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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – I- SOIL PROPERTIES – SCI1304

SOIL PROPERTIES

OBJECTIVES AND VALUES OF GEOTECHNICAL ENGINEERING:

Introduction:

The term "**soil**" can have different meanings, depending upon the field in which it is considered.

To a geologist, it is the material in the relative thin zone of the Earth's surface within which roots occur, and which are formed as the products of past surface processes. The rest of the crust is grouped under the term "rock".

To an engineer, it is a material that can be:

- **built on:** foundations of buildings, bridges
- **built in:** basements, culverts, tunnels
- **built with:** embankments, roads, dams
- **supported:** retaining walls

Soil Mechanics is a discipline of Civil Engineering involving the study of soil, its behaviour and application as an engineering material.

Soil Mechanics is the application of laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles, which are produced by the mechanical and chemical disintegration of rocks, regardless of whether or not they contain an admixture of organic constituents. Soil consists of a multiphase aggregation of solid particles, water, and air. This fundamental composition gives rise to unique engineering properties, and the description of its mechanical behavior requires some of the most classic principles of engineering mechanics.

Engineers are concerned with soil's mechanical properties: permeability, stiffness, and strength. These depend primarily on the nature of the soil grains, the current stress, the water content and unit weight.

Formation of soil:

In the Earth's surface, rocks extend upto as much as 20 km depth. The major rock types are categorized as igneous, sedimentary, and metamorphic.

- **Igneous rocks:** formed from crystalline bodies of cooled magma.
- **Sedimentary rocks:** formed from layers of cemented sediments.
- **Metamorphic rocks:** formed by the alteration of existing rocks due to heat from igneous intrusions or pressure due to crustal movement.

Soils are formed from materials that have resulted from the disintegration of rocks by various processes of physical and chemical weathering. The nature and structure of a given soil depends on the processes and conditions that formed it:

- **Breakdown** of parent rock: weathering, decomposition, erosion.
- **Transportation** to site of final deposition: gravity, flowing water, ice, wind.
- **Environment** of final deposition: flood plain, river terrace, glacial moraine, lacustrine or marine.
- **Subsequent conditions** of loading and drainage: little or no surcharge, heavy surcharge due to ice or overlying deposits, change from saline to freshwater, leaching, contamination.

All soils originate, directly or indirectly, from different rock types.

Soil types:

Soils as they are found in different regions can be classified into two broad categories:

(1) Residual soils

(2) Transported soils

Residual Soils

Residual soils are found at the same location where they have been formed. Generally, the depth of residual soils varies from 5 to 20 m.

Chemical weathering rate is greater in warm, humid regions than in cold, dry regions causing a faster breakdown of rocks. Accumulation of residual soils takes place as the rate of rock decomposition exceeds the rate of erosion or transportation of the weathered material. In humid regions, the presence of surface vegetation reduces the possibility of soil transportation.

As leaching action due to percolating surface water decreases with depth, there is a corresponding decrease in the degree of chemical weathering from the ground surface downwards. This results in a gradual reduction of residual soil formation with depth, until unaltered rock is found.

Residual soils comprise of a wide range of particle sizes, shapes and composition.

PHYSICAL PROPERTIES OF SOIL:

Features of the soil profile and the soil horizons are often described in the field in terms of the soil's physical properties. Horizons are defined based on difference in the physical properties. Soil physical properties affect the appearance and feel of a soil.

The major soil physical properties are:

- Soil Texture
- Soil Structure

- Soil Consistence/Soil Strength
- Soil Color
- Soil Permeability
- Soil Temperature

1. **<u>Soil texture:</u>**

Each soil separate represents a distinct physical size group. Mineral particles less than 2 millimeters in equivalent diameter and ranging between specified size limits. The names and sizes of the soil separates recognized in the United States are as follows

Very Coarse Sand	2.0 - 1.0 mm
Coarse Sand	1.0 - 0.5 mm
Medium Sand	0.5 - 0.25 mm
Fine Sand	0.25 - 0.10 mm
Very Fine Sand	0.10 - 0.05 mm
Silt	0.05 - 0.002 mm
Clay	0.002 mm

General classification is as follows:

Sand	2.0 - 0.05 mm
Silt	0.05 - 0.002 mm
Clay	<0.002 mm

2. **<u>Soil structure:</u>**

Structure is the arrangement of primary sand, silt and clay particles into secondary aggregates called peds or structural units which have distinct shapes and are easy to recognize. These differently shaped aggregates are called the structural type.

The five basic types of structural units are as follows:

a. Platy:

- Plate-like aggregates that form parallel to the horizons like pages in a book.
- This type of structure may reduce air, water and root movement.
- common structure in an E horizon and usually not seen in other horizons.

b. Blocky:

- Two types--angular blocky and subangular blocky
- These types of structures are commonly seen in the B horizon.

- Angular is cube-like with sharp corners while subangular blocky has rounded corners.

c. Prismatic:

- Vertical axis is longer than the horizontal axis. If the top is flat, it is referred to as prismatic.
- If the top is rounded, it is called columnar.

d. Granular:

Peds are round and pourous, spheroidal. This is usually the structure of A horizons.

e. Structureless

No observable aggregation or structural units.

- Single grain-sand
- Massive-solid mass without aggregates

3. <u>Soil Consistence</u>

Expresses cohesive and adhesive forces holding soil particles together; varies with moisture content. Describes the resistance of a soil at various moisture contents to mechanical stress or manipulation. It is described at three moisture levels:

a. Wet

Stickiness (non-sticky, slightly sticky, sticky,very sticky)
Plasticity (non-plastic, slightly plastic, plastic, very plastic)

b. Moist

Very friable, friable, firm ,very firm

c. Dry

Loose.soft. slightly hard,hard, very hard ,extremely hard

It **indicates** amount and type of clay material, condition for tillage and potential for compaction. Consistence is the resistence of the soil to deform or rupture. Soil consistence is the forces of cohesion and adhesion that are holding the peds together. It refers to the degree of plasticity and stickiness of the soil. Soil consistence is affected by the type and amount of clay that is in the soil.

Soil consistence indicates:

- where are zones that may restrict root growth and seedling emergence.
- whether a soil is likely to develop compacted zones; ruts, crusts, hardpans etc.

Determining soil consistence

- Fingers - squeeze aggregates or push fingers into the soil.
- Penetrometer - measures how hard it is to push into the soil. this would be the same effect as a plant root.
- Examine roots : J roots or a root mat indicate problems.

Sand has a very weak consistence, there is little force between the particles. This means that a car tire can easily push the sand apart and it is easy to get stuck.

Factors Affecting Soil Consistence

- Water Content
- Soil Texture
- Soil Density

3.Soil colour:

It is the most obvious and easily determined soil property It has little direct effect on the soil, but is an indicator of soil properties. However, there are many things we can tell about the soil by observing the color.

- Organic matter content; the more organic content the darker the soil color
- Soil color and soil temperature : dark colored soils absorb more heat so they warm up quicker and have higher soil temperatures.
- Soil color and parent material : generally dark parent material will develop into dark soils.
- Soil color and drainage:

  - soil drainage refers to the length of time a soil is waterlogged. Not how fast the soil is drained.

4. Soil permeability:

Permeability is the speed of air and water movement in a soil -- this is affected by texture and structure

1. if permeability is high : water moves quickly
2. if permeability is low : water moves slowly

Drainage is the frequency and duration of saturation. The time that the soil is waterlogged. -
- this is affected by landscape position and permeability

Another way to view this is; drainage refers to the amount of oxidation which has taken place in the soil and permeability

A clay could be very permeable, but in a low landscape position and be poorly drained.

PHASE RELATION OF SOIL:

Soil is not a coherent solid material like steel and concrete, but is a particulate material. Soils, as they exist in nature, consist of solid particles (mineral grains, rock fragments) with water and air in the voids between the particles. The water and air contents are readily changed by changes in ambient conditions and location.

As the relative proportions of the three phases vary in any soil deposit, it is useful to consider a soil model which will represent these phases distinctly and properly quantify the amount of each phase. A schematic diagram of the three-phase system is shown in terms of weight and volume symbols respectively for soil solids, water, and air. The weight of air can be neglected.

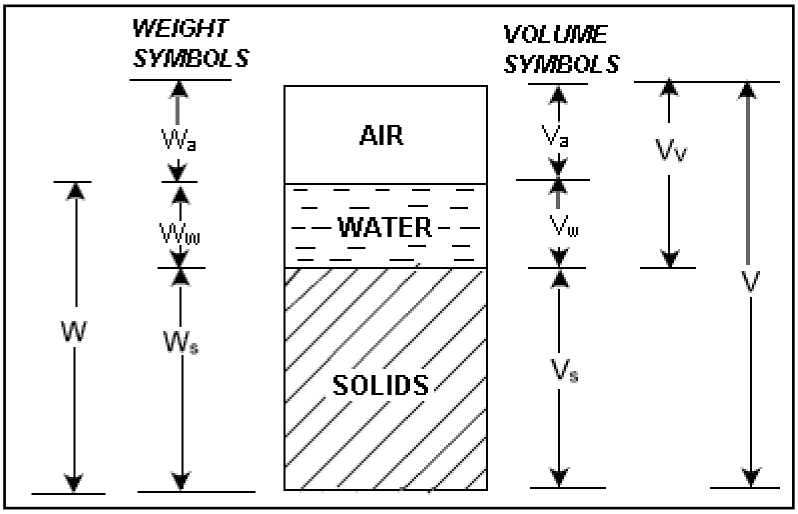


Fig-1 Phase relation of soil

The soil model is given dimensional values for the solid, water and air components. Total volume, $V = V_s + V_w + V_v$

Three phase system:

Soils can be partially saturated (with both air and water present), or be fully saturated (no air content) or be perfectly dry (no water content). In a saturated soil or a dry soil, the three-phase system thus reduces to two phases only, as shown.

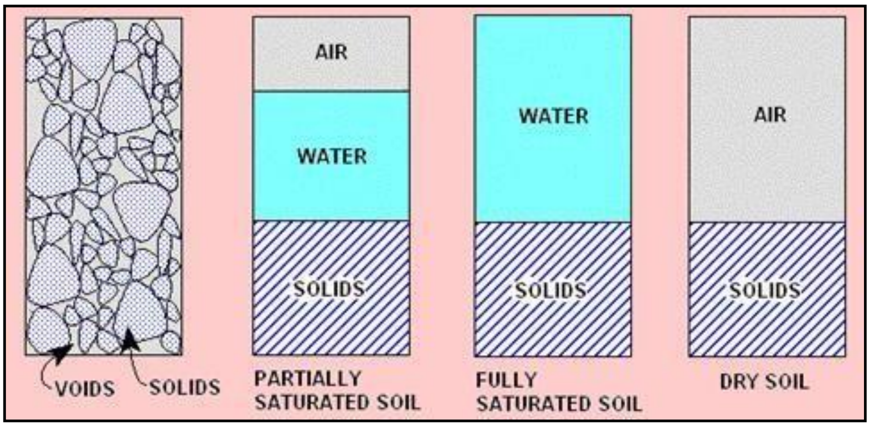


Fig-2 Phase relation of soil

For the purpose of engineering analysis and design, it is necessary to express relations between the weights and the volumes of the three phases.

The various relations can be grouped into:

1. Volume relations
2. Weight relations
3. Inter-relations

Volume relation :

As the amounts of both water and air are variable, the volume of solids is taken as the reference quantity. Thus, several relational volumetric quantities may be defined. The following are the **basic volume relations:**

1. Void ratio (e) is the ratio of the volume of voids (V_v) to the volume of soil solids (V_s), and is expressed as a decimal.

$$e = \frac{V_V}{V_S}$$

2. Porosity (n) is the ratio of the volume of voids to the total volume of soil (V), and is expressed as a percentage. $n = \frac{V_V}{V} x\,100$

Void ratio and porosity are inter-related to each other as follows:

$$e = \frac{n}{1-n} \quad \text{and} \quad n = \frac{e}{(1+e)}$$

3. The volume of water (V_w) in a soil can vary between zero (i.e. a dry soil) and the volume of voids. This can be expressed as the **degree of saturation (S)** in percentage.

$$S = \frac{V_W}{V_V} x\,100$$

For a dry soil, S = 0%, and for a fully saturated soil, S = 100%.

4. Air content (a_c) is the ratio of the volume of air (V_a) to the volume of voids.

$$a_c = \frac{V_a}{V_V}$$

5. Percentage air voids (n_a) is the ratio of the volume of air to the total volume.

$$n_a = \frac{V_a}{V} x\, 100 = n\ x\ a_c$$

Weight relation :

Density is a measure of the quantity of mass in a unit volume of material. Unit weight is a measure of the weight of a unit volume of material. Both can be used interchangeably. The units of density are ton/m³, kg/m³ or g/cm³. The following are the **basic weight relations:**

1. The ratio of the mass of water present to the mass of solid particles is called the **water content (w)**, or sometimes the **moisture content.**

$$w = \frac{W_W}{W_S}$$

Its value is 0% for dry soil and its magnitude can exceed 100%.

2. The mass of solid particles is usually expressed in terms of their **particle unit weight** (γ_s) or **specific gravity** (**G_s**) of the soil grain solids .

$$\gamma_s = \frac{W_s}{V_s} = G_s . \gamma_W$$

where γ_W = Unit weight of water

For most inorganic soils, the value of G_s lies between 2.60 and 2.80. The presence of organic material reduces the value of G_s.

3. Dry unit weight (γ_d) is a measure of the amount of solid particles per unit volume.

$$\gamma_d = \frac{W_s}{V}$$

4. Bulk unit weight $(\gamma_t \text{ or } \gamma)$ is a measure of the amount of solid particles plus water per unit volume.

$$\gamma_t = \gamma = \frac{(W_s + W_W)}{(V_s + V_V)}$$

5. Saturated unit weight (γ_{sat}) is equal to the bulk density when the total voids is filled up with water.

6. Buoyant unit weight (γ') or **submerged unit weight** is the effective mass per unit volume when the soil is submerged below standing water or below the ground water table.

$$\gamma' = \gamma_{sat} - \gamma_W$$

Inter relation :

It is important to quantify the state of a soil immediately after receiving in the laboratory and prior to commencing other tests. The water content and unit weight are particularly important, since they may change during transportation and storage.

Some physical state properties are calculated following the practical measurement of others.

For example, dry unit weight can be determined from bulk unit weight and water content.

The following are some inter-relations:

1. $$w = \frac{W_W}{W_s} = \frac{\gamma_W . V_W}{G_s . \gamma_W . V_s} = \frac{V_W}{G_s . V_s} = \frac{S . V_v}{G_s . V_s} = \frac{S . e}{G_s}$$

2. $$\gamma = \frac{(G_s + S . e) . \gamma_w}{1 + e}$$

3. $$\gamma = \frac{(1 + w) . G_s . \gamma_w}{1 + e}$$

4. $$\gamma_d = \frac{G_s . \gamma_w}{1 + e}$$

5. $$\gamma_d = \frac{\gamma}{1 + w}$$

6. $$\gamma' = \frac{[(G_s - 1) + (S - 1)e] \; x \; \gamma_w}{1 + e}$$

7. $$\gamma' = \frac{(G_s - 1) . \gamma_w}{1 + e}$$

Numerical:

1. A soil sample has a porosity of 40 percentage. The specific gravity of solids is 2.7. Calculate (a) voids ratio (b) Dry density (c) Unit weight is the soil is 50 % saturated (d) Unit weight if the soil is fully saturated.

Solution:

(a) $e = n/(1-n)$

$= 0.4/(1-0.4)$

$e = 0.67$

(b) $\gamma_d = G\gamma_w/(1+e)$

$= 15.89\ KN/m^3$

(c) $s = 0.5$

$es = Gw$

$w = eS/G$

$w = 0.124$

(d) $S = 1$

$w = eS/G$

$= 0.67 x 1/2.7$

$w = 0.247$

$\gamma_{sat} = \gamma_d(1+w)$

$\gamma_{sat} = 19.81\ KN/m^3$

DETERMINATION OF INDEX PROPERTIES:

Properties which are used in identification and classification of soil are called index properties of the soil

- Water content
- Specific gravity
- Particle distribution
- Consistency limits
- In-situ density
- Density index.

Methods for determining water content:

Water content of a soil sample can be determined from the following methods:

- Oven drying method
- Sand bath method
- Alcohol method
- Calcium carbide method
- Pycnometer method

- Radiation method
- Torsion balance method

GRAIN SIZE DISTRIBUTION :

It is also called mechanical analysis. It is used for the separation of soil into different fractions.

Methods to determine particle size distribution :

<u>Sieve analysis:</u>

Sieve size is denoted by the number of openings per square inch. Oven dried sample is separated into two fractions by sieving it through 4.75 mm IS sieve. Portion retained on 4.75 mm sieve is the gravel fraction which is used for coarse analysis. Portion passing through mm is used for fine analysis.

<u>Sedimentation analysis:</u>

For particles ≤ 0.05 mm (silt and clay fractions) sedimentation methods based on Stokes law are used to deduce particle size distribution. Soil particles settle in aqueous solution attaining terminal velocities proportional to their mass and size. The amount of suspended soil after a given settling time is used to determine particle size fractions. •The amount of soil in suspension is determined by either extracting a sample by the pipette method or from a direct hydrometer reading.

According to Stoke's law , velocity at which grains settle out of suspension , al other factors being equal ,is dependent on the weight and size of the grain assuming that the soil particles are spherical .

$$V = \frac{h}{t} = \frac{(\rho_s - \rho_1)d^2 g}{18\eta} \Rightarrow t = \frac{18\eta h}{(\rho_s - \rho_1)d^2 g}$$

Where V is the terminal velocity of the particles.Stokes' s law is valid only for grain sizes between 0.2 mm to 0.0002 mm .If the size is greater than 0.2 mm turbulency develops and stokes law becomes invalid. If the size is less than 0.0002 mm , particle will not settle properly and brownian movement takes place.

<u>PARTICLE SIZE DISTRIBUTION CURVE</u> :

It is also called as the grading curve.It reveals if the soil is coarse or fine grained soil.The distribution of particles of different sizes in a soil mass is called grading.

The results of mechanical analysis are generally presented by semi-logarithmic plots known as particle-size distribution curves. The particle diameters are plotted in log scale along the x axis , and the corresponding percent finer in arithmetic scale along the y axis.

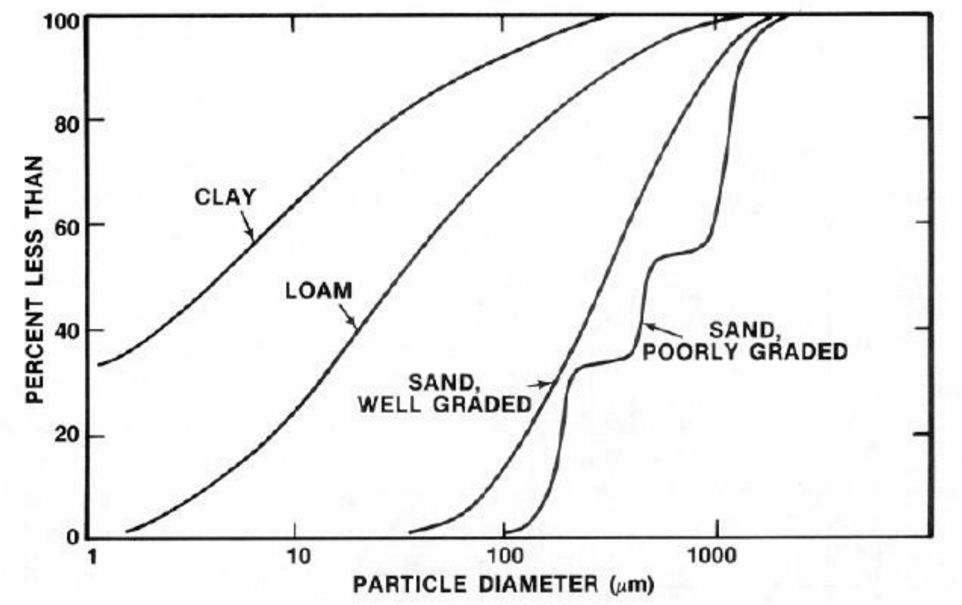


Fig -3 Particle size distribution curve

Three basic soil parameters can be determined from these grain-size distribution curves:

- Effective size
- Uniformity coefficient
- Coefficient of gradation

The diameter in the particle-size distribution curve corresponding to 10% finer is defined as the *effective size,* or *D*10.

Uniformity co efficient:

$C_u = D_{60} / D_{10}$

Where *D*60 is the diameter corresponding to 60% finer in the particle-size distribution. $C_u < 2$ – It is poorly or uniformly graded soil

$C_u > 6$ – It is well graded soil

Co efficient of gradation :

It is also called the co efficient of curvature. It represents the general shape of the particle size distribution curve.

$C_c = D_{30}^2 / (D_{10} \times D_{60})$

Where *D*30 is the diameter corresponding to 30% finer in the particle-size distribution.

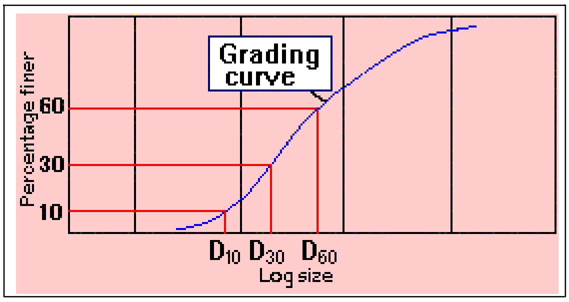


Fig-4 Grading curve

Based on type of distribution of various soil sizes, the sample is classified as poorly graded , well graded or gap graded soil.

Uniformly or poorly graded soil :

The particle-size distribution in which most of the soil grains are the same size is called a uniformly graded soil.

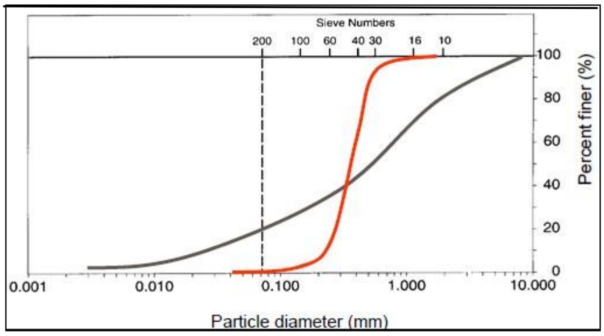


Fig- 5 Uniformly graded soil

Well graded soil:

A sample which has a good representation of particles of all sizes is called well graded soil.

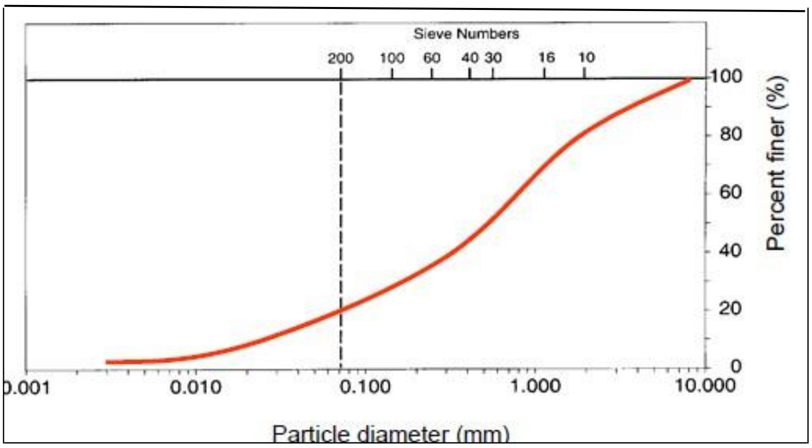


Fig- 6 Well graded soil

Skip or gap graded soil:

A sample in which some of the intermediate sizes of soil are missing is called skip graded soil.

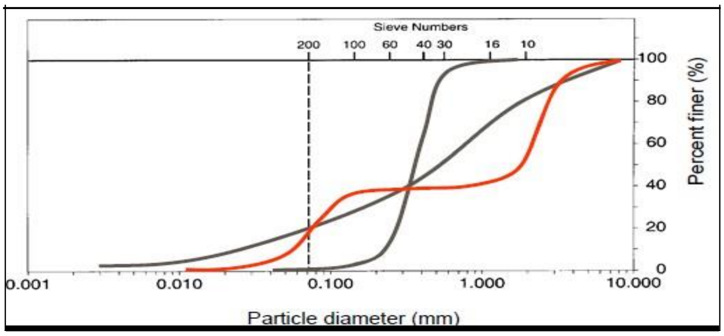


Fig-7 skip graded soil

<u>ATTERBERG'S LIMIT / CONSISTENCY LIMITS:</u>

Relative ease with which the soil can be deformed is called consistency. It denotes the degree of firmness of the soil which may be termed as soft, firm, stiff or hard.

Atterberg's limits:

Water contents at which soil mass passes from one state to another is called consistency limits. The various states through which the soil passes by are solid ,semi solid , plastic and liquid states.

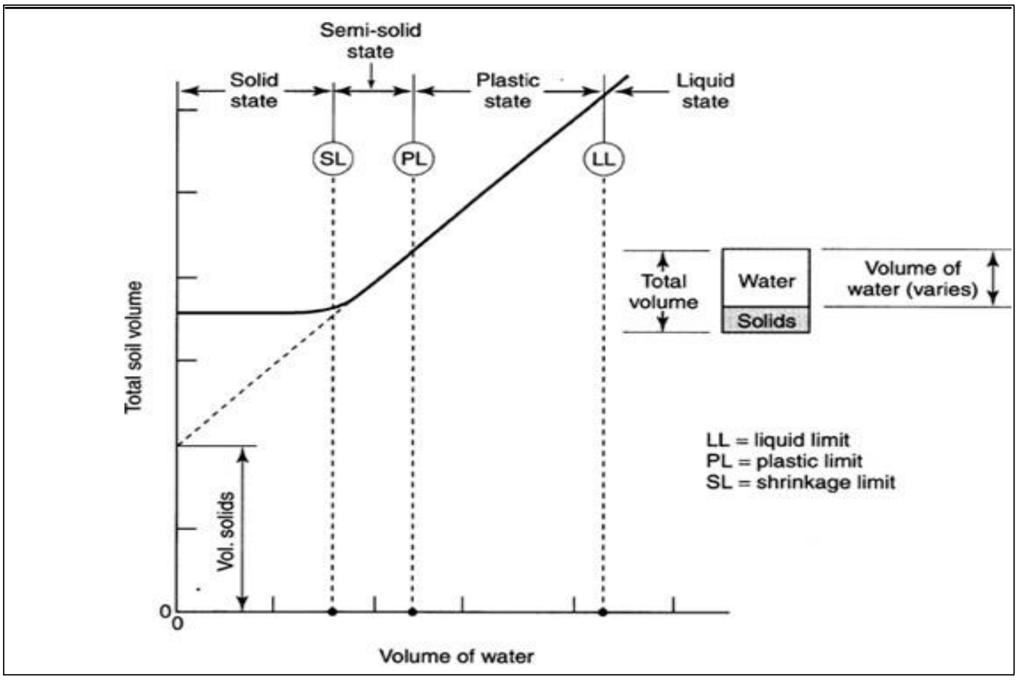


Fig –8 Consistency limits

Liquid limit (w_L):

It is defined as the minimum water content at which part of the soil cut by a groove of standard dimensions will flow together for a distance of half an inch under impact of 25 blows in the device.

Determination of liquid limit :

It is determined using Casagrande's apparatus. It consists of the following parts .

(i) Hard rubber base
(ii) Brass cup
(iii) Handle
(iv) Adjusting screws.

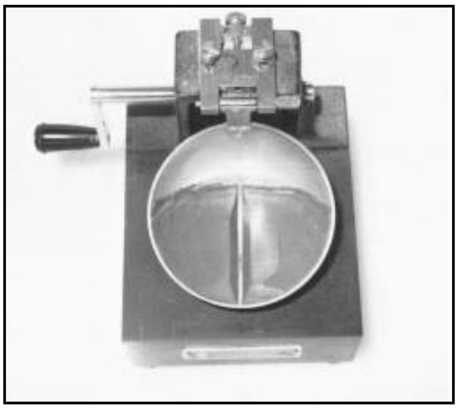


Fig -9 Casagrande apparatus

The handle is rotated at a rate of 2 rps. Number of blows are counted until two parts of the soil sample come into contact at the bottom of the groove along a distance of 10mm . Water content corresponding to 25 blows is called liquid limit.

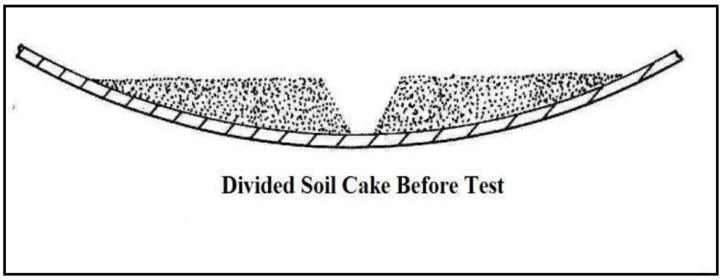


Fig –10 Soil sample before testing with groove

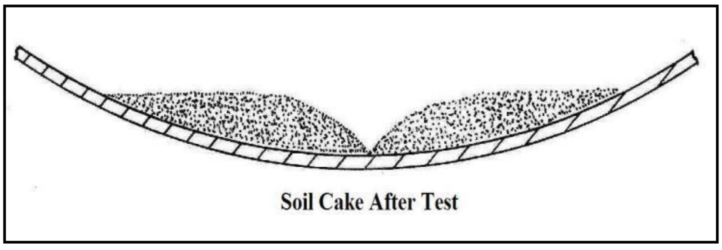


Fig –11 soil sample after test with groove closed.

