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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – I – STRUCTURAL ANALYSIS II – SCI1307

UNIT I

ROLLING LOADS

Rolling loads are those loads which roll over the given structural element from one endto another. You can see many live examples of rolling loads, like a train on the railway track, vehicles on the bridges or roads are rolling loads.

- The maximum moment occurs at a point of zero shears.
- For beams loaded with concentrated loads, the point of zero shears usually occurs under a concentrated load and so the maximum moment.
- Beams and girders such as in a bridge or an overhead crane are subject to moving concentrated loads, which are at fixed distance with each other.
- The problem here is to determine the moment under each load when each load is in a position to cause a maximum moment. The largest value of these moments governs the design of the beam

Now to analyze the given structural element for the rolling loads, which classify these rollingloads into the following classes:

(1) Single point rolling loads

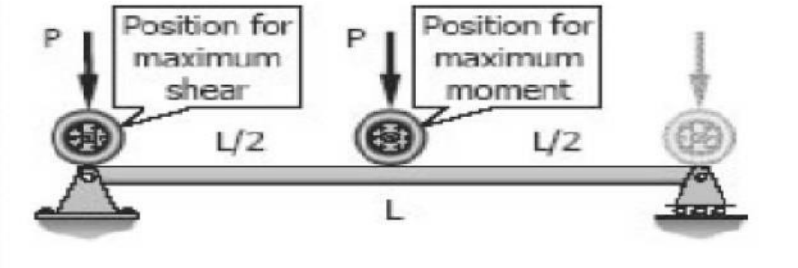
(2) Uniformly distributed rolling loads, - a) shorter than span b) Longer than span.

(3) Two point loads at a fixed distance apart.

(4) Several point loads at fixed distance apart.

- **Single point load:** If a single point concentrated load moves from one end of a girder toanother end of it, it becomes necessary to find out the maximum values of the shear forces and bending moments at every section of the girder to produce an economical and safe design.

Now if we analyse the girder then you will find that at any section on the girder ***maximumnegative shear*** force induces when the point load is just on the left of that section, and maximum.



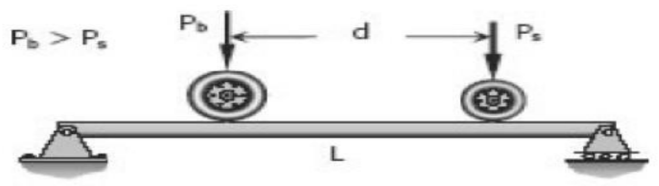
$$M_{max} = \frac{PL}{\;} \text{ and } V_{max} = P$$

Positive shear force is induced if the load is just on the right of the section.

Absolute maximum negative shear force is produced at the right end of the girder and absolutemaximum shear force is produced at the left end of the section.

Bending moment is positive for any position of the load, but maximum bending moment at a section occurs when the load is on the section itself and ***absolute maximum bending moment*** isproduced at the central section when load is also at the center.

- For two moving loads, the maximum shear occurs at the reaction when the larger load is over that support.
- The max moment is given as above.
- Where Ps is the smaller load, Pb is the bigger load, and P is the total load
- (P = Ps + Pb).

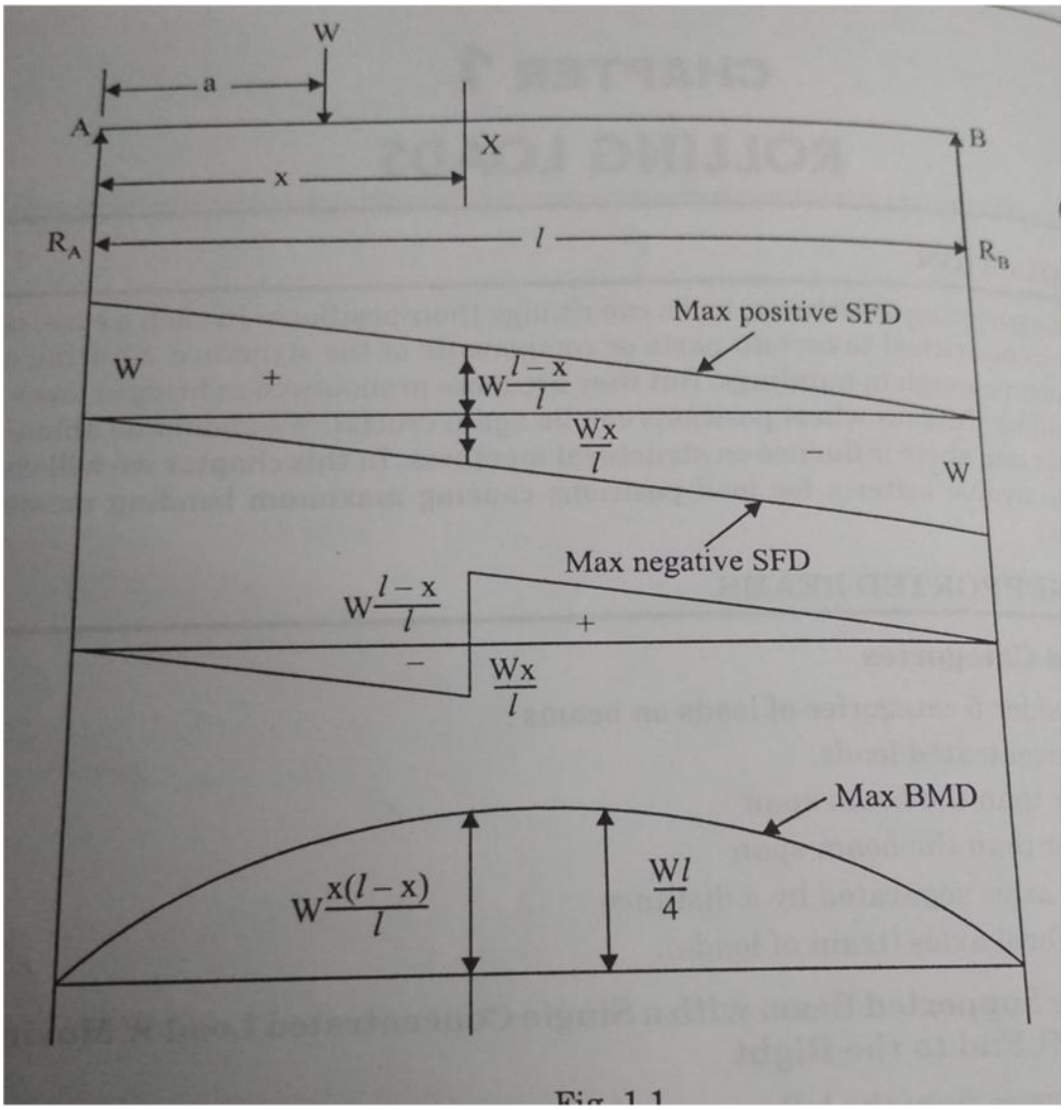


$$M_{max} = \frac{(PL - P_s d)^2}{4PL}$$

Three moving load

- In general, the bending moment under a particular load is a maximum when the center of the beam is midway between that load and the resultant of all the loads then on the span.
- With this rule, we compute the maximum moment under each load, and use the biggest of the moments for the design. Usually, the biggest of these moments occurs under the biggest load.
- The maximum shear occurs at the reaction where the resultant load is nearest. Usually, it happens if the biggest load is over that support and as many a possible of the remaining loads are still on the span.
- In determining the largest moment and shear, it is sometimes necessary to check the condition when the bigger loads are on the span and the rest of the smaller loads are outside.

SIMPLY SUPPORTED BEAM WITH A SINGLE CONCENTRATED LOAD W MOVING FROM THE LEFT END TO RIGHT



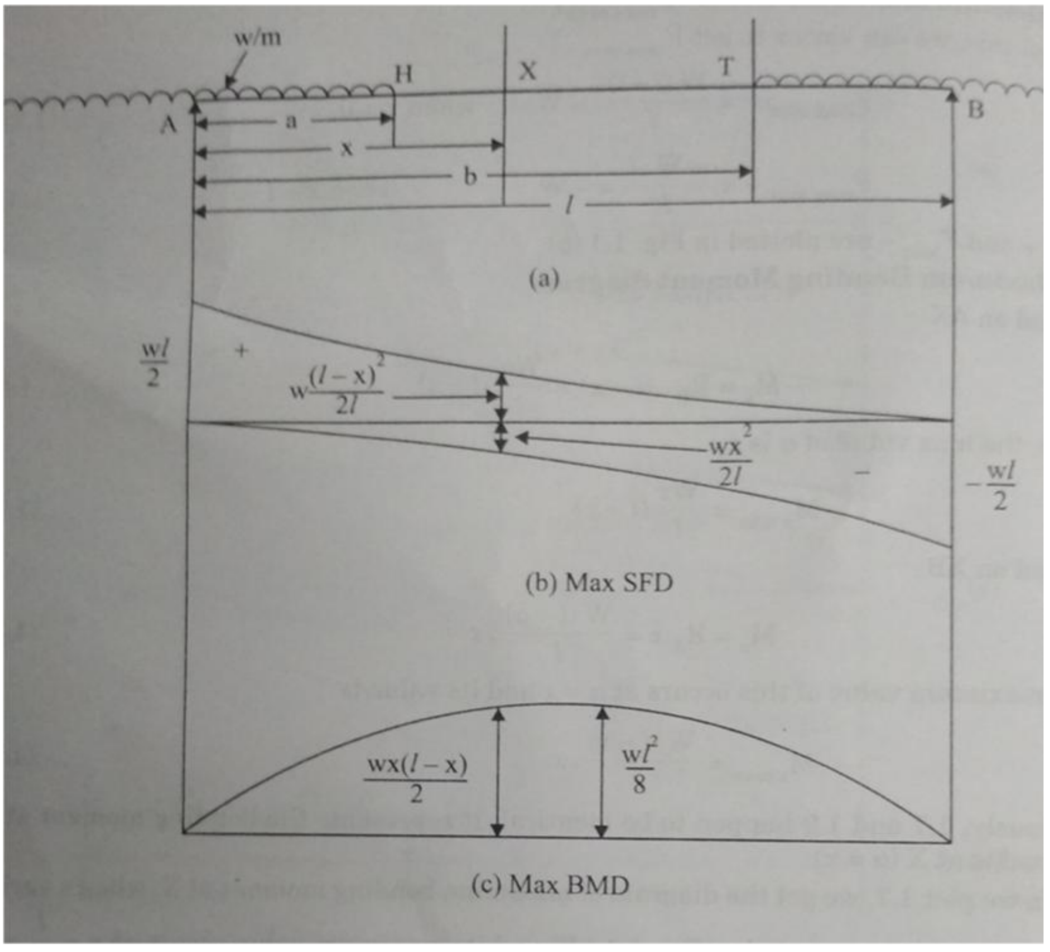
- Uniformly distributed Loads:

(a) Longer than span: When the given udl(uniformly distributed load) is longer than the given span, it is easy to find out the maximum negative shear force , positive shear force and bending moment values on the section.
Maximum negative shear force is induced when the udl is on the left part of the span and right end of the udl is just on the section itself.

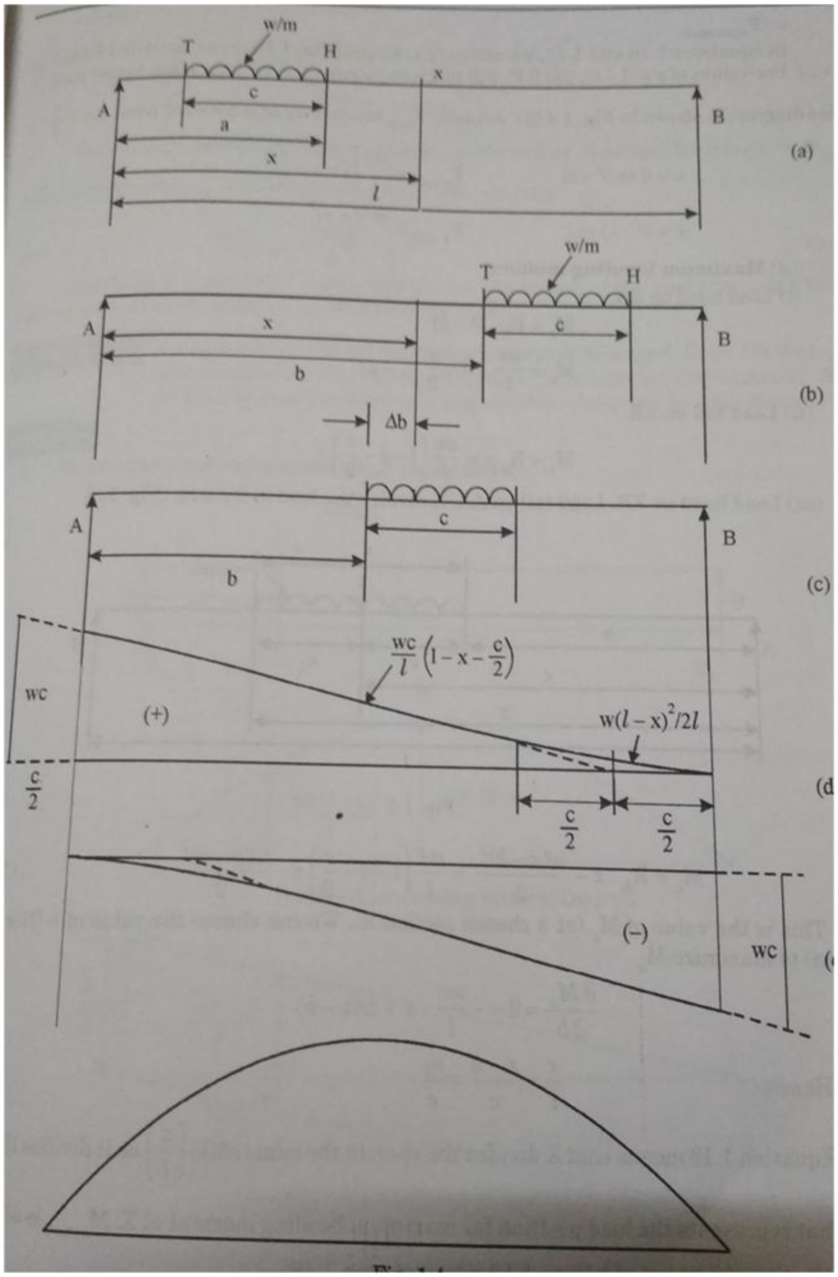
So, maximum negative shear force is induced at the right most end of the span when the end of the load is just to the left of the section.

Similarly, ***maximum positive shear*** force is induced at the left end of the girder when left end of the load is just on the right of the left end.

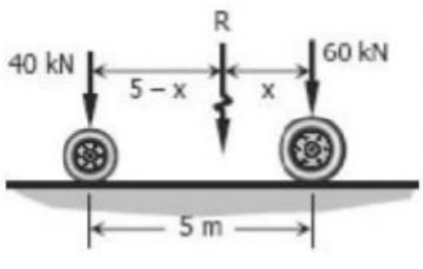
Maximum bending moment at a section is induced when whole of the span is loaded with the udl, and the value of the ***absolute maximum bending moment*** is induced at the center of the span and its value is given by (w.l^2)/8.



SIMPLY SUPPORTED BEAMS WITH A MOVING UDL SHORTER THAN THE SPAN

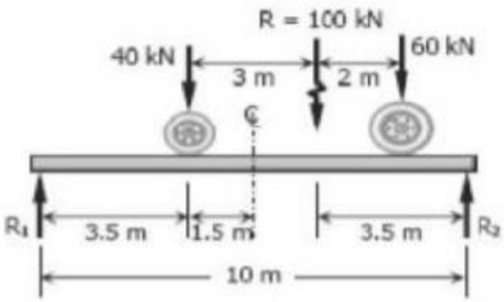


Problem: A truck with axle loads of 40 kN and 60 kN on a wheel base of 5 m rolls across a 10-m span. Compute the maximum bending moment and the maximum shearing force.



$x = 2$ m

For maximum moment under 40 k



$\Sigma M_{R2} = 0$

$10R_1 = 3.5(100)$

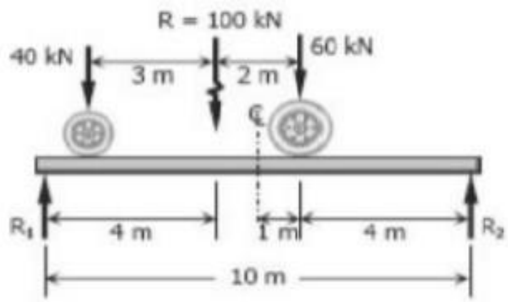
$R_1 = 35$ kN

$M_{\text{To the left of 40 kN}} = 3.5R_1$

$M_{\text{To the left of 40 kN}} = 3.5(35)$

$M_{\text{To the left of 40 kN}} = 122.5$ kN·m

For maximum moment under 60 kN wheel:



$\Sigma M_{R1} = 0$

$10R_2 = 4(100)$

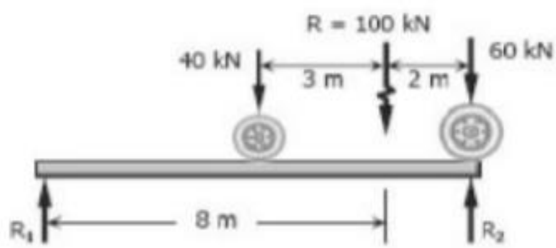
$R_2 = 40$ kN

$M_{\text{To the right of 60 kN}} = 4R_2$

$M_{\text{To the right of 60 kN}} = 4(40)$

$M_{\text{To the right of 60 kN}} = 160$ kN·m

Thus, $M_{max} = 160$ **kN·m**



The maximum shear will occur when the 60 kN is over a support.

