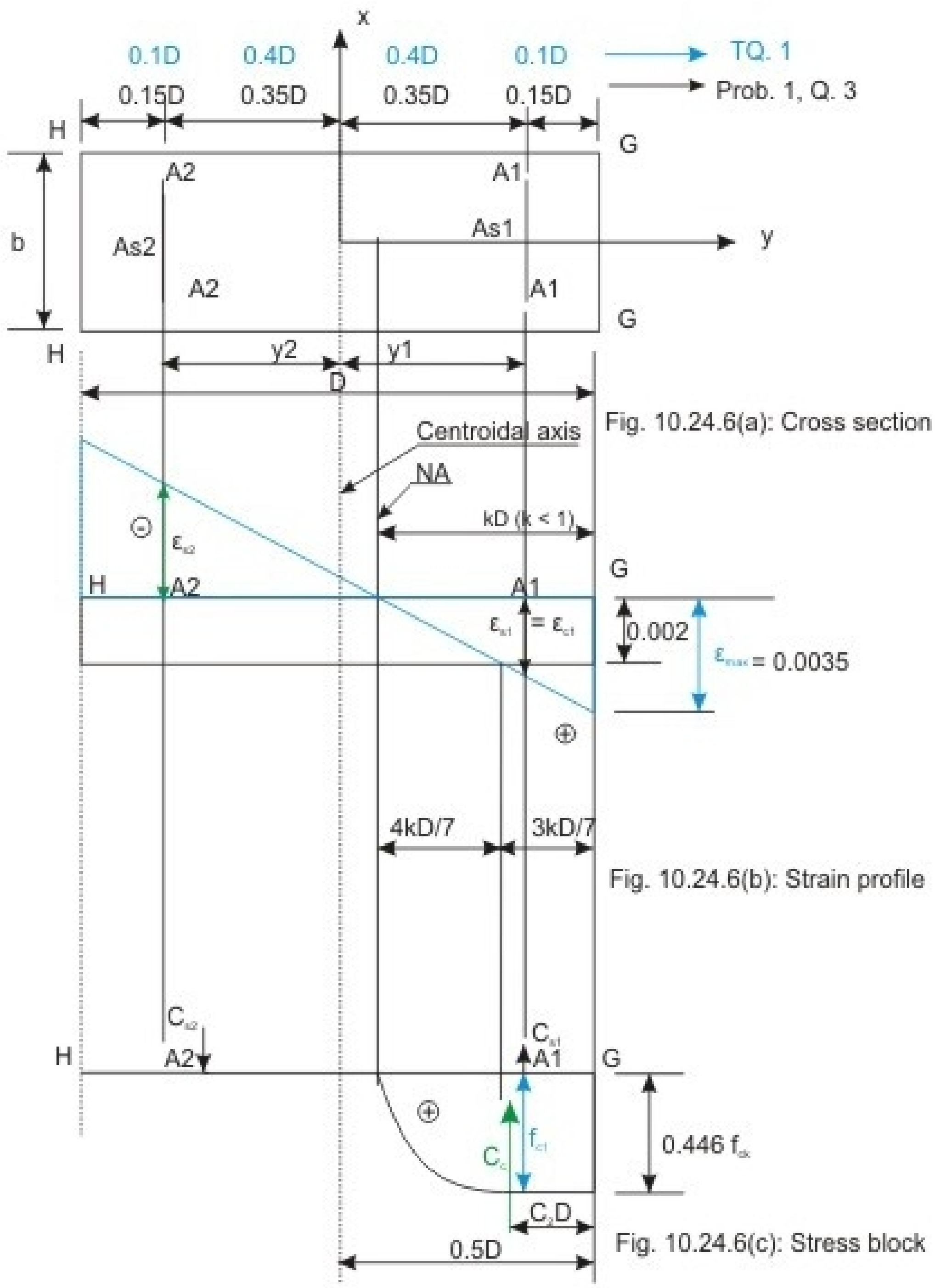


Fig. 10.24.6: Problem 1, Q.3 and TQ. 1 (step 4, $f_{s2} = f_{yd}$),final yield

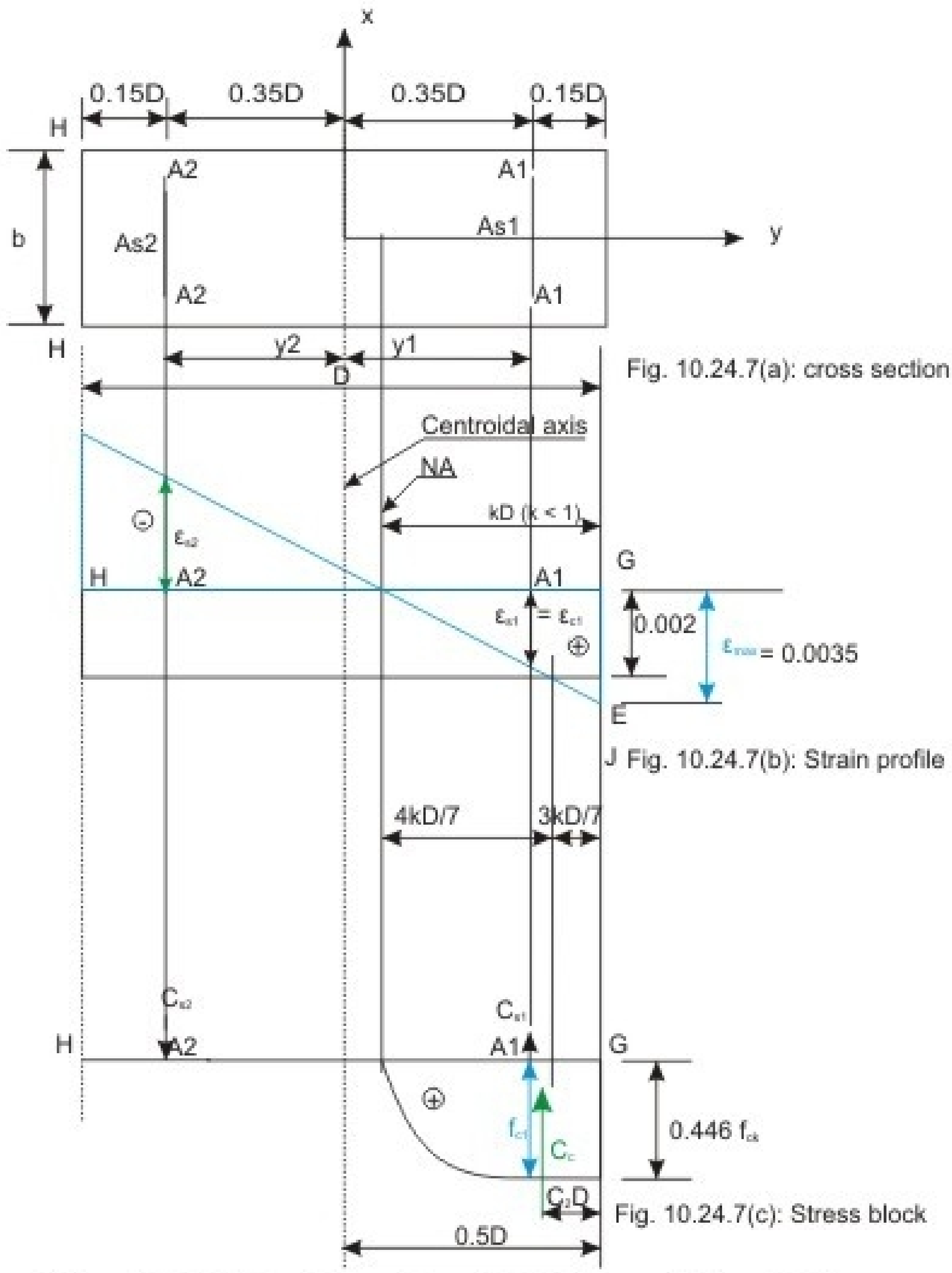


Fig. 10.24.7: Problem 1 and Q.3 (step 4, k = 0.25)

Step 4: When the neutral axis is within the section

In Step 4 of section 10.24.6, six different locations of neutral axis are mentioned; five of them (a to e) are specified by the magnitude of f_{s2} (tensile) of longitudinal steel and one of them is specified by the value of k = 0.25. The values of all the quantities are presented in Tables 10.6A and B, mentioning the source equation no., table no. etc.

Figures 10.24.2 to 10.24.7 present the respective strain profiles and the corresponding stress block separately for all six different locations of the neutral axis.

Table 10.6A Parameters and results of Problem 1 of Section 10.24.8

Given data: f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, p = 3 per cent, $p_1 = p_2$ = 1.5 per cent, d'/D = 0.15

Note: Units of f_{si}, f_{sc} and f_{ci} are in N/mm^2, (-) minus sign indicates tensile strain or stress.

Sl.No.	Given Description	k = 1.1	f_{s2} = 0	f_{s2} = -0.4 f_{yd}	f_{s2} = 0.8 f_{yd}
1	Sec. No.	10.24.7	10.24.7	10.24.7	10.24.7
2	Step No.	3	4	4	4
3	Fig. No.	10.24.1	10.24.2	10.24.3	10.24.4
4	$\varepsilon_{s1} = \varepsilon_{c1}$	0.002829	0.00288	0.00275	0.00263
5	$\varepsilon_{s2} = \varepsilon_{c2}$	0.000744	0.0	-0.00072	-0.00144
6	Table No. of f_{si} and f_{sc}	10.5	10.5	10.5	10.5
7	f_{s1}	352.407	352.871	351.669	348.392
8	f_{s2}	148.914	0.0	-144.42	-288.84
9	f_{sc}	NA	NA	NA	NA
10	Eq.Nos. of f_{ci}	10.23 and 10.27	10.34	10.34	10.34
11	f_{c1}	11.1	11.15	11.15	11.15

		5			
12	f_{c2}	6.75 7	0.0	0.0	0.0
13	Table No. of C_1 and C_2	10.4	NA	NA	NA
14	C_1	0.384	NA	NA	NA
15	C_2	0.443	NA	NA	NA
16	y_1/D	+0.3 5	+0.35	+0.35	+0.35
17	y_2/D	-0.35	-0.35	-0.35	-0.35
18	k	1.1	0.85	0.7046	0.6017
19	Eq.No. of $P_u/f_{ck}\, bD$	10.4 2	10.46	10.46	10.46
20	$P_u/f_{ck}\, bD$	0.6740	0.5110	0.3713	0.2457
21	Eq.No. of $M_u/f_{ck}\, bD^2$	10.4 4	10.48	10.48	10.48
22	$M_u/f_{ck}\, bD^2$	0.0643	0.1155	0.1536	0.1850

Table 10.6B Parameters and results of Problem 1 of Section 10.24.8

Given data: f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, p = 3 per cent, $p_1 = p_2$ = 1.5 per cent, d'/D = 0.15

Note: Units of f_{si}, f_{sc} and f_{ci} are in N/mm^2, (-) minus sign indicates tensile strain or stress.

Sl.No.	Given Description	$f_{s2} = -f_{yd}$ (Initial yield)	$f_{s2} = -f_{yd}$ (Final yield)	k = 0.25
1	Sec. No.	10.24.7	10.24.7	10.24.7
2	Step No.	4	4	4
3	Fig. No.	10.24.5	10.24.6	10.24.7
4	$\varepsilon_{s1} = \varepsilon_{c1}$	0.00256	0.00221	0.0014
5	$\varepsilon_{s2} = \varepsilon_{c2}$	-0.00180	-0.00380	-0.0084
6	Table No. of f_{si} and f_{sc}	10.5	10.5	10.5
7	f_{s1}	346.754	335.484	281.0
8	f_{s2}	-361.05	-361.05	-361.05
9	f_{sc}	NA	NA	NA
10	Eq.Nos. of f_{ci}	10.34	10.34	10.34
11	f_{c1}	11.15	11.15	10.146
12	f_{c2}	0.0	0.0	0.0
13	Table No. of C_1 and C_2	NA	NA	NA
14	C_1	NA	NA	NA

15	C_2	NA	NA	NA
16	y_1/D	+0.35	+0.35	+0.35
17	y_2/D	-0.35	-0.35	-0.35
18	k	0.5607	0.4072	0.25
19	Eq.No. of $P_u/f_{ck}\ bD$	10.46	10.46	10.46
20	$P_u/f_{ck}\ bD$	0.1866	0.1246	0.0353
21	Eq.No. of $M_u/f_{ck}\ bD^2$	10.48	10.48	10.48
22	$M_u/f_{ck}\ bD^2$	0.1997	0.1921	0.1680

Step 5: When the column behaves like a steel beam

For this case, the parameter ($M_u/f_{ck}\ bD^2$) is determined from Eq.10.50 using $p_1 = p_2 = 1.5$ per cent, $f_{ck} = 25$ N/mm^2, $f_y = 415$ N/mm^2, $y_1/D = 0.35$ and $y_2/D = -0.35$. Thus, we get

$(M_u/f_{ck}\, bD^2) = 0.15164$

Step 6: Final results of design chart

The values of ten pairs of ($P_u/f_{ck}\, bD$) and ($M_u/f_{ck}\, bD^2$) as obtained in steps 1 to 5 are presented in Sl. Nos. 1 to 10 of Table 10.6C. The design chart can be prepared by plotting these values.

Table 10.6C Final values of $P_u/f_{ck}\, bD$ and $M_u/f_{ck}\, bD^2$ of Problem 1 of Section 10.24.8

Sl. No.	Particulars about the point	$P_u/f_{ck}\, bD$	$M_u/f_{ck}\, bD^2$
1	$k =$	0.8259	0.0
2	Minimum eccentricity	0.7217	0.0361
3	$k = 1.1$	0.6740	0.0643
4	$f_{s2} = 0$	0.5110	0.1155
5	$f_{s2} = (-)0.4\, f_{yd}$	0.3713	0.1536
6	$f_{s2} = (-)0.8\, f_{yd}$	0.2457	0.1850
7	$f_{s2} = (-)\, f_{yd}$ (Initial yield)	0.1866	0.1997
8	$f_{s2} = (-)\, f_{yd}$ (Final yield)	0.1246	0.1921
9	$k = 0.25$	0.0353	0.1680
10	Steel Beam	0.0	0.1516

Design Charts of SP-16

SP-16 has three sets of design charts prepared by following the procedure explained in Lesson 24 for rectangular and circular types of cross-sections of columns. The three sets are as follows:

(i) Charts 27 to 38 are the first set of twelve charts for rectangular columns having symmetrical longitudinal steel bars in two rows (Fig.10.25.1) for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).

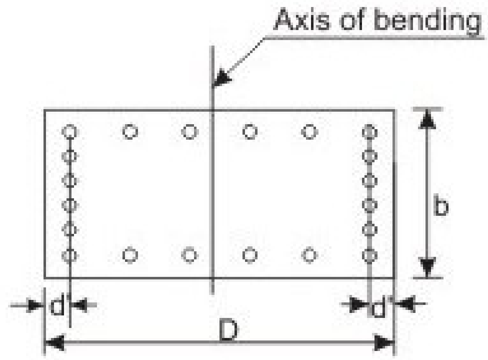


Fig.10.25.2: Rectangular column section - reinforcement distributed equally on four sides

(ii) Charts 39 to 50 are the second set of twelve charts for rectangular columns having symmetrical longitudinal steel bars (twenty numbers) distributed equally on four sides (in six rows, Fig.10.25.2) for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).

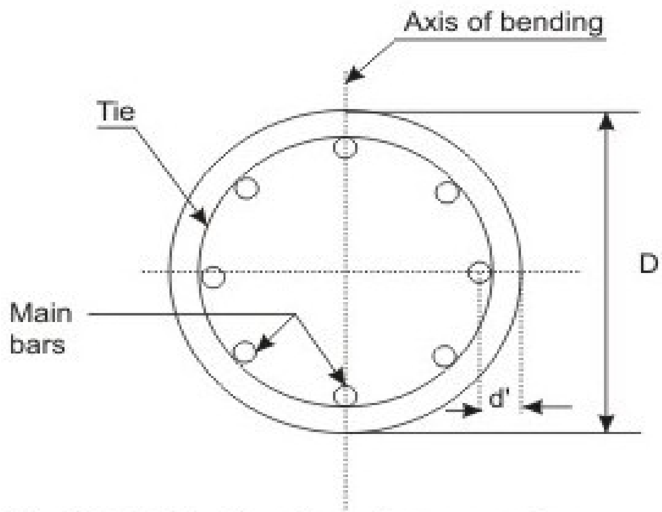


Fig.10.25.3: Circular column section - reinforcement uniformly distributed circumferencially

(iii) The third set of twelve charts, numbering from 51 to 62, are for circular columns having eight longitudinal steel bars of equal diameter and uniformly spaced circumferentially (Fig.10.25.3) for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).

All the thirty-six charts are prepared for M 20 grade of concrete only. This is a justified approximation as it is not worthwhile to have separate design charts for each grade of concrete.

Approximations and Limitations of Design Charts of SP-16

(i) Approximations

The following are the approximations of the design charts of SP-16:

(a) Grade of concrete

As mentioned in the earlier section, all the design chars of SP-16 assume the constant grade M 20 of concrete. However, each chart has fourteen plots having different values of the parameter p/f_{ck} ranging from zero to 0.26 at an interval of 0.02. The designer, thus, can make use of the actual grade of concrete by multiplying the p/f_{ck} obtained from the plot with the actual f_{ck} for the particular grade of concrete to partially compensate the approximation.

(b) The *d'/D* ratio

The three sets of charts have four fixed values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20). However, in the practical design, the *d'/D* ratio may be different from those values. In such situations intermediate values are determined by making linear interpolations.

(c) Equal distribution of twenty longitudinal steel bars on four sides of rectangular columns

In spite of the above consideration, the design charts may be used without significant error for any number of bars greater than eight provided the bars are distributed equally on four sides.

(d) Longitudinal bars in circular columns

Though the design charts are prepared considering eight bars uniformly placed circumferentially, they may generally be used for any number of bars greater than six, uniformly placed circumferentially.

(ii) Limitations

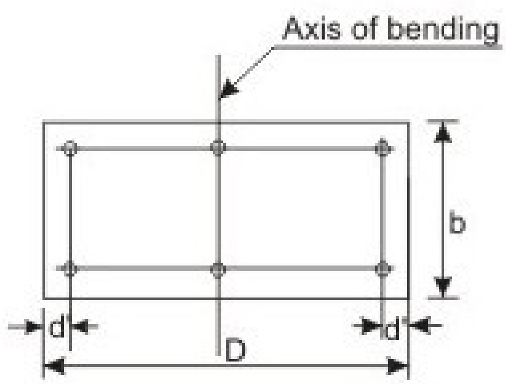


Fig.10.25.4: Six-bars arrangement

The following are the limitations of the design chars of SP-16:

(a) Longitudinal bars equally distributed on four sides of rectangular columns Twenty

bars, when equally placed on four sides, are placed in six rows avoiding any bar on the two axes. However, there will be bars on the axes for odd number of rows. A very common type is the 6-bar arrangement (Fig.10.25.4). Such arrangements, though symmetrical, are not covered in the design charts of SP-16. In such cases, the designer has to make his own assumptions judiciously in order to use the available charts of SP-16. Alternatively, he has to prepare the actual design chart depending on the bar arrangement to get accurate results.

(b) Unsymmetrical arrangement of longitudinal bars in rectangular cross-sections It is

not covered in the charts.

(c) Non-uniform placing of longitudinal bars in circularcross-sections It is

not covered in the charts.

(d) Cross-sections other than rectangular or circular like, I, T, H, X etc.

These are not covered in the charts.

The items under b, c and d, though rare, should be taken care of by preparing the respective design chars as and when needed.

(e) Concluding remarks

In spite of the above approximations and limitations, use of SP-16 has several advantages even by making some more approximations if the charts are not directly applicable. In the note of cl.39.5 of IS 456, the following is recommended, which is worth reproducing:

"The design of members subject to combined axial load and uniaxial bending will involve lengthy calculation by trial and error. In order to overcome these difficulties interaction diagrams may be used. These have been prepared and published by BIS in "SP-16 Design aids for reinforced concrete to IS 456'."

Accordingly, the use of SP-16 is explained in the following sections for the solutions of both analysis and design types of problems.