Flow index is determined by plotting number of blows on logarithmic scale on x axis and water content along y axis.

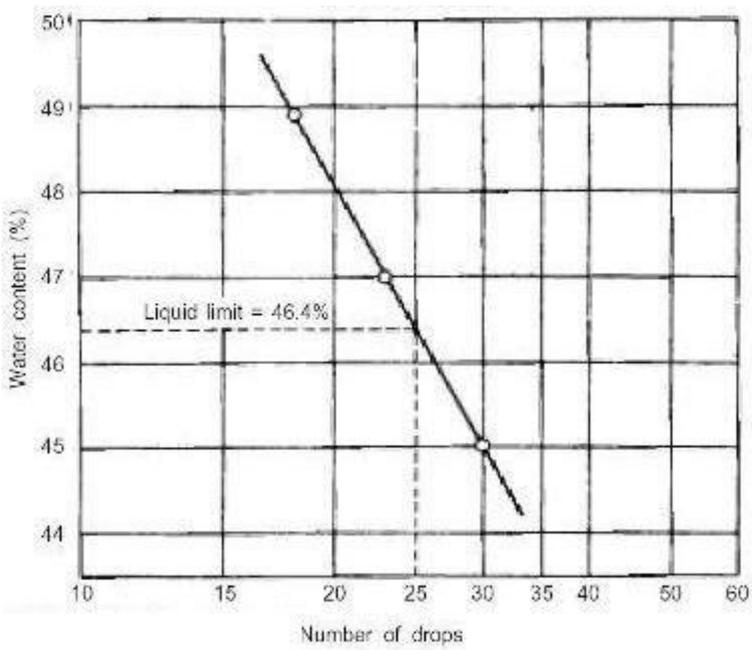


Fig-12 flow curve

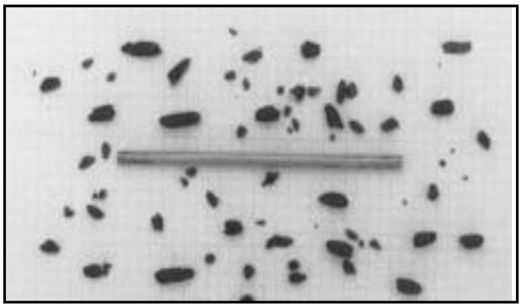
Slope of the flow curve gives the value of flow index.

$I_f = w_1 - w_2 / (\log N_2/N_1)$

Where w_1 is the water content corresponding to number of blows N_1 and w_2 is the water content corresponding to number of blows N_2.

Plastic limit(w_P):

Soil specimen passing through 425 microns is mixed with distilled water to form a plastic paste. It is rolled on a glass plate to make a thread of uniform diameter. Water content at which 3 mm diameter thread starts crumbling is called plastic limit of the soil.



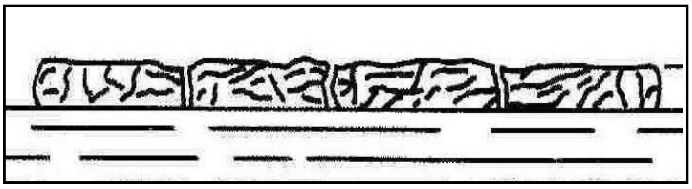


Fig –13 Crumbling of the soil particles.

Shrinkage limit:

It is the maximum water content at which reduction in water content will not cause decrease in volume of the soil mass. It is the lowest water content at which soil can still be saturated.

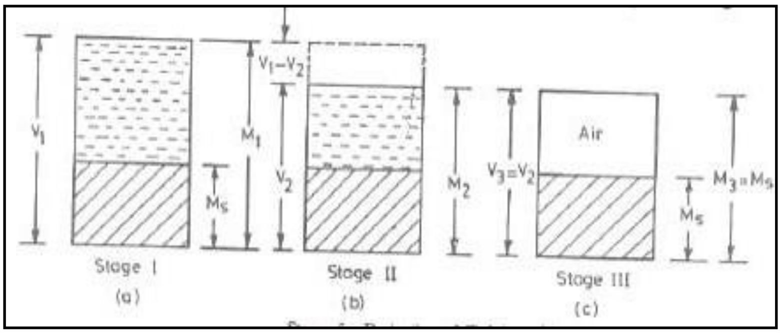


Fig –14 shrinkage limit

Shrinkage ratio:

Ratio of a given volume change expressed as a percentage of dry volume, to the corresponding change in water content above the shrinkage limit expressed in percentage.

S.R= $\{[(v_1-v_2)/v_d]/(w_1-w_2)\}$x 100

Where

v_1 is the volume of the soil at water content w_1 v_2 is

the volume of the soil at water content w_2. V_d is the

volume of the dry soil mass.

But $v_2=v_d$ and $w_2=w_s$

Hence

S.R= $\{[(v_1-v_d)/v_d]/(w_1-w_s)\}$x 100.

Plasticity index(I_P):

It refers to the range of water content over which the soil will remain in the plastic state. It is equal to the difference between liquid and plastic limit.

$I_P=w_L-w_P$

Consistency index(I_C):

It defines the firmness of the soil. It indicates the nearness of the water content to its plastic limit.

$I_C = \{(w_L - w)/I_P\} \times 100$

Where I_C is the consistency index

w_L is the liquid limit

w is the natural water content I_p

is the plasticity index.

Liquidity index(I_L):

It indicates the nearness of the soil to the liquid limit.

$I_L = \{(w - w_P)/I_P\} \times 100$

Where I_C is the liquidity index

w_P is the plastic limit

w is the natural water content

I_p is the plasticity index.

Activity of clay :

Ratio of plasticity index to percentage by weight of soil particles of diameter smaller than 2 microns present in the soil.

Sensitivity of clay:

Ratio of unconfined compressive strength in the natural or undisturbed state to that in remoulded state without change in water content

Thixotropy of clay:

When sensitive clays are used in construction ,they loose strength due to remoulding .With passage of time, strength again increases though not to the same level. This phenomenon of "strength loss-strength gain " with no change in volume or water content is called thixotropy.

Numerical:

1. The following test results were obtained for a fine-grained soil:
   W_L= 48% ; W_P = 26%
   Clay content = 55%
   Silt content = 35%
   Sand content = 10%
   In situ moisture content = 39% = w

Classify the soil, and determine its activity and liquidity index

Solution:

Plasticity index, $I_P = W_L - W_P = 48 - 26 = 22\%$

Liquid limit lies between 35% and 50%.

According to the Plasticity Chart, the soil is classified as CI, i.e. clay of intermediate plasticity.

$$\Rightarrow Activity = \frac{I_P}{Clay\ content} = \frac{22}{25} = 0.88$$

Liquidity index , $LI = \frac{w - W_P}{I_P} = \frac{39-26}{22} = 0.59$

The clay is of normal activity and is of soft consistency.

SOIL DESCRIPTION & ITS CLASSIFICATION FOR ENGINEERING PURPOSE :

<u>Purpose of classification:</u>

- This standard classifies soils from any geographic location into categories representing the results of prescribed laboratory tests to determine the particle- size characteristics, the liquid limit, and the plasticity index.
- The assigning of a group name and symbol(s) along with the descriptive information required in Practice can be used to describe a soil to aid in the evaluation of its significant properties for engineering use.
- The various groupings of this classification system have been devised to correlate in a general way with the engineering behavior of soils. This standard provides a useful first step in any field or laboratory investigation for geotechnical engineering purposes.
- This standard may also be used as an aid in training personnel in the use of Practice
- This standard may be used in combination with Practice when working with frozen soils.

In engineering point of view, the following classification of soil are available.

- Particle size classification
- Textural classification
- Highway Research Board (HRB) classification
- Unified soil classification and IS classification.

2. **<u>Particle size classification</u>** :

Soils are arranged according to grain size. Grain sizes are gravel, sand,silt and clay.

a. U.S Bureau of soil and Public Road Administration (PRA) system of United States.
b. International soil classification proposed at the International Congress at Washington.
c. M.I.T classification
d. Indian standard classification based on M.I.T system.

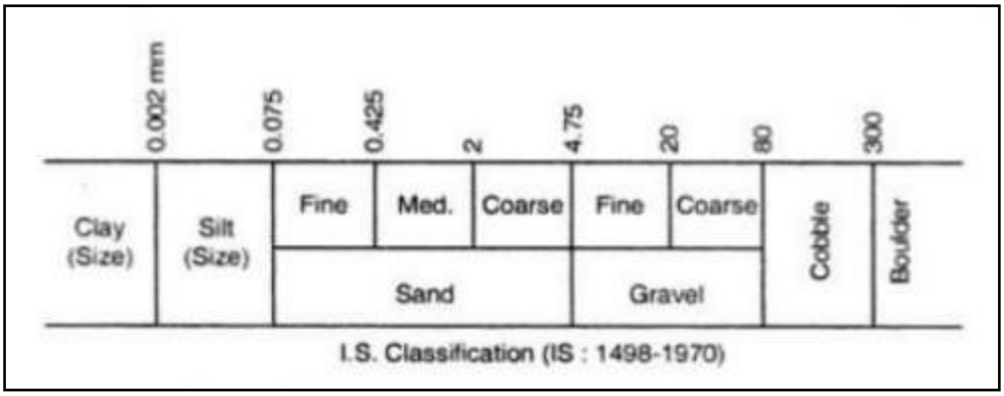


Fig-15 Indian standard classification

3. **Textural classification:**

Soils occuring in nature are composed of different percentage of sand, silt and clay size particles. Soil classification of composite soil exclusively based on particle size distribution is known as textural classification.

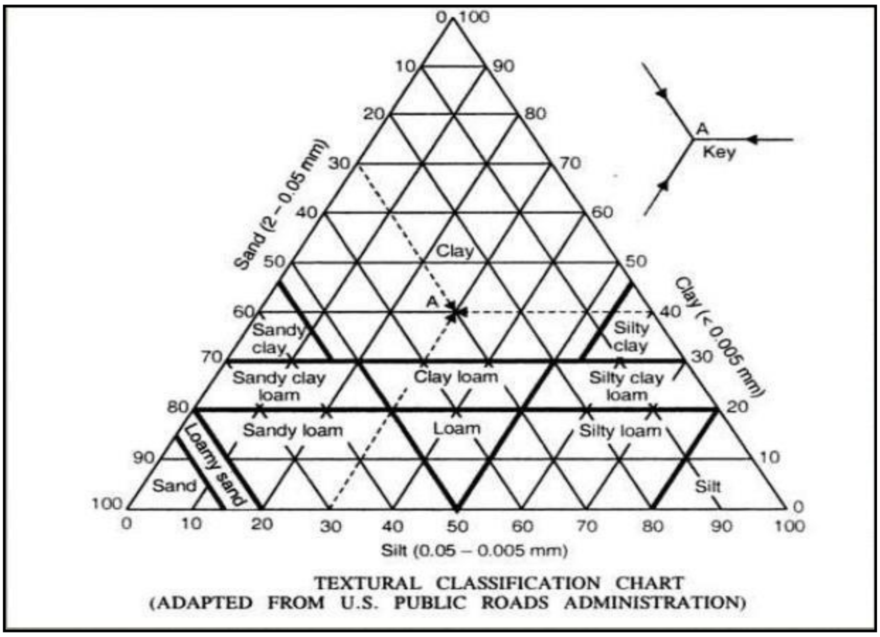


Fig- 16 Textural classification

4. **Highway Research Board classification:**

It is based on particle size composition and plasticity characteristics. It is mostly used for pavement construction. In this soil is divided into 7 groups from A1,A2 to A7.Group index is calculated to describe the performance of soil. It means rating the value of soil as a subgrade material within its own group. Higher the value of group index, poorer is the material.

Group index of a soil depends on

- Amount of soil passing 75 micron sieve.
- Liquid limit
- Plastic limit

$$\text{Group index} = 0.2\,a + 0.005\,ac + 0.01\,bd$$

where a = that portion of percentage passing 75 mircon sieve greater than 35 and not exceeding 75 expressed as a whole number (0 to 40)

b = that portion of percentage passing 75 micron sieve greater than 15 and not exceeding 55 expressed as a whole number (0 to 40)

c = that portion of the numerical liquid limit greater than 40 and not exceeding 60 expressed as positive whole number (0 to 20)

and d = that portion of the numerical plasticity index greater than 10 and not exceeding 30 expressed as a positive whole number (0 to 20).

Indian standard soil classification system :

The grain-size range is used as the basis for grouping soil particles into boulder, cobble, gravel, sand, silt or clay.

Very coarse soils	**Boulder size**		**> 300 mm**
	Cobble size		**80 - 300 mm**
Coarse soils	**Gravel size (G)**	***Coarse***	**20 - 80 mm**
		Fine	**4.75 - 20 mm**
	Sand size (S)	***Coarse***	**2 - 4.75 mm**
		Medium	**0.425 - 2 mm**
		Fine	**0.075 - 0.425 mm**
Fine soils	**Silt size (M)**		**0.002 - 0.075 mm**
	Clay size (C)		**< 0.002 mm**

Gravel, sand, silt, and clay are represented by **group symbols G, S, M, and C** respectively.

Fine-grained soils are those for which more than 50% of the material has particle sizes less than 0.075 mm. Clay particles have a flaky shape to which water adheres, thus imparting the property of plasticity.

A plasticity chart , based on the values of liquid limit (W_L) and plasticity index (I_P), is provided in ISSCS to aid classification. The 'A' line in this chart is expressed as

$I_P = 0.73 (W_L - 20)$.

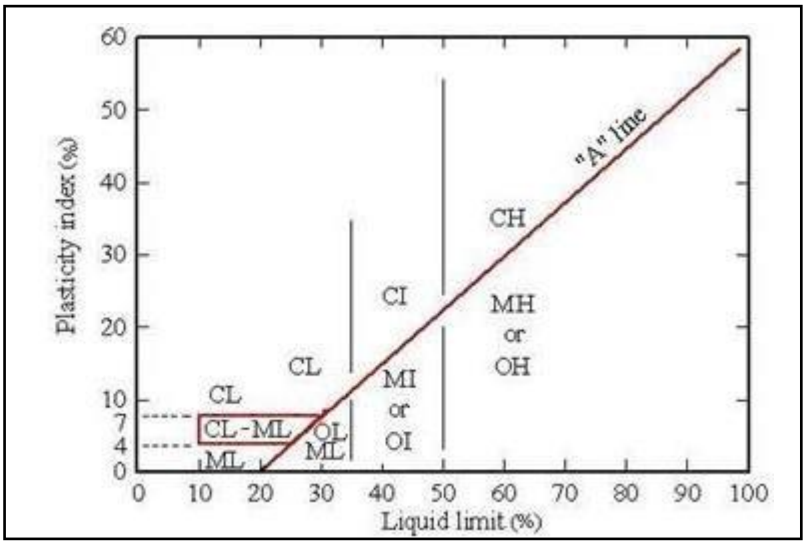


Fig- 17 plasticity chart

Depending on the point in the chart, fine soils are divided into **clays (C)**, **silts (M)**, or **organic soils (O)**. The organic content is expressed as a percentage of the mass of organic matter in a given mass of soil to the mass of the dry soil solids. Three divisions of plasticity are also defined as follows.

Table- Classification based on activity of soil

Low plasticity	**$W_L < 35\%$**
Intermediate plasticity	**$35\% < W_L < 50\%$**
High plasticity	**$W_L > 50\%$**

Group Symbol	Classification
Coarse soils	
GW	Well-graded GRAVEL
GP	Poorly-graded GRAVEL
GM	Silty GRAVEL
GC	Clayey GRAVEL
SW	Well-graded SAND
SP	Poorly-graded SAND
SM	Silty SAND
SC	Clayey SAND
Fine soils	
ML	SILT of low plasticity
MI	SILT of intermediate plasticity
MH	SILT of high plasticity
CL	CLAY of low plasticity
CI	CLAY of intermediate plasticity
CH	CLAY of high plasticity
OL	Organic soil of low plasticity
OI	Organic soil of intermediate plasticity
OH	Organic soil of high plasticity
Pt	Peat

Fig -18 Indian soil classification system

Problems for practice:

1. An undisturbed sample of soil has a volume of 100 cm^3 and mass of 190g.On oven drying for 24 hours, the mass is reduced to 160g.If the specific gravity is 2.68, determine the water content, voids ratio and degree of saturation of the soil.
2. A saturated sample of soil has a moisture content of 22.22% and specific gravity of 2.7.Determine the saturated unit weight and voids ratio.
3. A natural soil deposit has a bulk unit weight of 18.4 KN/m^3 and water content of 5%. Calculate the amount of water required to be added to 1 cubic meter of soil to raise the water content to 15%. Assume the voids ratio to remain constant. What will be the degree of saturation. Assume G=2.67.

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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – II- Soil Water and Flow Of Water – SCI1304

Soil Water and Flow of Water

When a load is applied to soil, it is carried by the water in the pores as well as the solid grains. The increase in pressure within the pore water causes **drainage** (flow out of the soil), and the load is transferred to the solid grains. The rate of drainage depends on the permeability of the soil.

The strength and compressibility of the soil depend on the stresses within the solid granular fabric. These are called effective stresses

The importance of effective stress

The principle of effective stress is fundamentally important in soil mechanics. It must be treated as the basic axiom, since soil behaviour is governed by it. Total and effective stresses must be distinguishable in all calculations: algebraically the *prime* should indicate effective stress, e.g. s´

Changes in water level *below* ground (water table changes) result in changes in effective stresses below the water table. Changes in water level *above* ground (e.g. in lakes, rivers, etc.) **do not** cause changes in effective stresses in the ground below.

Changes in effective stress

- Changes in strength
- Changes in volume

In some analyses it is better to work in *changes* of quantity, rather than in absolute quantities; the effective stress expression then becomes:

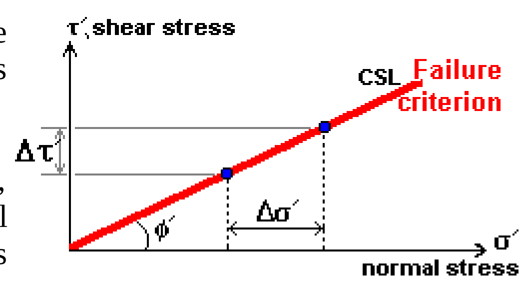
Ds´ = Ds - Du

If both total stress and pore pressure change by the same amount, the effective stress remains constant. A change in effective stress will cause: a change in strength and a change in volume.

Changes in strength

The critical shear strength of soil is proportional to the effective normal stress; thus, a change in effective stress brings about a change in strength.

Therefore, if the pore pressure in a soil slope increases, effective stresses will be reduced by Ds' and the critical strength of the soil will be reduced by Dt - sometimes leading to failure.



A seaside sandcastle will remain intact while damp, because the pore pressure is negative; as it dries, this pore pressure suction is lost and it collapses. Note: Sometimes a sandcastle will remain intact even when nearly dry because salt deposited as seawater evaporates slightly and cements the grains together.

Changes in volume

Immediately after the construction of a foundation on a fine soil, the pore pressure increases, but immediately begins to drop as drainage occurs.

The rate of change of effective stress under a loaded foundation, once it is constructed, will be the same as the rate of change of pore pressure, and this is controlled by the permeability of the soil.

Settlement occurs as the volume (and therefore thickness) of the soil layers change. Thus, settlement occurs rapidly in coarse soils with high permeabilities and slowly in fine soils with low permeabilities.

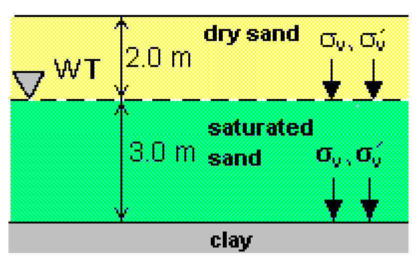
Calculating vertical stress in the ground

- Simple total and effective stresses
- Effect of changing water table
- Stresses under foundations
- Short-term and long-term stresses
- Steady-state seepage conditions

The worked examples here are designed to illustrate the principles and methods dealt with in *Pore pressure, effective stress and stresses in the ground*. The examples chosen are typical and simple.

Simple total and effective stresses

The figure shows soil layers on a site.
Unit weights are:
dry sand: g_d = 16 kN/m³
saturated sand: g_g = 20 kN/m³



(a) At the top of saturated sand (z = 2.0 m)

Vertical total stress	s_v = 16.0 x 2.0	= **32.0 kPa**
Pore pressure	u = 0	
Vertical effective stress	$s'_v = s_v$ - u	= **32.0 kPa**

(b) At the top of the clay (z = 5.0 m)

Vertical total stress s_v = 32.0 + 20.0 x 3.0 = **92.0 kPa**

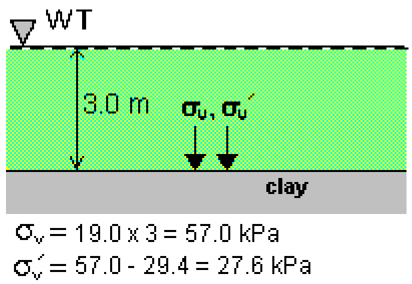
Pore pressure u = 9.81 x 3.0 = 29.4 kPa

Vertical effective stress $s'_v = s_v$ - u = 92.0 - 29.4 = **62.6 kPa**

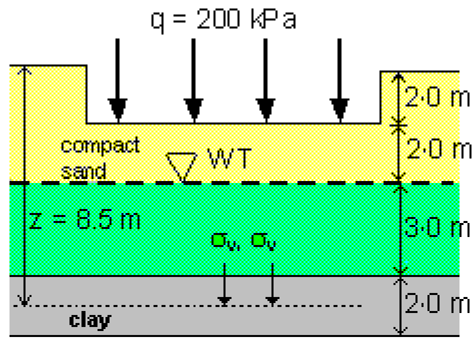
Effect of changing water table

The figure shows soil layers on a site. The unit weight of the silty sand is 19.0 kN/m³ both above and below the water table. The water level is presently at the surface of the silty sand, it may drop or it may rise. The following calculations show the effects of this:

Watertable



Stresses under foundations



From an initial state, the stresses under a foundation are first changed by excavation, i.e. vertical stresses are reduced. After construction the foundation loading increases stresses. Other changes could result if the water table level changed.

The figure shows the elevation of a foundation to be constructed in a homogeneous soil. The change in thickness of the clay layer is to be calculated and so the initial and final effective stresses are required at the mid-depth of the clay.

Unit weights: sand above WT = 16 kN/m³, sand below WT = 20 kN/m³, clay = 18 kN/m³.

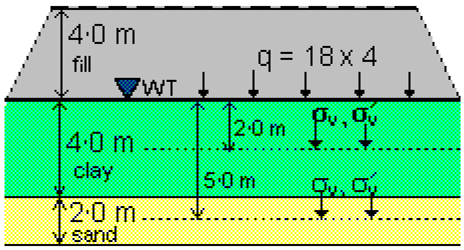
Calculations for

initial stresses

final stresses

Short-term and long-term stresses

- Initially, before construction
- Immediately after construction
- Many years after construction



The figure shows how an extensive layer of fill will be placed on a certain site.

The unit weights are:

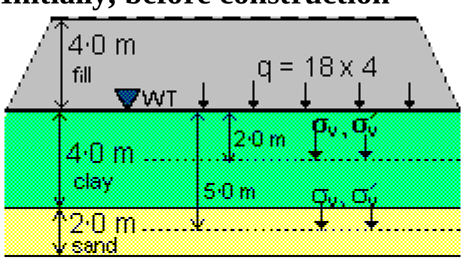
clay and sand = 20kN/m³ ,

rolled fill 18kN/m³ ,

assume water = 10 kN/m³.

Calculations are made for the total and effective stress at the mid-depth of the sand and the mid-depth of the clay for the following conditions: initially, before construction; immediately after construction; many years after construction.

Initially, before construction



Initial stresses at mid-depth of clay (z = 2.0m)

Vertical total stress

s_v = 20.0 x 2.0 = **40.0kPa**

Pore pressure

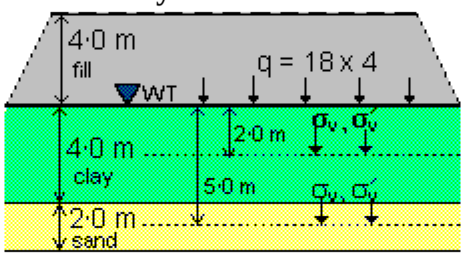
u = 10 x 2.0 = 20.0kPa

Vertical effective stress

s'_v = s_v - u = **20.0kPa**

Initial stresses at mid-depth of sand (z = 5.0 m)
Vertical total stress
s_v = 20.0 x 5.0 = **100.0 kPa**
Pore pressure
u = 10 x 5.0 = 50.0 kPa
Vertical effective stress
s'_v = s_v - u = **50.0 kPa**

Immediately after construction



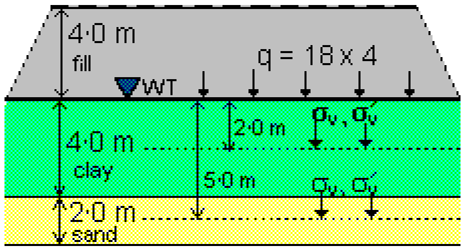
The construction of the embankment applies a surface surcharge: q = 18 x 4 = 72.0 kPa.

The sand is drained (either horizontally or into the rock below) and so there is no increase in pore pressure. The clay is undrained and the pore pressure increases by 72.0 kPa.

Initial stresses at mid-depth of clay (z = 2.0m)
Vertical total stress
s_v = 20.0 x 2.0 + 72.0 = **112.0kPa**
Pore pressure
u = 10 x 2.0 + 72.0 = 92.0 kPa
Vertical effective stress
s'_v = s_v - u = **20.0kPa**
(i.e. no change immediately)

Initial stresses at mid-depth of sand (z = 5.0m)
Vertical total stress
s_v = 20.0 x 5.0 + 72.0 = **172.0kPa**
Pore pressure
u = 10 x 5.0 = 50.0 kPa
Vertical effective stress
s'_v = s_v - u = **122.0kPa**
(i.e. an immediate increase)

Many years after construction



After many years, the excess pore pressures in the clay will have dissipated. The pore pressures will now be the same as they were initially.

Initial stresses at mid-depth of clay (z = 2.0 m)
Vertical total stress
s_v = 20.0 x 2.0 + 72.0 = **112.0 kPa**
Pore pressure
u = 10 x 2.0 = 20.0 kPa
Vertical effective stress
$s'_v = s_v$ - u = **92.0 kPa**
(i.e. a long-term increase)

Initial stresses at mid-depth of sand (z = 5.0 m)
Vertical total stress
s_v = 20.0 x 5.0 + 72.0 = **172.0 kPa**
Pore pressure
u = 10 x 5.0 = 50.0 kPa
Vertical effective stress
$s'_v = s_v$ - u = **122.0 kPa**
(i.e. no further change)

Steady-state seepage conditions

The figure shows seepage occurring around embedded sheet piling.
In steady state, the hydraulic gradient,
i = Dh / Ds = 4 / (7 + 3) = 0.4
Then the effective stresses are:
s'_A = 20 x 3 - 2 x 10 + 0.4 x 10 = 44 kPa
s'_B = 20 x 3 - 2 x 10 - 0.4 x 10 = 36 kPa

Introduction - Permeability.

Permeability of the soil quantitatively describes how easily water can flow through it. In loose soil, amount of pores within the soil grains is more. Water can flow easily through loose soils. However, in case of dense soil, amount of pores within the soil grains is less. Water can not flow easily through dense soils (as shown in Figure 17.1). Thus, permeability is high in case of loose soil whereas, it is low in case of dense soil.

