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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – I – INTRODUCTION-THEORYANDBEHAVIOUR – SCI1402

Introduction

Basic Concepts of Prestressing

Prestressed concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from external loads are counteracted to a desired degree. In reinforced concrete members, the prestress is commonly introduced by tensioning the steel reinforcement.

The earliest examples of wooden barrel construction by force-fitting of metal bands and shrink-fitting of metal tyres on wooden wheels indicate that the art of prestressing has been practised from ancient times. The tensile strength of plain concrete is only a fraction of its compressive strength and the problem of it being deficient in tensile strength appears to have been the driving factor in the development of the composite material known as “reinforced concrete”.

The development of early cracks in reinforced concrete due to incompatibility in the strains of steel and concrete was perhaps the starting point in the development of a new material like “prestressed concrete”. The application of permanent compressive stress to a material like concrete, which is strong in compression but weak in tension, increases the apparent tensile strength of that material, because the subsequent application of tensile stress must first nullify the compressive prestress. In 1904, Freyssinet[1] attempted to introduce permanently acting forces in concrete to resist the elastic forces developed under loads and this idea was later developed under the name of “prestressing”.

Historical Development

The present state of development in the field of prestressed concrete is due to the continuous research done by engineers and scientists in this field during the last 90 years.

In 1886, Jackson of San Francisco applied for a patent for “construction of artificial stone and concrete pavements”, in which prestress was introduced by tensioning the reinforcing rods set in sleeves. Dohring of Germany manufactured slabs and small beams in 1888, using embedded tensioned wires in concrete to avoid cracks.

The idea of prestressing to counteract the stresses due to loads was first put forward by the Austrian engineer, Mandl, in 1896. M Koenen, of Germany, further developed the subject by reporting, in 1907, on the losses of prestress due to elastic shortening of concrete. The importance of losses in prestressing due to shrinkage of concrete was first recognised by Steiner in the United States around 1908. In 1923, Emperger[2] of Vienna developed a method for making wire-bound reinforced concrete pipes by binding high-tensile steel wires on pipes at stresses ranging from 160 to 800 N/mm^2.

The use of unbonded tendons was first demonstrated by Dischinger, in 1928, in the construction of a major bridge of the deep-girder type, in which prestressing wires were placed inside the girder without any bond. Losses of prestress were compensated by the subsequent retensioning of the wires. Based on the exhaustive studies of properties of concrete and steel, Freyssinet demonstrated, in 1928, the advantages of using high-strength steel and concrete to account for the various losses of prestress due to creep and shrinkage of concrete.

The development of vibration techniques for the production of high-strength concrete and the invention of the double-acting jack for stressing high-tensile steel wires are considered to be the most significant contributions made by Freyssinet between 1928 and 1933.

The use of prestressed concrete spread rapidly from 1935 onwards and many long-span bridges were constructed between 1945 and 1950 in Europe and the United States. During the last 60 years, prestressed concrete has been widely used for the construction of long-span bridges, industrial shell roofs, marine structures, nuclear pressure vessels, water-retaining structures, transmission poles, railway sleepers and a host of other structures. In the words of Guyon[3]: “There is probably no structural problem to which prestress cannot provide a solution, and often a revolutionary one. Prestress is more than a technique; it is a general principle”.

Need for High-Strength Steel and Concrete

The significant observations which resulted from the pioneering research on prestressed concrete were:

Necessity of using high-strength steel and concrete.

Recognition of losses of prestress due to various causes.

The early attempts to use mild steel in prestressed concrete were not successful, as a working stress of 120 N/mm^2 in mild steel is more or 1ess completely lost due to elastic deformation, creep and shrinkage of concrete.

The normal loss of stress in steel is generally about 100 to 240 N/mm^2 and it is apparent that if this loss of stress is to be a small portion of the initial stress, the stress in steel in the initial stages must be very high, about 1200 to 2000 N/mm^2. These high stress ranges are possible only with the use of high-strength steel.

High-strength concrete is necessary in prestressed concrete, as the material offers high resistance in tension, shear, bond and bearing. In the zone of anchorages, the bearing stresses being higher, high-strength concrete is invariably preferred to minimise costs. High-strength concrete is less liable to shrinkage cracks, and has a higher modulus of elasticity and smaller ultimate creep strain, resulting in a smaller loss of prestress in steel. The use of high-strength concrete results in a reduction in the cross-sectional dimensions of prestressed concrete structural elements. With a reduced deadweight of the material, longer spans become technically and economically practicable.

Terminology

Various terms commonly used in the study of prestressed concrete are outlined in this section. The definitions detailed in this section largely comply with those recommended in the relevant Indian standard code of practise[4].

Tendon A stretched element used in a concrete member of structure to impart prestress to the concrete. Generally, high-tensile steel wires, bars cables or strands are used as tendons

Anchorage A device generally used to enable the tendon to impart and maintain prestress in the concrete. The commonly used anchorages are the Freyssinet, Magnel Blaton, Gifford-Udall, Leonhardt-Baur, LeeMcCall, Dywidag, Roebling and BBRV systems.

Pretensioning A method of prestressing concrete in which the tendons are tensioned before the concrete is placed. In this method, the prestress is imparted to concrete by bond between steel and concrete.

Post-tensioning A method of prestressing concrete by tensioning the tendons against hardened concrete. In this method, the prestress is imparted to concrete by bearing.

Bonded prestressed concrete Concrete in which prestress is imparted to concrete through bond between the tendons and surrounding concrete. Pretensioned members belong to this group.

Non-bonded prestressed concrete A method of construction in which the tendons are not bonded to the surrounding concrete. The tendons may be placed in ducts formed in the concrete members or they may be placed outside the concrete section.

Full prestressing Prestressed concrete in which tensile stresses in the concrete are entirely obviated at working loads by having sufficiently high prestress in the members.

Limited or partial prestressing The degree of prestress applied to concrete in which tensile stresses to a limited degree are permitted in concrete under working loads. In this case, in addition to tensioned steel, a considerable proportion of untensioned reinforcement is generally used to limit the width of cracks developed under service loads.

Moderate prestressing In this type, no limit is imposed upon the magnitude of the tensile stresses at working loads. According to Leonhardt[5], this form of construction is not really prestressed concrete but is to be regarded as reinforced concrete with reduced cracking and the sections should be analysed according to the rules of reinforced concrete, as a case of bending combined with axial force.

Axial prestressing Members in which the entire cross-section of concrete has a uniform compressive prestress. In this type of prestressing, the centroid of the tendons coincides with that of the concrete section.

Eccentric prestressing A section at which the tendons are eccentric to the centroid, resulting in a triangular or trapezoidal compressive stress distribution.

Concordant prestressing Prestressing of members in which the cables follow a concordant profile. In the case of statically indeterminate structures, concordant prestressing does not cause any change in the support reactions.

Non-distortional prestressing In this type, the combined effect of the degree of prestress and the deadweight stresses is such that the deflection of the axis of the member is prevented. In such cases, the moments due to prestress and deadweight exactly balance resulting only in an axial force in the member.

Uniaxial, biaxial and triaxial prestressing These terms refer to the cases where concrete is prestressed (i) in only one direction, (ii) in two mutually perpendicular directions, and (iii) in three mutually perpendicular directions.

Circular prestressing The term refers to prestressing in round members, such as tanks and pipes.

Transfer The stage corresponding to the transfer of prestress to concrete. For pretensioned members, transfer takes place at the release of prestress from the bulkheads; for post-tensioned members, it takes place after the completion of the tensioning process.

Supplementary or untensioned reinforcement Reinforcement in prestressed members not tensioned with respect to the surrounding concrete before the application of loads. These are generally used in partially prestressed members.

Transmission length The length of the bond anchorage of the prestressing wire from the end of a pretensioned member to the point of full steel stress.

Cracking load The load on the structural element corresponding to the first visible crack.

Creep in concrete Progressive increase in the inelastic deformation of concrete under sustained stress component.

Shrinkage of concrete Contraction of concrete on drying.

Relaxation in steel Decrease of stress in steel at constant strain.

Proof stress The tensile stress in steel which produces a residual strain of 0.2 per cent of the original gauge length on unloading.

Creep-coefficient The ratio of the total creep strain to elastic strain in concrete.

Cap cable A short curved tendon arranged at the interior supports of a continuous beam. The anchors are in the compression zone, while the curved portion is in the tensile zone.

Degree of prestressing A measure of the magnitude of the prestressing force related to the resultant stress occurring in the structural member at working load.

Debonding Prevention of bond between the steel wire and the surrounding concrete.

Advantages of Prestressed Concrete

Prestressed concrete offers great technical advantages in comparison with other forms of construction, such as reinforced concrete and steel. In the case of fully prestressed members, which are free from tensile stresses under working loads, the cross-section is more efficiently utilised when compared with a reinforced concrete section which is cracked under working loads. Within certain limits, a permanent dead load may be counteracted by increasing the eccentricity of the prestressing force in a prestressed structural element, thus effecting savings in the use of materials.

Prestressed concrete members possess improved resistance to shearing forces, due to the effect of compressive prestress, which reduces the principal tensile stress. The use of curved cables, particularly in long-span members, helps to reduce the shear forces developed at the support sections.

A prestressed concrete flexural member is stiffer under working loads than a reinforced concrete member of the same depth.

However, after the onset of cracking, the flexural behaviour of a prestressed member is similar to that of a reinforced concrete member.

The use of high-strength concrete and steel in prestressed members results in lighter and slender members than is possible with reinforced concrete. The two structural features of prestressed concrete, namely high-strength concrete and freedom from cracks, contributes to the improved durability of the structure under aggressive environmental conditions. Prestressing of concrete improves the ability of the material for energy absorption under impact loads. The ability to resist repeated working loads has been proved to be as good in prestressed as in reinforced concrete.

Prestressing Systems

The most widely used method for prestressing of structural concrete elements is longitudinal tensioning of steel by different tensioning devices. Prestressing by the application of direct forces between abutments is generally used for arches and pavements, while flat jacks are invariably used to impart the desired forces. For circular structures, such as tanks and pipes, it is a common practise to impart precompression in concrete by circular prestressing. With the development of expansive cements, prestress in concrete can be developed by chemical prestressing.