Use of Design Charts in the Analysis Type of Problems

In many situations, it becomes necessary to assess the safety of a column with known cross-section dimensions, and longitudinal and transverse steel reinforcing bars. The objective is to examine if the column can resist some critical values of P_u or M_u or pairs of P_u and M_u, as may be expected to be applied on the column. This is done by comparing if the given values of pair of P_u and M_u are less than the respective strength capacities pair of P_u and M_u. The word "given" shall be used in the suffix of pairs of P_u and M_u to indicate that they are the given values for which the column has to be examined. The strength capacities of the column, either P_u or M_u alone or pair of P_u and M_u, will not have any suffix. Thus, the designer shall assess (pair of P_u and M_u)$_{given}$ < pair of P_u and M_u, as strength capacities (10.53)

This type of problem is known as analysis type of problem. The three steps are given below while using design charts of SP-16 for solving such problems.

Step 1: Selection of the design chart

The designer has to select a particular design chart, specified by the chart number, from the known value of *d'/D* and the grade of steel for circular columns; and considering also the distribution of longitudinal steel bars equally on two or four sides for the rectangular columns.

Step 2: Selection of the particular curve

The designer shall select the particular curve out of the family of fourteen curves in the chart selected in Step 1. The selection of the curve shall be made from the value of p/f_{ck} parameter which is known.

Step 3: Assessment of the column

This can be done in any of the three methods selecting two of the three parameters as known and comparing the third parameter to satisfy Eq.10.53. The parameters are $P_u/f_{ck}\,bD$, $M_u/f_{ck}\,bD^2$ and p/f_{ck} for rectangular columns. For circular columns the breadth b shall be replaced by D (the diameter of the column).

Use of Design Charts in the Design Type of Problems

It is explained in sec.10.24.2 of Lesson 24 that the design of columns mainly involves with the determination of percentage of longitudinal steel p, either assuming or knowing the dimensions b and D, grades of concrete and steel, distribution of longitudinal bars in two or multiple rows and *d'/D* from the analysis or elsewhere. However, the designer has to confirm the assumed data or revise them, if needed. The use of design charts of SP-16 is explained below in four steps while designing columns:

Step 1: Selection of the design chart

As in step 1 of sec.10.25.4, the design chart is selected from the assumed values of the parameter as explained in step 1 of sec.10.25.4. The only difference is that, here the assumed parameter may be revised, if required.

Step 2: Determination of the percentage of longitudinal steel

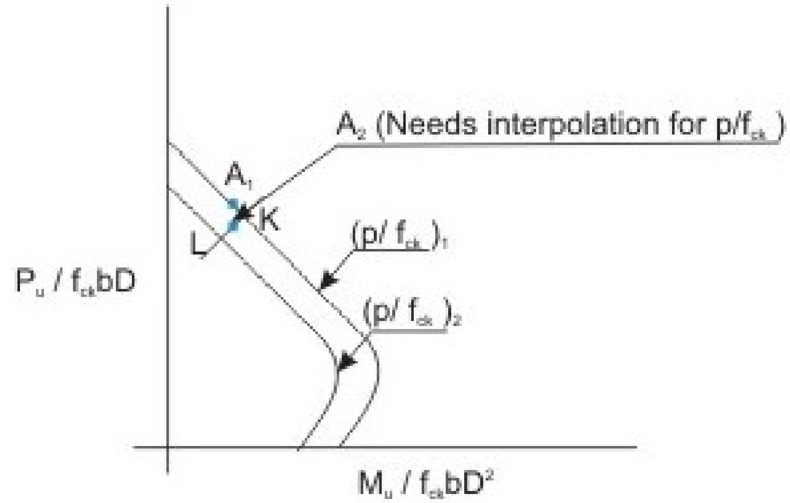


Fig.10.25.5: Determination of p/f_{ck} by linear interpolation (not to scale)

The two parameters ($P_u/f_{ck}\, bD$) and ($M_u/f_{ck}\, bD^2$) are known and the point A is located on the design chart with these two coordinates (Fig.10.25.5). The point may be like A1, on a particular curve of specified p/f_{ck}, or like A2, in between two such curves having two values of p/f_{ck}, the difference between the two values of p/f_{ck} is 0.02. In the first case, the corresponding p/f_{ck} is obtained directly as specified on the curve. While, in the second case, liner interpolation is to be done by drawing a line KL perpendicular to the two curves and passing through the point A2.

The percentage of longitudinal steel is obtained by multiplying the p/f_{ck}, so obtained, by the actual grade of concrete (which may be different from M 20 though the chart is prepared assuming M 20 only). Thus, percentage of longitudinal steel,

$$p = (p/f_{ck})\ (\text{Actual } f_{ck}) \quad (10.54)$$

This percentage of longitudinal steel (obtained from Eq.10.54) is a tentative value and shall be confirmed after finalizing the assumed data, i.e., *d'/D, b, D* etc.

Step 3: Design of transverse reinforcement

This should be done before confirming *d'/D* as the diameter of the lateral tie has a role in finalizing *d'*. The design of transverse reinforcement shall be done following the procedure explained in secs.10.21.8 and 10.21.9 of Lesson 21.

Step 4: Revision of the design, if required

If the value of d'/D changes in step 3 requiring any change of other dimension etc., the repetition of steps 1 to 3 are needed. Otherwise, the design is complete.

Illustrative Examples

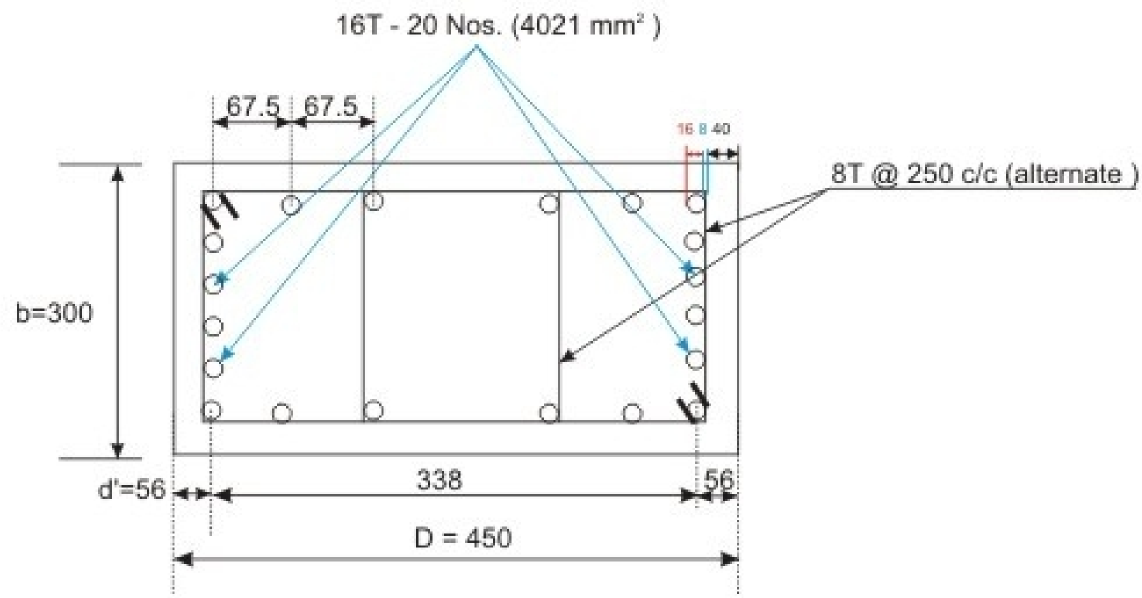


Fig. 10.25.6: Tied column of Problem 1

Problem 1:

Figure 10.25.6 shows a rectangular short reinforced concrete column using M 25 and Fe 415. Analyse the safety of the column when subjected to P_u = 1620 kN and M_u = 170 kNm.

Solution 1:

This is an analysis type of problems. The data given are: b = 300 mm, D = 450 mm, d' = 56 mm, A_{sc} = 4021 mm^2 (20 bars of 16 mm diameter), f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, P_u = 1620 kN and M_u = 170 kNm. So, we have d'/D = 56/450 = 0.1244, $P_u/f_{ck}bD$ = 0.48, $M_u/f_{ck}bD^2$ = 0.111934 and p/f_{ck} = 0.11914.

Step 1: Selection of design chart

From the given data: d'/D = 0.1244, f_y = 415 N/mm^2 and longitudinal steel bars are equally distributed on four sides, the charts selected are 44 (for d'/D =

0.1) and 45 (for d'/D = 0.15). Linear interpolation has to done with the values obtained from these two charts.

Step 2: Selection of the particular curve

From the given value of p/f_{ck} = 0.11914, the two curves having p/f_{ck} = 0.1 and 0.12 are selected from both the charts (No. 44 and 45). Here also, linear interpolation has to be done.

Step 3: Assessment of the column

In order to assess the column, we select the two given parameters p/f_{ck} and $P_u/f_{ck}bd^2$ to determine the third parameter $Mu/f_{ck}bD^2$ to compare its value with the given parameter $M_u/f_{ck}bD^2$. However, the value of $M_u/f_{ck}bD^2$ is obtained by doing linear interpolation two times: once with respect to p/f_{ck} and the second time with respect to d'/D. The results are furnished in Table 10.9 below:

Table 10.9: Values of $M_u/f_{ck}bD^2$ when $(P_u/f_{ck}bD^2)_{given}$ = 0.48 and $(p/f_{ck})_{given}$ = 0.11914; and d'/D = 0.1244

Sl. No.	p/f_{ck}	d'/D		
		0.1	0.15	0.1244
1	0.1	0.1*	0.09**	0.09512***
2	0.12	0.12*	0.107**	0.113656***
3	0.11914	0.1194***	0.10649***	0.1130941***

Note: * Values obtained from chart 44
** Values obtained from chart 45
*** Linearly interpolated values

Thus, the moment capacity of the column is obtained from the final value of $M_u/f_{ck}bD^2$ = 0.1130941 as

M_u = (0.1130941)(25)(300)(450)(450) Nmm = 171.762 kNm,

which is higher than the given M_u = 170 kNm. Hence, the column can be subjected to the pair of given P_u and M_u as 1620 kN and 170 kNm, respectively.

Problem 2:

Design a short spiral column subjected to P_u = 2100 kN and M_u = 187.5 kNm using M 25 and Fe 415. The preliminary diameter of the column may be taken as 500 mm.

Solution 2:

Step 1: Selection of design chart

With the given f_y = 415 N/mm^2 and assuming d'/D = 0.1, the chart selected for this problem is Chart 56.

Step 2: Determination of the percentage of longitudinal steel

With the given f_{ck} = 25 N/mm^2 and assuming the given D = 500 mm, we have:

$P_u/f_{ck}D^2$ = 2100000/25(500)(500) = 0.336, and

$M_u/f_{ck}D^3$ = 187.5(10^6)/25(500)(500)(500) = 0.06

The particular point A (Fig.10.25.5) having coordinates of $P_u/f_{ck}D^2$ = 0.336 and $M_u/f_{ck}D^3$ = 0.06 in Chart 56 gives: p/f_{ck} = 0.08. Hence, p = 0.08(25) = 2 per cent (see Eq.10.54).

A_{sc} = 0.02( )(500)(500)/4 = 3928.57 mm^2

Provide 8-25 mm diameter bars to have A_{sc} actually provided = 3927 mm^2. Marginally less amount of steel than required will be checked considering the enhancement of strength for spiral columns as stipulated in cl.39.4 of IS 456.

Step 3: Design of transverse reinforcement

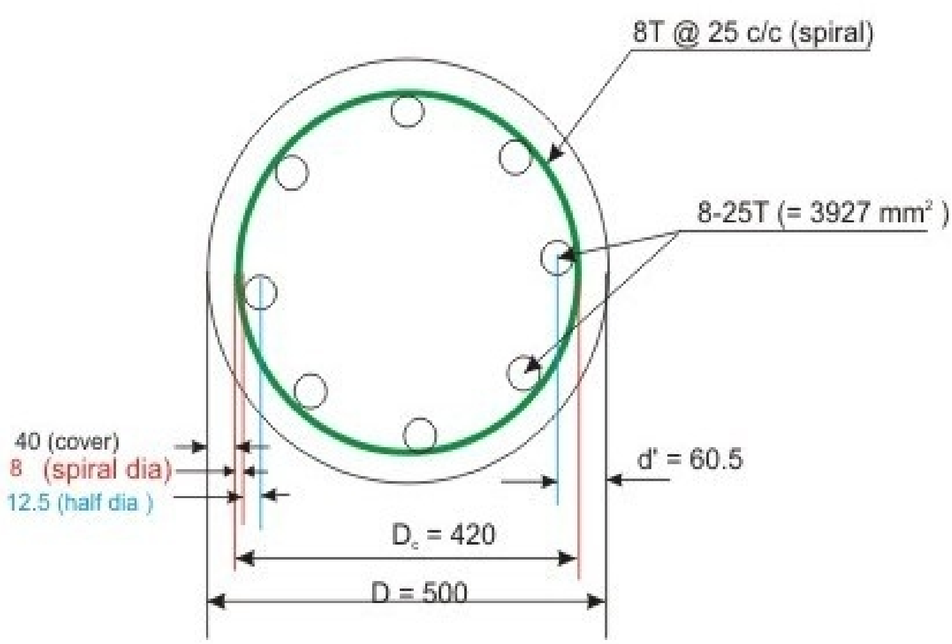


Fig. 10.25.7: Spiral column of Problem 2

The diameter of the helical reinforcement is taken as 8 mm (> 25 mm/4). The pitch p of the spiral is determined from Eq.10.11 of Lesson 22, which satisfies the stipulation in cl.39.4.1 of IS 456. From Eq.10.11, we have the pitch of the spiral p as:

$$p \leq 11.1(D_c - \phi_{sp})\, a_{sp} f_y/(D^2 - D_c^2)\, f_{ck} \quad \ldots\ (10.11)$$

where, D_c = 500 – 40 – 40 = 420 mm, D = 500 mm, f_{ck} = 25 N/mm², f_y = 415 N/mm², ϕ_{sp} = 8 mm and a_{sp} = 50 mm².

Using the above values in Eq.10.11, we have $p \leq 25.716$ mm. As per cl.26.5.3.2d1, regarding the pitch of spiral: $p \ngtr 420/6$ (= 70 mm), $p \nless 25$ mm and $p \nless 24$ mm. So, pitch of the spiral = 25 mm is o.k. Figure 10.25.7 presents the cross-section with reinforcing bars of the column.

Step 4: Revision of the design, if required

Providing 25 mm diameter longitudinal steel bars and 8 mm diameter spirals, we have d' = 40 + 8 + 12.5 = 60.5 mm. This gives d'/D = 60.5/500 = 0.121. In step 1, d'/D is assumed as 0.1. So, the revision of the design is needed.

However, as mentioned in step 2, the area of steel required is not provided and this may be offset considering the enhanced strength of the spiral column, as stipulated in cl.39.4 of IS 456.

We, therefore, assess the strength of the designed column, when d'/D = 0.121 and A_{sc} = 3927 mm², if it can be subjected to P_u = 2100 kN and M_u = 187.5 kNm.

For the purpose of assessment, we determine the capacity P_u of the column when M_u = 187.5 kNm. Further, the revised d'/D = 0.121 needs to interpolate the values from Charts 56 (for d'/D = 0.1) and 57 (for d'/D = 0.15). The value of p/f_{ck} = 0.08 and $M_u/f_{ck}bD^3$ = 0.06. Table 10.10 presents the results.

Table 10.10: Value of $P_u/f_{ck}bD^2$ when $M_u/f_{ck}D^3$ = 0.06 and p/f_{ck} = 0.08

Sl.No.	d'/D	$P_u/f_{ck}D^2$
1	0.1	0.336 (from Chart 56)
2	0.15	0.30 (from Chart 57)
3	0.121	0.32088 (Interpolated value)

From Table 10.10, thus, we get,

$P_u/f_{ck}D^2 = 0.32088$, which gives $P_u = (0.32088)(25)(500)(500) = 2005.5$ kN.

Considering the enhanced strength as 1.05 times as per cl.39.4 of IS 456, the actual capacity of this column is (1.05)(2005.5) = 2105 kN > 2100 kN.

Thus, the design is safe to carry P_u = 2100 kN and M_u = 187.5 kNm.

Biaxial Bending

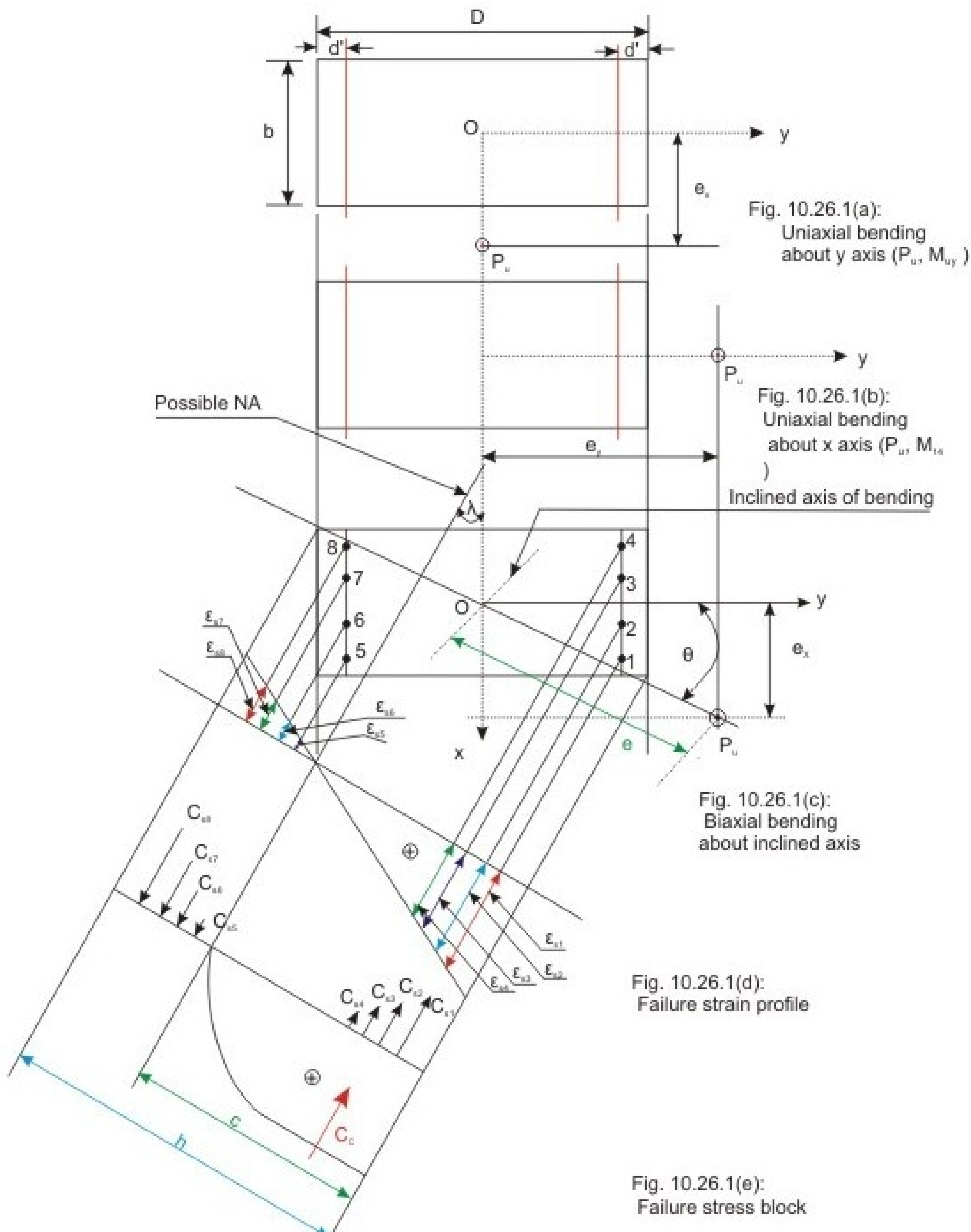


Fig. 10.26.1: Column under uniaxial and biaxial bending

Figures 10.26.1a and b present column section under axial load and uniaxial bending about the principal axes x and y, respectively. Figure 10.26.1c

presents the column section under axial load and biaxial bending. The eccentricities e_x and e_y of Fig.10.26.1c are the same as those of Fig.10.26.1a (for e_x) and Fig.10.26.1b (for e_y), respectively. Thus, the biaxial bending case (case
c) is the resultant of two uniaxial bending cases a and b. The resultant eccentricity e, therefore, can be written as (see Fig.10.26.1c):

$$e = (e_x^2 + e_y^2)^{1/2} \quad (10.55)$$

Designating the moments of cases a, b and c by M_{ux}, M_{uy} and M_u, respectively, we can write:

$$M_u = (M_{ux}^2 + M_{uy}^2)^{1/2} \quad (10.56)$$

and the resultant M_u is acting about an inclined axis, so that

$$\tan\theta = e_x/e_y = M_{uy}/M_{ux} \quad (10.57)$$

the angle of inclination θ is measured from y axis.

