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SCHOOL OF BIO AND CHEMICAL

DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – I-INSTRUMENTATION AND PROCESS CONTROL – SCHA1503

I Instrumentation

Temperature measurement: Thermocouples, Resistance thermometers, Optical and Radiation pyrometers. Pressure measurement: Use of manometers, Bourdon gauge, Bellows type gauge. Flow measurement: Variable area meters. Positive displacement type meters. Liquid level measurement: Direct and differential method, measurement in open and pressure vessels. Measurement of Viscosity, Conductivity, Humidity of gases and pH.

1.1 Instrumentation for temperature measurements

Temperature is one of the major physical parameter, which characterises the condition of substances involved in processes. In order to measure this parameter we need to choose an appropriate temperature scale and the unit of temperature.

Temperature scales, temperature units

Anders Celsius, the Swedish astronomer, devised a scale for measuring temperature, which later was named after his name. This scale has the symbol □*C*. The Celsius scale was based on two fixed and easily reproducible points:

- the ice point, ie the temperature of a mixture of ice and water in equilibrium with saturated air at a pressure 101325 Pa. This temperature was numbered 0 C;
- the steam point, ie the temperature of the water and steam in equilibrium at a pressure 101325 Pa. This temperature was numbered 100 C.

Later in 1954 this scale was redefined and was based on:a single fixed point - the triple point of water. This is the temperature at which solid, liquid and vapour phases of water exist together in equilibrium. The temperature of the triple point of water has the value of 0.01 °C; the ideal-gas temperature scale. On this scale the steam point was experimentally found to be equal to 100.00 °C.

The thermodynamic scale of temperature (or the absolute scale) was derived from the second law of thermodynamics. This scale is independent of any thermometric substance. The relation between the absolute scale and the Celsius scale is as follows:

$$T = \vartheta + 273.15,$$

where: T - temperature in the absolute scale, K;

ϑ - temperature in the Celsius scale, $^\circ C$.

The unit for the absolute scale is K - Kelvin, named after Lord Kelvin (William Thomson).

$1, K = \frac{1}{273.16}$ of the temperature at the triple point of water.

Table 1 Fixed points and corresponding temperatures for IPTS-68.

NN	Fixed points	Temperature, °C
1.	*Triple point of equilibrium-hydrogen (s+l+v)*	*-259.34*
2.	*Boiling point of equilibrium hydrogen (l+v) at 33.33 kPa*	*-256.108*
3.	*Normal boiling point of equilibrium hydrogen at 101325 Pa*	*-252.87*
4.	*Normal boiling point of neon*	*-246.048*
5.	*Triple point of oxygen*	*-218.789*
6.	*Normal boiling point of oxygen*	*-182.962*
7.	*Triple point of water*	*0.01*
8.	Normal boiling point of water	100
9.	*Normal freezing point of zinc (s+l) at 101325 Pa*	*419.58*
10.	*Normal freezing point of silver (s+l) at 101325 Pa*	*961.93*
11.	*Normal freezing point of gold (s+l) at 101325 Pa*	*1064.43*

In this table: *s* - solid; *l* - liquid; *v* - vapour.

The International Committee of Weights and Measures (CIPM) adopted a new International Temperature Scale on January 1, 1990. **Table 2** shows fixed point for ITS-90. ().

Table 2 Fixed points and corresponding temperatures for ITS-90.

No	Fixed points	Temperature, °C
1.	Normal boiling point of helium	-270.15 to --268.15
2.	*Triple point of equilibrium-hydrogen (s+l+v)*	*259.3467*
3.	*Boiling point of equilibrium hydrogen (l+v) at 33.33 kPa*	*≈-256.15*
4.	*Normal boiling point of equilibrium hydrogen at 101325 Pa*	*≈-252.85*
5.	Triple point of neon	-248.5939
6.	*Triple point of oxygen*	*-218.7916*
7.	Triple point of argon	-189.3442
8.	Triple point of mercury	-38.8344
9.	*Triple point of water*	*0.01*
10.	Melting point of gallium	29.7646
11.	Normal freezing point of indium (s+l) at 101325 Pa	156.5985
12.	Normal freezing point of tin (s+l) at 101325 Pa	231.928
13.	*Normal freezing point of zinc (s+l) at 101325 Pa*	*419.527*
14.	Normal freezing point of aluminium (s+l) at 101325 Pa	660.323
15.	*Normal freezing point of silver (s+l) at 101325 Pa*	*961.78*
16.	*Normal freezing point of gold (s+l) at 101325 Pa*	*1064.18*
17.	Normal freezing point of copper (s+l) at 101325 Pa	1084.62

Liquid-in-glass thermometers

These thermometers are used for temperature measurements from -200 to 750 □C. They are contact-type thermometers. Fig. 1 shows the principle of their design.

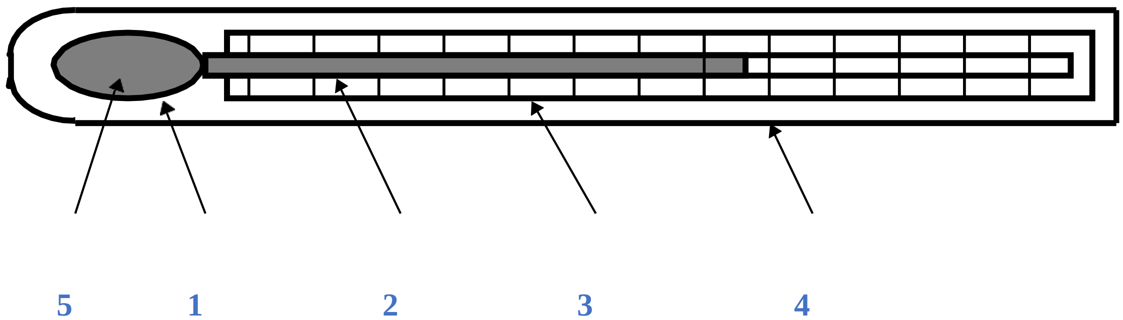


Fig.1. Liquid-in-glass thermometer

This thermometer consists of a glass bulb ***1***, which is connected with a glass capillary tube ***2***. A scale ***3*** in degrees of Celsius or Fahrenheit is placed behind the capillary tube. The bulb, the capillary tube and the scale are placed in a glass tube ***4*** to protect them against the damage. A thermometric liquid ***5*** fills the bulb and a part of the capillary tube. The operational principle of these thermometers is based on the difference between the volume expansion of liquids and glass with temperature.

The volumetric thermal expansion coefficient of glass is much less than that of liquids. The variation of temperature (up and down) of the bulb causes liquid in the system to expand or decrease its volume, respectively. As a result of such changes (the internal volume of the glass bulb and the glass capillary varies negligible), the length of the liquid column in the capillary tube goes up or down proportionally to the variation of temperature.

The type of thermometric liquid depends on the lower and upper limits of the measuring temperature range. Table 3 presents the most common types of liquids used in these types of thermometer.

Among these liquids *mercury* is the most widely used, because:

- mercury is easy obtainable with high chemical purity;
- mercury does not wet glass (this increases the accuracy of measurement/ reading);
- mercury remains in liquid state in a wide temperature range

Table .3. Types of thermometric liquids.

Liquid	**Temperature range, degC**	
	From	**To**
Mercury	-35	750
Toluene	-90	200
Ethanol	-80	70
Kerosene	-60	300
Petroleum Ether	-120	25
Pentane	-200	20

Among disadvantages inherent to mercury-in-glass thermometers we can mention the following:

- mercury is a poisonous element, which affects the central and peripheral nervous system, its vapour is the most toxic;
- small volumetric thermal expansion coefficient for mercury, therefore, mercury is used in thermometers with capillaries of small internal diameter;

The solidifying point of mercury, ie 38 □C, limits the lowest temperature that can be measured by mercury-in-glass thermometers. The upper temperature is determined by the temperature at which glass still retains its solid properties. This temperature is equal about 600 □C for glass, and about 750 C for silicon glass.

When air above mercury in the capillary is removed, a mercury-in-glass thermometer can be used at temperatures below 300 C, because the boiling temperature of mercury at atmospheric pressure is equal 356.9 C. In order to increase this temperature range it is necessary to increase the boiling temperature of mercury (saturation temperature). This can be achieved by increasing pressure in the capillary. Usually, the space above mercury in the capillary is filled by inert gas (such as nitrogen, argon) under pressure.

Liquid-in-glass thermometers with organic thermometric liquids are used for temperature measurements from -200 to 200 □C. One advantage of these thermometers is:

- a higher volume thermal expansion coefficient comparing with that for mercury (six times higher in average).

Disadvantage of thermometers with organic liquids is:

- these liquids wet glass, therefore, in order to increase the accuracy of measurement/reading, glass capillaries with bigger internal diameters (up to 1 mm) are used.

Advantages of liquid-in-glass thermometers are as follows:

- they are simple in design;
- they are relatively highly accurate in temperature measurement.

There are several disadvantages_inherent to liquid-in-glass thermometers

- they are fragile;
- it is difficult to perform readings due to low visibility of the scale;
- they are not capable of distance transmission of a measuring signal, thefore, they are used as locally placed devices;
- impossibility to repair;
- high values of time lag;
- low visibility of mercury in the capillary.

1.2 Thermocouples

Seebeck in 1821 discovered that thermal electromotive force (t.e.m.f.) is generated in a closed circuit of two wires made of dissimilar metals if two junction are at different temperatures. One junction is inserted into a measuring media, and it is called a hot or measuring junction. Another one, called a cold or reference junction, is kept either at 0 C or at ambient temperature and is connected to a measuring instrument (millivoltmeter). The electronic explanation of this phenomenon is as follows:the density of conduction electrons in two dissimilar metals is different. So, in the case when metals are brought into contact (welded together), the free (or conduction) electrons will flow from the metal with high their density to the metal with low density of the conduction electrons. As the result of this drift, a potential difference is produced in the boundary between these two metals. This potential difference will stop the flow of electrons. Since the metals are different, so they will differently respond to temperature variations. In other words, the variation of temperature will change the density and velocities of free electrons in two metals differently. This will cause the change in the magnitude of the thermal electromotive force.

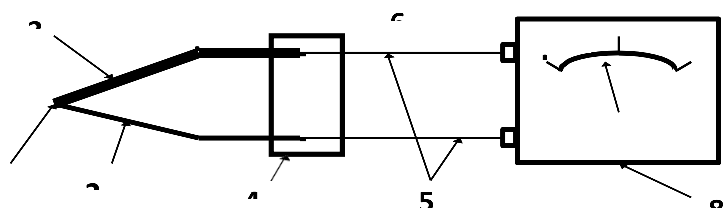


Figure 2 schematically shows a thermocouple and a measuring instrument.

The e.m.f. is proportional to the difference of temperatures between the two junctions. All tables, correlated t.e.m.f. of thermocouple (measured in mV) and temperature, are developed when the temperature of a cold junction is equal to 0 °C. T.e.m.f. is the function of temperature difference between the hot and the cold junctions:

1.3 Resistance temperature detectors

The principle of resistance temperature detectors (RTD) is based on the variation of electrical resistance of metals with temperature. For this purpose several metals are used, namely, platinum, copper, nickel. When temperature increases the resistance of these metals increases. Temperature function of resistance for metals in a narrow temperature interval can be expressed by a relationship:For metals this coefficient is positive. shows relationship between resistance of platinum and copper RTD and temperature.Platinum RTDs are used for temperature measurements from -220 to 850 ^oC (they are used as reference RTDs, as well), copper RTD - from -50 to 150 ^oC, and Nickel RTD - from -215 to 320^oC

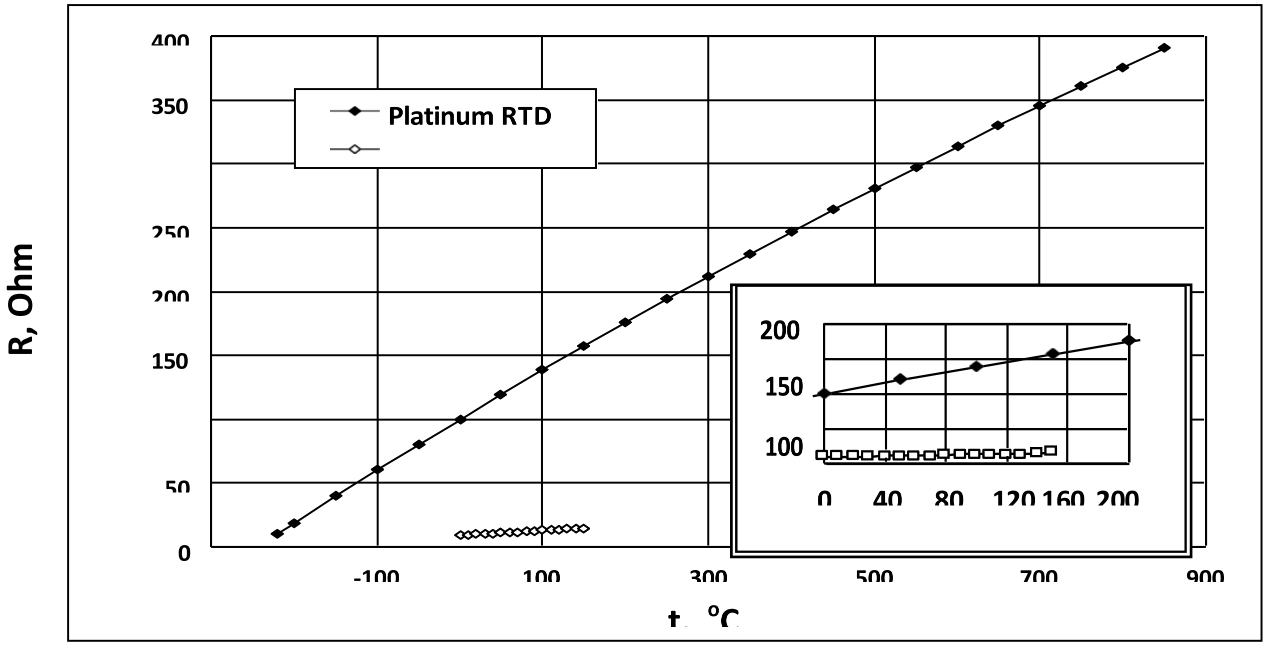


Figure 3**.**Resistance vs temperature for platinum and copper RTD.

Fig. 3 shows the assembly of RTDs. Sensitive elements of RTDs are made of a thin wire **1** with outside diameter equal to 0.025 mm (platinum RTD) and 0.1 mm (copper RTD) double wounded (non-inductive) on a micaceous or porcelain stem **2**. For mechanical strength the sensitive element is placed in the ceramic insulator tube **3** filled by extremely fine granular powder; extension wires are placed in the ceramic insulator **4**, and entire assembly is covered by a protective sheath of stainless steel **5**. The space between the sheath and ceramic insulator is filled by ceramic packing powder **6**. To avoid contact of sensitive element with environment, sensitive assembly is protected by high-temperature hermetic seal **7**. The contact between the wire of the sensitive element and the ceramic encapsulation permits a rapid speed of response.

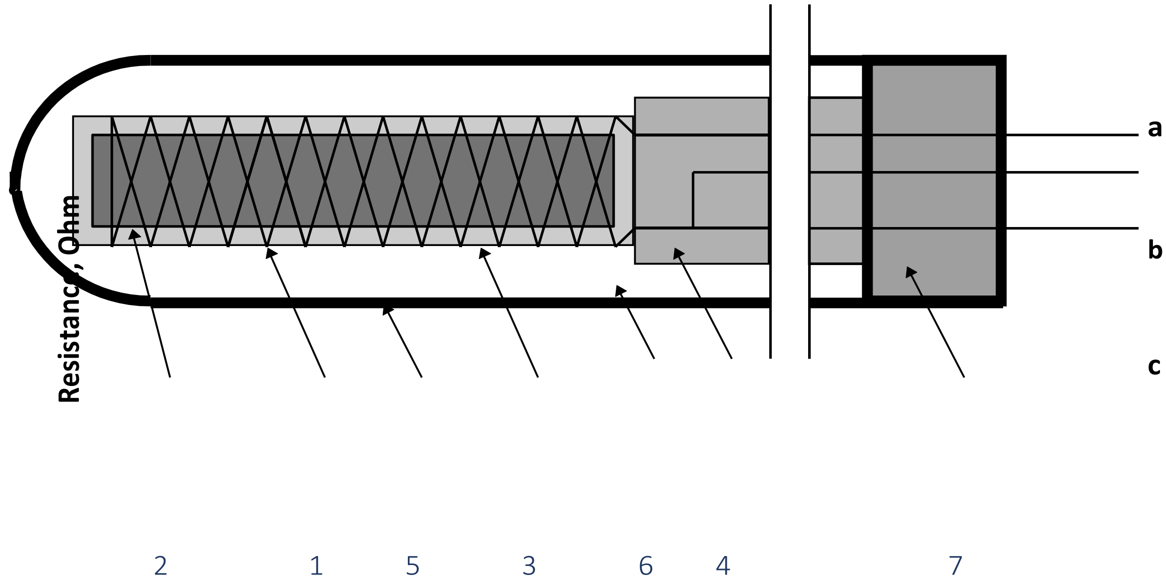


Figure 4. RTD assembly.

1.4 Thermistors

If semiconductors or heat-treated metallic oxides (oxides of cobalt, copper, iron, tin, titanium, etc.) are used as the materials for producing temperature sensitive elements, then these temperature transducers are called thermistors (the name is derived from the term of „thermally sensitive resistor"). These oxides are compressed into the desired shape from the specially formulated powder. After that, the oxides are heat-treated to recrystallise them. As the result of this treatment the ceramic body becomes dense. The leadwires are then attached to this sensor for maintaining electrical contact.

The following relationship applies to most thermistors:

Thermistors have negative thermal coefficient of electrical resistance. It means that when temperature increases the electrical resistance of thermistor decreases. They have greater resistance change (this is an advantage) compared with RTD in a given temperature range. For example, if we compare what change in resistance will be caused by variation of temperature in 1 □C for Platinum and Copper RTD and for thermistor in the temperature range from 273.15 to 423.15 K (ie, from 0 to 150 C),

Wheatstone bridge and resistance measuring constant current circuits, similar to that used in the case of RTDs, are used for resistance measurement of thermistors Despite their high sensitivity, thermistors have a worse accuracy and repeatability (this is the disadvantage) comparing with metallic RTDs. Since the resistance vs temperature function for thermistors is non-linear (although, some modern thermistors have a nearly linear relationship of *temperature vs resistance*), it is necessary to use prelinearisation circuits before interacting with related system instrumentation. In addition, due to the negative thermal coefficient of electrical resistance an inversion of the signal to positive form is required when interfacing with some analog or digital instrumentation. Therefore, thermistors are not widely used in process instrumentation field, at least at present. However, they have been well accepted in the food transportation industry, because they are small, portable and convenient. Another field of their growing application are heating and air-conditioning systems, where thermistors are used for checking the temperature in flow and return pipes.

1.5 Optical and radiation pyrometers

Pyrometer is a device which uses the relationship between the electromagnetic radiation emitted by a body and the temperature of this body. In order to better understand the phenomenon which forms the basis of pyrometry, it is useful to explain the concept of the blackbody, and the differences between it and real objects.

The term blackbody is ideal, and designates a body which radiates more electromagnetic energy for all wavelengths intervals than any other body of the same area and at the same temperature, and absorbs all the radiation it intercepts. Presents one of the classical blackbody model.

There are two types of pyrometers: optical (monochromatic or narrowband) and radiation (total radiation or broadband) pyrometers. The last devices originally were called radiation pyrometers, then radiation thermometers, and more recently infrared thermometers. However, the first their name (radiation pyrometers) is still widely used at present. These devices have high accuracy of □0.01 □C as a standard instruments, and from 0.5 to 1% for industrial purposes.

a). Optical pyrometers, sometimes referred to as brightness thermometers, generally involve wavelengths only in the visible part of the spectrum. When the temperature of the body increases, so does the intensity at any particular wavelength. If two bodies have the same temperature, then intensities of those two objects are equal. In this type of a pyrometer the intensity of a certain
b). wavelength of a heated body is compared with that of a heated platinum filament of a lamp

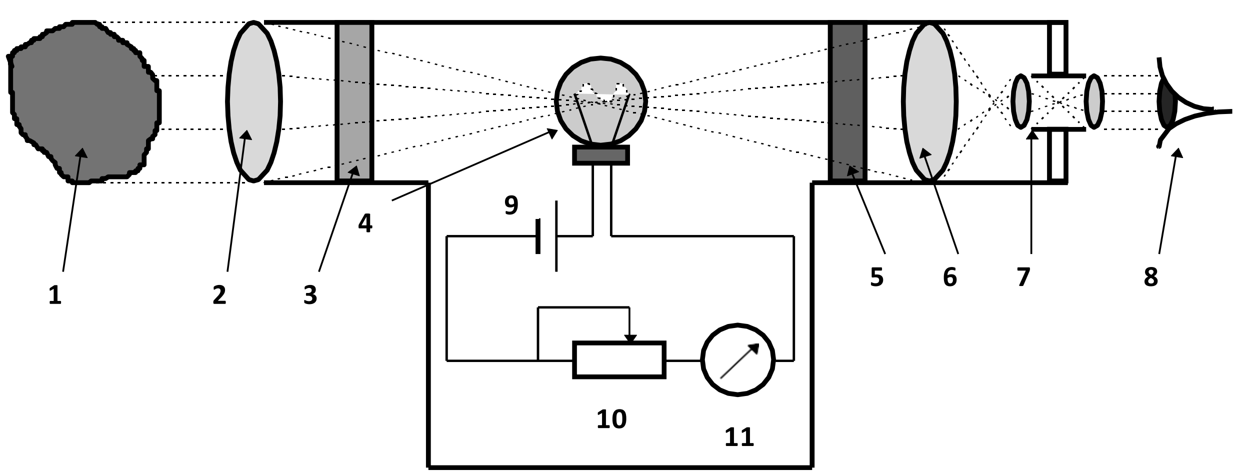


Figure 4 An optical pyrometer.

An object **1** which temperature is to be measured, emits electromagnetic radiation with intensity proportional to its absolute temperature. This radiation passes through lens **2** and red optical filter **3**. Optical filter picks out only the desired wavelength - red. Then radiation focuses on the platinum filament of a lamp **4**, and passes through another filter **5**, lens **6**, viewing system **7**. The viewer **8** sees the platinum filament superimposed on an image of the object **1**. When the temperature of the filament is low comparing with that of the object, the viewer sees the filament as a dark line on the bright background image of the object. The lamp **4** is connected in series with an electrical battery **9**, a variable resistor **10** and an ampermeter **11**. By reducing the resistance of the resistor an electrical current passing through the filament increases. So does the temperature of the filament and its brightness. For a certain value of an electrical current (corresponded to a certain value of an object temperature), the brightness of the platinum filament will match the brightness of the object **1**. At this setting the viewer cannot distinguish between the image of the object and the filament. At this time the measurement of temperature is performed. The scale of the ampermeter is calibrated in the units of temperature.