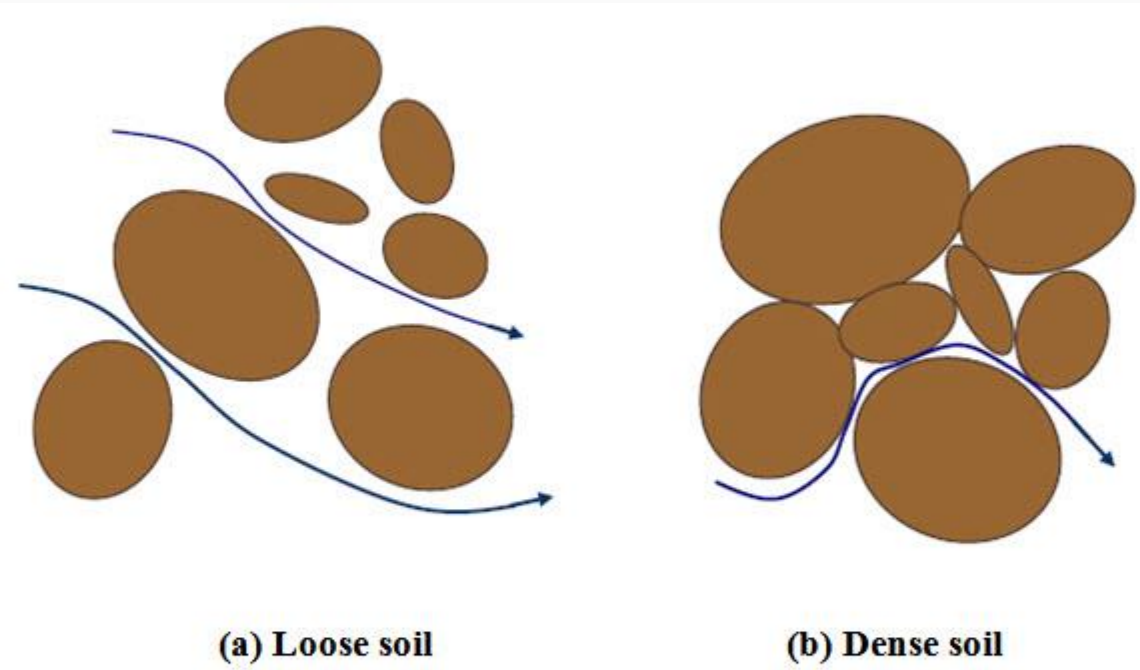


Fig.17.1. Flow of water a different type of soils

Darcy's Law

Velocity (***v***) of flow is proportional to the hydraulic gradient (***i***). Thus,

V = Ki (17.1)

where *k* is the coefficient of permeability (cm/sec) or hydraulic conductivity. If *i* is equal to one, the *v*= *k*. Thus, coefficient of permeability is velocity of water for unit hydraulic gradient. Hydraulic gradient can be expressed as: $i = \Delta h / L$, where Δh is the head loss and *L* is the length between two points (as shown in Figure 17.2).

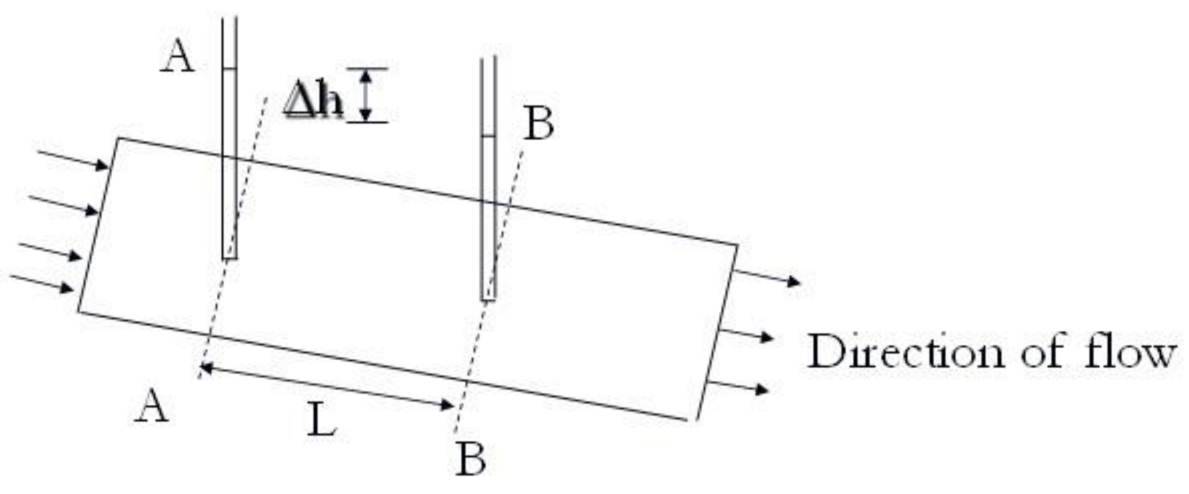


Fig.17.2. Flow of water

17.3. Factors affecting permeability

Some of the factors affecting permeability are:

- Grain Size
- Viscosity and temperature
- Void ratio
- Soil fabric of clay

Typical values of coefficient of permeability of various soils are presented as (Das, 1999):

Type of soil	Hydraulic conductivity (cm/sec)
Medium to coarse gravel	$>10^{-1}$
Coarse to fine sand	10^{-1} to 10^{-3}
Fine sand, silty sand	10^{-3} to 10^{-5}
Silt, clayey silt, silty clay	10^{-5} to 10^{-6}
Clays	10^{-7} or less

Introduction

Permeability is one of the important physical properties of soil as some of the major problems of soil mechanics are directly connected with it. Design of highways, airports, earth dams, construction of foundation below water – table, yield from a well, settlement of foundation etc. depend upon the permeability of soil. Hence to become a good soil engineer the knowledge of permeability is very essential. A material is said to be permeable if it contains continuous voids. Since such voids are contained in all soils including the stiffest clay, all these are permeable. Gravels are highly permeable and stiff clay is the least permeable soil.

Importance of Permeability:

The knowledge of permeability is important for the following engineering problems:

(i) Seepage through earthen dams and canals.

(ii) Unfit pressure under hydraulic structure and safety against piping

(iii) Rate of settlement of a saturated compressible soil layer.

(iv) Yield from a well and drainage of water logged agricultural land.

(v) Stability of upstream and downstream slopes of dams.

Definitions:

Permeability:

Permeability is the property of the soil which allows water to pass through its interconnecting voids.

Laminar Flow:

The flow in which all the particles of water move in parallel paths without crossing the path of other particles.

Turbulent Flow:

The flow in which all the particles of water move in zig-zag path.

Hydraulic Gradient:

The loos of hydaulic head per unit distance of flow is called hydraulic gradient. Consider a saturated flow through a uniform porous soil mass of length 'L' and let h_{P1} and h_{P2} be the piezometric head” or “Pressure head” at the entry and exit face respectively. Let $+Z_1$ and $-Z_2$ be the elevation head at the entry and exit face assuming the downstream water level as the datum line. The velocity head for flow through soil is negligible.

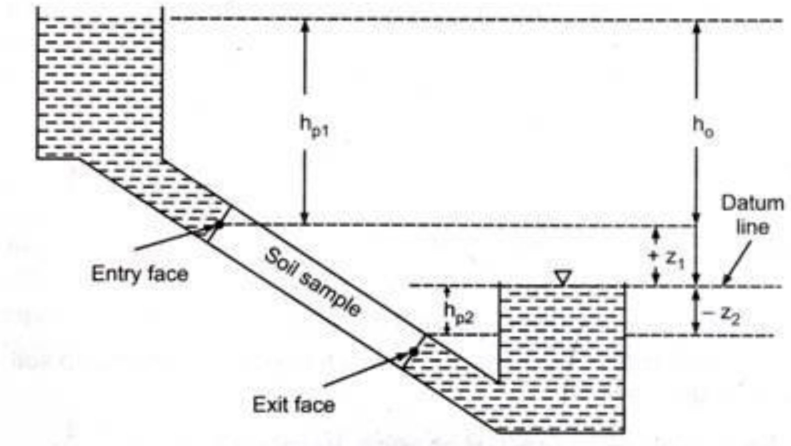


FIG. 4.1 Hydraulic gradient

Determination of Hydraulic Gradient:

The total head = Pressure head + Elevation head

The total head at the entry face,

$H_1 = hp_1 + Z_1$

The total head at the exits face

$H_2 = hp_2 - Z_2 = 0$

The total head difference

$H = h_1 - h_2 = h_{P1} + z_1 - 0 = h_{P1} + z_1$

[••• $hp_2 = Z_2$]

This total head difference is referred as the hydraulic head or ‘head loss’ or ‘head drop’. Any elevation can be selected for datum, as the base of elevation heads. The advantages of choosing the downstream water level as the datum is that the total head at the exits becomes zero and elevation of water in a piezometer at any point in soil measured above the datum line gives directly the hydraulic head

$h = h_p \pm z$

where h_p = Piezometric head

z = Elevation head

The loss of head per unit distance of flow (or along the length of flow) is called hydraulic gradient. It is denoted by 'I'

$I = h/L$

Where

h = head loss

L = length along the flow path over which head loss is h.

Darcy's Law:

In the mid-eighteenth century H. Darcy working in Paris studied experimentally the flow of water through soil. For laminar flow through saturated soil Darcy established experimentally that the rate of flow 'q' across a sectional area 'A' of soil is proportional to the hydraulic gradient".

$q = KiA$

Or $q/A = ki$

or $V = Ki$

where V = Flow velocity

K = Coefficient of permeability

i = Hydraulic gradient

Darcy's law is valid as long as the flow is laminar. It is applied to soil fraction finer than fine gravels.

Flow Velocity (or Discharge Velocity):

It is an apparent velocity being equal to average rate of flow across a unit gross area in the soil.

Rate of flow is the volume of water flowing per unit time.

Seepage Velocity:

Seepage velocity is the actual or true velocity with which water flows through soil voids.

Le A_v be the area of voids and

A be the gross area of soil perpendicular to the direction of flow. The rate of flow may be equated as $q = VA = A_V.V_S$

or $V_s = V\times A/A_V$

or $V_S = V/n$

The length of flow is same for both the case and n = Volume of voids/ Total volume]

Or $V_S = (1+e/e)\ V$

Where V Flow velocity

V_S = Seepage velocity

e = Void ratio

n = Porosity

As (1+e/e) is always greater than unity, Vg is a always greater than V.

Co-efficient of Permeability:

We know q = KIA (Darcy's law)

Putting A = 1 and I = 1 in equation we get

K =q

i.e. co-efficient of permeability, also known as hydraulic conductivity, may be defined as the rate of flow of water under laminar flow conditions through a unit cross-sectional area of a porous medium under a unit hydraulic gradient and standard temperature conditions (usually 27°C in India). The unit of K is similar to that of velocity i.e., either m/s or, cm/s etc.

The empirical relationship between K and D_{10} developed by Hazen (1911) for loose, clean sand is

$K = CD_{10}^2$

where K = co-efficient of permeability (cm/s)

C = Hazen's coefficient = 0.8 to 1.2 (1.0 is commonly used)

D_{10} = Effective size of soil

Co-efficient of Percolation:

The seepage velocity is also proportional to the hydraulic gradient.

$$\therefore \quad V_s \propto i$$

$$\text{or} \quad V_s = K_p i \qquad \ldots(4.1)$$

This coefficient of proportionality 'K_p' is called coefficeint of percolation

From Darcy's law

$$V = Ki \qquad \ldots(4.2)$$

DO YOU KNOW?

A typical value of k for natural sands is around 1×10^{-3} m/s.

From equations 4.1 and 4.2, we get

$$\frac{V_s}{K_p} = \frac{V}{K} = i$$

$$\text{or} \quad \frac{K_p}{K} = \frac{V_s}{V} = \frac{1}{n} \qquad \left[\text{We know } V_s = \frac{V}{n}\right]$$

$$\therefore \quad \boxed{K_p = \frac{K}{n}}$$

Factors Affecting Permeability:

Permeability can be obtained from the theoretical equation of Kozeny-Carman for flow through porous medium

$$K = CD_0^2\left(\frac{e^3}{1+e}\right)\frac{\gamma_w}{\eta} \qquad \ldots(4.3)$$

$K = CD^2_0\ (e^3+1+e)\gamma w\ /n$ ……………(4.3)

Where C = Composite shape factor

$D_{0=}$ Representative particle size

e = Void ratio

γ_w = Density of water,

n = Viscosity of water

The factors which affect the permeability are:

(i) Properties of pore fluid

(ii) Size and shape of particles

(iii) Void ratio of soil

(iv) Structural arrangement of soil particles

(v) Degree of saturation

(vi) Adsorbed water

(viii) Stratification

(i) Properties of pore fluid:

From equation 4.3 it is clear that the density and viscosity are the two physical properties of pore fluid (or water) which affects the permeability. The coefficient of permeability is directly proportional to density of water and inversely proportional to its viscosity. The value of density of water does not change much with the change in temperature but there is a great variation in viscosity. The viscosity decreases with increase in temperature and therefore permeability increase with increase in temperature.

(ii) Size and shape of particles:

Permeability of soil is directly proportional to the square of the particle size as shown in equation 4.3. This is most significant factor affecting permeability of soil as they decide the void ratio, size and shape of pores in a soil mass. A coarse soil has larger pore sizes and here greater K i.e. coefficient of permeability than the fine grained soils.

(iii) Void ratio of soil:

The marked influence of void ratio on permeability of soil as shown in the equation 4.3 has been experimentally verified.

K α e^3 / 1+e

From the above equation it is clear that K is directly proportional to the void ratio i.e. more the void ratio of soil more will be the permeability. A semi-log relationship also exists between K and e. A plot of log K (log scale) Vg e (linear scale) is approximately a straight line both coarse grained and fine grained soil.

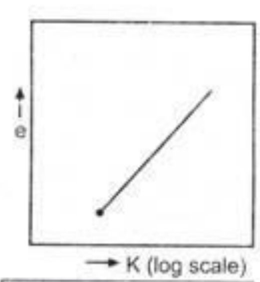


FIG. 4.2 Semi-log plotting of K V_s e.

(iv) Structural arrangement of soil particles:

The structural arrangement of the soil particles vary, at the same void ratio, depending upon the method of compaction of soil mass. The permeability of disturbed sample may be different from that of the undisturbed sample at the same void ratio. The effect of structural disturbance on permeability is much pronounced in fine grained soils.

(v) Degree of saturation:

The permeability of soil is observed to vary directly with the cube of the degree of saturation. Thus the more the saturated soil, more will be the permeability. However the pressure of entrapped air in soil pores obstructs the flow of water.

(vi) Adsorbed water:

Fine particles of clay are surrounded by films of adsorbed water. Forces of adsorption and development of diffuse ion-layer around the clay particles create immobilized hydrodynamic layers of water, thereby reducing the effective pore space available for seepage.

(vii) Stratification:

Layered soil posses different permeability characteristics. The permeability of the same soil is more when the flow is parallel to the layer than the permeability when the flow is perpendicular to the layer.

Method of Determination of Co-Efficient of Permeability:

The coefficient of permeability can be determined by the following methods:

(a) Laboratory methods [Direct methods]

(i) Constant head permeability test

(ii) Falling head test.

(b) Field methods

(i) Pumping out tests

(ii) Pumping in tests

(c) Indirect methods

(i) Computation from grain ($K = CD_{10}^2$) size

(ii) Horizontal capillarity test

(iii) Consolidated test data.

Constant Head Permeability Test:

The figure 4.3 shows diagrammatical representation of the test.

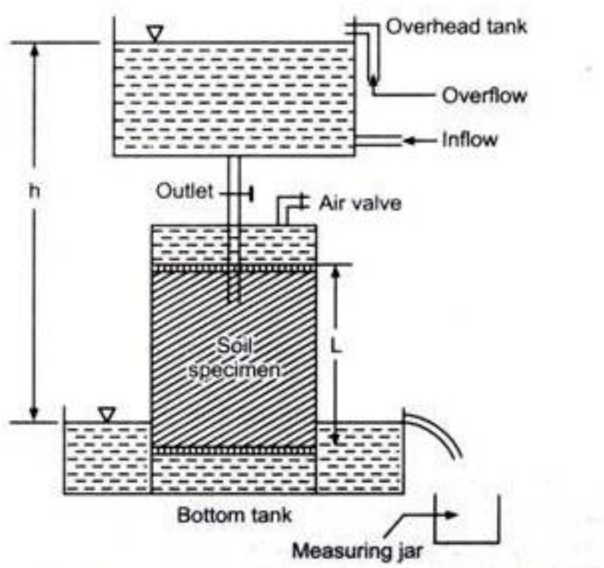


FIG. 4.3 (a) Constant head permeability test

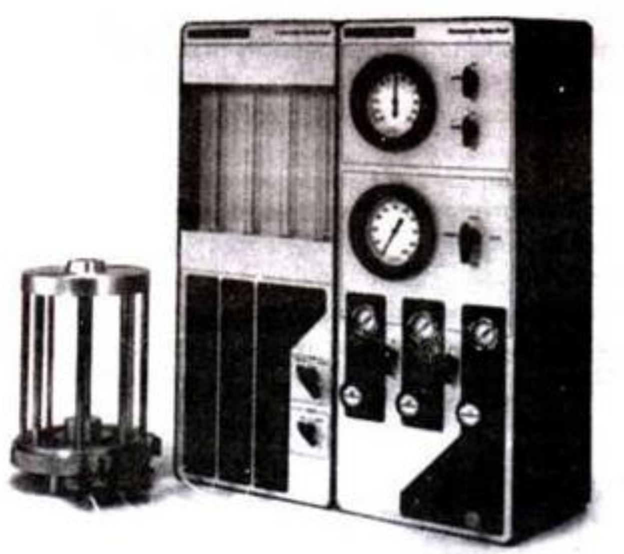


FIG. 4.3 (b) Permeameter (constant head) apparatus

Water flows from the overhead tank consists of three tubes: inlet, outlet and overflow tube. Constant head 'h' is maintained throughout the test. As the length of the soil sample 'L' is fixed throughout the test, the hydraulic gradient 'i' remains constant all through the test

We know I = h/L

Where h = difference of water level of overhead tank and bottom tank. If Q is the total quantity of flow in a time interval‘t’, we have form Darcy’s law.

$$q = \frac{Q}{t} = KiA$$

$$\therefore \quad K = \frac{Q}{tiA} = \frac{Q}{t} \times \frac{L}{h} \times \frac{1}{A} = \frac{QL}{thA}$$

where A = cross-sectional area of the sample

The measurement of Q is done after reaching the steady state. The test is repeated two or three times and the average value of Q is taken for the calculation of K. This test is suitable for coarse grained soil where a reasonable discharge can be collected in a given time.

1. Hydraulic head, h	in cm	6
2. Length of the sample, L	in cm	6
3. Hydraulic gradient i		1
4. Cross-sectional area of sample, A	cm^2	50
5. Time interval t	sec	600
6. Quantity of flow		
(a) Test I	ml	860
(b) Test II	ml	855
(c) Test III	ml	862
Average quantity of flow	ml	859
7. K	cm/s	2.86×10^{-2}
8. Test temp.	°C	32°
9. Permeability		2.57×10^{-2}

Falling Head Permeability Test:

Falling head test is suitable for less permeable soils. A stand pipe of known cross-sectional area ‘a’ is fitted with the permeameter and water is allowed to run down through this pipe. The water level in the stand pipe constantly falls as water flows. Observations are started after steady state of flow has reached. The head at any time‘t’ is equal to the difference in water levels in the stand pipe and the bottom tank.

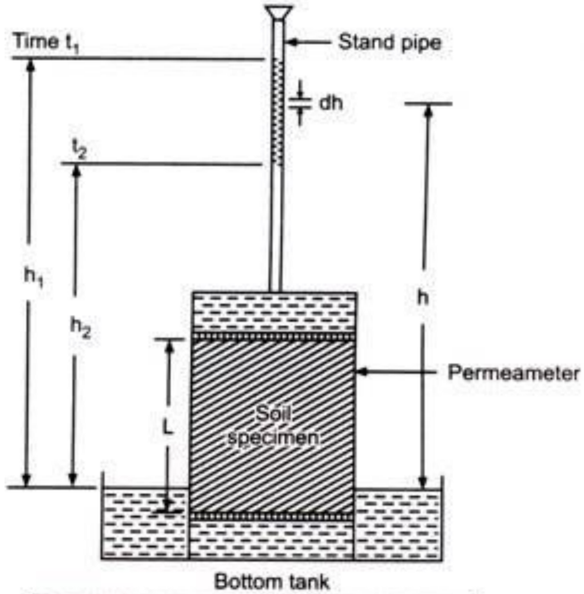


FIG. 4.4 Falling head permeability test

Let h_1 and h_2 be heads at time intervals t_1 and t_2 respectively ($t_1 > t_2$). Let h be the head at any intermediate time interval t and -dh be the change in the head in a smaller time interval 'dt' (minus sign has been used since h decreases as t increases). From Darcy's law, the rate of flow q is given by

$$q = \frac{(-dh \,.\, a)}{dt} = KiA$$

$$\frac{-dh}{dt} \,.\, a = \frac{KhA}{L} \qquad \left[\because i = \frac{h}{L} \text{ at time 't'}\right]$$

or $$\frac{-dh}{h} = \frac{KA}{aL} dt$$

Integrating between the two limits, we get

$$-\int_{h_1}^{h_2} \frac{dh}{h} = \frac{AK}{aL} \int_{t_1}^{t_2} dt$$

or $$\int_{h_2}^{h_1} \frac{dh}{h} = \frac{AK}{aL} \int_{t_1}^{t_2} dt$$

$$\log_e \left(\frac{h_1}{h_2}\right) = \frac{AK}{aL} (t_2 - t_1)$$

Denoting $t_2 - t_1 = t$, we get

$$\log_e \left(\frac{h_1}{h_2}\right) = \frac{AKt}{aL}$$

or $$K = \frac{aL}{At} \log_e \left(\frac{h_1}{h_2}\right) = 2.3 \frac{aL}{At} \log_{10} \left(\frac{h_1}{h_2}\right)$$

The laboratory observations consist of measurement of the heads h_1 and hg at two chosen time intervals t_1 and t_2. The averages of time intervals are taken for calculations.

Observation sheet for falling head permeability test:

1. Area of stand pipe (a)	cm^2	0.785
2. Cross-sectional area 'A' of the sample		50
3. Length of the sample (L)	cm	6
4. Initial head (h_1)	cm	40
5. Final head (h_2)	cm	20
6. Time interval		
(a) Test I	sec	56
(b) Test II		57
(c) Test III		55
Average time t	sec	56
7. K at test temp	cm/s	1.17×10^{-3}
8. Test temp.	°C	32
9. K at 27°C		1.05×10^{-3}

Table 4.1. Typical values of K for saturated soils

Soil type	Coefficient of permeability, cm/sec
Clean gravel	$1.0 - 100$
Sand gravel (mixture)	$10^{-2} - 10$
Clean sand (coarse)	$10^{-2} - 1.0$
Fine sand	$10^{-3} - 10^{-1}$
Silty sand	$10^{-3} - 10^{-2}$
Clay sand	$10^{-4} - 10^{-2}$
Silt	$10^{-8} - 10^{-3}$
Delhi silt	6×10^{-7}
Clay	$10^{-10} - 10^{-6}$
Boston blue clay	7×10^{-9}
London clay	1.5×10^{-11}

Table 4.2. Typical values of K as per BIS

S. No.	Soil type	K cm/sec	Drainage condition
1.	Clean gravel	$10^2 - 1.0$	Very good
2.	Clean sand, Clean sand and gravel mixture	$1.0 - 10^{-3}$	Good
3.	Very fine sand, organic and inorganic lits, mixture of sand, silt and clay, stratified clay deposits	$10^{-3} - 10^{-7}$	Poor
4.	Homogeneous, intact clays	$10^{-7} - 10^{-9}$	Practically impervious

Permeability of Stratified Soils:

Where a soil profile consists of a number of strata having different permeability, the equivalent or average permeability of the soil is different in direction parallel to and normal to, the strata. For flow parallel to layers the hydraulic gradient in each layer is the same and the total flow rate is the sum of flow rates in all the three layers.

$$q = q_1 + q_2 + q_3$$

$$K_x iH = K_1 iH_1 + K_2 iH_2 + K_3 iH_3$$

or

$$K_x H = K_1 H_1 + K_2 H_2 + K_3 H_3$$

$$K_x = \frac{K_1 H_1 + K_2 H_2 + K_3 H_3}{H}$$

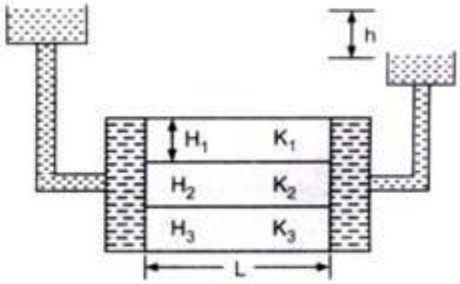


FIG. 4.5

Where K_x = Equivalent or average permeability in direction parallel to the layers. For flow normal to the layers the flow rate must be same in all layers for steady flow, and as the flow area 'A' is constant the flow velocity across layer is also the same

$$v = v_1 = v_2 = v_3$$

The total head loss 'h' is equal to the sum of the losses in the three layer.

$$h = h_1 + h_2 + h_3$$

$$iH = i_1 H_1 + i_2 H_2 + i_3 H_3$$

$$\frac{vH}{K_z} = \frac{v_1}{K_1}H_1 + \frac{v_2}{K_2}H_2 + \frac{v_3}{K_3}H_3$$

As $v = v_1 = v_2 = v_3$

We can write.

$$\frac{H}{K_z} = \frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}$$

or

$$K_z = \frac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}}$$

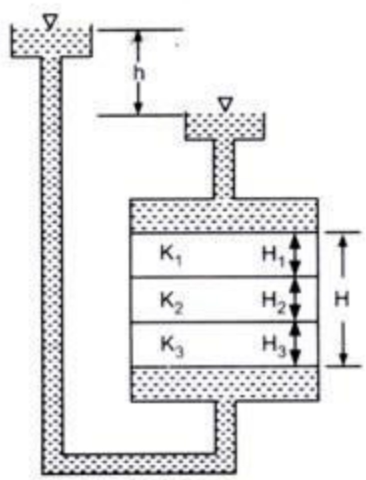


FIG. 4.6

Where K_z = equivalent permeability for flow normal to the layers. So the equivalent permeability for flow parallel to the strata is always greater than that for flow normal to the strata i.e., K_x is always greater than K_z.

Solved Example:

Example 1:

In a falling head permeability test on a specimen 6 cm high and 50 cm2 in cross-sectional area, the water level in the stand pipe, 0.8 cm^2 in sectional area, dropped from a height of 60 cm to 20 cm in 3 min 20 secs. Find the permeability.

Solution: Given $A = 50\ cm^2,\ L = 6\ cm$

$a = 0.8\ cm^2$

$h_1 = 60\ cm,\ h_2 = 20\ cm$

$t = 3\ min\ 20\ sec = 200\ sec$

At

$$K = 2.3\frac{aL}{At}\log_{10}\frac{h_1}{h_2} = 2.3\times\frac{0.8\times 6}{50\times 200}\log_{10}\frac{60}{20}$$

$$= 2.3\times\frac{4.8}{10^4}\log_{10}3$$

$$= \frac{2.3\times 4.8\times 0.477}{10^4}$$

$$= 5.27\times 10^{-4}\ cm/s \qquad [\because \log_{10}3 = 0.477] \qquad \textbf{Ans.}$$

Example 2:

During a constant head permeameter test, a flow Q of 160 cm^ is measured in 5 mins under a const, head of 15 cm. The specimen is 6 cm long and has a sectional area of 50 cm^2.The porosity n_1 of specimen is 42%. Determine the permeability, the flow velocity V and seepage velocity V_s. Estimate K_2 for $n_2 = 35\%$.

Solution: Given Q = 160 cm^3

L = 6 cm

$$A = 50\ cm^2$$
$$t = 5\ min = 300\ secs$$
$$h = 15\ cm$$
$$K = \frac{QL}{Ath} = \frac{160 \times 6}{50 \times 300 \times 15} = 4.27 \times 10^{-3}\ cm/s$$
$$V = \frac{Kh}{L} = \frac{4.27 \times 10^{-3} \times 15}{6} = 10.67 \times 10^{-2}\ cm/s$$
$$n_1 = 0.42,\ e_1 = 0.724$$
$$V_s = \frac{V}{n_1} = \frac{1.067 \times 10^{-2}}{0.42} = 2.54 \times 10^{-2}\ cm/s$$
$$n_2 = 0.35,\ e_2 = 0.538$$

We know $K \propto \dfrac{e^3}{1+e}$

or $K_2 \propto \dfrac{e_2^3}{1+e_2}$

Similarly $K_1 \propto \dfrac{e_1^3}{1+e_1}$

or
$$\frac{K_2}{K_1} = \frac{\dfrac{e_2^3}{1+e_2}}{\dfrac{e_1^3}{1+e_1}} = \frac{\dfrac{(0.538)^3}{1+0.538}}{\dfrac{(0.724)^3}{1+0.724}} = \frac{0.101}{0.220} = 0.459$$

$K_2 = 0.459 \times 4.27 \times 10^{-3} = 1.96 \times 10^{-3}$ cm/s **Ans.**

We can also calculate K_2 from the following relationship

$$K \propto e^2$$

$\therefore$ $K_1 = c\,e_1^2$

$K_2 = c\,e_2^2$

$\therefore$ $\dfrac{K_2}{K_1} = \dfrac{e_2^2}{e_1^2}$

$\therefore$ $K_2 = \dfrac{e_2^2}{e_1^2} \times K_1$

Example 3:

A sand deposit is made up of three horizontal layers of equal thickness. The permeability of the top and bottom layers is 2 x10^{-4} cm/s and that of middle layer is 3.2 x 10^{-2} cm/s. Find the equivalent permeability in the horizontal and vertical direction and their ratio.