This inclined resultant axis shall also be the principal axis if the column section including the reinforcing bars is axisymmetric. In such a situation, the biaxial bending can be simplified to a uniaxial bending with the neutral axis parallel to the resultant axis of bending.

The reinforced concrete column cross-sections are, in general, non- axisymmetric with reference to the longitudinal axis and, therefore, the neutral axis is not parallel to the resultant axis of bending (θ is not equal to λ in Fig.10.26.1c). Moreover, it is extremely laborious to find the location of the neutral axis with successive trials. However, failure strain profile and stress block can be drawn for a given location of the neutral axis. Figs.10.25.1d and e present the strain profile and stress block, respectively, of the section shown in Fig.10.25.1c.

Interaction Surface

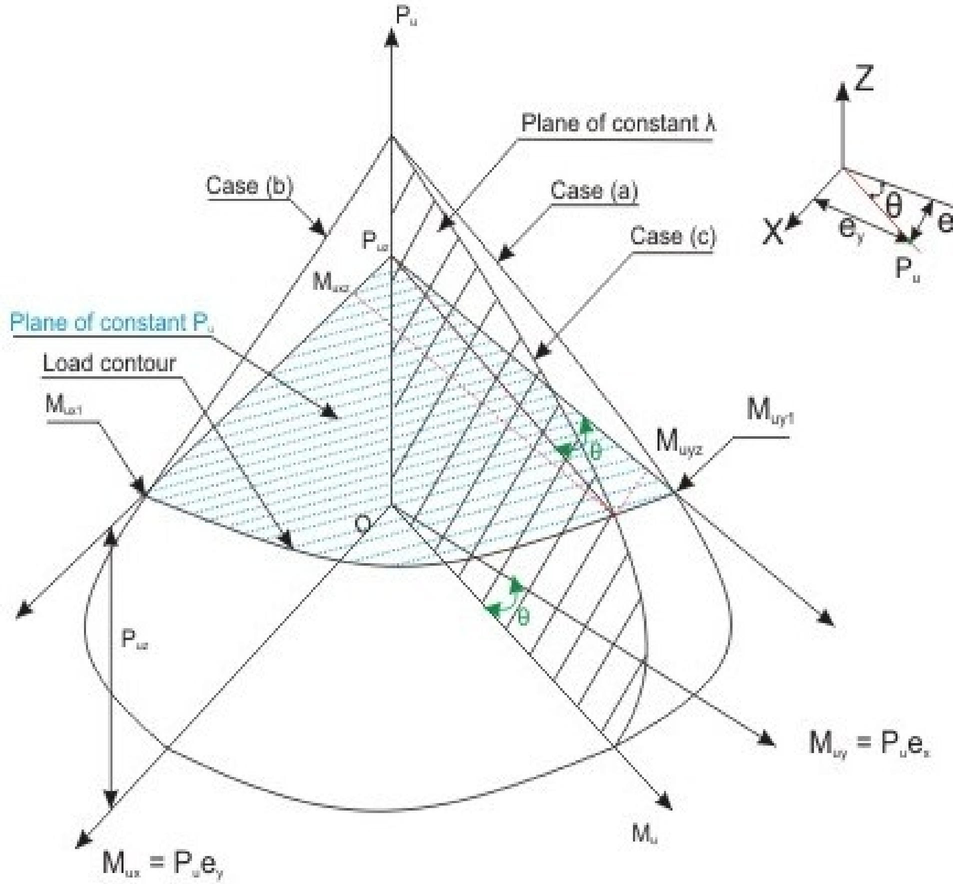


Fig. 10.26.2: Interaction diagram under axial load and biaxial bending

Figure 10.26.2 can be visualised as a three-dimensional plot of P_u-M_{ux}- M_{uy}, wherein two two-dimensional plots of P_u-M_{uy} and P_u-M_{us} are marked as case (a) and case (b), respectively. These two plots are the interaction curves for the columns of Figs.10.26.1a and b, respectively. The envelope of several interaction curves for different axes will generate the surface, known as interaction surface.

The interaction curve marked as case (c) in Fig.10.26.2, is for the column under biaxial bending shown in Fig.10.26.1c. The corresponding axis of bending is making an angle θ with the y axis and satisfies Eq.10.57. It has been explained in Lesson 24 that a column subjected to a pair of P and M will be safe if their respective values are less than P_u and M_u, given by its interaction curve. Extending the same in the three-dimensional figure of interaction surface, it is also acceptable that a column subjected to a set of P_u, M_{uy} and M_{ux} is safe if the set of values lies within the surface. Since P_u is changing in the direction of z, let us designate the moments and axial loads as mentioned below:

M_{uxz} = design flexural strength with respect to major axis xx under biaxial loading, when $P_u = P_{uz}$,

M_{uyz} = design flexural strength with respect to minor axis *yy* under biaxial loading, when $P_u = P_{uz}$,

M_{ux1} = design flexural strength with respect to major axis *xx* under uniaxial loading, when $P_u = P_{uz}$, and

M_{uy1} = design flexural strength with respect to minor axis *yy* under uniaxial loading, when $P_u = P_{uz}$.

The above notations are also shown in Fig.10.26.2.

All the interaction curves, mentioned above, are in planes perpendicular to *xy* plane. However, the interaction surface has several curves parallel to *xy* plane, which are planes of constant P_u. These curves are known as load contour, one such load contour is shown in Fig.10.26.2, when $P_u = P_{uz}$. Needless to mention that the load is constant at all points of a load contour. These load contour curves are also interaction curves depicting the interaction between the biaxial bending capacities.

Limitation of Interaction Surface

The main difficulty in preparing an exact interaction surface is that the neutral axis for the case (c) of Fig.10.26.1c will not, in general, be perpendicular to the line joining the loading point P_u and the centre of the column (Fig.10.26.1c). This will require several trials with c and λ , where c is the distance of the neutral axis and λ angle made by the neutral axis with the x axis, as shown in Fig.10.26.1c. Each trial will give a set of P_u, M_{ux} and M_{uy}. Only for a particular case, the neutral axis will be perpendicular to the line joining the load point P_u to the centre of the column. This search makes the process laborious. Moreover, several trials with c and λ , giving different values of h (see Fig.10.26.1c), may result in a failure surface with wide deviations, particularly as the value of P_u will be increasing.

Accordingly, the design of columns under axial load with biaxial bending is done by making approximations of the interaction surface. Different countries adopted different approximate methods. Clause 39.6 of IS 456 recommends one method based on Bresler's formulation, also known as "Load Contour Method", which is taken up in the following section. (For more information, please refer to: "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending", by B. Bresler, J. ACI, Vol.32, No.5, 1960, pp.481-490).

IS Code Method for Design of Columns under Axial Load and Biaxial Bending

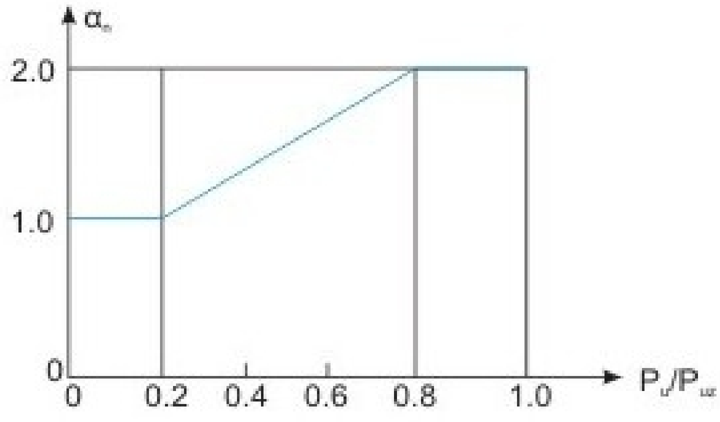


Fig. 10.26.3: Exponent α_n versus P_u / P_{uz}

IS 456 recommends the following simplified method, based on Bresler's formulation, for the design of biaxially loaded columns. The relationship between M_{uxz} and M_{uyz} for a particular value of $P_u = P_{uz}$, expressed in non-dimensional form is:

$$(M_{ux} / M_{ux1})^{\alpha n} + (M_{uy} / M_{uy1})^{\alpha n} \leq 1$$

(10.58)

where M_{ux} and M_{uy} = moments about x and y axes due to design loads, and

α_n is related to P_u/P_{uz}, (Fig.10.26.3), where

$P_{uz} = 0.45\, f_{ck}\, A_c + 0.75\, f_y\, A_{sc}$

$= 0.45\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$

(10.59)

where A_g = gross area of the section, and

A_{sc} = total area of steel in the section

M_{uxz}, M_{uyz}, M_{ux1} and M_{uy1} are explained in sec.10.26.3 earlier.

It is worth mentioning that the quantities M_{ux}, M_{uy} and P_u are due to external loadings applied on the structure and are available from the analysis, whereas M_{ux1}, M_{uy1} and P_{uz} are the capacities of the column section to be considered for the design.

Equation 10.58 defines the shape of the load contour, as explained earlier (Fig.10.26.2). That is why the method is also known as "Load Contour Method".

The exponent α_n of Eq.10.58 is a constant which defines the shape of the load contour and depends on the value of P_u. For low value of the axial load, the load contour is approximated as a straight line and, in that case, $\alpha_n = 1$. On the other hand, for high values of axial load, the load contour is approximated as a quadrant of a circle, when $\alpha_n = 2$. For intermediate load values, the value of α_n lies between 1 and 2. Chart 64 of SP-16 presents the load contour and Fig.10.26.3 presents the relationship between α_n and P_u/P_{uz}. The mathematical relationship between α_n and P_u/P_{uz} is as follows:

$\alpha_n = 1.0$, when $P_u/P_{uz} \leq 0.2$

$\alpha_n = 0.67 + 1.67\, P_u/P_{uz}$, when $0.2 < (P_u/P_{uz}) < 0.8$

$\alpha_n = 2.0$, when $(P_u/P_{uz}) \geq 0.8$

(10.60)

Solution of Problems using IS Code Method

The IS code method, as discussed in sec.10.26.5, can be employed to solve both the design and analysis types of problems. The only difference between the design and analysis type of problems is that a trial section has to be assumed including the percentage of longitudinal reinforcement in the design problems. However, these data are available in the analysis type of problems. Therefore, a guide line is given in this section for assuming the percentage of longitudinal reinforcement for the design problem. Further, for both types of problems, the eccentricities of loads are to be verified if they are more than the corresponding minimum eccentricities, as stipulated in cl.25.4 of IS 456. Thereafter, the relevant steps are given for the solution of the two types of problems.

(a) Selection of trial section for the design type of problems

As mentioned in sec.10.24.2(i) of Lesson 24, the preliminary dimensions are already assumed during the analysis of structure (mostly statically indeterminate). Thus, the percentage of longitudinal steel is the one parameter to be assumed from the given P_u, M_{ux}, M_{uy}, f_{ck} and f_y. Pillai and Menon (Ref. No. 4) suggested a simple way of considering a moment of approximately 15 per cent in excess (lower percentage up to 5 per cent if P_u/P_{uz} is relatively high) of the resultant moment

$$M_u = (1.15)(M_{ux}^2 + M_{uy}^2)^{1/2} \qquad (10.61)$$

as the uniaxial moment for the trial section with respect to the major principal axis xx, if $M_{ux} \geq M_{uy}$; otherwise, it should be with respect to the minor principal axis.

The reinforcement should be assumed to be distributed equally on four sides of the section.

(b) Checking the eccentricities e_x and e_y for the minimum eccentricities

Clause 25.4 of IS 256 stipulates the amounts of the minimum eccentricities and are given in Eq.10.3 of sec.10.21.11 of Lesson 21. However, they are given below as a ready reference.

$$e_{xmin} \geq \text{greater of } (l/500 + b/30) \text{ or } 20 \text{ mm}$$

$$e_{ymin} \geq \text{greater of } (l/500 + D/30) \text{ or } 20 \text{ mm} \quad \text{.... (10.3)}$$

where l, b and D are the unsupported length, least lateral dimension and larger lateral dimension, respectively. The clause further stipulates that for the biaxial bending, it is sufficient to ensure that the eccentricity exceeding the minimum value about one axis at a time.

(c) Steps for the solution of problems

The following are the steps for the solution of both analysis and design types of problems while employing the method recommended by IS 456.

(i) Verification of eccentricities

It is to be done determining $e_x = M_{ux}/P_u$ and $e_y = M_{uy}/P_u$ from the given data of P_u, M_{ux} and M_{uy}; and e_{xmin} and e_{ymin} from Eq.10.3 from the assumed b and D and given l.

(ii) Assuming a trial section including longitudinal reinforcement

This step is needed only for the design type of problem, which is to be done as explained in (a) above.

(iii) Determination of M_{ux1} and M_{uy1}

Use of design charts should be made for this. M_{ux1} and M_{uy1}, corresponding to the given P_u, should be significantly greater than M_{ux} and M_{uy}, respectively. Redesign of the section should be done if the above are not satisfied for the design type of problem only.

(iv) Determination of P_{uz} and α_n

The values of P_{uz} and α_n can be determined from Eqs.10.59 and 10.60, respectively. Alternatively, P_{uz} can be obtained from Chart 63 of SP-16.

(v) Checking the adequacy of the section

This is done either using Eq.10.58 or using Chart 64 of SP-16.

Illustrative Example

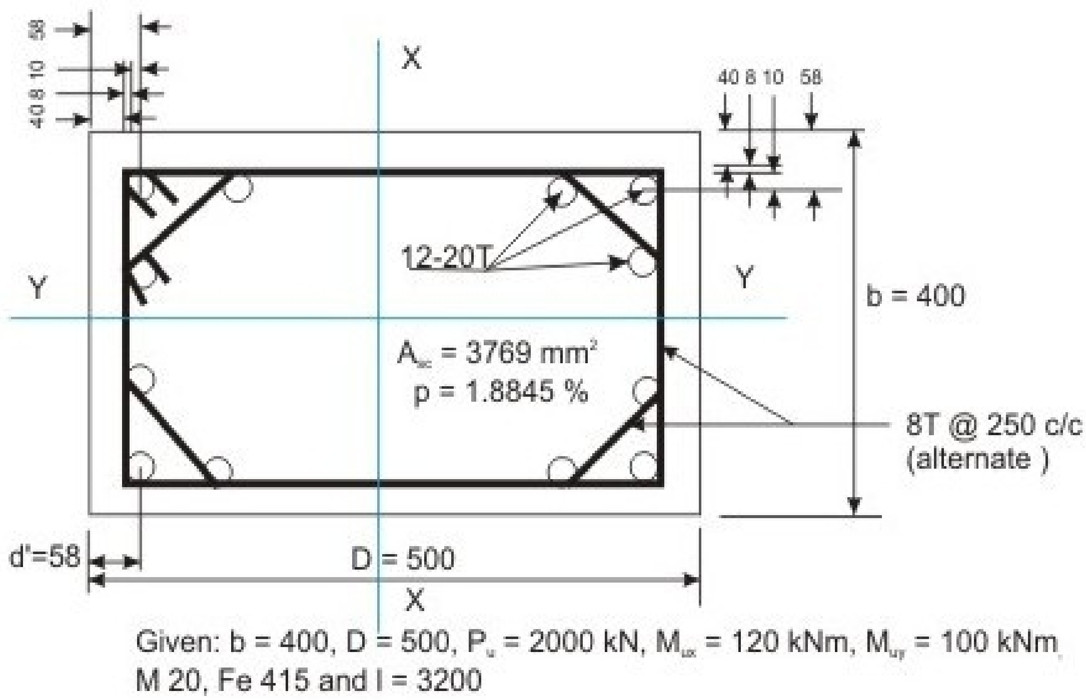


Fig. 10.26.4: Problem 1

Problem 1:

Design the reinforcement to be provided in the short column of Fig.10.26.4 is subjected to P_u = 2000 kN, M_{ux} = 130 kNm (about the major principal axis) and M_{uy} = 120 kNm (about the minor principal axis). The unsupported length of the column is 3.2 m, width b = 400 mm and depth D = 500 mm. Use M 25 and Fe 415 for the design.

Solution 1:

Step 1: Verification of the eccentricities

Given: l = 3200 mm, b = 400 mm and D = 500 mm, we have from Eq.10.3 of sec.10.26.6b, the minimum eccentricities are:

e_{xmin} = greater of (3200/500 + 400/30) and 20 mm = 19.73 mm or 20 mm = 20 mm

e_{ymin} = greater of (3200/500 + 500/30) and 20 mm = 23.07 mm or 20 mm = 23.07 mm

Again from P_u = 2000 kN, M_{ux} = 130 kNm and M_{uy} = 120 kNm, we have e_x = M_{ux}/P_u = 130(10^6)/2000(10^3) = 65 mm and e_y = M_{uy}/P_u = 120(10^6)/2000(10^3) = 60 mm. Both e_x and e_y are greater than e_{xmin} and e_{ymin}, respectively.

Step 2: Assuming a trial section including the reinforcement

We have b = 400 mm and D = 500 mm. For the reinforcement, $M_u = 1.15\,(M_{ux}^2 + M_{uy}^2)^{1/2}$, from Eq.10.61 becomes 203.456 kNm. Accordingly, we get

$P_u/f_{ck}bD$ = 2000(10^3)/(25)(400)(500) = 0.4

$M_u/f_{ck}bD^2$ = 203.456(10^6)/(25)(400)(500)(500) = 0.0814

Assuming d' = 60 mm, we have d'/D = 0.12. From Charts 44 and 45, the value of p/f_{ck} is interpolated as 0.06. Thus, p = 0.06(25) = 1.5 per cent, giving A_{sc} = 3000 mm^2. Provide 12-20 mm diameter bars of area 3769 mm^2, actual p provided = 1.8845 per cent. So, p/f_{ck} = 0.07538.

Step 3: Determination of M_{ux1} and M_{uy1}

We have $P_u/f_{ck}bD$ = 0.4 and p/f_{ck} = 0.07538 in step 2. Now, we get $M_{ux1}/f_{ck}bD^2$ from chart corresponding to d' = 58 mm (Fig.10.26.4) i.e., d'/D = 0.116. We interpolate the values of Charts 44 and 45, and get $M_{ux1}/f_{ck}bD^2$ = 0.09044. So, M_{ux1} = 0.0944(25)(400)(500(500)(10^{-6}) = 226.1 kNm.

For M_{ux1}, d'/b = 58/400 = 0.145. In a similar manner, we get M_{uy1} = 0.0858(25)(400)(400)(500)(10^{-6}) = 171.6 kNm.

As M_{ux1} and M_{uy1} are significantly greater than M_{ux} and M_{uy}, respectively, redesign of the section is not needed.

Step 4: Determination of P_{uz} and α_n

From Eq.10.59, we have P_{uz} = 0.45(25)(400)(500) + {0.75(415) - 0.45(25)}(3769) = 3380.7 kN.

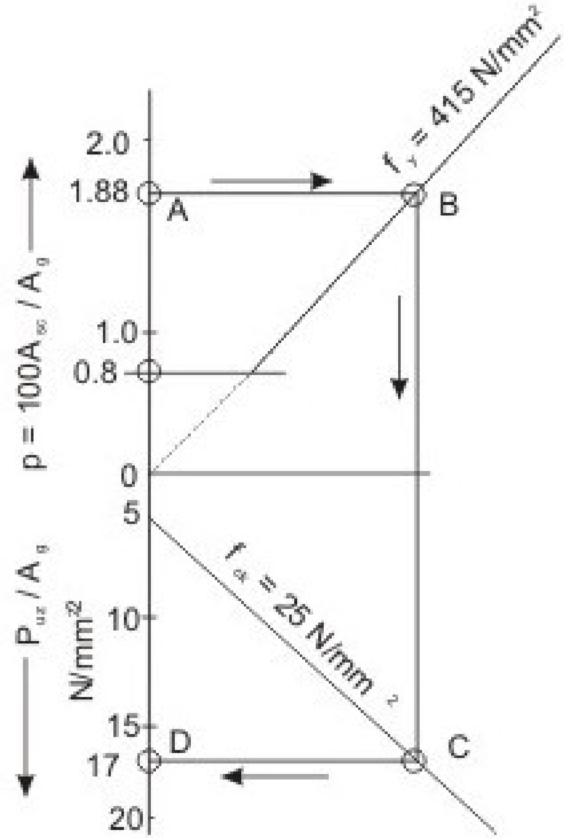


Fig. 10.26.5: Chart 63 of SP-16 in Problem 1
(not to scale)

Alternatively, Chart 63 may be used to find P_{uz} as explained. From the upper section of Chart 63, a horizontal line AB is drawn at p = 1.8845, to meet the Fe 415 line B (Fig.10.26.5). A vertical line BC is drawn from B to meet M 25 line at
C. Finally, a horizontal line CD is drawn from C to meet P_{uz}/A_g at 17. This gives P_{uz} = 17(400)(500) = 3400 kN. The difference between the two values, 19.3 kN is hardly 0.57 per cent, which is due to the error in reading the value from the chart. However, any one of the two may be employed.