Tensioning Devices

The various types of devices used for tensioning steel are grouped under four principal categories, namely:

Mechanical

Hydraulic

Electrical (thermal)

Chemical

The mechanical devices generally used include weights with or without lever transmission, geared transmission in conjunction with pulley blocks, screw jacks with or without gear drives and wire-winding machines. These devices are employed mainly for prestressing structural concrete components produced on a mass scale in factories.

Hydraulic jacks, being the simplest means of producing large prestressing forces, are extensively used as tensioning devices. Several commonly used patented jacks are due to Freyssinet, Magnel, Gifford Udall and Baur-Leonhardt for the range of 5–100 tonnes. Large hydraulic jacks for forces in the range of 200–600 tonnes have also been developed by Baur–Leonhardt. It is important that during the tensioning operation, the force applied should be accurately measured. In most of the jacks, calibrated pressure gauges directly indicate the magnitude of force developed during the tensioning of the wires.

Electrical devices have been successfully used in erstwhile USSR since 1958 for tensioning of steel wires and deformed bars. The steel wires are electrically heated and anchored before placing concrete in the moulds. This method is often referred to as ‘thermo-electric prestressing’.

In the chemical method, expanding cements are used and the degree of expansion is controlled by varying the curing conditions. Since the expansive action of cement while setting is restrained, it induces tensile forces in tendons and compressive stresses in concrete.

Pretensioning Systems

In the pretensioning system, the tendons are first tensioned between rigid anchor-blocks cast on the ground or in a column or unit-mould-type pretensioning bed, prior to the casting of concrete in the moulds. A typical column-type pretensioning bed is shown in Fig. 3.1. The tendons comprising

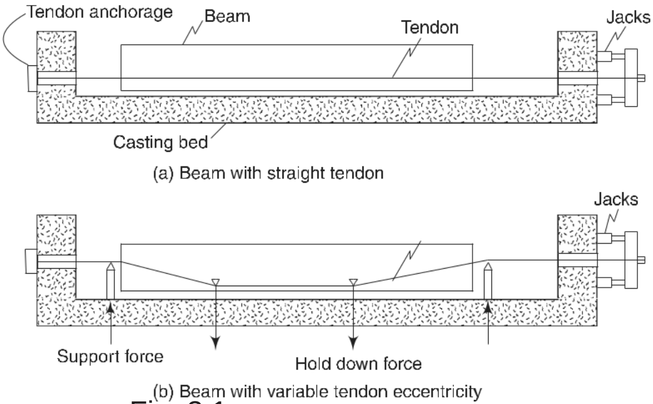
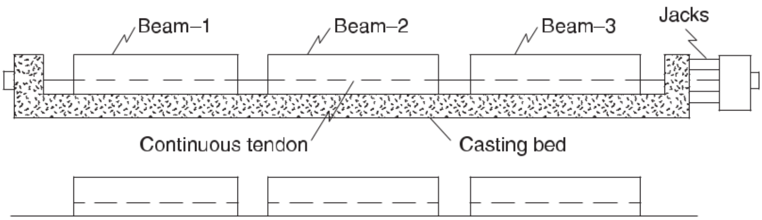


Fig. 3.1 **Methods of pretensioning**

individual wires or strands are stretched with constant eccentricity as shown in Fig. 3.1(a) or variable eccentricity as shown in Fig. 3.1(b) with tendon anchorage at one end and jacks at the other. With the forms in place, the concrete is cast around the stressed tendon.

High early strength concrete is often used in a factory to facilitate early stripping and reuse of moulds. When the concrete attains sufficient strength, the jacking pressure is released. The high-tensile wires tend to shorten but are checked by the bond between concrete and steel. In this way, the prestress is transferred to the concrete by bond, mostly near the ends of the beam, and no special anchorages are required in pretensioned members.

For mass production of pretensioned elements, the long-line process developed by Hoyer is generally used in a factory. In this method, the tendons are stretched between two bulk heads several hundred metres apart so that a number of similar units may be cast along the same group of tensioned wires as shown in Fig. 3.2. The tension is applied by hydraulic jacks or by a movable stressing machine. The wires or strands when tensioned singly or in groups are generally anchored to the abutments by steel wedges.



Post-Tensioning Systems

Principles of Post-tensioning

In post-tensioning, the concrete units are first cast by incorporating ducts or grooves to house the tendons. When the concrete attains sufficient strength, the high-tensile wires are tensioned by means of jack bearing on the end face of the member and anchored by wedges or nuts. The forces are transmitted to the concrete by means of the end anchorages and, when the cable is curved, through the radial pressure between the cable and the duct. The space between the tendons and the duct is generally grouted after the tensioning operation.

Most of the commercially patented prestressing systems are based on the following principles of anchoring the tendons:

Wedge action producing a frictional grip on the wires.

Direct bearing from rivet or bolt heads formed at the end of the wires.

Looping the wires around the concrete.

Post-tensioning Anchorages

The Freyssinet system of post-tensioning anchorages which was developed in 1939, gave impetus to the various new systems devised over the years. At present, according to Abeles4, there are over 64 patented post-tensioning systems used worldwide. Some of the more prominent systems are compiled in Table 3.1, based on the work of Abeles4 and Bennett5.

The post-tensioning systems based on wedge-action include the Freyssinet, Gifford–Udall, Anderson and Magnel–Blaton anchorages.

The Freyssinet anchorage system, which is widely used in Europe and India, consists of a cylinder with a conical interior through which the high-tensile wires pass and against the walls of which the wires are wedged by a conical plug lined longitudinally with grooves to house the wires . The main advantage of the Freyssinet system is that a large number of wires or strands can be simultaneously tensioned using the double-acting hydraulic jack as shown in Fig.

.

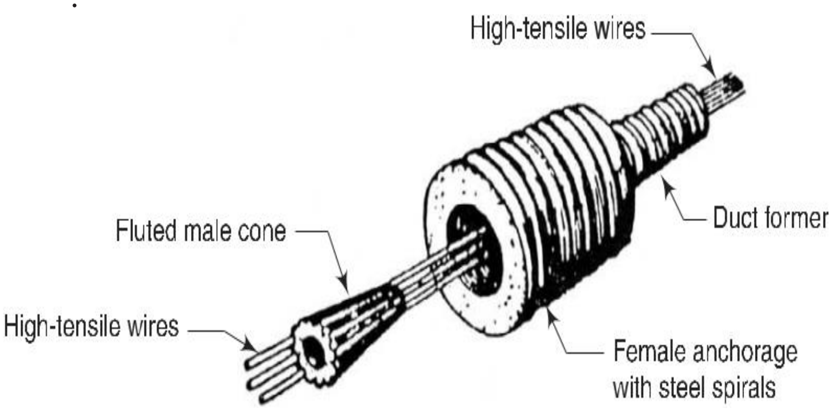


Fig. **Freyssinet anchorage**

The Gifford-Udall (CCL) system developed in UK, consists of steel split-cone and cylindrical female-cone anchorages to house the high-tensile wires bearing against steel plates as shown in Fig.. Each wire is tensioned separately and anchored by forcing a sleeve wedge into a cylindrical grip resting against a bearing plate. The ducts are generally formed by metal sheaths cast into the concrete member.

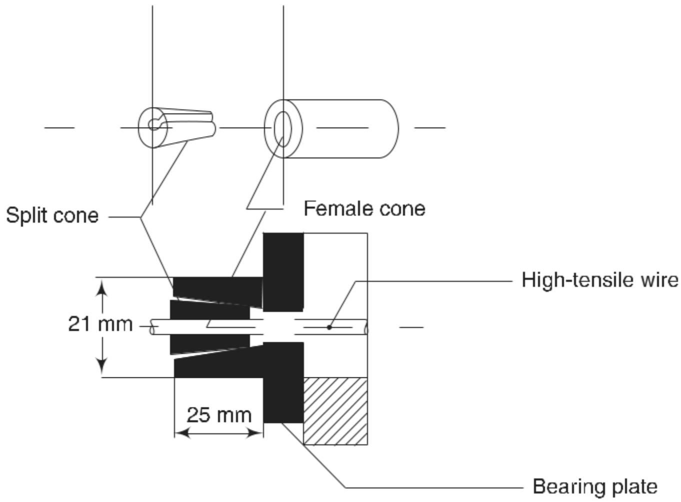


Fig. Gifford Udall system

Thermo-Electric Prestressing

The method of prestressing by heated tendons, achieved by passing an electric current in the high-tensile wires, is generally referred to as 'thermo-electric prestressing'. In the erstwhile USSR, the electrothermic method8 has been widely used since 1958 for pretensioning bar reinforcements of precast units.

The process consists of electrically heating the bars to a temperature of 300–400°C within 3–5 min. The bars undergo an elongation of about 0.4–0.5 per cent. The bars, after cooling, try to shorten but are checked by the fixed anchors at the two ends as shown in Fig. 3.12. The cooling period is reckoned to be 12–15 min.

By this process, it is possible to induce initial stresses of 500–600 N/mm2 in the tendons. The concrete is placed in moulds only after the temperature of the wires falls below 90°C. In the erstwhile USSR9, this method has been found to be more economical than conventional mechanical devices.

Thermo-electric prestressing has also been adopted in Germanyl0 for the tensioning of oval-section-ribbed wires with an ultimate tensile strength of 1600 N/mm2. A temperature of about 460°C was necessary to induce an initial stress of 55 per cent of the ultimate tensile strength; the heating time being 40–90 s at 30 V and 300–1100 A. Empirical relations for the estimation of the current, voltage and power requirements of the transformer are reported by Graduck11.

Chemical Prestressing

Self-stressing or chemical prestressing of concrete was made possible by the development of expanding cements by Lossier12 of France in 1944. Generally, expanding cements consist of 75 per cent Portland cement, 15 per cent high alumina cement and 10 per cent gypsum, which result in the formation of calcium sulphoaluminate. The linear expansion of the cement is about 3–4 per cent. Mikhailov13 reported that expansive cements have been used for prestressing purposes in the erstwhile USSR since 1949. The degree of expansion can be controlled by varying the curing conditions.

Since the expansion of the concrete is restrained by high-tensile steel wires, the compressive stresses that develop in concrete and steel wires are subjected to tensile stresses. Investigations by Lin and Klein14 have established that it is possible to obtain initial compressive stresses in concrete of 4–6.5 N/mm2, which may be reduced to 3–6 N/mm2 after shrinkage and creep. Tensile stresses of up to 850 N/mm2 were developed in steel by the expansion of concrete. Results of laboratory investigations of several types of chemically prestressed elements, such as beams, slabs, frames, columns, pipes and hyperbolic paraboloid shells, have demonstrated the feasibility of chemical prestressing.