Solution: Given $H_1 = H_3$

$K_1 = K_3 = 2 \times 10^{-4}$ cm/s

$K_x = 3.2 \times 10^{-2}$

$$K_x = \frac{K_1H_1 + K_2H_2 + K_3H_3}{H}$$

$$= \frac{2\times10^{-4}\times H_1 + 3.2\times10^{-2}\times H_1 + 2\times10^{-4}\times H_1}{3H_1}$$

$$[\because H_1 = H_2 = H_3]$$

$K_x = 1.08 \times 10^{-2}$ cm/s **Ans**

$$K_z = \frac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}}$$

$$= \frac{3H_1}{\frac{H_1}{2\times10^{-4}} + \frac{H_1}{3.2\times10^{-2}} + \frac{H_1}{2\times10^{-4}}}$$

$= 2.99 \times 10^{-4}$ cm/s

$\frac{K_x}{K_z} = 36.1$ **Ans.**

Example 4:

Calculate the value of coefficient of permeability of soil with their effective dia 0.5 mm. **Solution:**

We have Hazen's correlation $K = CD^2_{10}$ cm/s

C = 1.0

D_{10} = 0.5 mm

K = ?

K = 1.0 X $(0.5)^2$ cm/s = 0.25 cm/s Ans.

Example 5:

A soil sample was tested in a constant head permeameter. The diameter and length of the sample was 3 cm and 15 cm respectively. Under a head of 30 cm, the discharge was found to be 80 cc in 15 minutes.

Calculate:

(i) Coefficient of permeability

(ii) Type of soil used in the test

Solution: Given

$$d = 3 \text{ cm}$$
$$L = 15 \text{ cm}$$
$$h = 30 \text{ cm}$$
$$Q = 80 \text{ c.c.}$$
$$t = 15 \times 60 = 900 \text{ secs}$$
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.07 \text{ cm}^2$$
$$i = \frac{h}{L} = \frac{30}{15} = 2$$

(i) We know

$$K = \frac{Q}{tiA} = \frac{80}{900 \times 2 \times 7.07}$$
$$= 6.29 \times 10^{-3} \text{ cm/sec} \quad \textbf{Ans.}$$

(ii) The value of K lies between 10^{-1} to 10^{-1}. The soil consists of fine gravel coarse, medium and fine sands.

Example 6:

A soil sample 5 cm in length and 60 cm in cross-sectional area, water perculates through the sample in 10 minutes is 480 ml under a constant head of 40 cm. Weight of oven dried sample is 498 gm and specific gravity of soil = 2.65.

Calculate:

(i) Coefficient of permeability

(ii) Seepage velocity.

Solution : Given :

$$Q = 480 \text{ ml}$$
$$L = 5 \text{ cm}$$
$$A = 60 \text{ cm}^2$$
$$h = 40 \text{ cm}$$
$$W_d = 498 \text{ gm}$$
$$G = 2.65$$
$$t = 10 \times 60 = 600 \text{ secs}$$
$$i = \frac{h}{L} = \frac{40}{5} = 8$$

(i) We know $K = \frac{Q}{tiA} = \frac{480}{600 \times 8 \times 60} = 1.67 \times 10^{-3}$ cm/s **Ans.**

(ii) Discharge velocity, $V = \frac{q}{A} = \frac{Q}{tA} = \frac{480}{600 \times 60} = 1.33 \times 10^{-2}$ cm/s

Seepage velocity, $V_s = \frac{V}{n}$ ...(i)

where $n = \frac{e}{1+e}$

and $e = \frac{G\gamma_w}{\gamma_d} - 1$ ...(ii)

$$\gamma_d = \frac{W_d}{A \times L} = \frac{498}{60 \times 5} = 1.66 \text{ gm/c.c}$$

Putting the value of γ_d in (*ii*) we get

$$e = \frac{2.65 \times 1}{1.66} - 1 = 0.596$$

$\therefore$ $n = \frac{0.596}{1 + 0.596} = 0.373$

Putting the value of V and n in (*i*) we get

$$V_s = \frac{1.33 \times 10^{-2}}{0.373} = 3.56 \times 10^{-2} \text{ cm/s} \quad \textbf{Ans.}$$

EXAMPLE 7:

The coefficient of permeability of a soil sample in found to be 1 X 10^{-3} cm/s at a void ratio of 0.4. Estimate its permeability at a void ratio of 0.6. Solution: We know that:

K α e^3 / 1+e

$$\Rightarrow \qquad \frac{K_2}{K_1} = \frac{e_2^3}{e_1^3} \times \frac{1+e_1}{1+e_2}$$

Given $K_1 = 1 \times 10^{-3}$ cm/s

$e_1 = 0.4$

$K_2 = ?,\ e_2 = 0.6$

Putting the value of K_1, e_1 and e_2 in (*i*) we get

$$\frac{K_2}{1 \times 10^{-3}} = \frac{(0.6)^3}{(0.4)^3} \times \frac{1+0.4}{1+0.6}$$

$$\therefore \qquad K_2 = \frac{0.216}{0.064} \times \frac{1.4}{1.6} \times 1 \times 10^{-3} = 2.953 \times 10^{-3} \text{ cm/s} \quad \textbf{Ans.}$$

Example 8:

If during a permeability test on a soil sample with falling head permeameter, equal time intervals are noted for drops of head from h_1 and h_2 and again from h_1 to h_2, find a relationship between h_1, h_2 and h_3.

Solution: For falling head from h_1 and h_2

$$K = 2.3 \frac{aL}{At} \log_{10}\left(\frac{h_1}{h_2}\right) \qquad \ldots(i)$$

For falling head from h_2 to h_3

$$K = 2.3 \frac{aL}{At} \log_{10}\left(\frac{h_2}{h_3}\right) \qquad \ldots(ii)$$

a, L, A and t are same for both the tests.

From (*i*) and (*ii*) we get

$$2.3 \frac{aL}{At} \log_{10}\left(\frac{h_1}{h_2}\right) = 2.3 \frac{aL}{At} \log_{10}\left(\frac{h_2}{h_3}\right)$$

$$\log_{10}\left(\frac{h_1}{h_2}\right) = \log_{10}\left(\frac{h_2}{h_3}\right)$$

$$\frac{h_1}{h_2} = \frac{h_2}{h_3}$$

$$h_2^2 = h_1 h_3$$

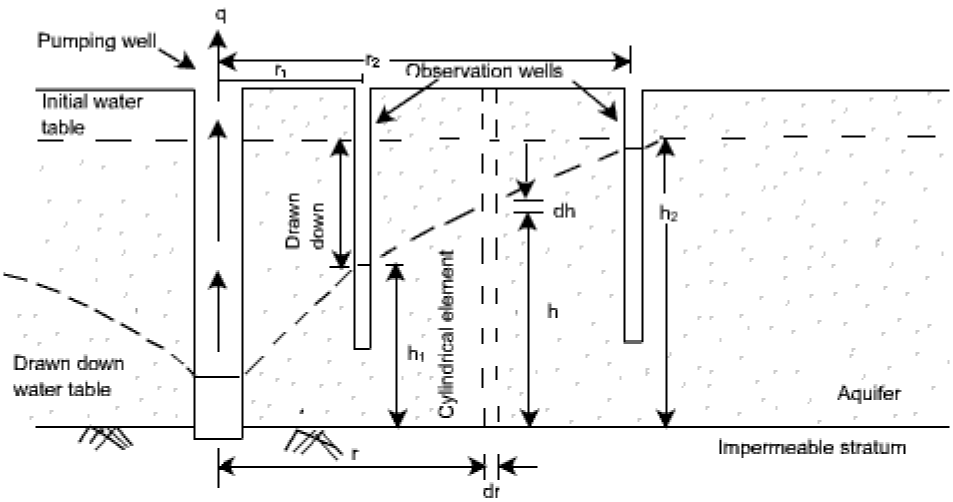
$$\therefore \qquad h_2 = \sqrt{h_1 h_3} \quad \textbf{Ans.}$$

Field or *in-situ* measurement of permeability avoids the difficulties involved in obtaining and setting up undisturbed samples in a permeameter. It also provides information about bulk permeability, rather than merely the permeability of a small sample.

A field permeability test consists of pumping out water from a main well and observing the resulting drawdown surface of the original horizontal water table from at least two observation wells. When a steady state of flow is reached, the flow quantity and the levels in the observation wells are noted.

Two important field tests for determining permeability are: Unconfined flow pumping test, and confined flow pumping test.

Unconfined Flow Pumping Test



In this test, the pumping causes a drawdown in an unconfined (i.e. open surface) soil stratum, and generates a radial flow of water towards the pumping well. The steady-state heads h_1 and h_2 in observation wells at radii r_1 and r_2 are monitored till the flow rate **q** becomes steady.

The rate of radial flow through any **cylindrical surface** around the pumping well is equal to the amount of water pumped out. Consider such a surface having radius **r**, thickness **dr** and height **h.** The hydraulic gradient is

$$i = \frac{dh}{dr}$$

Area of flow, $A = 2\pi r h$

From Darcy's Law,

$$q = k.i.A$$
$$= k.\frac{dh}{dr}.2\pi rh$$

Arranging and integrating,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} \frac{2r}{q}.K.h.dh$$

$$k = \frac{q.\log_e(\frac{r_2}{r_1})}{r(h_2^2 - h_1^2)}$$

1. Effective Stress Equation

The effective stress equation is a simple, but very important equation in geotechnical engineering.

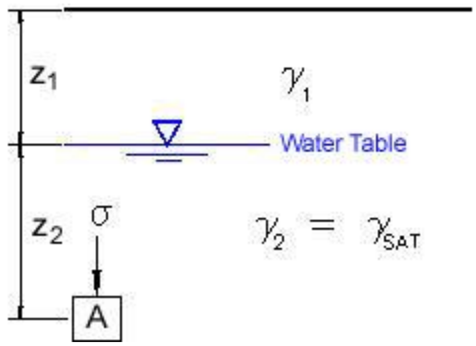
$$\sigma = \sigma' + u \qquad \begin{aligned} \sigma &= \text{Total Stress} \\ u &= \text{Neutral Stress} \\ \sigma' &= \text{Effective Stress} \end{aligned}$$

The total stress in a soil is divided into two parts: (1) the neutral stress, which is the pressure in the water and air in the voids, and (2) the effective stress, which is the stress in the solid portion of the soil. Neutral stresses may be determined from the principles of fluid mechanics. Effective stresses may be determined from the principles of mechanics of materials.

2. Stresses in Saturated Soil Without Seepage

In a saturated soil all the void spaces are filled with water and the degree of saturation is 100%. Seepage refers to the flow of water through a soil. Therefore in the following situation, no water is flowing, and the soil is completely saturated. In general, the soil below a water table is considered to be completely saturated.

A. A Soil Profile



B. Calculation of Stresses at Point A for the Soil Profile Above

The total stress at A is equal to the weight of all the material above the point, and may be calculated from the sum of the depths of the layers multiplied by the soil unit weights for the respective layers. The neutral stress is equal to the depth below the water table multiplied by the unit weight of water.

$$\sigma = \text{Total Stress} = z_1\gamma_1 + z_2\gamma_2$$
$$u = \text{Neutral Stress} = z_2\gamma_w$$

The effective stress may be calculated from the effective stress equation.

$$\sigma' = \text{Effective Stress} = \sigma - u$$
$$\sigma' = z_1\gamma_1 + z_2\gamma_2 - z_2\gamma_w$$
$$\sigma' = z_1\gamma_1 + z_2(\gamma_2 - \gamma_w)$$

C. Submerged Unit Weight

The submerged unit weight of a soil is the apparent unit weight of a soil below the water table, and may be determined by subtracting the unit weight of water from the saturated unit weight of the soil.

$$\gamma' = \text{Submerged Unit Weight} = \gamma_{sat} - \gamma_w$$

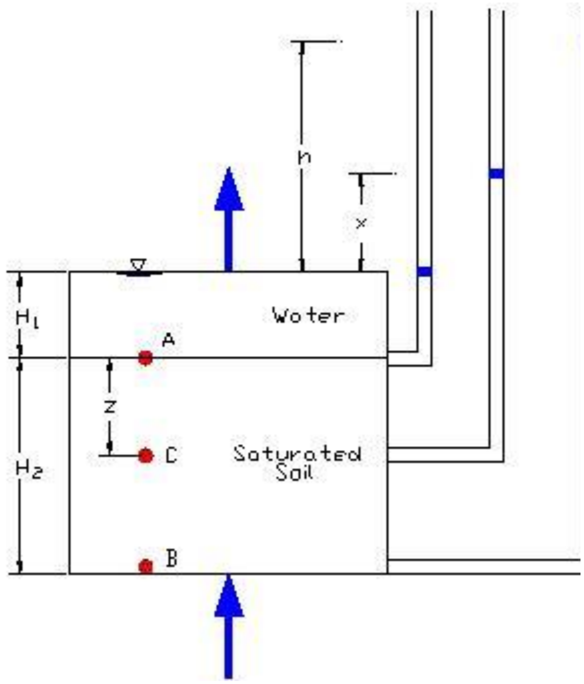
Thus, for situations with no flow of water, the effective stress may be determined by the sum of the layer depths multiplied by the respective soil unit weights, using moist unit weights above the water table, and submerged unit weights below the water table.

$$\sigma' = z_1\gamma_1 + z_2\gamma'_2$$

3. Stresses in Saturated Soil with Upward Seepage

A. The Situation

Water is flowing upward through the soil shown. The stresses in the soil are calculated at points A, B, and C.



Stresses At Point A

$$\sigma = H_1\gamma_W$$
$$u = H_1\gamma_W$$
$$\sigma' = \sigma - u = 0$$

Stresses At Point B

$$\sigma = H_1\gamma_W + H_2\gamma_{SAT}$$
$$u = (H_1 + H_2 + h)\gamma_W$$
$$\sigma' = \sigma - u = H_2(\gamma_{SAT} - \gamma_W$$
$$\sigma' = H_2\gamma' - h\gamma_W$$

Stresses At Point C

$$\sigma = H_1\gamma_W + z\gamma_{SAT}$$
$$u = (H_1 + z + iz)\gamma_W$$
$$\sigma' = \sigma - u = z\gamma' - iz\gamma_W$$

B. Effective Stress Equation for Upward Flow Through Soil

The equation for effective stress at point C is a general equation that applies for upward flow through soil.

$$\sigma' = z\gamma' - iz\gamma_W$$

$$z = \text{Depth Below The Soil Surface}$$

$$i = \text{Hydraulic Gradient}$$

$$\gamma' = \gamma_{SAT} - \gamma_W$$

C. The "Quick" Condition and the Critical Hydraulic Gradient

The "quick" condition may occur in sandy soils where there is an upward flow of water. The soil appears to be boiling and loses its shear strength. The condition may also occur in cohesive soils subjected to excess porewater pressure. In this case the surface of the soil may heave and crack and water will flow up through the cracks. The soil state necessary for the "quick" condition is for the effective stress to equal zero and the hydraulic gradient to be a critical value.

$$\sigma' = 0 = z\gamma' - iz\gamma_W$$

$$z\gamma' = iz\gamma_W$$

$$i_c = \frac{\gamma'}{\gamma_W} = \text{Critical Hydraulic Gradient}$$

4. Stresses in Saturated Soil with Downward Seepage

The equation for effective stress in soil where the seepage is downward may be developed in a fashion similar to the case of upward flow.

$$\sigma' = z\gamma' + iz\gamma_W$$

5. Summary of Effective Stress Conditions

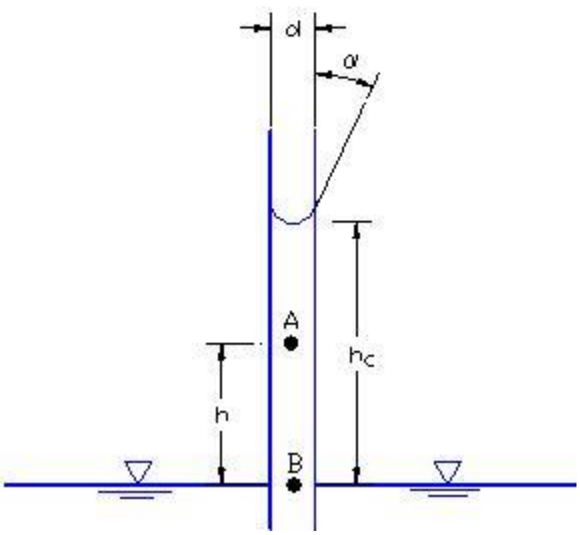
Condition	Hydraulic Gradient	Effective Stress
No Seepage	0	$z\gamma'$
Upward Seepage	i	$\sigma' = z\gamma' - iz\gamma_W$
Downward Seepage	i	$\sigma' = z\gamma' + iz\gamma_W$

Quick Condition	$\frac{\gamma'}{\gamma_w}$	0

7. Capillary Water in Soils

A. Capillary Water in a Tube

To help understand capillary water in soil, it is helpful to review capillary action in a small glass tube inserted into water. Water will rise in the tube and form a meniscus as shown in the diagram. The height the capillary rise is inversely proportional to the inside diameter of the tube.



$$\mathbf{h_c = \frac{4T\cos\alpha}{d\gamma_w}}$$

$\mathbf{h_c}$ **= capillary rise**

$\mathbf{d}$ **= tube diameter**

$\mathbf{T}$ **= surface tension**

$\boldsymbol{\alpha}$ **= contact angle**

$\boldsymbol{\gamma_w}$ **= unit weight of water**

B. Pressure in Capillary Water in a Tube

The pressure in the water at point B in the diagram above is 0, and it may be shown that the pressure at point A in the capillary water is negative and depends on the height above the surface of the water surrounding the tube. The equation for the water pressure at point A is:

$$\mathbf{u_A = -h\gamma_w}$$

C. Capillary Water in Soil

In soil water is able to rise above the water table by moving through the inter-connected void spaces by capillary action. The height of capillary rise depends primarily on soil particle size, becoming higher as the size gets smaller. Thus, capillary action may be significant for silt and clay size particles. Capillary rise may be estimated from:

$$h_c = \frac{C}{eD_{10}}$$

h_c = capillary rise

e = void ratio

C = coefficient = 10 mm^2 to 50 mm^2

D_{10} = effective size of soil particles

The porewater pressure in the capillary zone will be negative and may be determined from:

$$u = -\left(\frac{S}{100}\right)\gamma_w h$$

u = water pressure in capillary water

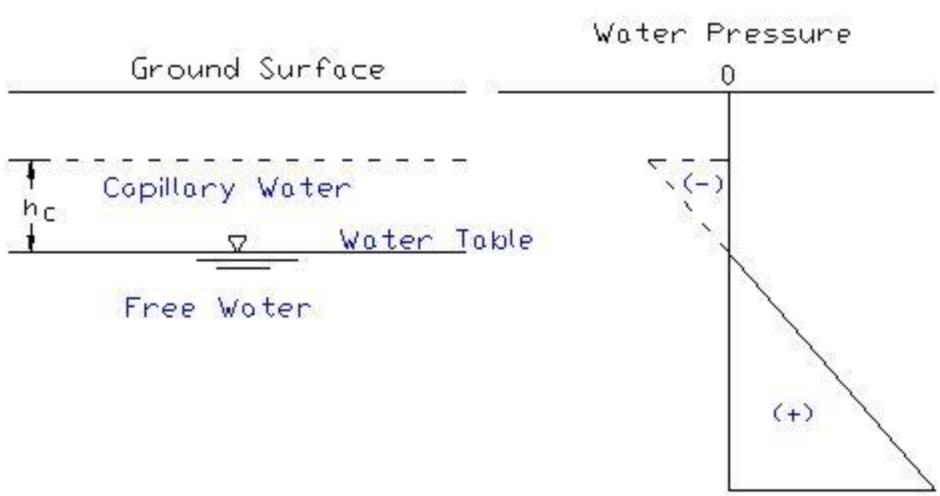
S = degree of saturation

h = height above the water table

γ_w = unit weight of water

8. A Summary of Water in Soil

- The porewater pressure at the water table is zero.
- Below the water table the soil is saturated and the water is called free water. The water pressure is positive (compression) and increases with depth below the water table.
- Above the water table up to the height of capillary rise is the capillary zone and the water is called capillary water. The water pressure is negative (less than atmospheric) in the capillary zone.
- These observations are illustrated in the figures below.



9. Piping, Filters, and Geosynthetics

- Erosion in Soil. -- Moving surface water may pick up unprotected soil and transport it downstream. Silty soils are especially subject to erosion by moving water.
- Piping. -- Piping is underground erosion of soil. As groundwater moves it may pick up soil if the velocity is fast enough. As the soil is transported a "pipe" is formed below the ground surface. As the "pipe" gets larger, the velocity increases, and more soil is transported. This process may lead to failure and collapse of the soil structure.
- Soil Filters. -- A soil filter is a certain gradation of soil particle sizes that allow water to pass through, but prohibit the soil particles from passing through the filter. The purpose of a filter is to prevent piping or underground erosion.
- Geosynthetic Filters. -- Geosynthetic filters are polymer fabrics placed in the soil that allow water to flow through, but prevent the soil particles from passing. They serve a similar purpose as a soil filter.

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SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT – III- Stress Distribution – SCI1304

STRESS DISTRIBUTION

1.0 Stress Distribution in soil media

For many problems of practical interest, it is necessary to estimate settlements under conditions in which the induced stress varies spatially. The first step in the analyses of such problems usually involves estimations of the initial states of stress in the soil and of the changes in these stresses during loading and as the soil again approaches equilibrium. Two methods of analysis have commonly been followed.

The stresses within a soil mass due to its own weight are known as geostatic stresses. Let us take the soil mass to be bounded by the horizontal plane (ground surface) xy, and the z-axis be directed downwards. Under this condition, the soil mass is said to be semi-infinite. Where there is no external loading, the ground plane becomes a principal plane since it is devoid of any shear loading. $\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$, we get $\sigma_z = \gamma z$

γ = Unit weight of soil

σ_z = Vertical stress at the point within and soil massat a depth z below ground surface

VERTICAL STRESS INCREASE (σ_z)IN SOIL

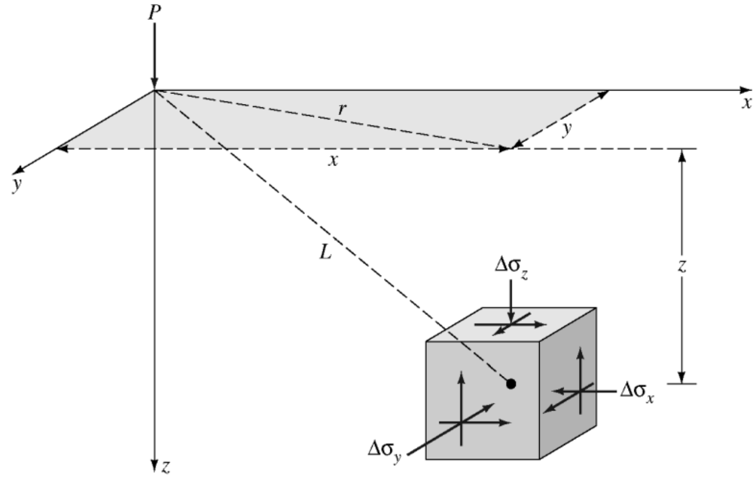
BOUSSINESQ EQUATION (POINT LOADING)

Boussinesq (1885) solved the problem of stress distribution in soil due to a concentrated load acting at the ground surface. The following assumption are made in the solutions by the theory of elasticity.

1. The soil mass is an elastic medium, for which the modulus of elasticity E is constant.
2. The soil mass is homogeneous, that is , all its constituent parts or elements are similar.
3. The soil mass is isotropic, that it has identical elastic properties in all directions through any point of it.
4. The soil mass is semi- infinite, that it extend infinitely in all directions below ground surface.

   Let a point load P act at the ground surface, at a point O which may taken as the origin of the x, y and z axes as shown. And the Table shows the variation of influence factor with respect to r/z

Vertical Stress Increase in soil for Point Load



$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

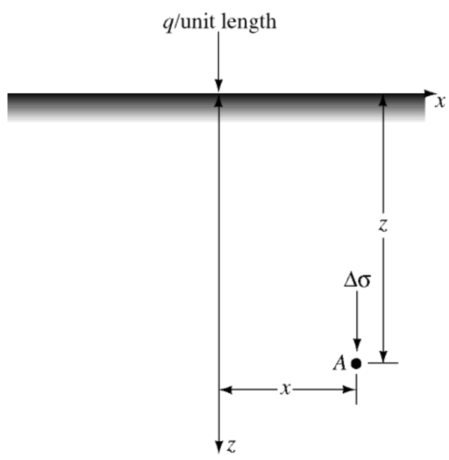
$$I_1 = \frac{3}{2\pi}\frac{1}{\left[(r/z)^2+1\right]^{5/2}}$$

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{\left[(r/z)^2+1\right]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

$\Delta\sigma_z$ = Change in vertical stress
P = Point Load
I_1 = Boussinesq influence factor

r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

Vertical Increase in soil for Line Loading



$$\Delta\sigma = \frac{2qz^3}{\pi(x^2+z^2)^2}$$

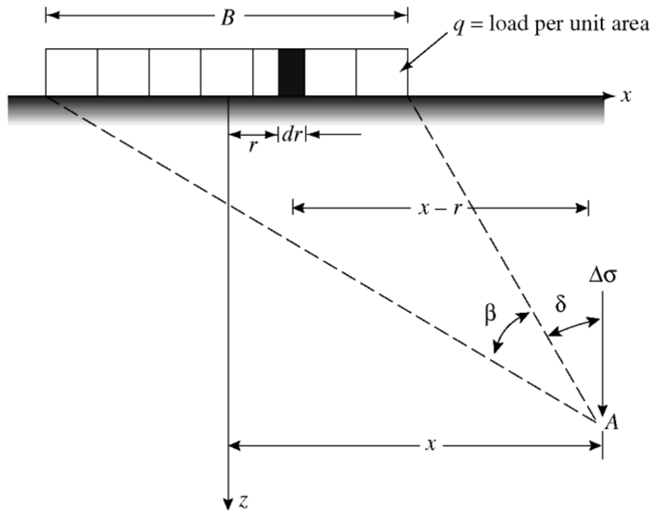
or

$$\frac{\Delta\sigma}{(q/z)} = \frac{2}{\pi\left[\left(\frac{x}{z}\right)^2+1\right]^2}$$

$\Delta\sigma$ =Change in Vertical Stress
q = Load per Unit Length
z = Depth

x = Distance from Line Load

Vertical Increase in soil for Strip Loading



$$\Delta\sigma = \frac{q}{\pi}\left[\beta + \sin\beta\cos(\beta + 2\delta)\right]$$

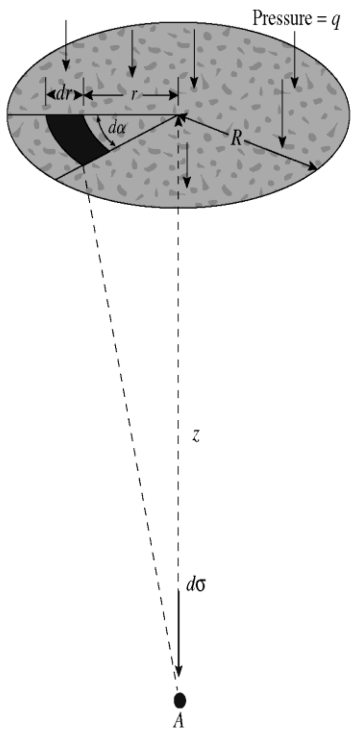
Δσ =Change in Vertical Stress
q = Load per Unit Area
z = Depth

x = Distance from Line Load

Table :Variation of $\Delta\sigma/q$ with $2z/B$ and $2x/B$

	2x/B				
2z/B	**0**	**0.5**	**1.0**	**1.5**	**2.0**
0	1.000	1.000	0.500	—	—
0.5	0.959	0.903	0.497	0.089	0.019
1.0	0.818	0.735	0.480	0.249	0.078
1.5	0.668	0.607	0.448	0.270	0.146
2.0	0.550	0.510	0.409	0.288	0.185
2.5	0.462	0.437	0.370	0.285	0.205
3.0	0.396	0.379	0.334	0.273	0.211
3.5	0.345	0.334	0.302	0.258	0.216
4.0	0.306	0.298	0.275	0.242	0.205
4.5	0.274	0.268	0.251	0.226	0.197
5.0	0.248	0.244	0.231	0.212	0.188

Vertical Stress Increase in soil for Circular Loading.



$$\Delta\sigma = q\left\{1-\frac{1}{\left[(R/z)^2+1\right]^{3/2}}\right\}$$

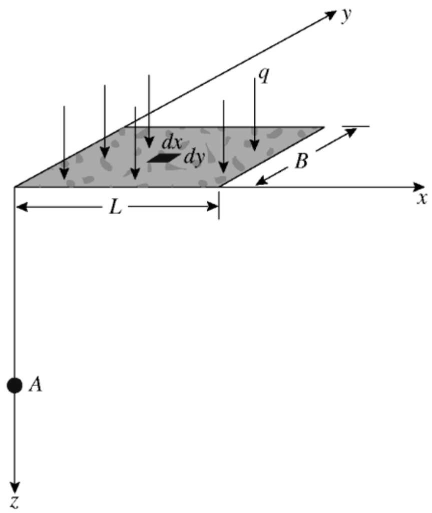
$\Delta\sigma$ =Change in Vertical Stress
q = Load per Unit Area
z = Depth

R= Radius

Table: Variation of $\Delta\sigma/q$ with z/R

z/R	$\Delta\sigma/q$	z/R	$\Delta\sigma/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

Vertical Stress Increase in soil for Rectangular Loading.