Now, the value of α_n is obtained from Eq.10.60 for P_u/P_{uz} = 2000/3380.7 = 0.5916, i.e., 0.2 < P_u/P_{uz} < 0.8, which gives, α_n = 0.67 + 1.67 (P_u/P_{uz}) = 1.658.

Alternatively, α_n may be obtained from Fig.10.26.3, drawn to scale.

Step 5: Checking the adequacy of the section

Using the values of M_{ux}, M_{ux1}, M_{uy}, M_{uy1} and α_n in Eq.10.58, we have $(130/226.1)^{1.658} + (120/171.6)^{1.658} = 0.9521 < 1.0$. Hence, the design is safe.

Alternatively, Chart 64 may be used to determine the point (M_{ux}/M_{ux1}), (M_{uy}/M_{uy1}) is within the curve of P_u/P_{uz} = 0.5916 or not.

Here, M_{ux}/M_{ux1} = 0.5749 and M_{uy}/M_{uy1} = 0.6993. It may be seen that the point is within the curve of P_u/P_{uz} = 0.5916 of Chart 64 of SP-16.

Step 6: Design of transverse reinforcement

As per cl.26.5.3.2c of IS 456, the diameter of lateral tie should be > (20/4) mm diameter. Provide 8 mm diameter bars following the arrangement shown in Fig.10.26.4. The spacing of lateral tie is the least of :

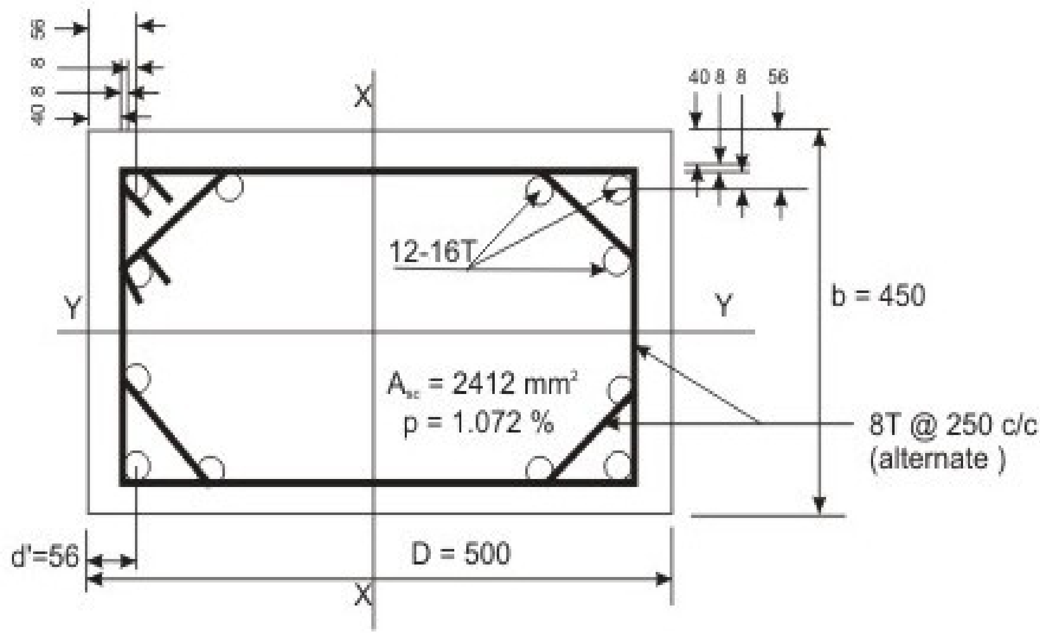
(a) 400 mm = least lateral dimension of column,

(b) 320 mm = sixteen times the diameter of longitudinal reinforcement (20 mm),

(c) 300 mm

Accordingly, provide 8 mm lateral tie alternately @ 250 c/c (Fig.10.26.4).

Q.5:



Given: b = 450, D = 500, P_u = 1600 kN, M_{ux} = 120 kNm, M_{uy} = 100 kNm, M 20, Fe 415, l = 3200, A_{sc} = 2412 mm², tie 8T @ 250 c/c (alternate)

Fig. 10.26.6: Q.5

Analyse the safety of the short column of unsupported length 3.2 m, b = 450 mm, D = 500 mm, as shown in Fig.10.26.6, having 12-16 mm diameter bars as longitudinal reinforcement and 8 mm diameter bars as lateral tie @ 250 mm c/c, when subjected to P_u = 1600 kN, M_{ux} = 120 kNm and M_{uy} = 100 kNm. Use M 25 and Fe 415.

A.5:

Step 1: Verification of the eccentricities

From the given data: l = 3200 mm, b = 450 mm and D = 500 mm,

e_{xmin} = 3200/500 + 450/30 = 21.4 > 20 mm, so, 21.4 mm

e_{ymin} = 3200/500 + 5000/30 = 23.06 > 20 mm, so, 23.06 mm

$e_x = M_{ux}/P_u = 120(10^3)/1600 = 75$ mm

$e_y = M_{uy}/P_u = 100(10^3)/1600 = 62.5$ mm

So, the eccentricities e_x and e_y are >> e_{xmin} and e_{ymin}.

Step 2: Determination of M_{ux1} and M_{uy1}

Given data are: b = 450 mm, D = 500 mm, f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, P_u = 1600 kN, M_{ux} = 120 kNm, M_{uy} = 100 kNm and A_{sc} = 2412 mm^2 (12- 16 mm diameter bars).

We have p = (100)(2412)/(450)(500) = 1.072 per cent, and d'/D = 56/500 = 0.112, d'/b = 56/450 = 0.124, $P_u/f_{ck}bD$ = 1600/(25)(450)(500) = 0.2844 and p/f_{ck} = 1.072/25 = 0.043. We get $M_{ux1}/f_{ck}bD^2$ from Charts 44 and 45 as 0.09 and 0.08, respectively. Linear interpolation gives $M_{ux1}/f_{ck}bD^2$ for d'/D = 0.112 as 0.0876. Thus,

$$M_{ux1} = (0.0876)(25)(450)(500)(500) = 246.376 \text{ kNm}$$

Similarly, interpolation of values (0.09 and 0.08) from Charts 44 and 45, we get $M_{uy1}/f_{ck}db^2$ = 0.085 for d'/b = 0.124. Thus

$$M_{uy1} = (0.085)(25)(500)(450)(450) = 215.156 \text{ kNm}$$

Step 3: Determination of P_{uz} and α_n

From Eq.10.59, P_{uz} = 0.45(25)(450)(500) + {0.75(415) - 0.45(25)}(2412) = 3254.85 kN. This gives P_u/P_{uz} = 1600/3254.85 = 0.491574.

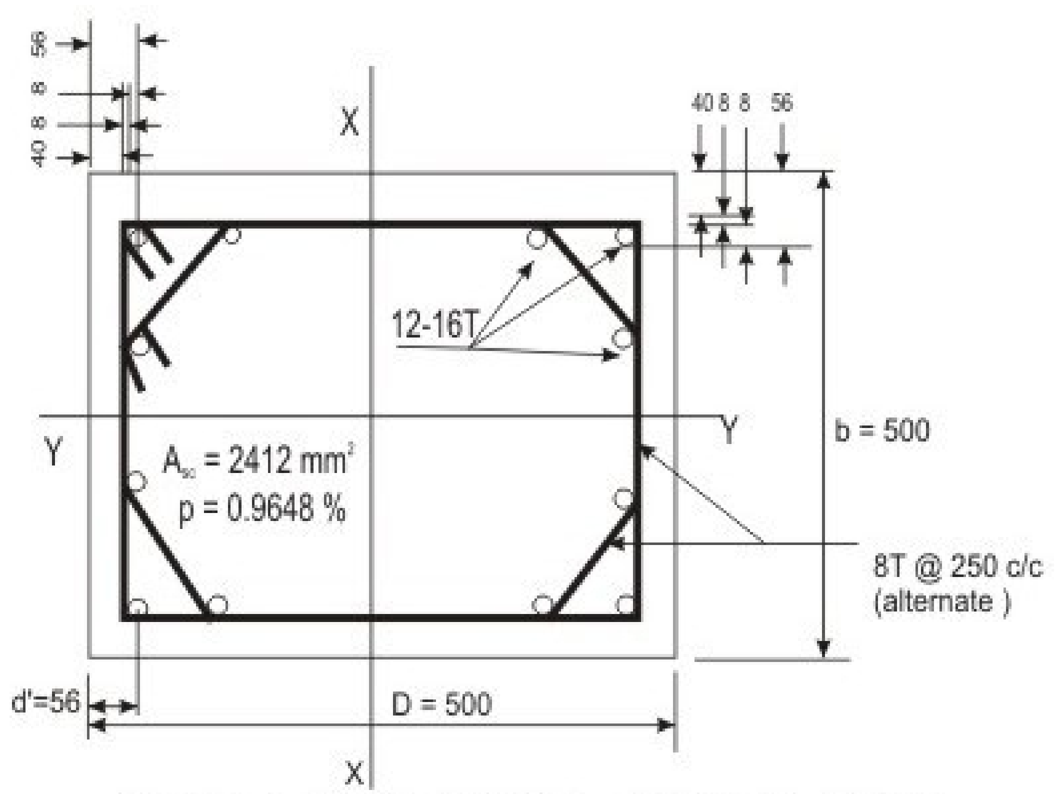
From Eq.10.60, α_n = 0.67 + 1.67(P_u/P_{uz}) = 0.67 + 1.67(0.491574) = 1.4909.

Step 4: Checking the adequacy of the section

From Eq.10.58, we have: $(120/246.376)^{1.4909}$ + $(100/215.156)^{1.4909}$ = 0.6612 < 1.

Hence, the section is safe to carry P_u = 1600 kN, M_{ux} = 120 kNm and M_{uy} = 100 kNm.

TQ.1:



Given: b = D = 500, P_u = 1800 kN, M_{ux} = 160 kNm, M_{uy} = 150 kNm,
M 20, Fe 415, l = 3500, A_{sc} = 2412 mm^2, tie 8T @ 250 c/c (alternate)

Fig. 10.26.7: TQ.1

Analyse the safety of the short square column of unsupported length = 3.5 m, b = D = 500 mm, as shown in Fig.10.26.7, with 12-16 mm diameter bars as longitudinal reinforcement and 8 mm diameter bars as lateral tie @ 250 mm c/c, when subjected to P_u = 1800 kN, M_{ux} = 160 kNm and M_{uy} = 150 kNm.

A.TQ.1:

Step 1: Verification of the eccentricities

From the given data: l = 3500 mm, b = D = 500 mm, we have

e_{min} in both directions (square column) = (3500/500) + (500/30) = 23.67 mm

e_x = 160(10^3)/1800 = 88.88 mm and e_y = 150(10^3)/1800 = 83.34 mm

Therefore, e_x and e_y >> e_{min}.

Step 2: Determination of M_{ux1} and M_{uy1}

We have the given data: $b = D = 500$ mm, $f_{ck} = 25$ N/mm^2, $f_y = 415$ N/mm^2, $P_u = 1800$ kN, $M_{ux} = 160$ kNm, $M_{uy} = 150$ kNm and $A_{sc} = 2412$ mm^2 (12-16 mm diameter bars).

The percentage of longitudinal reinforcement $p = 241200/(500)(500) = 0.9648$ per cent, and $d'/D = 56/500 = 0.112$ and $p/f_{ck} = 0.9648/25 = 0.03859$. Linear interpolation of values of $M_{ux1}/f_{ck}bD^2$ from Charts 44 and 45 for $d'/D =$
0.112 is obtained as 0.08. Thus,

$M_{ux1} = (0.08)(25)(500)(500)(500) = 250$ kNm

$M_{uy1} = M_{ux1} = 250$ kNm (square column)

Step 3: Determination of P_{uz} and α_n

From Eq.10.59,

$P_{uz} = 0.45(25)(500)(500) + \{0.75(415) - 0.45(25)\}(2415) = 3536.1$ kN.

$P_u/P_{uz} = 1800/3536.1 = 0.509$.

From Eq.10.60, $\alpha_n = 0.67 + 1.67(0.509) = 1.52$.

Step 4: Checking the adequacy of the section

From Eq.10.58, we have: $(160/250)^{1.52} + (150/250)^{1.52} = 0.967 < 1$.

Hence, the section can carry $P_u = 1800$ kN, $M_{ux} = 160$ kNm and $M_{uy} = 150$ kNm.

Introduction to Slender and short columns

Slender and short are the two types of columns classified on the basis of slenderness ratios as mentioned in sec.10.21.5 of Lesson 21. Columns having both l_{ex}/D and l_{ey}/b less than twelve are designated as short and otherwise, they are slender, where l_{ex} and l_{ey} are the effective lengths with respect to major and minor axes, respectively; and D and b are the depth and width of rectangular columns, respectively. Short columns are frequently used in concrete structures, the design of such columns has been explained in Lessons 22 to 26, loaded concentrically or eccentrically about one or both axes. However, slender columns are also becoming increasingly important and popular because of the following reasons:

(i) the development of high strength materials (concrete and steel),

(ii) improved methods of dimensioning and designing with rational and reliable design procedures,

(iii) innovative structural concepts – specially, the architect's expectations for creative structures.

Accordingly, this lesson explains first, the behaviour of slender elastic columns loaded concentrically. Thereafter, reinforced concrete slender columns loaded concentrically or eccentrically about one or both axes are taken up. The design of slender columns has been explained and illustrated with numerical examples for easy understanding.

Concentrically Loaded Columns

It has been explained in Lessons 22 to 26 that short columns fail by reaching the respective stresses indicating their maximum carrying capacities. On the other hand, the slender or long columns may fail at a much lower value of the load when sudden lateral displacement of the member takes place between the ends. Thus, short columns undergo material failure, while long columns may fail by buckling (geometric failure) at a critical load or Euler's load, which is much less in comparison to that of short columns having equal area of cross-section. The buckling load is termed as Euler's load as Euler in 1744 first obtained the value of critical load for various support conditions. For more information, please refer to Additamentum, "De Curvis elasticis", in the "Methodus inveiendi Lineas Curvas maximi minimive proprietate gaudentes" Lausanne and Geneva, 1744. An English translation of this work is given in Isis No.58, Vol.20, p.1, November 1933.

The general expression of the critical load P_{cr} at which a member will fail by buckling is as follows:

$$P_{cr} = \pi^2 EI /(kl)^2$$

where E is the Young's modulus I is the moment of inertia about the axis of bending, l is the unsupported length of the column and k is the coefficient whose value depends on the degree of restraints at the supports. Expressing moment of inertia $I = Ar^2$, where A is the area of cross-section of the column and r is the radius of gyration, the above equations can be written as,

$$P_{cr} = \pi^2 EA /(kl/r)^2$$

(10.62)

Thus, P_{cr} of a particular column depends upon kl/r or slenderness ratio. It is worth mentioning that kl is termed as effective length l_e of the column.

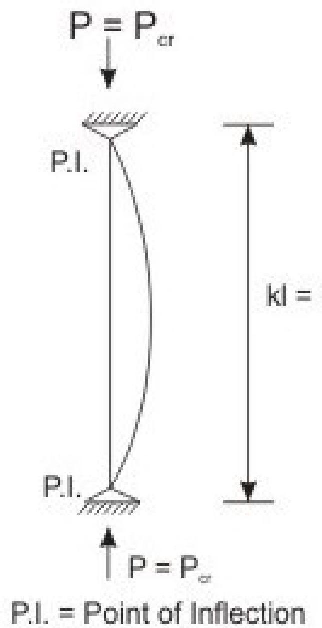


Fig. 10.27.1: Column - hinged at both ends, (k = 1)

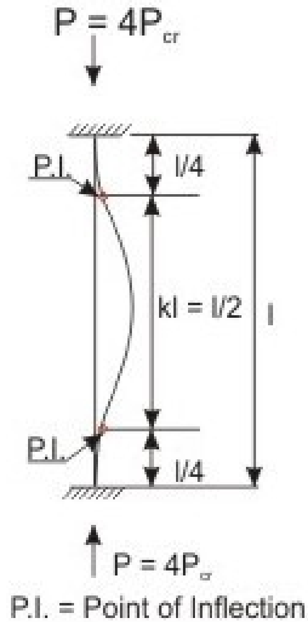


Fig. 10.27.2: Column - fixed at both ends, (k = 0.5)

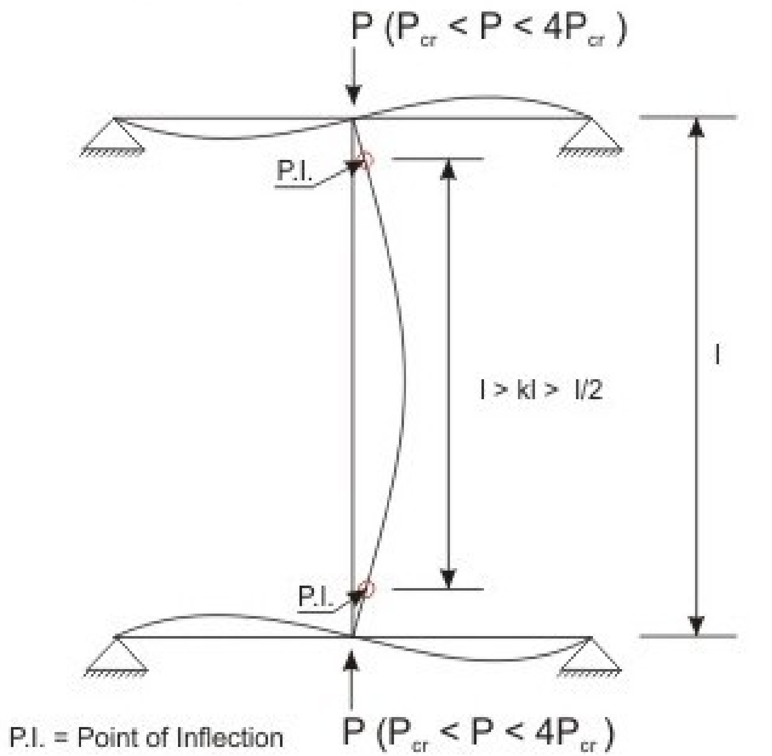


Fig.10.27.3: Column supported on cross-beams (0.5 < k < 1)

Figures 10.27.1 and 2 show two elastic slender columns having hinge supports at both ends and fixed supports against rotation at both ends, respectively. Figure 10.27.3 presents a column of real structure whose end supports are not either hinged or fixed. It has supports partially restrained against rotation by the top and bottom beams. Each of the three figures shows the respective buckled shape, points of inflection *PI*s (points of zero moment), the distance between the *PI*s and the value of *k*. All the three columns, having supports at both ends, have the *k* values less than one or at most one. By providing supports at both ends, one end of the column is prevented from undergoing lateral movement or sidesway with respect to the other end.