The lower temperature limit for optical pyrometers is determined by the temperature at which objects become visible in red (about 225 C). However, there are devices which are able to measure even lower temperatures down to -50 C. The upper limit varies from 600 to 3000 C, and is limited by the melting point of the platinum filament. An accuracy is typically varied from 0K to 10 K.

Radiation pyrometers, being very simple and cheap, use an exponential. The lower limits for radiation pyrometers vary from 0 to 600 °C, the upper limits vary from 1000 to 1900 °C. The accuracy varies from ±0.5 to ±5 K, depending on cost. They are widely used for temperature measurements in metal production facilities, glass industries, semiconductor processes, etc.

1.6 Pressure measurement

The construction of a bourdon tube gauge, construction elements are made of brass. Many techniques have been developed for the measurement of pressure and vacuum. Instruments used to measure pressure are called pressure gauges or vacuum gauges.

A manometer could also be referring to a pressure measuring instrument, usually limited to measuring pressures near to atmospheric. The term *manometer* is often used to refer specifically to liquid column hydrostatic instruments.

- **Absolute pressure** is zero referenced against a perfect vacuum, so it is equal to gauge pressure plus atmospheric pressure.
- **Gauge pressure** is zero referenced against ambient air pressure, so it is equal to absolute pressure minus atmospheric pressure. Negative signs are usually omitted.
- **Differential pressure** is the difference in pressure between two points.

The zero reference in use is usually implied by context, and these words are only added when clarification is needed. Tire pressure and blood pressure are gauge pressures by convention, while atmospheric pressures, deep vacuum pressures, and altimeter pressures must be absolute

Differential pressures are commonly used in industrial process systems. Differential pressure gauges have two inlet ports, each connected to one of the volumes whose pressure is to be monitored. In effect, such a gauge performs the mathematical operation of subtraction through mechanical means, obviating the need for an operator or control system to watch two separate gauges and determine the difference in readings.

Pressure Units						
	pascal (Pa)	**bar** (bar)	**technical atmosphere** (at)	**atmosphere** (atm)	**torr** (Torr)	**pound-force per square inch** (psi)
1 Pa	≡ 1 N/m^2	10^{-5}	1.0197×10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	100,000	≡ 10^6 dyn/cm^2	1.0197	0.98692	750.06	14.5037744
1 at	98,066.5	0.980665	≡ 1 kgf/cm^2	0.96784	735.56	14.223
1 atm	101,325	1.01325	1.0332	≡ 1 atm	760	14.696
1 torr	133.322	1.3332×10^{-3}	1.3595×10^{-3}	1.3158×10^{-3}	≡ 1 Torr; ≈ 1 mmHg	19.337×10^{-3}
1 psi	6,894.76	68.948×10^{-3}	70.307×10^{-3}	68.046×10^{-3}	51.715	≡ 1 lbf/in^2

Example reading: 1 Pa = 1 N/m^2 = 10^{-5} bar = 10.197×10^{-6} at = 9.8692×10^{-6} atm, etc.

1.7 Hydrostatic

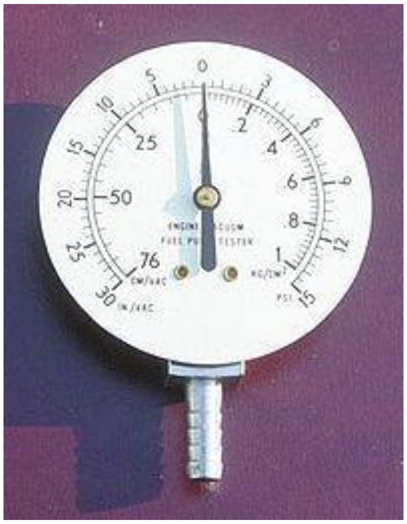
Hydrostatic gauges (such as the mercury column manometer) compare pressure to the hydrostatic force per unit area at the base of a column of fluid. Hydrostatic gauge measurements are independent of the type of gas being measured, and can be designed to have a very linear calibration. They have poor dynamic response.

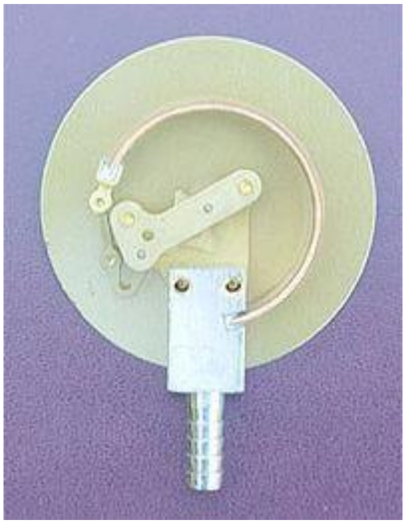
A single-limb liquid-column manometer has a larger reservoir instead of one side of the U-tube and has a scale beside the narrower column. The column may be inclined to further amplify the liquid movement. Based on the use and structure following type of manometers are used

1. Simple Manometer
2. Micromanometer
3. Differential manometer
4. Inverted differential manometer

1.8 Bourdon

A Bourdon gauge uses a coiled tube, which, as it expands due to pressure increase causes a rotation of an arm connected to the tube.





Indicator Side with card and dial Mechanical Side with Bourdon tube

The pressure sensing element is a closed coiled tube connected to the chamber or pipe in which pressure is to be sensed. As the gauge pressure increases the tube will tend to uncoil, while a reduced gauge pressure will cause the tube to coil more tightly. This motion is transferred through a linkage to a gear train connected to an indicating needle. The needle is presented in front of a card face inscribed with the pressure indications associated with particular needle deflections. In a barometer, the Bourdon tube is sealed at both ends and the absolute pressure of the ambient atmosphere is sensed. Differential Bourdon gauges use two Bourdon tubes and a mechanical linkage that compares the readings.

In the following pictures the transparent cover face has been removed and the mechanism removed from the case. This particular gauge is a combination vacuum and pressure gauge used for automotive diagnosis:

- the left side of the face, used for measuring manifold vacuum, is calibrated in centimetres of mercury on its inner scale and inches of mercury on its outer scale.
- the right portion of the face is used to measure fuel pump pressure and is calibrated in fractions of 1 kgf/cm² on its inner scale and pounds per square inch on its outer scale.

1.9 Diaphragm

A pile of pressure capsules with corrugated diaphragms in an aneroid barograph.

A second type of aneroid gauge uses the deflection of a flexible membrane that separates regions of different pressure. The amount of deflection is repeatable for known pressures so the pressure can be determined by using calibration. The deformation of a thin diaphragm is dependent on the difference in pressure between its two faces. The reference face can be open to atmosphere to measure gauge pressure, open to a second port to measure differential pressure, or can be sealed against a vacuum or other fixed reference pressure to measure absolute pressure. The deformation can be measured using mechanical, optical or capacitive techniques. Ceramic and metallic diaphragms are used.

For absolute measurements, welded pressure capsules with diaphragms on either side are often used.

Shape:

- Flat
- corrugated
- flattened tube
- capsule

1.10 Bellows

In gauges intended to sense small pressures or pressure differences, or require that an absolute pressure be measured, the gear train and needle may be driven by an enclosed and sealed bellows chamber, called an **aneroid**, which means "without liquid". (Early barometers used a column of liquid such as water or the liquid metal mercury suspended by a vacuum.) This bellows configuration is used in aneroid barometers (barometers with an indicating needle and dial card), altimeters, altitude recording barographs, and the altitude telemetry instruments used in weather balloon radiosondes. These devices use the sealed chamber as a reference pressure and are driven by the external pressure. Other sensitive aircraft instruments such as air speed indicators and rate of climb indicators (variometers) have connections both to the internal part of the aneroid chamber and to an external enclosing chamber.

Secondary transducer

- resistive (strain gauge)
- inductive
- capacitive - The deflection of the piston is often one half of a capacitor, so that when the piston moves, the capacitance of the device changes. This is a common way (with proper calibrations) to get a very precise, electronic reading from a manometer, and this configuration is called a **capacitive manometer vacuum gauge**.

"This is also called a capacitance manometer, in which the diaphragm makes up a part of a capacitor. A change in pressure leads to the flexure of the diaphragm, which results in a change in capacitance. These gauges are effective from 10^{-3} Torr to 10^{-4} Torr." [Nonsense! MKS sells capacitance gages over the range 0.01 – 155,000 Torr!]

1.11 Ionization gauge

Ionization gauges are the most sensitive gauges for very low pressures (high vacuums, AKA "hard" vacuums). They sense pressure indirectly by measuring the electrical ions produced when the gas is bombarded with electrons. Fewer ions will be produced by lower density gases. The calibration of an ion gauge is unstable and dependent on the nature of the gases being measured, which is not always known. They can be calibrated against a McLeod gauge which is much more stable and independent of chemistry.

Most ion gauges come in two types: hot cathode and cold cathode, a third type exists which is more sensitive and expensive known as a spinning rotor gauge, but is not discussed here. In the hot cathode version an electrically heated filament produces an electron beam. The electrons travel through the gauge and ionize gas molecules around them. The resulting ions are collected at a negative electrode. The current depends on the number of ions, which depends on the pressure in the gauge. Hot cathode gauges are accurate from 10^{-3} Torr to 10^{-10} Torr. The principle behind cold cathode version is the same, except that electrons are produced in a discharge created by a high voltage electrical discharge. Cold Cathode gauges are accurate from 10^{-2} Torr to 10^{-9} Torr. Ionization gauge calibration is very sensitive to construction geometry, chemical composition of gases being measured, corrosion and surface deposits. Their calibration can be invalidated by activation at atmospheric pressure or low vacuum. The composition of gases at high vacuums will usually be unpredictable, so a mass spectrometer must be used in conjunction with the ionization gauge for accurate measurement.

MEASUREMENT OF LIQUID LEVEL

In industry, usually vast quantities of liquids such as water, solvents, chemicals, etc. are used in a number of industrial processes. Liquid level measurements are *made to ascertain the quantity of liquid held in a container or vessel*. The liquid level affects both pressure and rate of flow in and out of the container and therefore its measurement and / or control becomes quite important in a variety of processes encountered in modern manufacturing plants. Liquid level measurements can be broadly classified as:

1. direct methods and

2. indirect methods

Direct Liquid Level Measurements

In these methods, the actual liquid level is directly measured by means of a simple mechanical type of device.

Dip-stick Method

This is a commonly used method for determining the liquid level is dipping a graduated rod in a liquid. Boatmen usually dip the oars in the canal / river to know the depth of water at a particular place. Similarly, a dip-stick is used to measure the level of oil in a car engine or the height of fuel oil in a uniformly shaped storage tank. This method, though quite economical, is not very accurate specially for moving fluids. Further, it is not possible to get continuous on-line observations in industrial processes.

Sight Glass Method

The sight glass or piezo-meter tube is graduated glass tube mounted on the side of the liquid containing vessel for providing a visual indication of the liquid level (Fig. 26.1). Since the liquids keep level, therefore the rise or fall of the liquid level in a tank / vessel results in a corresponding change in the level indicated by the sight tube.

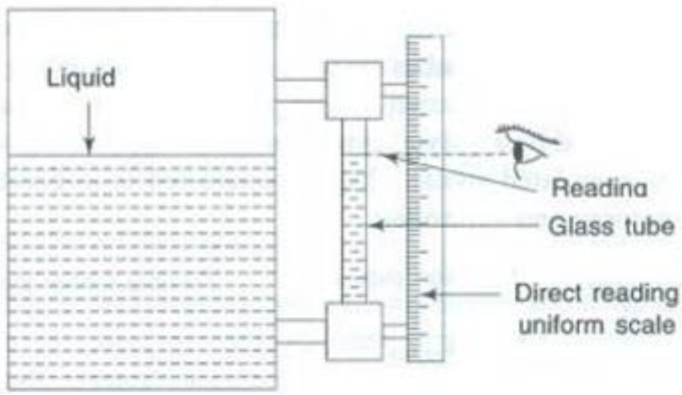


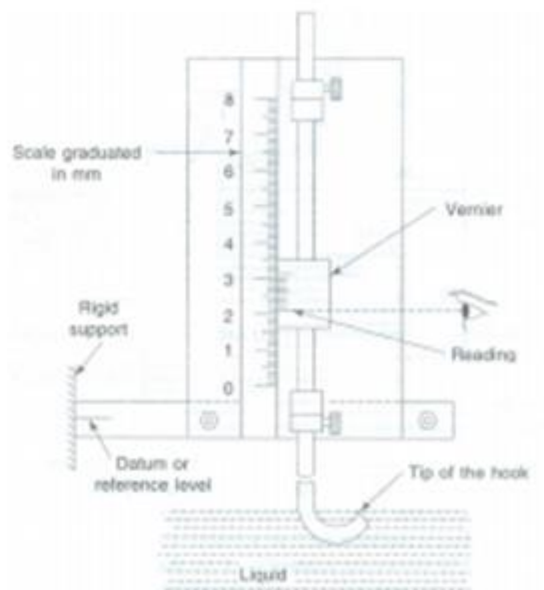
Fig. Sight glass level gate

Sight tubes are usually made of toughened glass and are provided with metallic protecting covers around them. Further, the diameter of such tubes is neither too large to change the tank / vessel level, nor too small to cause capillary action in the tube.

The measurement of liquid level with this device is simple and direct for clean and coloured liquids. However, it is rather unsuitable for dirty, viscous and corrosive liquids. Further, an operator is required to record the liquid levels with this device.

Hook Gauge

Sometimes it becomes necessary to accurately measure very small changes in liquid level in open tanks / containers. In a large tank / reservoir, a small change in level would mean large volumetric changes. For such applications, a simple hook gage is quite suitable. The schematic arrangement of this gauge is shown in Fig. In this device, a vertical tubular rod is provided with a vernier scale to be clamped at a suitable height at the upper end and a V-shaped hook at the lower end. This rod moves in a guide bracket fixed to a rigid body at the datum or reference level and has a main graduated scale in it. The movable rod is brought downwards so that the hook is first pushed below the surface of the liquid. It is then gradually raised until the top of the hook breaks through the surface of the liquid. The movable rod is then clamped and the level is read off the scale. The device is accurate up to ± 0.1 mm, the least count of the instrument. Further, the device is manually operated and does not lend itself to automatic reading.



A floating body, because of its buoyancy, would always follow the varying liquid level. Therefore, float-operated devices are capable of giving continuous, direct liquid level measurements. The floats generally used are hollow metal spheres, cylindrical ceramic floats or / disc shaped floats of synthetic materials. The top of the float is usually made sloping so that any solid suspensions in the liquid do not settle on the float and change its weight. Float gauges are sufficiently accurate when properly calibrated after installation. Further, a proper correction is required if there is a change in the liquid density due to a change in temperature.

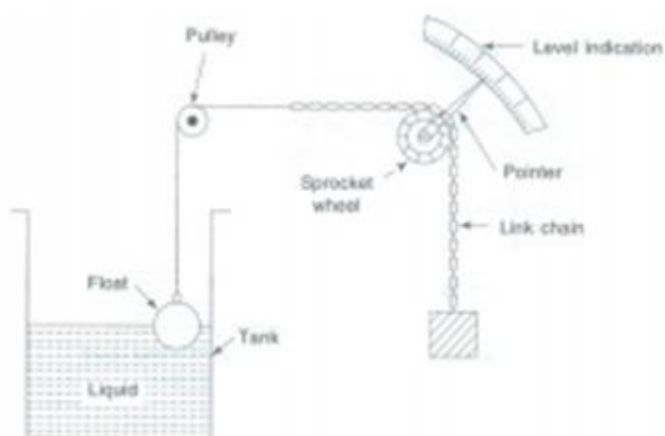


Fig. . Float and chain liquid level gauge

Figure illustrates a typical float-and-chain liquid level gauge generally used for directly measuring the liquid level in open tanks. The instrument consists of a float, a counter weight and a flexible connection that may be a chain or a thin metallic perforated tape. The counter weight keeps the chain / tape taut as the liquid rises or falls with any changes in the liquid level. The chain / perforated tape link is wound on a gear or sprocket wheel to which the pointer is attached. Any movement of this wheel would indicate on a suitably calibrated scale the level of the liquid in the tank.

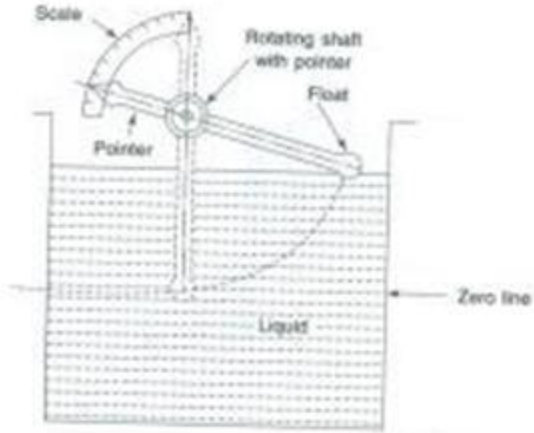
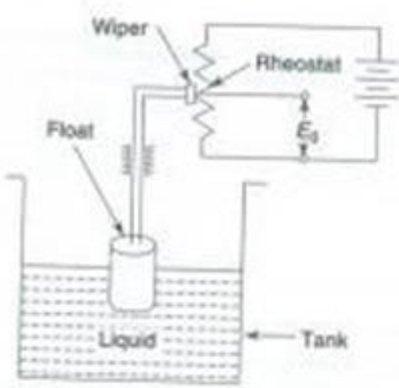
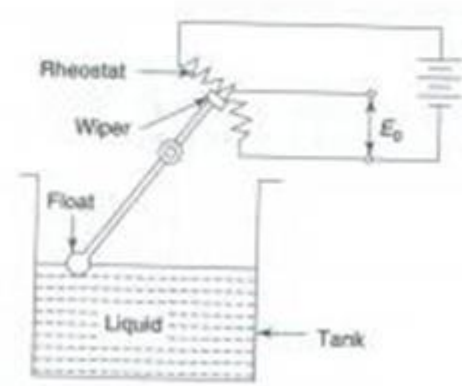


Fig. Float-and-shaft liquid level gauge

Further, there are a number of float-operated schemes with electrical read-outs. In these, the float acts as a primary transducer that converts liquid level variation into a suitable displacement. This displacement is sensed by the secondary transducer such as a resistive type of potentiometric device, inductive type of LVDT, etc. Figure , shows the schematic of the float-actuated rheostatic (resistive) device. The float displacement actuates the arm which causes the slider to move over the resistive element of a rheostat. The circuit resistance changes and this resistance change is directly proportional to the liquid level in the tank.



(a) Linear potentiometer liquid level gauge



(b) Rotary potentiometer liquid

The hydrostatic pressure created by a liquid is directly related to the height of the liquid column ($p = \rho g h$). Therefore, a pressure gauge is installed at the bottom or on the side of the tank containing the liquid (Fig.). The rise and fall of the liquid level causes a corresponding increase or decrease in the pressure p which is directly proportional to the liquid level h. The dial or scale of the pressure gauge is calibrated in the units of level measurement. These gauges function smoothly when the liquids are clean and non-corrosive. For corrosive liquids with solid suspensions, diaphragm seals between the fluid and the pressure gauge are generally employed.

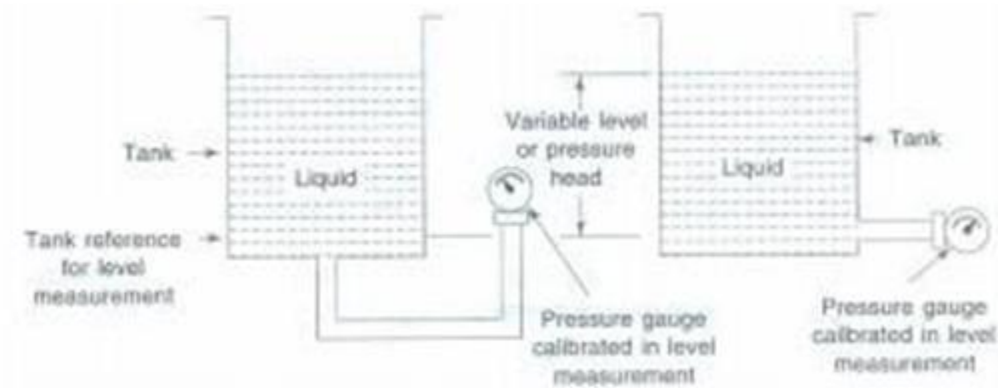


Fig.. . Typical arrangements of hydrostatic pressure type level measuring devices

Bubbler or Purge Technique for Level Measurement

In this method, the air pressure in a pneumatic pipeline is so regulated that the air pressure in the bubbler tube, shown in Fig. , is very slightly in excess over that of the hydrostatic pressure at the lowermost end of the bubbler tube. The bubbler tube is installed vertically in the tank with its lowermost open end at zero level. The other end of the tube is connected to a regulated air supply and a pressure gauge. The air supply in the bubbler tube is so adjusted that the pressure is just greater than the pressure exerted by the liquid column in the tank. This is achieved by adjusting the air pressure regulator until bubbles can be seen slowly leaving the open end of the tube. Sometimes a small air flow meter is fitted in the line to control / check the excessive flow of air. When the air flow is small and the density of the fluid is uniform, then gauge pressure is directly proportional to the height of the liquid level in the open tank. In practice, the gauge is directly calibrated in the units of liquid level and if the tank is uniformly shaped, then the calibration may be in the units of volume.

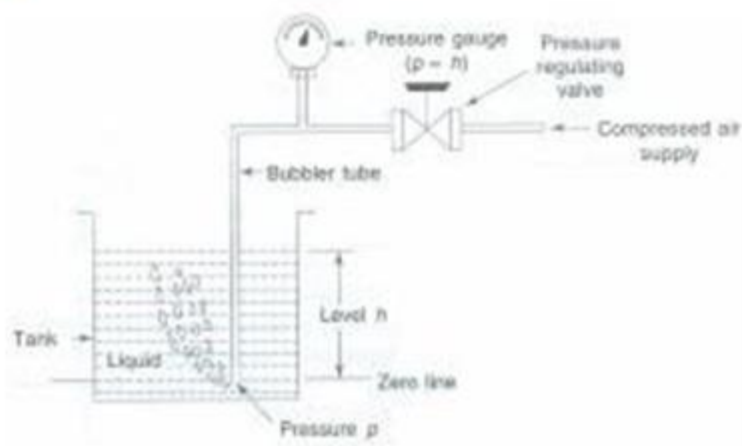


Fig. Bubbler or purge type of liquid level meter

Measurement of Viscosity

The device used for measurement of viscosity is known as *viscometer* and it uses the basic laws of laminar flow. The principles of measurement of some commonly used viscometers are discussed here;

Rotating Cylinder Viscometer: It consists of two co-axial cylinders suspended co-axially as shown in the Fig. 7.1.1. The narrow annular space between the cylinders is filled with a liquid for which the viscosity needs to be measured. The outer cylinder has the provision to rotate while the inner cylinder is a fixed one and has the provision to measure the torque and angular rotation. When the outer cylinder rotates, the torque is transmitted to the inner stationary member through the thin liquid film formed between the cylinders. Let r_1 and r_2 be the radii of inner and outer cylinders, h be the depth of immersion in the inner cylinder in the liquid and $t\left(=r_2-r_1\right)$ is the annular gap between the cylinders. Considering N as the speed of rotation of the cylinder in rpm, one can write the expression of shear stress (τ) from the definition of viscosity (μ), as given below;

$$\tau = \mu \frac{du}{dy} = \mu \left(\frac{2\pi r_2 N}{60t} \right)$$

This shear stress induces viscous drag in the liquid that can be calculated by measuring the toque through the mechanism provided in the inner cylinder.

$$T = \text{shear stress} \times \text{area} \times \text{radius} = \mu \left(\frac{2\pi r_2 N}{60t} \right) (2\pi r_1 h) r_1$$

$$\text{or, } \mu = \frac{15tT}{\pi^2 r_1^2 r_2 h N} = \frac{T}{CN}$$

Here, C is a constant quantity for a given viscometer.