$\Sigma M_{R1} = 0$

$10R_2 = 100(8)$

$R_2 = 80$ kN

Thus, $V_{max} = 80$ **kN**

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UNIT – II – STRUCTURAL ANALYSIS II – SCI1307

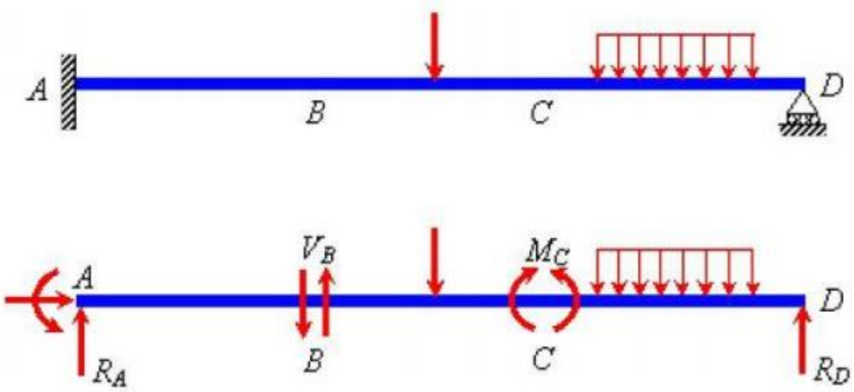
UNIT-II

INFLUENCE LINESDETERMINATE AND NDETERMINATESTRUCTURES

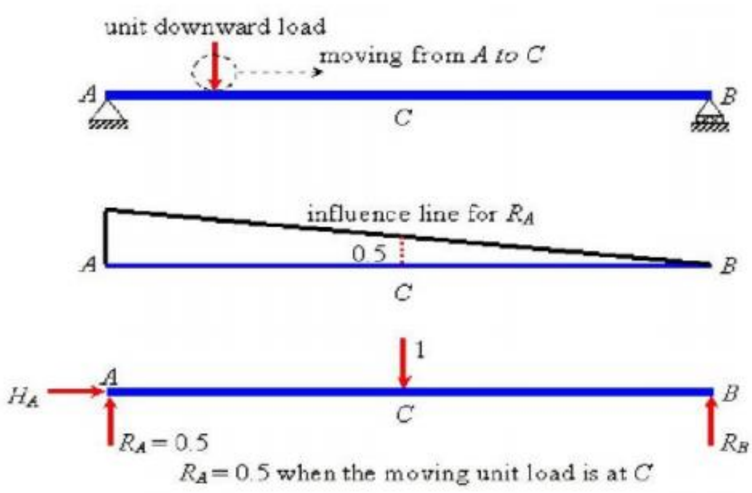
2.1 Definitions of influence line

- An influence line is a diagram whose ordinates, which are plotted as a function of distance along the span, give the value of an internal force, a reaction, or a displacement at a particular point in a structure as a unit load move across the structure.
- An influence line is a curve the ordinate to which at any point equals the value of some particular function due to unit load acting at that point.
- An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a unit concentrated force moves over the member.

For example, we can construct influence lines for (shear force at B) or (bending moment at) or (vertical reaction at support D) and each one will help us calculate the corresponding response parameter for different sets of loading on the beam AD (Figure 2).



An influence line is a diagram which presents the variation of a certain response parameter due to the variation of the position of a unit concentrated load along the length of the structural member. Let us consider that a unit downward concentrated force is moving from point A to point B of the beam shown in Figure 3a. We can assume it to be a wheel of unit weight moving along the length of the beam. The magnitude of the vertical support reaction at A will change depending on the location of this unit downward force. The influence line for (Figure3b) gives us the value of for different locations of the moving unit load. From the ordinate of the influence line at C, we can say that when the unit load is at point C.

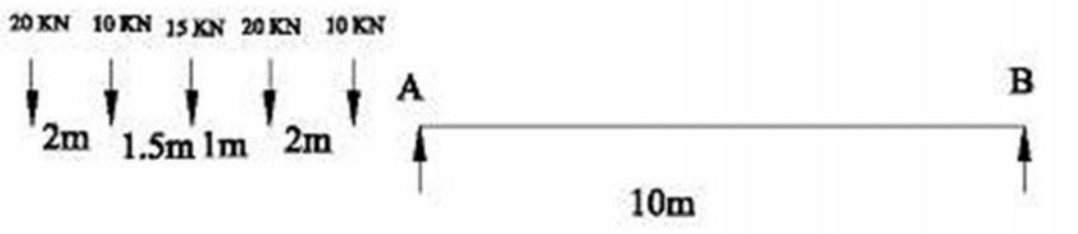


Thus, an influence line can be defined as a curve, the ordinate to which at any abscissa gives the value of a particular response function due to a unit downward load acting at the point in the structure corresponding to the abscissa. The next section discusses how to construct influence lines using methods of equilibrium.

2 Construction of Influence Lines using Equilibrium Methods The most basic method of obtaining influence line for a specific response parameter is to solve the static equilibrium equations for various locations of the unit load. The general procedure for constructing an influence line is described below. 1. Define the positive direction of the response parameter under consideration through a free body diagram of the whole system. 2..For a particular location of the unit load, solve for the equilibrium of the whole system and if required, as in the case of an internal force, also for a part of the member to obtain the response parameter for that location of the unit load.This gives the ordinate of the influence line at that particular location of the load. 3. Repeat this process for as many locations of the unit load as required to determine the shape of the influence line for the whole length of the member. It is often helpful if we can consider a generic location (or several locations) x of the unit load. 4. Joining ordinates for different locations of the unit load throughout the length of the member, we get the influence line for that particular response parameter. The following three examples show how to construct influence lines for a support reaction, a shear force and a bending moment for the simply supported beam AB.

Draw the influence line for (vertical reaction at A) of beam AB in Fig.1

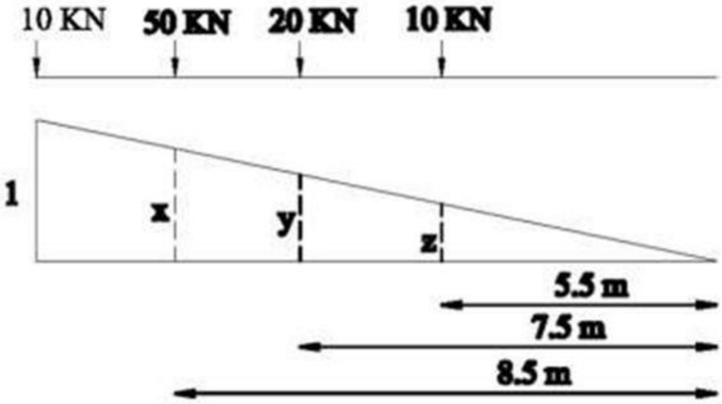
1) *A system of concentrated load, role beam left to right, s.s beam span of 10m and 10 KNload leading*



Find 1.Absolute max +ve S.F

2. .Absolute max -ve S.F

3..Absolute max BM

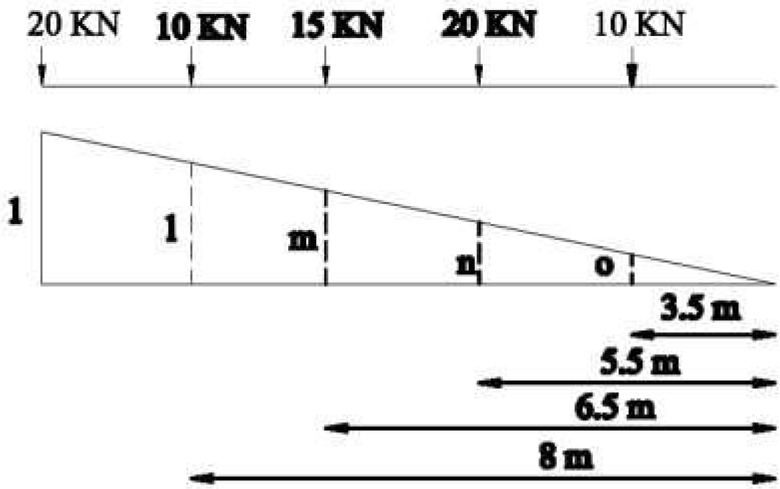


Solution

1. Absolute max +ve S.F

Using the similar triangle method and we get the x, y & z values

X	=	0.85 m
Y	=	0.75 m
Z	=	0.55 m
S.F	=	(10×1)+(15×0.83)+(20×0.75)+(10×0.55)
	=	43.25 KN



Using the similar triangle method and we get the l, m, n & o valuesL=0.8 m

M = 0.65 m

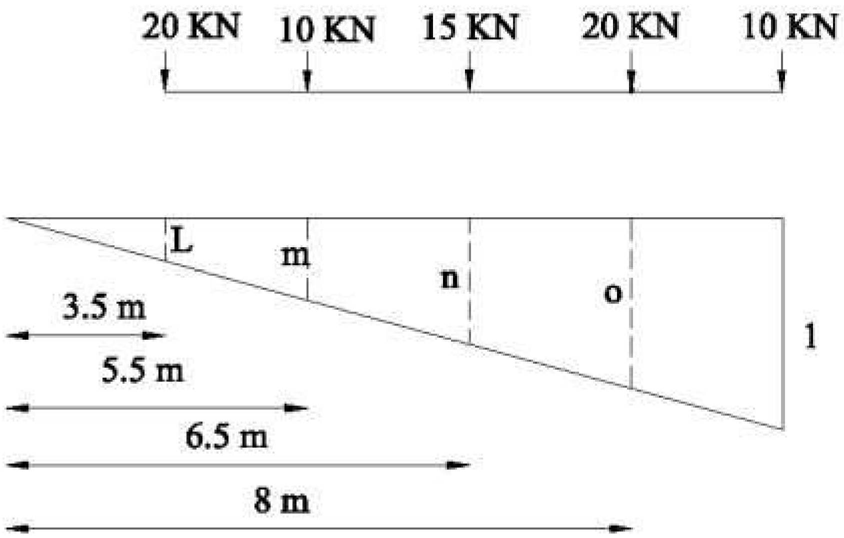
N = 0.55 m

O = 0.35 m

S.F = (20×1)+(10×0.8)+(15×0.65)+(20×0.55)+(10×0.35)

= 52.25 KN

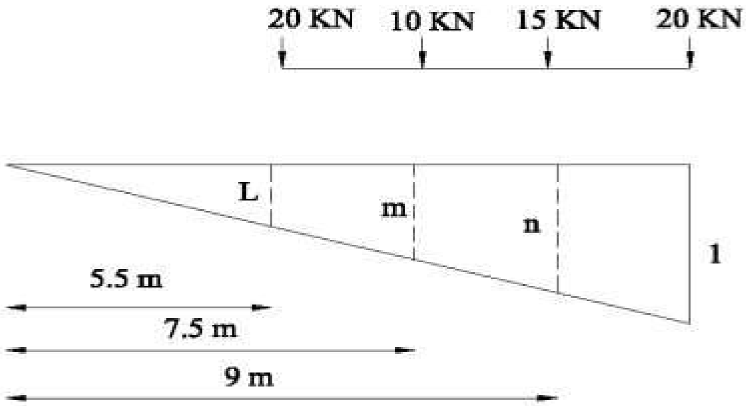
Absolute max -ve S.F



Using the similar triangle method and we get the l, m, n & o values

L = 0.35 m

M	=	0.55 m
N	=	0.7 m
O	=	0.8 m
S.F	=	(10×1)+(20×0.8)+(15×0.7)+(10×0.55)+(20×0.35)
	=	- 49 KN



Using the similar triangle method and we get the l,m, & n valuesL=0.55

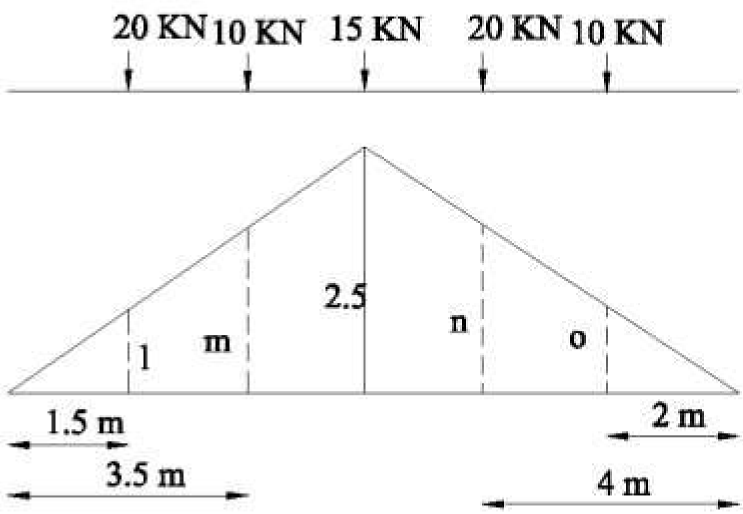
m

M=0.75 m

N=0.85 m

S.F=-((20×1)+(15×0.9)+(10v0.75)+(20×0.55))=-52 KN

(iii) Absolute max BM



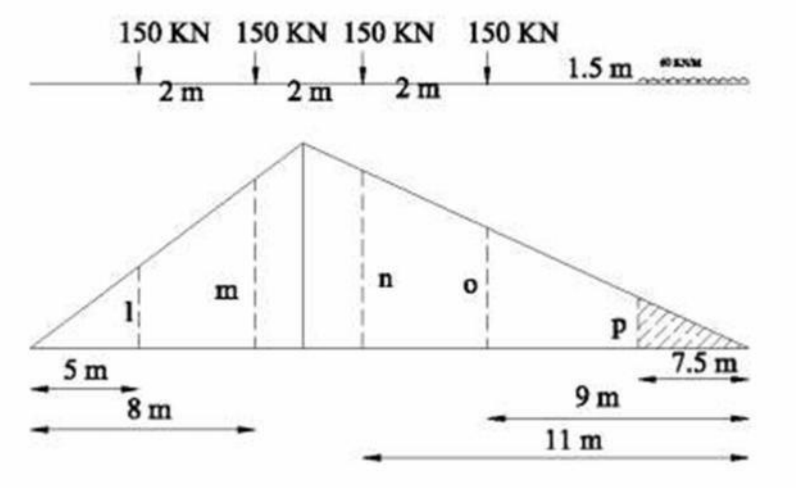
Using the similar triangle method and we get the l, m, n & o values

L	=	0.75 m
M	=	1.75 m
N	=	2 m
O	=	1 m
Max BM	=	(20×0.75) +(10×1.75)+(15×2.5)+(20×2)+(10×1)
	=	22.75 KN

2) *The four equal loads of 150 KN ,each equally spaced at apart 2m and UDL of 60 KN/mat a distance of 1.5m from the last 150 KN loads cross a girder of 20m from span R to L.Using influence line ,calculate the S.F and BM at a section of 8m from L.H.S support when leading of 150KN 5m from L.H.S.*

Solution

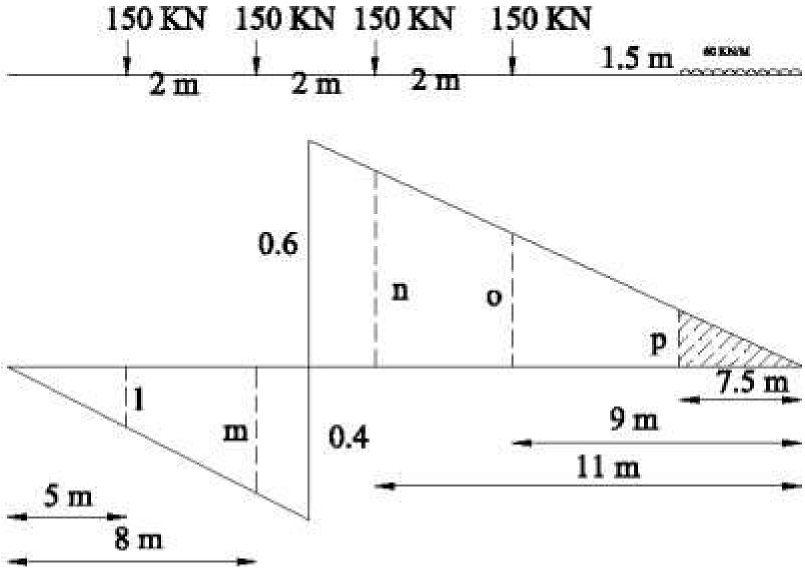
(i) Max BM



L	=	3 m
M	=	4.2 m
N	=	4.4 m
0	=	3.6 m
P	=	3 m
A	=	11.25 m^2
BM	=	(150×3)+(150×4.2)+(150×4.4)+(150×306)+(60×11.25)
	=	2955 KNm

ii) Shear Force

Compute maximum end shear for the given beam loaded with moving loads as shown inFigure



L	=	0.25 m,
M	=	0.3 m,
N	=	0.55 m,
O	=	0.45 m,
P	=	0.375 m
SF	=	((150×0.25)+(150×0.35)+(150×0.55)+(150×0.45)+(60×1.41))
	=	144. KN

Where do you get rolling loads in practice?