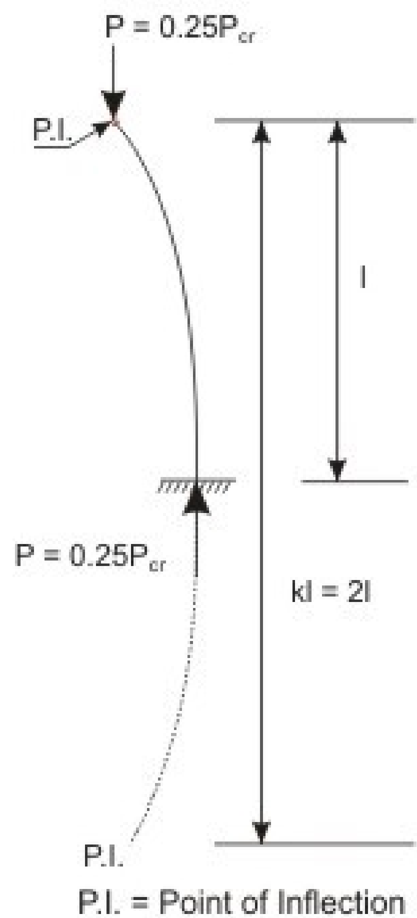


Fig. 10.27.4: Column - fixed at one end and free at other end, (k = 2)

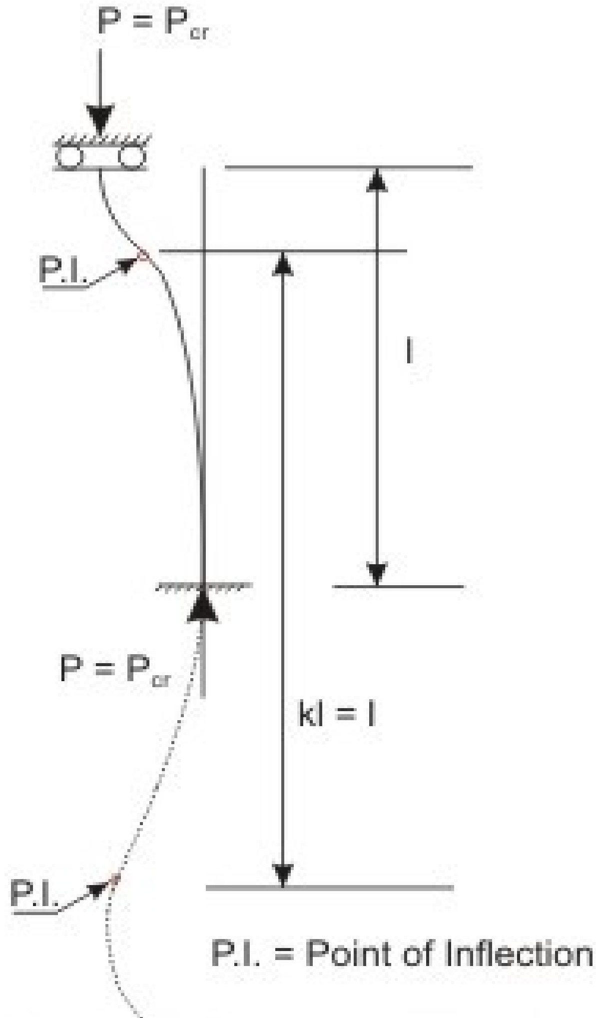


Fig. 10.27.5: Column rotationally fixed at both end, (k = 1)

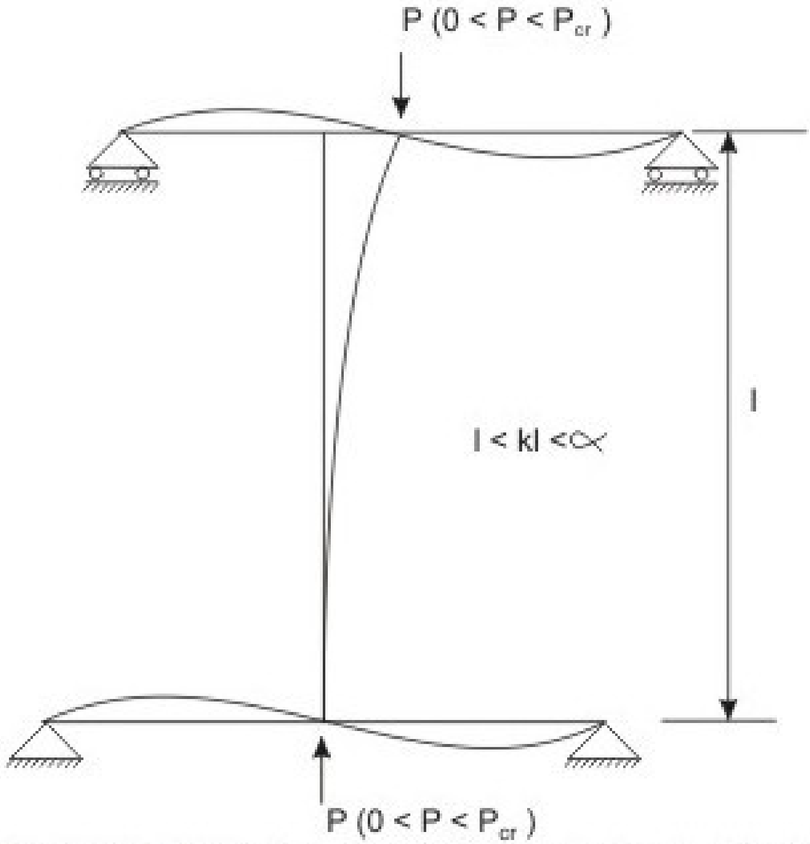


Fig.10.27.6: Column partially restrained at both ends
($1 < k < \infty$)

However, cantilever columns are entirely free at one end, as shown in Fig.10.27.4. Figure 10.27.5 shows another type of column, rotationally fixed at both ends but one end can move laterally with respect to the other. Like that of Fig.10.27.3, a real column, not hinged, fixed or entirely free but restrained by top and bottom beams, where sideway can also take place. Each of these three figures, like those of Figs.10.27.1 to 3, presents the respective buckled shape, points of inflection (*PI*s), if any, the distance between the *PI*s and the value of *k*. All these columns have the respective *k* values greater than one or at least one.

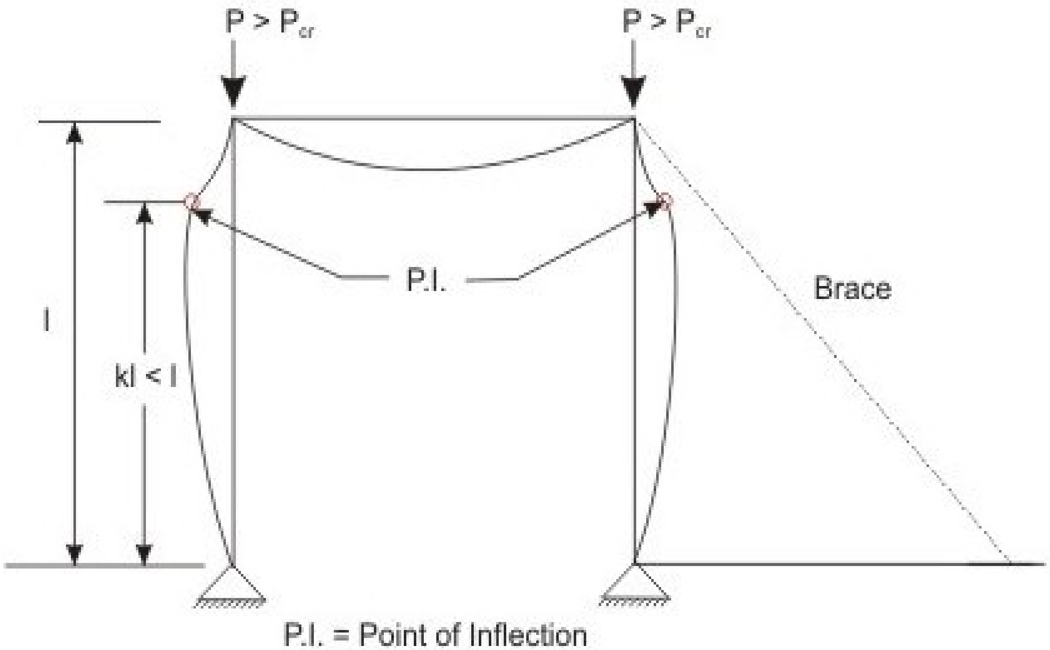


Fig. 10.27.7: Braced portal frame (k < 1)

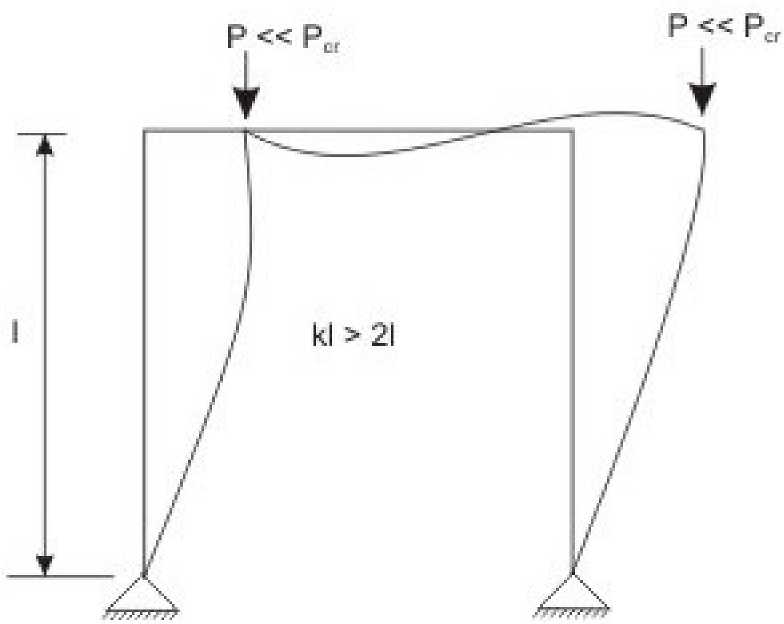


Fig. 10.27.8: Unbraced portal frame (k > 2)

Figures 10.27.7 and 8 present two reinforced concrete portal frames, a typical reinforced concrete rigid frame. Columns of Fig.10.27.7 are prevented from sidesway and those of Fig.10.27.8 are not prevented from sidesway, respectively, when subjected to concentric loadings. The buckled configuration of the frame, prevented from sidesway (Fig.10.27.7) is similar to that of Fig.10.27.3, except that the lower ends of the portal frame are hinged. One of the two points of inflection (*PI*s) is at the lower end of the column, while the other *PI* is slightly below the upper end of the column, depending on the degree of restraint. The value of k for such a frame is thus less than 1. The critical load is, therefore, slightly more than P_{cr} of the hinge-hinge column of Fig.10.27.1. The buckled configuration of the other portal frame of Fig.10.27.8, where sidesway is not prevented, is similar to the column of Fig.10.27.4 when it is made upside down, except that the upper end is not fixed but partially restrained by the supporting beam. In this case, the value of k exceeds 2, depending on the degree of restraint. One of the two *PI*s is at the bottom of the column. The critical load of the column of Fig.10.27.8 is much less than that of the column of Fig.10.27.1.

Table 10.14: Critical loads in terms of P_{cr} of hinge-hinge column and effective lengths $l_e = kl$ of elastic and reinforced concrete columns with different boundary conditions and for a constant unsupported length l

Sl. No.	Support conditions	Critical load P_{cr}	Effective length $l_e = kl$	Fig. No.
(A) Elastic single columns				
1.	Hinged at both ends, no sidesway	P_{cr}	l	10.27.1
2.	Fixed against rotation at both ends – no sidesway	$4P_{cr}$	$0.5\ l$	10.27.2
3.	Partially restrained against rotation by top and bottom cross- beams, no sidesway	Between P_{cr} and $4P_{cr}$	$l > kl > l/2$	10.27.3
4.	Fixed at one end and entirely free at other end – sidesway not prevented	$0.25\ P_{cr}$	$2\ l$, one *PI* is on imaginary extension	10.27.4
5.	Rotationally fixed at both ends – sidesway not prevented	P_{cr}	l, one *PI* is on imaginary extension	10.27.5

6.	Partially restrained against rotation at both ends – sidesway not prevented	Between zero and slightly less than P_{cr} *	$l < kl <$	10.2 7.6
(B) Reinforced concrete columns				
7.	Hinged portal frame – no sidesway	$> P_{cr}$	$kl < l$	10.2 7.7
8.	Hinged portal frame – sidesway not prevented	$<< P_{cr}$	$kl > 2\, l$	10.2 7.8

Notes: 1. Buckled shapes are half sine wave between two points of inflection (*PI*s).

2. * The critical load is slightly less than P_{cr} of hinge-hinge column (Sl.No.1), when cross-beams are very rigid compared to columns, i.e., the case under Sl.No.6 approaches the case under Sl.No.1.

The critical load is zero when cross-beams are very much flexible compared to columns, i.e., the case under Sl.No.6 approaches to hinge-hinge column of Sl.No.1, allowing sidesway. In that case, it becomes unstable and hence, carries zero load.

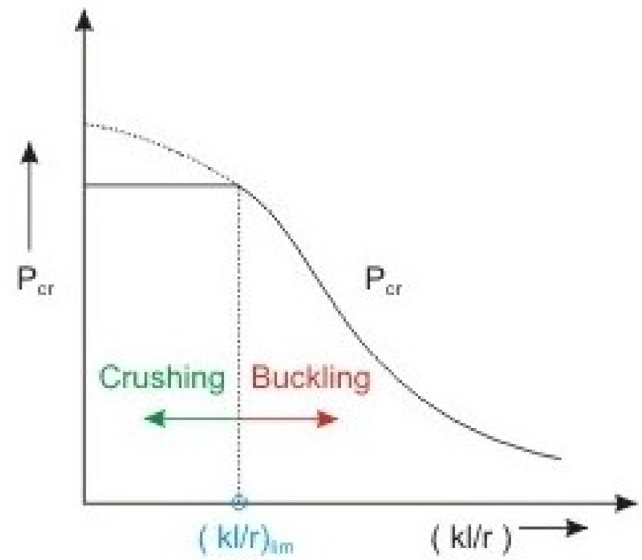


Fig. 10.27.9: Effect of slenderness on strength

Table 10.14 presents the critical load in terms of that of hinge-hinge column P_{cr} and effective lengths l_e (equal to the distance between two points of inflection PIs = kl) of elastic and reinforced concrete columns for a constant value of the unsupported length l.

The stress-strain curve of concrete, as shown in Fig.1.2.1 of Lesson 2, reveals that the initial tangent modulus of concrete E_c is much higher than E_t (tangent modulus at higher stress level). Taking this into account in Eq.10.62, Fig.10.27.9 presents a plot of buckling load P_{cr} versus kl/r. It is evident from the plot that the critical load is reducing with increasing slenderness ratio. For very short columns, the limiting factored concentric load estimated from Eq.10.39 of Lesson 24 will be found to be less than the critical load, determined from Eq.10.62. The column, therefore, will fail by direct crushing and not by buckling. We can also find out the limiting value of kl/r when the crushing load and the buckling load are the same. The $(kl/r)_{lim}$ is shown in Fig.10.27.9. The limiting value of kl/r also indicates that a column having kl/r more than $(kl/r)_{lim}$ will fail by buckling, while columns having any value of *kl/r* less than $(kl/r)_{lim}$ will fail by crushing of concrete.

The following are the observations of the discussions about the concentrically loaded columns:

1. As the slenderness ratio *kl/r* increases, the strength of concentrically loaded column decreases.

2. The effective length of columns either in single members or parts of rigid frames is between 0.5*l* and *l*, if the columns are prevented from sidesway by bracing or otherwise. The actual value depends on the degree of end restraints.

3. The effective length of columns either in single members or parts of rigid frames is always greater than one, if the columns are not prevented from sidesway. The actual value depends on the degree of end restraints.

4. The critical load of braced frame against sidesway is always significantly larger than that of the unbraced frame.

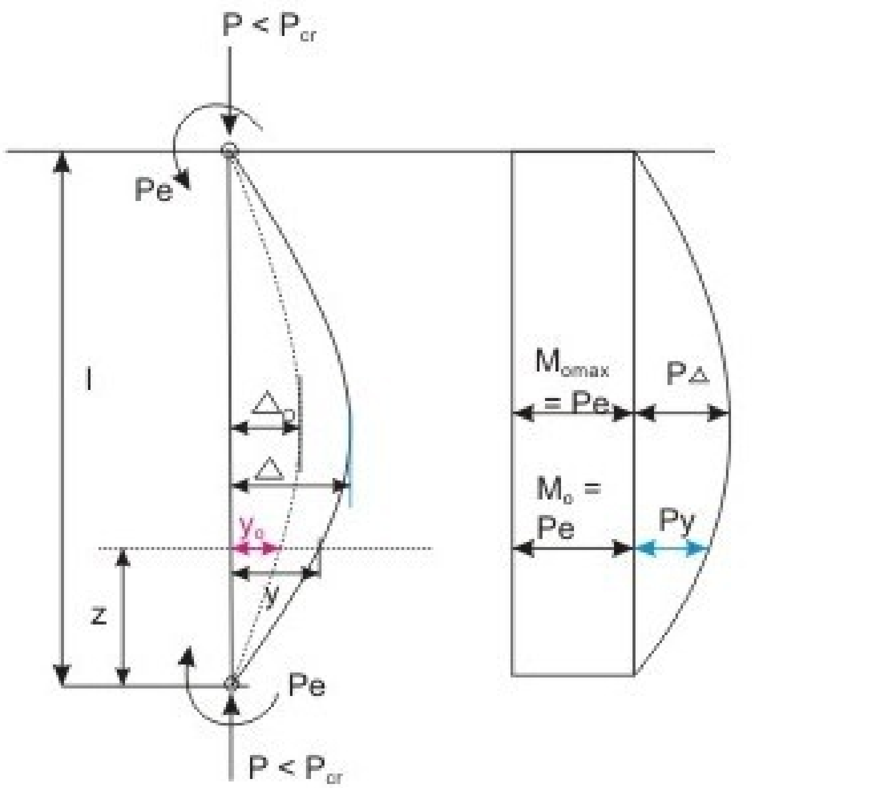


Fig. 10.27.10(a): Deflections Fig. 10.27.10(b): Moments

Fig. 10.27.10: Column bent in single curvature, (H = 0)

Slender Columns under Axial Load and Uniaxial Moment

(A) Columns bent in single curvature

Figure 10.27.10a shows a column bent in single curvature under axial load P less than its critical load P_{cr} with constant moment Pe. The deflection profile marked by dotted line is due to the constant moment. However, there will be additional moment of Py at a distance z from the origin (at the bottom of column) which will deflect the column further, as shown by the solid line. The constant moment Pe and additional moment Py are shown in Fig.10.27.10b. Thus, the total moment becomes

$$M = M_o + Py \qquad = P(e + y)$$

(10.63)

The maximum moment is $P(e + \Delta)$ at the mid-height of the column. This, we can write

$$M_{max} = M_o + P\Delta \qquad = P(e + \Delta)$$

(10.64)

This is known as $P - \Delta$ effect.

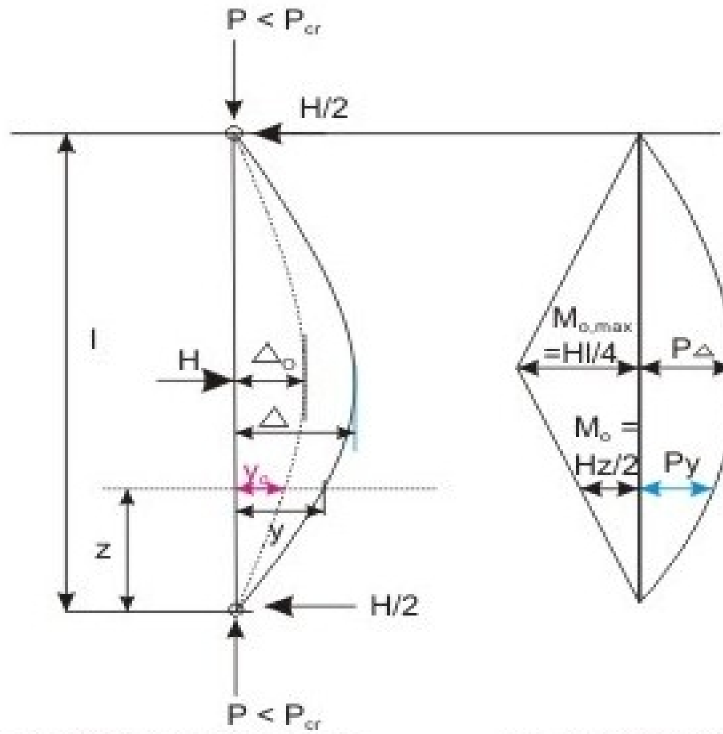


Fig. 10.27.11(a): Deflections Fig. 10.27.11(b): Moments

Fig. 10.27.11: Column bent in single curvature, (H = H)

Figure 10.27.11a shows another column whose bending is caused by a transverse load *H*. The bending moment at a distance *z* from the origin (bottom of the column) is *Hz/*2 causing deflection of the column marked by dotted line in the figure. The axial load *P*, less than its critical load $P_{cr,}$ causes additional moment resulting in further deflection, marked by solid line in the figure. This additional deflection produces additional moment of *Py* at a section *z* from the origin. The two bending moment diagrams are shown in Fig.10.27.11b. Here again, the total moment is

$$M = M_o + Py = Hz/2 + Py$$

(10.65)

The maximum moment at the mid-height of the column is

$$M_{max} = M_{o,max} + P\,\Delta \qquad = Hl/4 + P\,\Delta$$

(10.66)

The total moment in Eqs.10.63 and 10.65 consists of the moment M_o that acts in the presence of *P* and the additional moment caused by *P* (= *Py*). The deflections *y* can be computed from y_o, the deflections without the axial load from the expression

$$y = y_o[1/\{1 - (P/P_{cr})\}] \quad (10.67)$$

From Eq.10.64, we have

$$M_{max} = M_o + P\Delta \qquad = M_o + P\,\Delta_o[1/\{1 - (P/P_{cr})\}]$$

(10.68)

Equation 10.68 can be written as

$$M_{max} = M_o \frac{1 + \psi(P/P_{cr})}{1 - (P/P_{cr})}$$

(10.69)

where ψ depends on the type of loading and generally varies between ± 0.20.