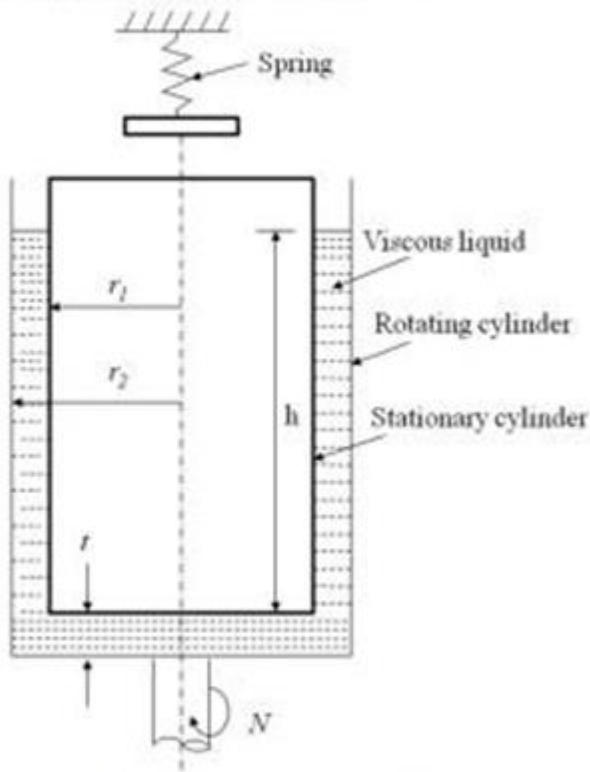


Fig. : Schematic nomenclature of a rotating cylinder viscometer.

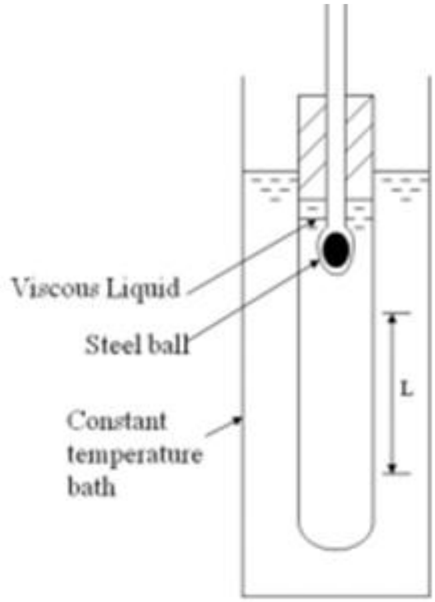


Fig. Schematic diagram of a falling sphere viscometer.

A perfectly smooth spherical ball is allowed to fall vertically through the liquid by virtue of its own weight (W). The ball will accelerate inside the liquid, until the net downward force is zero i.e. the submerged weight of the ball (F_B) is equal to the resisting force (F_R) given by *Stokes' law*. After this point, the ball will move at steady velocity which is known as ***terminal velocity***. The equation of motion may be written as below;

$$F_B + F_R = W \quad \Rightarrow \frac{\pi}{6} D^3 w_l + F_R = \frac{\pi}{6} D^3 w_s$$

where, w_l and w_s are the specific weights of the liquid and the ball, respectively. If the spherical ball has the diameter D that moves at constant fall velocity V in a fluid having viscosity μ, then using Stokes' law, one can write the expression for resisting force (F_R).

Substituting Eq and solving for μ,

$$\mu = \frac{D^2}{18V}(w_s - w_l) \text{ where } V = \frac{L}{t}$$

The constant fall velocity can be calculated by measuring the time (t) taken by the ball to fall through a distance (L). It should be noted here that the falling sphere viscometer is applicable for the Reynolds number below 0.1 so that wall will not have any effect on the fall velocity.

<u>Capillary Tube Viscometer</u>: This type of viscometer is based on laminar flow through a circular pipe. It has a circular tube attached horizontally to a vessel filled with a liquid whose viscosity has to be measured. Suitable head (h_f) is provided to the liquid so that it can flow freely through the capillary tube of certain length (L) into a collection tank as shown in Fig. . The flow rate (Q) of the liquid having specific weight w_l can be measured through the volume flow rate in the tank. The *Hagen-Poiseuille* equation for laminar flow can be applied to calculate the viscosity (μ) of the liquid.

$$\mu = \left(\frac{\pi}{128}\right)\frac{w_l h_f d^4}{QL}$$

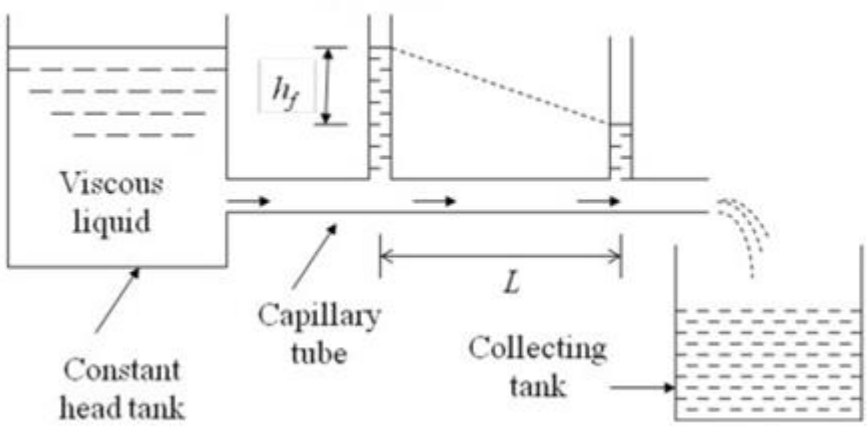


Fig. . Schematic diagram of a capillary tube viscometer.

Saybolt and Redwood Viscometer: The main disadvantage of the capillary tube viscometer is the errors that arise due to the variation in the head loss and other parameters. However, the *Hagen-Poiseuille* formula can be still applied by designing a efflux type viscometer that works on the principle of vertical gravity flow of a viscous liquid through a capillary tube. The *Saybolt viscometer* has a vertical cylindrical chamber filled with liquid whose viscosity is to be measured (Fig.). It is surrounded by a constant temperature bath and a capillary tube (length 12mm and diameter 1.75mm) is attached vertically at the bottom of the chamber. For measurement of viscosity, the stopper at the bottom of the tube is removed and time for 60ml of liquid to flow is noted which is named as *Saybolt seconds*. So, Eq. can be used for the flow rate (Q) is calculated by recording the time (*Saybolt seconds*) for collection of 60ml of liquid in the measuring flask. For calculation purpose of kinematic viscosity (ν), the simplified expression is obtained as below;

$$\nu = \frac{\mu}{\rho} = 0.002t - \frac{1.8}{t}; \text{ where, } \nu \text{ in Stokes and } t \text{ in seconds}$$

A *Redwood viscometer* is another efflux type viscometer (Fig. that works on the same principle of *Saybolt viscometer*. Here, the stopper is replaced with an orifice and *Redwood seconds* is defined for collection of 50ml of liquid to flow out of orifice. Similar expressions can be written for *Redwood viscometer*. In general, both the viscometers are used to compare the viscosities of different liquid. So, the value of viscosity of the liquid may be obtained by comparison with value of time for the liquid of known viscosity.

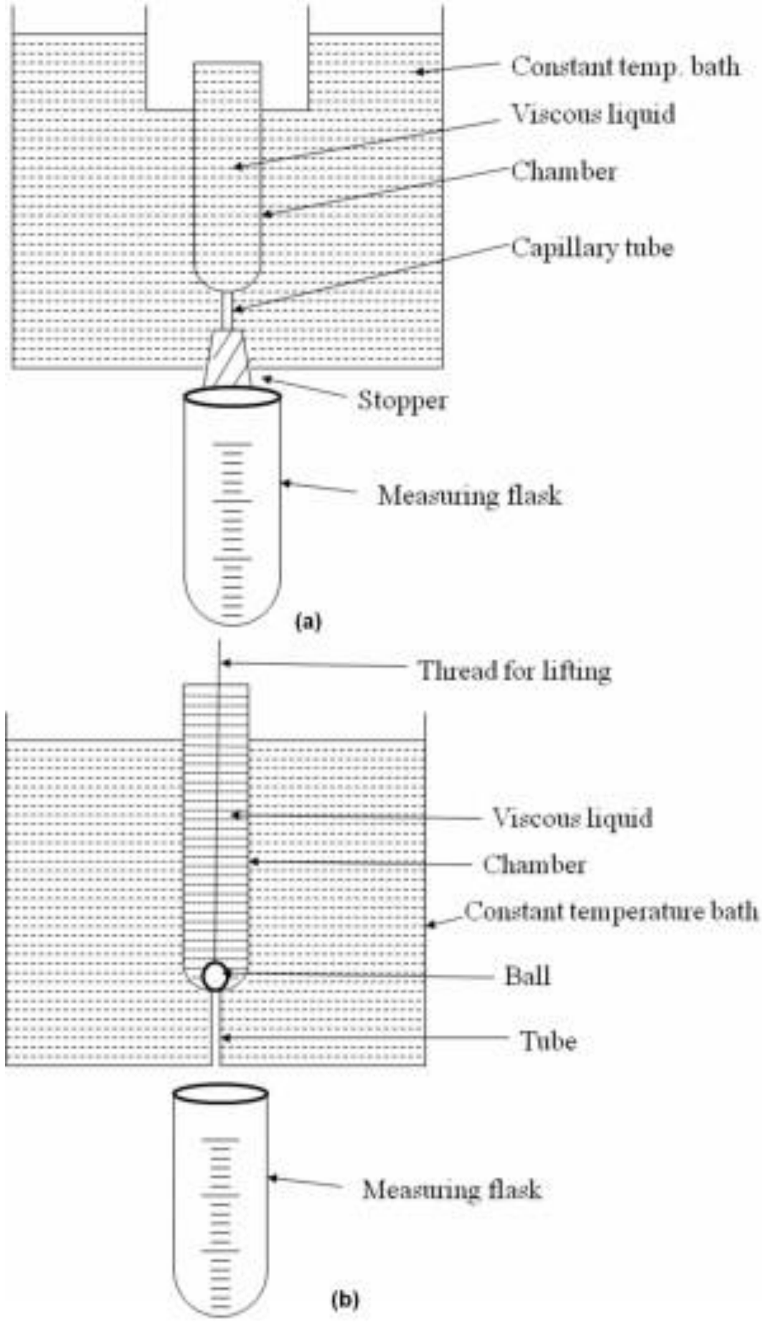


Fig. 7.1.4: Schematic diagram: (a) Saybolt viscometer; (b) Redwood viscometer.

CONDUCTIVITY METER

The basic principle of conductivity measurement is the same with all methods: the instrument generates an electric voltage across the measured solution. An electric current flows whose value depends on the conductivity. Depending on the method or application, the instrument either maintains the voltage signal constant and records the change in electric current, or maintains the current value constant and evaluates the voltage change.

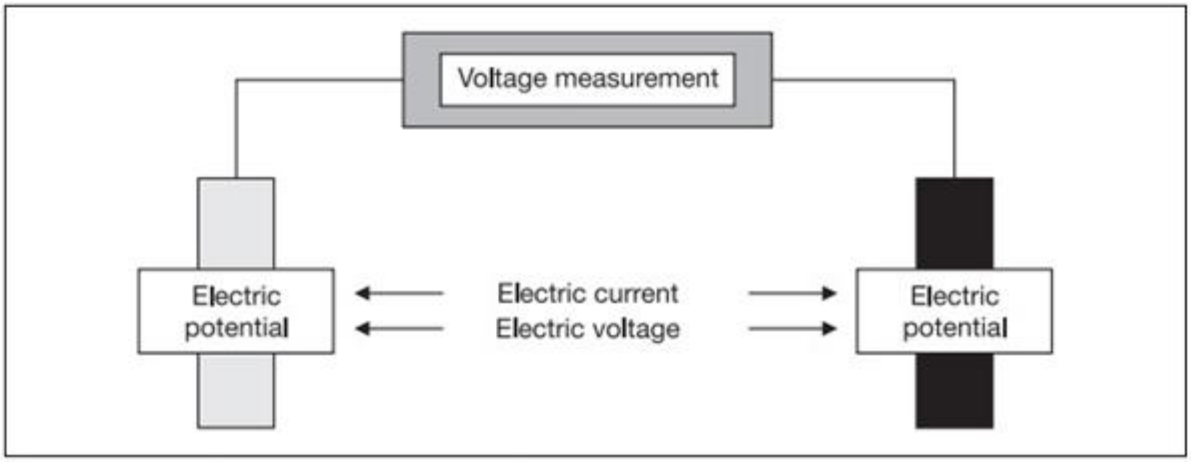


Diagram of a conductivity measuring cell

Both measurement principles are based on Ohm´s law:

$$R = \frac{U}{I} = \frac{1}{G} \qquad (1)$$

R: electrical resistance
I: electrical current
U: electrical voltage, or rearranged for the conductance
G: conductivity

$$\gamma = \frac{I}{U} \bullet k' \qquad (2)$$

γ: (specific) electrical conductance
I: electrical current
U: electrical voltage
k': cell constant

At constant voltage, the current increases proportionally with the conductance. At constant current, the voltage decreases with increasing conductance. It is clear from Ohm's law that conductivity measurements really concern resistance measurements. The conductance value I/U is obtained from the reciprocal of the resistance.

1.2.2 The cell constant

Both the resistance and the conductivity depend on the dimensions of the electrical conductor. The length and surface area of the conductor determine the cell constant.

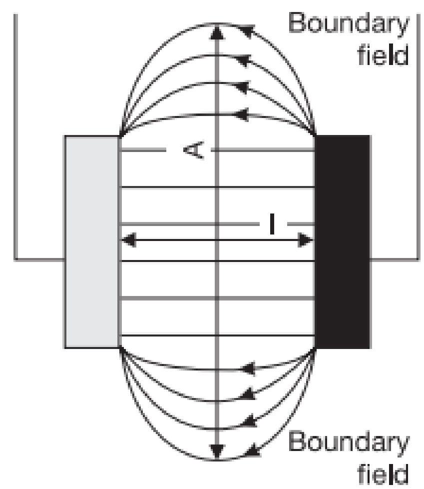


Fig. 3: Schematic representation of the active area

With a short length of conductor the electrodes are close together. The smaller the distance between the electrodes, the lower is the resistance of the measured solution. The influence of the electrodes on the ions increases.

A large conductor surface area is synonymous with large electrode surface areas. The bigger the surface area of the electrodes, the smaller is the resistance of the measured solution. As the surface area increases, more and more ions come within the range of influence of the electrodes.

The surface area of the electrical conductor is normally larger than the electrode surface area. The electrodes not only affect ions that are directly between the electrode surfaces, but also those in the boundary fields. The influence of the electrode boundary fields on the ions decreases with the distance. The boundary fields can be limited by the construction of the cell or the location, e. g. pipe internal diameters.

These influences are taken into account in the cell constant:

These influences are taken into account in the cell constant:

$$k' = \frac{l}{A} \qquad (3)$$

k': cell constant
l: length of the conductor
A: surface area of the conductor

The (specific) electrical conductivity is therefore a substance-specific variable, which, in contrast to the conductance G, does not depend on geometric factors. The precise value of the cell constant is obtained from a calibration with a reference solution with a reference solution that has a known temperature-dependent conductivity.

1.2.3 AC voltage

The electric current between the electrodes depends on the movement of the ions in the measured solution. During measurement, the ions move towards the electrode that is oppositely charged at the time. Each ion that reaches one of the electrode surfaces balances out a part of the voltage between the electrodes. As it is no longer mobile, it blocks the current flow. This effect (polarization) can be counteracted by an AC voltage. Because of the constant reversal of polarity, only a small quantity of ions reach the electrodes, and then only for a short period. The more ions the solution contains, that is the higher the conductivity, the higher must be the frequency that the instrument uses to prevent polarization of the electrodes.

Modern instruments (e. g. JUMO dTRANS Lf 01) match the measurement frequency to the range in order to achieve optimal measurement results.

1.2.4 Measurement principles

- Conductive principle:
  2-electrode measuring cells,
  4-electrode measuring cells
- Inductive principle

2-electrode measuring cells
This is the simplest design for a conductivity measuring cell. The 2-electrode measuring cell is perfectly adequate for general industrial measurement. This cell consists of two electrodes and a housing that fixes the two electrodes. A constant AC voltage is applied between the two electrodes. The current flowing through the measured solution is the measurement signal.

With this type of cell, the cell constant and the nature of the electrode surface depend on the purpose the cell is to be used for.

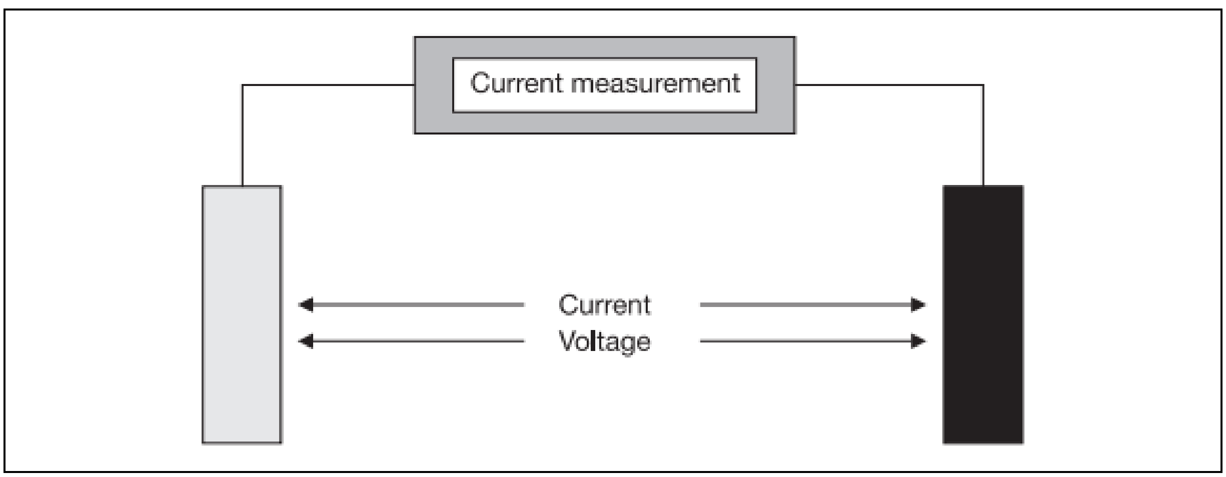


Fig. 4: **Diagram of a 2-electrode measuring cell**

A small cell constant means a large measurement signal. This effect is desirable for small conductivity values. At higher measured values it can overload the instrument. The optimal cell constant is thus all a question of the range.

Application examples	Conductivity range	Cell constant
Distillate, condensate, high-purity, fully-desalinated water	$\gamma < 10 \mu S/cm$	$k' \leq 0.1 cm^{-1}$
Ground, surface and drinking water	$\gamma < 10 - 10{,}000 \mu S/cm$	$k' \leq 1 cm^{-1}$
Sea water and saline solutions	$\gamma > 10 mS/cm$	$k' \leq 10 cm^{-1}$

Table 3: Cell constants for various conductivity ranges with application examples

With measuring cells with a constant $k' \leq 0.1 cm^{-1}$ the electrode surfaces are smooth, to improve the adjustment behavior at low conductivity values.

Measuring cells with a constant k' > 0.1 have rough electrode surfaces to reduce the tendency to polarize at higher conductivity values.

4-electrode measuring cells

The measuring cells contain two pairs of electrodes. One pair measures the current, the other measures the voltage applied across the measured solution.

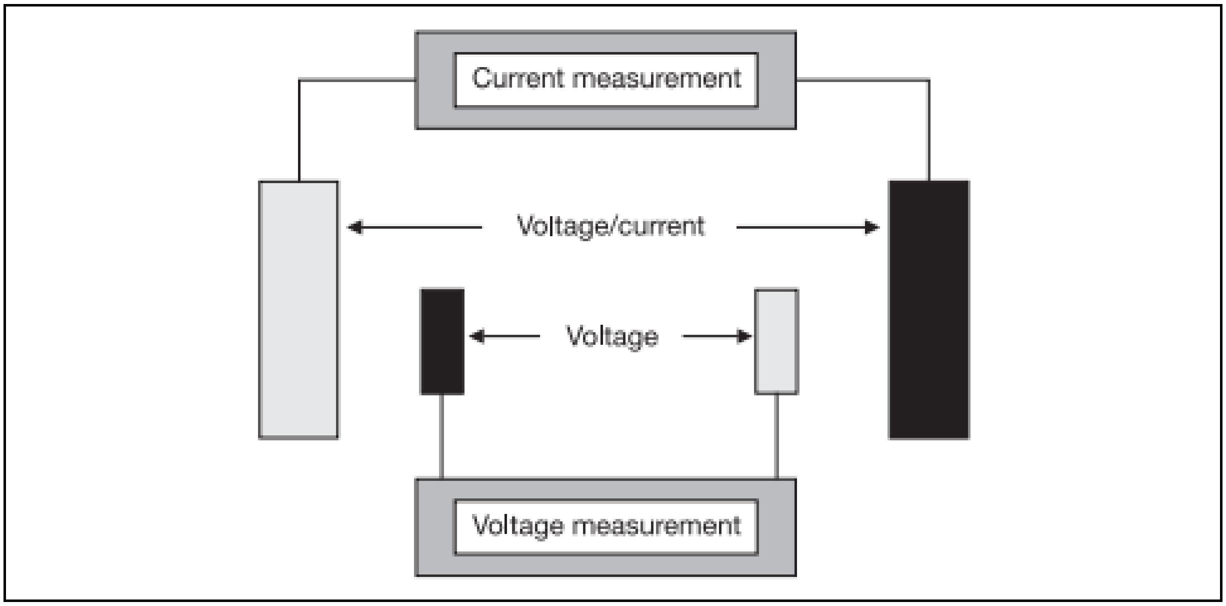


Fig. 5: Diagram of a 4-electrode measuring cell

The advantage of 4-electrode measuring cells is their insensitivity to interfering resistances due to long connecting cables, contaminants, or due to polarization, for example. These effects lead to low readings as they reduce the voltage that the electrodes apply to the measured solution. The second pair of electrodes determines the voltage across the measured solution. The instrument can take account of an interfering resistance by means of an electronic adjustment based on the measured current/voltage values of the two electrode pairs.