It has been found that structural elements ideally suited for chemical prestressing include pipes, thin walls and slabs, shells15, folded plates and composite columns16, as well as precast beams and columns17.

In the present state of art, chemical prestressing can be applied to structural elements and systems in which the optimum amount of prestress is relatively low. The method is not suited for high degrees of prestress and high percentages of steel where mechanical prestressing can be conveniently used.

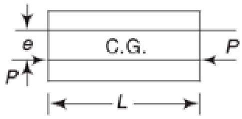
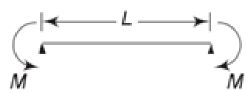
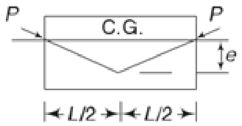
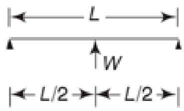
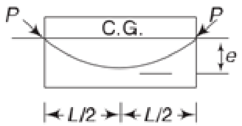
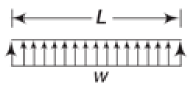
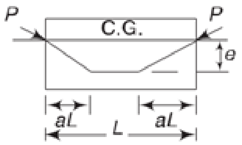
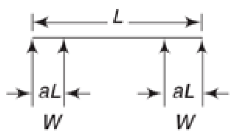
Concept of Load Balancing

It is possible to select suitable cable profiles in a prestressed concrete member such that the transverse component of the cable force balances the given type of external loads. This can be readily illustrated by considering the free body of concrete, with the tendon replaced by forces acting on the concrete beam as shown in Fig. 4.18 and Table 4.1.

The various types of reactions of a cable upon a concrete member depend upon the shape of the cable profile. Straight portions of the cable do not induce any reactions except at the ends, while curved cables result in uniformly distributed loads. Sharp angles in a cable induce concentrated loads. The concept of loading-balancing1 is useful in selecting the tendon profile, which can supply the most desirable system of forces in concrete.

In general, this requirement will be satisfied if the cable profile in a prestressed member corresponds to the shape of the bending moment diagram resulting from the external loads. Thus, if the beam supports two concentrated loads, the cable should follow a trapezoidal profile. If the beam supports uniformly distributed loads, the corresponding tendon should follow a parabolic profile. The principle of load balancing is further amplified with the following examples.

Table 4.1 **Tendon profiles and equivalent loads in prestressed concrete beams**

Tendon Profile	Equivalent Moment or Load	Equivalent Loading	Camber
	$M = Pe$		$\frac{ML^2}{8EI}$
	$W = \frac{4Pe}{L}$		$\frac{WL^3}{48EI}$
	$W = \frac{8Pe}{L^2}$		$\frac{5WL^4}{384EI}$
	$W = \frac{Pe}{aL}$		$\frac{a(3 - 4a^2)WL^3}{24EI}$

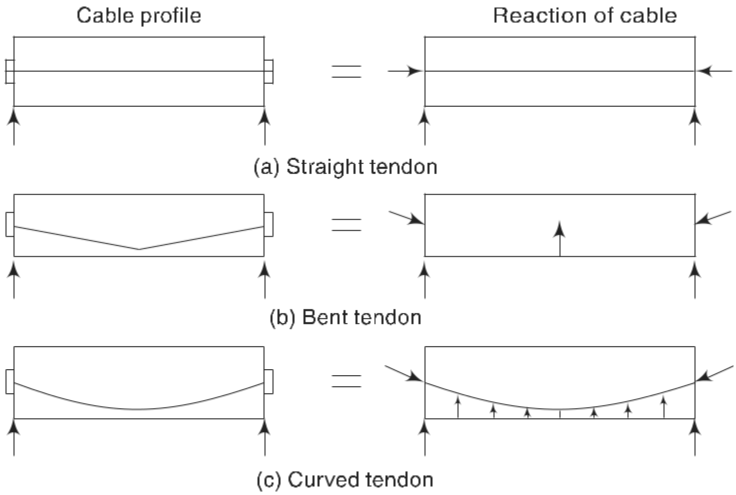


Fig. 4.18 **Reactions of cable on beam**

Losses of Prestress

Nature of Losses of Prestress

The initial prestress in concrete undergoes a gradual reduction with time from the stage of transfer due to various causes. This is generally referred to as 'loss of prestress'. A reasonably good estimate of the magnitude of loss of prestress is necessary from the point of view of design. The different types of losses encountered in the pretensioning and post-tensioning systems are compiled in Table 5.1.

Table 5.1 Types of losses of prestress

S. No.	Pretensioning	S. No.	Post-tensioning
1.	Elastic deformation of concrete	1.	No loss due to elastic deformation if all the wires are simultaneously

			tensioned. If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete
2.	Relaxation of stress in steel	2.	Relaxation of stress in steel
3.	Shrinkage of concrete	3.	Shrinkage of concrete
4.	Creep of concrete	4.	Creep of concrete
		5.	Friction
		6.	Anchorage slip

In addition to these, there may be losses of prestress due to sudden changes in temperature, especially in steam curing of pretensioned units. The rise in temperature causes a partial transfer of prestress (due to the elongation of the tendons between adjacent units in the long line process) which may cause a large amount of creep if the concrete is not properly cured. If there is a possibility of a change of temperature between the times of tensioning and transfer, the corresponding loss should be allowed for in the design.

Loss due to Elastic Deformation of Concrete

The loss of prestress due to elastic deformation of concrete depends on the modular ratio and the average stress in concrete at the level of steel.

If fc = prestress in concrete at the level of steel

Es = modulus of elasticity of steel

Ec = modulus of elasticity of concrete

$$\alpha_c = \frac{E_s}{E_c} = \text{modular ratio}$$

Strain in concrete at the level of steel = $\left(\frac{f_c}{E_c}\right)$

Stress in steel corresponding to this strain = $\left(\frac{f_c}{E_c}\right)E_s$

∴ Loss of stress in steel = $\alpha_c f_c$

If the initial stress in steel is known, the percentage loss of stress in steel due to the elastic deformation of concrete can be computed.

Example 5.1 A pretensioned concrete beam of rectangular cross-section, 150 mm wide and 300 mm deep, is prestressed by eight high tensile wires of 7 mm diameter located at 100 mm from the soffit of the beam. If the wires are tensioned to a stress of 1100 N/mm^2, calculate the percentage loss of stress due to elastic deformation assuming the modulus of elasticity of concrete and steel as 31.5 and 210 kN/mm^2.

Solution.

Area of eight steel wires = A_p = (8 × 38.48) = 308 mm^2

Prestressing force = P = [(1100 × 308)/1000] = 45000 mm^2

Area of concrete section = A = (150 × 300) = 45000 mm^2

Second moment of area = I = [(150 × 330^3)/12] = [33.75 × 10^7] mm^4

Modular ratio = α_c = [E_s/E_c] = [210/31.5] = 6.66

Eccentricity = 50 mm

Stress in concrete at the level of steel =

$$f_c = \left[\frac{338.8 \times 10^3}{45000}\right] + \left[\frac{338.8 \times 10^3 \times 50 \times 50}{33.75 \times 10^7}\right] = 10.02 \text{ N/mm}^2$$

Loss of stress due to elastic deformation = $(\alpha_c \times f_c)$ = (6.66 × 10.2) = 66.73 N/mm^2

Percentage loss of stress in steel = [(66.73 × 100)/1100] = 6.06%

Loss of Prestress due to Shrinkage of Concrete

The shrinkage of concrete in prestressed members results in a shortening of tensioned wires and hence contributes to the loss of stress. The shrinkage of concrete is influenced by the type of cement and aggregates and the method of curing used. Use of high-strength concrete with low water-cement ratios results in a reduction in shrinkage and consequent loss of prestress. The primary cause of drying shrinkage is the progressive loss of water from concrete. The rate of shrinkage is higher at the surface of the members. The differential shrinkage between the interior and surface of large members may result in strain gradients leading to surface cracking. Hence, proper curing is essential to prevent shrinkage cracks in prestressed members.In the case of pretensioned members, generally moist curing is resorted to in order to prevent shrinkage until the time of transfer. Consequently, the total residual shrinkage strain will be larger in pretensioned members after transfer of prestress in comparison with post-tensioned members, where a portion of shrinkage will have already taken place by the time of transfer of stress.

This aspect has been considered in the recommendations made by the Indian standard code (IS: 1343) for the loss of prestress due to the shrinkage of concrete and is detailed below.As outlined in Section 2.1.4, the total shrinkage strain (ecs) comprises of two components, the drying shrinkage strain (ecd) and the autogenous shrinkage strain (eca), expressed as,

ecs = (ecd+ eca)

The loss of stress in steel due to the shrinkage of concrete is estimated by the relation,

Loss of stress = (ecs ¥ Es), where Es = modulus of elasticity of steel

The value of the total shrinkage strain can be evaluated with the available data of the drying and autogenous shrinkage strain using Tables 2.3, 2.4 and 2.5. In contrast to the Indian Standard Code IS: 1343-2012, the British code lists the drying shrinkage values for humidity’s varying from 20 to 100 per cent.

Loss of Prestress due to Creep of Concrete

The initial stress in the tendons gradually reduces due to the creep of concrete. The factors influencing the creep of concrete are presented in detail in Section 2.1.5. In designing prestressed concrete members, a knowledge of the magnitude of loss of prestress due to creep is necessary. Various national codes recommend the creep coefficient method for estimating the loss of prestress.

Creep Coefficient Method

If ϕ_o = creep coefficient
ε_c = creep strain
ε_e = elastic strain
m = modular ratio
E_s = modulus of elasticity of steel
E_c = modulus of elasticity of concrete
f_c = stress in concrete

$$\text{Creep coefficient} = \left(\frac{\text{creep strain}}{\text{elastic strain}}\right) = \left(\frac{\varepsilon_c}{\varepsilon_e}\right)$$

$$\therefore \quad \varepsilon_c = \phi_o\, \varepsilon_e = \phi_o (f_c/E_c)$$

Hence, loss of stress $= (\varepsilon_c \cdot E_s) = \phi_o (f_c/E_c)\, E_s = (\phi_o \cdot f_c \cdot m)$.