Shifting of load positions is common enough in buildings. But they are more pronounced in bridges and in gantry girders over which vehicles keep rolling.

Name the type of rolling loads for which the absolute maximum bending moment occurs atthe midspan of a beam.

Single concentrated load

udl longer than the span

udl shorter than the span

Also when the resultant of several concentrated loads crossing a span, coincides with a concentrated load then also the maximum bending moment occurs at the centre of the span.

What is meant by absolute maximum bending moment in a beam?

When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section.

The maximum of these bending moments will usually occur near or at the midspan.

The maximum of maximum bending moments is called the absolute maximum bending moment.

Where do you have the absolute maximum bending moment in a simply supported beamwhen a series of wheel loads cross it?

When a series of wheel loads crosses a simply supported beam, the absolute maximum bending moment will occur near midspan under the load Wcr , nearest tomidspan (or the heaviest load).

If Wcr is placed to one side of midspan C, the resultant of the load system R shall be on the other side of C; and Wcr and R shall be equidistant from C.

Now the absolute maximum bending moment will occur under Wcr .

If Wcr and R coincide, the absolute maximum bending moment will occur at midspan.

What is the absolute maximum bending moment due to a moving udl longer than the spanof a simply supported beam?

When a simply supported beam is subjected to a moving udl longer than the span, the absolute maximum bending moment occurs when the whole span is loaded.

Mmax max = $wl^2/\,8$

State the location of maximum shear force in a simple beam with any kind of loading.

In a simple beam with any kind of load, the maximum positive shear force occurs at theleft hand support and maximum negative shear force occurs at right hand support.

What is meant by maximum shear force diagram?

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions.

When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram.

This diagram yields the 'design shear' for each cross section.

What is meant by influence lines?

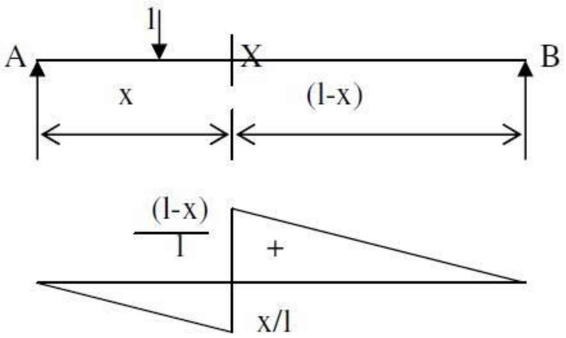
An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

What are the uses of influence line diagrams?

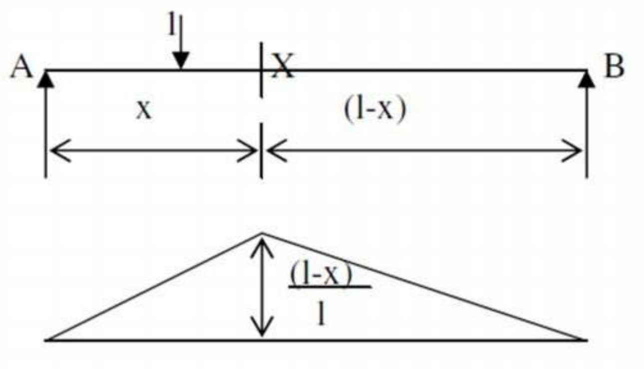
Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

Draw the influence line diagram for shear force at a point X in a simply supported beamAB of span 'l' m.



Draw the ILD for bending moment at any section X of a simply supported beam and mark the ordinates.



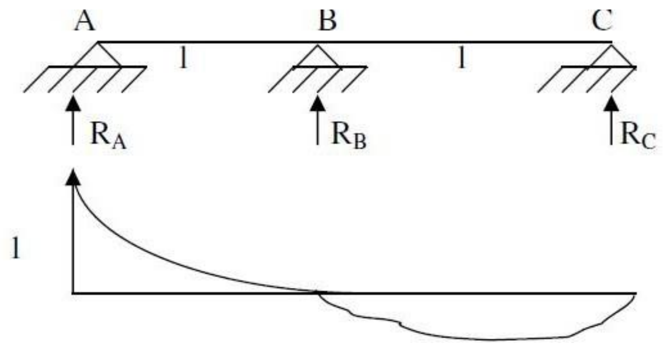
What do you understand by the term reversal of stresses?

- In certain long trusses the web members can develop either tension or compression depending upon the position of live loads.
- This tendancy to change the nature of stresses is called reversal of stresses.

State Muller-Breslau principle.

- Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,
- We remove from the structure the resistant to that force quantity and
- We apply on the remaining structure a unit displacement corresponding to that force quantity.
- The resulting displacements in the structure are the influence line ordinates sought.

State Maxwell-Betti' s theorem.



- In a linearly elastic structure in static equilibrium acted upon by either of two systems of external forces, the virtual work done by the first system of forces in undergoing the displacements caused by the second system of forces is equal to the virtual work done by the second system of forces in undergoing the displacements caused by the first system of forces.
- Maxwell Betti's theorem helps us to draw influence lines for structures.

What is the necessity of model analysis?

- When the mathematical analysis of problem is virtually impossible.
- Mathematical analysis though possible is so complicated and time consuming that the model analysis offers a short cut.
- The importance of the problem is such that verification of mathematical analysis by an actual test is essential.

Define similitude.

- Similitude means similarity between two objects namely the model and the prototype with regard to their physical characteristics:

- Geometric similitude is similarity of form
- Kinematic similitude is similarity of motion
- Dynamic and/or mechanical similitude is similarity of masses and/or forces.

State the principle on which indirect model analysis is based.

- The indirect model analysis is based on the Muller Breslau principle.
- Muller Breslau principle has lead to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.,
- To get the influence line for any force quantity,
    - (i) remove the resistant due to the force,
    - (ii) apply a unit displacement in the direction
    - (iii) plot the resulting displacement diagram.
- This diagram is the influence line for the force.

What is the principle of dimensional similarity?

- Dimensional similarity means geometric similarity of form.
- This means that all homologous dimensions of prototype and model must be in some constant ratio.

What is Begg' s deformeter?

- Begg's deformeter is a device to carry out indirect model analysis on structures.
- It has the facility to apply displacement corresponding to moment, shear or thrust at any desired point in the model.
- In addition, it provides facility to measure accurately the consequent displacements all over the model.

Name any four model making materials.

- Perspex,
- plexiglass,
- acrylic,
- plywood,
- sheet araldite
- bakelite
- Micro-concrete,
- mortar and plaster of paris

What is 'dummy length' in models tested with Begg's deformeter.

- Dummy length is the additional length (of about 10 to 12mm) left at the extremities of the model to enable any desired connection to be made with the gauges.

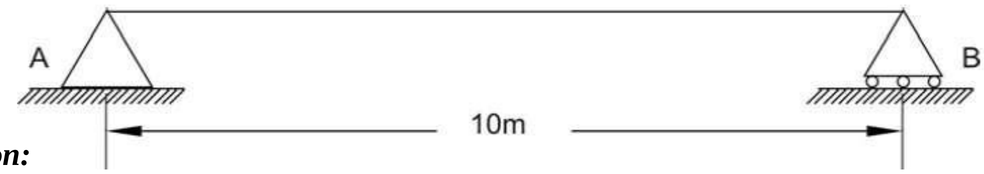
What are the three types of connections possible with the model used with Begg's deformeter.

- Hinged connection
- Fixed connection
- Floating connection

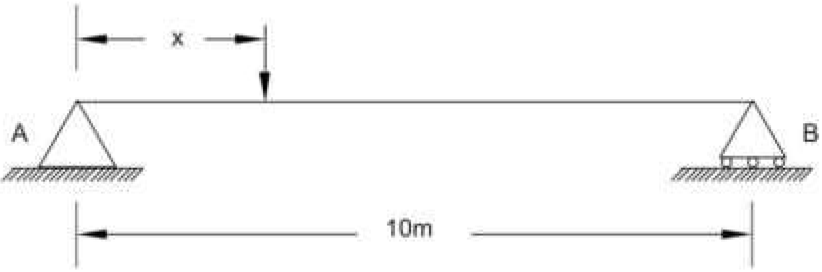
What is the use of a micrometer microscope in model analysis with Begg's deformeter.

- Micrometer microscope is an instrument used to measure the displacements of any point in the x and y directions of a model during tests with Begg's deformeter.

Construct the influence line for the reaction at support B for the beam of span 10 m. Thebeam structure is shown in Figure



Solution:



- A unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A.
- Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows

$$\Sigma MA = 0:$$

$$R_B \times 10 - 1 \times 2.5 = 0 \Rightarrow R_B = 0.25$$

- Similarly, the load can be placed at 5.0, 7.5 and 10 m away from support A and reaction R_B can be computed and ta bulated as given below.

X	**R_B**
0	0
2.5	0.25

5	0.5
7.5	0.75
10	1

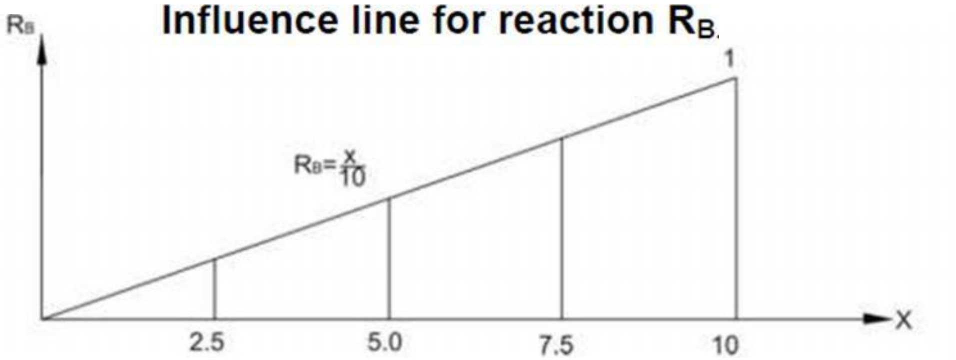
- Graphical representation of influence line for RB is shown in Figure

Influence Line Equation:

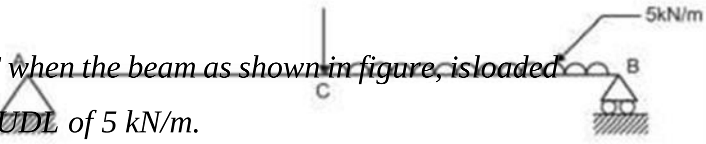
- When the unit load is placed at any location between two supports from support A at distance x then the equation for reaction RB can be written as

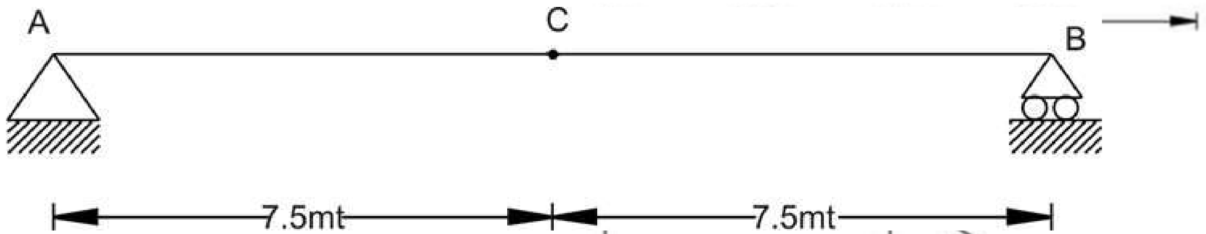
  Σ MA = 0:

  RB x 10 – x = 0 ⇒ RB = x/10



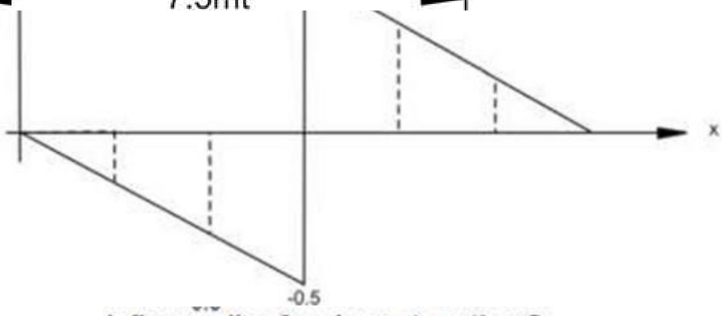
Find the maximum positive live shear at point C when the beam as shown in figure, isloaded with a concentrated moving load of 10 kN and UDL of 5 kN/m.





Concentrated load:

- the maximum live shear force at C will be when the concentrated load 10 kN is located just before C or just after C.
- Our aim is to find positive live shear and hence, we will put 10 kN just after C.
- In that case, Vc = 0.5 x 10 = 5 kN.



Influence line for shear at section C.

UDL:

- the maximum positive live shear force at C willbe when the

UDL 5 kN/m is acting between

x = 7.5 and x = 15.

Vc = [0.5 x (15 –7.5) (0.5)] x 5 = 9.375

Total maximum Shear at C:

(Vc) max = 5 + 9.375 = 14.375.

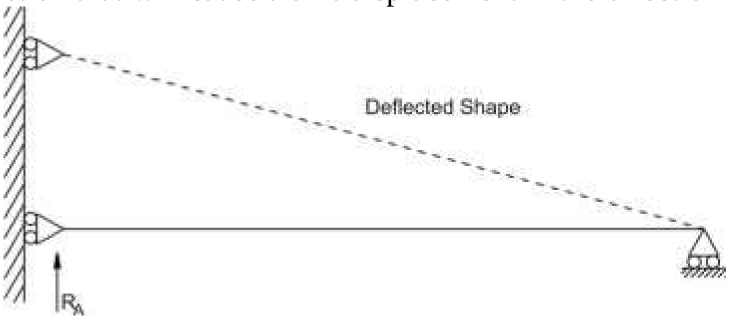
Overhang beam

Muller Breslau Principle for Qualitative Influence Lines

- In 1886, Heinrich Müller Breslau proposed a technique to draw influence lines quickly.
- The Müller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

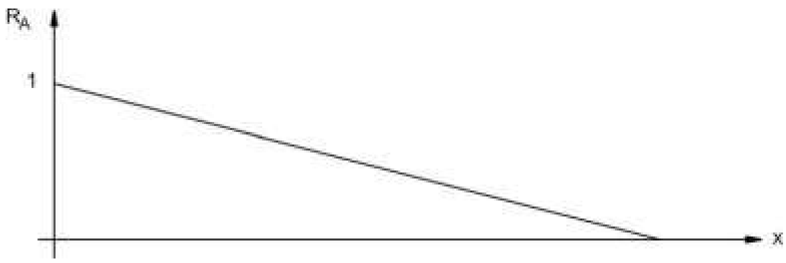
Procedure:

- First of all remove the support corresponding to the reaction and apply a force in the positive direction that will cause a unit displacement in the direction of R_A



Deflected shape of beam

- The resulting deflected shape will be proportional to the true influence line for the support reaction at A.

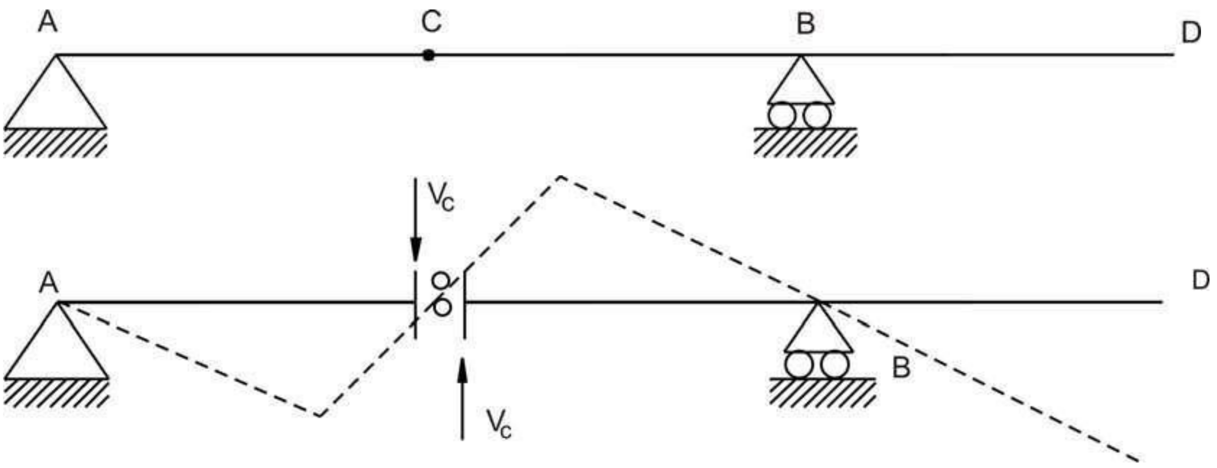


Influence line for support reaction A

- The deflected shape due to a unit displacement at A is shown in above Figure:1 and matches with the actual influence line shape as shown in Figure 3.