The conductivity meter calculates the conductivity value in accordance with the equation quoted earlier:

$$\gamma = \frac{I}{U} \bullet k' \quad (2)$$

4. Measurement of pH

pH is a measure of hydrogen ion concentration in aqueous solution. It is an important parameter to determine the quality of water. The pH value is expressed as:

$$pH = \frac{1}{\log_{10} C}$$

Where C is the concentration of H^+ ions in a solution. In pure water, the concentration of H^+ ions is 10^{-7} *gm/ltr* at 25° C. So the pH value is

$$pH = \frac{1}{\log_{10} 10^{-7}} = 7.$$

The advantage of using pH scale is that the activities of all strong acids and bases can be brought down to the scale of 0-14. The pH value of acidic solutions is in the range 0-7 and alkaline solutions in the range 7-14.

The pH value of a solution is measured by using pH electrode. It essentially consists of a pair of electrodes: *measuring* and *reference* electrode, both dipped in the solution of unknown pH. These two electrodes essentially form two *half-cells;* the total potential developed is the difference between the individual electric potential developed in each half cell. While the potential developed in the reference cell is constant, the measuring cell potential is dependent on the hydrogen ion concentration of the solution and is governed by *Nernst's equation:*

$$E = E_0 + \frac{RT}{nF}\ln(aC)$$

Where:

E= e.m.f of the half cell
E_0= e.m.f of the half cell under saturated condition
R= Gas constant (8.314 $J/{}^0C$)
T= Absolute temperature (K)
N= valance of the ion
F= Faraday Constant = 96493 C
a= Activity co-efficient $(0 \le a \le 1)$; for a very dilute solution, $a \to 1$
C= molar concentration of ions.

Measuring Electrode

The measuring electrode is made of *thin sodium ion selective glass*. A potential is developed across the two surfaces of this glass bulb, when dipped in aqueous solution. This potential is sensitive to the H^+ ion concentration, having a sensitivity of 59.2 *mv/pH* at 25^0C. Fig. 7 shows the basic schematic of a measuring probe. The buffer solution inside the glass bulb has a constant H^+ ion concentration and provides electrical connection to the lead wire.

Reference Electrode

The basic purpose of a reference electrode is to provide continuity to the electrical circuit, since the potential across a single half cell cannot be measured. With both the measuring and reference cells dipped in the same solution, the potential is measured across the two lead wires. A reference electrode should satisfy the following basic requirements:

(i) The potential developed should be independent of H^+ ion concentration.
(ii) The potential developed should be independent of temperature
(iii) The potential developed should not change with time.

Considering all these requirements, two types of reference electrodes are commonly used: (i) Calomel (Mercury-Mercurous Chloride) and (ii) Silver-Silver Chloride. The construction of a Calomel reference electrode is shown in Fig. 8. The electrical connection is maintained through the *salt bridge*.

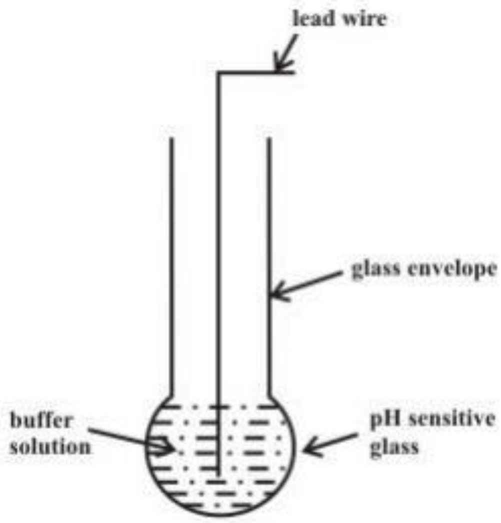


Fig. 7 Measuring Electrode

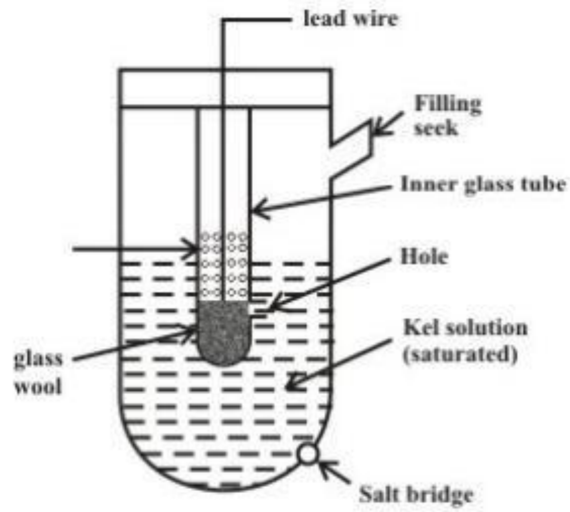


Fig. 8 Reference Electrode

Sometimes the reference and measuring electrodes are housed together, as shown in Fig. 9. This type of electrode is known as *Combination Electrode*. The reference electrode used in this case is Silver-Silver Chloride. The combination is dipped in the solution whose pH is to be measured and the output voltage is the difference between the e.m.f.s generated by the measuring glass electrode and the reference electrode.

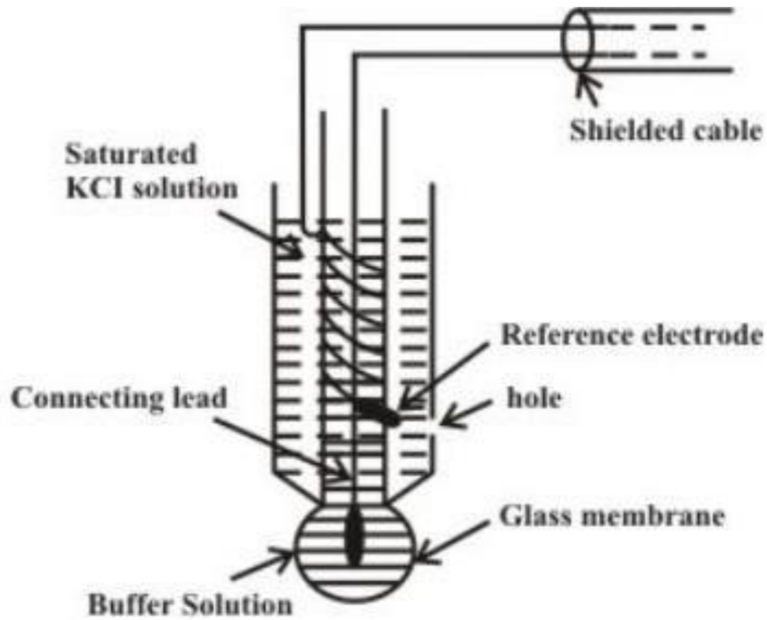


Fig. 9 Combination type pH electrode

Measuring scheme

The sensitivity of pH probe is around 59.2mv/pH at 25^0C. This sensitivity should be sufficient for measurement of voltage using ordinary electronic voltmeters. But, that is not the case; special measuring circuits are required for measurement of pH voltage. This is because of the fact that the internal resistance of the pH probe as a voltage source is very high, in the order of 10^8-10^9 Ω. This is because of the fact; the electrical path between the two lead wires is completed through the glass membrane. As a result, the input resistance for of the measuring device must be at least ten times electrode resistance of the electrode. FET-input amplifier circuits are normally used for amplifying the voltage from the pH probe. Not only that, the insulation resistance between the leads must also be very high. They are normally provided with moisture resistance insulation coating.

The voltage in the pH probe is temperature dependent, as evident from Nernst equation. As a result suitable temperature compensation scheme should also be provided in the measuring scheme.

1.12 Humidity Measurement Methods

Most of the above humidity generation methods utilise some form of negative feedback to control the relative or absolute humidity in the wet air they output. In order for such a system to work, it is necessary to accurately measure the air's humidity. Humidity can either be measured directly; or indirectly, by measuring the dew-point and the temperature, and thus inferring the humidity. Some methods, such as the two-pressure method, assume saturation and evaluate resulting humidities using measurements of just temperature and pressure. Temperature and pressure measurements can be performed by any standard apparatus including thermometer or pyrometer, manometer or Piezoelectric respectively.

Psychrometer (Wet- and Dry-Bulb Hygrometer)

The simplest type of hygrometer is a Psychrometer. This constitutes a pair of temperature sensors. These are placed in a wet air stream. One of the thermometers (the wet bulb) has a wet wick around it; evaporation causes this thermometer to read a lower temperature than the other. The difference in temperatures, and the absolute temperature of the wet thermometer can be used to calculate the relative humidity of the air.

$$p_w = p_s - AP(T_D - T_W)$$

- p_w is the experimental partial vapor pressure of water
- p_s is the saturation vapor pressure at temperature T_w

· A is the psychrometer constant (typically, values of A, for T_w above 0°C, are around 5×10^{-4} to 10^{-3})

- P is the experimental pressure
- T_D is the dry bulb temperature
- T_W is the wet bulb temperature

Alternatively, for a given Psychrometer, the values obtained can be looked-up in a reference table.

Mechanical Hygrometers

Mechanical hygrometers utilise the change in dimensions of various porous materials (such as wet paper and hair) as they absorb/exude water vapor. This change in dimensions can be used to move a needle or dial; which, when calibrated, will give the relative humidity.

Electrical Hygrometers

Electrical impedance sensors measure the changes in electrical capacitance or resistance of a hygroscopic material. The material will absorb or desorb water depending on the partial vapor pressure in the atmosphere around it, thus changing its electrical properties. These sensors measure relative humidity. While capacitive hygrometers can withstand condensation, resistive ones usually cannot.

Dew-Point Sensors

Dew-point sensors measure the temperature of the surrounding air, and the dew-point of a small sample thereof. The humidity of the air is then inferred from the dew-point temperature and the initial air temperature. Optical sensors cool a surface (usually a mirror) until condensation starts to form. This indicates that the surface is at the air's dew-point. Other methods of measuring dew-point include measuring the oscillation rate of a quartz crystal. When condensation forms on the crystal, this rate will change.

References:

1. Eckman D.P., Industrial Instrumentation, 2nd Edition, Wiley Eastern Ltd, 1996.
2. Burns G.W., Scroger M.G., Strouse G.F., Croarkin M.C., Guthrie W.F. Temperature- Electromotive Force Reference Functions and Tables for the Letter-Designated Thermocouple Types Based on the ITS-90

SCHOOL OF BIO AND CHEMICAL

DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – II-INSTRUMENTATION AND PROCESS CONTROL – SCHA1503

II Basic Concepts of Process Control

Laplace transformation and its application in process control. Response of first order systems, Examples of first order systems, Process Dynamics of linear open systems. Second order and first order systems in series higher order systems Transfer Function Step, Ramp, Pulse and Sinusoidal inputs and Linearization

2.1 Laplace transformation

- Laplace transformation is a technique for solving differential equations.
- Here differential equation of time domain form is first transformed to algebraic equation of frequency domain form.
- After solving the algebraic equation in frequency domain, the result then is finally transformed to time domain form to achieve the ultimate solution of the differential equation.
- In other words it can be said that the Laplace transformation is nothing but a shortcut method of solving differential equation.

First Let f(t) be the function of t, time for all $t \geq 0$

Then the Laplace transform of f(t), F(s) can be defined as

$$\mathbf{f(s) = \int_0^{\infty} f(t) e^{-st}\, dt}$$

Where $$\mathbf{f(s) = L\{f(t)\}}$$

Provided that the integral exists. Where the Laplace Operator, $s = \sigma + j\omega$; will be real or complex $j = \sqrt{(-1)}$

Find the laplace transform of a function

$$f(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \qquad \mathbf{f(s) = \int_0^{\infty} f(t) e^{-st}\, dt}$$

Substitute **f(t)=1**

$$\mathbf{f(s) = \int_0^{\infty} 1 . e^{-st}\, dt} \qquad = \frac{e^{-st}}{s}\Big|_{t=0}^{\infty} = \mathbf{0 - \left(-\frac{1}{s}\right) = \frac{1}{s}}$$

•

Find the laplace transform of a function

$$f(t)=\begin{cases} 0 & t<0 \\ \mathbf{e^{-at}} & t>0 \end{cases}$$

$$\mathbf{f(s)=\int_0^\infty f(t)e^{-st}\,dt}$$

Substitute **f(t)=e -at**

$$\mathbf{f(s)=\int_0^\infty e^{-at}e^{-st}\,dt} \quad =\int_0^\infty \mathbf{e^{-(s+a)t}\,dt}$$

$$=\frac{e^{-(s+a)t}}{s+a}\int_{t=0}^{\infty}=\mathbf{0-\left(-\frac{1}{s+a}\right)=\frac{1}{s+a}}$$

Find the laplace transform of a function

$$f(t)=\begin{cases} 0 & t<0 \\ \mathbf{t} & t>0 \end{cases}$$

$$\mathbf{f(s)=\int_0^\infty f(t)e^{-st}\,dt}$$

Substitute **f(t)=t**

$$\mathbf{f(s)=\int_0^\infty te^{-st}\,dt}$$

$$y(s)=\frac{1}{s}\left|-\frac{1}{s}e^{-st}\right|\int_0^\infty=\frac{1}{s^2}$$

$$\int_0^\infty u.dv=uv\int_0^\infty-\int_0^\infty v.du$$

$$u=t:du=dt$$

$$dv=e^{-st}dt;\ v=-\frac{e^{-st}}{s}$$

$$y(s)=\left[t\times-\frac{e^{-st}}{s}\right]\int_0^\infty-\int_0^\infty-\frac{e^{-st}}{s}dt$$

Laplace transforms can be used in process control for:

1. Solution of differential equations (linear)
2. Analysis of linear control systems (frequency response)
3. Prediction of transient response for different inputs

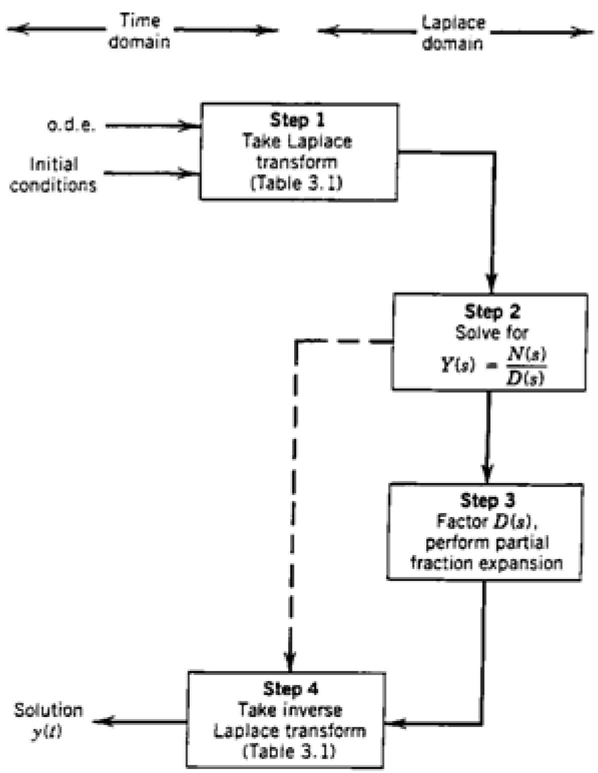


Figure The general procedure for solving an ordinary differential equation using Laplace transforms.

Applications of Laplace Transform

- It is used to convert complex differential equations to a simpler form having polynomials.
- It is used on to convert derivatives into multiple of domain variable and then convert the polynomials back to the differential equation using Inverse Laplace transform.
- It is used in the telecommunication field to send signals both the sides of the medium. For example, when the signals are sent through phone then they are first converted into a time-varying wave and then super-imposed on the medium.
- It is also used for many engineering tasks such as Electrical Circuit Analysis, Digital Signal Processing, System Modelling, etc.

2.2 Response of first order systems

- Response of 1^{st} order system when the input is unit step -

$$\frac{Y(s)}{X(s)} = \frac{k}{\tau s + 1}$$

In the above transfer function, the power of 's' is the one in the denominator. That is why the above transfer function is of the first order, and the system is said to be the first order system.

It is a system whose dynamic behavior is described by a first order differential equation.

The transfer function is defined as the ratio of the output and the input in the Laplace domain with zero initial condition.

- Standard form of first order transfer functions

$$\frac{Y(s)}{X(s)} = \frac{K}{\tau s + 1}$$

The important characteristics of the standard form are as follows:

- The denominator must be of the form $\tau s + 1$
- The coefficient of the s term in the denominator is the system time constant τ
- The numerator is the steady-state gain K.

Linear system

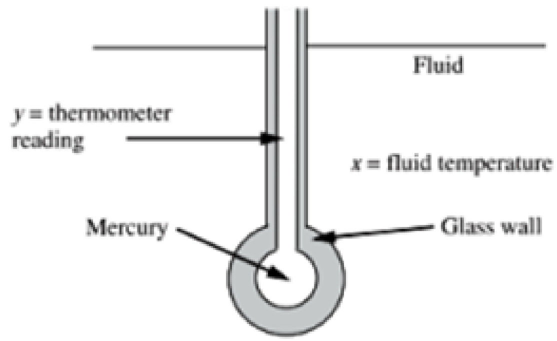
- In a time domain, a linear system is modeled by a linear differential equation
- n^{th} order system

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + a_{n-2} \frac{d^{n-2} y}{dt^{n-2}} + \cdots \dots \dots \dots \dots . + a_1 \frac{dy}{dt} + a_0\, y = bu(t)$$

- Assumptions
  - the coefficient of the differential equations are constant
  - The output y is equal to the state x

2.3 Mercury thermometer

- A mercury thermometer: consider the mercury thermometer shown in the figure



Cross view of thermometer

Basic assumptions:

1. All the resistance to heat transfer resides in the film surrounding the bulb (conduction resistance is neglected).
2. All the thermal capacity is in the mercury.
3. The mercury assumes a uniform temperature throughout.
4. The glass wall containing the mercury does not expand or contract during the transient response.
5. Constant properties.

Unsteady state heat balance

Input – Output = Rate of change

Surrounding Temperature

$$hA(x-y)-0=mc_p\frac{dy}{dt}$$

Heat capacity of mercury

Film coefficient of heat transfer

Mass of mercury in bulb

Surface area of bulb for heat transfer

Mercury temperature

Steady state balance $\quad hA(x_s-y_s)=0$

Unsteady state balance – steady state balance

$$hA[(x-x_s)-(y-y_s)] = mc_p\frac{d(y-y_s)}{dt}$$

Substitute

X= $(x-x_s)$

Y= $(y-y_s)$

By substituting

$$hA[X - Y] = mc_p \frac{dY}{dt}$$

$$[X - Y] = \frac{mc_p}{hA} \frac{dY}{dt}$$

Taking Laplace transform

$$[X(s) - Y(s)] = \frac{mc_p}{hA} sY(s)$$

- Rearranging

$$\frac{Y(s)}{X(s)} = \frac{1}{\underbrace{\frac{mc_p}{hA}}_{\tau} s + 1}$$

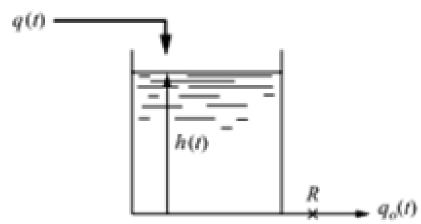
By substituting

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s + 1}$$

The time constant is a measure to how fast be the system response. The smaller is the time constant, the more responsive is the system. τ also called "dead time" or "dynamic lag"

Liquid level

A tank of uniform cross sectional area A, the inlet flow is q and the outlet is q_o. The liquid level in the tank is h and the tank has a linear flow resistance at the outlet is R



Liquid level system

- Basic assumptions:

a) Constant density

b) Linear resistance

c) Constant cross sectional area

Unsteady state mass balance

Input – Output = Rate of change

$$q - q_o = A\frac{dh}{dt} \longrightarrow 1$$

- Flow head equation $$Q_o = \frac{H}{R}$$

- Steady state mass balance

$$q_s - q_{os} = 0 \longrightarrow 2$$

Subtract the steady state equation from the unsteady state one (1)-(2)

$$Q - Q_o = A\frac{dH}{dt} \longrightarrow 3$$

$$Q = q - q_s$$
$$Q_o = q_o - q_{os}$$
$$H = h - h_s$$

- Taking the transform of the resulting equation

$$Q(s) - Q_o(s) = AsH(s)$$

$$Q_o = \frac{H}{R}$$

$$Q(s) - \frac{H(s)}{R} = AsH(s)$$

$$\frac{H(s)}{Q(s)} = \frac{R}{ARs+1}$$

$$\frac{H(s)}{Q(s)} = \frac{R}{\tau s+1}$$

$$\tau = AR$$

2.4 Response of first order systems to some common forcing functions

Step response ($X(t) = A\ u(t);\ Y(t) = ??$)

$$\frac{Y(s)}{X(s)} = \frac{K}{\tau s+1}$$

Step input $$X(s) = \frac{A}{s}$$

$$Y(s) = \frac{A}{s}\frac{K}{\tau s+1} = \frac{C_1}{s} + \frac{C_2}{\tau s+1}$$

$$Y(s) = \frac{KA/\tau}{s(s+\frac{1}{\tau})} = \frac{C_1}{s} + \frac{C_2}{s+\frac{1}{\tau}}$$

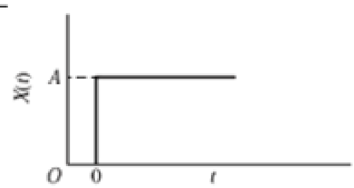
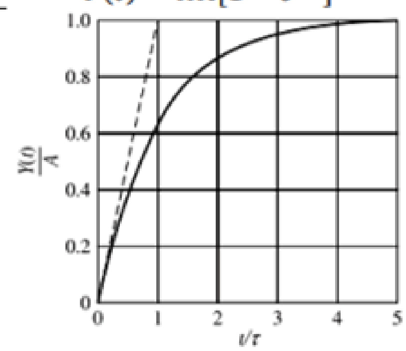
$$C_1 = \frac{KA\tau}{s\left(s+\frac{1}{\tau}\right)} \times S \int_{S=0} = KA$$

$$C_2 = \frac{KA/\tau}{s\left(s+\frac{1}{\tau}\right)} \times \left(s+\frac{1}{\tau}\right) \int_{S=-\frac{1}{\tau}} = -KA$$