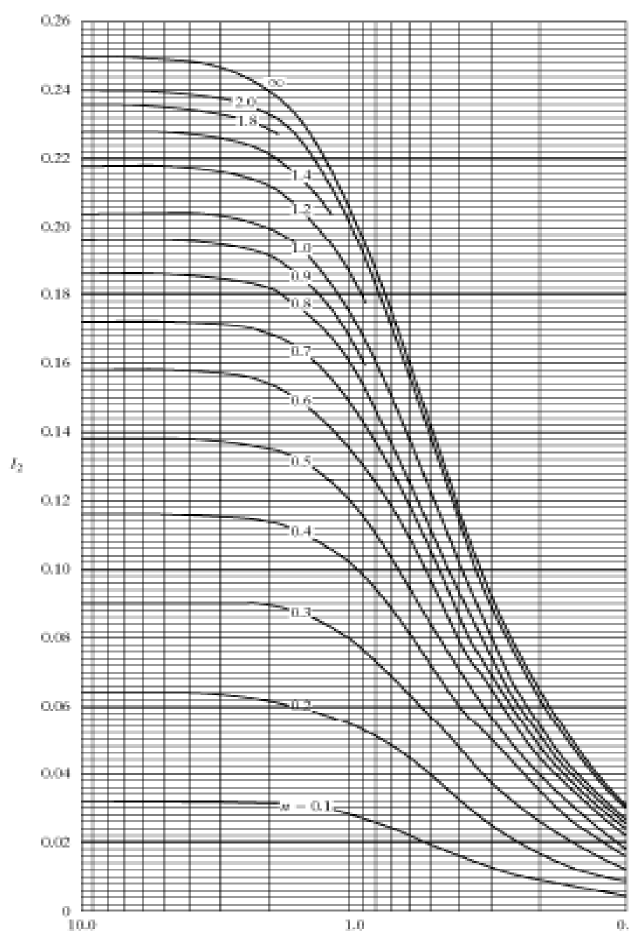


$$\Delta\sigma = \int d\sigma = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dxdy)}{2\pi(x^2+y^2+z^2)^{5/2}} = qI_2$$

$$I_2 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$

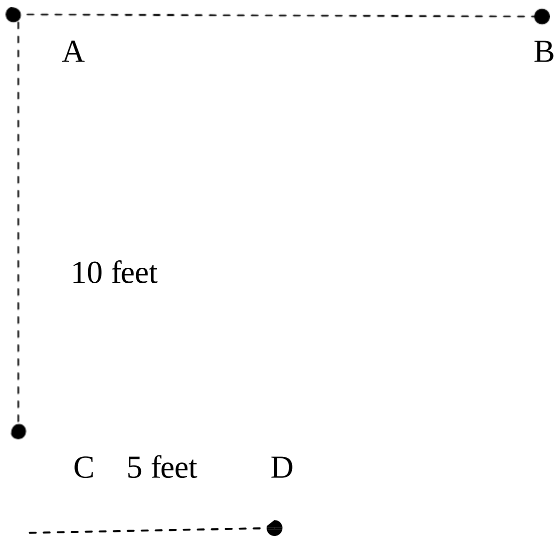
m = B / z ; n = L / z

Fig : Variation with m and n



Problem 1: *Stress increase at a point from several surface point loads*

Point loads of 2000, 4000, and 6000lbs act at points A, B and C respectively, as shown below. Determine the increase in vertical stress at a depth of 10 feet below point D.



Solution.

Using the Boussinesq z= 10

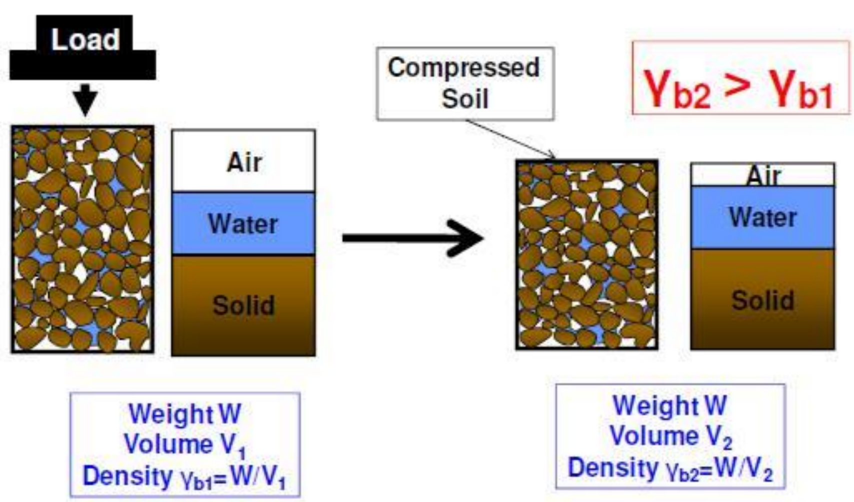
Increase in the load at:	***P*** (lbs)	***r*** (ft)	***z*** (ft)	***r/z***	***I1***	***p*** (psf)
Δp fromA	2,000	$(10^2+5^2)^{1/2}$= 11.18	10	1.12	0.0626	1.25
Δp fromB	4,000	$(10^2+5^2)^{1/2}$= 11.18	10	1.12	0.0626	2.50
Δp fromC	6,000	5	10	0.50	0.2733	16.40
					Total=*20.2psf*	

COMPACTION

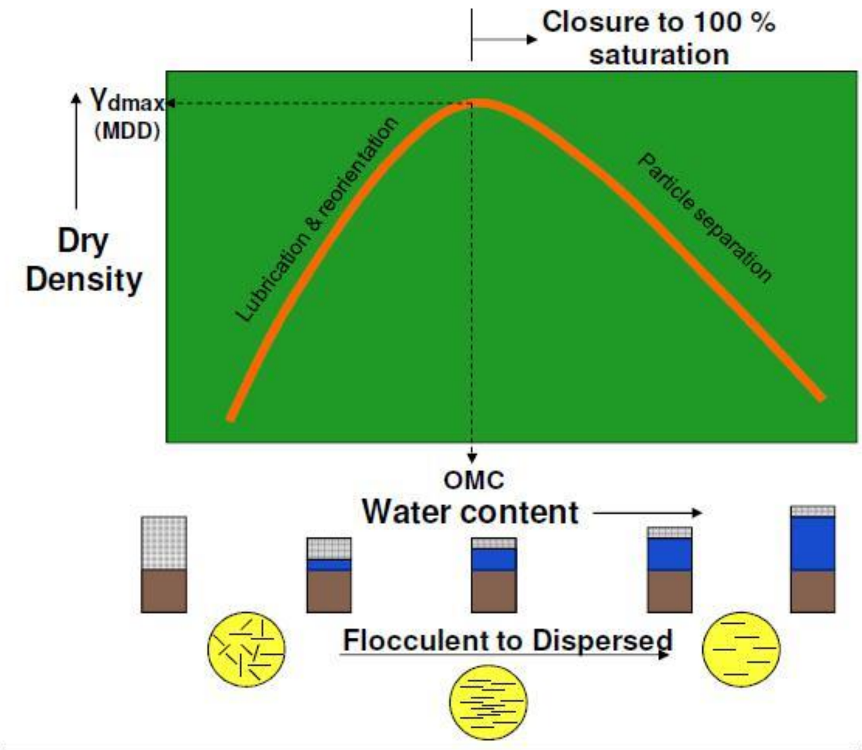
Compaction is the process of increasing the bulk density of a soil or aggregate by driving out air. For any soil, at a given compactive effort, the density obtained depends on the moisture content. An "Optimum Moisture Content" exists at which it will achieve a maximum density. Compaction is the method of mechanically increasing the density of soil. The densification of soil is achieved by reducing air void space. During compaction, air content reduces, but not water content It is not possible to compact saturated soil. It should be noted that higher the density of soil mass, stronger, stiffer, more durable will be the soil mass.

Hence, Compaction

1) Increases density
2) Increases strength characteristics
3) Increases load-bearing capacity
4) Decreases undesirable settlement
5) Increases stability of slopes and embankments
6) Decreases permeability
7) Reduces water seepage
8) Reduces Swelling & Shrinkage
9) Reduces frost damage
10) Reduces erosion damage
11) Develops high negative pore pressures (suctions) increasing effective stress



Mechanism of Compaction-



Optimum Moisture Content (OMC) is the moisture content at which the maximum possible dry density is achieved for a particular compaction energy or compaction method. The corresponding dry density is called Maximum Dry Density (MDD). Water is added to lubricate the contact surfaces of soil particles and improve the compressibility of the soil matrix. It should be noted that increase in water content increases the dry density inmost soils up to one stage (Dry side). Water acts as lubrication. Beyond this level, any further increase in water (Wet side) will only add more void space, there by reducing the dry density. Hence OMC indicates the boundary between the dry side and wet side. Hence the compaction curve as shown in figure indicates the initial upward trend up to OMC and the down ward trend.

Reasons for the shape of curve

1. On dry side of OMC, clayey soil shows high suction, lumps are difficult to break or compact.
2. Increasing the water content reduces suction, softens lumps, lubricates the grains for easy compaction.
3. As water content increases, lubrication improves compaction resulting in higher dry density.
4. Now nearly impossible to drive out the last of the air – further increase in water content results in reduced dry density (curve follows down parallel to the maximum possible density curve – the Zero Air Voids curve)
5. MDD and OMC depend on the compaction energy and are not unique soil properties.
6. For sand, suction at low water contents also prevents compaction (but not if completely dry)
7. In cohesionless soils, MDD is achieved either when completely dry, or when completely saturated.
8. At low water content, grains are held together by suction (water at grain contacts only)
9. This prevents compaction.
10. Laboratory test for MDD on sand requires fully saturated sample, and involves vibration

Percent Air Voids:

$$\gamma_d = \frac{(1-n_a)G\gamma_\omega}{1+\omega G}$$

Factors affecting Compaction-

1. Water Content

2. Amount of Compaction

3. Method of Compaction

4. Type of Soil

5. Addition of Admixtures

Effect of Water Content-

1. With increase in water content, compacted density increases up to a stage, beyond which compacted density decreases.

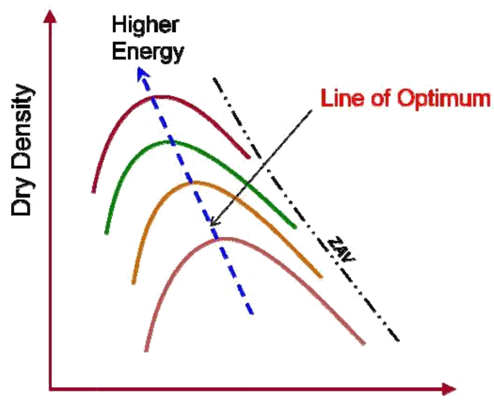
2. The maximum density achieved is called MDD and the corresponding water content is called OMC.

3. At lower water contents than OMC, soil particles are held by the force that prevents the development of diffused double layer leading to low inter-particle repulsion.

4. Increase in water results in expansion of double layer and reduction in net attractive force between particles. Water replaces air in void space

5. Particles slide over each other easily increasing lubrication, helping in dense packing.

6. After OMC is reached, air voids remain constant. Further increase in water, increases the void space, thereby decreasing dry density.



Effect of Amount of Compaction-

1. As discussed earlier, effect of increasing compactive effort is to increase MDD And reduce OMC (Evident from Standard & Modified Proctor's Tests).

2. However, there is no linear relationship between compactive effort and MDD.

Effect of Method of Compaction-

The dry density achieved by the soil depends on the following characteristics of compacting method.

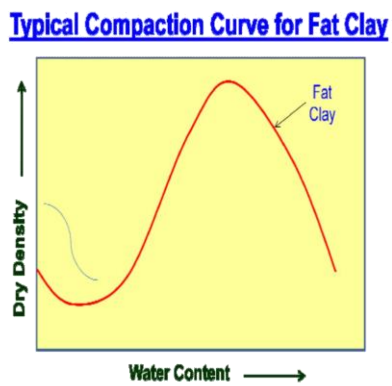
Weight of compacting equipment

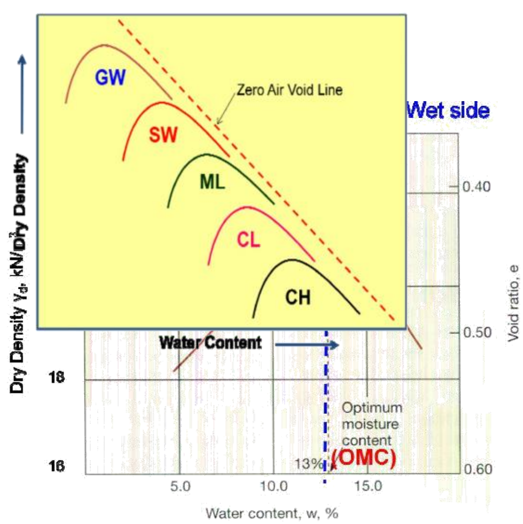
Type of compaction

Area of contact of Time of exposure of these approaches will yield different compactive effort. Further, suitability of a particular method depends on type of soil.

Effect of Type of Soil

Maximum density achieved depends on type of Coarse grained soil achieves higher density at lower water content and fine grained soil achieves lesser density,but at higher water content.





Effect of Addition of Admixtures-

1. Stabilizing agents are the admixtures added to soil.
2. The effect of adding these admixtures is to stabilize the soil.
3. In many cases they accelerate the process of densification.

Effect of compaction on soil properties-

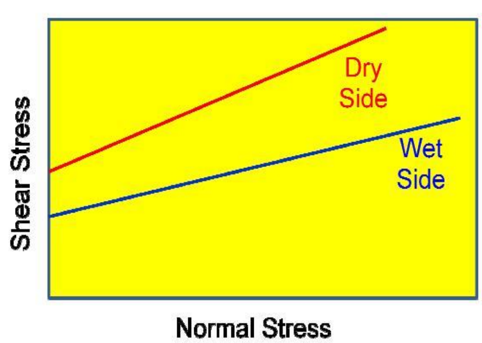
1. Density
2. Shear strength
3. Permeability
4. Bearing Capacity
5. Settlement
6. Soil Structure
7. Pore Pressure
8. Stress Strain characteristics
9. Swelling & Shrinkage

Influence on Density:

Effect of compaction is to reduce the voids by expelling out air. This results in increasing the dry density of soil mass.

Influence on Shear strength:

Increase the number of contacts resulting in increased shear strength, especially in granular soils. In clays, shear strength depends on dry density, moulding water content, soil structure, method of compaction, strain drainage condition etc. Shear strength of cohesive soils compacted dry of optimum (flocculated structure) will be higher than those compacted wet of optimum (dispersed structure).



Effect of compaction on permeability

1. Increased dry density, reduces the void space, thereby reducing permeability.
2. At same density, soil compacted dry of optimum is more permeable.
3. At same void ratio, soil with bigger particle size is more permeable.
4. Increased compactive effort reduces permeability.

Effect on Bearing Capacity

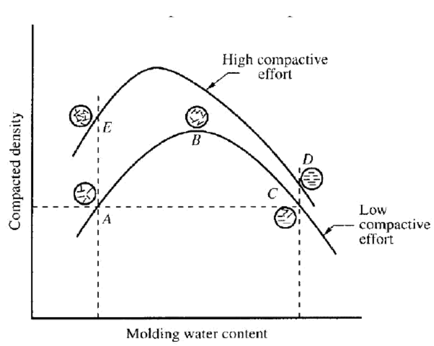
1. Increase in compaction increases the density and number of contacts between soil particles.
2. This results in increased
3. Hence bearing capacity increases which is a function of density and

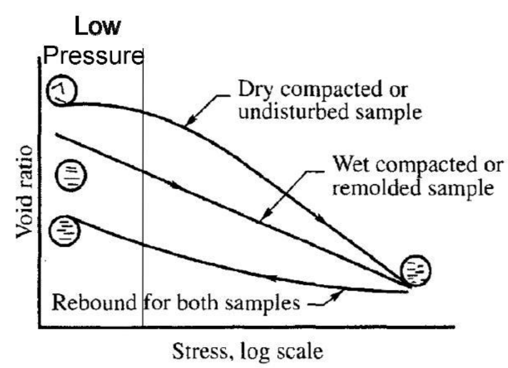
Effect on Settlement

1. Compaction increases density and decreases void ratio.
2. This results in reduced settlement.
3. Both elastic settlement and consolidation settlement are reduced.
4. Soil compacted dry of optimum experiences greater compression than that compacted wet of optimum.

Effect on Compressibility

Optimum shows more compressibility than that on dry side. But at higher pressure, behavior is similar.





Effect on Soil Structure

In fine grained soil On dry side of optimum, the structure is flocculated. The particles repel and density is less.

1. Addition of water increases lubrication and transforms the structure into dispersed structure In coarse grained soil, single grained structure is maintained

In composite soil, behaviour depends on composition.

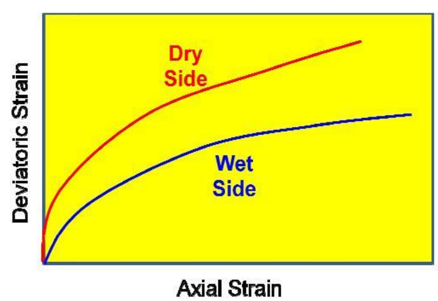
Effect on Pore Pressure

1. Clayey soil compacted dry of optimum develops less pore water pressure than that compacted wet of optimum at the same density at low strains.

2. However, at higher strains the effect is the same in both the cases.

Effect on Stress Strain Characteristics:

The strength and modulus of elasticity of soil on the dry side of optimum will always be better than on the wet side for the same density. Soil compacted dry of optimum shows brittle failure and that compacted on wet side experiences increased strain.



FIELD TEST

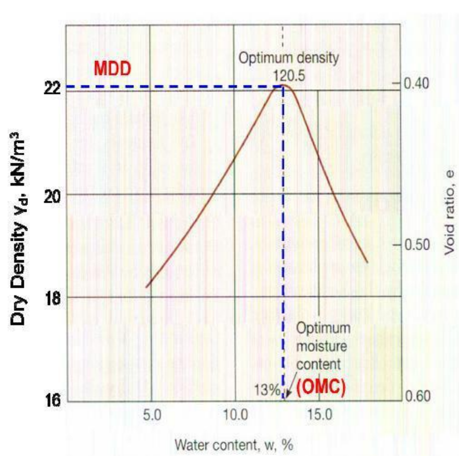
Standard Proctor's Compaction Test

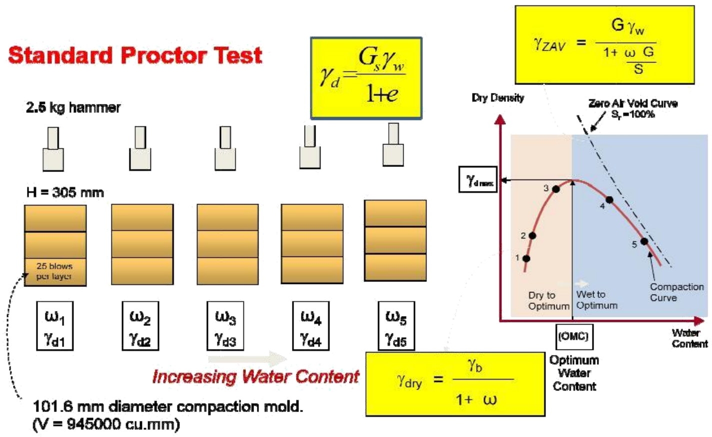
Apparatus

1. Cylindrical metal mould with detachable base plate (having internal diameter 101.6 mm, internal height 116.8 mm and internal volume 945000 mm^3)
2. Collar of 50 mm effective height
3. Rammer of weight 2.5 kgf (25 N) with a height of fall of 304.8 mm

Procedure

1. About 3 kg of dry soil, with all lumps pulverized and passing through 4.75 mm sieve is taken.
2. The quantity of water to be added in the first trial is decided. (Less for Corse grained soil and more for Fine grained soil).
3. Mould without base plate & collar is weighed
4. The inner surfaces of mould, base plate and collar are greased.
5. Water and soil are thoroughly mixed.
6. Soil is placed in mould and compacted in three uniform layers, with 25 blows in each layer. Blows are maintained uniform and vertical and height of drop is controlled.
7. After each layer, top surface is scratched to maintain integrity between layers.
8. The height of top layer is so controlled that after compaction, soil slightly protrudes in to collar.
9. Excess soil is scrapped.
10. Mould and soil are weighed (W)
11. A representative sample from the middle is kept for the determination of water content.
12. The procedure is repeated with increasing water content.
13. The number of trials shall be at least 6 with a few after the decreasing trend of bulk density.





Modified Compaction Test

In early days, compaction achieved in field was relatively less. With improvement in knowledge and technology, higher compaction became a necessity in field. Hence Modified Compaction Test became relevant. It was developed during World War II by the U.S. Army Corps of Engineeri raft.

6.4 Distinction between Standard & Modified Compaction

Standard Proctor Test	Modified Proctor Test
305 mm height of drop	450 mm height of drop
25 N hammer	45 N hammer
25 blows/layer	25 blows/layer
3 layers	5 layers
Mould size: 945 ml	Mould size: 945 ml
Energy 605160 N-mm per m^3	Energy 2726000 N-mm per m^3

Types of field Compaction Equipment:

1. Smooth Wheeled Steel Drum Rollers
2. Pneumatic Tyred Rollers

3. Sheepsfoot Rollers
4. Impact Rollers
5. Vibrating Rollers
6. Hand Operated vibrating plate & rammer compactors

Degree of Compaction

Relative compaction or degree of compaction

$$R.C. = \frac{\gamma_{d-field}}{\gamma_{d\,\max-laboratory}} \times 100\%$$

Correlation between relative compaction & relative density $R.C. = 80 + 0.2D_r$

It is a statistical result based on 47 soil samples.
Typical required R.C. >= 95%

Problem : 1 : Find the optimum moisture content (OMC).

A Standard Proctor test has yielded the values shown below. Determine:

a) The maximum dry unit weight and its OMC; remember V=1/30ft^3.

b) The moisture range for 93% of maximum dry unit weight.

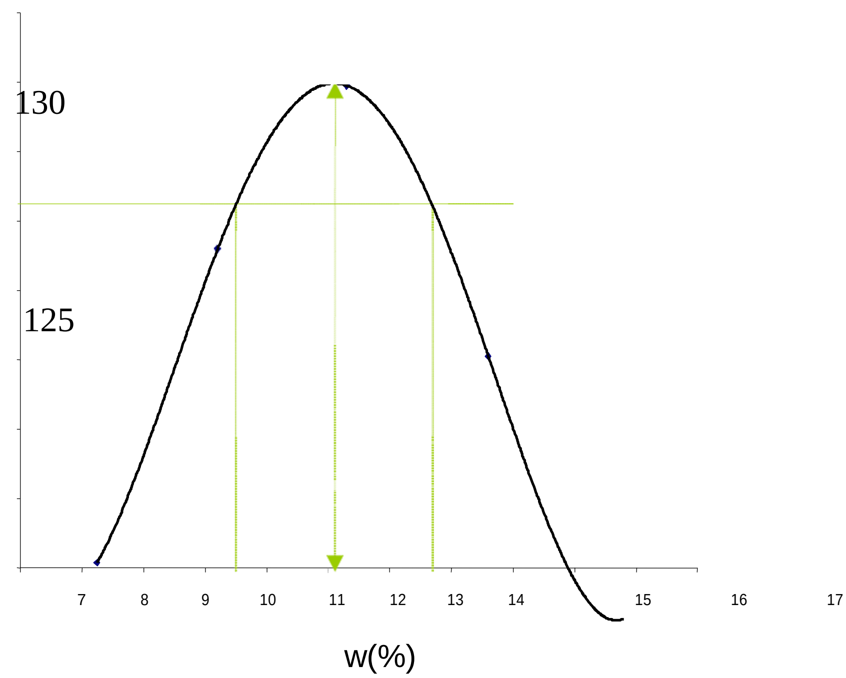
No	*Weight of wet soil(lb)*	*Moisture%*
1	3.26	8.24
2	4.15	10.20
3	4.67	12.30
4	4.02	14.60
5	3.36	16.80

Solution:

$$\gamma = \frac{W}{V} \text{ and } \gamma_d = \frac{\gamma}{1+w}$$

Formulas used for the calculations:

W(lb)	*w(%)*	$\gamma\,(lb/ft^3)$	$\gamma_d\,(lb/ft^3)$
3.26	8.24	97.8	90.35
4.15	10.20	124.5	113.0
4.67	12.30	140.1	124.8
4.02	14.60	120.6	105.2
3.36	16.80	100.8	86.30



Compaction–02: ***Find maximum dry unit weight in SI units.***

Using the table shown below:

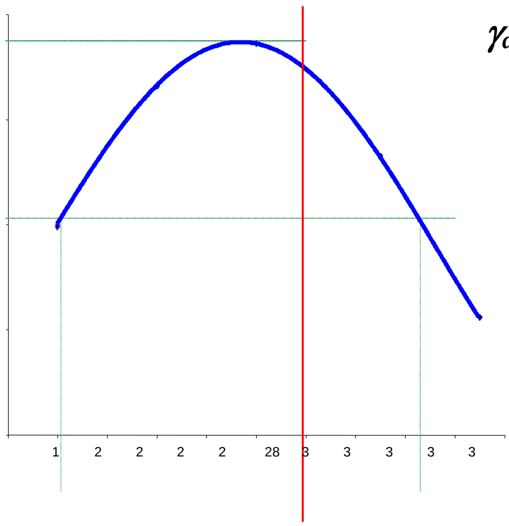
a) Estimate the maximum dry weight of a sample of road base material, tested under Standard Proctor ASTMD-698 (all weights shown are in Newton).

b) What is the appropriate moisture range when attaining 95% of Standard Proctor? $V=9.44\times10^{-3}$ m^3

Trial	1	2	3	4	5
No.W(New	14.5	15.6	16.3	16.4	16.1
ω(%)	20	24	28	33	37

Solution:

Trial No.	1	2	3	4	5
$\gamma = \frac{W}{V}\left(kN/m^3\right)$	15.4	16.5	17.3	17.4	17.1
$\gamma_d = \frac{\gamma}{1+\omega}\left(kN/m^3\right)$	12.8	13.3	13.5	13.1	12.5



γ_{dmax}*=13.5kN/m*^{3}

OMC = 28%

Problem 3: What is the saturation S at the OMC?

The results of a Standard Compaction test are shown in the table below:

$\omega(\%)$	**6.2**	**8.1**	**9.8**	**11.5**	**12.3**	**13.2**
γ (kN/m^3)	**16.9**	**18.7**	**19.5**	**20.5**	**20.4**	**20.1**
$\gamma_d = \frac{\gamma}{1+\omega}$	**15.9**	**17.3**	**17.8**	**18.4**	**18.2**	**17.8**

a)Determine the maximum dry unit weight and the OMC.

b)What is the dry unit weight and moisture range at 95% RC (Relative Compaction)?

c)Determine the degree of saturation at the maximum dry density if G_s=2.70.

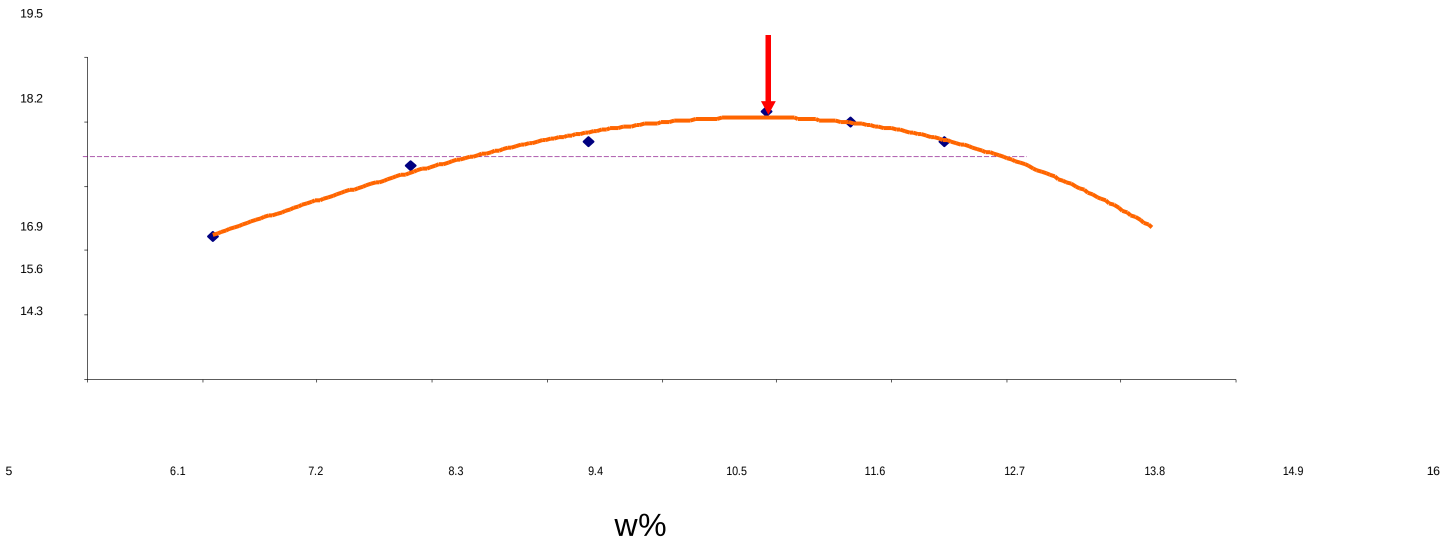
<u>Solution:</u>

a) γ_{dmax}=18.4kN/m^3,

b) OMC = 11.5%b)

c) $\gamma_{dat95\%}$=(0.95)(18.4
) 7.5 kN/m^3

The moisture range w for 95% RC is from8.75% to 13.75%.