The Indian and British codes recommend the values of 70 year creep coefficient for varying humidity, age at loading and the notional size of the member. The value of the creep coefficient varies from a minimum of 1.0 to a maximum of 5.8. For grades of concrete from M-30 to M-60 generally used in prestressed members, the values of creep coefficient listed in Table 2.7 are useful in computing the loss of stress due to creep of concrete.

Loss of Prestress due to Relaxation of Stress in Steel

The various factors which influence the phenomenon of creep in steel have been discussed in Section 2.2.4. Most of the codes provide for the loss of stress due to relaxation of steel as a percentage of the initial stress in steel. The Indian Standard Code recommends a value varying from 0 to 90 N/mm2 for stress in wires varying from 0.5fpu to 0.8fpu. The loss of prestress due to relaxation of steel recommended in British and Indian codes is compiled in Table 5.2. Temporary over-stressing by 5–10 per cent for a period of 2 min. is sometimes used to reduce this loss as in the case of drawn wires.

However, over-stressing does not appear to be beneficial for stabilised wires2 which, as a result of heat treatment, have 0.1 per cent proof stress in excess of 85 per cent of the tensile strength, since such wires suffer very little permanent deformation when over-stressed.

Loss of Prestress due to Friction

In the case of post-tensioned members, the tendons are housed in ducts preformed in concrete. The ducts are either straight or follow a curved profile depending upon the design requirements. Consequently, on tensioning the curved tendons, loss of stress occurs in the post-tensioned members due to friction between the tendons and the surrounding concrete ducts. The magnitude of this loss is of the following types:

Loss of stress due to the curvature effect3, which depends upon the tendon form or alignment which generally follows a curved profile along the length of the beam.

Loss of stress due to the wobble effect4, which depends upon the local deviations in the alignment of the cable. The wobble or wave effect is the result of accidental or unavoidable misalignment, since ducts or sheath cannot be perfectly located to follow a predetermined profile throughout the length of the beam.

Loss due to Anchorage Slip

In most post-tensioning systems, when the cable is tensioned and the jack is released to transfer prestress to concrete, the friction wedges, employed to grip the wires, slip over a small distance before the wires are firmly housed between the wedges. The magnitude of slip depends upon the type of wedge and the stress in the wires. In systems where the tendons are looped around concrete anchorage blocks, as in the case of Leonhardt–Baur system, loss of stress may take place due to the wires biting into the anchorage. When anchor plates are employed, it may be necessary to allow for the small settlement of the plate into the end of the concrete member.

The loss during anchoring, which occurs with wedge-type grips, is normally allowed for on the site by over-extending the tendon in the prestressing operation by the amount of the draw-in before anchoring. However, this method is satisfactory, provided the momentary over-stress does not exceed the prescribed limits of 80–85 per cent of the ultimate tensile strength of the wire.

Problem

A pretensioned beam, 200 mm wide and 300 mm deep, is prestressed by 10 wires of 7 mm diameter initially stressed to 1200 N/mm2, with their centroids located 100 mm from the soffit. Find the maximum stress in concrete immediately after transfer, allowing only for elastic shortening of concrete.If the concrete undergoes a further shortening due to creep and shrinkage while there is a relaxation of five per cent of steel stress, estimate the final percentage loss of stress in the wires using the Indian Standard Code IS: 1343 regulations, and the following data:

$$E_s = 210 \text{ kN/mm}^2$$

$$E_c = 5700\,(f_{cu})^{1/2}$$

$$f_{cu} = 42 \text{ N/mm}^2$$

Creep coefficient $(\phi) = 1.6$

Total residual shrinkage strain $= (3 \times 10^4)$

Solution.

$$A_c = (6 \times 10^4) \text{ mm}^2$$

$$E_c = 5700(42)^{1/2} = 36900 \text{ N/mm}^2$$

$$I = 45 \times 10^7 \text{ mm}^4$$

$$\alpha_e = (E_s/E_c) = 5.7$$

$$P = (1200)(10 \times 38.5) = (462 \times 10^3) \text{ N} = 462 \text{ kN}$$

Stress in concrete at the level of steel is given by

$$f_c = \left[\frac{462 \times 10^3}{6 \times 10^4} + \frac{(462 \times 10^3 \times 50)50}{45 \times 10^7}\right] = 10.3 \text{ N/mm}^2$$

Loss of stress due to elastic deformation of concrete

$$= (5.7 \times 10.3) = 58.8 \text{ N/mm}^2$$

Force in wires immediately after transfer = (1200 – 58.8) 38.5

$$= 440\,000 \text{ N} = 440 \text{ kN}$$

Stress in concrete at the level of steel is given by

$$f_c = \left[\frac{440 \times 10^3}{6 \times 10^4} + \frac{(440 \times 10^3 \times 50)50}{45 \times 10^7}\right] = 978 \text{ N/mm}^2$$

Type of losses of prestress

1. Elastic deformation	$= 58.8 \text{ N/mm}^2$
2. Creep of concrete = $(1.6 \times 9.78 \times 5.7)$	$= 89.2 \text{ N/mm}^2$
3. Shrinkage of concrete = $(3 \times 10^{-4})(210 \times 10^3)$	$= 63.0 \text{ N/mm}^2$
4. Relaxation of steel stress = (5/100) 1200	$= 60.0 \text{ N/mm}^2$
Total loss	$= 271.0 \text{ N/mm}^2$
Final stress in wires = (1200 – 271.0)	$= 929.0 \text{ N/mm}^2$
Percentage loss = $\left(\frac{271.0}{1200} \times 100\right)$	$= 22.58\%$

A prestressed concrete beam, 200 mm wide and 300 mm deep, is prestressed with wires (area = 320 mm2) located at a constant eccentricity of 50 mm and carrying an initial stress of 1000 N/mm2. The span of the beam is 10 m. Calculate the percentage loss of stress in wires if (a) the beam is pretensioned, and (b) the beam is post-tensioned, using the following data:

Solution.

$E_s = 210 \text{ kN/mm}^2$ and $E_c = 35 \text{ kN/mm}^2$

Relaxation of steel stress = 5 per cent of the initial stress

Shrinkage of concrete = 300×10^{-6} for pretensioning and 200×10^{-6} for post-tensioning.

Creep coefficient = 1.6

Slip at anchorage = 1 mm

Frictional coefficient for wave effect = 0.0015 per m

$$\text{Prestressing force} = \left[\frac{(320 \times 1000)}{1000}\right] = 320 \text{ kN}$$

$$\text{Cross-sectional area, } A = (6 \times 10^4)\text{ mm}^2$$

$$\text{Modular ratio} = \alpha_e = \left(\frac{210}{35}\right) = 6$$

$$I = \left[\frac{(200 \times 300^3)}{12}\right] = (45 \times 10^7)\text{ mm}^4$$

Stress in concrete at the level of steel

$$= \left[\frac{(320 \times 10^3)}{(6 \times 10^4)} + \frac{(320 \times 10^3 \times 50 \times 50)}{(45 \times 10^7)}\right] = 7 \text{ N/mm}^2$$

The various losses are compiled in the tabular statement as follows:

Losses of Stress

Type of Loss	*Pretensioned Beam* (N/mm^2)	*Post-tensioned Beam* (N/mm^2)
1. Elastic deformation of concrete	$(6 \times 7) = 42.00$	—
2. Relaxation of stress in steel	5% of 1000 = 50.00	5% of 1000 = 50.00
3. Creep of concrete	$(1.6 \times 7 \times 6) = 67.20$	$(1.6 \times 7 \times 6) = 67.20$
4. Shrinkage of concrete	$(300 \times 10^{-6} \times 210 \times 10^3)$ $= 63.00$	$(200 \times 10^{-6} \times 210 \times 10^3)$ $= 21.00$
5. Slip at anchorage	—	$(1 \times 210 \times 10^3) / (10 \times 1000)$ $= 21.00$
6. Friction effect	—	$(1000 \times 0.0015 \times 10)$ 15.00
7. Total loss of stress	222.20	195.20
8. Percentage loss of stress	22.2%	19.52%

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DEPARTMENT OF CIVIL ENGINEERING

UNIT – II – DESIGN FOR FLEXURE AND SHEAR – SCI1402

Balanced sections,under reinforced section and over reinforced section

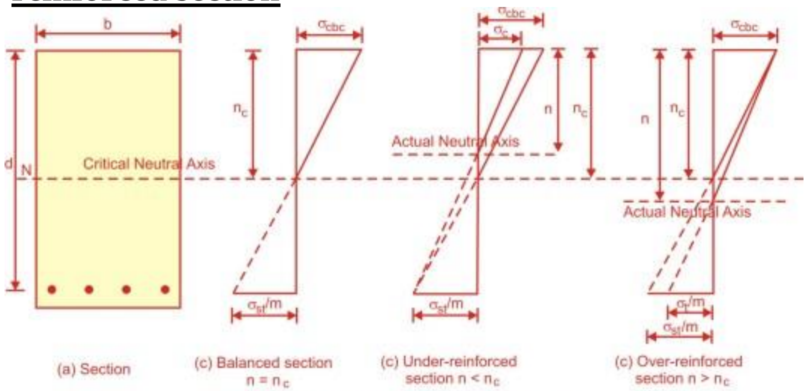


Fig. 2.6. Different types of R.C.C. beam sections.

Balanced sections,under reinforced section and over reinforced section

Balanced Sections

A balanced sections is that in which stress in concrete and steel reach their permissible value at the same time. This means that stress diagram is as shown in Fig. 2.6(b). *The percentage of steel corresponding to this section is called as balanced steel and the neutral axis is called as critical neutral axis nc*

$m.\sigma cbc\sigma st=ncd-nc$

For a balanced sections, the moment of resistance is calculated as under :

$MB=\sigma cbc2b.nc(d-nc3)=Rbd2$

Under Reinforced Section

In an under reinforced section, the percentage of steel provided is less than that provided in balanced section. So the actual neutral axis will shift upwards i.e., nc > n as shown in Fig. 2.6(c). In under reinforced section, the stress in steel first reaches it permissible value, while the concrete is under stressed. The moment of resistance of this section is calculated as

$M_r = \sigma_{st}.A_{st}(d - \frac{n}{3})$

The various features of under reinforced section are as follows :

(i) Steel is fully stressed while concrete not (i.e., stress in steel is σ_{st} (permissible) but stress in concrete is less than σ_{cbc}

(ii) The actual neutral axis lies above the critical neutral axis ($n < n_c$).