Since P/P_{cr} is always less than one, we can ignore ψ (P/P_{cr}) term of Eq.10.69, to have

$$M_{max} = M_o/\{1 - (P/P_{cr})\}$$

(10.70)

where $1/\{1 - (P/P_{cr})\}$ is the moment magnification factor. In both the cases above (Figs.10.27.10 and 11), a direct addition of the maximum moment caused by transverse load or otherwise, to the maximum moment caused by P gives the total maximum moment as that is the most unfavourable situation. However, this is not the case for situation taken up in the following.

(B) Columns bent in double curvature

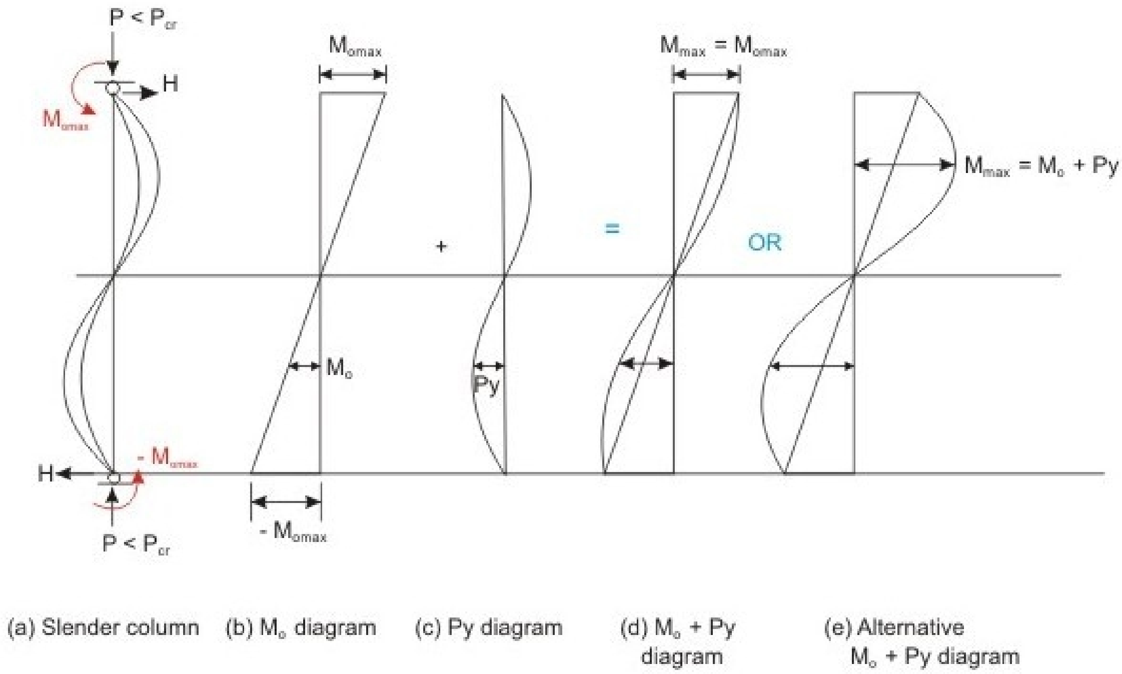


Fig. 10.27.12: Slender column under axial load and bending, bent in double curvature

Figure 10.27.12a shows a column subjected to equal end moment of opposite signs. From the moment diagrams M_o and Py (Figs.10.27.12b and c), it is clear that though M_o moments are maximum at the ends, the Py moments are maximum at some distance from the ends. The total moment can be either as shown in d or in e of Fig.10.27.12. In case of Fig.10.27.12d, the maximum moment remains at the ends and in Fig.10.27.12e, the maximum moment is at some distance from the ends, where M_o is comparatively smaller than $M_{o\ max}$ at the ends. Accordingly, the total maximum moment is moderately higher than $M_{o\ max}$.

From the above, it is evident that the moment M_o will be magnified most strongly if the section of $M_{o\ max}$ coincides with the section of maximum value of y, as in the case of column bent in single curvature of Figs.10.27.10 and 11. Similarly, if the two moments are unequal but of same sign as in Fig.10.27.10, the moment M_o will be magnified but not so much as in Fig.10.27.10. On the other hand, if the unequal end moments are of opposite signs and cause bending in double curvature, there will be little or no magnification of M_o moment.

This dependence of moment magnification on the relative magnitudes of the two moments can be expressed by modifying the earlier Eq.10.70 as

$$M_{max} = M_o\, C_m/\{1 - (P/P_{cr})\} \quad (10.71)$$

where $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$ (10.72)

The moment M_1 is smaller than M_2 and M_1/M_2 is positive if the moments produce single curvature and negative if they produce double curvature. It is further seen from Eq.10.72 that $C_m = 1$, when $M_1 = M_2$ and in that case, Eq.10.71 becomes the same as Eq.10.70.

For the column of Fig.10.27.12a, the deflections caused by M_o are magnified when axial load P is applied. The deflection can be obtained from

$$y = y_o\, [1/\{1 - (P/4P_{cr})\}] \quad (10.73)$$

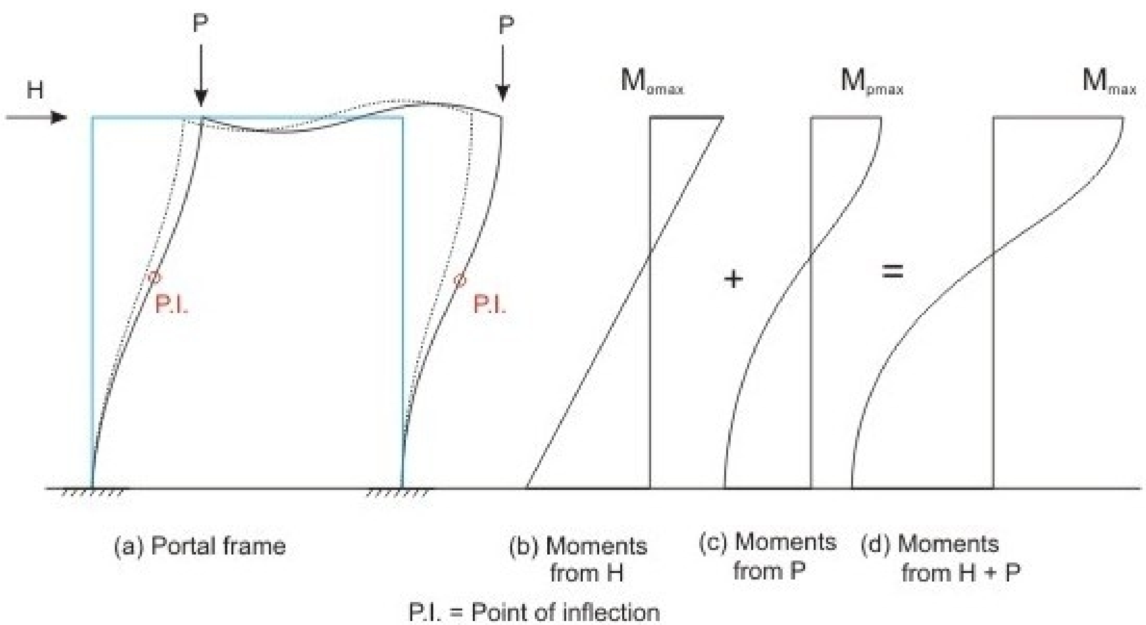


Fig. 10.27.13: Fixed portal frame - laterally unbraced

(C) Portal frame laterally unbraced and braced

Here, the sidesway can occur only for the entire frame simultaneously. A fixed portal frame, shown in Fig.10.27.13a, is under horizontal load H and compression force P. The moments due to H and P and the total moment diagrams are shown in Fig.10.27.13b, c and d, respectively. The deformations of the frame due to H are shown in Fig.10.27.13a by dotted curves, while the solid curves are the magnified deformations. It is observed that the maximum values of positive and negative M_o are at the ends of the column where the maximum values of positive and negative moments due to *P* also occur. Thus, the total moment shall be at the ends as the two effects are fully additive.

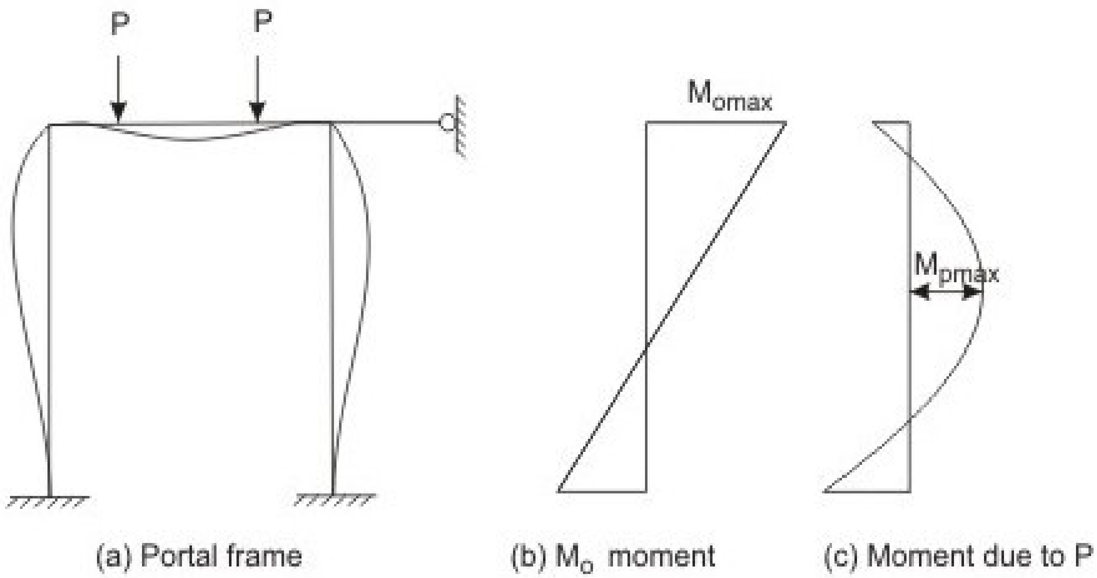


Fig. 10.27.14: Fixed portal frame - laterally braced

Figure 10.27.14a shows a fixed portal frame, laterally braced so that no sidesway can occur. Figures 10.27.14b and c show the moments M_o and due to *P*. It is seen that the maximum values of the two different moments do not occur at the same location. As a result, the magnification of the moment either may not be true or shall be small.

(D) Columns with different slenderness ratios

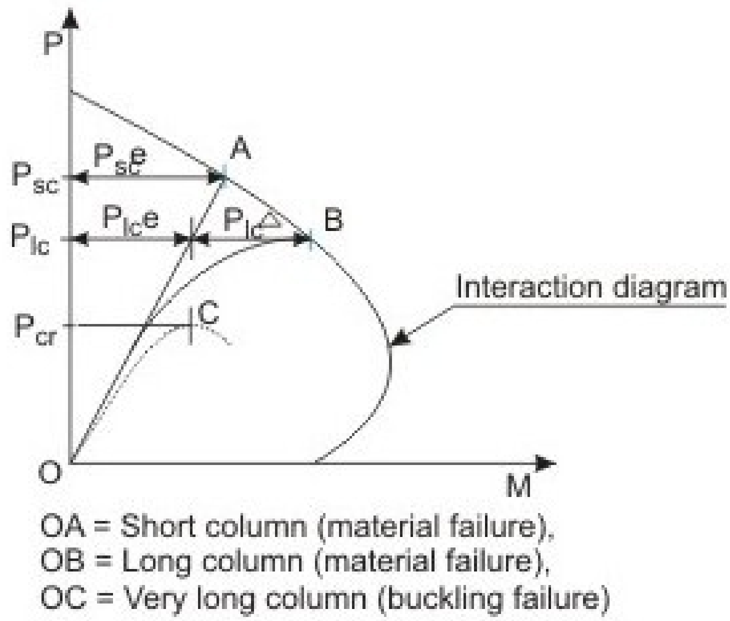


Fig. 10.27.15: Behaviour of slender column

Figure 10.27.15 shows the interaction diagram of P and M at the mid- height section of the column shown in Fig.10.27.10. Three loading paths OA, OB and OC are also shown in the figure for three columns having the same cross- sectional area and the eccentricity of loads but with different slenderness ratios. The three columns are loaded with increasing P and M (at constant eccentricity) up to failure. The loading path OA is linear indicating $\Delta = 0$, i.e., for a very short column. It should be noted that Δ should be theoretically zero only when either the effective length or the eccentricity is zero. In a practical short column, however, some lateral deflection shall be there, which, in turn will cause additional moment not more than five per cent of the primary moment and may be neglected. The loading path OA terminates at point A of the interaction diagram, which shows the failure load P_{sc} of the short column with moment $M_{sc} = P_{sc}\, e$. The short column fails by crushing of concrete at the mid-height section. This type of failure is designated as material failure, either a tension failure or a compression failure depending on the location of the point A on the interaction curve.

The load path OB is for a long column, where the deflection Δ caused by increasing value of P is significant. Finally, the long column fails at load P_{lc} and moment $M_{lc} = P_{lc}(e + \Delta)$. The loading path OB further reveals that the secondary moment $P_{lc}\,\Delta$ is comparable to the primary moment $P_{lc}\, e$. Moreover, the failure load and the primary moment of the long column P_{lc} and $P_{lc}\, e$, respectively, are less than those of the short column (P_{sc} and $P_{sc}\, e$, respectively), though both the columns have the same cross-sectional areas and eccentricities but different slenderness ratios. Here also, the mid-height section of the column undergoes material failure, either a compression failure or a tension failure, depending on the location of the point B on the interaction diagram.

The loading path OC, on the other hand, is for a very long column when the lateral deflection Δ is so high that the slope of the path dP/dM at C is zero. The column is so slender that the failure is due to buckling (instability) at a comparatively much low value of the load P_{cr}, though this column has the same cross-sectional area and the eccentricity of load as of the other two columns. Such instability failure occurs for very slender columns, specially when they are not braced.

The following points are summarised from the discussion made in sec.10.27.3.

1. Additional deflections and moments are caused by the axial compression force P in columns. The additional moments increase with the increase of kl/r, when other parameters are equal.

2. Laterally braced compression members and bent in single curvature have the same or nearby locations of the maxima of both M_o and Py. Thus, being fully additive, they have large moment magnification.

3. Laterally braced compression members and bent in double curvature have different locations of the maxima of both M_o and Py. As a result, the moment magnification is either less or zero.

4. Members of frames not braced laterally, the maxima of M_o and Py mostly occur at the ends of column and cause the maximum total moment at the ends of columns only. Additional moments and additional deflections increase with the increase of kl/r.

Effective Length of Columns

Annex E of IS 456 presents two figures (Figs.26 and 27) and a table (Table 26) to estimate the effective length of columns in frame structures based on a research paper, "Effective length of column in multistoreyed building" by R.H. Wood in The Structural Engineer Journal, No.7, Vol.52, July 1974. Figure 26 is for columns in a frame with no sway, while Fig.27 is for columns in a frame with sway. These two figures give the values of k (i.e., l_e/l) from two parameters β_1 and β_2 which are obtained from the following expression:

$$\beta = \sum K_c / \sum K_c + \sum K_b$$

(10.74)

where K_c and K_b are flexural stiffnesses of columns and beams, respectively. The quantities β_1 and β_2 at the top and bottom joints A and B, respectively, are determined by summing up the K values of members framing into a joint at top and bottom, respectively. Thus as β_1 and β_2 for the frame shown in Fig.10.27.16 are follows:

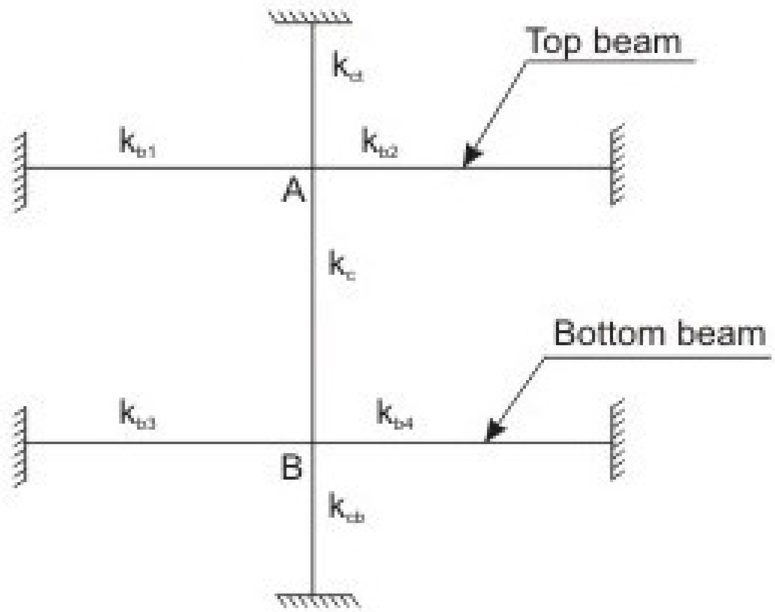


Fig. 10.27.16: Stiffness of columns in Wood's chart

$$\beta_1 = (K_c + K_{ct})/(K_c + K_{ct} + K_{b1} + K_{b2}) \quad (10.75)$$

$$\beta_2 = (K_c + K_{cb})/(K_c + K_{cb} + K_{b3} + K_{b4}) \quad (10.76)$$

However, assuming idealised conditions, the effective length in a given plane may be assessed from Table 28 in Annex E of IS 456, for normal use.

Determination of Sway or No Sway Column

Clause E-2 of IS 456 recommends the stability index Q to determine if a column is a no sway or sway type. The stability index Q is expressed as:

$$Q = \sum P_u \Delta_u / H_u h_z \quad (10.77)$$

where $\sum P_u$ = sum of axial loads on all columns in the storey,

Δ_u = elastically computed first-order lateral deflection,

H_u = total lateral force acting within the storey, and

h_z = height of the storey.

The column may be taken as no sway type if the value of Q is ≤ 0.4, otherwise, the column is considered as sway type.

Design of Slender Columns

The design of slender columns, in principle, is to be done following the same procedure as those of short columns. However, it is essential to estimate the total moment i.e., primary and secondary moments considering P- Δ effects. These secondary moments and axial forces can be determined by second-order rigorous structural analysis – particularly for unbraced frames. Further, the problem becomes more involved and laborious as the principle of superposition is not applicable in second-order analysis.

However, cl.39.7 of IS 456 recommends an alternative simplified method of determining additional moments to avoid the laborious and involved second- order analysis. The basic principle of additional moment method for estimating the secondary moments is explained in the next section.

Additional Moment Method

In this method, slender columns should be designed for biaxial eccentricities which include secondary moments (Py of Eq.10.63 and 10.65) about major and minor axes. We first consider braced columns which are bent symmetrically in single curvature and cause balanced failure i.e., $P_u = P_{ub}$.

(A) Braced columns bent symmetrically in single curvature and undergoing balanced failure

For braced columns bent symmetrically in single curvature, we have from Eqs.10.63 and 10.65,

$$M = M_o + Py = M_o + P\,e_a = M_o + M_a \qquad (10.78)$$

where P is the factored design load P_u, M are the total factored design moments M_{ux} and M_{uy} about the major and minor axes, respectively; M_o are the primary factored moments M_{oux} and M_{ouy} about the major and minor axes, respectively; M_a are the additional moments M_{ax} and M_{ay} about the major and minor axes, respectively and e_a are the additional eccentricities e_{ax} and e_{ay} along the minor and major axes, respectively. The quantities M_o and P of Eq.10.78 are known and hence, it is required to determine the respective values of e_a, the additional eccentricities only.