γ_{dmax}=13.5kN/m^3 OMC=11.5%

$$S = \frac{\dfrac{wG_s\gamma_{dmax}}{\gamma_w}}{G_s - \dfrac{\gamma_{dmax}}{\gamma_w}} = \frac{\dfrac{(0.115)(2.70)(18.4)}{(9.8)}}{2.7 - \dfrac{18.4}{9.8}} = 0.71$$

Saturation S=71%

CALIFORNIA BEARING RATIO TEST

AIM:

To determine the California Bearing Ratio value of the given soil specimen.

APPARATUS:

1. CBR Test apparatus

MACHINE DESCRIPTION:

The laboratory CBR apparatus consists of a mould 150 mm diameter with a base plate and a collar, a loading frame with the cylindrical plunger of 50 mm diameter and dial gauges for measuring the expansion on soaking and the penetration values.

PROCEDURE:

1. The specimen is filled in the mould in 3 layers and compacted by a rammer.
2. The specimen in the mould is subjected to 4 soaking and the swelling and water absorption values are noted.
3. The surcharge weight is placed on the top of the specimen in the mould and the assembly is placed under the plunger of the loading frame.
4. The load values are noted corresponding to penetration values of 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 10.0 and 12.5 mm.
5. The load-penetration graph is plotted.
6. From the graph, loads corresponding to 2.5 and 5.0 mm penetration values are noted.
7. The CBR value is calculated using the formula given below for 2.5mm and 5.0 mm penetration values.
8. Normally the CBR value at 2.5 mm penetration which is higher than at 5.0 mm is reported as CBR value of the material.
9. If the CBR value obtained from the test at 5.0 mm penetration is higher than at 2.5 mm, the test is to be repeated for checking.

FORMULA USED:

$$\text{CBR}\ (\%) = \frac{\text{Load sustained by the specimen at 2.5 or 5.0 mm penetration}}{\text{Load sustained by the standard aggregates at the Corresponding penetration level}} \times 100$$

TABULATION:

S.No.	Penetration (mm)	Load (kg)

RESULT: The CBR value of the given soil specimen is

SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

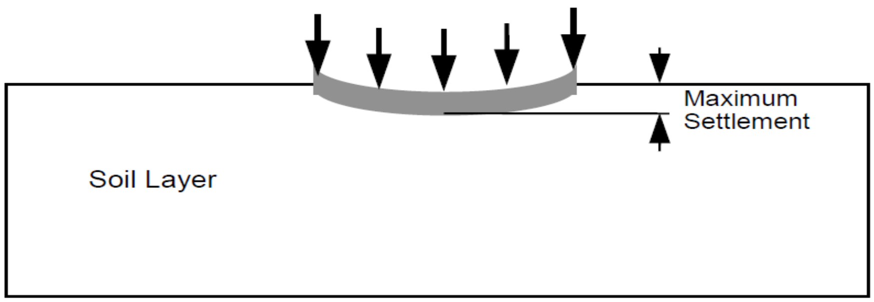
UNIT – IV- Consolidation of Soil– SCI1304

CONSOLIDATION OF SOIL

INTRODUCTION

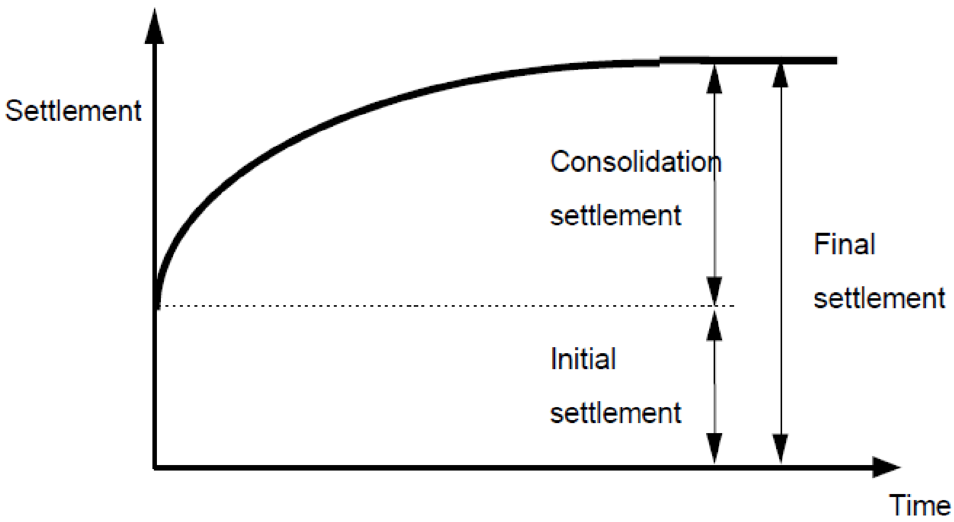
An important task in the design of foundations is to determine the settlement; this is shown

Schematically in Figure 1.



The skeletal soil material and the pore water are relatively incompressible and any change in volume can only occur due to change in the volume of the voids. For the volume of the voids to change, pore water must flow into or out of a soil element. Because this cannot happen instantaneously when a load is first applied to a soil there cannot be any immediate change in its volume.

For one-dimensional conditions with no lateral strain this implies that there is no immediate vertical strain and hence that the excess pore pressure is equal to the change in vertical stress. However, under more general conditions both lateral (or horizontal) and vertical strains can occur. Immediately after load is applied there will be no change in volume, but the soil deformations will result in an initial settlement. This is said to occur under undrained conditions because no pore water has been able to drain from the soil. With time the excess pore pressures generated during the undrained loading will dissipate and further lateral and vertical strains will occur. Ultimately the settlement will reach its long term or drained value.



Spring Analogy model

The consolidation process is often explained with an idealized system composed of a spring, a container with a hole in its cover, and water. In this system, the spring represents the compressibility or the structure itself of the soil, and the water which fills the container represents the pore water in the soil.

On figure 3, the tube on the left of the container shows the water pressure in the container.

1. The container is completely filled with water, and the hole is closed. (Fully saturated soil)
2. A load is applied onto the cover, while the hole is still unopened. At this stage, only the later resists the applied load. (Development of excessive pore water pressure)
3. As soon as the hole is opened, water starts to drain out through the hole and the spring shortens. (Drainage of excessive pore water)
4. After some time, the drainage of water no longer occurs. Now, the spring alone resists the applied load. (Full dissipation of excessive pore water pressure. End of consolidation)

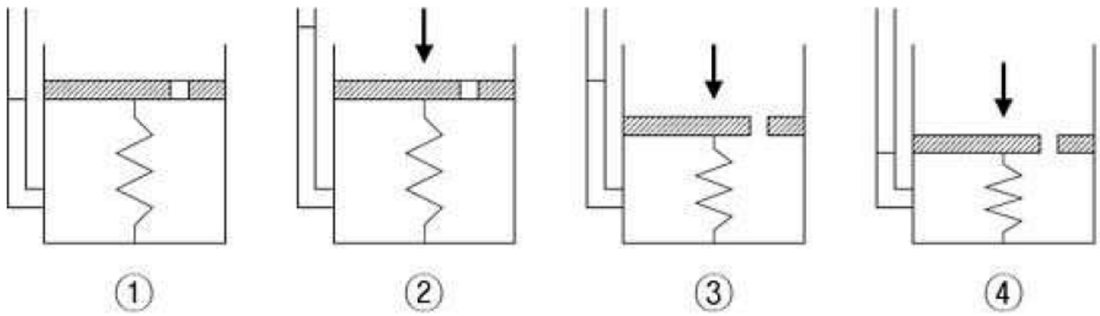
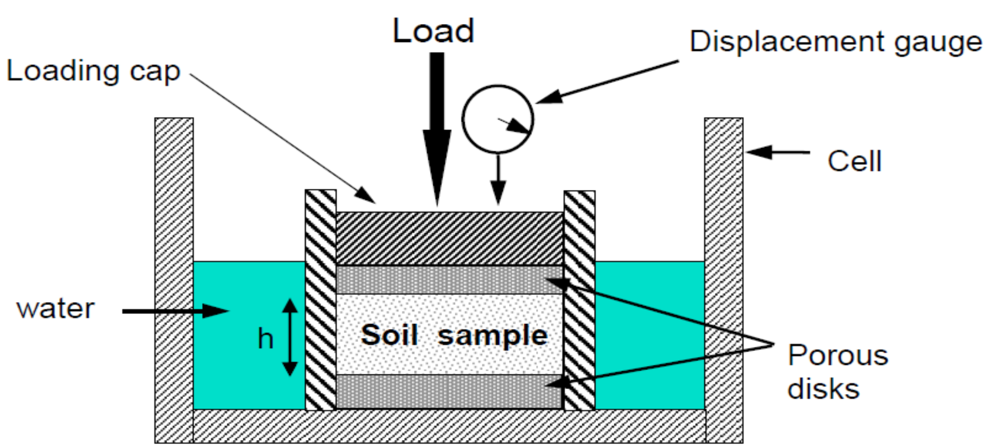


Figure 3 Process of Consolidation

The Oedometer or Consolidometer Laboratory Test

The behaviour of soil during one-dimensional loading can be tested using a device called an oedometer4, which is shown schematically in Fig. The one-dimensional condition in which the vertical strain, □zz ≠□0, and the lateral strains, □xx = □yy = 0 is also referred to as confined compression.

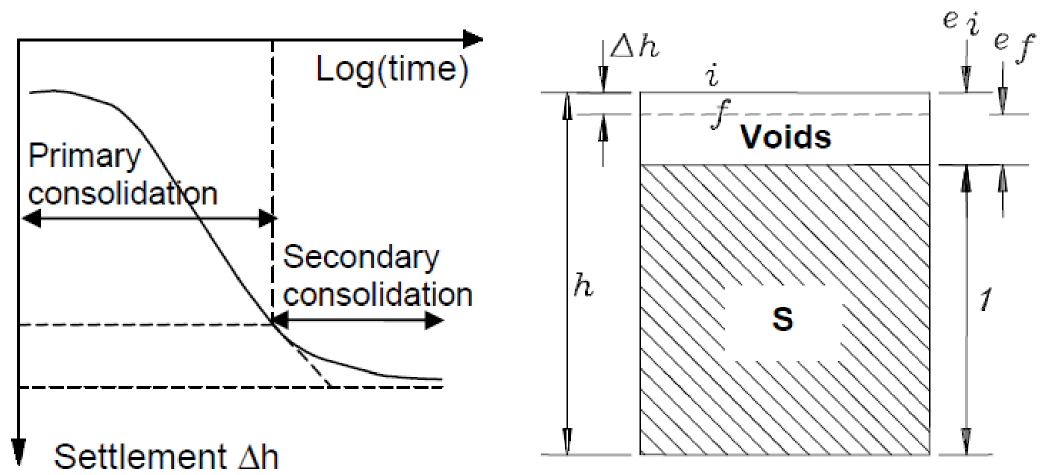


The following points may be noted:

The soil is loaded under conditions of no lateral strain (expansion), as the soil fits tightly into a relatively rigid ring.

Uncontrolled drainage is provided at the top and bottom of the specimen by porous discs (two way drainage). In more sophisticated Oedometer apparatus control of drainage is possible.

A vertical load is applied to the specimen and a record of the settlement versus time is made. The load is left on until primary consolidation ceases (usually 24 hours although this depends on the soil type, impermeable clays may take longer) shown in the graph



Primary consolidation is caused by drainage of excessive pore water. Secondary consolidation is caused by creep, is behaviour of the clay-water system, compression of organic matter, and other processes. In sand, settlement caused by secondary consolidation is negligible, but in peat, it is very significant.

The load is then increased (usually by a factor of 2, so the vertical stresses might be e.g. 20, 40, 80, 160 kPa). When the maximum load is reached, the soil is unloaded in several increments. If desired reloading can be carried out. At each step, time-settlement records are made.

It is conventional to plot the void ratio versus the logarithm of the effective stress in examining the behaviour of soil, rather than plotting the relationship between effective stress and strain as is often done in materials testing. The reason for this is that the relationship between effective stress and voids ratio is fundamental to an understanding of soil behaviour.

$$\frac{\Delta h}{h} = \frac{e_f - e_i}{1 + e_i} \ .$$

Thus the final voids ratio can be determined by measuring Δ□□□and the initial voids ratio e_i

$$e_f = \frac{\Delta h(1+e_i)}{h} + e_i$$

Consolidation Settlement Formulas

$$S_{total} = S_r + S_c$$

$$S_r = S_{rebound} = \frac{C_r}{1+e_0}\mathrm{H}_0\log(\frac{\sigma'_2}{\sigma'_1}) = \frac{C_r}{1+e_0}\mathrm{H}_0\log(\frac{\sigma'_\mathrm{c}}{\sigma'_1})$$

$$\text{note : for this case } \sigma'_1 = 0$$

$$\mathrm{S_c} = \frac{C_c}{1+e_0}\mathrm{H}_0\log(\frac{\sigma'_2}{\sigma'_1}) = \frac{C_c}{1+e_0}\mathrm{H}_0\log(\frac{\sigma'_\mathrm{c}+\Delta\sigma}{\sigma'_c})$$

Over Consolidation Ratio

$$OCR = \frac{\sigma'_c}{\sigma'}$$

where :

OCR = Over Consolidation Ratio

σ'_c = preconsolidation pressure of a specimen

σ' = present (field) effective overburben pressure

Settlement Analysis

$$S_{total} = S_r + S_c$$

$$S_r = \frac{C_r}{1+e_0} \mathrm{H}_0 \log(\frac{\sigma'_2}{\sigma'_1}) = \frac{C_r}{1+e_0} \mathrm{H}_0 \log(\frac{\sigma'_c}{\sigma'})$$

$$\mathrm{S_c} = \frac{C_c}{1+e_0} \mathrm{H}_0 \log(\frac{\sigma'_2}{\sigma'_1}) = \frac{C_c}{1+e_0} \mathrm{H}_0 \log(\frac{\sigma'_c + \Delta\sigma}{\sigma'_c})$$

$$\text{If OCR} = 1 \text{ then } \mathrm{S_r} = 0$$

Theory of One-Dimensional Consolidation

The theory for the time rate of one-dimensional consolidation was first proposed by Terzaghi (1925).

The underlying assumption in the derivation of the mathematical equations are as follows:

1. The clay layer is homogeneous.
2. The clay layer is saturated.
3. The compression of the soil layer is due to the change in volume only, which, in turn, is due to the squeezing out of water from the void spaces.
4. Darcy's law valid.
5. Deformation of soil occurs only in the direction of the load application.
6. The coefficient of consolidation is constant during the consolidation

With the above assumptions, let us consider a clay layer of thickness as shown in **Figure.** The layer is located between two highly permeable sand layers.

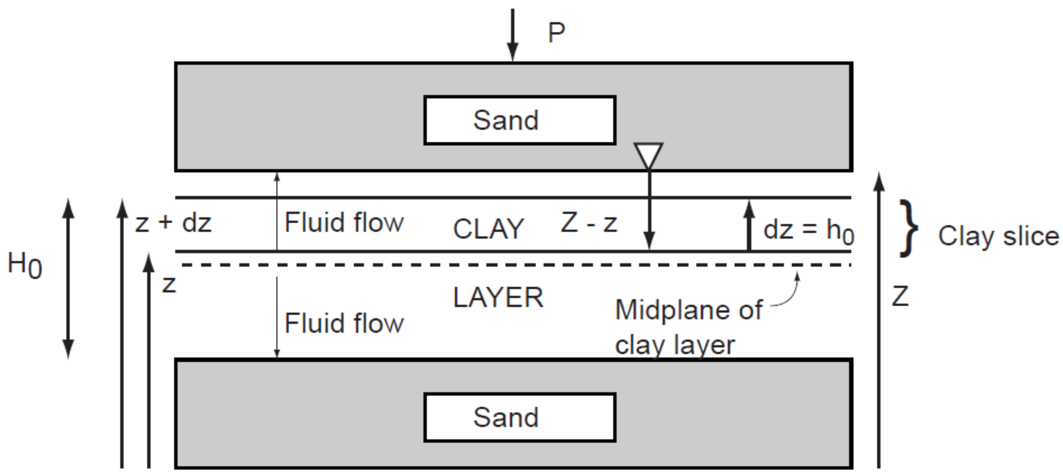
TERZAGHI'S 1-D CONSOLIDATION EQUATION (40)

I Main Topics

A The one-dimensional consolidation equation analog to heat flow

C Calculating consolidation for double-sided drainage

II The one-dimensional consolidation equation analog to heat flow



Saturated clay layer (of thickness H_0) with double drainage

At time t=0, a pressure P is applied to the top of our sand-clay-sand sandwich. The increase in load will be initially born by the water; the water pressure in all the layers goes up. This excess pore pressure (water pressure above the hydrostatic [equilibrium] level) will dissipate quickly in the sand layers because the water flows rapidly through the high-permeability sand (it flows sideways in a "violation" of our one-dimensional assumptions). The excess pore pressure will dissipate slowly in the clay. The water in the clay has to flow vertically because of the long, slow horizontal flow path in the clay. Experience shows that the sand will consolidate little. Individual sand grains are stiff, and their volumes change little with the applied loads. Unless the sand grains change their packing, the collective volume of the sand won't change significantly either. The clay, however, will consolidate significantly.

We start by looking at how the water flows, using Darcy's Law.

$$Q = -k\, i\, A \tag{40.1}$$

Q=discharge; k=conductivity; i=head gradient; A=Area

<u>Dimensions</u>

Q: L^3/t k: L/t i: L/L (dimensionless) A: L^2

$$q = Q/A = flux = -ki \tag{40.2}$$

q = unit discharge (discharge/unit area). Dimensions of velocity.

Now we investigate the head gradient, which drives fluid flow:

$$i = \frac{\partial H}{\partial z} = \frac{\partial\left(z + [u/\rho g]\right)}{\partial z} = \frac{\partial\left(z + \left[(u_{hydrostatic} + u_{excess})/\rho g\right]\right)}{\partial z} \tag{40.3}$$

The excess pore pressure (u_{excess}) in (40.3) is the difference between the actual pore pressure (u) and hydrostatic pressure ($u_{hydrostatic}$). The rate of change in the elevation head (z) and the hydrostatic pressure head cancel each other out <u>exactly</u>:

$$\frac{\partial\left(z + [u_{hydrostatic}/\rho g]\right)}{\partial z} = \frac{\partial\left(z + [\rho g(Z-z)/\rho g]\right)}{\partial z} = \frac{\partial\left(z + (Z-z)\right)}{\partial z} = \frac{\partial Z}{\partial z} = 0$$

Without this cancellation, then water at the bottom of a still swimming pool might flow to the top of the pool! So equation (40.3) simplifies:

$$i = \frac{\partial H}{\partial z} = \frac{\partial\left(z + \left[(u_{hydrostatic} + u_{excess})/\rho g\right]\right)}{\partial z} = \frac{1}{\rho g}\frac{\partial u_e}{\partial z} \tag{40.4}$$

The head gradient at the base of the slice (i.e., at elevation "z") is:

$$i_1 = \frac{1}{\rho g}\frac{\partial u_e}{\partial z} \tag{40.5}$$

The head gradient at the top of the slice (i.e., at elevation "z+dz") is:

$$i_2 = \frac{1}{\rho g}\frac{\partial}{\partial z}(u_e + \Delta u_e) = \frac{1}{\rho g}\frac{\partial}{\partial z}\left(u_e + \frac{\partial u_e}{\partial z}dz\right) = \frac{1}{\rho g}\left(\frac{\partial u_e}{\partial z} + \frac{\partial^2 u_e}{\partial z^2}dz\right) \tag{40.6}$$

The change in unit discharge [i.e<u>, the net flow of water out of the clay slice (per unit area</u>)] reflects the water <u>loss</u> in the slice (per unit area):

$$\Delta q = q_2 - q_1 = -k(i_2 - i_1) = \frac{-k}{\rho g}\frac{\partial^2 u_e}{\partial z^2}dz \tag{40.7}$$

Two comments. First, we have held k, the hydraulic conductivity, constant - is that OK? Second, the right hand side of (40.7) looks somewhat like one side of the heat equation. What about the left side? The change in unit discharge of the slice times the area (A) of the slice gives the water volume loss with respect to time:

$$\Delta q \; A = \frac{\partial V_{water}}{\partial t} \qquad \left(\frac{L}{T} L^2 = \frac{L^3}{T} \right) \tag{40.8}$$

The water volume loss is also the void volume loss in the clay. If both sides of (40.8) are divided by the area A, then the right side is the change in void volume/unit area with respect to time, or in other words, the height change of the slice with respect to time. The height change, in turn, is the product of the vertical strain (ε_z) and the original slice height h_0 = dz, so

$$\Delta q = \frac{\partial (V_{water}/A)}{\partial t} = \frac{\partial (\Delta h)}{\partial t} = \frac{\partial (\Delta h/h_0)dz}{\partial t} = \frac{(\partial \varepsilon_z)dz}{\partial t} \tag{40.9}$$

Let's substitute this into the left side of equation (40.7):

$$\frac{(\partial \varepsilon_z)dz}{\partial t} = \frac{-k}{\rho g} \frac{\partial^2 u_e}{\partial z} dz \tag{40.10a}$$

$$\frac{\partial \varepsilon_z}{\partial t} = \frac{-k}{\rho g} \frac{\partial^2 u_e}{\partial z^2} \tag{40.10b}$$

This looks even more like the heat equation, but note that $\varepsilon_z \neq u_e$.

Equation (40.10b) expresses how the vertical strain changes with time relative to the second partial derivative of the excess pore pressure with respect to position. We seek to find how the vertical strain changes as a function of the effective stress. The coefficient of compressibility (m_v), also known as the coefficient of volume change, is defined as the change in volumetric strain divided by the change in effective stress. For our 1-D case:

$$m_v = \frac{\Delta V / V_0}{\Delta \sigma'} = \frac{\Delta h / h_0}{\Delta \sigma'} \tag{40.11}$$

In our case here, the change in effective stress is exactly opposite to the change in the excess pore pressure (i.e., the increase in load picked up by the soil skeleton equals the decrease in the excess water pressure). So:

$$m_v = \frac{\Delta h / h_0}{\Delta\sigma'} = \frac{-\Delta h / h_0}{\Delta u_e} = \frac{-\varepsilon_z}{u_e - u_0} \qquad (40.12)$$

Solving for the strain, which appears on the left side of (40.10b), gives

$$\varepsilon_z = -m_v (u_e - u_0) \qquad (40.13)$$

Inserting (40.13) into (40.10b) yields

$$\frac{\partial (m_v (u_e - u_0))}{\partial t} = \frac{k}{\rho g} \frac{\partial^2 u_e}{\partial z^2} \qquad (40.14)$$

Now differentiate the left side of (40.14), noting that u_0 is a constant:

$$m_v \frac{\partial (u_e - u_0)}{\partial t} = m_v \left(\frac{\partial u_e}{\partial t} - \frac{\partial u_0}{\partial t} \right) = m_v \frac{\partial u_e}{\partial t} = \frac{k}{\rho g} \frac{\partial^2 u_e}{\partial z^2} \qquad (40.15)$$

This can be simplified by grouping all the constants:

$$\frac{\partial u_e}{\partial t} = \frac{k}{m_v \rho g} \frac{\partial^2 u_e}{\partial z} = C_v \frac{\partial^2 u_e}{\partial z} \qquad (40.16)$$

The term Cv is called the coefficient of consolidation

Terzaghis Limitation is only applicable for Shallow Foundation.

Problems

1. For a laboratory consolidation test on a soil specimen that is drained on both sides, the following were obtained:

   Thickness of the clay specimen = 25 mm

   $P_1 = 50\ kN/m^2$; $e_1 = 0.92$

   $P_2 = 120\ kN/m^2$; $e_2 = 0.78$

Time for 50% consolidation = 2.5 min. Determine the soil permeability for the

$$a_v = \frac{\Delta e}{\Delta \sigma'} = \frac{0.92 - 0.78}{120 - 50} = 0.002 \ m^2/kN$$

$$H = \frac{25\ mm}{2} = \frac{0.025\ m}{2} = 0.0125\ m$$

$$t_{50} = 25 \text{ min}$$

$$c_v = \frac{T_{50}.H^2}{t_{50}}$$

$$= \frac{0.197\ x\ (0.0125)^2}{2.5}$$

$$= 1.23\ x\ 10^{-5}\ m^2/\text{min}$$

$$k = \frac{c_v.a_v.\gamma_w}{1+e_0}$$

$$= \frac{1.23\ x\ 10^{-5}\ x\ 0.002\ x\ 9.81}{1+0.92}$$

$$= 1.26\ x\ 10^{-7}\ m/\text{min}$$

loading range.

2 A 3 m thick clay layer in the field under a given surcharge will undergo 7 cm of total primary consolidation.If the first 4 cm of settlement takes 90 days, calculate the time required for the first 2 cm of settlement.

Solution:

Total consolidation = 7 cm
For 4 cm settlement, U_1 = 4/7 x100 = 57.14%
For 2 cm settlement, U_2 = 2/7 x 100 = 28.57%
t_1 = 90 days.

For,

$U \le 60\%$,

$$\frac{c_v . t}{H^2} = T \propto U^2$$

$$\therefore \frac{t_1}{t_2} = \frac{U_1^2}{U_2^2}$$

$$\therefore t_2 = \frac{U_2^2}{U_1^2} \, x \, t_1$$

$$= \frac{(28.57)^2}{(57.14)^2} \, x \, 90$$

$$= 22.5 \, days$$

3. A 3 m thick layer of saturated clay in the field under a surcharge loading will achieve 90% consolidation in 75 days in double drainage conditions. Find the coefficient of consolidation of the clay.

Solution:

As the clay layer has two-way drainage, H = 1.5 m = 150 cm
t_{90} = 75 days = 75 x 24 x 60 x 60 seconds
For 90% consolidation (U = 90%)

$$T_{90} = \frac{c_v . t_{90}}{H^2} = 0.848$$

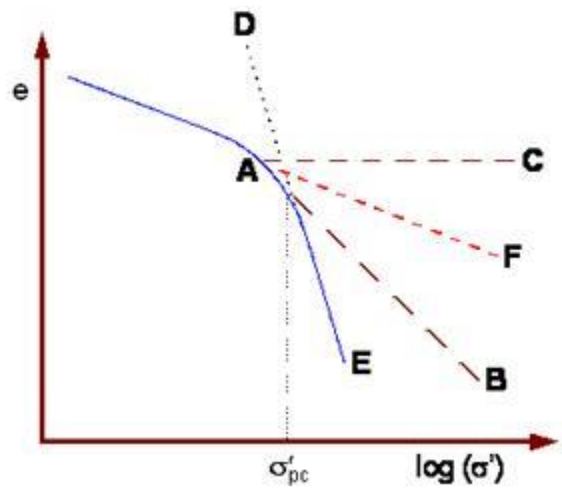
$$\therefore c_v = \frac{T_{90} . H^2}{t_{90}}$$

$$= \frac{0.848 \, x \, (150)^2}{75 \, x \, 24 \, x \, 60 \, x \, 60}$$

$$= 2.94 \, x \, 10^{-3} \, cm^2 / s$$

Determination of Pre-consolidation pressure

it is possible to determine the pre-consolidation stress that the soil had experienced. The soil sample is to be loaded in the laboratory so as to obtain the void ratio - effective stress relationship. Empirical procedures are used to estimate the pre-consolidation stress, the most widely used being **Casagrande's construction** which is illustrated.



The steps in the construction are:

- Draw the graph using an appropriate scale.
- Determine the point of maximum curvature **A.**
- At **A,** draw a tangent **AB** to the curve.
- At **A,** draw a horizontal line **AC.**
- Draw the extension **ED** of the straight line portion of the curve.
- Where the line **ED** cuts the bisector **AF** of angle **CAB**, that point corresponds to the preconsolidation stress.

Calculation of coefficient of consolidation from laboratory test results

1. Logarithm-of-time method
2. Square-root-of-time method
3. Su's maximum slope method
4. Sivaram and Swamee's computational method

Calculation of coefficient of consolidation from laboratory test results. For a given load increment, the coefficient of consolidation Cv can be determined from laboratory observations of time vs. dial reading. Two graphical procedures are commonly used for this: the logarithm of-time method proposed by Casagrande and Fadum (1940), and the square-root-of-time method proposed by Taylor (1942). There are also two other useful methods, which were proposed by Su (1958) and Sivaram and Swamee (1977). Each of these four methods is described below.

Logarithm-of-time method

1. Plot the dial readings for sample deformation for a given load increment against time on semi log graph paper as shown in Fig.