- THEREFORE $$Y(s) = \frac{KA}{s} - \frac{KA}{s+\frac{1}{\tau}}$$

- TAKING INVERSE LAPLACE TRANSFORM

$$Y(t) = KA[1 - e^{-\frac{t}{\tau}}]$$

Input X(t)	Output Y(t)
$X(t) = A\,u(t)$	$Y(t) = KA[1 - e^{-\frac{t}{\tau}}]$
	 Ultimate value = KA Maximum value = KA

Ramp response $(X(t) = At\, u(t); Y(t) = ??)$
where A is the slope of the ramp function

$$\frac{Y(s)}{X(s)} = \frac{K}{\tau s + 1}$$

$$X(s) = \frac{A}{s^2}$$

$$Y(s) = \frac{KA}{s^2(\tau s + 1)} = \frac{\frac{KA}{\tau}}{s^2\left(s + \frac{1}{\tau}\right)} = \frac{C_1}{s^2} + \frac{C_2}{s} + \frac{C_3}{\left(s + \frac{1}{\tau}\right)}$$

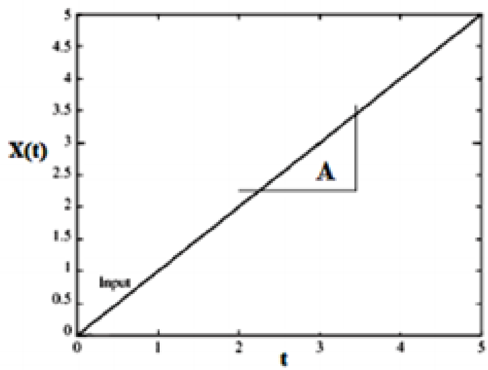
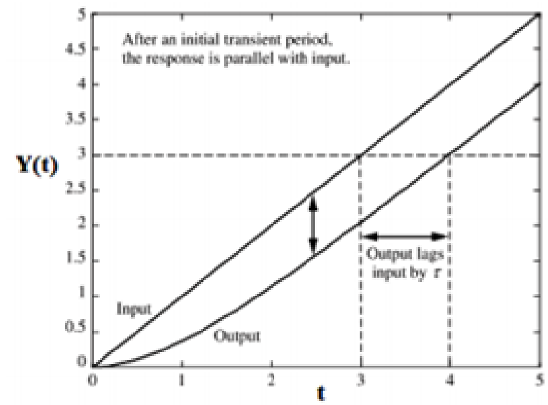
Solving by partial fractions

- $C_1 = KA$
- $C_2 = -KA\tau$
- $C_3 = KA\tau$

$$Y(s) = \frac{KA}{s^2} - \frac{KA\tau}{s} + \frac{KA\tau}{\left(s + \frac{1}{\tau}\right)}$$

$$Y(t) = KAt - KA\tau + KA\tau\, e^{-\frac{t}{\tau}}$$

$$Y(t) = KAt - KA\tau\left(1 - e^{-\frac{t}{\tau}}\right)$$

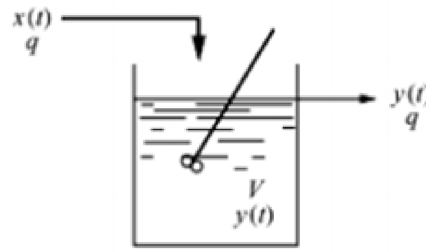
<table>
<tr><th>Input X(t)</th><th>Output Y(t)</th></tr>
<tr><td>$X(t) = At\,u(t)$</td><td>$Y(t) = KAt - KA\tau\left(1 - e^{-\frac{t}{\tau}}\right)$</td></tr>
<tr><td>
</td><td>

Ultimate value = ∞
Maximum value = ∞</td></tr>
</table>

2.5 Mixing Process

- Consider the mixing process shown in the above figure in which a stream of solution containing dissolved salt flows at a constant volumetric flow rate (q) into a tank of constant holdup volume V.
- The concentration of the salt in the entering stream x (mass of salt/volume) varies with time.
- It is desired to determine the transfer function relating the outlet concentration y to the inlet concentration x ($Y(s)\,/X(s)$).

- **Basic assumptions:**
- Constant density
- Constant holdup
- Perfect mixing (outlet concentration equal the concentration inside the tank



Mixing process

- Unsteady state mass balance

$$qx - qy = \frac{d(Vy)}{dt} = V\frac{dy}{dt}$$

- Steady state mass balance

$$qx_s - qy_s = 0$$

- Subtract the steady state equation from the unsteady state one

$$qX(t) - qY(t) = V\frac{dY}{dt}$$

- Taking the transform of the resulting equation

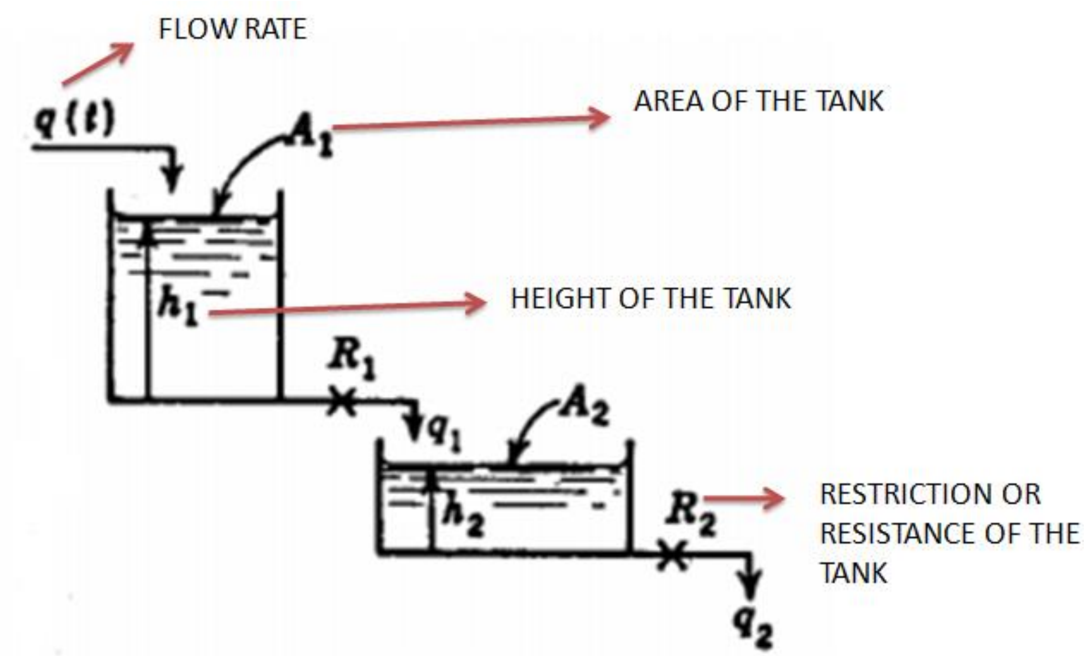
$$qX(s) - qY(s) = VsY(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\frac{V}{q}s + 1}$$

$$\tau = \frac{V}{q}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s + 1}$$

2.6 Non-Interacting system



- UNSTEADY MASS BALANCE EQUATION

$$q - q_1 = A_1 \frac{dh_1}{dt} \longrightarrow (1)$$

- FLOW HEAD EQUATION

$$q_1 = \frac{h_1}{R_1} \longrightarrow (2)$$

- STEADY STATE MASS BALANCE EQUATION

$$q_s - q_{1s} = A_1 \frac{dh_{1s}}{dt} \longrightarrow (3)$$

- FLOW HEAD EQUATION

- $$q_{1s} = \frac{h_{1s}}{R_1} \longrightarrow (4)$$

- Unsteady state- steady state

(1)-(3)

$$(q - q_s) - (q_1 - q_{1s}) = A_1 \frac{d(h_1 - h_{1s})}{dt} \longrightarrow (5)$$

(2)-(4)

$$q_1 - q_{1s} = \frac{h_1 - h_{1s}}{R_1} \longrightarrow (6)$$

ASSUMPTIONS

$$q - q_s = Q$$

$$q_1 - q_{1s} = Q_1$$

$$h_1 - h_{1s} = H_1 \longrightarrow (7)$$

- SUBSTITUTE (7) IN (5) & (6)

$$Q - Q_1 = A_1 \frac{dH_1}{dt} \longrightarrow (8)$$

$$Q_1 = \frac{H_1}{R_1} \longrightarrow (9)$$

- TAKING L.T

$$Q(S) - Q_1(S) = A_1 S H_1(S) \longrightarrow (10)$$

$$Q_1(S) = \frac{H_1(S)}{R_1} \longrightarrow (11)$$

- SUBSTITUTE (11) IN (10)

$$Q(S) - \frac{H_1(S)}{R_1} = A_1 S H_1(S) \longrightarrow (12)$$

- REARRANING

$$Q(S) = H_1(S)[A_1 S + \frac{1}{R_1}] \longrightarrow (13)$$

$$\frac{H_1(S)}{Q(S)} = \frac{1}{A_1 S + \frac{1}{R_1}} \longrightarrow (14)$$

$$\boxed{\frac{H_1(S)}{Q(S)} = \frac{R_1}{A_1 R_1 S + 1}} \longrightarrow (15)$$

- UNSTEADY MASS BALANCE EQUATION

$$q_1 - q_2 = A_2 \frac{dh_2}{dt} \longrightarrow (16)$$

- FLOW HEAD EQUATION

$$q_2 = \frac{h_2}{R_2} \longrightarrow (17)$$

- STEADY STATE MASS BALANCE EQUATION

$$q_{1s} - q_{2s} = A_2 \frac{dh_{2s}}{dt} \longrightarrow (18)$$

- FLOW HEAD EQUATION

$$q_{2s} = \frac{h_{2s}}{R_2} \longrightarrow (19)$$

- Unsteady state- steady state

  (16)-(18)

$$(q_1 - q_{1s}) - (q_2 - q_{2s}) = A_2 \frac{d(h_2 - h_{2s})}{dt} \longrightarrow (20)$$

(17)-(19)

$$q_2 - q_{2s} = \frac{h_2 - h_{2s}}{R_2} \longrightarrow (21)$$

ASSUMPTIONS

$$q_1 - q_{1s} = Q_1$$

$$q_2 - q_{2s} = Q_2$$

$$h_2 - h_{2s} = H_2$$

- SUBSTITUTE (22) IN (20) & (21)

$$Q_1 - Q_2 = A_2 \frac{dH_2}{dt} \longrightarrow (23)$$

$$Q_2 = \frac{H_2}{R_2} \longrightarrow (24)$$

- TAKING L.T

$$Q_1(S) - Q_2(S) = A_2 S H_2(S) \longrightarrow (25)$$

$$Q_2(S) = \frac{H_2(S)}{R_2} \longrightarrow (26)$$

- SUBSTITUTE (11) IN (10)

$$Q_1(S) - \frac{H_2(S)}{R_2} = A_2 S H_2(S) \longrightarrow (27)$$

- REARRANING

$$Q_1(S) = H_2(S)[A_2 S + \frac{1}{R_2}] \longrightarrow (28)$$

$$\frac{H_2(S)}{Q_2(S)} = \frac{1}{A_2 S + \frac{1}{R_2}} \longrightarrow (29)$$

$$\boxed{\frac{H_2(S)}{Q_1(S)} = \frac{R_2}{A_2 R_2 S + 1}} \longrightarrow (30)$$

- Multiplty (15)x(30)

$$\frac{H_1(S)}{Q(S)} X \frac{H_2(S)}{Q_1(S)} = \frac{R_1}{A_1 R_1 S + 1} X \frac{R_2}{A_2 R_2 S + 1} \longrightarrow (31)$$

- We Know that $Q_1(S) = \frac{H_1(S)}{R_1}$

$$\frac{\cancel{H_1}(S)}{Q(S)} X \frac{\cancel{R_1} . H_2(S)}{\cancel{H_1}(S)} = \frac{\cancel{R_1}}{A_1 R_1 S + 1} X \frac{R_2}{A_2 R_2 S + 1} \longrightarrow (32)$$

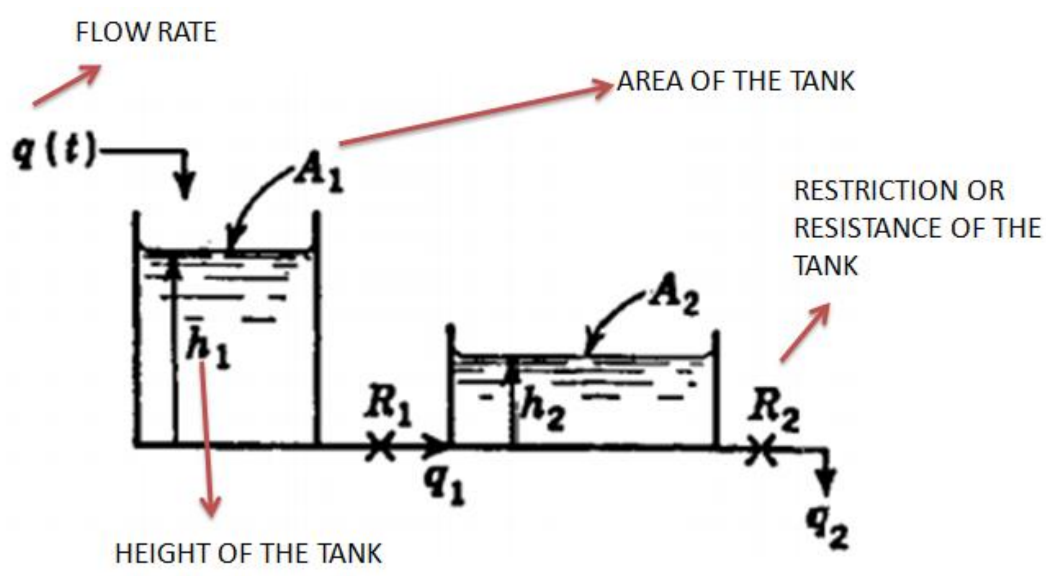
$$\frac{H_2(S)}{Q(S)} = \frac{R_2}{(A_1R_1S+1)(A_2R_2S+1)}$$

SUBSTITUTE

$$\tau_1 = A_1R_1 \qquad \tau_2 = A_2R_2$$

$$\boxed{\frac{H_2(S)}{Q(S)} = \frac{R_2}{(\tau_1 s+1)(\tau_2 s+1)}}$$

2.7 INTERACTING SYSTEM



- UNSTEADY MASS BALANCE EQUATION

TANK 1

$$q - q_1 = A_1 \frac{dh_1}{dt} \longrightarrow (1)$$

TANK 2

$$q_1 - q_2 = A_2 \frac{dh_2}{dt} \longrightarrow (2)$$

- FLOW HEAD EQUATION

TANK 1

$$q_1 = \frac{h_1 - h_2}{R_1} \longrightarrow (3)$$

TANK 2

$$q_2 = \frac{h_2}{R_2} \longrightarrow (4)$$

- STEADY STATE MASS BALANCE EQUATION

TANK 1

$$q_s - q_{1s} = A_1 \frac{dh_{1s}}{dt} \longrightarrow (5)$$

TANK 2

$$q_{1s} - q_{2s} = A_2 \frac{dh_{2s}}{dt} \longrightarrow (6)$$

- STEADY STATE FLOW HEAD EQUATION

TANK 1

$$q_{1S} = \frac{h_{1S} - h_{2S}}{R_1} \longrightarrow (7)$$

TANK 2

$$q_{2s} = \frac{h_{2s}}{R_2} \longrightarrow (8)$$

- SUBTRACT (1)-(5)

$$(q - q_s) - (q_1 - q_{1s}) = A_1 \frac{d(h_1 - h_{1s})}{dt} \longrightarrow \quad (9)$$

- (2)-(6)

$$(q_1 - q_{1s}) - (q_2 - q_{2s}) = A_2 \frac{d(h_2 - h_{2s})}{dt} \longrightarrow \quad (10)$$

- SUBTRACT (3)-(7)

$$q_1 - q_{1S} = \frac{(h_1 - h_{1S}) - (h_2 - h_{2S})}{R_1} \longrightarrow \quad (11)$$

(4)-(8)

$$q_2 - q_{2s} = \frac{h_2 - h_{2s}}{R_2} \longrightarrow \quad (12)$$

ASSUMPTIONS

$$q - q_s = Q$$

$$q_1 - q_{1s} = Q_1$$

$$q_2 - q_{2s} = Q_2 \longrightarrow (13)$$

$$h_1 - h_{1s} = H_1$$

$$h_2 - h_{2s} = H_2$$

- SUBSTITUTING

$$Q - Q_1 = A_1 \frac{dH_1}{dt} \longrightarrow (14)$$

$$Q_1 - Q_2 = A_2 \frac{dH_2}{dt} \longrightarrow (15)$$

$$Q_1 = \frac{H_1 - H_2}{R_1} \longrightarrow (16)$$

$$Q_2 = \frac{H_2}{R_2} \longrightarrow (17)$$

- TAKING LAPLACE TRANSFORM

$$Q(S) - Q_1(S) = A_1 S H_1(S) \longrightarrow (18)$$

$$Q_1(S) - Q_2(S) = A_2 S H_2(S) \longrightarrow (19)$$

$$Q_1(S) = \frac{H_1(S) - H_2(S)}{R_1} \longrightarrow (20)$$

$$Q_2(S) = \frac{H_2(S)}{R_2} \longrightarrow (21)$$

- SUBSTITUTION
- FROM (19) $\quad Q_1(S) = Q_2(S) + A_2 S H_2(S) \longrightarrow (22)$
- SUBSTITUTE (22) in (18)

$$Q(S) - Q_2(S) - A_2 S H_2(S) = A_1 S H_1(S) \longrightarrow (23)$$

- SUBSTITUTE (21) in (23)

$$Q(S) - \frac{H_2(S)}{R_2} - A_2 S H_2(S) = A_1 S H_1(S) \longrightarrow (24)$$

- Find $H_1(s)$ from (20) and substitute in(24)

$$R_1 Q_1(S) + H_2(S) = H_1(S) \longrightarrow \text{(25)}$$

$$Q(S) - \frac{H_2(S)}{R_2} - A_2 S H_2(S) = A_1 S[R_1 Q_1(S) + H_2(S)] \longrightarrow \text{(26)}$$

- REARRANGING

$$R_2 Q(S) - H_2(S) - A_2 R_2 S H_2(S) = A_1 S R_1 R_2 Q_1(S) + A_1 R_2 S H_2(S)$$

$$\longrightarrow \text{(27)}$$

- Substitute the value of $Q_1(S)$

$$Q_1(S) = Q_2(S) + A_2 S H_2(S)$$

IN EQN (27)

$$R_2 Q(S) - H_2(S) - A_2 R_2 S H_2(S) = A_1 S R_1 R_2 [Q_2(S) + A_2 S H_2(S)] + A_1 R_2 S H_2(S)$$

$$\longrightarrow \text{(28)}$$

SUB $Q_2(S) = \dfrac{H_2(S)}{R_2}$ IN (28)

$$R_2Q(S) - H_2(S) - A_2R_2SH_2(S) = A_1SR_1\cancel{R_2}[\frac{H_2(S)}{\cancel{R_2}} + A_2SH_2(S)] + A_1R_2SH_2(S)$$

REARRANING

$$R_2Q(S) - H_2(S) - A_2R_2SH_2(S) = A_1SR_1H_2(S) + A_1R_1A_2R_2S^2H_2(S) + A_1R_2SH_2(S)$$

$$R_2Q(S) = A_1SR_1H_2(S) + A_1R_1A_2R_2S^2H_2(S) + A_1R_2SH_2(S) + H_2(S) + A_2R_2SH_2(S)$$

$$R_2Q(S) = H_2(S)[A_1S\,R_1 + A_1R_1A_2R_2S^2 + A_1R_2S + 1 + A_2R_2S]$$

$$\boxed{\frac{H_2(S)}{Q(S)} = \frac{R_2}{[A_1S\,R_1 + A_1R_1A_2R_2S^2 + A_1R_2S + 1 + A_2R_2S]}}$$

$$\boxed{\frac{H_2(S)}{Q(S)} = \frac{R_2}{\tau_1 s + \tau_1\tau_2 s^2 + A_1R_2 s + 1 + \tau_2 s]}}$$

Substitute

$$\tau_1 = A_1R_1 \qquad \tau_2 = A_2R_2$$

References

1. Coughanowr D.R and Koppel L.M.,Process Systems Analysis and Control, 3 rd Edition,McGraw Hill,NewYork,1991.
2. Vyas.R.P., Process control and instrumentation, 2nd Edition, Central Techno Publications, Nagpur 2005

SCHOOL OF BIO AND CHEMICAL

DEPARTMENT OF CHEMICAL ENGINEERING

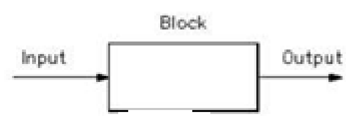
UNIT – III-INSTRUMENTATION AND PROCESS CONTROL – SCHA1503

III Linear Closed Loop Systems

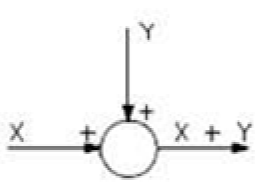
Development of a block diagram, Controllers and Final control element Closed loop control systems. Block Diagram for feedback control systems, Servo and Regulator problems. Principles of Pneumatic and Electronic controllers. Transportation Lag Transient response of closed loop control systems, Control valve PI, P, PID control.

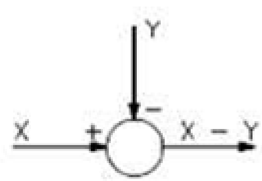
3.1 Development of a block diagram

- A block diagram is a pictorial representation of the cause and effect relationship between the input and output of a physical system.
- A block diagram provides a means to easily identify the functional relationships among the various components of a control system.
- The simplest form of a block diagram is the block and arrows diagram.
- It consists of a single block with one input and one output.
- The block normally contains the name of the element or the symbol of a mathematical operation to be performed on the input to obtain the desired output.
- Arrows identify the direction of information or signal flow.

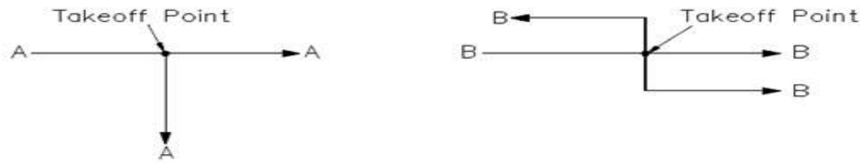


- Although blocks are used to identify many types of mathematical operations, operations of addition and subtraction are represented by a circle, called a *summing point*.
- A summing point may have one or several inputs.
- Each input has its own appropriate plus or minus sign.
- A summing point has only one output and is equal to the algebraic sum of the inputs.