(iii) The percentage of steel is less than the balanced section hence the section is economical.

(iv) Ductile failure.

(v) The moment of resistance is less than balanced section.

In under reinforced section, the failure is ductile because steel fails first and sufficient warning is given before collapse. Due to ductile failure and economy, the under-reinforced sections are preferred by designers.

Over Reinforced Section

In an over reinforced section the percentage of steel provided is greater than the balanced section. So the actual neutral axis shift downward i.e., $n > n_c$ [Fig. 2.6(d)]. In this section, stress in concrete reaches its permissible value while steel is not fully stressed. Concrete is brittle and it fails by crushing suddenly. As steel is not fully utilised, the over reinforced section is uneconomical (steel is much costlier than concrete). The various features of over reinforced s section are :

(i) Concrete is fully stressed while steel is not (i.e., the stress in concrete is at its permissible value σ_{cbc} but stress in steel is less than σ_{st}).

(ii) The actual neutral axis is below the critical neutral axis i.e., $n > n_c$.

(iii) The percentage of steel is more than the balanced section, so the section is uneconomical.

(iv) Sudden failure.

(v) The moment of resistance of over-reinforced section is calculated as

$$Mr=12\sigma cbcb.n(d-n3)$$

A beam bends under bending moment, resulting in a small curvature. At the outer face (tensile face) of the curvature the concrete experiences tensile stress, while at the inner face (compressive face) it experiences compressive stress.

Definition of beam

Singly reinforced beam

A singly reinforced beam is one in which the concrete element is only reinforced near the tensile face and the reinforcement, called tension steel, is designed to resist the tension.

Doubly reinforced beam

A doubly reinforced beam is one in which besides the tensile reinforcement the concrete element is also reinforced near the compressive face to help the concrete resist compression. The latter reinforcement is called compression steel. When the compression zone of a concrete is inadequate to resist the compressive moment (positive moment), extra reinforcement has to be provided if the architect limits the dimensions of the section.

Under-reinforced beam

An under-reinforced beam is one in which the tension capacity of the tensile reinforcement is smaller than the combined compression capacity of the concrete and the compression steel (under-reinforced at tensile face). When the reinforced concrete element is subject to increasing bending moment, the tension steel yields while the concrete does not reach its ultimate failure condition. As the tension steel yields and stretches, an “under-reinforced” concrete also yields in a ductile manner, exhibiting a large deformation and warning before its ultimate failure. In this case the yield stress of the steel governs the design.

Over-reinforced beam

An over-reinforced beam is one in which the tension capacity of the tension steel is greater than the combined compression capacity of the concrete and the compression steel (over-reinforced at tensile face). So the “over-reinforced concrete” beam fails by crushing of the compressive-zone concrete and before the tension zone steel yields, which does not provide any warning before failure as the failure is instantaneous.

balanced-reinforced beam

A balanced-reinforced beam is one in which both the compressive and tensile zones reach yielding at the same imposed load on the beam, and the concrete will crush and the tensile steel will yield at the same time. This design criterion is however as risky as over-reinforced concrete, because failure is sudden as the concrete crushes at the same time of the tensile steel yields, which gives a very little warning of distress in tension failure.

Steel-reinforced concrete moment-carrying elements should normally be designed to be under-reinforced so that users of the structure will receive warning of impending collapse.

Characteristic strength

The characteristic strength is the strength of a material where less than 5% of the specimen shows lower strength.

Design strength or nominal strength

The design strength or nominal strength is the strength of a material, including a material-safety factor. The value of the safety factor generally ranges from 0.75 to 0.85 in Permissible stress design.

The assumptions made in the Theory of Simple Bending are as follows:

- The material of the beam that is subjected to bending is homogenous (same composition throughout) and isotropic(same elastic properties in all directions).
- The beams have a symmetrical cross section and they are subjected to bending only in the plane of symmetry.

- The beam is made up of a number of fibers that run longitudinally to each other and are all straight initially. On bending, they do so in the form of circular arcs, with a common centre of curvature.
- The effect of Shear stresses is neglected. The bam is subjected to pure bending.
- No warping of the cross section takes place. That is, transverse sections through the beam taken normal to the axis of the beam ramin plane after the beam is subjected to bending.
- The dimensions of the beam are very small as compared to the radius of curvature of the beam.

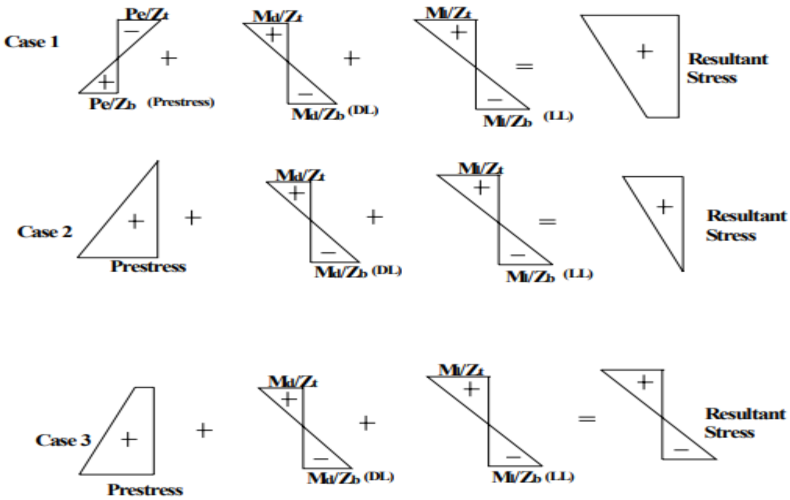
Analysis of Prestress Member Basic assumption

1. Concrete is a homogenous material.
2. Within the range of working stress, both concrete & steel behave elastically, notwithstanding the small amount of creep, which occurs in both the materials under the sustained loading.
3. A plane section before bending is assumed to remain plane even after bending, which implies a linear strain distribution across the depth of the member.

Analysis of prestress member The stress due to prestressing alone are generally combined stresses due to the action of direct load bending from an eccentrically applied load. The following notations and sign conventions are used for the analysis of prestress members.

PÆPrestressing force (Positive when compressive)
eÆEccentricity of prestressing force M = PeÆMoment AÆCross-sectional area of the concrete member IÆSecond moment of area of the section about its centroid Zt Zb , ÆSection modulus of the top & bottom fibre respectively , top bot f f ÆPrestress in concrete developed at the top & bottom fibres t b y , y ÆDistance of the top & bottom fibre from the centroid of the section rÆRadius of gyration (i) Concentric

tendon In this case, the load is applied concentrically and a compressive stress of magnitude (P/A) will act through out the section. Thus the stress will generate in the section as shown in the figure below



Analysis of Members under Flexure Introduction Similar to members under axial load, the analysis of members under flexure refers tothe evaluation of the following.

1) Permissible prestress based on allowable stresses at transfer.

2) Stresses under service loads. These are compared with allowable stresses underservice conditions.

3) Ultimate strength. This is compared with the demand under factored loads.

4) The entire load versus deformation behaviour. The analyses at transfer and under service loads are presented in this section. The evaluation of the load versus deformation behaviour is required in special type of analysis.

Assumptions The analysis of members under flexure considers the following.

1. Concrete is a homogeneous elastic material.

2. With in the range of working stress, both concrete & steel behave elastically, not with standing the small amount of creep, which occurs in both the materials under the sustained loading.

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UNIT – III – DEFLECTION AND DESIGN OF ANCHORAGE ZONE – SCI1402

Importance of Control of Deflections

The philosophy of design, termed “limit state approach,” adopted by the Russian code in 1954 and the American and British codes in 1971, requires a proper knowledge of the behaviour of structural concrete members at the multiple limit states, of which deflection forms an important criterion for the safety of the structure. It is the general practise, according to various national codes, that structural concrete members should be designed to have adequate stiffness to limit deflections, which may adversely affect the strength or serviceability of the structure at working loads.

Suitable control on deflectional1 is very essential for the following reasons:

Excessive sagging of principal structural members is not only unsightly, but at times, also renders the floor unsuitable for the intended use.

Large deflections under dynamic effects and under the influence of variable loads may cause discomfort to the users.

Excessive deflections are likely to cause damage to finishes, partitions and associated structures.

In recent years, damage to partitions and finishes has been the most important consequence of excessive deflections. A field survey conducted by Mayer2 in Germany, revealed over 80 examples of damage to partition walls, of which 21 had estimated deflections within the prescribed code-limits. The survey also indicated that a maximum limit on deflection should be specified in addition to a limiting deflection–span ratio, since it was recognised that as the span increases, the former limitation is likely to control. For a reasonably accurate assessment of deflections, it is very essential to consider the various factors which influence them.

Factors Influencing Deflections

The deflections of prestressed concrete members are influenced by the following salient factors:

Imposed load and self-weight

Magnitude of the prestressing force

Cable profile

Second moment of area of cross-section

Modulus of elasticity of concrete

Shrinkage, creep and relaxation of steel stress

Span of the member

Fixity conditions

In the precracking stage, the whole cross-section is effective and the deflections in this stage are computed by using the second moment of area of the gross concrete section. The computation of short-term or instantaneous deflections, which occur immediately after transfer of prestress and on application of loads, is conveniently done by using Mohr’s theorems.

In the post-cracking stage, a prestressed concrete beam behaves in a manner similar to that of a reinforced concrete beam and the computation of deflections at this stage is made by considering moment–curvature relationships which involve the section properties of the cracked beam.

In both cases, the effect of creep and shrinkage of concrete is to increase the long-term deflections under sustained loads, which is estimated by using empirical methods that involve the use of effective (long-term) modulus of elasticity or by multiplying short-term deflections by suitable factors

Mohr's Theorems

Short-term or instantaneous deflections of prestressed members are governed by the bending moment distribution along the span and the flexural rigidity of the members. Mohr's moment area theorems3 are readily applicable for the estimation of deflections due to the prestressing force, self-weight and imposed loads. Consider Fig. 6.1 in which the beam AB is subjected to a bending moment distribution due to the prestressing force or self-weight or imposed loads. ACB is the centre line of the deformed structure under the system of given loads.