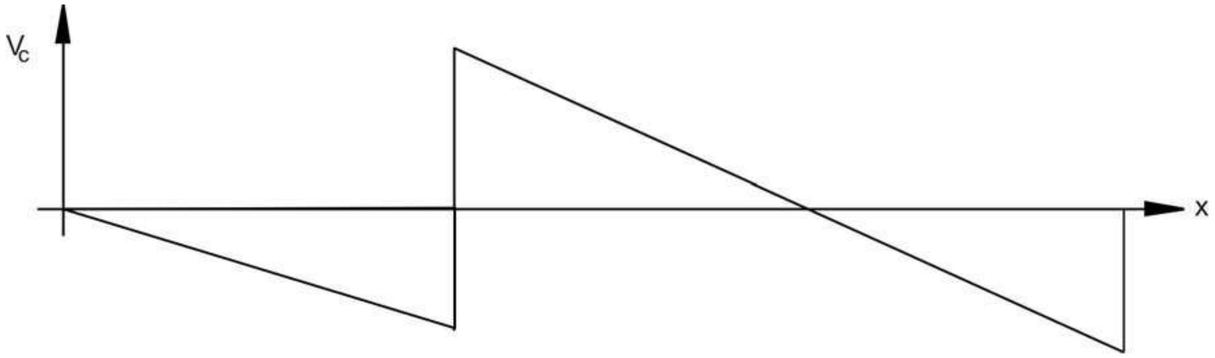
✸ Note that the deflected shape is linear, i.e., the beam rotates as a rigid body without any curvature. This is true only for statically determinate systems.

Overhang beam



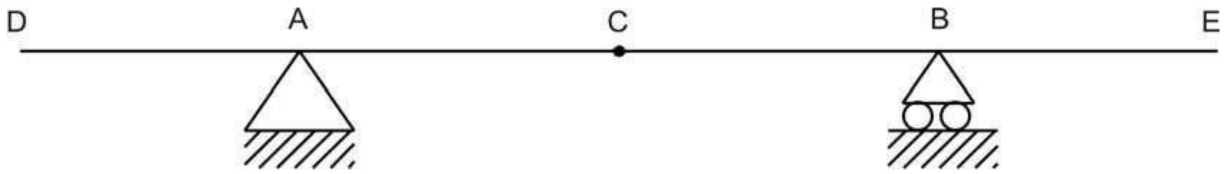
Deflected shape of beam

- Now apply a force in the positive direction that will cause a unit displacement in the direction of V_C.
- The resultant deflected shape is shown above Figure. Again, note that the deflected shape is linear.



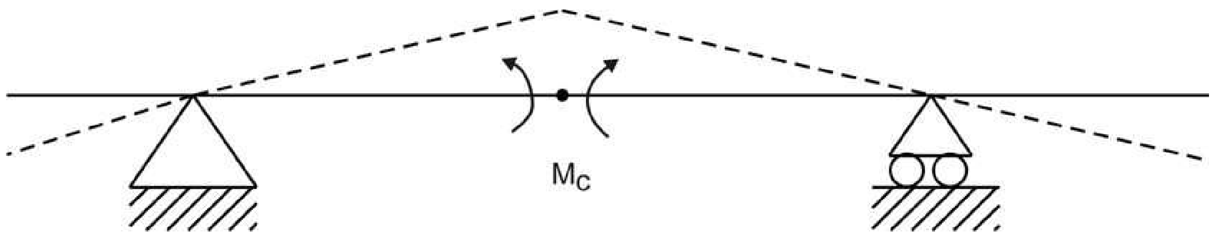
Influence line for shear at section C

Overhang beam - 2

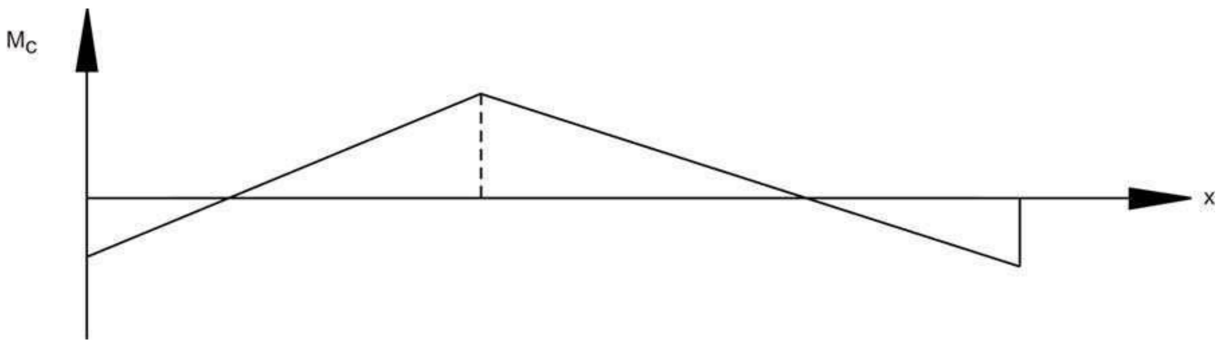


Beam structure

- ✸ To construct influence line for moment, we will introduce hinge at C and that will only permit rotation at C.
- ✸ Now apply moment in the positive direction that will cause a unit rotation in the direction of Mc.
- ✸ The deflected shape due to a unit rotation at C is shown in Figure and matches with the actual shape of the influence line as shown in Figure 3.



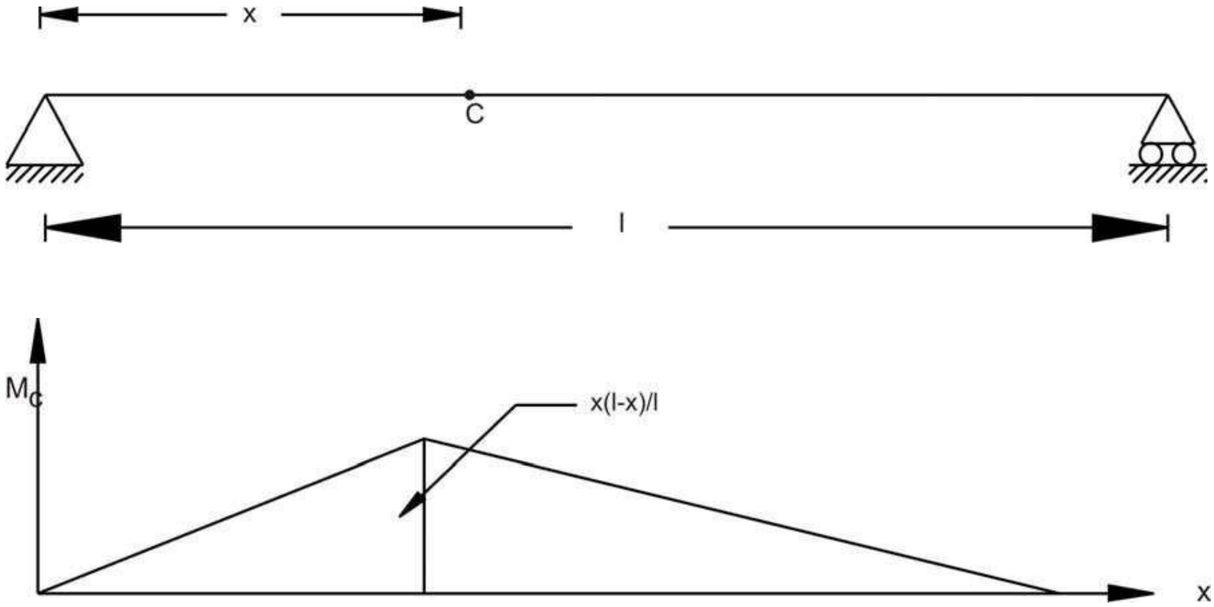
Deflected shape of beam



Influence line for moment at section C

Maximum shear in beam supporting UDLs

UDL longer than the span



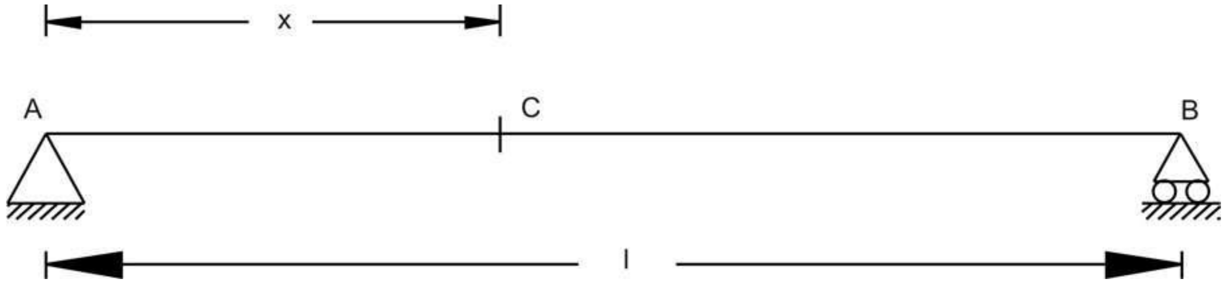
Influence line for moment at section C

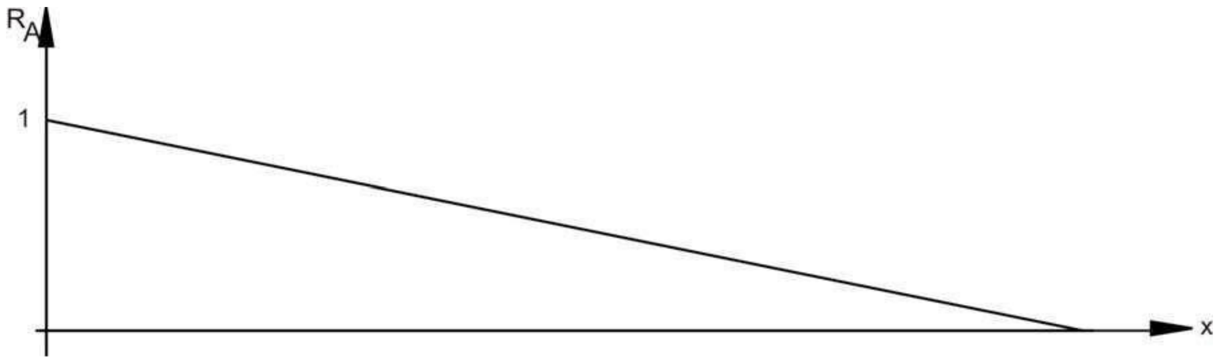
$$w \times \frac{1}{2} \times l \times \frac{x(l-x)}{l} = -\frac{wx(l-x)}{2}$$

Suppose the section C is at mid span, then maximum moment is given by

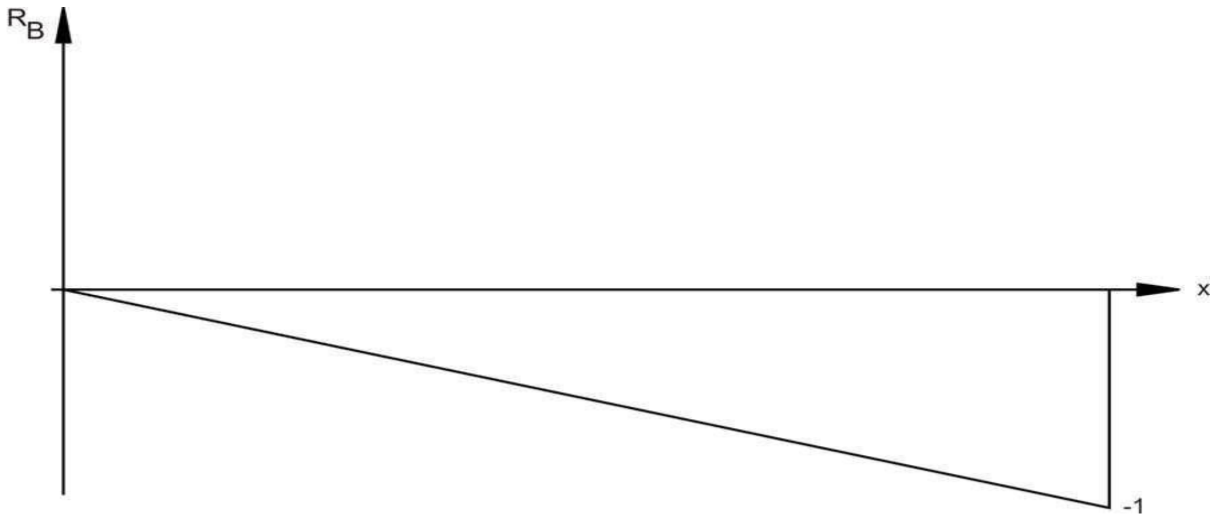
$$\frac{w \times \frac{l}{2} \times \frac{l}{2}}{2} = \frac{wl^2}{8}$$

UDL longer than the span

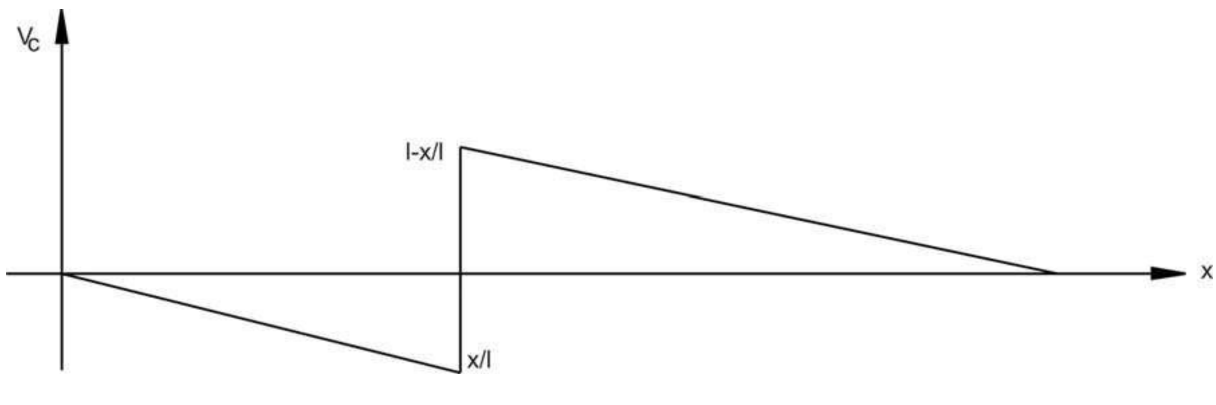




Influence line for support reaction at A



Influence line for support reaction at B



Influence line for shear at section C

$$R_A = w \times \frac{1}{2} \times l \times 1 = \frac{wl}{2}$$

$$R_B = -w \times \frac{1}{2} \times l \times 1 = \frac{-wl}{2}$$

Maximum negative shear is given by

$$= -\frac{1}{2} \times x \times \frac{x}{l} \times w = -\frac{wx^2}{2l}$$

Maximum positive shear is given by

$$= \frac{1}{2} \times \left(\frac{l-x}{l}\right) \times (l-x) \times w = -\frac{w(l-x)^2}{2l}$$

Problem : Draw the ILD for shear force and bending moment for a section at 5m from the left hand support of a simply supported beam, 20m long. Hence, calculate the maximum bending moment and shear force at the section, due to a uniformly distributed rolling load of length 8m and intensity 10 kN/m run.

(a) Maximum bending moment:

Maximum bending moment at a D due to a udl shorter than the span occurs when the section divides the load in the same ratio as it divides the span.

In the above fig. $\frac{A_1 D}{B_1 D} = \frac{AD}{BD} = 0.25, \quad A_1 D = 2M, B_1 D = 6M$

Ordinates:

Ordinate under A_1 = (3.75/5)*3 = 2.25

Ordinate under B_1 = (3.75/15)*9 = 2.25

Maximum bending moment = Intensity of load * Area of ILd under the load

$$= 10 * \frac{(3.75 + 2.25) * 8}{2}$$

At D, M_{max} = 240 kNm

(b) **Maximum positive shear force**

Maximum positive shear force occurs when the tail of the UDL is at D as it traverses from left to right.

Ordinate under B_1 = $\frac{0.75}{15} * (15 - 8) = 0.35$

Maximum positive shear force = Intensity of load * Area of ILD under load

$$= 10 * \frac{(0.75 + 0.35) * 8}{2}$$

SF_{max} = + 44 kNm

(c) **Maximum negative shear force**

Maximum negative shear force occurs when the head of the UDL is at D as it traverses from left to right.