- A *takeoff point* is used to allow a signal to be used by more than one block or summing point



3.2 Process Control

- **Process Control** is the active changing of the process based on the results of process monitoring.
- **Input variables**: which denote the effect of the surroundings on the chemical process
- **Output variables**: which denote the effect of the process on the surrounding
- **Controlled variables -** these are the variables which quantify the performance or quality of the final product, which are also called output variables.
- **Manipulated variables -** these input variables are adjusted dynamically to keep the controlled variables at their set-points.
- **Disturbance variables -** these are also called "load" variables and represent input variables that can cause the controlled variables to deviate from their respective set points.
- **Set-point change -** implementing a change in the operating conditions. The set-point signal is changed and the manipulated variable is adjusted appropriately to achieve the new operating conditions. Also called servomechanism (or "servo") control.
- **Disturbance change -** the process transient behavior when a disturbance enters, also called regulatory control or load change. A control system should be able to return each controlled variable back to its set-point.

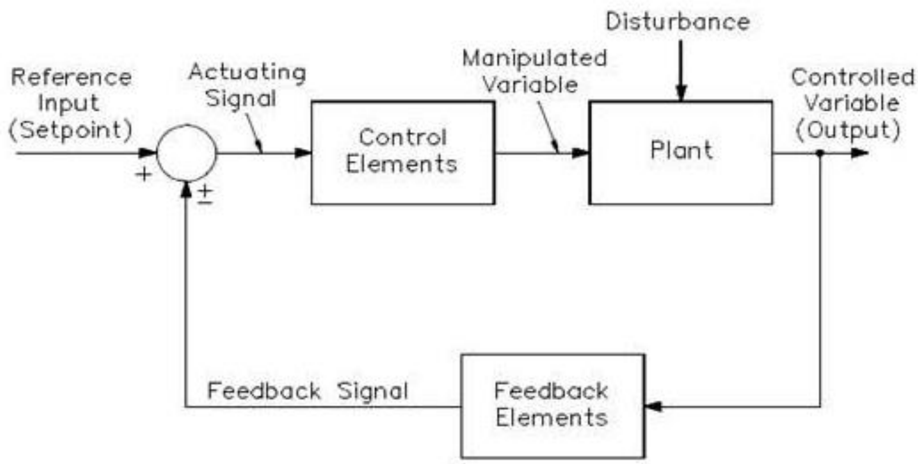


Fig 1.Feedback Control System Block Diagram

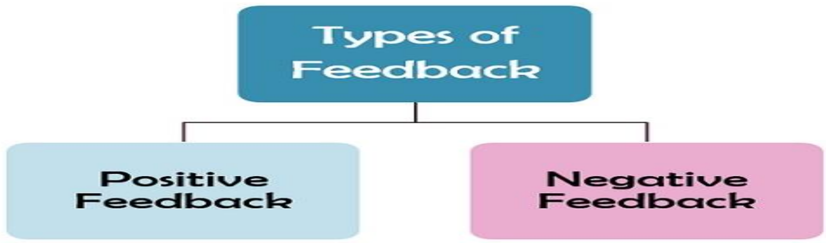
- The plant is the system or process through which a particular quantity or condition is controlled. This is also called the controlled system.
- The control elements are components needed to generate the appropriate control signal applied to the plant. These elements are also called the "controller."
- The feedback elements are components needed to identify the functional relationship between the feedback signal and the controlled output.
- The reference point is an external signal applied to the summing point of the control system to cause the plant to produce a specified action. This signal represents the desired value of a controlled variable and is also called the "setpoint."
- The controlled output is the quantity or condition of the plant which is controlled. This signal represents the controlled variable.
- The feedback signal is a function of the output signal. It is sent to the summing point and algebraically added to the reference input signal to obtain the actuating signal.
- The actuating signal represents the control action of the control loop and is equal to the algebraic sum of the reference input signal and feedback signal. This is also called the "error signal."
- The manipulated variable is the variable of the process acted upon to maintain the plant output (controlled variable) at the desired value.
- The disturbance is an undesirable input signal that upsets the value of the controlled output of the plant.

The main characteristics of **Closed-loop Control** as being:

- To reduce errors by automatically adjusting the systems input.
- To improve stability of an unstable system.

- To increase or reduce the systems sensitivity.
- To enhance robustness against external disturbances to the process.
- To produce a reliable and repeatable performance.

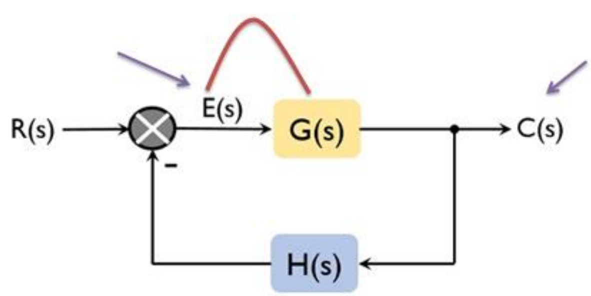
Feedback in any circuit can be generally of 2 types:



- **Positive Feedback**: The type of feedback in a control system in which the input signal and the feedback signal are in phase with each other is known as a positive feedback system.In these systems, the reference input gets added with the feedback signal thereby increasing the gain of the overall system.
- **Negative Feedback**: In the case of negative feedback, the input signal and the feedback signal show out-of-phase relationship wrt each other. Thus the applied input signal and the feedback signal are subtracted to get the error signal. This leads to a reduction in the overall gain of the system.

3.3 Transfer Function of Closed-Loop Control System

- Transfer function indicates the behaviour of the system as it is defined as the mathematical relation between the input and output of the system.
- The gain of the system defines the ratio of output to input. Thus we can say the output of the system is the product of transfer function and input.



$$C(s) = E(s) \,.\, G(s)$$

$$E(s) = R(s) - H(s)C(s)$$

Substitute E(s) in C(s)

$$C(s) = [R(s) - H(s)C(s)].\, G(s)$$

$$C(s) = R(s)\, G(s) - H(s)C(s)\, G(s)$$

By rearranging

$$R(s)\, G(s) = C(s) + H(s)C(s)\, G(s)$$

$$R(s)\, G(s) = C(s).[1 + G(s)\, H(s)]$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{[1 + G(s)\, H(s)]}}$$

For Negative Feedback

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{[1 - G(s)\, H(s)]}}$$

For Positive Feedback

Advantages

- The closed-loop system is more accurate than the open-loop system because of controlling through the output signal.
- These types of systems are less affected by noise and other environmental disturbances.
- It provides a high-frequency range of operation.
- These are more flexible as compared to the open-loop system.

Disadvantages

- The addition of the feedback elements leads to the generation of complex structures.
- Closed-loop systems are not economical.

- The problem of instability in output is a crucial factor of the closed-loop system as the presence of feedback causes timely variation in the system's output.

3.4 Types of Controllers

Let us classify the controllers. There are mainly two **types of controllers** and they are written below: **Continuous Controllers:** The main feature of continuous controllers is that the controlled variable (also known as the manipulated variable) can have any value within the range of controller's output. Now in the continuous controller's theory, there are three basic modes on which the whole control action takes place and these modes are written below. We will use the combination of these modes in order to have a desired and accurate output.

1. **Proportional controllers**.
2. **Integral controllers**.
3. **Derivative controllers**.

Combinations of these three controllers are written below:

4. Proportional and integral controllers.
5. Proportional and derivative controllers.

Now we will discuss each of these modes in detail.

Proportional Controllers

We cannot use **types of controllers** at anywhere, with each type controller, there are certain conditions that must be fulfilled. With **proportional controllers** there are two conditions and these are written below:

Deviation should not be large, it means there should be less deviation between the input and output.

Deviation should not be sudden.

Now we are in a condition to discuss proportional controllers, as the name suggests in a proportional controller the output (also called the actuating signal) is directly proportional to the error signal. Now let us analyze proportional controller mathematically. As we know in proportional controller output is directly proportional to error signal, writing this mathematically we have,

$$A(t) \propto e(t)$$

Removing the sign of proportionality we have,

$$A(t) = K_p \times e(t)$$

Where K_p is proportional constant also known as controller gain. It is recommended that K_p should be kept greater than unity. If the value of K_p is greater than unity, then it will amplify the error signal and thus the amplified error signal can be detected easily.

Advantages of Proportional Controller

Now let us discuss some advantages of proportional controller.

Proportional controller helps in reducing the steady state error, thus makes the system more stable.

Slow response of the over damped system can be made faster with the help of these controllers.

Disadvantages of Proportional Controller

Now there are some serious disadvantages of these controllers and these are written as follows:

1. Due to presence of these controllers we some offsets in the system.
2. Proportional controllers also increase the maximum overshoot of the system.

Integral Controllers

As the name suggests in **integral controllers** the output (also called the actuating signal) is directly proportional to the integral of the error signal. Now let us analyze integral controller mathematically. As we know in an integral controller output is directly proportional to the integration of the error signal, writing this

$$A(t) \propto \int_0^t e(t)dt$$

mathematically we have,Removing the sign of proportionality we have,

$$A(t) = K_i \times \int_0^t e(t)dt$$

Where K_i is integral constant also known as controller gain. Integral controller is also known as reset controller.

Advantages of Integral Controller

Due to their unique ability they can return the controlled variable back to the exact set point following a disturbance that’s why these are known as reset controllers.

Disadvantages of Integral Controller

It tends to make the system unstable because it responds slowly towards the produced error.

Derivative Controllers

We never use **derivative controllers** alone. It should be used in combinations with other modes of controllers because of its few disadvantages which are written below:

1. It never improves the steady state error.
2. It produces saturation effects and also amplifies the noise signals produced in the system.

Now, as the name suggests in a derivative controller the output (also called the actuating signal) is directly proportional to the derivative of the error signal. Now let us analyze derivative controller mathematically. As we know in a derivative controller output is directly proportional to the derivative of the error signal, writing this mathematically we have, $A(t) \propto \frac{de(t)}{dt}$

Removing the sign of proportionality we have,

$$A(t) = K_d \times \frac{de(t)}{dt}$$

Where K_d is proportional constant also known as controller gain. Derivative controller is also known as rate controller.

Advantages of Derivative Controller

The major advantage of derivative controller is that it improves the transient response of the system.

Proportional and Integral Controller

As the name suggests it is a combination of proportional and an integral controller the output (also called the actuating signal) is equal to the summation of proportional and integral of the error signal. Now let us analyze proportional and integral controller mathematically. As we know in a proportional and integral controller output is directly proportional to the summation of proportional of error

$$A(t) \propto \int_0^t e(t)dt + A(t) \propto e(t)$$

and integration of the error signal, writing this mathematically we have,
Removing the sign of proportionality we have,

$$A(t) = K_i \int_0^t e(t)dt + K_p e(t)$$

Where K_i and k_p proportional constant and integral constant respectively.

Advantages and disadvantages are the combinations of the advantages and disadvantages of proportional and integral controllers.

Proportional and Derivative Controller

As the name suggests it is a combination of proportional and a derivative controller the output (also called the actuating signal) is equals to the summation of proportional and derivative of the error signal. Now let us analyze proportional and derivative controller mathematically. As we know in a proportional and derivative controller output is directly proportional to summation of proportional of error and

$$A(t) \propto \frac{de(t)}{dt} + A(t) \propto e(t)$$

differentiation of the error signal, writing this mathematically we have, Removing the sign of proportionality we have,

$$A(t) = K_d \frac{de(t)}{dt} + K_p e(t)$$

Where K_d and k_p proportional constant and derivative constant respectively.

Advantages and disadvantages are the combinations of advantages and disadvantages of proportional and derivative controllers

PID control

PID control stands for proportional plus derivative plus integral control. PID control is a feedback mechanism which is used in control system. This type of control is also termed as three term control. By controlling the three parameters - proportional, integral and derivative we can achieve different control actions for specific work. PID is considered to be the best controller in the control system family. Nicholas Minorsky published the theoretical analysis paper on PID controller. For PID control the actuating signal consists of proportional error signal added with derivative and integral of the error signal. Therefore, the actuating signal for PID control is –

$$e_a(t) = e(t) + T_d \frac{de(t)}{dt} + K_i \int e(t)dt$$

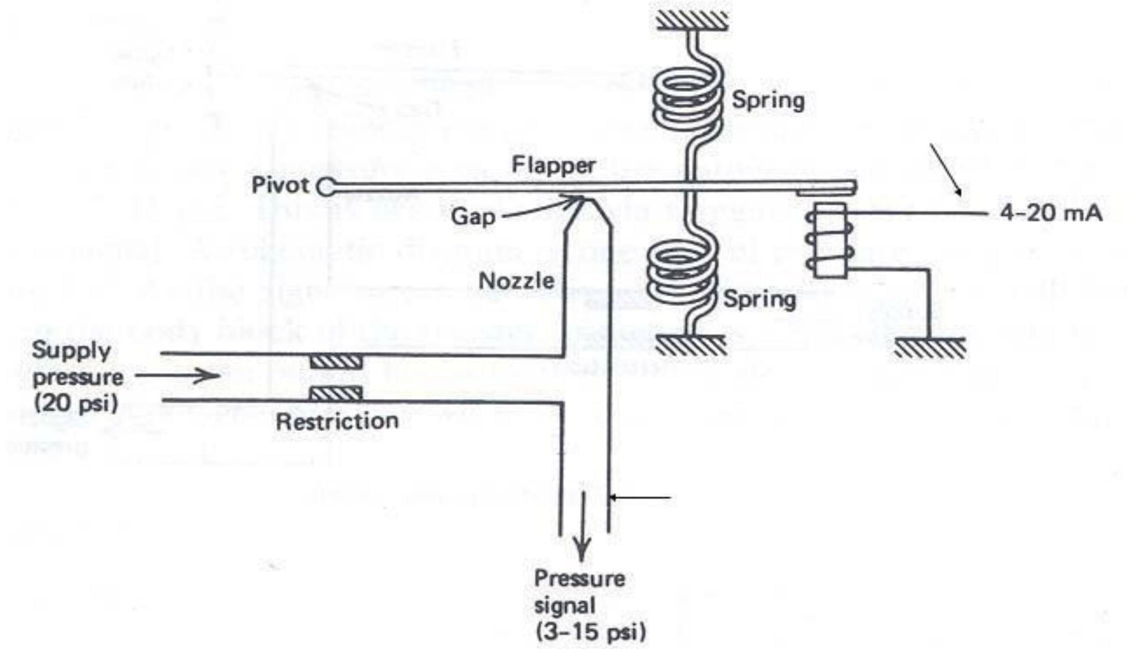
The Laplace transform of the actuating signal incorporating PID control is

$$E_a(s) = E(s) + sT_d E(s) + \frac{K_i}{s} E(s)$$

$$or,\ E_a(s) = E(s)\left[1 + sT_d + \frac{K_i}{s}\right]$$

3.5 Pneumatic controllers

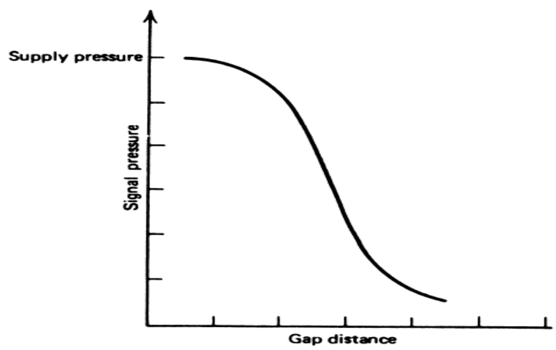
Flapper nozzle arrangement



The 4-to-20 milliamp range for the electronic control system, the pneumatic system has a range, 3 to 15 pounds per square inch (psi).

- A regulated supply of pressure, usually 20 psi, provides a source of air through the restriction. The nozzle is open at the end where the gap exists between the nozzle and flapper, and air escapes in this region.
- If the flapper moves down and closes off the nozzle opening so that no air leaks, the signal pressure will rise to the supply pressure. As the flapper moves away, the signal pressure will drop because of leaking air.

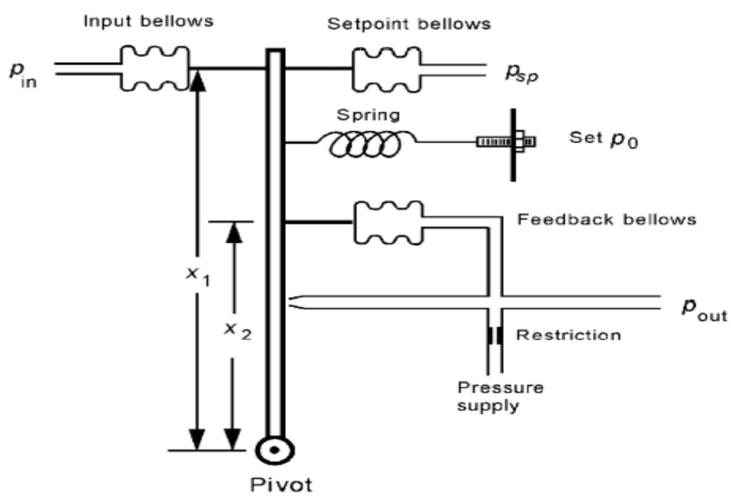
- Finally, when the flapper is far away, the pressure will stabilize at some value determined by the maximum leak through the nozzle.
- The pneumatic PID controller can go proportional only (P), proportional-derivative (PD), proportional-integral (PI), or proportional-integral-derivative (PID).



Signal pressure versus gap distance

Proportional Mode

A proportional mode of operation can be achieved with the system shown in Figure



- Operation is understood by noting that if the input pressure increases, then the input bellows forces the flapper to rotate to close off the nozzle When this happens, the output pressure increases so that the feedback bellows exerts a force to balance that of the input bellows. A balance condition then occurs when torques exerted by each about the pivot are equal, or

$$(p_{out} - p_0)A_2x_2 = (p_{in} - p_{sp})A_1x_1$$

This equation is solved to find the output pressure

$$p_{out} = \frac{x_1}{x_2}\frac{A_1}{A_2}(p_{in} - p_{sp}) + p_0$$

where
p_0 = pressure with no error
p_{in} = input pressure (Pa)
A_1 = input and setpoint bellows effective area (m^2)
x_1 = level arm of input (m)
p_{out} = output pressure (Pa)
A_2 = feedback bellows effective area (m^2)
x_2 = feedback lever arm (m)
p_{sp} = setpoint pressure

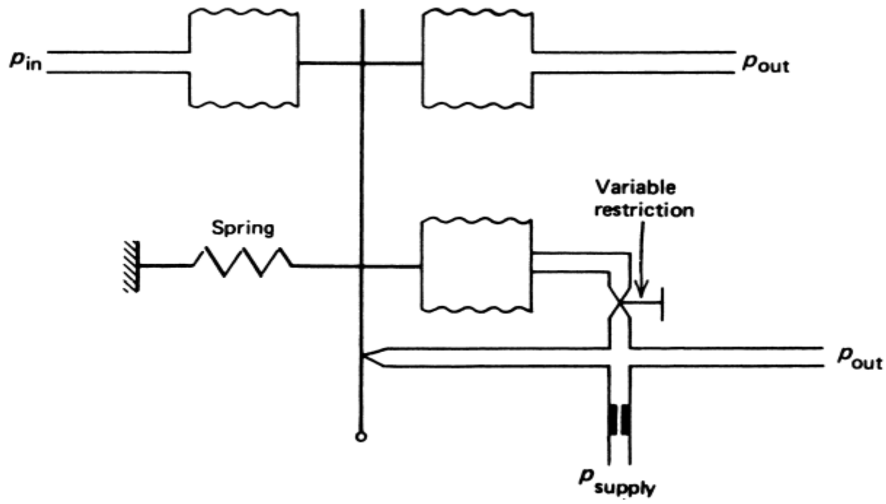
- This relation is based on the notion of torque equaling force times lever arm, and the fact that a pressure in a bellows produces a force that is effectively the pressure times bellows area, much like a diaphragm.
- Equation displays the standard response of a proportional mode in that output is directly proportional to input. The gain in this case is given by

$$K_p = \left(\frac{x_1}{x_2}\right)\left(\frac{A_1}{A_2}\right)$$

- Because the bellows are usually of fixed geometry, the gain is varied by changing the lever arm length.
- In this simple representation, the gain is established by the distance between the bellows.
- If this separation is changed, the forces are no longer balanced, and for the same pressure a new controller output will be formed, corresponding to the new gain.

Proportional-Derivative

- This controller action can be accomplished pneumatically by the method shown in Figure

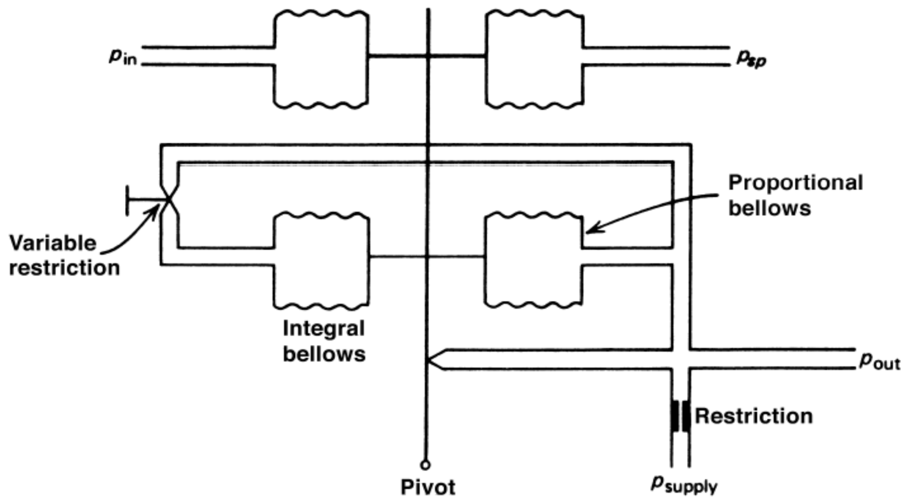


- A variable restriction is placed on the line leading to the balance bellows.
- Thus, as the input pressure increases, the flapper is moved toward the nozzle with no impedance, because the restrictions prevent an immediate response of the balance bellows.
- Thus, the output pressure rises very fast and then, as the increased pressure leaks into the balance bellows, decreases as the balance bellows moves the flapper back away from the nozzle.
- Adjustment of the variable restriction allows for changing the derivative time constraint.