The deflection of symmetrically loaded and simply supported beams at the mid-span point are directly obtained from the second moment area theorem since the tangent is horizontal at this point. More complicated problems involving unsymmetrical loading may be solved by combining both the moment area theorems.

Effect of Tendon Profile on Deflections

In most of the cases of prestressed beams, tendons are located with eccentricities towards the soffit of beams to counteract the sagging bending moments due to transverse loads. Consequently, the concrete beams deflect upwards (camber) on the application or transfer of prestress. Since the bending moment at every section is the product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of the BMD. The method of computing deflections of beams with different cable profiles is outlined as follows:

Straight Tendons Figure 6.2 shows a beam with a straight tendon at a uniform eccentricity below the centroidal axis.

If upward deflections are considered as negative and

P = effective prestressing force

e = eccentricity

L = length of the beam

Deflections due to Self-Weight and Imposed Loads

At the time of transfer of prestress, the beam hogs up due to the effect of prestressing. At this stage, the self-weight of the beam induces downward deflections, which further increase due to the effect of imposed loads on the beam.

If g = self-weight of the beam/m

q = imposed load/m (uniformly distributed), the downward deflection is computed as,

$$a = \frac{5(g+q)L^4}{384EI}$$

Deflections due to concentrated live loads can be directly computed by using Mohr's theorems.

Problems

The deck of a prestressed concrete culvert is made up of a slab 500 mm thick. The slab is spanning over 10.4 m and supports a total uniformly distributed load comprising the dead and live loads of 23.5 kN/m2. The modulus of elasticity of concrete is 38 kN/mm2. The concrete slab is prestressed by straight cables each containing 12 high-tensile wires of 7 mm diameter stressed to 1200 N/mm2 at a constant eccentricity of 195 mm. The cables are spaced at 328 mm intervals in the transverse direction. Estimate the instantaneous deflection of the slab at centre of span under prestress and the imposed loads

Solution.

Considering 1 m width of cable, the properties of the cross-section are computed.

Thickness of slab = d = 500 mm Width of slab = b = 1000 mm

Span of the slab = L = 10.4 m

Second moment of area = $I = \left(\frac{bd^3}{12}\right) = \left(\frac{1000 \times 500^3}{12}\right) = (1041 \times 10^7)\,\text{mm}^4$

Force in each cable = $\left(\frac{12 \times 38.5 \times 1200}{1000}\right)$ = 554 kN

Spacing of cables in the transverse direction = 328 mm

Hence, the prestressing force per metre width of slab is computed as,

$P = \left(\frac{1000 \times 554}{328}\right)$ = 1689 kN Eccentricity = e = 195 mm

Total uniformly distributed load on the beam

$= w = 33.5$ kN/m = 0.0335 kN/mm

Deflection due to prestressing force = $a_p = \left(\frac{-PeL^2}{8EI}\right)$

$$= -\left(\frac{1689 \times 195 \times (10.4 \times 1000)^2}{8 \times 38 \times 1041 \times 10^7}\right)$$

= –11.25 mm (Upwards)

Deflection due to loads = $a_L = \left(\frac{5wL^4}{384EI}\right) = \left(\frac{5 \times 0.0335 \times (10.4 \times 1000)^4}{384 \times 38 \times 1041 \times 10^7}\right)$

= 12.90 mm (downwards)

Resultant deflection = (12.90 – 11.25) = 1.65 mm (downwards).

Codes of Practice

It is the general practise in most of the codes to safeguard against excessive deflections under serviceability limit states, either indirectly by prescribing a minimum span to depth ratio for the member, or directly by specifying a maximum permissible deflection expressed as a fraction of the span.

The recommendations of the Indian Standard Code (IS: 1343)20 with regard to the limit state of deflection are as follows:

The final deflection, due to all loads including the effects of temperature, creep and shrinkage should normally not exceed span/250.

A prestressed concrete beam of rectangular section 120 mm wide and 300 mm deep, spans over 6 m. The beamis prestressed by a straight cable carrying an effective force of 180 kN at aneccentricity of 50 mm. If it supports an imposed load of 4 kN/m and the modulus of elasticity of concrete is 38 kN/mm2, compute the deflection at the following stages and check whether they comply with the IS code specifications:

a)Upward deflection under (prestress + self-weight).

b)Final downward deflection under (prestress + self-weight + imposed load) including the effects of creep and shrinkage. Assume the creep coefficient to be 1.80.

Solution.

$P = 180$ kN — Self-weight, $g = 0.86$ N/mm

$e = 50$ mm — Imposed load, $q = 4$ N/mm

$I = (27 \times 10^7)$ mm^4 — $E_c = 38$ kN/mm^2

$L = 6000$ mm

Deflection due to the prestressing force

$$= \left(\frac{PeL^2}{8EI}\right) = \left(\frac{180 \times 50 \times 6000^2}{8 \times 38 \times 27 \times 10^7}\right) = 4.0 \text{ mm (upward)}$$

Deflection due to the self-weight of the beam

$$= \left(\frac{5gL^4}{384EI}\right) = \left(\frac{5 \times 0.86 \times 6000^4}{384 \times 38 \times 10^3 \times 27 \times 10^7}\right) = 1.4 \text{ mm}$$

Deflection due to self-weight and live load

$$= \left[\frac{5(g+q)L^4}{384EI}\right] = \left[\frac{5 \times 4.86 \times 6000^4}{384 \times 38 \times 10^3 \times 27 \times 10^7}\right] = 8.0 \text{ mm}$$

(a) Deflection due to (prestress + self-weight) $= (-4.0 + 1.4) = -2.6$ mm

Permissible upward deflection as per draft IS: 1343

$$= \left(\frac{\text{span}}{300}\right) = \left(\frac{6000}{300}\right) = 20 \text{ mm}$$

Hence, the upward deflection is within permissible limits.

(b) Deflection due to (prestress + self-weight + live load) including effects of creep and shrinkage

$$= (-4.0 + 8.0)(1 + \phi) = (4.0)(1 + 1.8) = 11.2 \text{ mm}$$

Permissible downward deflection as per IS: 1343

$$= \left(\frac{\text{span}}{250}\right) = \left(\frac{6000}{250}\right) = 24 \text{ mm}$$

Hence, the final downward deflection is within permissible limits.

Example 6.8 A concrete beam with a symmetrical I-section has flange width and depth of 200 mm and 60 mm, respectively. The thickness of the web is 80 mm and the overall depth is 400 mm. The beam is prestressed by a cable carrying a force of 1000 kN. The span of the beam is 8 m. The centre line of the cable is 150 mm from the soffit of the beam at the centre of span, linearly varying to 250 mm at the supports. Compute the initial deflection at mid-span due to prestress and the self-weight of the beam, assuming $E_c = 38\ \text{kN/mm}^2$. Compare the deflection with the limiting deflection permitted in IS: 1343 ($D_c = 24\ \text{kN/m}^3$).

Solution.

Self-weight of the beam, $g = 1.12$ kN/m $= 0.00112$ kN/mm

$$\text{Prestressing force } P = 1000\ \text{kN},\ I = (847 \times 10^6)\ \text{mm}^4$$

$$e_1 = 50\ \text{mm},\ e_2 = 50\ \text{mm},\ L = 8000\ \text{mm}$$

Deflection due to self-weight

$$= \left(\frac{5 \times 0.00112 \times 8000^4}{384 \times 38 \times 847 \times 10^6}\right) = 1.86\ \text{mm (downward)}$$

Deflection due to the prestressing force

$$= \left[\frac{Pe_2 L^2}{8EI} - \frac{P(e_1 + e_2)L^2}{12EI}\right]$$

$$= \left(\frac{1000 \times 8000^2}{38 \times 847 \times 10^6}\right)\left[\frac{50}{8} - \frac{(50 + 50)}{12}\right]$$

$$= -4.1\ \text{mm (upward)}$$

$\therefore$ Deflection due to (prestress + self-weight)

$$= (-4.1 + 1.86) = -2.24\ \text{mm (upward)}$$

Maximum permissible deflection according to IS: 1343

$$= \left(\frac{\text{span}}{300}\right) = \left(\frac{8000}{300}\right) = 26.6\ \text{mm (upward)}$$

Hence, the actual deflection is within permissible limits.

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UNIT – IV – COMPOSITE BEAMS AND CONTINUOUS BEAMS – SCI1402

Transmission of Prestressing force by bond

In a pretensioned system, when a wire is released from its temporary anchorage on the prestressing bed, the end of the wire swells as a result of the recovery of the lateral contraction and develops a wedge effect. This is to enable the prestressing force to become zero at the end of the wire. This is generally referred to as the Hoyer Effect[1]. The swelling of the wire is only a few thousandths of a millimetre, but it nevertheless produces considerable radial pressures on the concrete, giving rise to large frictional forces.

The transmission of prestressing force from steel to concrete is generally through a bond comprising (i) adhesion, (ii) friction, and (iii) shearing resistance (dilatancy). At intermediate points along the length of a beam, the bond stress is resisted by adhesion, while in the transfer zone the tendons invariably slip and sink into the concrete, destroying most of the adhesion. Consequently, the bond stresses are due to the friction and shearing resistance. The distribution of bond stress, stress in steel and concrete in the transmission zone are shown in Fig. 9.1. The maximum bond stress is reached in the zone of transverse compression. When the bond stress is zero, the stress in steel and concrete reach their maximum values, and uniform stress distribution is prevalent from this section. The length needed for achieving this is termed as *transmission length*.

Transmission Length

The length required at the ends of a pretensioned member for the build-up of stress in concrete is of great importance, particularly in short pretensioned units, since it controls the working bending moment and the shear force allowable on the section. The transmission length depends mainly on the diameter and surface characteristics of the wire, the elastic properties of steel and concrete, and the coefficient of friction between steel and concrete.