2. Plot two points, P and Q on the upper portion of the consolidation curve which correspond to time t_1 and t_2 , respectively. Note that $t_2 = 4t_1$.

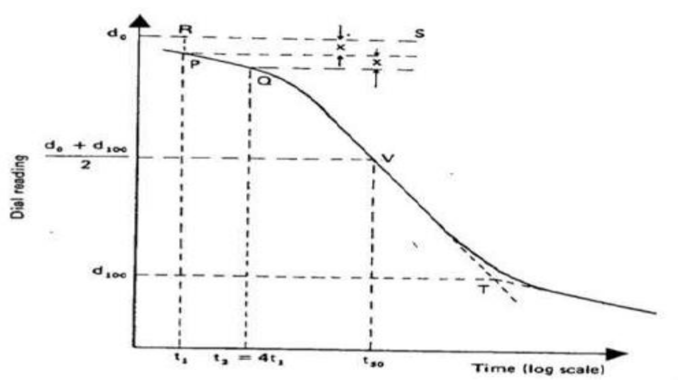
3. The difference of dial readings between P and Q is equal to x. locate point R, which is at a distance x above point P.

4. Draw the horizontal line RS. The dial reading corresponding to this line is d_0 , which corresponds to 0% consolidation.

5. Project the straight-line portions of the primary consolidation and the secondary consolidation to intersect at T. the dial reading corresponding to T is $d100$, i.e., 100% primary consolidation.

6. Determine the point V on the consolidation curve which corresponds to a dial reading of $(d_0 + d_{100})/2 = d_{50}$. The time corresponding to the point V is $t50$ $i.e.$, time for 50% consolidation.

7. Determine Cv from the equation $T_v = C_v\ t/H^2$. The value of T_v for U_{av} = 50% is 0.197



Logarithm-of-time method for determination of C_v

$$C_v = \frac{.0197H^2}{t_{50}}$$

Square-root-of-time method

1. Plot the dial reading and the corresponding *square-root-of-time t*

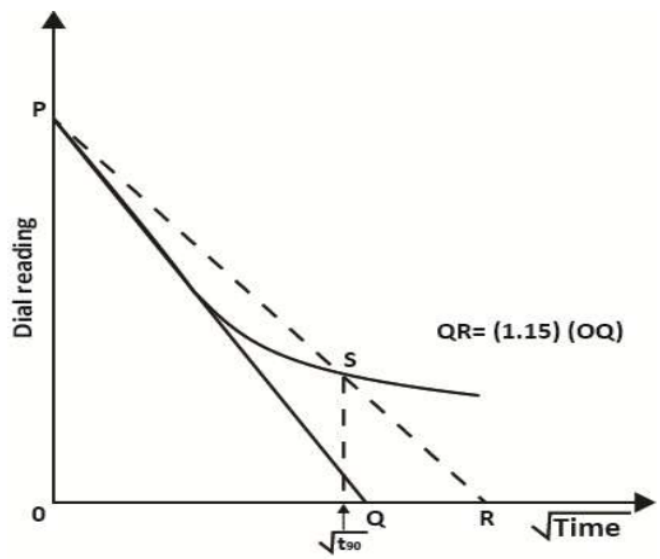
2. Draw the tangent *PQ* to the early portion of the plot.

3. Draw a line *PR* such that *OR* = (1.15) (*OQ*).

4. The abscissa of the point *S* (i.e., the intersection of *PR* and the consolidation curve) will give t_{50}(i.e., the square-root-of-time for 90% consolidation).

5. The value of T_v for Uav = 90% *is* 0.848. so,

$$C_v = \frac{0.848 H^2}{t_{50}}$$



SCHOOL OF BUILDING AND ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

UNIT –V- SHEAR STRENGTH AND STABILITY – SCI1304

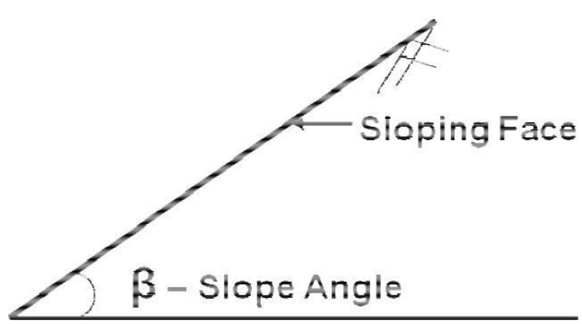
SHEAR STRENGTH AND STABILITY

Introduction:

An exposed ground surface that stands at an angle (β) with the horizontal is called slope. Slopes are required in the construction of highway and railway embankments, earth dams, levees and canals. These are constructed by sloping the lateral faces of the soil because slopes are generally less expensive than constructing walls. Slopes can be natural or man made. When the ground surface is not horizontal a component of gravity will try to move the sloping soil mass downwards. Failure of natural slopes (landslides) and man-made slopes has resulted in much death and destruction. Some failures are sudden and catastrophic; others are widespread; some are localized. Civil Engineers are expected to check the safety of natural and slopes of excavation. Slope stability analysis consists of determining and comparing the shear stress developed along the potential rupture surface with the shear strength of the soil. Attention has to be paid to geology, surface drainage, groundwater, and the shear strength of soils in assessing slope stability.

In this chapter, we will discuss simple methods of slope stability analysis from which one will be able to:

- Understand the forces and activities that provoke slope failures.
- Understand the effects of seepage on the stability of slopes.
- Estimate the stability of slopes with simple geometry for different types of soils.



Man made slopes are used in

- Highways
- Railways
- Earth Dams
- River Training works

Slope Failure Triggering Mechanisms

- Intense Rain-Fall
- Water-Level Change Seepage Water Flow Volcanic Eruption
- Earthquake Shaking Human activity

Causes of Slope failure

1. **Erosion:** The wind and flowing water causes erosion of top surface of slope and makes the slope steep and thereby increase the tangential component of driving force.
2. **Steady Seepage:** Seepage forces in the sloping direction add to gravity forces and make the slope susceptible to instability. The pore water pressure decrease the shear strength. This condition is critical for the downstream slope.
3. **Sudden Drawdown:** in this case there is reversal in the direction flow and results in instability of side slope. Due to sudden drawdown the shear stresses are more due to saturated unit weight while the shearing resistance decreases due to pore water pressure that does not dissipate quickly.
4. **Rainfall:** Long periods of rainfall saturate, soften, and erode soils. Water enters into existing cracks and may weaken underlying soil layers, leading to failure, for example, mud slides.
5. **Earthquakes:** They induce dynamic shear forces. In addition there is sudden buildup of pore water pressure that reduces available shear strength.
6. **External Loading:** Additional loads placed on top of the slope increases the gravitational forces that may cause the slope to fail.

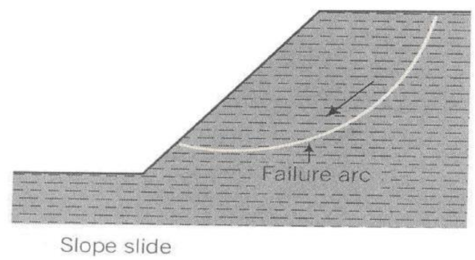
7. **Construction activities at the toe of the slope:** Excavation at the bottom of the sloping surface will make the slopes steep and there by increase the gravitational forces which may result in slope failure

Types of failure

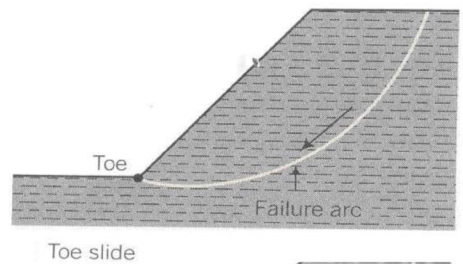
Broadly slope failures are classified into 3 types as

1. Face (Slope) failure
2. Toe failure
3. Base failure

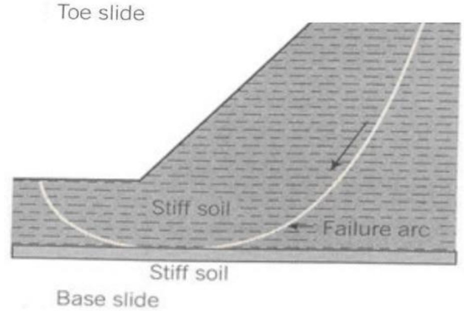
1. **Face (Slope) Failure:** This type of failure occurs when the slope angle ( ) is large and when the soil at the toe portion is strong.



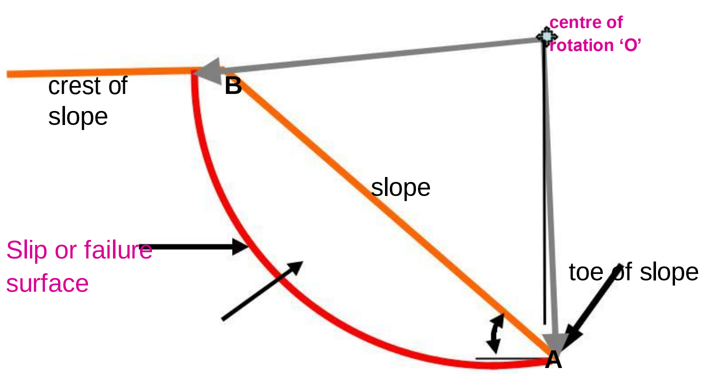
2. **Toe Failure:** In this case the failure surface passes through the toe. This occurs when the slope is steep and homogeneous.



3. **Base Failure:** In this case the failure surface passes below the toe. This generally occurs when the soil below the toe is relatively weak and soft.



Definition of Key Terms



Slip or failure zone: It is a thin zone of soil that reaches the critical state or residual state and results in movement of the upper soil mass.
Slip plane or failure plane or slip surface or failure surface: It is the surface of sliding.

Sliding mass: It is the mass of soil within the slip plane and the ground surface.

Slope angle : It is the angle of inclination of a slope to the horizontal. The slope angle is sometimes referred to as a ratio, for example, 2:1 (horizontal: vertical).

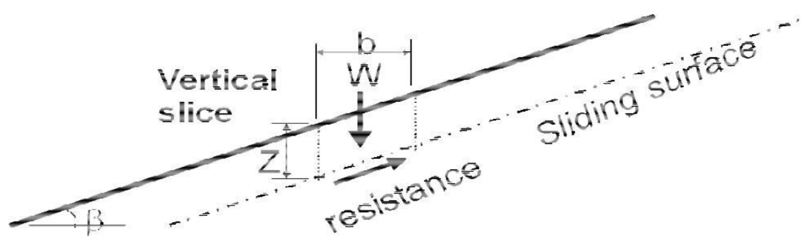
Stability Analysis consists of

- Determination of the potential failure surface.
- Forces that tend to cause slip.
- Forces that tend to restore (stabilize)
- Determination of the available margin of safety.

Types of Slopes

1. **Infinite Slopes**
2. **Finite Slopes**

Infinite slopes: They have dimensions that extend over great distances and the soil mass is inclined to the horizontal.



Infinite Slope

Finite slopes:
A finite slope is one with a base and top surface, the height being limited. The inclined faces of earth dams, embankments and excavation and the like are all finite slopes.

Factor of safety

Factor of safety of a slope is defined as the ratio of average shear strength (τ_f) of a soil to the average shear stress (τ_d) developed along the potential failure surface.

$$FS = \frac{\tau_f}{\tau_d}$$

FS = Factor of safety

τ_f = average shear strength of the soil

τ_d = average shear stress developed along the potential surface.

Shear Strength:-

Shear strength of a soil is given by

$$\tau_f = c + \sigma \tan\phi$$

Where, c = cohesion

= angle of internal friction

= Normal stress on the potential failure surface

Similarly, the mobilized shear strength is given by

$$\tau_d = c_d + \sigma \tan\phi_d$$

c_d and ϕ_d are the cohesion and angle of internal friction that develop along the potential failure surface.

$$FS = \frac{c + \sigma \tan\phi}{c_d + \sigma \tan\phi_d}$$

FS w.r.t cohesion is

$$F_c = \frac{c}{c_d}$$

$$F_\phi = \frac{\tan\phi}{\tan\phi_d}$$

When F_c = F it gives Factor of safety w.r.t strength

$$\frac{c}{c_d} = \frac{\tan\phi}{\tan\phi_d}$$

Then F_s = F_c =F

When FS = 1, then the slope is said to be in a state of failure.

Infinite Slopes:

Infinite slopes have dimensions that extended over great distances and the soil mass is inclined to the horizontal. If different strata are present strata boundaries are assumed to be parallel to the surface. Failure is assumed to occur along a plane parallel to the surface.

Infinite Slope in layered soils

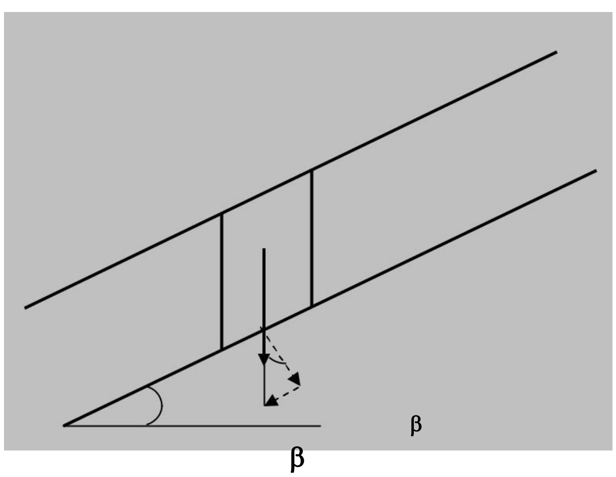
3 cases of stability analysis of infinite slopes are considered

Case (i) Cohesionless soil

Case (ii) Cohesive soil

Case (iii) Cohesive-frictional soil.

Infinite slopes in cohesionless soils



Infinite slope in cohesion less soil

Consider an infinite slope in a cohesionless soil inclined at an angle to the horizontal as shown. Consider an element 'abcd' of the soil mass.

Let the weight of the element be W.

The component of W parallel to slope = T = W Sin

The component of W perpendicular to slope = N = W Cos

The force that causes slope to slide = T = W Sin

The force that restrains the sliding of the slope = $\sigma \tan\phi$

= N $\tan\phi$ = W Cos $\tan\phi$

The factor of safety against sliding failure is

$$FS = \frac{\text{Restraining force}}{\text{Sliding force}} = \frac{W \cos\beta \tan\phi}{W \sin\beta}$$

$$FS = \frac{\tan\phi}{\tan\beta}$$

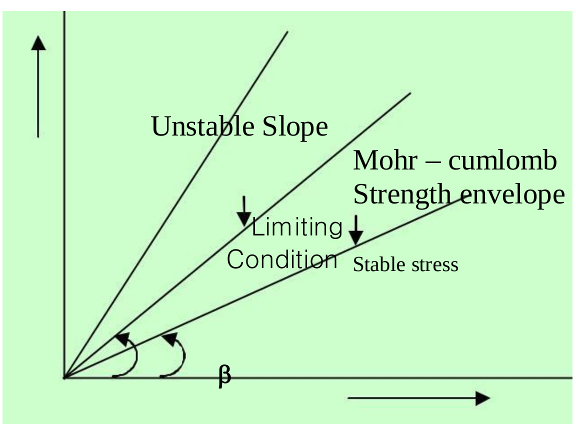
under the limiting equilibrium FS =1

$$\tan\beta = \tan\phi$$

$$\beta = \phi$$

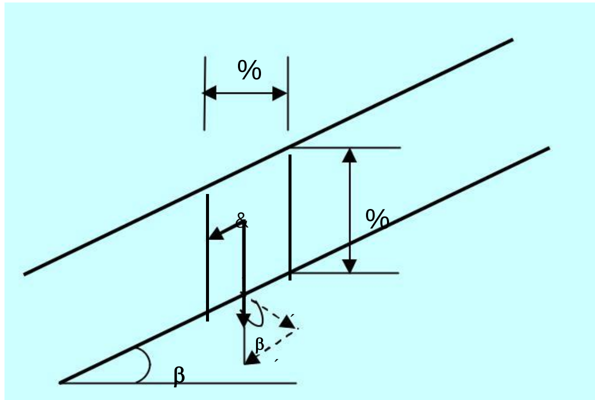
The maximum inclination of an infinite slope in cohesion less soil for stability is equal to the angle of internal friction of the soil".

The limiting angle of inclination for stability of an infinite slope in cohesion less soil is as shown below.



Normal stress Vs. Shear stress indicating the limiting condition of a slope

Effect of seepage when seepage force is parallel to the slope



Infinite slope in cohesionless soil under steady seepage

Consider an infinite slope AB with steady seepage parallel to the sloping surface. Due to this seepage force (J_s) acts in the direction of seepage

Seepage force $Js = i\,\gamma_w b_j Z_j$

i = Sinβ (Since seepage is parallel to the slope)

$$N' = W'Cos\beta = \gamma' b_j Z_j Cos\beta$$

$$T = W'Sin\beta + Js = \gamma' b_j Z_j Sin\beta + \gamma_w b_j Z_j Sin\beta$$

$$= (\gamma' + \gamma_w)\, b_j Z_j Sin\beta = \gamma_{sat}\, b_j Z_j Sin\beta$$

$$FS = \frac{Restraining\ force}{Sliding\ force} = \frac{N' \tan\phi'}{T}$$

$$FS = \frac{\gamma' b_j Z_j Cos\beta \tan\phi'}{\gamma_{sat}\, b_j Z_j Sin\beta} = \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta}$$

At limit equilibrium Fs = 1

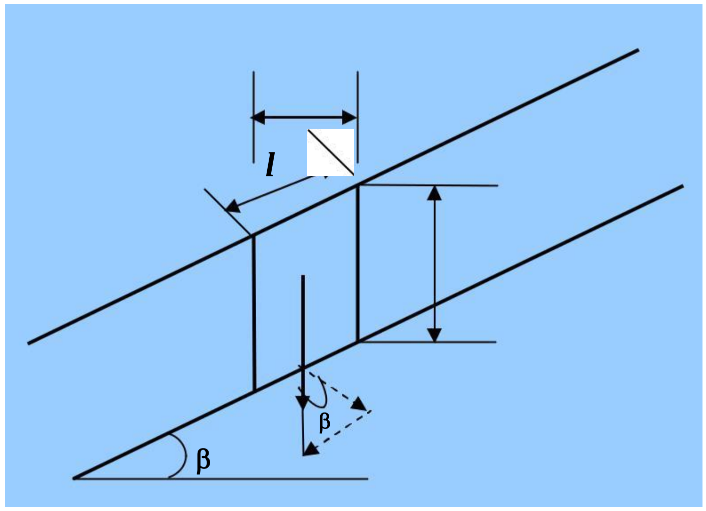
$$\tan\beta = \frac{\gamma'}{\gamma_{sat}} \tan\phi'$$

$$\frac{\gamma'}{\gamma_{sat}} = \frac{1}{2}$$ (approximately)

$$\therefore \tan\beta = \frac{1}{2}\tan\phi'$$

The seepage parallel to the slope reduces the limiting slope angle in coarse grained soil by one-half of the friction angle".

Infinite slope in pure cohesive soil



Free body diagram of slice in infinite slope of pure cohesive soil

The normal stress on the failure surface is $\boldsymbol{\sigma_n = \frac{N}{l \times 1}}$

$$\mathrm{Cos}\beta = \frac{b}{l}\,; \quad l = \frac{b}{\mathrm{Cos}\beta}$$

$$\sigma_n = \frac{W\,Cos\beta}{\frac{b}{Cos\beta}} = \frac{W}{b}Cos^2\beta$$

$$W = \gamma Z\,b$$

$$\sigma_n = \frac{\gamma Z\,b}{b}Cos^2\beta = \gamma Z\,Cos^2\beta$$

$$\sigma_n = W\,Cos^2\beta$$

The shear stress $_d = \frac{T}{l \times 1} = \frac{W\,\mathrm{Sin}}{l \times 1} = \frac{W\,\mathrm{Sin}}{\frac{b}{\mathrm{Cos}}}$

$$\tau_d = \frac{W}{b}\,\mathrm{Sin}\,\beta\,\mathrm{Cos}\,\beta$$

$$\tau_d = \frac{\gamma Z b}{}\,\mathrm{Sin}\beta\mathrm{Cos}\beta$$

b

$$\tau_d = \gamma\, Z \operatorname{Sin}\beta \operatorname{Cos}\beta$$

The Mohr Coulomb shear stress

$$\tau_f = c + \sigma \tan\phi = c \quad \text{(cohesive soil)}$$

$$FS = \frac{\tau_f}{\tau_d} = \frac{c}{\gamma Z\, Sin\beta\, Cos\beta}$$

$$FS = \frac{c}{\gamma Z\, Sin\beta\, Cos\beta}$$

At critical conditions FS =1

$$Z_c = \frac{c}{\gamma\, Sin\beta\, Cos\beta}$$

For a given slope angle 'Z_c' is directly proportional to the cohesion and inversely proportional to unit weight

$\frac{c}{\gamma\, Z_c}$ is a dimensionless quantity called the stability number denoted by S_n

$$S_n = \frac{c}{\gamma\, Z_c}$$

we know

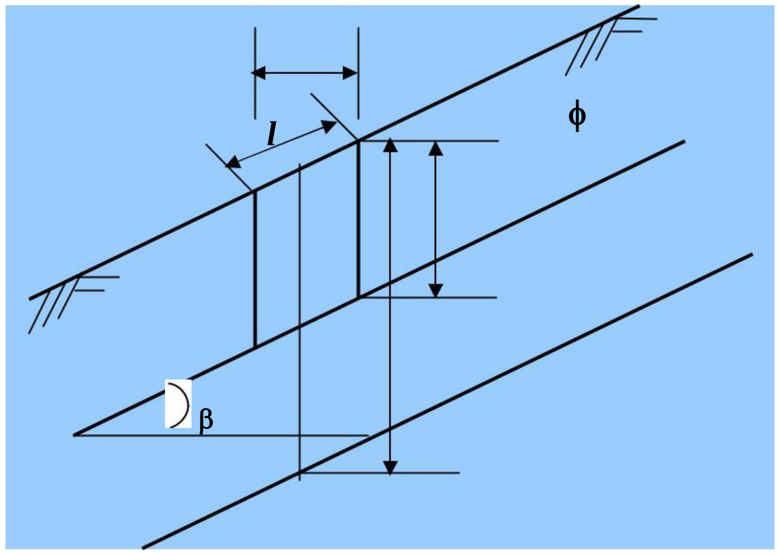
$$FS = \frac{c}{\gamma Z\, Sin\beta\, Cos\beta}$$

$$Sin\beta\, Cos\beta = \frac{c}{\gamma Z_c}$$

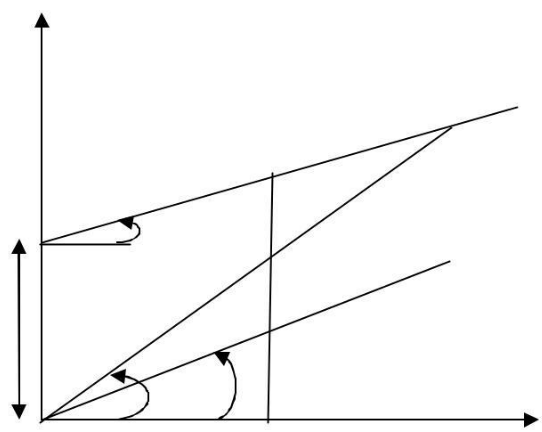
$$FS = \frac{c}{\gamma Z\, \frac{c}{\gamma Z_c}} = \frac{Z_c}{Z}$$

$$FS = \frac{Z_c}{Z}$$

Infinite slope in cohesive frictional soil



Infinite slope in c-ϕ soil



Relation between strength envelope and angle of repose

Consider an infinite slope in c-ϕ soil as shown with slope angle β

The strength envelope for the c-ϕ soil is $\tau_f = c + \sigma_{nf} \tan\phi$. If the slope angle β is less then ϕ, slope will be stable for any depth.

When the slope angle β > ϕ, the slope will be stable upto a depth $Z=Z_c$ corresponding to point P. The point P corresponds to the depth at which the shear stress mobilized will be equal to the available shear strength.

For all depths less than that represented by point P shearing stress will be less than the shear strength and the slope will be stable

$\tau = c + \sigma \tan \phi$

$\tau_f = c + \sigma \tan \phi$

At P:

$$\sigma_{nf} = \gamma Z_c Cos^2 \beta$$

$\tau_f = c + \gamma Z_c Cos^2 \beta \tan\phi$ ----- 1

the developed shear stress is

$\tau_d = \gamma Z_c Sin\beta Cos\beta$ ----- 2

Equating 1 and 2

$\gamma Z_c Sin\beta Cos\beta = c + \gamma Z_c Cos^2 \beta \tan\phi$

$\gamma Z_c (Sin\beta Cos\beta - Cos^2 \beta \tan\phi) = c$

$$\gamma Z_c Cos^2 \beta \frac{Sin\beta}{Cos\beta} - \tan \phi = c$$

$\gamma Z_c Cos^2 \beta \left(\tan \beta - \tan \phi\right) = c$

$$Z_c = \frac{c}{\gamma Cos^2 \beta (\tan \beta - \tan\phi)}$$

Therefore the critical depth Z_c is proportional to cohesion for a given value of slope angle (β) and friction angle (ϕ)

Therefore $\frac{c}{\gamma Z_c} = Cos^2 \beta (\tan \beta - \tan\phi)$

The term $\frac{c}{\gamma Z_c} = S_n$ Stability number (S_n)

For all depth $Z < Z_c$

$$FS = \frac{Shearingstrength}{shearingstress} = \frac{c + \gamma Z Cos^2 \beta \tan\phi}{\gamma Z Sin\beta Cos\beta}$$

Finite Slopes

A finite slope is one with a base and top surface, the height being limited. The inclined faces of earth dams, embankments, excavation and the like are all finite slopes.

Investigation of the stability of finite slopes involves the following steps

a) assuming a possible slip surface,

b) studying the equilibrium of the forces acting on this surface, and

c) Repeating the process until the worst slip surface, that is, the one with minimum margin of safety is found.

Methods:-

I. Total stress analysis for purely cohesive soil.

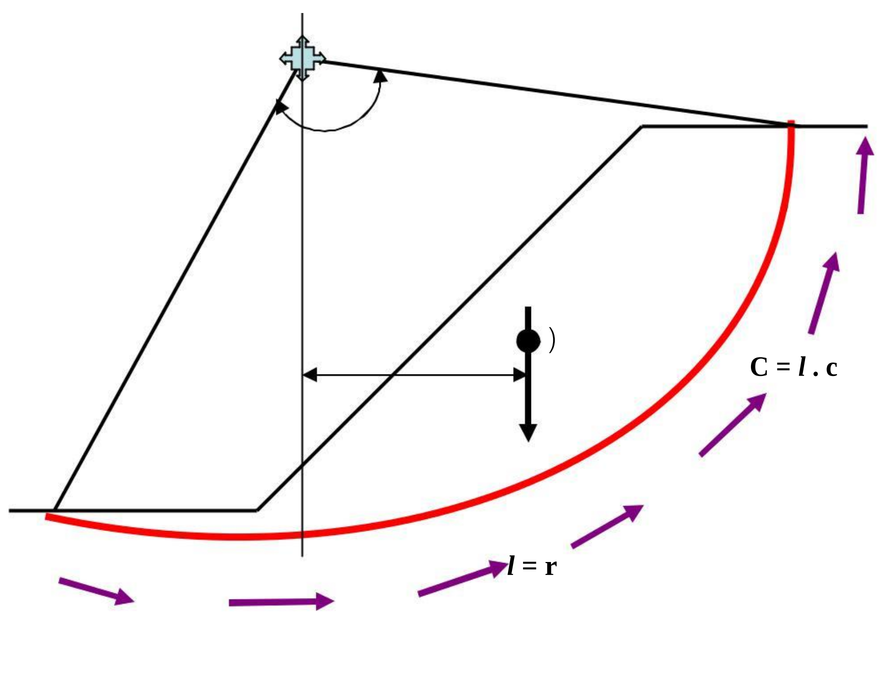
II. Total stress analysis for cohesive –frictional (c-ϕ) soil – (Swedish method of slices or Method of slices)

III. Effective stress analysis for conditions of steady seepage, rapid drawdown and immediately after construction.

IV. Friction circle method

V. Taylor's method.

Total stress analysis for pure cohesive soil



Finite slope in cohesive soils

The analysis is based on total stresses, it is also called $\phi = 0$ analysis. It gives the stability of an embankment immediately after construction. It is assumed that the soil has no time to drain and the shear strength parameters used are obtained from undrained conditions with respect to total stresses. These may be obtained from either unconfined compression test or an undrained triaxial test without pore pressure measurements.

Let AB be a trial slip surface in the form of a circular arc of radius 'r' with respect to center of rotation 'O' as shown in Fig

Let `W' be the weight of the soil within the slip surface

Let `G' be the position of its centre of gravity.

The exact position of G is not required and it is only necessary to ascertain the position of the line of action of W, this may be obtained by dividing the failure plane into a set of vertical slices and taking moments of area of these slices about any convenient vertical axis.

The shearing strength of the soil is c, since $\phi = 0°$.