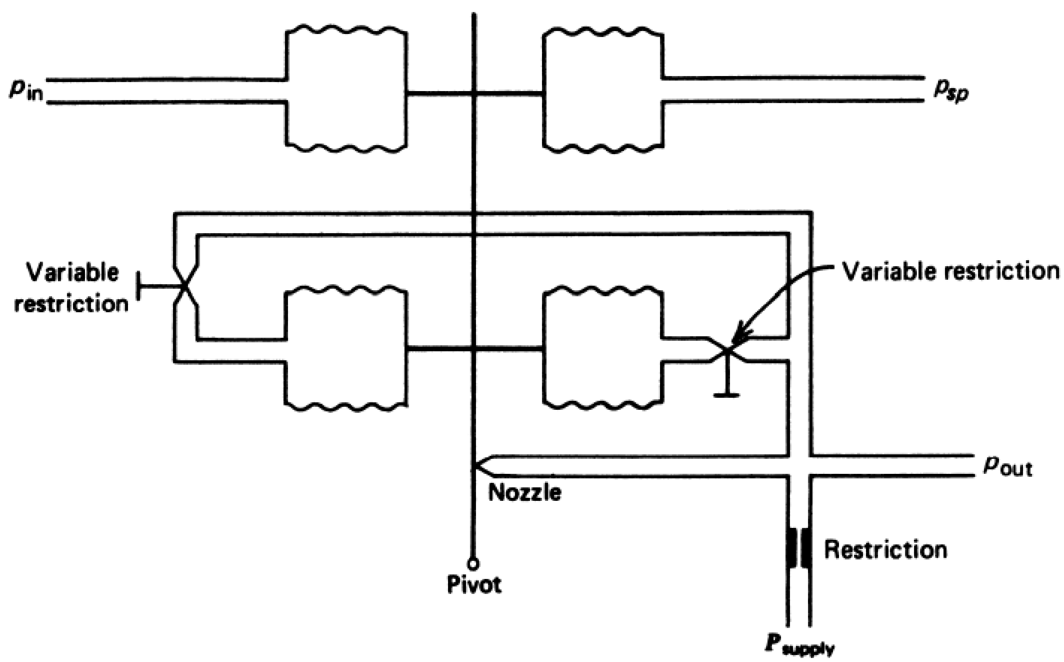
Proportional-Integral

- This control mode is also implemented using pneumatics by the system shown in Figure
- In this case, an extra bellows with a variable restriction is added to the proportional system.
- Suppose the input pressure shows a sudden increase.
- This drives the flapper toward the nozzle, increasing output pressure until the proportional bellows balances the input as in the previous case.
- The integral bellows is still at the original output pressure, because the restriction prevents pressure changes from being transmitted immediately.
- As the increased pressure on the output bleeds through the restriction, the integral bellows slowly moves the flapper closer to the nozzle, thereby causing a steady increase in output pressure (as dictated by the integral mode).

- The variable restriction allows for variation of the leakage rate, and hence the integration time.



Pneumatic three-mode (PID) controller



- The three-mode controller is actually the most common type produced, because it can be used to accomplish any of the previous modes by setting of restrictions.
- This device is shown in Figure and, as can be seen, it is simply a combination of the three systems presented.

- By opening or closing restrictions, the three-mode controller can be used to implement the other composite modes.
- Proportional gain, reset time, and rate are set by adjustment of bellows separation and restriction size.

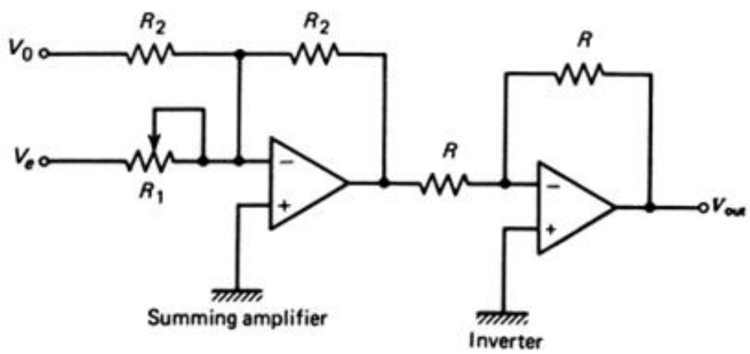
3.6 ELECTRONIC CONTROLLERS

Proportional Mode

- Implementation of this mode requires a circuit that has a response given by

$$p = K_P e_p + p_0$$

where
p = controller output 0–100%
K_P = proportional gain
e_p = error in percent of variable range
p_0 = controller output with no error



$$V_{out} = \left(\frac{R_2}{R_1}V_e + \frac{\cancel{R_2}}{\cancel{R_2}}V_o\right)$$

$$V_{out} = \left(\frac{R_2}{R_1}V_e + V_o\right)$$

$$V_{out} = G_P V_e + V_0$$

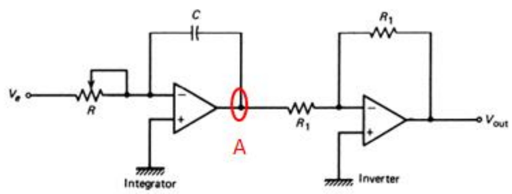
where
V_{out} = output voltage
$G_P = R_2/R_1$ = gain
V_e = error voltage
V_0 = output with zero error

Integral Mode

the integral mode was characterized by

$$p(t) = K_I \int_0^t e_p \, dt + p(0)$$

where $p(t)$ = controller output in percent of full scale
K_I = integration gain (s^{-1})
e_p = deviations in percent of full-scale variable value
$p(0)$ = controller output at $t = 0$



$$A = -\frac{1}{RC}\int V_e$$

$$V_{out} = G_I \int_0^t V_e \, dt + V_{out}(0)$$

$$G_I = \frac{K_I \,\%\, of\ output\ range\ for\ sec}{1\%\ of\ input\ range}$$

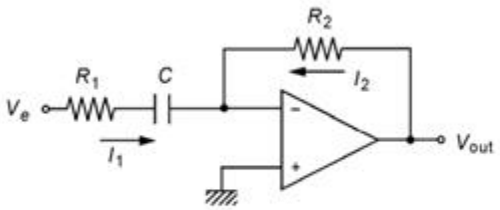
V_{out} = output voltage
$G_I = 1/RC$ = integration gain
V_e = error voltage
$V_{out}(0)$ = initial output voltage

Derivative Mode

- The derivative mode is never used alone because it cannot provide a controller output when the error is zero.
- It is implemented with op amps so it can be combined with other modes. The control mode equation was given earlier as

$$p(t) = K_D \frac{de_p}{dt}$$

where p = controller output in percent of full output
K_D = derivative time constant (s)
e_p = error in percent of full-scale range



$$V_{out} = -RC\frac{dV_e}{dt}$$

$$V_{out} + R_1C\frac{dV_{out}}{dt} = -R_2C\frac{dV_e}{dt}$$

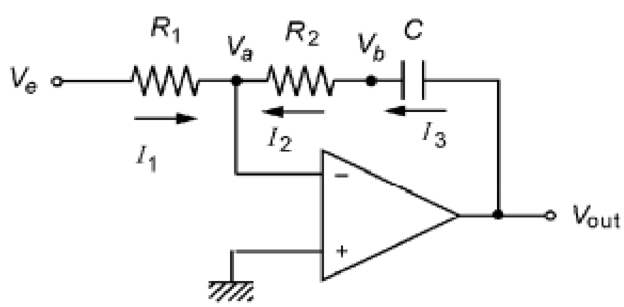
$$G_D = R_2C.$$

$$G_D = \frac{K_D\,\%\ of\ output\ range}{1\%\ of\ input\ range\ per\ second}$$

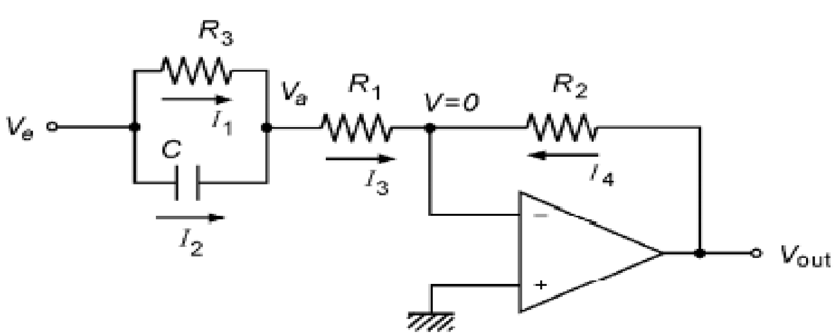
Proportional-Integral

In order to solve, we integrate this equation to eliminate the derivative on V_{out}. After some algebra and solving for V_{out}, the result is

$$V_{out} = -\frac{R_2}{R_1}V_e - \left(\frac{R_2}{R_1}\right)\frac{1}{R_2C}\int_0^t V_e\,dt \;+ V(0)$$



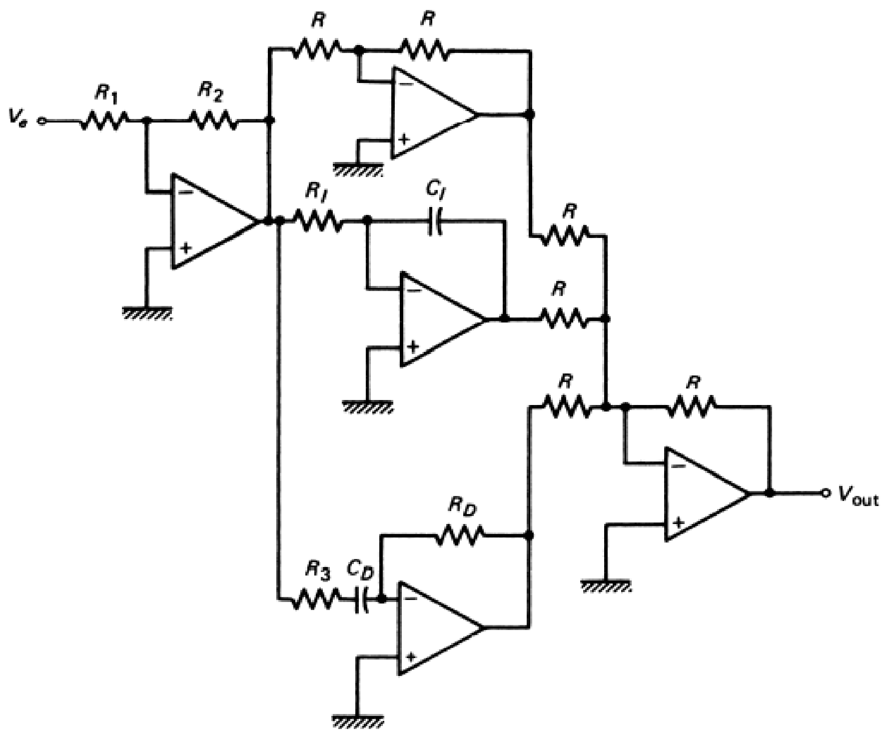
PD-MODE



After rearranging and some more algebra, this reduces to the equation

$$V_{\text{out}} + \frac{R_1}{R_1 + R_3} R_3 C \frac{dV_{\text{out}}}{dt} = -\frac{R_2}{R_1 + R_3} V_e - \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_e}{dt}$$

PID (Three-Mode)

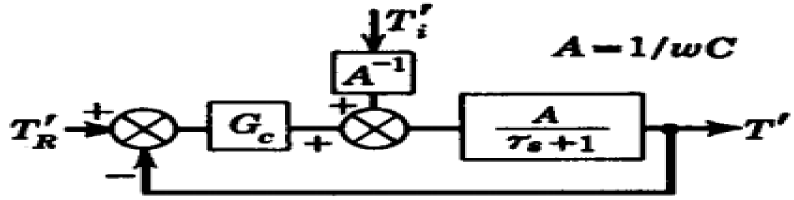


$$-V_{\text{out}} = \left(\frac{R_2}{R_1}\right) V_e + \left(\frac{R_2}{R_1}\right) \frac{1}{R_I C_I} \int V_e \, dt + \left(\frac{R_2}{R_1}\right) R_D C_D \frac{dV_e}{dt} + V_{\text{out}}(0)$$

where R_3 has been chosen from $2\pi f_{\max} R_3 C_D = 0.1$ for stability.

$$G_p = \frac{R_2}{R_1}, \qquad G_D = R_D C_D, \qquad G_I = \frac{1}{R_I C_I}$$

3.7 Transient response of closed loop control systems



Proportional Control for Set-Point Change (Servo Problem)

- For proportional control, $G_c = K_c$

$$\frac{T'}{T_R'} = \frac{K_cA/(\tau s + 1)}{1 + K_cA/(\tau s + 1)} = \frac{K_cA}{\tau s + 1 + K_cA}$$

This may be rearranged in the form of a first-order lag to give

$$\frac{T'}{T_R'} = \frac{A_1}{\tau_1 s + 1}$$

where $\tau_1 = \frac{\tau}{1 + K_cA}$

$$A_1 = \frac{K_cA}{1 + K_cA} = \frac{1}{1 + 1/K_cA}$$

Proportional Control for Load Change (Regulator Problem)

$$\frac{T'}{T_i'} = \frac{AA^{-1}/(\tau s + 1)}{1 + K_cA/(\tau s + 1)} = \frac{1}{\tau s + 1 + K_cA}$$

This may be arranged in the form of the first-order lag; thus

$$\frac{T'}{T_i'} = \frac{A_2}{\tau_1 s + 1}$$

where $A_2 = \frac{1}{1 + K_cA}$

$$\tau_1 = \frac{\tau}{1 + K_cA}$$

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DEPARTMENT OF CHEMICAL ENGINEERING

UNIT - IV-INSTRUMENTATION AND PROCESS CONTROL – SCHA1503

IV Stability Analysis

Stability, Stability criterion Routh test for stability ; Routh-Hurwitz criterion, Root-Locus analysis. Introduction to frequency response of closed-loop systems, Frequency response Substitution rule Bode diagrams Bode stability criterion. Gain and Phase margins Ziegler Nichols control settings and Transient response Nyquist diagram

4.1 Routh Hurwitz Stability Criterion.

After reading the theory of network synthesis, we can easily say that any pole of the system lies on the right hand side of the origin of the s plane, it makes the system unstable. On the basis of this condition A. Hurwitz and E.J.Routh started investigating the necessary and sufficient conditions of stability of a system. We will discuss two criteria for stability of the system. A first criterion is given by A. Hurwitz and this criterion is also known as **Hurwitz Criterion for stability** or **Routh Hurwitz Stability Criterion**.

Hurwitz Criterion

With the help of characteristic equation, we will make a number of Hurwitz determinants in order to find out the stability of the system. We define characteristic equation of the system as

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots\cdots\cdots\cdots\cdots a_{n-1} s^1 + a_n$$

Now there are n determinants for n^{th} order characteristic equation. Let us see how we can write determinants from the coefficients of the characteristic equation. The step by step procedure for k^{th} order characteristic equation is written below: Determinant one : The value of this determinant is given by |a1| where a1 is the coefficient of s^{n-1} in the characteristic equation. Determinant two : The value of this determinant is given by

$$\begin{bmatrix} a_1 & a_3 \\ a_0 & a_2 \end{bmatrix}$$

Here number of elements in each row is equal to determinant number and we have determinant number here is two. The first row consists of first two odd coefficients and second row consists of first two even coefficients. Determinant three : The value of this determinant is given by

$$\begin{bmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{bmatrix}$$

Here number of elements in each row is equal to determinant number and we have determinant number here is three. The first row consists of first three odd coefficients, second row consists of first three even coefficients and third row consists of first element as zero and rest of two elements as first two odd coefficients. Determinant four: The value of this determinant is given by,

$$\begin{bmatrix} a_1 & a_3 & a_5 & a_7 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{bmatrix}$$

Here number of elements in each row is equal to determinant number and we have determinant number here is four. The first row consists of first three four coefficients, second row consists of first four even coefficients, third row consists of first element as zero and rest of three elements as first three odd coefficients the fourth row consists of first element as zero and rest of three elements as first three even coefficients. By following the same procedure we can generalize the determinant formation. The general form of determinant is given below:

$$\begin{bmatrix} a_1 & a_3 & a_5 & a_7 & \cdot & \cdot & \cdot & a_{2k-1} \\ a_0 & a_2 & a_4 & a_6 & \cdot & \cdot & \cdot & a_{2k-2} \\ 0 & a_1 & a_3 & a_5 & \cdot & \cdot & \cdot & a_{2k-3} \\ 0 & a_0 & a_2 & a_4 & \cdot & \cdot & \cdot & a_{2k-4} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & ak \end{bmatrix}$$

Now in order to check the stability of the above system, calculate the value of each determinant. The system will be stable if and only if the value of each determinant is greater than zero, i.e. the value of each determinant should be positive. In all the other cases the system will not be stable.

Routh Stability Criterion

This criterion is also known as modified Hurwitz Criterion of stability of the system. We will study this criterion in two parts. Part one will cover necessary condition for stability of the system and part two will cover the sufficient condition for the stability of the system. Let us again consider the characteristic equation of the system as

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots\cdots\cdots\cdots a_{n-1} s^1 + a_n$$

1) Part one (necessary condition for stability of the system): In this we have two conditions which are written below: (a) All the coefficients of the characteristic equation should be positive and real. (b) All the coefficients of the characteristic equation should be non zero. 2)Part two (sufficient condition for stability of the system): Let us first construct routh array. In order to construct the routh array follow these steps: (a) The first row will consist of all the even terms of the characteristic equation. Arrange them from first (even

term) to last (even term). The first row is written below: a_0 a_2 a_4 a_6............ (b) The second row will consist of all the odd terms of the

characteristic equation. Arrange them from first (odd term) to last (odd term). The first row is written below: a_1 a_3 a_5 a_7 (c) The elements of third row can be calculated as:

(1) First element : Multiply a0 with the diagonally opposite element of next column (i.e. a3) then subtract this from the product of a_1 and a_2 (where a_2 is diagonally opposite element of next column) and then finally divide the result so obtain with a_1. Mathematically we write as first element

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

(2) Second element : Multiply a_0 with the diagonally opposite element of next to next column (i.e. a_5) then subtract this from the product of a_1 and a_4 (where a_4 is diagonally opposite element of next to next column) and then finally divide the result so obtain with a_1. Mathematically we write as second element

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

Similarly, we can calculate all the elements of the third row. (d) The elements of fourth row can be calculated by using the following procedure: **(1) First element :** Multiply b_1 with the diagonally opposite element of next column (i.e. a_3) then subtract this from the product of a_1 and b_2 (where b_2 is diagonally opposite element of next column) and then finally divide the result so obtain with b_1. Mathematically we write as first element

$$c_1 = \frac{a_1 b_2 - b_1 a_3}{b_1}$$

(2) Second element : Multiply b_1 with the diagonally opposite element of next to next column (i.e. a_5) then subtract this from the product of a_1 and b_3 (where b_3 is diagonally opposite element of next to next column) and then finally divide the result so obtain with a_1. Mathematically we write as second element

$$c_2 = \frac{a_1 b_3 - b_1 a_5}{b_1}$$

Similarly, we can calculate all the elements of the fourth row. Similarly, we can calculate all the elements of all the rows. Stability criteria if all the elements of the first column are positive then the system will be stable. However if anyone of them is negative the system will be unstable. Now there are some special cases related to Routh Stability Criteria which are discussed below:

(1) Case one: If the first term in any row of the array is zero while the rest of the row has at least one non zero term. In this case we will assume a very small value (ε) which is tending to zero in place of zero. By replacing zero with (ε) we will calculate all the elements of the Routh array. After calculating all the elements we will apply the limit at each element containing (ε). On solving the limit at every element if we will get positive limiting value then we will say the given system is stable otherwise in all the other condition we will say the given system is not stable.

(2) Case second : When all the elements of any row of the Routh array are zero. In this case we can say the system has the symptoms of marginal stability. Let us first understand the physical meaning of having all the elements zero of any row. The physical meaning is that there are symmetrically located roots of the characteristic equation in the s plane. Now in order to find out the stability in this case we will first find out auxiliary equation. Auxiliary equation can be formed by using the elements of the row just above the row of zeros in the Routh array. After finding the auxiliary equation we will differentiate the auxiliary equation to obtain elements of the zero row. If there is no sign change in the new routh array formed by using auxiliary equation, then in this we say the given system is limited stable. While in all the other cases we will say the given system is unstable.

4.2 Root-Locus analysis

The root locus technique was introduced by **W.R.Evans** in 1948 for the analysis of control systems. The root locus technique is a powerful tool for adjusting the location of closed loop poles to achieve the desired system performance by varying one or more system parameters.

Consider the open loop transfer function of system $G(s) = \dfrac{K}{s\,(s+p_1)\,(s+p_2)}$

The closed loop transfer function of the system with unity feedback is given by,

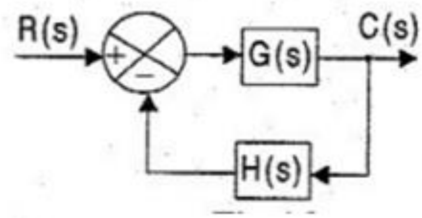
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K}{s\,(s+p_1)\,(s+p_2)}}{1+\dfrac{K}{s\,(s+p_1)\,(s+p_2)}} = \frac{K}{s\,(s+p_1)\,(s+p_2)+K}$$

The denominator polynomial of C(s)/R(s) is the characteristic equation of the system. The characteristic equation is given by,

$$s\,(s+p_1)\,(s+p_2)+K=0\,.$$

For the single loop system shown in fig

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$



The Characteristic equation is,

$$1+G(s)\,H(s) = 0$$

$$\therefore\ G(s)\,H(s) = -1$$

The equation can be converted to two Evans conditions given below,

$$|G(s)H(s)| = 1 \qquad \text{magnitude criterion}$$

$$\angle G(s)H(s) = \pm 180^{\circ}(2q+1), \qquad \text{where } q = 0,1,2,3,\ \ldots.. \qquad \text{angle criterion.}$$

The magnitude criterion states that $s = s_a$ will be a point on root locus if for that value of s,

$|G(s)\ H(s)| = 1$.

RULES FOR CONSTRUCTION OF ROOT LOCUS

Rule 1 : The root locus is symmetrical about the real axis.

Rule 2 : Each branch of the root locus originates from an open-loop pole corresponding to $K = 0$ and terminates at either on a finite open loop zero (or open loop zero at infinity) corresponding to $K = \infty$. The number of branches of the root locus terminating on infinity is equal to $n-m$, (i.e., the number of open loop poles minus the number of finite zeros)

Rule 3 : Segments of the real axis having an odd number of real axis open-loop poles plus zeros to their right are parts of the root locus.

Rule 4 : The $n-m$ root locus branches that tend to infinity, do so along straight line asymptotes making angles with the real axis given by,

$$\phi_A = \frac{180^{\circ}(2q+1)}{n-m} \ ; \quad q = 0,1,2,\ldots..,\ n-m.$$

Rule 5 : The point of intersection of the asymptotes with the real axis is at $s = \sigma_A$ where,

$$\sigma_A = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n - m}$$

Rule 6 : The breakaway and breakin points of the root locus are determined from the roots of the equation $dK/ds = 0$. If r numbers of branches of root locus meet at a point, then they break away at an angle of $\pm 180°/r$.

Rule 7 : The angle of departure from a complex open-loop pole is given by,

$$\phi_p = \pm 180° \, (2q + 1) + \phi \, ; \qquad q = 0, 1, 2, ...$$

where ϕ is the net angle contribution at the pole by all other open loop poles and zeros. Similarly the angle of arrival at a complex open loop zero is given by,

$$\phi_z = \pm 180° \, (2q + 1) + \phi \, ; \qquad q = 0, 1, 2, ...$$

where ϕ is the net angle contribution at the zero by all other open-loop poles and zeros.

Rule 8 : The points of intersection of root locus branches with the imaginary axis can be determined by use of the Routh criterion. Alternatively they can be evaluated by letting $s = j\omega$ in the characteristic equation and equating the real part and imaginary part to zero, to solve for ω and K. The values of ω are the intersection points on imaginary axis and K is the value of gain at the intersection points.