Based on the wedge action, Hoyer developed an expression for computing the transmission length, which is given by,

$$L_t = \frac{\phi}{2\mu}(1+v_c)\left(\frac{\alpha_e}{v_s} - \frac{f_{pi}}{E_c}\right)\left(\frac{f_{pe}}{2f_{pi} - f_{pe}}\right)$$

where L_t = transmission length
ϕ = wire diameter
μ = coefficient of friction between steel and concrete
v_c = Poisson's ratio for concrete
v_s = Poisson's ratio for steel
α_e = modular ratio (E_s/E_c)
E_c = modulus of elasticity of concrete
f_{pi} = initial stress in steel
f_{pe} = effective stress in steel

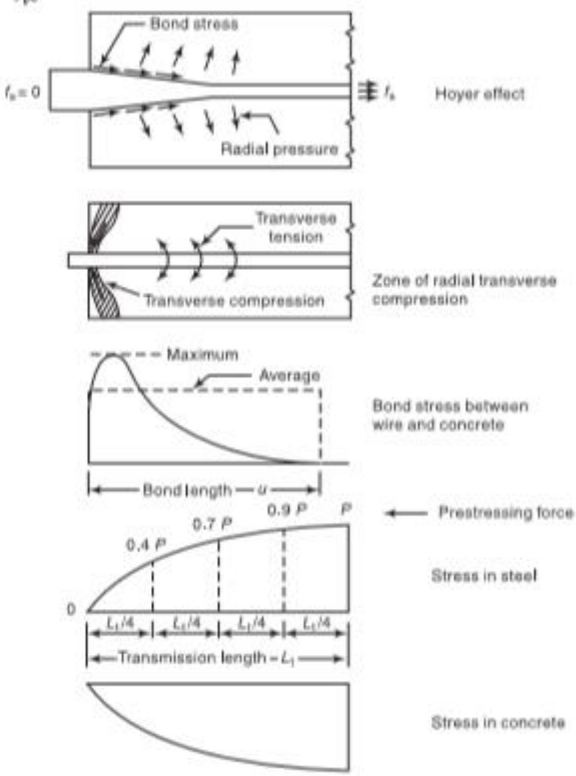

Fig. 9.1 *Distribution of bond stresses*

Example 9.4 A pretensioned beam of 8 m span has a symmetrical I-section. The flanges are 200 mm wide and 60 mm thick. The web thickness is 80 mm and the overall depth of girder is 400 mm. The member is prestressed by 8 wires of 5 mm diameter located on the tension side such that the effective eccentricity is 90 mm. The initial stress in the wires is 1280 N/mm^2 and the cube strength of concrete at transfer is 42 N/mm^2.

(a) Determine the maximum vertical tensile stress developed in the transfer zone.

(b) Design suitable mild steel reinforcement, assuming the permissible stress in steel as 140 N/mm^2.

Solution. The distribution of 5 mm diameter high-tensile wires in the cross-section is shown in Fig. 9.4, along with the longitudinal stress distribution at a distance equal to the transmission length from the end face of the member.

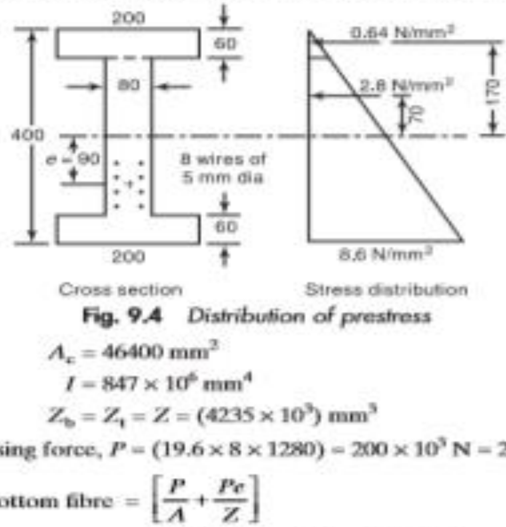

Fig. 9.4 *Distribution of prestress*

$$A_c = 46400 \text{ mm}^2$$

$$I = 847 \times 10^6 \text{ mm}^4$$

$$Z_b = Z_t = Z = (4235 \times 10^3) \text{ mm}^3$$

Total prestressing force, $P = (19.6 \times 8 \times 1280) = 200 \times 10^3 \text{ N} = 200 \text{ kN}$

$$\text{Stress at the bottom fibre} = \left[\frac{P}{A} + \frac{Pe}{Z}\right]$$

$$= \left[\frac{200 \times 10^3}{464 \times 10^2} + \frac{200 \times 10^3 \times 90}{4235 \times 10^3}\right]$$

$$= 8.6 \text{ N/mm}^2 \text{ (compression)}$$

$$\text{Stress at the top fibre} = \left[\frac{P}{A} - \frac{Pe}{Z}\right] = \left[\frac{200 \times 10^3}{464 \times 10^3} - \frac{200 \times 10^3 \times 90}{4235 \times 10^3}\right] = 0$$

Taking into consideration all the forces above the centroidal axis of the section, External moment due to prestressing force = 0

The internal moment due to the distribution of prestress developed is obtained as,

$$M = [(200 \times 60 \times 0.64 \times 170) + (140 \times 80 \times 2.8 \times 70)] = 351 \times 10^4 \text{ N mm}$$

Transmission length,

$$L_t = \sqrt{\frac{\sqrt{f_{cu}} \times 10^3}{0.0235}} \text{ for 5 mm-diameter wires}$$

$$L_t = \sqrt{\frac{\sqrt{42} \times 10^3}{0.0235}} = 525 \text{ mm}$$

Maximum vertical tensile stress near the end face

$$= \left(\frac{10M}{b_w h L_t}\right) = \left(\frac{10 \times 351 \times 10^4}{80 \times 400 \times 525}\right) = 2.09 \text{ N/mm}^2$$

Area of vertical reinforcement A_{sv}

$$= \left(\frac{2.5M}{f_s h}\right) = \left(\frac{2.5 \times 351 \times 10^4}{140 \times 400}\right) = 158 \text{ mm}^2$$

Provide three bars of 6 mm-diameter stirrups (two-legged) in the transfer zone.

Example 9.5 A post-tensioned prestressed concrete rectangular beam, 240 mm wide and 500 mm depth, is grouted before the application of live loads. The steel consists of three tendons, each made up of 12 numbers of 7 mm-diameter wires encased in a thin metallic hose of 30 mm diameter with an effective cover of 50 mm. The modulus of elasticity of steel and concrete are 210 and 35 kN/mm^2, respectively. The beam spans 10 m and supports two concentrated loads of 250 kN each at the third points. Compute the unit bond stress,

(a) between each wire and grout; and
(b) between the hose and the concrete

Solution.

Maximum shear force in the beam, $V = 250$ kN

Second moment of area, $I = \left(\frac{(240 \times 500^3}{12}\right) = (25 \times 10^8) \text{ mm}^4$

Modular ratio, $\alpha_e = (E_s/E_c) = 6$

$y = 200$ mm

(a) Bond stress between each wire and grout is given by,

$$= \left(\frac{Vy\alpha_e \Phi}{4I}\right) = \left(\frac{250 \times 10^3 \times 200 \times 6 \times 7}{4 \times 25 \times 10^8}\right) = 0.21 \text{ N/mm}^2$$

(b) Bond stress between the hose and the concrete is calculated as,

Area of steel in one hose, $A_s = (12 \times 38.5) = 462 \text{ mm}^2$

Hose diameter = 30 mm

Hose circumference = $(\pi \times 30) = 94$ mm

Bond stress between the hose and the concrete

$$= \left(\frac{V\alpha_e A_s y}{\Sigma u I}\right) = \left(\frac{250 \times 10^3 \times 6 \times 462 \times 200}{94 \times 25 \times 10^8}\right) = 0.59 \text{ N/mm}^2$$

Example 10.1 The end block of a prestressed concrete beam, rectangular in section, is 100 mm wide and 200 mm deep. The prestressing force of 100 kN is transmitted to concrete by a distribution plate, 100 mm wide and 50 mm deep, concentrically located at the ends. Calculate the position and magnitude of the maximum tensile stress on the horizontal section through the centre and edge of the anchor plate. Compute the bursting tension on these horizontal planes.

Solution. Given data:

$$P = 100 \text{ kN}$$
$$h = 200 \text{ mm}$$
$$b = 100 \text{ mm}$$

Direct stress, $f_h = \left(\dfrac{100 \times 10^3}{200 \times 100}\right) = 5 \text{ N/mm}^2$

Normally, the vertical stress f_v and the principal tensile stress are critical at $x = 0.5\,h$. Referring to Fig. 10.5,

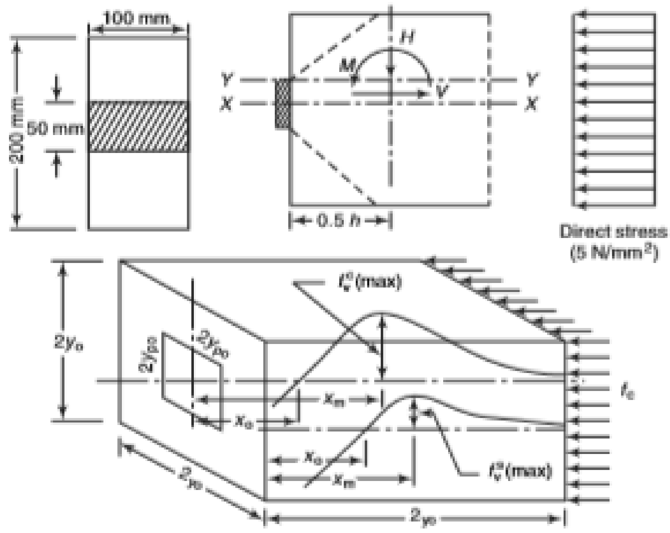

Fig. 10.5 *Forces acting on the end block*

For section XX

At $\dfrac{x}{h} = 0.5$ from Table 10.1

$$K_1 = -5.00$$
$$K_2 = 2.00$$
$$K_3 = 1.25$$

$$M = \left[(5 \times 100 \times 100)\left(\frac{100}{2}\right) - \left(\frac{100 \times 10^3}{2}\right)\left(\frac{50}{4}\right)\right] = 1875 \times 10^3 \text{ N mm}$$

$$V = 0 \text{ and } H = 0$$

$$\therefore \quad f_v = -5\left(\frac{1875 \times 10^3}{100 \times 200^2}\right) = -2.35 \text{ N/mm}^2$$

$$f_h = +5 \text{ N/mm}^2$$

The principal tensile stress (acting at $0.5\,h = 100$ mm from the end) is given by,

$$f_{min} = \left(\frac{5-2.35}{2}\right) - \frac{1}{2}\sqrt{(5+2.35)^2 + 0} = -2.35 \text{ N/mm}^2$$