Maximum negative shear force = Intensity of load * Area of ILD under the load

= 10(1/2*0.25*5)

Negative SF_{max} = 6.25 kN.

1. Two point loads of 100 kN and 200 kN spaced 3m apart cross a girder of span 15m from left to right with the 100 kN load loading. Draw the influence line for shear force and bending moment and find the value of maximum shear force and bending moment at a section, 6m from the left hand support. Also, find the absolute maximum moment due to the given load system.
2. .A train of 5 wheel loads crosses a simply supported beam of span 22.5 m. Using influence lines, calculate the maximum positive and negative shear forces at mid span and absolute maximum bending moment anywhere in the span
3. A girder a span of 18mis simply supported at the ends. It is traversed by a train of loads as shown in fig. The 50 kN load loading. Find the maximum bending moment which can occur (i) under the 200 kN load (ii) Under 50 kN load, using influence line diagrams

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UNIT –III – STRUCTURAL ANALYSIS II – SCI1307

UNIT III

Flexibility Matrix Method

INTRODUCTION

These are the two basic methods by which an indeterminate skeletal structure is analyzed. In these methods flexibility and stiffness properties of members are employed. These methods have been developed in conventional and matrix forms. Here conventional methods are discussed. Suitable number of releases. The number of releases required is equal to statically indeterminacys. Introduction of releases results in displacement discontinuities at these releases under the externally applied loads. Pairs of unknown biactions (forces and moments) are applied at these releases in order to restore the continuity or compatibility of structure.

The computation of these unknown biactions involves solution of? linear simultaneous equations. The number of these equations is equal to staticalindeterminacy s. After the unknown biactions are computed all the internal forces can be computed in the entires tructure using equations of equilibrium and free bodies of members. The required displacements can also be computed using methods of displacement computation.

Inflexibility methods in unknowns are forces at the releases the method is also called force method. Since computation of displacement is also required at releases for imposing conditions of compatibility the method is also called compatibility method. In computation of displacements use is made of flexibility properties, hence, the method is also called flexibility method.

DETERMINATE AND INDETERMINATE STRUCTURAL SYSTEMS

If skeletal structure is subjected to gradually increasing loads, without distorting the initial geometry of structure, that is, causing small displacements, the structure is said to be stable. Dynamic loads and buckling or instability of structural system are not considered here. If for the stable structure it is possible to find the internal forces in all the members constituting the structure and supporting reactions at all the supports provided from statically equations of equilibrium only, the structure is said to be determinate.

If it is possible to determine all the support reactions from equations of equilibrium alone the structure is said to be externally determinate externally indeterminate. If structure is externally determinate but itis not possible to determine all internal forces then structure is said tobe internally indeterminate. Therefore a structural system may be:

(1) Externally indeterminate but internally determinate(2)Externally determinate but internally indeterminate(3)Externally and internally indeterminate (4)Externally and internally determinate

DETERMINATEVs INDETERMINATESTRUCTURES.

Determinate structures can be solving using conditions of equilibrium alone (H=0; V=0 ;M=0). No other conditions are required.

Indeterminate structures cannot be solved using conditions of equilibrium because (?H=0; ?V=0;?M=0). Additional conditions are required for solving such structures. Usually matrix methods are adopted.

Flexibility Matrix Method

A systematic development of consistent deformation method is also known as flexibility matrix method or force matrix method. In this method, the basic unknowns to be determined are redundant forces. Hence, the degree of static indeterminacy of the structure is calculated first and then coordinate number is assigned to each redundant force direction. Thus, if F1, F2, …, Fn are the redundant forces in the coordinate direction 1, 2, …, n respectively. If the restraints to the entire redundant are removed, the resulting structure is called as basic determinate structure or Released structure. From the principle of superposition, the net displacement at any point in a statically determinate structure is the sum of the displacement in basic determinate structure due to the applied loads and redundant forces.

$$\Delta_1 = \Delta_{1L} + \delta_{11}F_1 + \delta_{12}F_2 + \ldots + \delta_{1n}F_n$$

$$\Delta_2 = \Delta_{2L} + \delta_{21}F_1 + \delta_{22}F_2 + \ldots + \delta_{2n}F_n$$

The above equation can be expressed in matrix form as

$$[\Delta]=[\Delta_L]+[\delta][F] \text{ or}$$

$$[D_Q]=[D_{QL}]+[F][Q]$$

where, DQ = Displacement corresponding to action in original structure
DQL = Displacement corresponding to action in released structure
F = Flexibility coefficient factor matrix
Q = Unknown redundant force matrix.

$$D_{QL} = \int_0^L \frac{Mm_i}{EI} d_x$$

$$F_{ij} = \int_0^L \frac{m_i m_j}{EI} d_x$$

In the indeterminate structure, the final displacements [Δ] are either zero or known values. The solution for [F] from above equation gives all the redundant forces. Then, bending moment, shear forces at any required point can be calculated by using equations of statics.

Application

1 Analysis of pin jointed plane trusses
2. Analysis of continuous beams
3. Analysis of rigid jointed rectangular portal frame

Steps for the solution of Indeterminate Beams by Flexibility Method

1. Determine the degree of static indeterminacy (DI)
2. Choose the redundant
3. Assign the coordinates to the redundant force direction
4. Remove restraints to redundant forces and get basic determinate structure
5. Determine the deflections in coordinate directions due to given loading in the basic determinate structure
6. Determine the flexibility matrix
7. Apply the compatibility conditions.
8. Calculate the redundant forces
9. Calculate member forces, shear forces and bending moment
10.Draw SFD, BMD

Problem : Analyse the continuous beam shown in fig by flexibility matrix method. Flexural rigidity is constant throughout.

Solution

1) Degree of static indeterminacy = R-3 = 4-3 = 1

Let $Q_1 = R_B$

2) Displacement analysis

2) Displacement analysis

Zone	Origin	Limit	M fig. 3.1(d)	$m_1$fig. 3.1(e)
BA	B	0-3	$-\frac{10x^2}{2}$	x

3) Superposition principle

$$[D_Q] = [D_{QL}] + [F][Q]$$
$$[0] = [D_{QL1}] + [F_{11}][Q_1]$$

$$D_{QL1} = \int_0^3 \frac{Mm_i}{EI} d_x = \int_0^3 \frac{-10x^2}{2EI} x d_x = -\frac{101.25}{EI}$$

$$F_{11} = \int_0^3 \frac{m_i m_i}{EI} d_x = \int_0^3 \frac{xx}{EI} d_x = \frac{9}{EI}$$

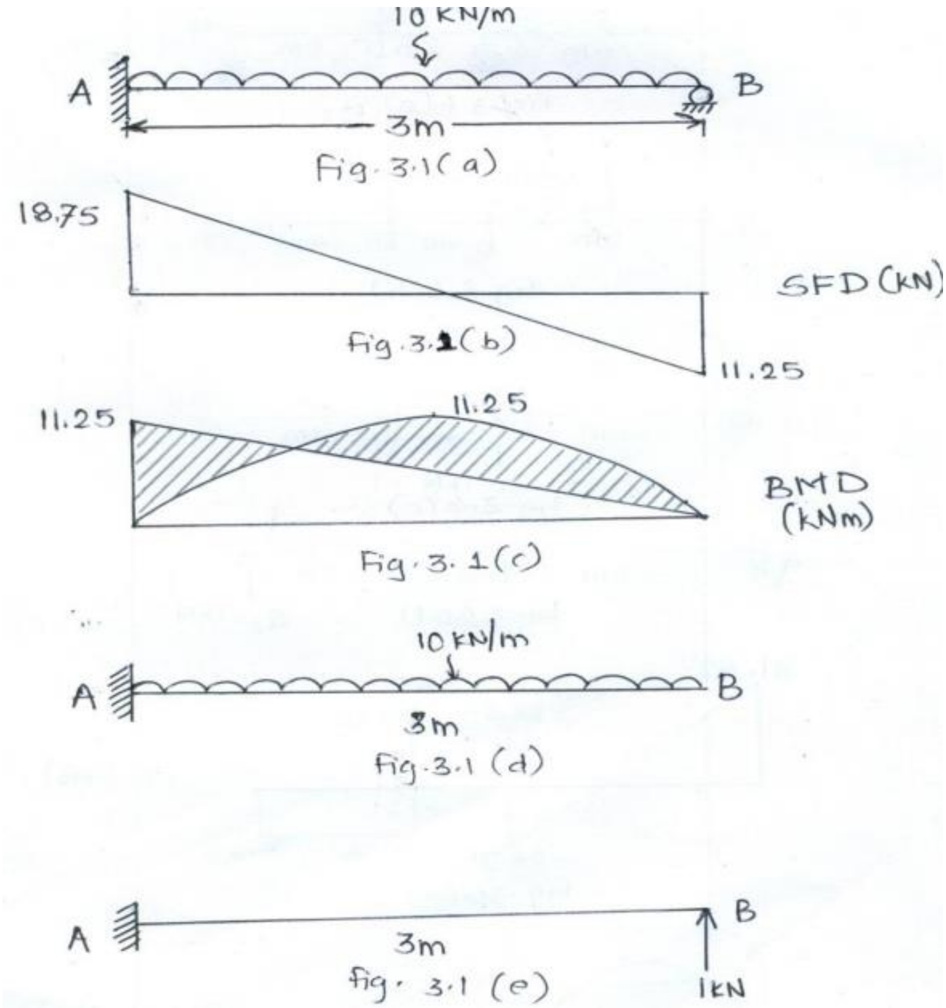
$$\therefore 0 = -\frac{101.25}{EI} + \frac{9}{EI} Q_1$$
$$\therefore Q_1 = 11.25kN \uparrow$$
$$\therefore R_B = 11.25kN \uparrow$$

Reaction calculations

Ha=0

All vertical force equal to zero, then

$$R_A + 11.25 - 30 = 0$$
$$R_A = 18.75kN \uparrow$$
$$\sum M @ A = 0$$
$$M_A - 10 \times 3 \times 1.5 + 11.25 \times 3 = 0$$
$$\therefore M_A = 11.25kNm$$



Problem : Analyse the continuous beam shown in figure 3.2(a) by flexibility matrix method.

Solution

1) Degree of static indeterminacy = R-3 = 4-3 = 1

Let $Q_1 = R_B$

2) Displacement analysis

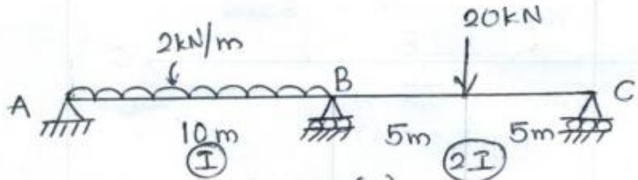


Fig. 3.2 (a)

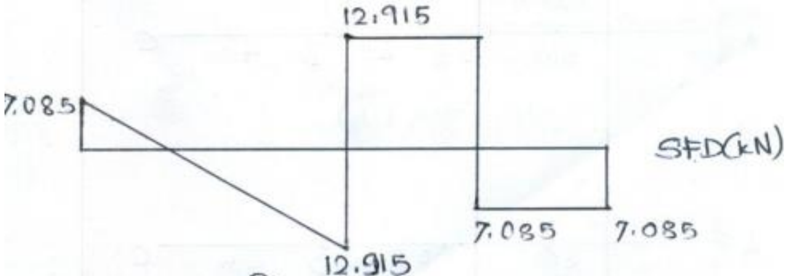


Fig. 3.2(b)

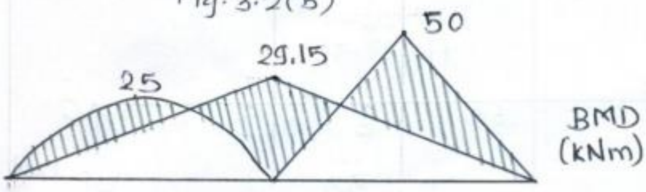


Fig 3.2 (c)

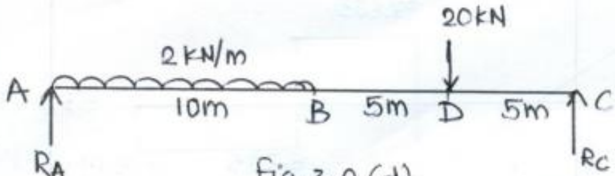
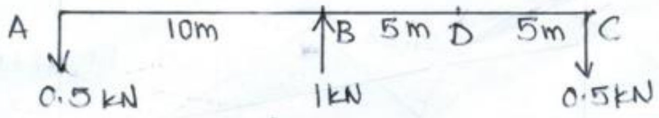


Fig. 3.2 (d)



2) Displacement analysis

Zone	Origin	Limit	M fig. 3.2(d)	$m_1$fig. 3.2(e)
CD	C	0-5	$20x$	$-0.5x$
DB	C	5-10	$20x-20(x-5)$	$-0.5x$
BA	C	10-20	$20x-20(x-5)-(x-10)^2$	$-0.5x+(x-10)$

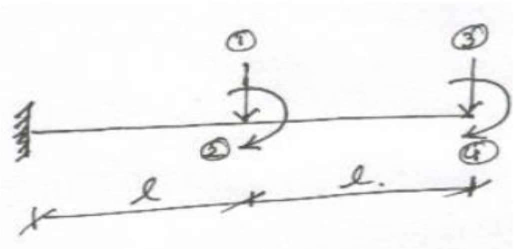
End moments

$M_{AB} = M_{CB} = 0$

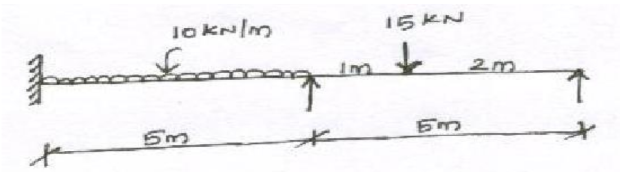
$M_{BA} = -29.15kNm$

$M_{BC} = 29.15kNm$

1. Differentiate between determinate and indeterminate structures.
2. Define the term Degrees of freedom.
3. State the principle of superposition of forces.
4. Differentiate between determinate and indeterminate structures.
5. Define the term Degrees of freedom.
6. State the principle of superposition of forces.
7. Derive the element flexibility matrix for the cantilever element shown in fig.



8 Analyze the continuous beam shown in fig by flexibility method. Draw the bending Moment diagram. EI is Constant.



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UNIT –IV – STRUCTURAL ANALYSIS II – SCI1307

UNIT-IV

STIFFNESS MATRIX METHOD

INTRODUCTION

The given indeterminate structure is first made kinematic ally determinate by introducing constraints at the nodes. The required number of constraints is equal to degrees of freedom at the nodes that is kinematic indeterminacy. The kinematically determinate structure comprises of fixed ended members, hence, all nodal displacements are zero. These results in stress resultant discontinuities at these nodes under the action of applied loads or in other words the clamped joints are not in equilibrium.

In order to restore the equilibrium of stress resultants at the nodes the nodes are imparted suitable unknown displacements. The number of simultaneous equations represent ting joint equilibrium of forces is equal to kinematic indeterminacy. Solution of these equations gives unknown nodal displacements. Using stiffness properties of members, the member end forces arecomputed and hence the internal forces throughout the structure.

Since nodal displacements are unknowns, the method is also called displacement method. Since equilibrium conditions are applied at the joints the method is also called equilibrium method. Since stiffness properties of members are used the method is also called stiffness method.

DETERMINATE AND INDETERMINATE STRUCTURAL SYSTEMS

If skeletal structure is subjected to gradually increasing loads, without distorting the initial geometry of structure, that is, causing small displacements, the structure is said to be stable. Dynamic loads and buckling or instability of structural system are not considered here. If for the stable structure it is possible to find the internal forces in all the members constituting the structure and supporting reactions at all the supports provided from statically equations of equilibrium only, the structure is said to be determinate.

If it is possible to determine all the support reactions from equations of equilibrium alone the structure is said to be externally determinate externally indeterminate. If structure is externally determinate but itis not possible to determine all internal forces then structure is said tobe internally indeterminate. Therefore a structural system may be:

(1) Externally indeterminate but internally determinate
(2) Externally determinate but internally indeterminate
(3) Externally and internally indeterminate (4) Externally and internally determinate

PROPERTIES OF THE STIFFNESS MATRIX

The properties of the stiffness matrix are:

- � It is asymmetric matrix
- � The sum of elements in any column must be equal to zero.
- � It is an unstable element therefore the determinant is equal to zero.