Let us consider the columns of Figs.10.27.10 and 11 showing Δ as the maximum deflection at the mid-height section of the columns. The column of Fig.10.27.10, having a constant primary moment M_o, causes constant curvature ϕ, while the column of Fig.10.27.11, having a linearly varying primary moment with a maximum value of $M_{o\ max}$ at the mid-height section of the column, has a linearly varying curvature with the maximum curvature of ϕ_{max} at the mid-height section the column. The two maximum curvatures can be expressed in terms of their respective maximum deflection Δ as follows:

The constant curvature (Fig.10.27.10) (10.79) $$\phi_{max} = 8\Delta/l_e^2$$

The linearly varying curvature (Fig.10.27.11) $$\phi_{max} = 12\Delta/l_e^2$$ (10.80)

where l_e are the respective effective lengths kl of the columns. We, therefore, consider the maximum ϕ as the average value lying in between the two values of Eqs.10.79 and 80 as

$$\phi_{max} = 10\Delta/l_e^2$$

(10.81)

Accordingly, the maximum additional eccentricities e_a, which are equal to the maximum deflections Δ , can be written as

$$e_a = \Delta = \phi\, l_e^2/10$$

(10.82)

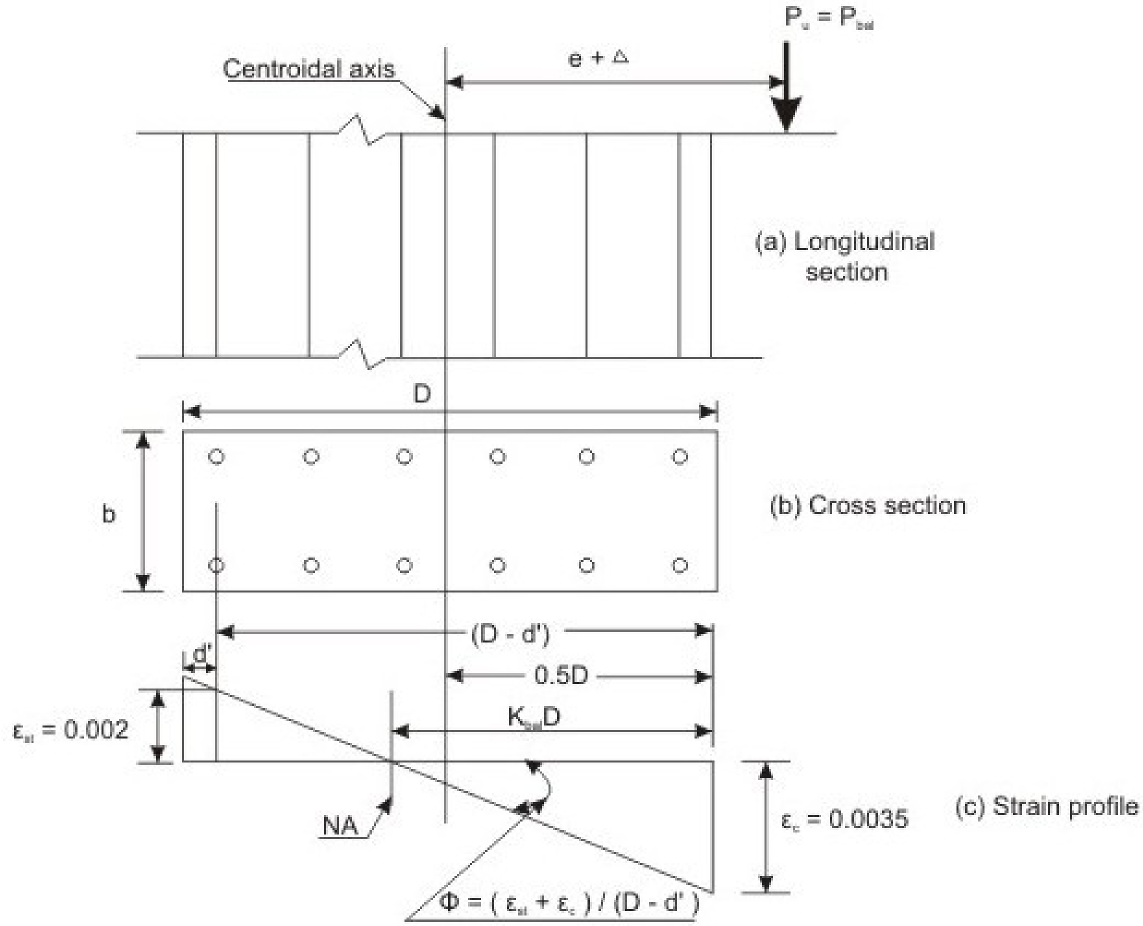


Fig. 10.27.17: Maximum curvature at mid-height section when $P_u = P_{bal}$

Assuming the column undergoes a balanced failure when $P_u = P_{ub}$, the maximum curvature at the mid-height section of the column, shown in Figs.10.27.17a and b, can be expressed as given below, assuming (i) the values of ε_c = 0.0035, ε_{st} = 0.002 and d'/D = 0.1, and (ii) the additional moment capacities are about eighty per cent of the total moment.

ϕ = eighty per cent of {(0.0035 + 0.002)/0.9D} (see Fig.10.27.17c) or ϕ = 1/200D (10.83)

Substituting the value of ϕ in Eq.10.82,

$$e_a = D(l_e/D)^2/2000 \quad (10.84)$$

Therefore, the additional moment M_a can be written as,

$$M_a = Py = P\,\Delta = Pe_a = (PD/2000)\,(l_e/D)^2 \quad (10.85)$$

Thus, the additional moments M_{ax} and M_{ay} about the major and minor axes, respectively, are:

$$M_{ax} = (P_u D/2000)\,(l_{ex}/D)^2$$
(10.86)

$$M_{ay} = (P_u b/2000)\,(l_{ey}/b)^2$$
(10.87)

where P_u = axial load on the member,

l_{ex} = effective length in respect of the majoraxis,

l_{ey} = effective length in respect of the minoraxis,

D = depth of the cross-section at right angles to the major axis, and

b = width of the member.

Clause 39.7.1 of IS 456 recommends the expressions of Eqs.10.86 and 87 for estimating the additional moments M_{ax} and M_{ay} for the design. These two expressions of the additional moments are derived considering the columns to be braced and bent symmetrically undergoing balanced failure. Therefore, proper modifications are necessary for different situations like braced columns with unequal end moments with the same or different signs, unbraced columns and columns causing compression failure i.e., when $P_u > P_{ub}$.

(B) Braced columns subjected to unequal primary moments at the two ends

For braced columns without any transverse loads occurring in the height, the primary maximum moment ($M_{o\,max}$ of Eq.10.64), with which the additional moments of Eqs.10.86 and 87 are to be added, is to be taken as:

$$M_{o\,max} = 0.4\,M_1 + 0.6\,M_2$$

(10.88)

and further $M_{o\,max} \geq 0.4\,M_2$

(10.89)

where M_2 is the larger end moment and M_1 is the smaller end moment, assumed to be negative, if the column is bent in double curvature.

To eliminate the possibility of total moment $M_{u\ max}$ becoming less than M_2 for columns bent in double curvature (see Fig.10.27.12) with M_1 and M_2 having opposite signs, another condition has been imposed as

$$M_{u\ max} \geq M_2$$

(10.90)

The above recommendations are given in notes of cl.39.7.1 of IS 456.

(C) Unbraced columns

Unbraced frames undergo considerable deflection due to P- Δ effect. The additional moments determined from Eqs.10.86 and 87 are to be added with the maximum primary moment $M_{o\ max}$ at the ends of the column. Accordingly, we have

$$M_{o\ max} = M_2 + M_a$$

(10.91)

The above recommendation is given in the notes of cl.39.7.1 of IS 456.

(D) Columns undergoing compression failure ($P_u > P_{ub}$)

It has been mentioned in part A of this section that the expressions of additional moments given by Eqs.10.86 and 10.87 are for columns undergoing balanced failure (Fig.10.27.17). However, when the column causes compression failure, the e/D ratio is much less than that of balanced failure at relatively high axial loads. The entire section may be under compression causing much less curvatures. Accordingly, additional moments of Eqs.10.86 and 10.87 are to be modified by multiplying with the reduction factor k as given below:

(i) For $P_u > P_{ubx}$: $k_{ax} = (P_{uz} - P_u)/(P_{uz} - P_{ubx})$
(10.92)

(ii) For $P_u > P_{uby}$: $k_{ay} = (P_{uz} - P_u)/(P_{uz} - P_{uby})$
(10.93)

with a condition that k_{ax} and k_{ay} should be ≤ 1 (10.94)

where P_u = axial load on compression member

P_{uz} is given in Eq.10.59 of Lesson 26 and is,

$$P_{uz} = 0.45\, f_{ck} A_c + 0.75\, f_y A_{st} \qquad \ldots \ (10.59)$$

P_{ubx}, P_{uby} = axial loads with respect to major and minor axes, respectively, corresponding to the condition of maximum compressive strain of 0.0035 in concrete and tensile strain of 0.002 in outermost layer of tension steel.

It is seen from Eqs.10.92 and 10.93 that the values of k (k_{ax} and k_{ay}) vary linearly from zero (when $P_u = P_{uz}$) to one (when $P_u = P_{ub}$). Since Eqs.10.92 and 10.93 are not applicable for $P_u < P_{ub}$, another condition has been imposed as given in Eq.10.94.

The above recommendations are given in cl.39.7.1.1 of IS 456.

The following discussion is very important for the design of slender columns.

Additional moment method is one of the methods of designing slender columns as discussed in A to D of this section. This method is recommended in cl.39.7 of IS 456 also. The basic concept here is to enhance the primary moments by adding the respective additional moments estimated in a simple way avoiding laborious and involved calculations of second-order structural analysis. However, these primary moments under eccentric loadings should not be less than the moments corresponding to the respective minimum eccentricity, as stipulated in the code. Hence, the primary moments in such cases are to be replaced by the minimum eccentricity moments. Moreover, all slender columns, including those under axial concentric loadings, are also to be designed for biaxial bending, where the primary moments are zero. In such cases, the total moment consisting of the additional moment multiplied with the modification factor, if any, in each direction should be equal to or greater than the respective moments under minimum eccentricity conditions. As mentioned earlier, the minimum eccentricity consideration is given in cl.25.4 of IS 456.

Illustrative Example

The following illustrative example is taken up to explain the design of slender columns. The example has been solved in step by step using (i) the equations of Lessons 21 to 27 and (ii) employing design charts and tables of SP- 16, to compare the results.

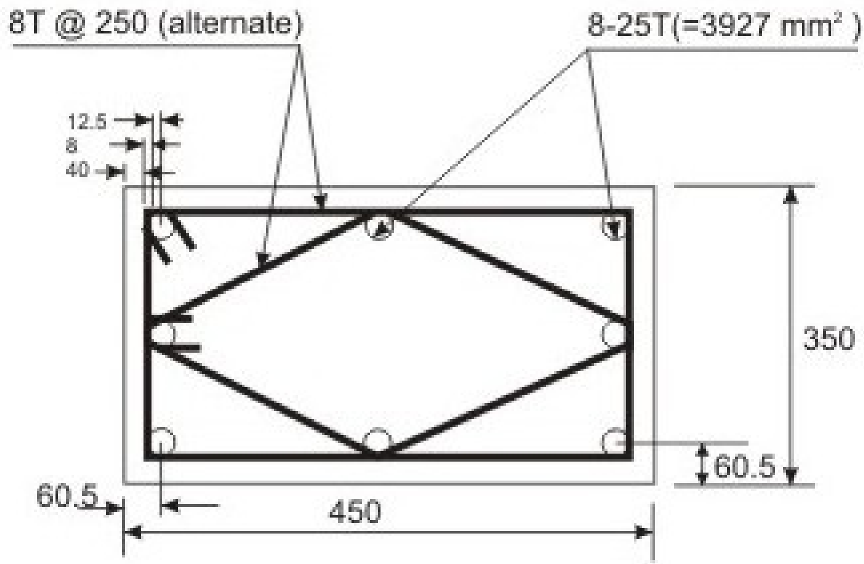


Fig. 10.27.18: Problem 1

Problem 1:

Determine the reinforcement required for a braced column against sidesway with the following data: size of the column = 350 x 450 mm (Fig.10.27.18); concrete and steel grades = M 30 and Fe 415, respectively; effective lengths l_{ex} and l_{ey} = 7.0 and 6.0 m, respectively; unsupported length l = 8 m; factored load P_u = 1700 kN; factored moments in the direction of larger dimension = 70 kNm at top and 30 kNm at bottom; factored moments in the direction of shorter dimension = 60 kNm at top and 30 kNm at bottom. The column is bent in double curvature. Reinforcement will be distributed equally on four sides.

Solution 1:

Step 1: Checking of slenderness ratios l_{ex}/D =

7000/450 = 15.56 > 12, l_{ey}/b =

6000/350 = 17.14 > 12.

Hence, the column is slender with respect to both the axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson21)

$e_{x\,min} = l/500 + D/30 = 8000/500 + 450/30 = 31.0 > 20$ mm

$e_{y\,min} = l/500 + b/30 = 8000/500 + 350/30 = 27.67 > 20$ mm

M_{ox} (Min. ecc.) = $P_u(e_{x\ min})$ = (1700) (31) (10^{-3}) = 52.7 kNm

M_{oy} (Min. ecc.) = $P_u(e_{y\ min})$ = (1700) (27.67) (10^{-3}) = 47.04 kNm

Step 3: Additional eccentricities and additional moments

Method 1: Using Eq. 10.84

$e_{ax} = D(l_{ex}/D)^2/2000$ = (450) $(7000/450)^2/2000$ = 54.44 mm e_{ay} =

$b(l_{ex}/b)^2/2000$ = (350) $(6000/350)^2/2000$ = 51.43 mm $M_{ax} = P_u(e_{ax})$

= (1700) (54.44) (10^{-3}) = 92.548 kNm

$M_{ay} = P_u(e_{ay})$ = (1700) (51.43) (10^{-3}) = 87.43 kNm

Method 2: Table I of SP-16

For l_{ex}/D = 15.56, Table I of SP-16 gives:

e_{ax}/D = 0.1214, which gives e_{ax} = (0.1214) (450) = 54.63 mm For l_{ey}/D = 17.14,

Table I of SP-16 gives:

e_{ay}/b = 0.14738, which gives e_{ay} = (0.14738) (350) = 51.583 mm

It is seen that values obtained from Table I of SP-16 are comparable with those obtained by Eq. 10.84 in Method 1.

Step 4: Primary moments and primary eccentricities (Eqs.10.88 and 89)

$M_{ox} = 0.6M_2 - 0.4M_1 = 0.6(70) - 0.4(30)$ = 30 kNm, which should be ≥ 0.4 M_2 (= 28 kNm). Hence, o.k.

$M_{oy} = 0.6M_2 - 0.4M_1 = 0.6(60) - 0.4(30)$ = 24 kNm, which should be ≥ 0.4 M_2 (= 24 kNm). Hence, o.k.

Primary eccentricities:

$e_x = M_{ox}/P_u$ = (30/1700) (10^3) = 17.65 mm

$e_y = M_{oy}/P_u$ = (24/1700) (10^3) = 14.12 mm

Since, both primary eccentricities are less than the respective minimum eccentricities (see Step 2), the primary moments are revised to those of Step 2. So, M_{ox} = 52.7 kNm and M_{oy} = 47.04 kNm.

Step 5: Modification factors

To determine the actual modification factors, the percentage of longitudinal reinforcement should be known. So, either the percentage of longitudinal reinforcement may be assumed or the modification factor may be assumed which should be verified subsequently. So, we assume the modification factors of 0.55 in both directions.

Step 6: Total factored moments

$M_{ux} = M_{ox}$ + (Modification factor) (M_{ax}) = 52.7 + (0.55) (92.548)

= 52.7 + 50.9 = 103.6 kNm

$M_{uy} = M_{oy}$ + (Modification factor) (M_{ay}) = 47.04 + (0.55) (87.43)

= 47.04 + 48.09 = 95.13 kNm

Step 7: Trial section (Eq.10.61 of Lesson 26)

The trial section is determined from the design of uniaxial bending with P_u = 1700 kN and M_u = 1.15 $(M_{ux}^2 + M_{uy}^2)^{1/2}$. So, we have M_u = (1.15){(103.6)^2 + (95.13)^2}$^{1/2}$ = 161.75 kNm. With these values of P_u (= 1700 kN) and M_u (= 161.75 kNm), we use chart of SP-16 for the d'/D = 0.134. We assume the diameters of longitudinal bar as 25 mm, diameter of lateral tie = 8 mm and cover = 40 mm, to get d' = 40 + 8 + 12.5 = 60.5 mm. Accordingly, d'/D = 60.5/450 = 0.134 and d'/b = 60.5/350 = 0.173. We have:

$P_u/f_{ck}\,bD$ = 1700(10^3)/(30)(350)(450) = 0.3598

$M_u/f_{ck}\,bD^2$ = 161.75(10^6)/(30)(350)(450)(450) = 0.076

We have to interpolate the values of p/f_{ck} for d'/D = 0.134 obtained from Charts 44 (for d'/D = 0.1) and 45 (d'/D = 0.15). The values of p/f_{ck} are 0.05 and 0.06 from Charts 44 and 45, respectively. The corresponding values of p are 1.5 and 1.8 per cent, respectively. The interpolated value of p for d'/D = 0.134 is 1.704 per cent, which gives A_{sc} = (1.704)(350)(450)/100 = 2683.8 mm^2. We use 4-25 + 4-20 (1963 + 1256 = 3219 mm^2), to have p provided = 2.044 per cent giving p/f_{ck} = 0.068.

Step 8: Calculation of balanced loads P_b

The values of P_{bx} and P_{by} are determined using Table 60 of SP-16. For this purpose, two parameters k_1 and k_2 are to be determined first from the table.

We have p/f_{ck} = 0.068, d'/D = 0.134 and d'/b = 0.173. From Table 60, k_1 = 0.19952 and k_2 = 0.243 (interpolated for d'/D = 0.134) for P_{bx}. So, we have: $P_{bx}/f_{ck}bD = k_1 + k_2\,(p/f_{ck})$ = 0.19952 + 0.243(0.068) = 0.216044, which gives P_{bx} = 0.216044(30)(350)(450)(10^{-3}) = 1020.81 kN.

Similarly, for P_{by}: d'/b = 0.173, p/f_{ck} = 0.068. From Table 60 of SP-16, k_1 = 0.19048 and k_2 = 0.1225 (interpolated for d'/b = 0.173). This gives $P_{by}/f_{ck}bD$ = 0.19048 + 0.1225(0.068) = 0.19881, which gives P_{by} = (0.19881)(30)(350)(450)(10^{-3}) = 939.38 kN.

Since, the values of P_{bx} and P_{by} are less than P_u, the modification factors are to be used.

Step 9: Determination of P_{uz}

Method 1: From Eq.10.59 of Lesson 26

$$P_{uz} = 0.45\, f_{ck}\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$$

$$= 0.45(30)(350)(450) + \{0.75(415) - 0.45(30)\}(3219) = 3084.71 \text{ kN}$$

Method 2: Using Chart 63 of SP-16

We get P_{uz}/A_g = 19.4 N/mm^2 from Chart 63 of SP-16 using p = 2.044 per cent. Therefore, P_{uz} = (19.4)(350)(450)(10^{-3}) = 3055.5 kN, which is in good agreement with that of Method 1.

Step 10: Determination of modification factors

Method 1: From Eqs.10.92 and 10.93

$$k_{ax} = (P_{uz} - P_u)/(P_{uz} - P_{ubx}) \qquad \ldots\ (10.92)$$

or k_{ax} = (3084.71 – 1700)/(3084.71 – 1020.81) = 0.671 and

$$k_{ay} = (P_{uz} - P_u)/(P_{uz} - P_{uby}) \qquad \ldots\ (10.93)$$ or

k_{ay} = (3084.71 – 1700)/(3084.71 – 939.39) = 0.645

The values of the two modification factors are different from the assumed value of 0.55 in Step 5. However, the moments are changed and the section is checked for safety.

Method 2: From Chart 65 of SP-16

From Chart 65 of SP-16, for the two parameters, P_{bx}/P_{uz} = 1020.81/3084.71 = 0.331 and P_u/P_{uz} = 1700/3084.71 = 0.551, we get k_{ax} = 0.66.
Similarly, for the two parameters, P_{by}/P_{uz} = 939.38/3084.71 = 0.3045 and P_u/P_{uz} = 0.551, we have k_{ay} = 0.65. Values of k_{ax} and k_{ay} are comparable with those of Method 1.