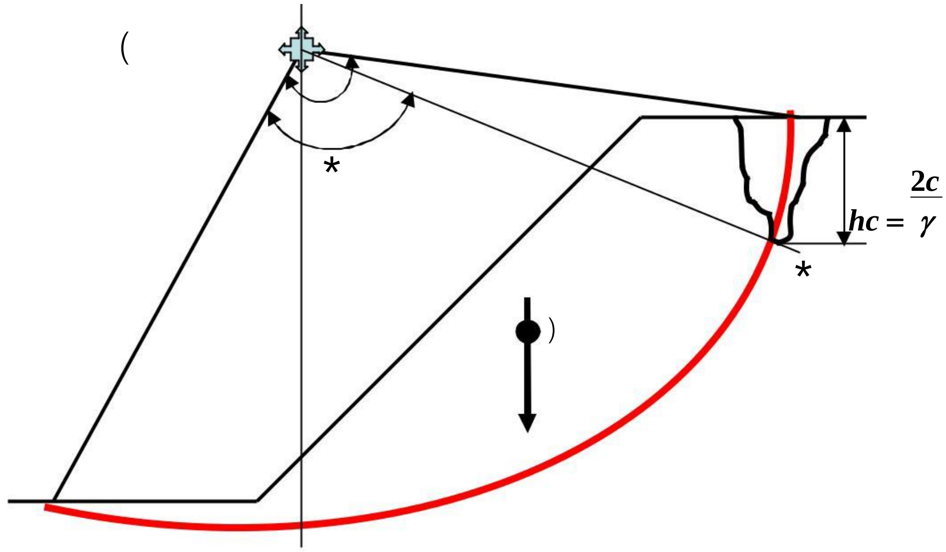
The restoring moment (along the slip surface) = $c\,l\,r$

$$= c\,r\,\theta\,r = c\,r^2\,\theta$$

The driving moment =W.e

$$\text{Factor of safety, FS} = \frac{\text{Restoring moment}}{\text{Driving moment}} = \frac{c.r^2.\theta}{W.e}$$

Effect of Tension cracks on Stability



Infinite slope with tension crack on top

In case of cohesive soil when the slope is on the verge slippage there develops a tension crack at the top of the slope as shown in Fig 11. the depth of tension crack is

$$hc = \frac{2c}{\gamma}$$

Where, c = cohesion

γ = unit weight

There is no shear resistance along the crack. The failure arc reduces from Arc AB to Arc AB' and the angle reduces to ´.

For computation of FS we have to

1. Use ´ instead of in the restoring moment component.
2. Consider the full weight `W' of the soil within the sliding surface AB to compensate for filling of water in the crack in the driving moment component

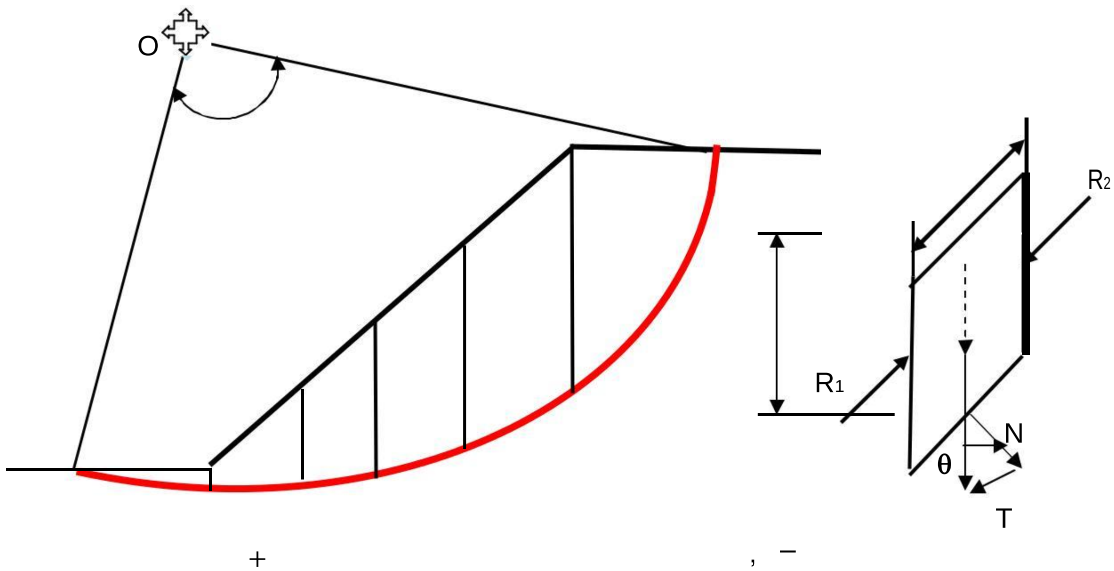
$$FS = \frac{c.r.\theta'}{W.e}$$

“Tension crack reduces F.S due to decrease in restoring moment”

The effect of tension cracks are

1. It modifies the slip surface and reduces the length of the slip surface.
2. It is usually filled with water and produces hydrostatic pressure along the depth.
3. It acts as channel for water to flow into underlying soil layers, inducing seepage forces.
4. It reduces the factor of safety.

The Swedish method of slices for a cohesive –frictional (c-ϕ) soil



Method of slices in c-ϕ soils

For a c-ϕ soils the undrained strength envelope shows both c and ϕ values. The total stress analysis can be adopted.

The procedure is follows

1. Draw the slope to scale
2. A trail slip circle such as AB with radius `r' is drawn from the center of rotation O.
3. Divide the soil mass above the slip surface into convenient number of slices (more than 5 is preferred)
4. Determine the area of each slice A1, A2, -------, An

$$A = \text{width of the slice X mid height}$$

$$= b \text{ X } Z$$

5. Determine the total weight W including external load if any as

$$W = \gamma\ b\ Z = \gamma\ A$$

Where, γ = unit weight

b = width of slice Z

= height of slice.

The forces on a typical slice are given in Fig 12.

The reactions R_1 and R_2 on the sides of the slice are assumed equal and therefore do not have any effect on stability.

6. The weight W of the slice is set ±off at the base of the slice. The directions of its normal component `N' and the tangential component `T' are drawn to complete the vector triangle.

   $N = W \cos\delta$, $T = W \sin\delta$

7. The values of N and T are scaled off for each of the slices
8. The values of `N' and `T' are tabulated and summed up as shown in the following table.
9. The factor of safety is calculated as follows

$$Sliding\ moment = r \quad T\ (reckoned \quad positive\ if\ clockwise\ )$$

$$\mathrm{Re}\ storing\ moment = r\ (c\ r\ \theta + \quad N \tan\phi)\ (reckoned\ positive\ if\ counterclockwise)$$

$$Factor\ of\ safety,\ FS = \frac{(c\ r\ \theta + \ N \tan\phi)}{T}$$

Note: The tangential components of a few slices at the base may cause restoring moments.

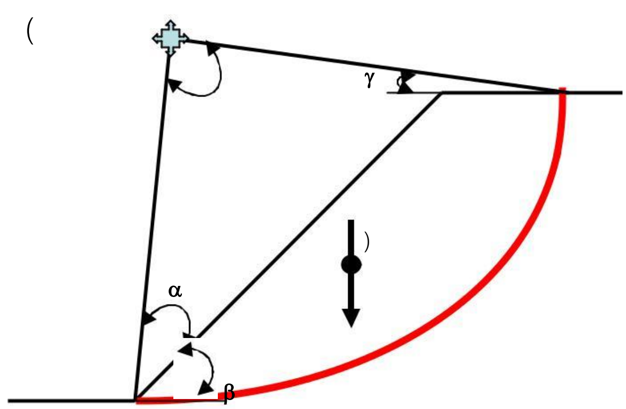
Normal and tangential components of various slices in the slope

Slice No.	Area m^2	Weight W (kN)	Normal component N (kN) $N = W \cos\delta$	Tangential components T (kN) $T = W \sin\delta$
1				
2				
3				
			Sum, Σ N = ___ kN	Sum, Σ T = ___ kN

10. Repeat step 2 to 9 by considering various trial slip circles and calculate FS for each of these slip circles. The slip circle with a minimum FS is called critical slip circle.

Critical Slip Circle by Fellenius Direction angles

In case of slopes in homogeneous cohesive soil deposits, the centre of a critical circle can be directly located by using Fellenius direction angles. Fellenius (1936) has given direction angles α and β for various slopes as shown below



Fellenius direction angles in finite slope

Table 2: Fellenius direction angles for locating critical slip circle

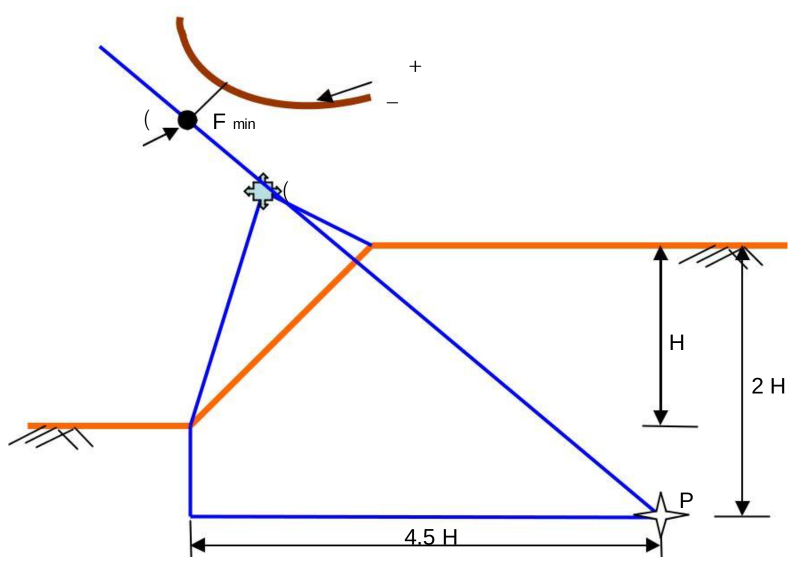
Slope	Angle α	Angle γ
1:1	28ë	37ë
1:1.5	26ë	35ë
1:2	25ë	35ë
1:3	25ë	35ë
1:5	25ë	37ë

For any given slope the corresponding direction angles α and γ are set out from the base and the top as shown in Fig 13. The point of intersection of these two lines is the centre of critical circle.

After locating the centre of critical circle the method of slices can be adopted to obtain minimum F.S.

Critical Slip circle in C- ϕ Soils

In case of c-ϕ soils the procedure for locating critical slip surface is slightly different and is as given below



Location of critical circle in c-ϕ soil

1. Locate point O_1 the centre of Fellenius circle
2. Locate point P at 2H below the top surface of the slope and 4.5H from the toe of the slope as shown in Fig. 14.
3. Extend backwards the line PO_1 beyond O_1
4. Construct trial slip circles with centres located on the extended portion of the line PO_1
5. For each of these trial slip circles find the F.S by the method of slices.
6. Plot the F.S for each of these trial slip circles from their respective centres and obtain a curve of factor of safety.
7. Critical slip circle is the one that has a minimum F.S.

Effective Stress Analysis

When the pore water pressures exist in the embankment due to seepage, sudden drawdown or due to any other reason, then stability should be computed based on effective stress analysis

$$\sigma = \sigma' + u \; ; \; \sigma' = \sigma - u$$

Stability during steady seepage

When seepage occurs at a steady rate through an earth dam or embankment it represents critical condition for the stability of slope.

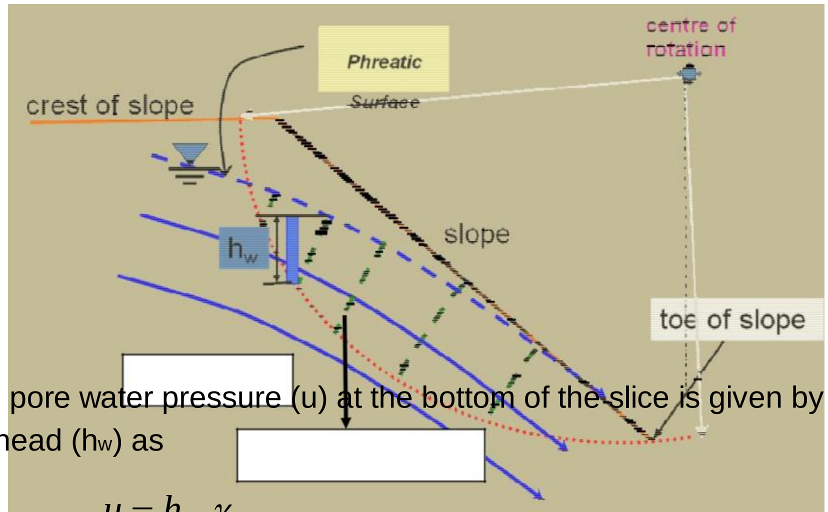
When seepage occurs pore water pressure (u) develops and this will reduce the effective stress which in turn decreases the shear strength along the failure surface.

The following procedure is adopted to obtain stability

1. Draw the C/S of the slope
2. Draw the potential failure surface
3. Divide the soil mass into slices
4. Calculate the weight W and the corresponding normal and tangential components for all the slices in the usual way

In addition

For the given slope construct flow net (network of equipotential and flow lines) as shown



The average pore water pressure (u) at the bottom of the slice is given by the piezometric head (h_w) as

$$u = h_W \gamma_W$$

h_W = piezometric head above the base of the slice

The total force due to pore water pressure at the bottom of the slice

$$U = u \gamma_W$$

Tabulate all the values as shown below

Slice No.	Width	Area (m^2)	Weight 'W' (kN)	Normal component 'N' (kN)	Tangential component 'T' (kN)	Pore water pressure (u)	Total force due to pore pressure (U)
				Σ N = ___	Σ T = ___		Σ U = ___

The F.S is computed as

$$Factor\ of\ Safety,\ FS = \frac{(c'\, r\theta + \tan\phi' \quad (N-U))}{T}$$

c´ and ϕ´ - Shear parameters based on effective stress analysis obtained from drained shear tests.

If the flownet is not constructed then F.S may be computed as

$$Factor\ \ of\ Safety\ ,\ FS = \frac{(c'\ r\ \theta + \tan\ \phi' \ \ (N'))}{T}$$

$$N' = W' \cos\delta = bZ\,\gamma' \cos\delta$$

N´= Weight of slice computed from effective unit weight

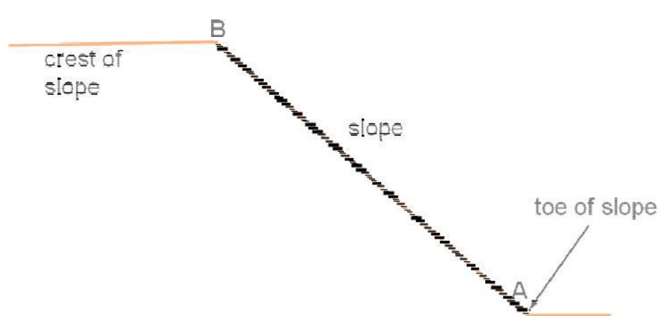
T = W Sinδ = b Z γ_{sat} Sinδ - Weight of slice computed from saturated unit weight.

Friction Circle Method

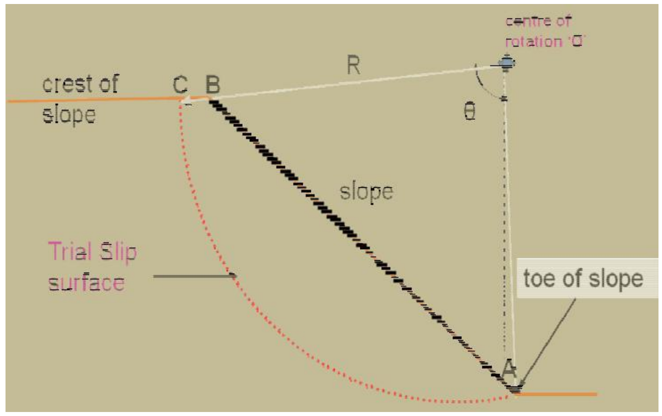
This method uses total stress based limit equilibrium approach. In this method the equilibrium of the resultant weight `w', the reaction `p' due to frictional resistance and the cohesive force `c' are considered. The magnitude direction and line of action of `w', the line of action of the reaction force `p' and the cohesive force `c' being known the magnitude of p and c are determined by considering the triangle of forces. The F.S. w. r. t. cohesion and friction is evaluated.

The procedure is as follows:

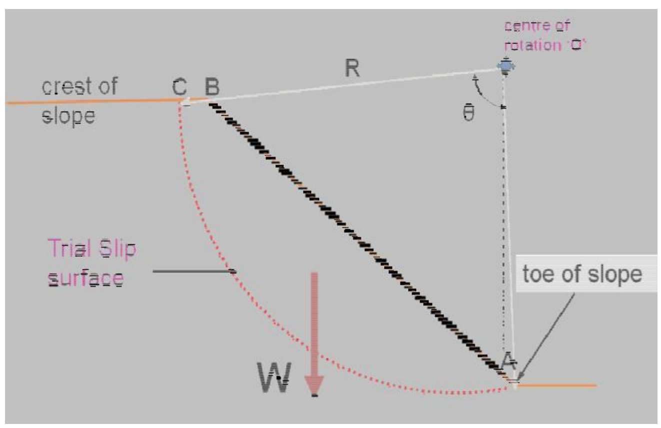
1. Consider a slope shown in Figure



2. Draw a trial circular slip surface (Arc AC) from the toe as shown with `O' as centre and `R' as radius



3. Find the centroid of the sliding mass ABCA and calculate it's weight `W'

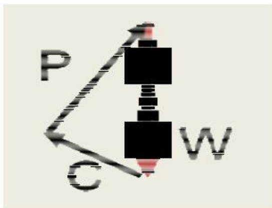


4. For analysis the following 3 forces are considered

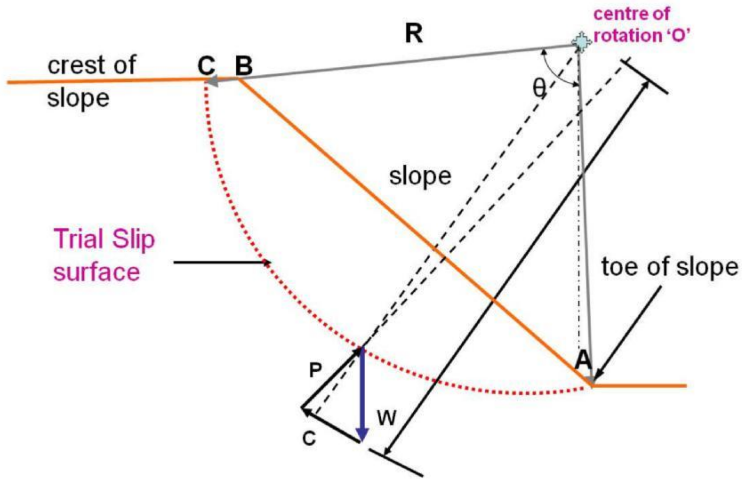
   The weight W of the sliding soil mass

   The total reaction P due to frictional resistance

   The total cohesive force C mobilized along the slip surface

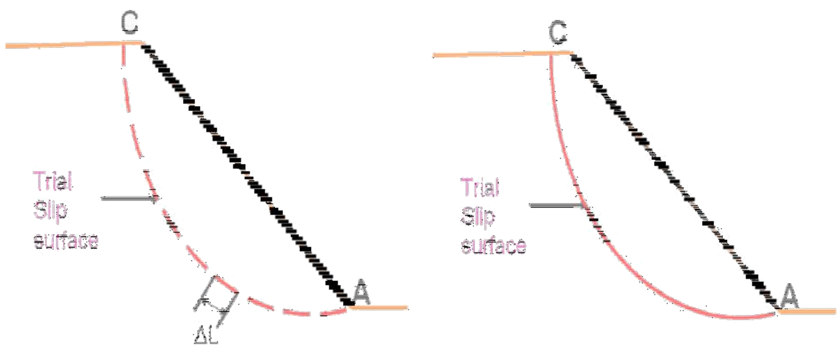


Force Triangle

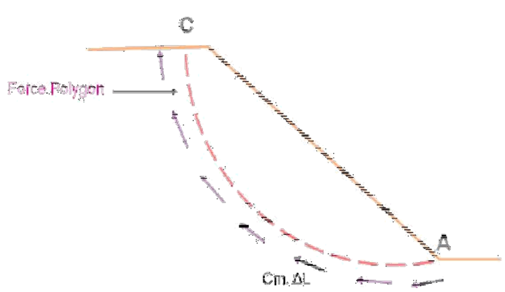


Resultant cohesive force C and its point of application

5. Let the slip circle be considered to be made up of a number of elementary arcs each of length L

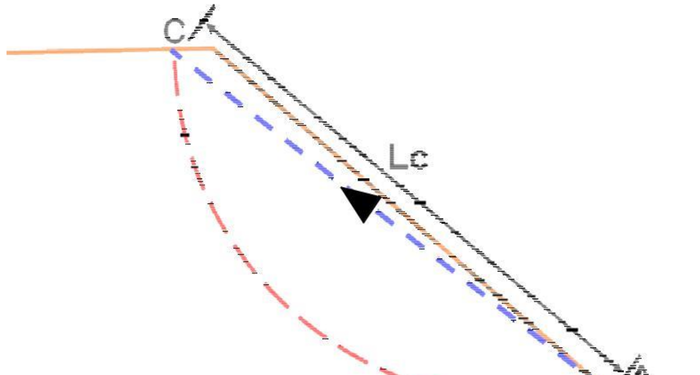


6. Let the cohesive force acting along this element opposing the sliding of soil be C_mX L. C_m is the mobilized cohesion.



The total cohesive force along the arc AC forms a force polygon

7. The closing side (AC) of the polygon represents the magnitude and direction of resultant cohesive force.



The length of the chord AC be Lc and the length of arc AC be L. The magnitude of resultant force C = Cm X Lc

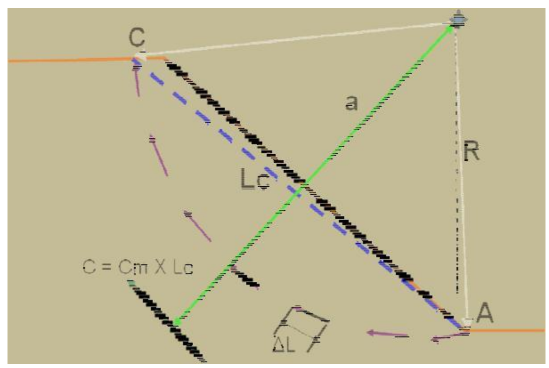
8. The position of resultant can be obtained by Varignon's theorem

$$C a = C_m \Delta L R C_m$$

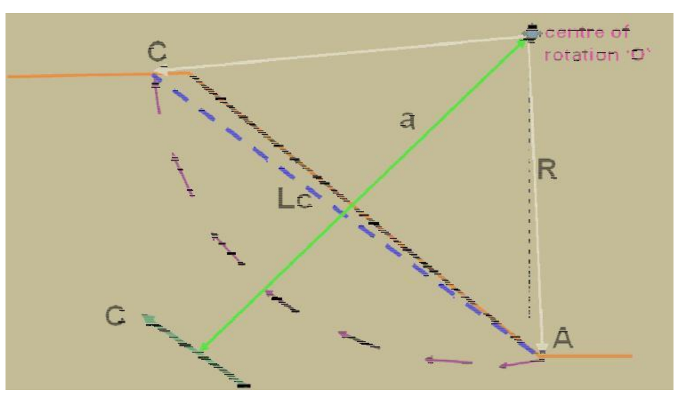
$$L_c a = C_m L R$$

$$a = \frac{L}{L_c} R$$

$$L > L_c \quad \therefore \quad a > R$$

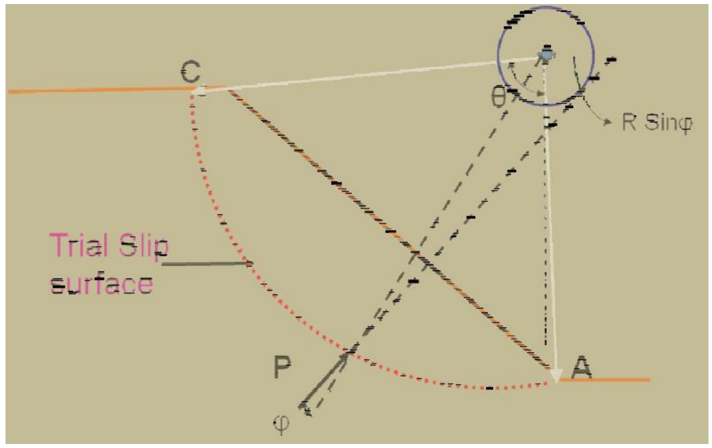


9. The cohesive forces CmX L along the slip circle can be replaced by their resultant C = Cm X Lc acting parallel to chord AC at a distance a>R from the centre of rotation as shown

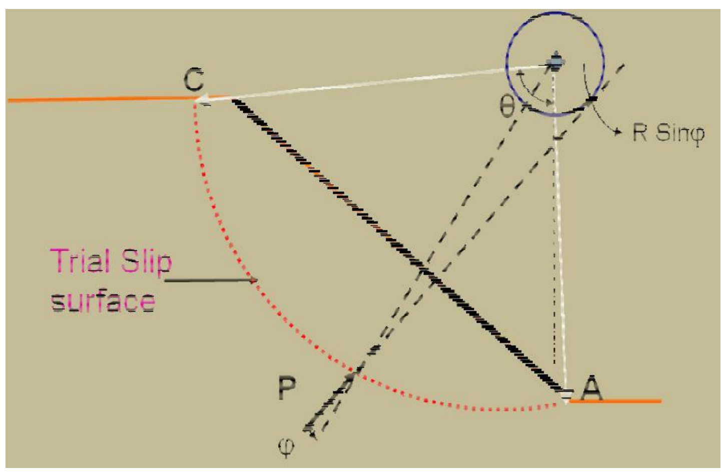


Reaction 'P' due to Frictional Resistance

10. On mobilization of frictional resistance. Let P be the soil reaction opposing the sliding of soil mass as shown. P is inclined at an angle to the normal at the point of action as shown

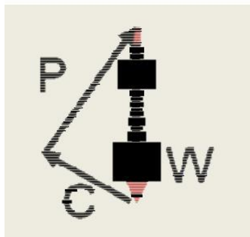


The line of action of P will pass as tangent to a circle of radius R Sin drawn with `O' as centre called "Friction Circle" or - Circle

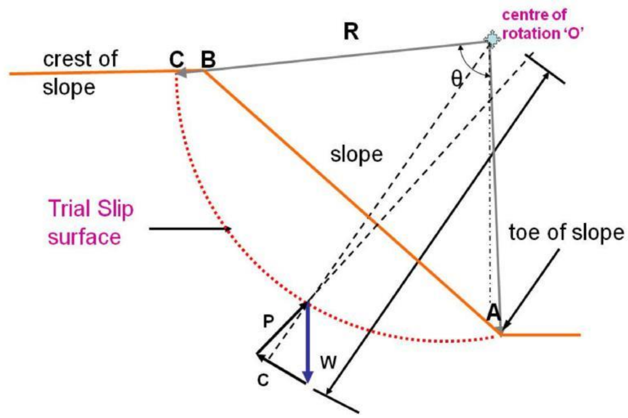


The three forces considered for analysis are:

- The weight `W' drawn as vertical passing through the centroid of sliding mass (ABCA)
- The resultant cohesive force `C' drawn parallel to the chord AC at a distance `a' from the centre `O'
- The resultant reaction `P' passing through the point of intersection of `W' and `C' and tangential to friction circle



Force Triangle



By knowing the magnitude and direction of `W' and the direction and line of action of other forces the force triangle can be completed. Measuring the magnitude of C the F.S. is computed as shown below

$$\text{The mobilized cohesion} \quad C_m = \frac{C}{L_c}$$

F.S w.r.t cohesion is given by

$$F.S = \frac{C_u}{C_m}$$

The minimum F.S is obtained by locating the critical slip circle

Computation of F.S w.r.t Strength

1. Assume a trial F.S w.r.t friction as F
2. Draw a friction circle with a reduced radius R Sin $_m$

   Where $\tan\varphi_m = \dfrac{\tan\varphi_u}{F\varphi}$
3. Carry out the friction circle analysis and find the FS w.r.t cohesion

   $$F_c = \frac{C_u}{C_m}$$
4. If F_c = F -------------- it represents F.S w.r.t strength
5. Otherwise choose different F and repeat the procedure till F_c = F

Stability Number

In a slope the component of the self weight (γ) causes instability and the cohesion contributes to stability. The maximum height (H_c) of a slope is directly proportional to unit cohesion (C_u) and inversely proportional to unit weight (γ). In addition, H_c is also related to friction angle (ϕ_u) and slope angle β.

This can be expressed as $H_c = \frac{C_u}{\gamma} f(\phi_u, \beta)$

When the term $f(\phi_u, \beta)$ is dimensionless then equation above is dimensionally balanced

Taylor (1937) expressed $f(\phi_u, \beta)$ as a reciprocal of a dimensionless number called "Stability Number" (Sn) popularly called as Taylor's stability Number.

$$f(\phi_u, \beta) = \frac{1}{S_n}$$

$$\therefore H_c = \frac{C_u}{\gamma S_n}$$

$$S_n = \frac{C_u}{\gamma H_c}$$