ELEMENT AND GLOBAL STIFFNESS MATRICES

Local co ordinates

In the analysis for convenience we fix the element coordinates coincident with the memberaxis called element (or) local coordinates (coordinates defined along the individual member axis)

Global co ordinates

It is normally necessary to define a coordinate system dealing with the entire structure is called system on global coordinates (Common coordinate system dealing with the entire structure)

Transformation matrix

The connectivity matrix which relates the internal forces Q and the external forces R is known as the force transformation matrix. Writing it in a matrix form,

$$\{Q\} = [b]\{R\}$$

Where Q=member force matrix/vector, b=force transformation matrix R = external force/load matrix/ vector

ANALYSIS OF CONTINUOUS BEAMS

Fig 2.1 Cantilever Beam

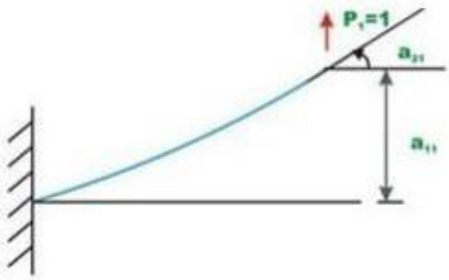


Fig 2.2 Cantilever Beam with unit load along P_1

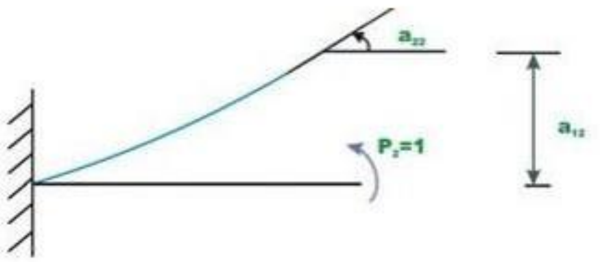


Fig 2.3 Cantilever Beam with unit Moment along P_2

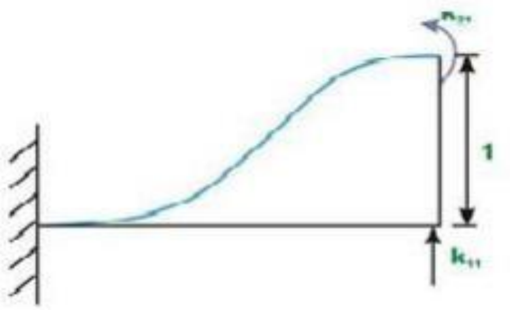


Fig 2.4 Cantilever Beam with unit Displacement along U_1

u_1 and u_2 by stiffness method. This is done in following steps:

1. In the first step, make all the unknown displacements equal to zero by altering the boundary conditions as shown in Fig.23.4b. On this restrained /kinematically determinate structure, apply all the external loads except the joint loads and calculate the reactions corresponding to unknown joint displacements u_1 and u_2. Since, in the present case, there are no external loads other than the joint loads, the reactions $(R_L)_1$ and $(R_L)_2$ will be equal to zero. Thus,

$$\begin{Bmatrix} (R_L)_1 \\ (R_L)_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

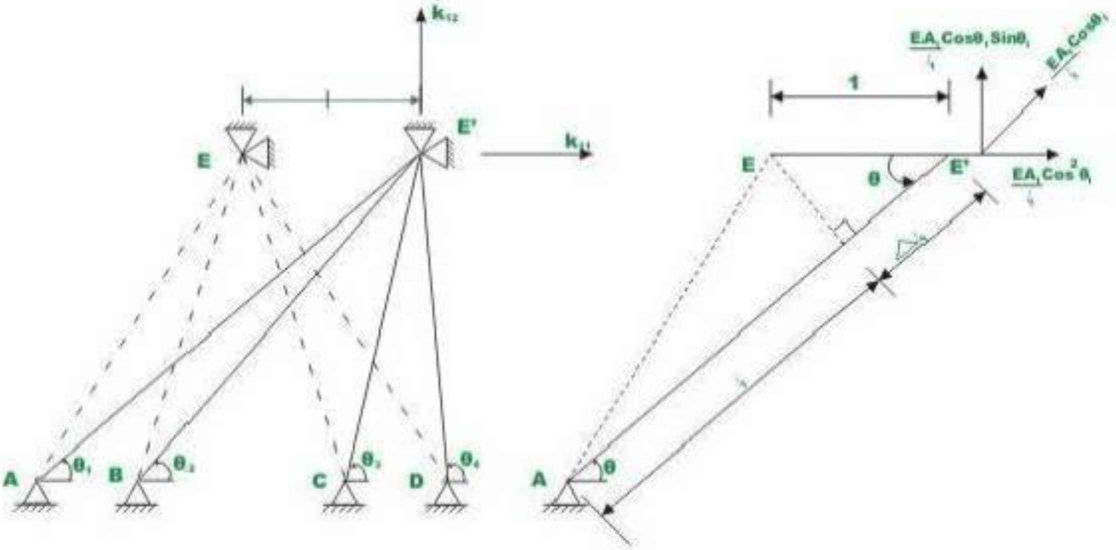


Fig 2.7Unit Displacement along U

In the next step, give a unit displacement along u_2 holding displacement along u_1 equal to zero and calculate reactions at E corresponding to unknown displacements u_1 and u_2 in the kinematically determinate structure. The corresponding reactions are denoted by k_{12} and k_{22} as shown in The joint E gets displaced to E' when a unit vertical displacement is given to the joint as shown in the figure. Thus, the new length of the member AE' is $l_1 + \Delta l_1$. From the geometry, the elongation Δl_1 is given by

3. Joint forces in the original structure corresponding to unknown displacements u_1 and u_2 are

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

Now the equilibrium equations at joint E states that the forces in the original structure are equal to the superposition of (i) reactions in the kinematically restrained structure corresponding to unknown joint displacements and (ii) reactions in the restrained structure due to unknown displacements themselves. This may be expressed as,

$$F_1 = (R_L)_1 + k_{11}u_1 + k_{12}u_2$$
$$F_2 = (R_L)_2 + k_{21}u_1 + k_{22}u_2$$

The systematic development of slope deflection method in the matrix form has lead to Stiffness matrix method. The method is also called Displacement method. Since the basic unknowns are the displacement at the joint.

The stiffness matrix equation is given by

$[\Delta]\,[K] = \{[P] - [P_L]\}$

$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

Where,

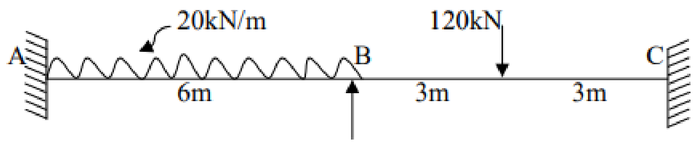
[P] = Redundant in matrix form

[F] = Stiffness matrix

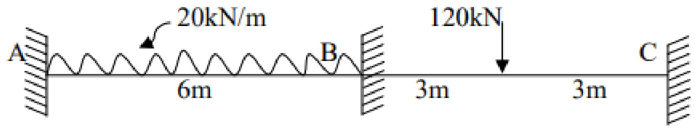
[P] = Final force at the joints in matrix form

$[P_L]$= force at the joints due to applied load in matrix form

Problem : Analyse the continuous beam by Stiffness method Sketch the BMD



Kinematic Indeterminacy KI = 1 (θ_B)

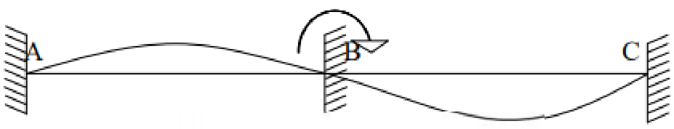


$[P_L] = M_{FBA} + M_{FBC}$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) = \frac{20\,X 6^2}{12} - \frac{120\,X 6}{8}$$

$[P_L]$ = -30kN-m

Apply unit displacement at joint B.



$$[K] = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{6} + \frac{4EI}{6} = 1.33EI \qquad (\theta=1)$$

By condition of equilibrium at joint B

$[P] = 0$

$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

$= \frac{1}{K}\{[P] - [P_L]\}$

$\theta_B = \frac{1}{1.33EI}\{[0] - [-30]\} = \frac{22.56}{EI}$

Slope deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A - \theta_B)$

$= -60 + \frac{2EI}{6}(2\theta_A + \frac{22.5}{EI})$ (θ_A = 0 due to fixity at support A)

$M_{AB} = -52.5$kN-m

$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$

$= 60 + \frac{2EI}{6}(2 X \frac{22.5}{EI} + \theta_A)$

$M_{BA} = 75.04$kN-m

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$

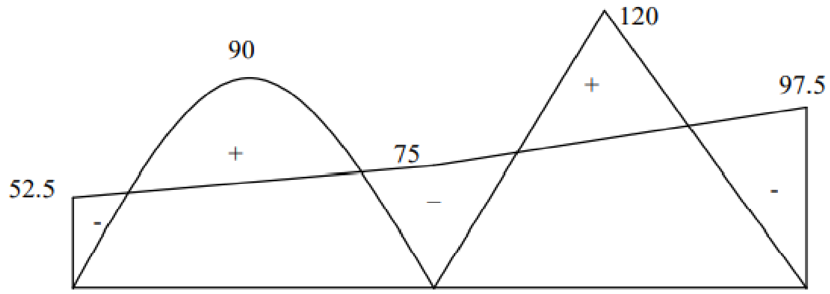
$= -90 + \frac{2EI}{6}(2 X \frac{22.5}{EI} + 0)$

$M_{BC} = -75$kN-m

$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$

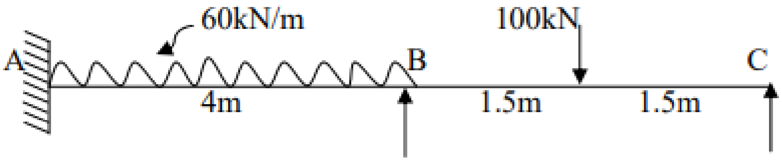
$= 90 + \frac{2EI}{6}(0 + \frac{22.5}{EI})$

$M_{CB} = 97.52$kN-m

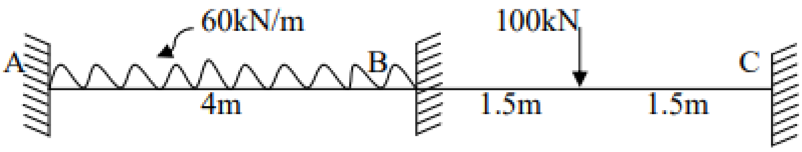


BMD

Problem: Analyse the continuous beam by Stiffness method Sketch the BMD



Kinematic Indeterminacy KI = 2 (θ_B & θ_C)



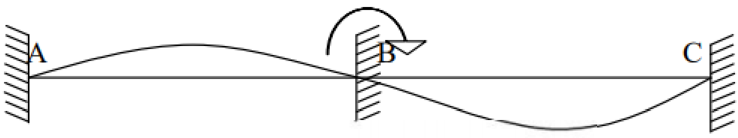
$[P_{1L}] = M_{FBA} + M_{FBC}$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) = \frac{60 X 4^2}{12} - \frac{100 X 3}{8} = 42.5\text{kN-m}$$

$[P_{2L}] = M_{FCB} = \frac{wl}{8} = \frac{100 X 3}{8} = 37.5\text{kN-m}$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 42.5 \\ 37.5 \end{pmatrix} \text{kN-m}$$

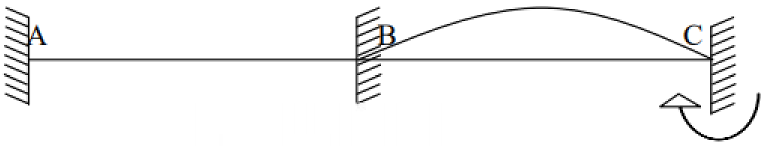
Apply unit displacement at joint B.



$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{4} + \frac{4EI}{3} = 2.33EI \qquad (\theta=1)$

$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$

Apply unit displacement at joint B.



$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$

$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$

By condition of equilibrium at joint B

$[P] = 0$

$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -11.88 \\ -22.19 \end{bmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A - \theta_B)$$

$$= -80 + \frac{2\,EI}{4}\left(2\theta_A - \frac{11.88}{EI}\right) \qquad (\theta_A = 0 \text{ due to fixity at support A})$$

$M_{AB} = -85.94$kN-m

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 80 + \frac{2\,EI}{4}\left(2 \text{ X } \frac{-11.88}{EI} + \theta_A\right)$$

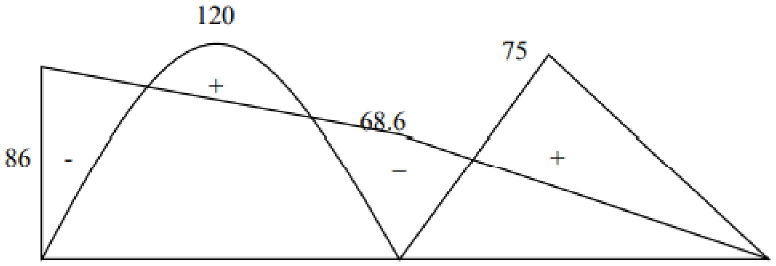
$M_{BA} = 68.12$kN-m

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -37.5 + \frac{2EI}{6}\left(2 \text{ X } \frac{-11.88}{EI} + \frac{-22.9}{EI}\right)$$

$M_{BC} = -68.6$kN-m

$M_{CB} = 0$

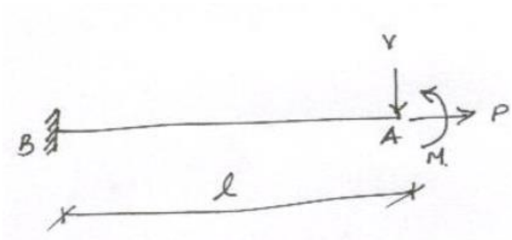


BMD

1. What is stiffness coefficient?
2. Define kinematic indeterminacy.
3. What is axial stiffness?
4. In what way stiffness matrix differ from flexibility matrix

1. Using stiffness method draw bending moment diagram of the continuous beam ABCD (All the supports are freely supported). Span AB (12 m) carried uniformly distributed load of 4kN/m Span BC (12 m) carried central concentrated load of 12 kN. Span CD (12 m) carried uniformly distributed load of 2 kN/m.

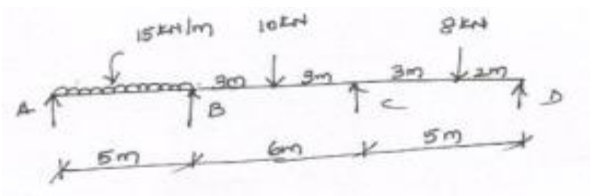
2. Derive the element stiffness matrix for the cantilever element shown in fig. Assume uniform EI



3. Set up stiffness matrix or stiffness influence coefficients for a beam element of uniform flexural rigidity considering the following displacement.

4. Set up the element stiffness matrix for a beam element subjected to clockwise moments at the two ends. Assume uniform EI.

5. Analyze the given continuous beam using displacement method? Draw BMD



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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT –V – STRUCTURAL ANALYSIS II – SCI1307

UNIT –V

PLASTIC ANALYSIS OF STRUCTURES

1. Statically indeterminate axial problems

In these analyses we used superposition often, knowing that for a linearly elastic structure it was valid. However, an elastic analysis does not give information about the loads that will actually collapse a structure. An indeterminate structure may sustain loads greater than the load that first causes a yield to occur at any point in the structure.

In fact, a structure will stand as long as it is able to find redundancies to yield. It is only when a structure has exhausted all of its redundancies will extra load causes it to fail. Plastic analysis is the method through which the actual failure load of a structure is calculated, and as willbe seen, this failure load can be significantly greater than the elastic load capacity.

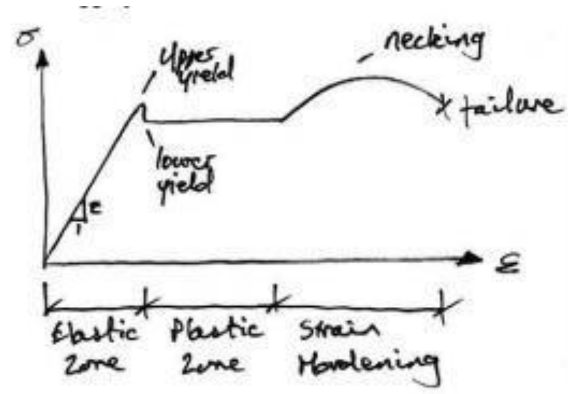
To summarize this, Prof. Seande Courcy (UCD) used to say: 'a structure only collapses when it has exhausted all means of standing'.

Before analyzing complete structures, we review material and cross section behavior beyond theelastic limit.