Therefore, the total splitting tension, assuming parabolic distribution of stress as shown in Fig. 10.6, is given by,

$$F_{bst} = [(2/3 \times 150 \times 2.35)100] = 23500 \text{ N}$$

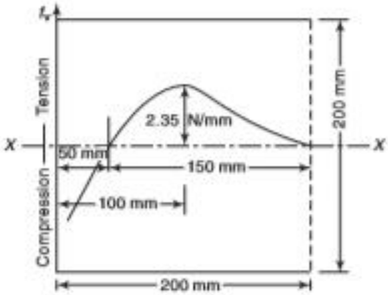

Fig. 10.6 *Distribution of tensile stress*

For section YY (passing through edge of plate)
Stresses at $x = 0.5h = 100$ mm from end

$$M = (100 \times 75 \times 5 \times 75/2) = 14 \times 10^5 \text{ N mm}$$

$$V = -(100 \times 75 \times 5) = -37500 \text{ N (acting towards the end of beam)}$$

$$H = 0$$

$$f_v = -5\left(\frac{14 \times 10^5}{100 \times 200^2}\right) + 0 = -1.75 \text{ N/mm}^2$$

$$\tau = 1.25\left(\frac{-37500}{100 \times 200}\right) = -2.35 \text{ N/mm}^2$$

$$f_h = +5 \text{ N/mm}^2$$

Principal tensile stress

$$= \left(\frac{5-1.75}{2}\right) - \frac{1}{2}\sqrt{(5+1.75)^2 + 4(-2.35)^2} = -2.475 \text{ N/mm}^2$$

Angle of inclination of the plane of principal stress with respect to the vertical plane is,

$$\tan 2\theta = \left(\frac{2\tau}{f_v - f_h}\right) = \left(\frac{-2 \times 2.35}{-1.75 - 5.0}\right) = 0.7$$

$\therefore \quad 2\theta = 35°$ and $\theta = 17.5°$

Tensile stress component in the vertical direction

$$= (2.475 \times \sec 17.5°) = 2.6 \text{ N/mm}^2$$

Bursting tension, $\quad F_{bst} = (2/3 \times 150 \times 2.6)100 = 26000$ N (on axis YY)

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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – V – MISCELLANEOUS STRUCTURES – SCI1402

Prestressed tension member:

Prestressed tension member have the following three specific behaviors',

1.the member can be considered as essentially made of concrete which is put under uniform compression , so that it can carry tension produced by internal pressure or external loads, if the concrete has no cracked , it is able to carry a total tensile force equal to the total effective pre compression plus the tensile capacity of the concrete itself.

2.the member can be considered as essentially made of high tensile steel which is prolonged to reduce the deflection under load, from the view point, the ultimate strength of the member is dependent upon the tensile strength of the steel, but the usable strength is often limited by excessive elongation of the steel following the cracking of the concrete.

3.the member can be considered as a combined steel and concrete member whose strains and stresses before cracking can be evaluated, assuming elastic behavior and taking into account the effect of shrinkage and creep.

The design criteria for prestressed concrete tanks

It is to resist the hoop tension and moments developed are based on the considerations of desirable load factors against cracking and collapse.

It is desirable to have at least a minimum load factor of 1.2 against cracking and 2 againstultimate collapse as per IS code.

It is desirable to have at least a minimum load factor of 1.25 against cracking and 2.5 against ultimate collapse as per BS code.

The principal compressive stress in concrete should not exceed one-third of the characteristic cube strength.

When the tank is full, there should be a residual compressive stress of at least 0.7 N/mm^2. When the tank is empty, the allowable tensile stress at any point is limited to 1 N/mm2. The maximum flexural stress in the tank walls should be assumed to be numerically equal to 0.3 times the hoop compression.

Circular prestressing.

When the prestressed members are curved in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing.

The tanks classified based on the joint.

a.Tank wall with fixed base.

b.Tank wall with hinged base.

c.Tank wall with sliding base.

The different types of joints used between the slabs of prestressed concrete tanks

1.Movement joint

2.Expansion joint

3.Construction Joint

4.Temporary Open Joints.

The stress induced in concrete due to circular prestressing

The circumferential hoop compression stress is induced in concrete by prestressing counterbalances the hoop tension developed due to the internal fluid pressure.

The □ advantages of prestressing water tanks

Water storage tanks of large capacity are invariably made of prestressed concrete.

Square tanks are used for storage in congested urban and industrial sites where

land space is a major constraint.

This shape is considerable reduction in the thickness of concrete shell.

The efficiency of the shell action of the concrete is combined with the prestressing at the edges.

The functions of water stopper (water bar) in water tank construction

The base slab is subdivided by joints which are sealed by water stops.

The reinforcement in the slab should be well distributed to control the cracking of theslab due to shrinkage and temperature.

The design criteria for prestressed concrete pipes

Circumferential prestressing, winding with or without longitudinal prestressing. Handling stresses with or without longitudinal prestressing.

Condition in which a pipe is supported by saddles at extreme points with full water loadbut zero hydrostatic pressure.

Full working pressure conforming to the limit state of serviceability. The first crack stagecorresponding to the limit state of local damage.

prestressed cylinder and non-cylinder pipe.Prestressed cylinder pipe:

It is developed by the Lock Joint Company.

A welded cylinder of 16 gauge steel is lined with concrete inside and steelpipe wrapped with a highly stressed wire.

Tubular fasteners are used for the splices and for end fixing of the wireand pipe is finished with a coating of rich mortar.

It is suitable upto 1.2 m diameter.

Prestressed non-cylinder pipe:

It is developed by Lewiston Pipe Corporation.

At first concrete is cast over a tensioned longitudinal reinforcement.

A concrete pipes after curing are circumferentially stressed by means of a spiral wire.

The types of prestressed concrete pipes construction

Monolyte construction

Two stage construction..

Non-cylinder pipes:

The design principles are used for determining the minimum thickness of concrete required and the pitch of circumferential wire winding on the pipe.

Cylinder pipes:

The design principles of cylinder pipes are similar to those of the non-cylinder pipes except that the required thickness of concrete is computed by considering the equivalent area of the light gauge steel pipe embedded in the concrete.

The advantages of prestressed concrete piles

High load and moment carrying capacity.

Standardization in design for mass production.

Excellent durability under adverse environmental conditions. Crack free characteristicsunder handling and driving.

Resistance to tensile loads due to uplift. Combined load moment capacity.

Prestressed concrete poles:

The effect of prestressing force in concrete poles.

It should be reduced in proportion to the cross section by the techniques of debonding or dead ending or looping some of the tendons at mid height.

The types of loadings that act on prestressed concrete poles.

Bending due to wind load on the cable and on the exposed face. Combined bending and torsion due to eccentric snapping of wires. Maximum torsion due to skew snapping of wires. Bending due to failure of all the wires on one side of the pole. Handing and erection stresses.

The importance of shrinkage in composite construction

The time dependent behavior of composite prestressed concrete beams depends upon the presence of differential shrinkage and creep of the concretes of web and deck, in addition to other parameters, such as relaxation of steel, presence of untensioned steel, and compressionsteel etc.

The advantages of partially prestressed concrete poles

Resistance to corrosion in humid and temperature climate and to erosion in desert areas.

Easy handling due to less weight than other poles.

Easily installed in drilled holes in ground with or without concrete fill.

Lighter because of reduced cross section when compared with reinforced concrete poles.

Fire resisting, particularly grassing and pushing fire near ground line.

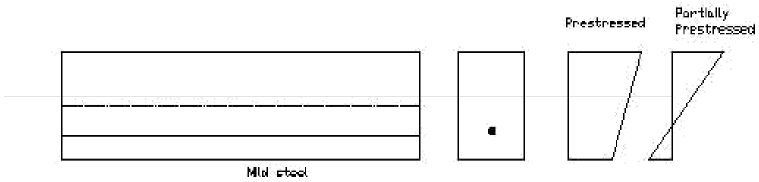
The advantages of prestressed concrete over R.C.C concrete.

The use of high strength concrete and steel in prestressed members results in lighter and slender members than is possible with reinforced concrete.

The effectiveness of carrying external loads is only by the section above the neutral axis is reinforced concrete but the entire cross section is effective isprestressed concrete.

Partial prestressing.

Under the working load, if the cross section is subjected to no tension after prestressing then it is known as fully prestressed. under working loads even after the pretsress is apply. If there is some tension. It is known as partial prestressing . Normally the tension portion is reinforced with mild steel reinforcement. This untensioned reinforcement is required so as to resist differential shrinkage temperature effects and handling stresses.



Limited prestressing

It leads to a reduction in the cost of stressing, sheathing and grouting. The use of high yield strength deformed bars is generally believed to offer better crack control andhigh ultimate strength.

Objectives of partial prestressing:

Partial prestressing may be adopted to serve one or more of the following objectives,

Better distribution of stress

Reinforce regions of peak moment

Control on excessive camber

Control of crack width

Increase of ductility and rotation capacity

Limit state collapse condition

Methods of achieving partial prestressing

Partial prestress may be obtained by any of the following measures,

By using less high tensile steel for prestressing, this will save steel, but will also decrease the ultimate strength, which is almost directly proportional to the amount ofsteel.

By using the same amount of high tensile steel, but leaving some non- prestressed. This will save some tensioning and anchorage, and may increase resilience at the sacrifice of earlier cracking and slightly smaller ultimate strength.

By using same amount of steel , but tensioning them to a lower level. The effects of this are similar to those, but no end anchorage are saved.

By using less prestressed steel and adding some mild steel for reinforcing. This will give the desired ultimate strength and will result in greater resilience at the expense of earlier cracking.

Merits

Camber of bridge deck is better controlled

Savings in the amount of prestressing steel.

Greater resilience in the structure is possible.

Economical utilization of mild steel.

Demerits

Earlier appearance of cracks

Greater deflections under overloads

Higher principal tensile stress under working loads.

Slight decrease in ultimate flexural strength for the same amount of steel.

<u>The advantages of partial prestressing.</u>

Limited tensile stresses are permitted in concrete under service loads with controls on the maximum width of cracks and depending upon the type of prestressing and environmental condition.

Untensioned reinforcement is required in the cross-section of a prestressed member for various reasons, such as to resist the differential shrinkage, temperature effects and handling stresses.