Step 11: Total moments incorporating modification factors

$M_{ux} = M_{ox}$ (from Step 4) + (k_{ax}) M_{ax} (from Step 3)

= 52.7 + 0.671(92.548) = 114.8 kNm

$M_{uy} = M_{oy}$ (from Step 4) + k_{ay} (M_{ay}) (from Step 3)

= 47.04 + (0.645)(87.43) = 103.43 kNm.

Step 12: Uniaxial moment capacities

The two uniaxial moment capacities M_{ux1} and M_{uy1} are determined as stated: (i) For M_{ux1}, by interpolating the values obtained from Charts 44 and 45, knowing the values of $P_u/f_{ck}bD$ = 0.3598 (see Step 7), p/f_{ck} = 0.068 (see Step 7), d'/D = 0.134 (see Step 7), (ii) for M_{uy1}, by interpolating the values obtained from Charts 45 and 46, knowing the same values of $P_u/f_{ck}bD$ and p/f_{ck} as those of (i) and d'/D = 0.173 (see Step 7). The results are given below:

(i) $M_{ux1}/f_{ck}bD^2$ = 0.0882 (interpolated between 0.095 and 0.085)
(ii) $M_{uy1}/f_{ck}bb^2$ = 0.0827 (interpolated between 0.085 and 0.08) So,

we have, M_{ux1} = 187.54 kNm and M_{uy1} = 136.76 kNm.

Step 13: Value of α_n

Method 1: From Eq.10.60 of Lesson 26

We have P_u/P_{uz} = 1700/3084.71 = 0.5511. From Eq.10.60 of Lesson 26, we have α_n = 0.67 + 1.67 (P_u/P_{uz}) = 1.59.

Method 2: Interpolating the values between (P_u/P_{uz}) = 0.2 and 0.6

The interpolated value of α_n = 1.0 + (0.5511 – 0.2)/0.6 = 1.5852. Both the values are comparable. We use α_n = 1.5852.

Step 14: Checking of column for safety

Method 1: From Eq.10.58 of Lesson 26

We have in Lesson 26:

$$(M_{ux}/M_{ux1})^{\alpha_n} + (M_{uy}/M_{uy1})^{\alpha_n} \leq 1 \qquad \dots (10.58)$$

Here, putting the values of M_{ux}, M_{ux1}, M_{uy}, M_{uy1} and α_n, we get:

$(114.8/187.54)^{1.5452} + (103.43/136.76)^{1.5852} = 0.4593 + 0.6422 = 1.1015$. Hence, the section or the reinforcement has to be revised.

Method 2: Chart 64 of SP-16

The point having the values of $(M_{ux}/M_{ux1}) = 114.8/187.54 = 0.612$ and $(M_{uy}/M_{uy1}) = 103.43/136.76 = 0.756$ gives the value of P_u/P_z more than 0.7. The value of P_u/P_{uz} here is 0.5511 (see Step 13). So, the section needs revision.

We revise from Step 7 by providing 8-25 mm diameter bars (= 3927 mm^2, p = 2.493 per cent and $p/f_{ck} = 0.0831$) as the longitudinal reinforcement keeping the values of b and D unchanged. The revised section is checked furnishing the repeated calculations from Step 8 onwards. The letter R is used before the number of step to indicate this step as revised one.

Step R8: Calculation of balanced loads P_b

Table 60 of SP-16 gives $k_1 = 0.19952$, and $k_2 = 0.243$. We have $p/f_{ck} = 0.0831$ now. So, P_{bx} = $\{0.19952 + (0.243)(0.0831)\}$ $(30)(350)(450)(10^{-3})$ = 1038.145 kN. Similarly, $k_1 = 0.19048$, $k_2 = 0.1225$ and $p/f_{ck} = 0.0831$ give P_{by} = $\{0.19048 + (0.1225)(0.0831)\}$ $(30)(350)(450)(10^{-3}) = 948.12$ kN.

The values of P_{bx} and P_{by} are less than P_u (= 1700 kN). So, modification factors are to be incorporated.

Step R9: Determination of P_{uz} (Eq. 10.59 of Lesson 26)

$P_{uz} = 0.45(30)(350)(450) + \{0.75(415) – 0.45(30)\}(3927) = 3295.514$ kN.

Step R10: Determination of modification factors (Eqs.10.92 and 10.93)

$k_{ax} = (3295.514 – 1700)/(3295.514 – 1038.145) = 0.707$

$k_{ay} = (3295.514 – 1700)/(3295.514 – 948.12) = 0.68$

Step R11: Total moments incorporating modification factors

M_{ux} = 52.70 + 0.707(92.548) = 118.13 kNm

M_{uy} = 47.04 + 0.68(87.43) = 106.49 kNm

Step R12: Uniaxial moment capacities

Using Charts 44 and 45 for M_{ux1} and Charts 45 and 46 for M_{uy1}, we get (i) the coefficient 0.1032 (interpolating 0.11 and 0.10) and (ii) the coefficient 0.0954 (interpolating 0.1 and 0.09) for M_{ux1} and M_{uy1}, respectively.

M_{ux1} = (0.1032)(30)(350)(450)(450)(10^{-6}) = 219.429 kNm

M_{uy1} = (0.0954)(30)(450)(350)(350)(10^{-6}) = 157.77 kNm

Step R13: Value of α_n **(Eq.10.60 of Lesson 26)**

P_u/P_{uz} = 1700/3295.514 = 0.5158 which gives

α_n = 1 + (0.5158 – 0.2)/0.6 = 1.5263

Step R14: Checking of column for safety (Eq.10.58 of Lesson 26)

$(118.13/219.424)^{1.5263} + (106.49/157.77)^{1.5263} = 0.3886 + 0.5488 = 0.9374 < 1.0$

Hence, the revised reinforcement is safe. The section is shown in Fig.10.27.18.

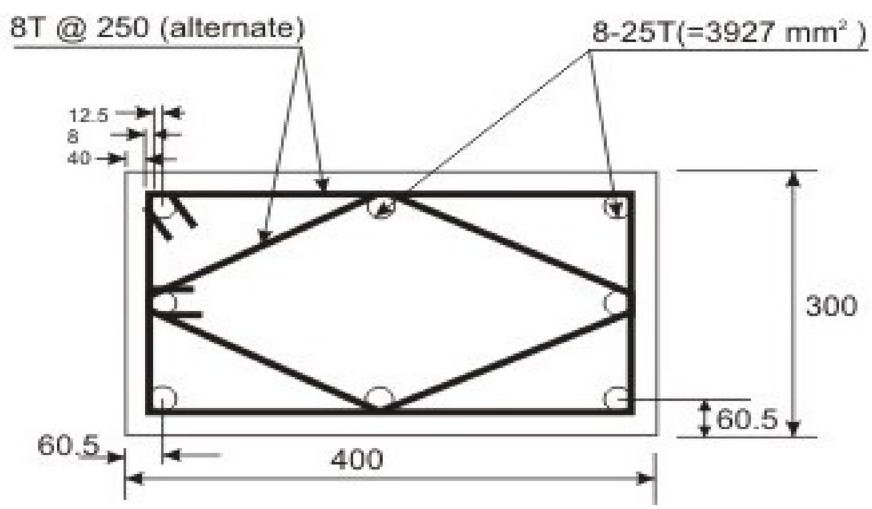


Fig. 10.27.19: Q.7

Q.7: Check the column of Fig.10.27.19, if subjected to an axial factored load of P_u = 1500 kN only when the unsupported length of the column = l = 8.0 m, l_{ex} = l_{ey} = 6.0 m, D = 400 mm, b = 300 mm, using concrete of M 20 and steel grade in Fe 415.

A.7: Solution:

Step 1: Slenderness ratios

L_{ex}/D = 6000/400 = 15 > 12

L_{ey}/b = 6000/300 = 20 > 12

The column is slender about both the axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson 21)

$e_{x\,min} = l/500 + D/30$ = 8000/500 + 400/30 = 29.33 mm > 20 mm

$e_{y\,min}$ = 8000/500 + 300/30 = 26 mm > 20 mm

M_x due to min. ecc. = P_u ($e_{x\,min}$) = 1500(29.33) = 43.995 kNm

M_y due to min. ecc. = P_u ($e_{y\,min}$) = 1500(26.0) = 39.0 kNm

Step 3: Primary moments

Since the column is concentrically loaded, the primary moments are zero. Therefore, the additional moments must be greater than the respective moments due to minimum eccentricity.

Step 4: Additional eccentricities and moments (Eq.10.84)

$e_{ax} = D(l_{ex}/D)^2/2000 = 400(6000/400)^2/2000$ = 45 mm > $e_{x\,min}$ (= 29.23 mm)

$e_{ay} = b(l_{ey}/b)^2/2000 = 300(6000/300)^2/2000$ = 60 mm > $e_{y\,min}$ (= 26 mm)

Step 5: Calculation of balance loads P_{bx} and P_{by}

Given A_{sc} = 3927 mm^2 (8 bars of 25 mm diameter give p = 3.2725 per cent. So, p/f_{ck} = 0.1636. Using 8 mm diameter lateral tie, d' = 40 + 8 + 12.5 =

60.5 mm giving $d'/D = 60.5/400 = 0.15125 \cong 0.15$ and $d'/b = 60.5/300 = 0.2017 \cong 0.20$.

From Table 60 of SP-16, we get $k_1 = 0.196$ and $k_2 = 0.061$. Thus, we have:

$P_{bx} = \{0.196 + (0.061)(0.1636)\}(20)(300)(400)(10^{-3}) = 494.35$ kN

Similarly, for P_{by}: $k_1 = 0.184$ and $k_2 = -0.011$, we get

$P_{by} = \{0.184 - (0.011)(0.1636)\}(20)(300)(400)(10^{-3}) = 437.281$ kN

Since, P_{bx} and P_{by} are less than P_u (= 1500 kN), modification factors are to be incorporated.

Step 6: Determination of P_{uz} (Eq.10.59 of Lesson 26)

$P_{uz} = 0.45(20)(300)(400) + \{0.75(415) – 0.45(20)\}(3927)(10^{-3}) = 2266.94$ kN

Step 7: Determination of modification factors

$k_{ax} = (2266.94 – 1500)/(2266.94 – 494.35) = 0.433$ and

$k_{ay} = (2266.94 – 1500)/(2266.94 – 437.281) = 0.419$

Step 8: Additional moments and total moments

$M_{ax} = 1500(0.433)(45) = 29.2275$ kNm

$M_{ay} = 1500(0.419)(60) = 37.71$ kNm

Since, primary moments are zero as the column is concentrically loaded, the total moment shall consist of the additional moments. But, as both the additional moments are less than the respective moment due to minimum eccentricity, the revised additional moments are: $M_{ax} = 43.995$ kNm and $M_{ay} = 39.0$ kNm, which are the total moments also.

Thus, we have:

$M_{ux} = 43.995$ kNm, $M_{uy} = 39.0$ kNm and $P_u = 1500$ kN.

Step 9: Uniaxial moment capacities

We have, $P_u/f_{ck}\ bD$ = {1500/(20)(300)(400)}(1000) = 0.625, p/f_{ck} = 0.1636 and

d'/D = 0.15 for M_{ux1}; and d'/b = 0.2 for M_{uy1}. The coefficients are 0.11 (from Chart 45) and 0.1 (from Chart 46) for M_{ux1} and M_{uy1}, respectively. So, we get,

$M_{ux1} = 0.11(20)(300)(400)(400)(10^{-6}) = 225.28$ kNm, and

$M_{uy1} = 0.1(20)(300)(300)(400)(10^{-6}) = 72.0$ kNm

Step 10: Value of α_n **(Eq.10.60 of Lesson 26)**

Here, P_u/P_{uz} = 1500/2266.94 = 0.6617. So, we get

α_n = 1.0 + (0.4617/0.6) = 1.7695

Step 11: Checking the column for safety (Eq.10.58 of Lesson 26)

$$(M_{ux} / M_{ux1})^{\alpha_n} + (M_{uy} / M_{uy1})^{\alpha_n} \le 1$$

Here, $(43.995/225.28)^{1.7695} + (39.0/72.0)^{1.7695}$ = 0.0556 + 0.3379 = 0.3935 < 1

Hence, the column is safe to carry P_u = 1500 kN.

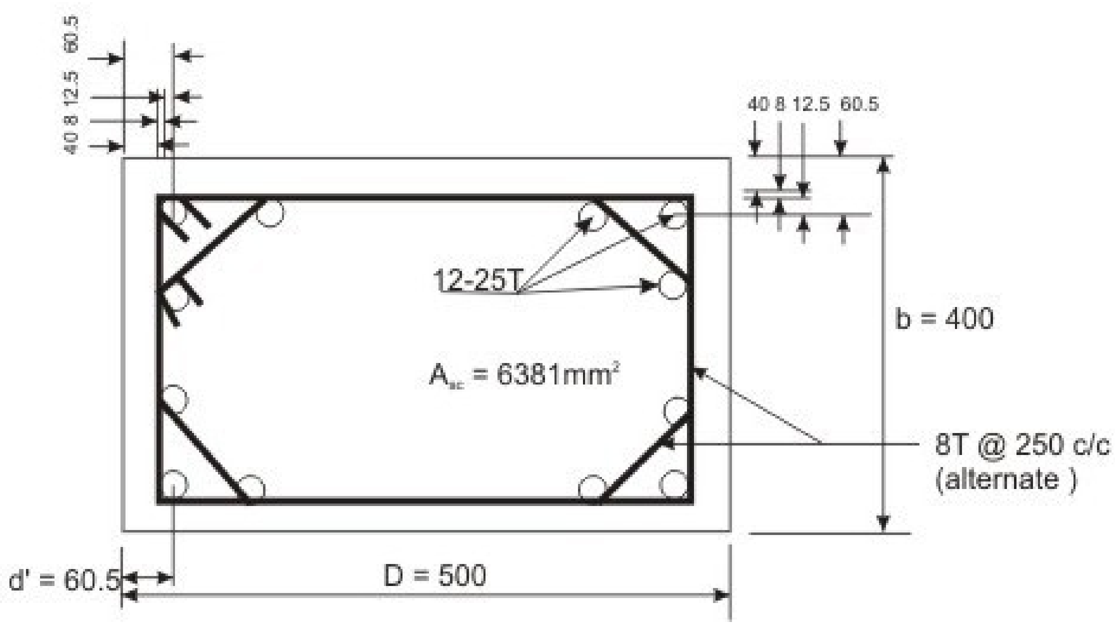


Fig. 10.27.20: TQ.1

TQ.1: Determine the primary, additional and total moments of the column shown in Fig.10.27.20 for the three different cases:

(i) When the column is braced against sidesway and is bent in single curvature.

(ii) When the column is braced against sidesway and is bent in double curvature.

(iii) When the column is unbraced.

Use the following data: P_u = 2000 kN, concrete grade = M 20, steel grade = Fe 415, unsupported length l = 8.0 m, l_{ex} = 7.0 m, l_{ey} = 6.0 m, A_{sc} = 6381 mm^2 (12-25 mm diameter bars), lateral tie = 8 mm diameter @ 250 mm c/c, d' = 60.5 mm, D = 500 mm and b = 400 mm. The factored moments are: 70 kNm at top and 40 kNm at bottom in the direction of larger dimension and 60 kNm at top and 30 kNm at bottom in the direction of shorter dimension.

A.TQ.1: Solution

The following are the common steps for all three cases.

Step 1: Slenderness ratios

l_{ex}/D = 7000/500 = 14 > 12 and l_{ey}/b = 6000/400 = 15 > 12 The column is slender about both axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson 21)

$e_{x\,min} = l/500 + D/30$ = 8000/500 + 500/30 = 32.67 mm > 20 mm, and

$e_{y\,min} = l/500 + b/30$ = 8000/500 + 400/30 = 29.34 mm > 20 mm

M_x (min. ecc.) = 2000(32.67)(10^{-3}) = 65.34 kNm, and

M_y (min. ecc.) = 2000(29.34)(10^{-3}) = 58.68 kNm

Step 3: Additional eccentricities and moments due to additional eccentricities (Eq.10.84)

$e_{ax} = D(l_{ex}/D)^2/2000 = 500(7000/500)^2/2000$ = 49 mm > $e_{x\,min}$ (= 32.67 mm)

$e_{ay} = b(l_{ey}/b)^2/2000 = 400(6000/400)^2/2000 = 45$ mm $> e_{y\ min}$ (= 29.34 mm)

$M_{ax} = P_u(e_{ax}) = (2000)(49)(10^{-3}) = 98$ kNm, and

$M_{ay} = P_u(e_{ay}) = (2000)(45)(10^{-3}) = 90$ kNm

Step 4: Calculation of balanced loads

Using $d'/D = 0.121$ and $p/f_{ck} = 3.1905/20 = 0.159525$ in Table 60 of SP- 16, we have $k_1 = 0.20238$ and $k_2 = 0.2755$ (by linear interpolation). This gives

$P_{bx} = \{0.20238 + 0.2755(0.159525)\}(20)(400)(500)(10^{-3}) = 983.32$ kN

Similarly, $d'/b = 0.15125$ and $p/f_{ck} = 0.159525$ in Table 60 of SP-16 gives $k_1 = 0.1957$ and $k_2 = 0.198625$ (by linear interpolation). So, we get

$P_{by} = \{0.1957 + 0.198625(0.159525)\}(20)(400)(500)(10^{-3}) = 909.54$ kN

Both P_{bx} and P_{by} are smaller than P_u (= 2000 kN). Hence, modification factors are to be incorporated.

Step 5: Calculation of P_{uz} (Eq.10.59 of Lesson 26)

$P_{uz} = 0.45\, f_{ck}\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$

$= 0.45(20)(400)(500) + \{0.75(415) - 0.45(20)\}(6381) = 3728.66$ kN

Step 6: Modification factors and revised additional moments (Eqs.10.92 and 10.93)

$k_{ax} = (3728.66 - 2000)/(3728.66 - 983.32) = 0.6297$, and

$k_{ay} = (3728.66 - 2000)/(3728.66 - 909.54) = 0.6132$

The revised additional moments are:

$M_{ax} = 98(0.6297) = 61.71$ kNm, and

$M_{ay} = 90(0.6132) = 55.19$ kNm

Now, the different cases are explained.

Case (i): Braced column in single curvature

Primary moments = $0.4 M_1 + 0.6 M_2$, but should be equal to or greater than $0.4 M_2$ and moment due to minimum eccentricities. So, we get,

M_{ox} = largest of 58 kNm, 28 kNm and 65.34 kNm = 65.34 kNm

M_{oy} = largest of 48 kNm, 24 kNm and 58.68 kNm = 58.68 kNm

Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm (incorporating the respective modification factors).

Total moments = $M_{ux} = M_{ox} + M_{ax}$ = 65.34 + 61.71 = 127.05 kNm >

65.34 kNm (moment due to minimum eccentricity), and

$M_{uy} = M_{oy} + M_{ay}$ = 58.68 + 55.19 = 113.87 kNm > 58.68 kNm (moment due to minimum eccentricity).

Case (ii): Braced column in double curvature

Primary moments = $- 0.4 M_1 + 0.6 M_2$, but should be equal to or greater than $0.4M_2$ and the moment due to minimum eccentricity. So, we get,

M_{ox} = largest of 26 kNm, 28 kNm and 65.34 kNm = 65.34 kNm M_{oy} = largest of 24 kNm, 24 kNm and 58.68 kNm = 58.68 kNm Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm

Final moments = $M_{ux} = M_{ox} + M_{ax}$ = 65.34 + 61.71 = 127.05 kNm >

65.34 kNm (moment due to minimum eccentricity), and

M_{uy} = 58.68 + 55.19 = 113.87 kNm > 58.68 kNm (moment due to minimum eccentricity).

Case (iii): Unbraced column

Primary moments = M_2 and should be greater than or equal to moment due to minimum eccentricity.

M_{ox} = 70 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{oy} = 60 kNm > 58.68 kNm (moment due to minimum eccentricity). Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm

Final moments = $M_{ux} = M_{ox} + M_{ax}$ = 70.0 + 61.71 = 131.71 kNm >

65.34 kNm (moment due to minimum eccentricity), and

$M_{uy} = M_{oy} + M_{ax} = 60.0 + 55.19 = 115.19$ kNm > 58.68 kNm (moment due to minimum eccentricity).