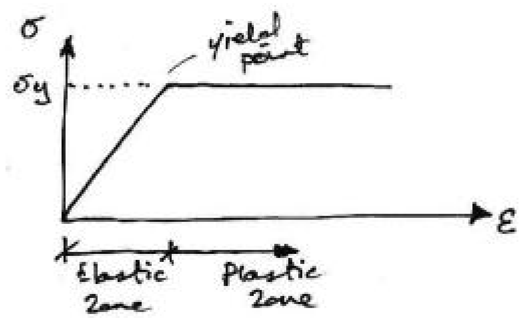
2. Beams in pure bending

2.1. Material Behavior

A uniaxial tensile stress on a ductile material such as mild steel typically provides the followinggraph of stress versus strain:



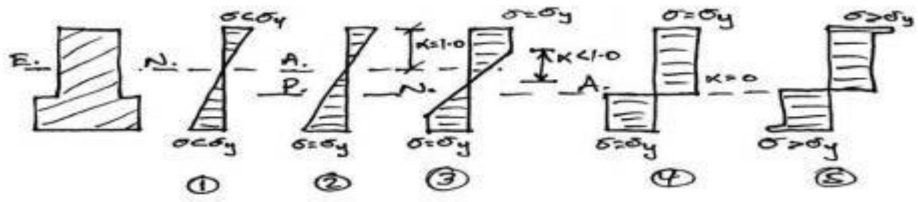
As can be seen, the material can sustain strains farinexcess of the strain at which yield occurs before failure. This property of the material is called its ductility. Though complex models do exist to accurately reflect the above real behavior of the material, the most common, and simplest, model is the idealized stress-strain curve. This is the curve for an ideal elastic-plastic material (which doesn't exist), and the graph is:



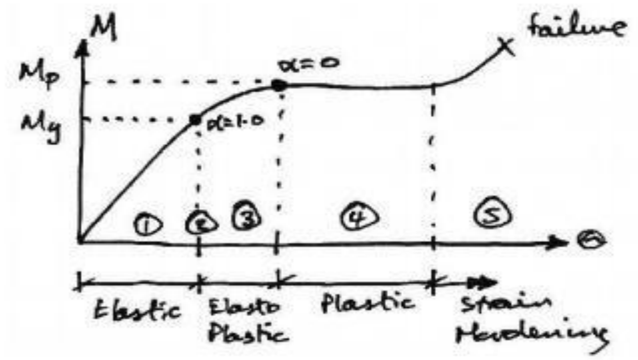
As can be seen, once the yield has been reached it is taken that an indefinite amount of strain can occur. Since so much post-yield strain is modeled, the actual material (or cross section) must also be capable of allowing such strains. That is, it must be sufficiently ductile for the idealized stress-strain curve to bevalid. Next we consider the behavior of across section of an ideal elastic- plastic material subject to bending. In doing so, we seek the relationship between applied moment and the rotation (or more accurately, the curvature) of across section.

2.2. Moment-Rotation Characteristics of General Cross Section

We consider an arbitrary cross-section with a vertical plane of symmetry, which is also the plane of loading. We consider the cross section subject to an increasing bending moment, and assess thestresses at each stage.



Cross sectionandStresses



Stage1- Elastic Behaviour

The applied moment causes stresses over the cross-section that are all less than the yield stress of the material.

Stage2-Yield Moment

The applied moment is just sufficient that the yield stress of the material is reached at the outer most fibre(s) of the cross-section. All other stresses in the cross section are less than the yield stress. This is limit of applicability of an elastic analysis and of elastic design. Since all fibres are elastic, the ratio of the depth of the elastic to plastic regions,

Stage3- Elasto-Plastic Bending

The moment applied to the cross section has been increased beyond the yield moment. Since by the idealized stress-strain curve the material cannot sustain a stress greater than yield stress, the fibres at the yield stress have progressed inwards towards the centre of the beam. Thus over the cross section the reisanelastic core and a plastic region. The ratio of the depth of the elastic core to the plastic region is .

Since extra moment is being applied and no stress is bigger than the yield stress, extra rotation of the section occurs: the moment-rotation curve losses its linearity and curves, giving more rotation per unit moment (i.e. looses stiffness).

Stage4- Plastic Bending

The applied moment to the cross section is such that all fibres in the cross section are at yield stress. This is termed the Plastic Moment Capacity of the section since there are no fibres at an elastic stress, Also note that the full plastic moment requires an infinite strain at the neutral axis And so is physically impossible to achieve. However, it is closely approximated in practice. Any attempt at increasing the moment at this points imply results in more rotation, once the cross-section has sufficient ductility. There fore in steel members the cross section classification must be plastic and in concrete members the section must be under-reinforced.

Stage5-Strain Hardening

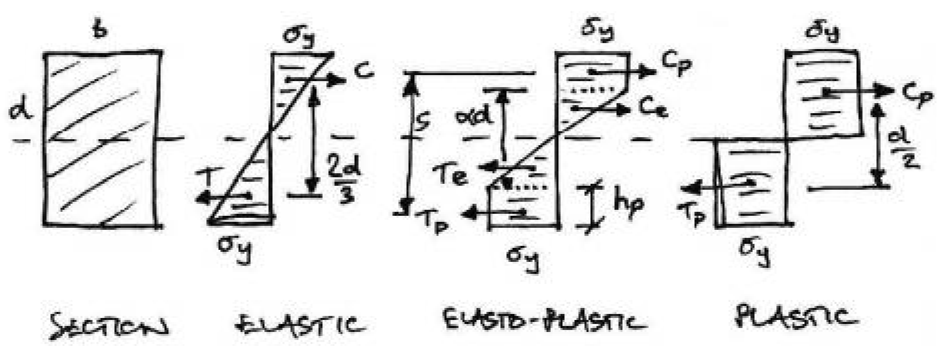
Due to strain hardening of the material, a small amount of extra moment can be sustained.

The above moment-rotation curve represents the behavior of across section of a regular elastic-plastic material. However, it is usually further simplified as follows:

With this idealized moment-rotation curve, the cross section linearly sustains moment up to the plastic moment capacity of the section and then yields in rotation an indeterminate amount. Again, to use this idealization, the actual section must be capable of sustaining large rotations- that is it must be ductile.

Analysis of Rectangular Cross Section

Since we now know that across section can sustain more load than just the yield moment, we are interested in how much more. In other words we want to find the yield moment and plastic moment, and we do so for a rectangular section. Taking the stress diagrams from those of the moment-rotation curve examined previously, we have:



3. Shape Factor

Thus the ratio of elastic to plastic moment capacity is:

$$\frac{M_P}{M_Y} = \frac{\sigma_Y \cdot S}{\sigma_Y \cdot Z} = \frac{S}{Z}$$

This ratio is termed the *shape factor*,*f*, and is a property of across section alone. For a rectangular cross-section, we have:

$$f = \frac{S}{Z} = \frac{bd^2/4}{bd^2/6} = 1.5$$

And so a rectangular section can sustain 50% more moment than the yield moment, before a plastic hinge is formed. Therefore the shape factor is a good measure of the efficiency of across section In bending. Shape factors for some other cross sections are

Rectangle: $f = 1.5$, as above;

Circle: $f = 1.698$;

Diamond: $f = 2.0$;

Steel I-beam: f is between 1.12 and 1.15.

4. Plastic Hinge

Note that once the plastic moment capacity is reached, the section can rotate freely- that is, it behaves like a hinge, except with moment of M patthe hinge. This is termed a *plastic hinge*, and is the basis for plastic analysis. At the plastic hinge stresses remain constant, but strains and hence rotations can increase.

4.1. Methods of Plastic Analysis

1. The Incremental Method

This is probably the most obvious approach: the loads on the structure are incremented until the first plastic hinge forms. This continues until sufficient hinges have formed to collapse the structure. This is a labour-intensive, 'brute-force', approach, but one that is most readily suited for computer implementation.

2. The Equilibrium (or Statical) Method

In this method, free and reactant bending moment diagrams are drawn. These diagrams are over laid to identify the likely locations of plastic hinges. This method therefore satisfies the equilibrium criterion first leaving the two remaining criterion to derived therefrom.

3. The Kinematic (or Mechanism) Method

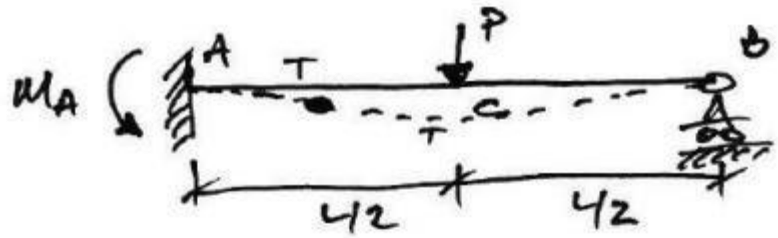
In this method, a collapse mechanism is first postulated. Virtual work equations are then written for this collapse state, allowing the calculations of the collapse bending moment diagram. This method satisfies the mechanism condition first, leaving the remaining two criteria to be derived there from.

We will concentrate mainly on the Kinematic Method, but introduce now the Incremental Method to illustrate the main concepts.

4.1.1. Incremental Method

Example1- Propped Cantilever

We now assess the behavior of a simple statically indeterminate structure under increasing load. We consider a propped cantilever with mid-span point load:



From previous analyses we know that:

$$M_A = \frac{3PL}{16} \qquad M_C = \frac{5PL}{32}$$

We will take the span to be $L = 1\text{ m}$ and the cross section to have the following capacities:

$$M_Y = 7.5\text{ kNm} \qquad M_P = 9.0\text{ kNm}$$

Further, we want this beam to be safe at a working load of 32 kN, so we start there.

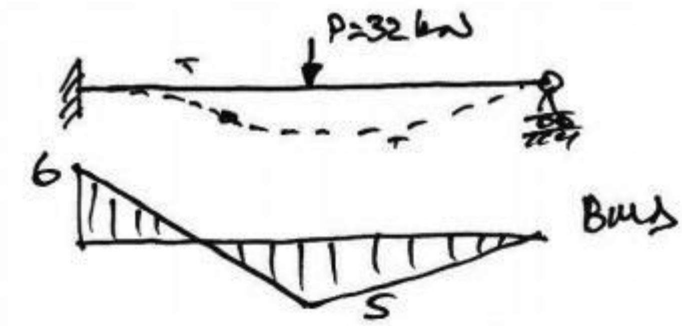
We will also look at the deflections for better understanding of the behaviour. To do this, we will take $EI = 10\text{ kNm}^2$.

Load of 32 kN

At this value of load the BMD is as shown, with:

$$M_A = \frac{3(32)(1)}{16} = 6\text{kNm} \qquad M_c = \frac{5(32)(1)}{32} = 5\text{ kNm}$$

Since the peak moments are less than the yield moments, we know that yield stress has not been reached at any point in the beam. Also, the maximum moment occurs at *A* and so this point will first reach the yield moment.



The corresponding deflection under the point load is:

$$\delta_c = \frac{7PL^3}{768EI} = \frac{7(32)(1^3)}{768(10)} = 29.17 \text{ mm}$$

The rotation at A is, of course, zero.

The load factor before yielding occurs, based on the maximum moment (at A) and the yield moment is 7.5/6 = 1.25. Thus a load of 1.25 × 32 = 40 kN will cause yielding.

4.1.2. <u>Equilibrium Method</u>

<u>Introduction</u>

To perform this analysis we generally follow the following steps:

1.Find a primary structure by removing redundant until the structure is statically determinate;

2. Draw the primary (or free) bending moment diagram;

3. Draw the reactant BMD for each redundant, as applied to the primary structure;

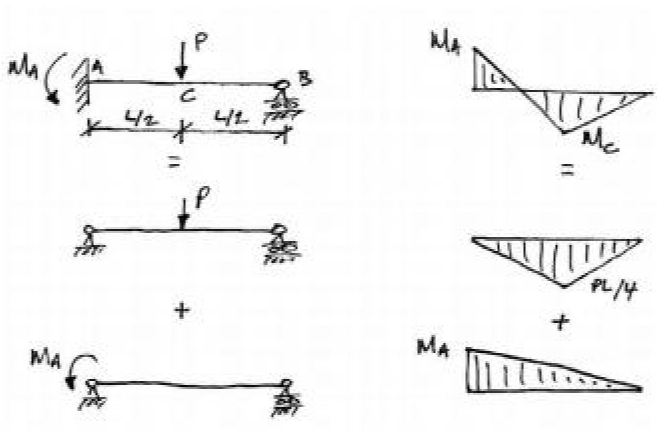
4. Construct a composite BMD by combing the primary and reactant BMDs;
5. Determine the equilibrium equations from the composite BMD;

6. Choose the points where plastic hinges are likely to form and introduce into the equilibrium equations;

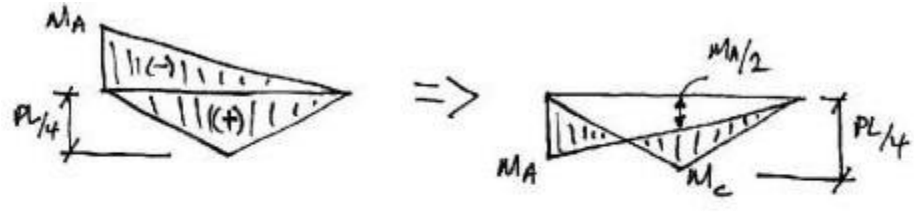
7. Calculate the collapse load factor, or plastic moment capacity as required.

For different possible collapse mechanisms, repeat steps 6 and 7, varying the hinge locations. We now apply this method to the Illustrative Example previously analyzed.

Steps 1 to 3 of the Equilibrium Method are illustrated in the following diagram:



For Step 4, in constructing the Composite BMD, we arbitrarily choose tension on the underside of the beam as positive. By convention in the Equilibrium Method, instead of drawing the two BMDs on opposite sides (as is actually the case), the reactant BMD is drawn 'flipped' over the line and subtracted from the primary BMD: the net remaining area is the final BMD. This is best explainedby illustration below:



As may be seen from the composite diagram, M_A, can actually have any value (for example, if a rotational spring support existed at A), once overall equilibrium of the structure is maintained through the primary (or free) BMD ordinate of $PL/4$.

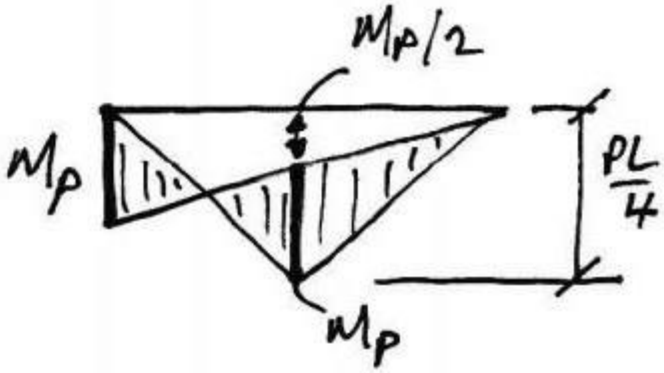
For Step 5, from the diagram, the equilibrium equation is:

$$M_C = \frac{PL}{4} - \frac{M_A}{2}$$

For Step 6, we recognize that there are two hinges required to collapse the structure and identify the peak moments from the diagram as being at A and C. Thus these are the likely hinge locations. Setting $M_A = M_C = M_P$ in the equilibrium equation gives:

$$M_P = \frac{PL}{4} - \frac{M_P}{2}$$

This is equivalent to drawing the following diagram:



ForStep7, wesolvethis equationforthecollapseload:

$$\frac{3}{2}M_P = \frac{PL}{4}$$
$$P = \frac{6M_P}{L}$$

For our particular example, $L = 1$ m, $M_P = 9$ kNm, and $P = 32\lambda$. Thus:

$$32\lambda = \frac{6(9)}{1}$$

And so the collapse load factor is:

$$\lambda_C = 1.6875$$

Which is the same as the results previously found.

Kinematic Method Using Virtual Work -

Introduction

Probably the easiest way to carry out a plastic analysis is through the Kinematic Method using virtual work. To do this we allow the presumed shape at collapse to be the compatible displace mentset, and the external loading and internal bending moments to be the equilibrium set. We can then equate external and internal virtual work, and solve for the collapse load factor for that supposed mechanism.

Remember:

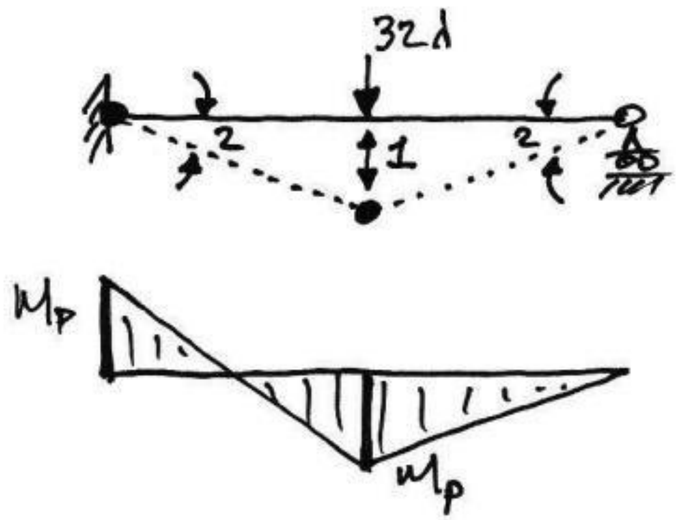
Equilibrium set: the internal bending moments at collapse;

Compatibleset: the virtual collapsed configuration (see below).

Note that in the actual collapse configuration the members will have elastic deformation in between the plastic hinges. However, since a virtual displacement does not have to be real, only compatible, we will choose to ignore the elastic deformations between plastic hinges, and take the members to be straight between them.

5. <u>**Collapse Mechanism**</u>

So for our previous beam, we know that we require two hinges for collapse (one more than its degree of redundancy), and we think that the hinges will occur under the points of peak moment, *A* and *C*. Therefore impose a unit virtual displacement at *C* and relate the corresponding virtual rotations of the hinges using,



Notice that the collapse load is the working load times the collapse load factor. So:

$$\delta W_e = \delta W_I$$

$$(32\lambda)(1) = \underbrace{(M_P)(2)}_{\text{At } A} + \underbrace{(M_P)(4)}_{\text{At } C}$$

$$32\lambda = 6M_P$$

$$\lambda = \frac{6(9)}{32} = 1.6875$$

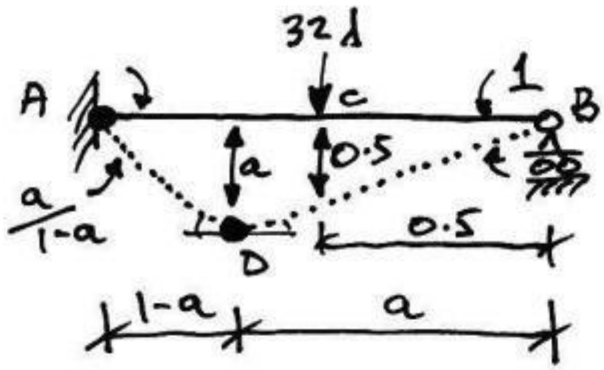
since $M_P = 9$ kNm. This result is as found before.

Other Collapse Mechanisms

For the collapse mechanism looked at previously, it seemed obvious that the plastic hinge in the span should be beneath the load. But why? Using virtual work we can examine any possible collapse mechanism. So let's consider the following collapse mechanism and see why the plastic hinge has to be located beneath the load.

Plastic Hinge between *A* and *C*:

Imposing a unit virtual deflection at *B*, we get the following collapse mechanism:



And so the virtual work equation becomes:

$$\delta W_e = \delta W_I$$

$$(32\lambda)(0.5) = \underbrace{(M_P)\left(\frac{a}{1-a}\right)}_{\text{At } A} + \underbrace{(M_P)\left(\frac{a}{1-a}+1\right)}_{\text{At } D}$$

$$16\lambda = M_P\left[\frac{2a+(1-a)}{1-a}\right]$$

And since $M_P = 9$ kNm:

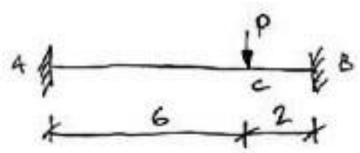
$$\lambda_{1<a\leq 0.5} = \frac{9}{16}\left[\frac{a+1}{1-a}\right]$$

And so we see that the collapse load factor for this mechanism depends on the position of the plastic hinge in the span.

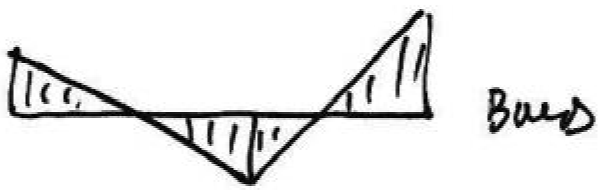
Plastic Analysis of Beams

Example 2-Fixed-Fixed Beam with Point Load

For the following beam, find the load at collapse, given that $M_p = 60$ kNm



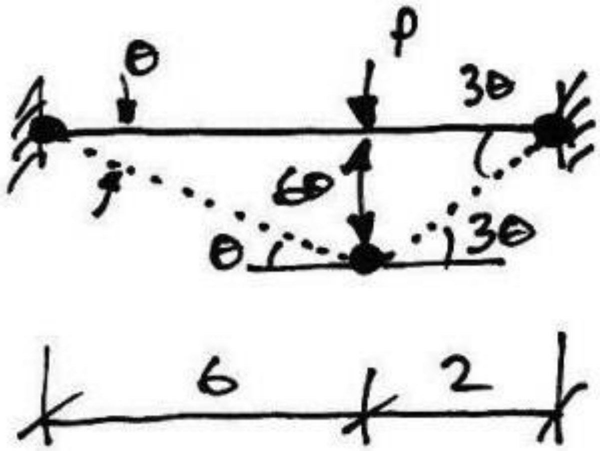
To start the problem, we examine the usual elastic BMD to see where the plastic hinges are likely to form:



We also need to know how many hinges are required. This structure is 3? statically indeterminate and so we might expect the number of plastic hinges required to be 4. However, since one of the indeterminacies is horizontal restraint, removing it would not change the bending behavior of the beam.

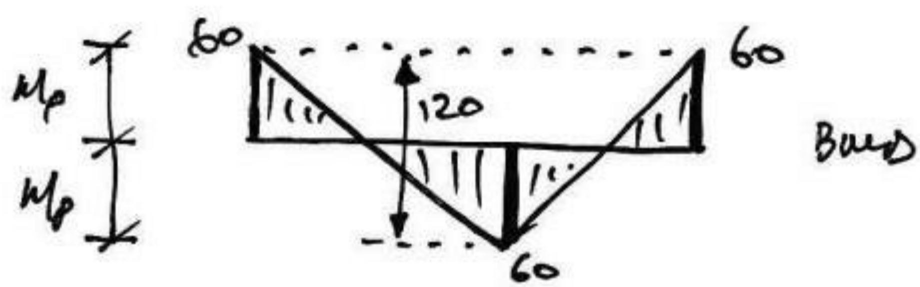
Thus for a bending collapse only 2 indeterminacies apply and so it will only take 3 plastic hinges to cause collapse. So looking at the elastic BMD, we'll assume a collapse mechanism with the 3 plastic hinges at the peak moment locations: *A*, *B*, and *C*.

Next, we impose a virtual rotation of θ to the plastic hinge at A and using the $S = R\theta$ rule, relate all other displacements to it, and then apply the virtual work equation:



$$\delta W_e = \delta W_i$$
$$P(6\theta) = \underbrace{M_P(\theta)}_{At\ A} + \underbrace{M_P(\theta + 3\theta)}_{At\ C} + \underbrace{M_P(3\theta)}_{At\ B}$$
$$6P\theta = 8M_P\theta$$
$$P = \frac{8}{6}M_P$$

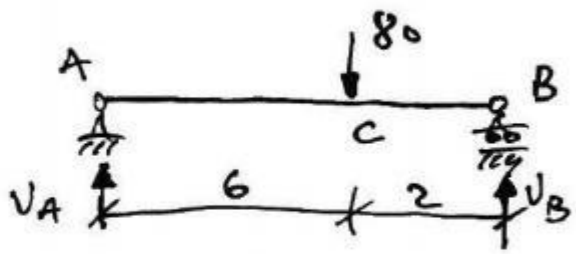
Since $M_P = 60$ kNm the load required for collapse is $P = 80$ kN and so the collapse BMD for this mechanism is:



Weneedtocheckthatthisisthecorrectsolutionusingthe Uniqueness Theorem:

1. Equilibrium:

We'll check that the height of the free BMD is 120 kNm as per the collapse BMD:



$$\sum M \text{ about } A = 0 \quad \therefore 80 \cdot 6 - 8V_B = 0 \quad \therefore V_B = 60 \text{ kN}$$

Thus, using a free body diagram of *CB*:

$$\sum M \text{ about } C = 0 \quad \therefore M_C - 2V_B = 0 \quad \therefore M_C = 120 \text{ kNm}$$

And so the applied load is in equilibrium with the free BMD of the collapse BMD.

2. Mechanism:

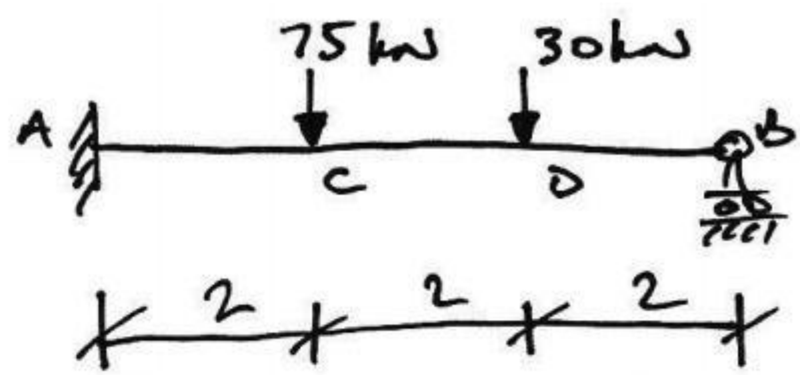
From the proposed collapse mechanism it is apparent that the beam is a mechanism.

3. Yield:

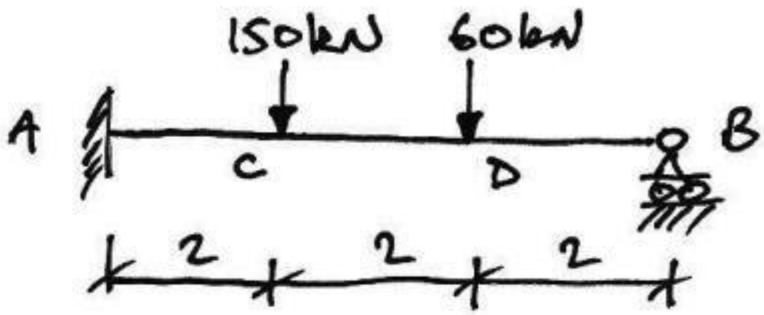
From the collapse BMD it can be seen that now here is exceeded. PM Thus the solution meets the three conditions and so, by the Uniqueness Theorem, is the correct solution.

Example 3- Propped Cantilever with Two Point Loads

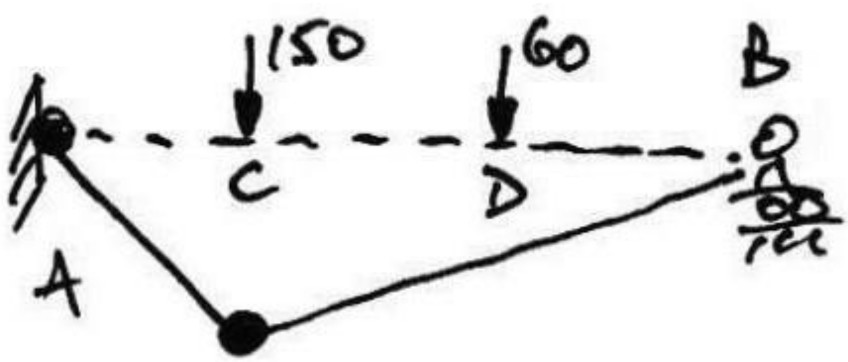
For the following beam, for a load factor of 2.0, find the required plastic moment capacity:



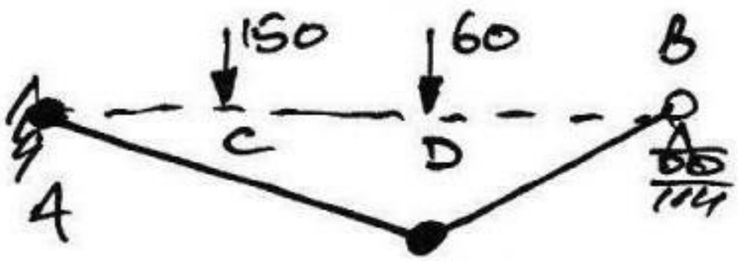
Allowingfortheloadfactor,weneedtodesignthe beamfor thefollowingloads:



Once again we try to picture possible failure mechanisms. Since maximum moments occur underneath point loads, there are two real possibilities:



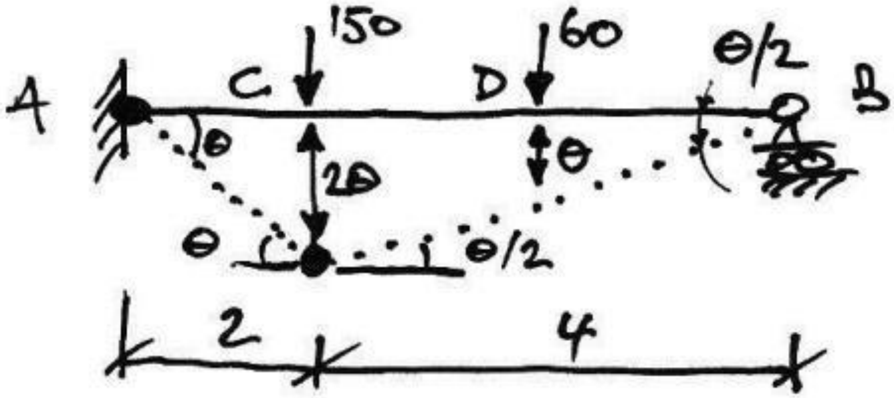
Mechanism-1



Mechanism-2

Therefore, we analyse both and apply the Upper bound Theorem to find the design plastic moment capacity.

Mechanism1: Plastic Hingeat *C*:



$$\delta W_e = \delta W_I$$

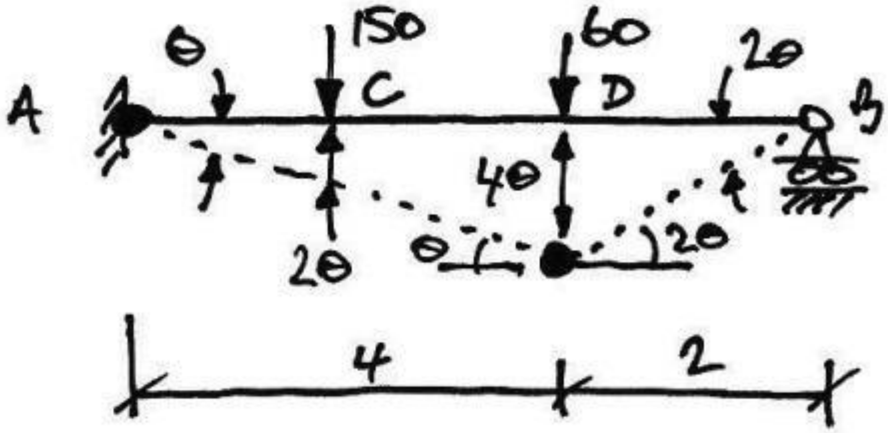
$$75\lambda(2\theta) + 30\lambda(\theta) = \underbrace{M_p(\theta)}_{\text{At } A} + \underbrace{M_p\left(\theta + \frac{\theta}{2}\right)}_{\text{At } C}$$

$$180\lambda\theta = \frac{5}{2}M_p\theta$$

$$M_p = \frac{2}{5}\cdot 180\lambda$$

But the load factor, $\lambda = 2.0$, giving $M_p = 144$ kNm.

Mechanism2:PlasticHingeat*D*:



$$\delta W_e = \delta W_I$$

$$75\lambda(2\theta) + 30\lambda(4\theta) = \underbrace{M_P(\theta)}_{\text{At } A} + \underbrace{M_P(\theta + 2\theta)}_{\text{At } D}$$

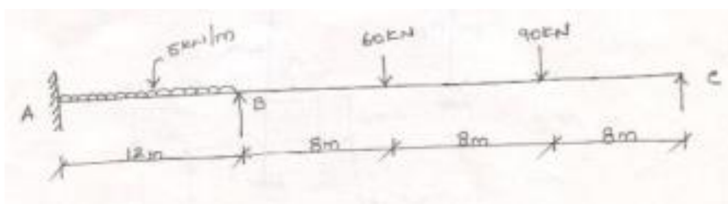
$$270\lambda\theta = 4M_P\theta$$

$$M_P = \frac{270}{4}\lambda$$

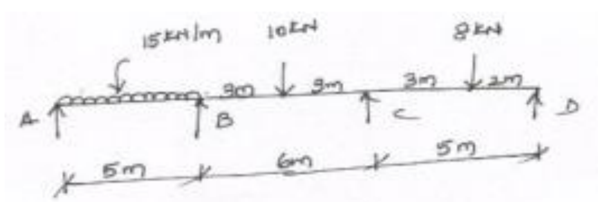
Using $\lambda = 2.0$ then gives $M_P = 135$ kNm.

So by the application of the Upperbound theorem for the design plastic capacity, we choose $M_P = 144$ kNm as the design moment and recognize Mechanism 1 to be the correct failure mechanism. We check this by the Uniqueness Theorem:

1. Draw the stress-strain diagram for mild steel.
2. List the assumptions in plastic theory.
3. Define load factor.
4. sketch an idealized stress strain relation for structural steel.
5. Define the term “Shape factor”.
6. Define: Plastic moment.
7. Determine the collapse load for a propped cantilever with UDL.
8. A beam ABC of span “L” is fixed at ends A&C and carries a point load at a distance L/4 from the left end. Find the value of the load at collapse if the left half of the beam has 2Mp and the right half has 1Mp.
9. A continuous beam ABC is loaded as shown in fig. Determine the required plastic moment if the load factor is3.2.



10. Determine the value of Mp for the continuous beam shown in fig. The moment capacity of the beam AB, BC and CD are Mp, 2Mp, 2Mp respectively.



11. Find the collapse load Wc for the continuous beam shown in fig. The bean has uniform plastic moment Mp.