Rule 9 : The open-loop gain K at any point $s = s_a$ on the root locus is given by,

$$K = \frac{\prod_{i=1}^{n} |s_a + p_i|}{\prod_{i=1}^{m} |s_a + z_i|} = \frac{\text{Product of vector lengths from open loop poles to the point } s_a}{\text{Product of vector lengths from open loop zeros to the point } s_a}$$

PROCEDURE FOR CONSTRUCTING ROOT LOCUS

Step 1 : Locate the poles and zeros of G(s)H(s) on the s-plane. The root locus branch starts from open loop poles and terminates at zeros.

Step 2 : Determine the root locus on real axis.

Step 3 : Determine the asymptotes of root locus branches and meeting point of asymptotes with real axis.

Step 4 : Find the breakaway and breakin points.

Step 5 : If there is a complex pole then determine the angle of departure from the complex pole. If there is a complex zero then determine the angle of arrival at the complex zero.

Step 6 : Find the points where the root loci may cross the imaginary axis.

Step 7 : Take a series of test points in the broad neighbourhood of the origin of the s-plane and adjust the test point to satisfy angle criterion. Sketch the root locus by joining the test points by smooth curve.

Step 8 : The value of gain K at any point on the locus can be determined from magnitude condition.

The value of K at a point $s = s_a$, is given by,

$$K = \frac{\text{product of length of vectors from poles to the point, } s = s_a}{\text{product of length of vectors from finite zeros to the point, } s = s_a}$$

EXPLANATION FOR THE VARIOUS STEPS IN THE PROCEDURE FOR CONSTRUCTING ROOT LOCUS

Step 1 : Location of poles and zeros

Draw the real and imaginary axis on an ordinary graph sheet and choose same scales both on real and imaginary axis.

The poles are marked by cross "**X**" and zeros are marked by small circle "**o**". The number of root locus branches is equal to number of poles of open loop transfer function. The origin of a root locus is at a pole and the end is at a zero.

Let, n = number of poles
m = number of finite zeros

Now, **m** root locus branches ends at finite zeros. The remaining **n–m** root locus branches will end at zeros at infinity.

Step 2 : Root locus on real axis

In order to determine the part of root locus on real axis, take a test point on real axis. If the total number of poles and zeros on the real axis to the right of this test point is odd number, then the test point lies on the root locus. If it is even then the test point does not lie on the root locus.

Step 3 : Angles of asymptotes and centroid

If n is number of poles and m is number of finite zeros, then $n-m$ root locus branches will terminate at zeros at infinity.

These n–m root locus branches will go along an asymptotic path and meets the asymptotes at infinity. Hence number of asymptotes is equal to number of root locus branches going to infinity. The angles of asymptotes and the centroid are given by the following formulae.

$$\text{Angles of asymptotes} = \frac{\pm 180\,(2q+1)}{n-m}$$

where, $q = 0, 1, 2, 3, \ldots\ldots(n-m)$

$$\text{Centroid (meeting point of asymptote with real axis)} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

Step 4 : Breakaway and Breakin points

The breakaway or breakin points either lie on real axis or exist as complex conjugate pairs. If there is a root locus on real axis between 2 poles then there exist a breakaway point. If there is a root locus on real axis between 2 zeros then there exist a breakin point. If there is a root locus on real axis between pole and zero then there may be or may not be breakaway or breakin point.

Let the characteristic equation be in the form,

$$B(s) + K\,A(s) = 0$$

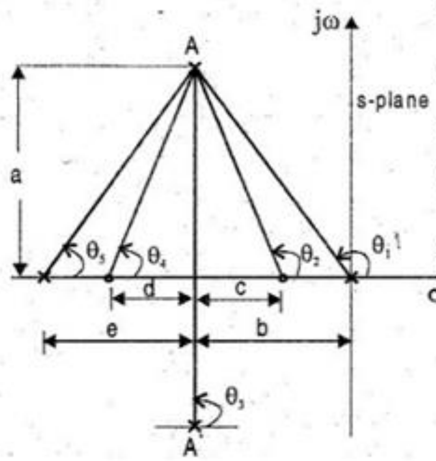
$$\therefore K = \frac{-B(s)}{A(s)}$$

The breakaway and breakin point is given by roots of the equation $dK/ds = 0$. The roots of $dK/ds = 0$ are actual breakaway or breakin point provided for this value of root, the gain K should be positive and real.

Step 5 : Angle of Departure and angle of arrival

$$\left.\begin{array}{l}\text{Angle of Departure}\\ \text{(from a complex pole A)}\end{array}\right\} = 180° - \begin{pmatrix}\text{Sum of angles of vector to the}\\ \text{complex pole A from other poles}\end{pmatrix} + \begin{pmatrix}\text{Sum of angles of vectors to the}\\ \text{complex pole A from zeros}\end{pmatrix}$$

Note : *The angles can be calculated as shown in fig 4.9 or they can be measured using protractor.*



$$\theta_1 = 180° - \tan^{-1}\frac{a}{b}$$

$$\theta_2 = 180° - \tan^{-1}\frac{a}{c}$$

$$\theta_3 = 90°$$

$$\theta_4 = \tan^{-1}\frac{a}{d}$$

$$\theta_5 = \tan^{-1}\frac{a}{e}$$

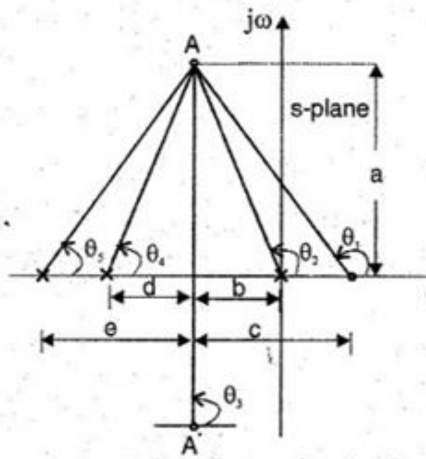
Example:

Consider the two complex conjugate poles A and A* shown in fig 4.9.(If poles are complex then they exist only as conjugate pairs)

$$\left.\begin{array}{c}\text{Angle of departure}\\ \text{at pole A}\end{array}\right\} = 180° - (\theta_1 + \theta_3 + \theta_5) + (\theta_2 + \theta_4)$$

$$\left.\begin{array}{c}\text{Angle of departure}\\ \text{at pole A *}\end{array}\right\} = -[\text{Angle of departure at pole A}]$$

$$\left.\begin{array}{l}\text{Angle of arrival at a}\\ \text{complex zero A}\end{array}\right\} = 180° - \begin{pmatrix}\text{Sum of angles of vectors to the}\\ \text{complex zero A from all other zeros}\end{pmatrix} + \begin{pmatrix}\text{Sum of angles of vectors to the}\\ \text{complex zero A from poles}\end{pmatrix}$$



$$\theta_1 = 180° - \tan^{-1}\frac{a}{b}$$

$$\theta_2 = 180° - \tan^{-1}\frac{a}{c}$$

$$\theta_3 = 90°$$

$$\theta_4 = \tan^{-1}\frac{a}{d}$$

$$\theta_5 = \tan^{-1}\frac{a}{e}$$

Calculation of angle of arrival

Example:

Consider the two complex conjugate zeros B and B* as shown in fig 4.10.(If zeros are complex then they exist only as conjugate pairs)

$$\left.\begin{matrix}\text{Angle of arrival}\\ \text{at zero B}\end{matrix}\right\} = 180° - (\theta_1 + \theta_3) + (\theta_2 + \theta_4 + \theta_5)$$

$$\left.\begin{matrix}\text{Angle of arrival}\\ \text{at zero B*}\end{matrix}\right\} = -[\text{Angle of arrival at zero B}]$$

Step 6 : Point of intersection of root locus with imaginary axis

The point where the root loci intersects the imaginary axis can be found by following three methods.

1. By Routh Hurwitz array.
2. By trial and error approach.
3. Letting $s = j\omega$ in the characteristic equation and separate the real part and imaginary part. Two equations are obtained : *one by equating real part to zero and the other by equating imaginary part to zero.* Solve the two equations for ω and **K**. The values of ω gives the points where the root locus crosses imaginary axis. The value of K gives the value of gain K at there crossing points. Also this value of K is the limiting value of K for stability of the system.

Step 7 : Test points and root locus

Choose a test point. Using a protractor roughly estimate the angles of vectors drawn to this point and adjust the point to satisfy angle criterion. Repeat the procedure for few more test points. Sketch the root locus from the knowledge of typical sketches and the informations obtained in steps 1 through 6.

4.3 BODE PLOT

The Bode plot is a frequency response plot of the sinusoidal transfer function of a system. A Bode plot consists of two graphs. One is a plot of the magnitude of a sinusoidal transfer function versus log ω. The other is a plot of the phase angle of a sinusoidal transfer function versus log ω.

The Bode plot can be drawn for both open loop and closed loop system. Usually the bode plot is drawn for open loop system. The standard representation of the logarithmic magnitude of open loop transfer function of $G(j\omega)$ is $20 \log |G(j\omega)|$ where the base of the logarithm is 10. The unit used in this representation of the magnitude is the decibel,usually abbreviated as db. The curves are drawn on semilog paper, using the log scale (abcissa) for frequency and the linear scale (ordinate) for either magnitude (in decibels) or phase angle (in degrees).

The main advantage of the bode plot is that multiplication of magnitudes can be converted into addition. Also a simple method for sketching an approximate log-magnitude curve is available.

The step by step procedure for plotting the magnitude plot is given below

Step1 : Convert the transfer function into Bode form or time constant form. The Bode form of the transfer function is

$$G(s)=\frac{K(1+sT_1)}{s(1+sT_2)\left(1+\frac{s^2}{\omega_n^2}+2\zeta\frac{s}{\omega_n}\right)} \xrightarrow{s=j\omega} G(j\omega)=\frac{K(1+j\omega T_1)}{j\omega(1+j\omega T_2)\left(1-\frac{\omega^2}{\omega_n^2}+j2\zeta\frac{\omega}{\omega_n}\right)}$$

Step 2 : List the corner frequencies in the increasing order and prepare a table as shown below.

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec

In the above table enter K or $K/(j\omega)^n$ or $K(j\omega)^n$ as the first term and the other terms in the increasing order of corner frequencies. Then enter the corner frequency, slope contributed by each term and change in slope at every corner frequency.

Step 3 : Choose an arbitrary frequency ω_l which is lesser than the lowest corner frequency. Calculate the db magnitude of K or $K/(j\omega)^n$ or $K(j\omega)^n$ at ω_l and at the lowest corner frequency.

Step 4 : Then calculate the gain (db magnitude) at every corner frequency one by one by using the formula,

Gain at ω_y = change in gain from ω_x to ω_y + Gain at ω_x

$$= \left[\text{Slope from } \omega_x \text{ to } \omega_y \times \log\frac{\omega_y}{\omega_x}\right] + \text{Gain at } \omega_x$$

Step 5 : Choose an arbitrary frequency ω_h which is greater than the highest corner frequency. Calculate the gain at ω_h by using the formula in step 4.

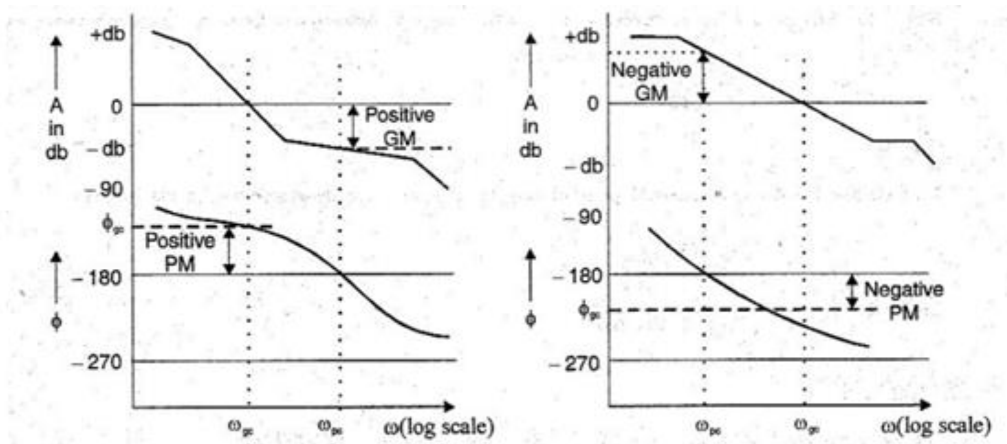
Step 6 : In a semilog graph sheet mark the required range of frequency on x-axis (log scale) and the range of db magnitude on y-axis (ordinary scale) after choosing proper units.

***Note :** The magnitude plot obtained above is an approximate plot. If an exact plot is needed then appropriate corrections should be made at every corner frequencies.*

PROCEDURE FOR PHASE PLOT OF BODE PLOT

The phase plot is an exact plot and no approximations are made while drawing the phase plot. Hence the exact phase angles of $G(j\omega)$ are computed for various values of ω and tabulated. The choice of frequencies are preferably the frequencies chosen for magnitude plot. Usually the magnitude plot and phase plot are drawn in a single semilog - sheet on a common frequency scale.

Take another y-axis in the graph where the magnitude plot is drawn and in this y-axis mark the desired range of phase angles after choosing proper units. From the tabulated values of ω and phase angles, mark all the points on the graph. Join the points by a smooth curve.



DETERMINATION OF GAIN MARGIN AND PHASE MARGIN FROM BODE PLOT

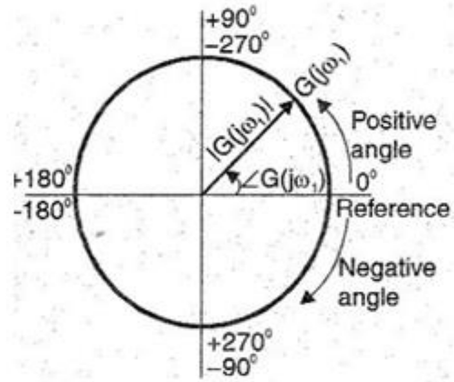
The gain margin in db is given by the negative of db magnitude of $G(j\omega)$ at the phase cross-over frequency, ω_{pc}. The ω_{pc} is the frequency at which phase of $G(j\omega)$ is -180°. If the db magnitude of $G(j\omega)$ at ω_{pc} is negative then gain margin is positive and vice versa.

Let ϕ_{gc} be the phase angle of $G(j\omega)$ at gain cross over frequency ω_{gc}. The ω_{gc} is the frequency at which the db magnitude of $G(j\omega)$ is zero. Now the phase margin, γ is given by, $\gamma = 180^\circ + \phi_{gc}$. If ϕ_{gc} is less negative than -180° then phase margin is positive and vice versa.

4.4 Nyquist diagram or Polar Plot

The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity. Thus the polar plot is the locus of vectors $|G(j\omega)| \angle G(j\omega)$ as ω is varied from zero to infinity. The polar plot is also called *Nyquist plot.*

The polar plot is usually plotted on a polar graph sheet. The polar graph sheet has concentric circles and radial lines. The circles represent the magnitude and the radial lines represent the phase angles. Each point on the polar graph has a magnitude and phase angle. The magnitude of a point is given by the value of the circle passing through that point and the phase angle is given by the radial line passing through that point. In polar graph sheet a positive phase angle is measured in anticlockwise from the reference axis (0°) and a negative angle is measured clockwise from the reference axis (0°).



In order to plot the polar plot, magnitude and phase of $G(j\omega)$ are computed for various values of ω and tabulated. Usually the choice of frequencies are corner frequencies and frequencies around corner frequencies. Choose proper scale for the magnitude circles. Fix all the points on polar graph sheet and join the points by smooth curve. Write the frequency corresponding to each point of the plot.

Alternatively, if $G(j\omega)$ can be expressed in rectangular coordinates as,

$$G(j\omega) = G_R(j\omega) + jG_I(j\omega)$$

where, $G_R(j\omega)$ = Real part of $G(j\omega)$; $G_I(j\omega)$ = Imaginary part of $G(j\omega)$.

4.5 Ziegler Nichols control

•Ziegler and Nichols also developed another method of controller setting assignment that has come to be associated with their name. This technique, also called the ultimate cycle method, is based on adjusting a closed loop until steady oscillations occur.

•Controller settings are then based on the conditions that generate the cycling.

•The particular method is accomplished through the following steps:

1. Reduce any integral and derivative actions to their minimum effect.

2. Gradually begin to increase the proportional gain while providing periodic small disturbances to the process. (These are to “jar” the system into oscillations.)

3. Note the critical gain, , at which the dynamic variable just begins to exhibit steady cycling—that is, oscillations about the setpoint.

4. Note the critical period, , of these oscillations measured in minutes. This method can be used for systems without self-regulation. Now, from the critical gain and period, the settings of the controller are assigned as follows:

- This method can be used for systems without self-regulation. Now, from the critical gain and period, the settings of the controller are assigned as follows:
- Ultimate gain $K_c = \frac{1}{M}$
- M is the amplitude ratio
- Ultimate period $T_c = \frac{2\pi}{\omega_{c0}}$
- Crossover frequency ω_{c0}

Proportional-Integral If proportional-integral action is used in the process-control loop, then the settings are determined from

$$\left.\begin{aligned} K_P &= 0.45K_c \\ T_I &= T_c/1.2 \end{aligned}\right\}$$

Three-Mode The three-mode controller requires proportional gain, integral time, and derivative time. These are determined for nominal response as

$$\left.\begin{aligned} K_P &= 0.6K_c \\ T_I &= T_c/2.0 \\ T_D &= T_c/8 \end{aligned}\right\}$$

References

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2. Nagoorkani, Control systems

SCHOOL OF BIO AND CHEMICAL

DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – V-INSTRUMENTATION AND PROCESS CONTROL – **SCHA1503**

V Control Valves & Advanced control systems

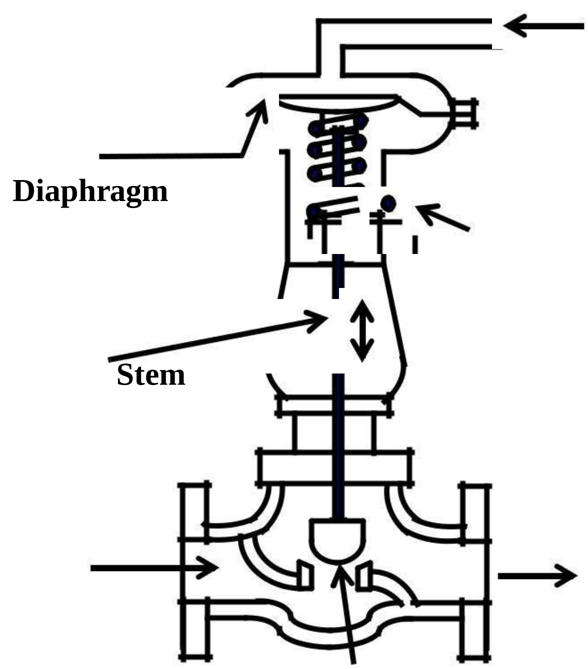
Control valves, Valve sizing Characteristics Control valve construction, Valve positioning Power unit Transducers Electric to Pneumatic and pneumatic to electric types Transfer function matrix Dead time compensation. Introduction to advanced control systems, Cascade control, Feed forward control and Feedback control.

5.1 Control Valve -Introduction

The control action in any control loop system, is executed by the final control element. The most common type of final control element used in chemical and other process control is the control valve. A control valve is normally driven by a diaphragm type pneumatic actuator that throttles the flow of the manipulating variable for obtaining the desired control action. A control valve essentially consists of a plug and a stem. The stem can be raised or lowered by air pressure and the plug changes the effective area of an orifice in the flow path. A typical control valve action can be explained using Fig. When the air pressure increases, the downward force of the diaphragm moves the stem downward against the spring.

Classifications

Control valves are available in different types and shapes. They can be classified in different ways; based on: (a) action, (b) number of plugs, and (c) flow characteristics.

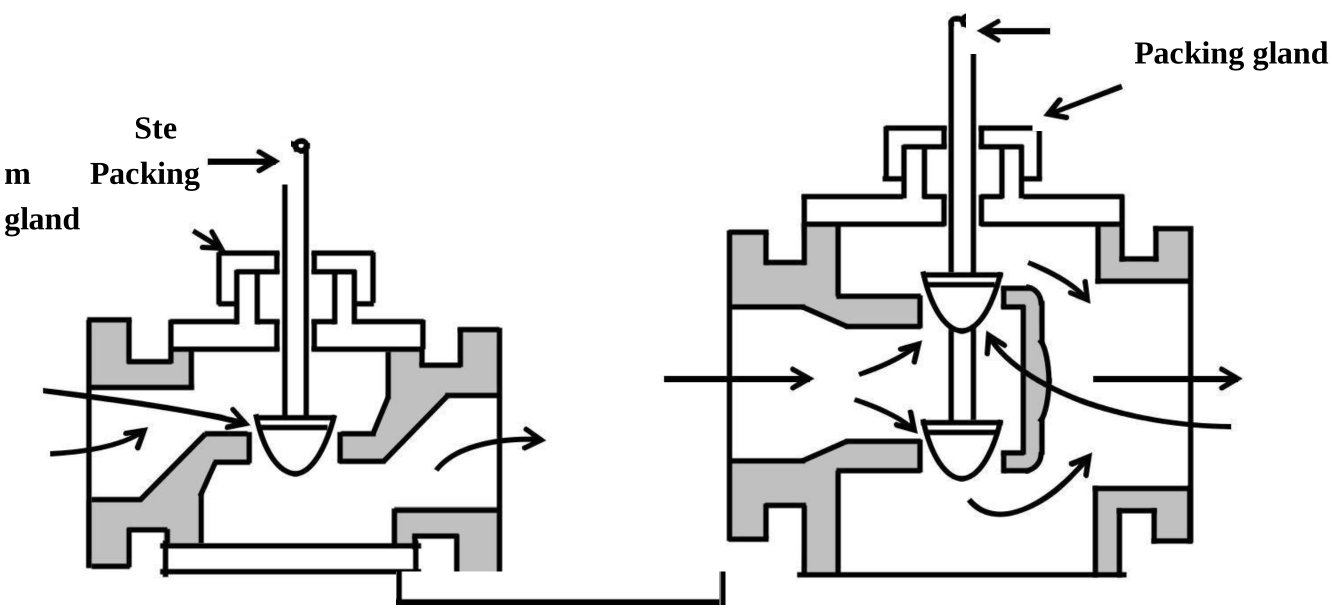


(a) Action: Control valves operated through pneumatic actuators can be either (i) air to open, or (ii) air to close. They are designed such that if the air supply fails, the control valve will be either fully open, or fully closed, depending upon the safety requirement of the process. For example, if the valve is used to control steam or fuel flow, the valve should be shut off completely in case of air failure. On the other hand, if the valve is handling cooling water to a reactor, the flow should be maximum in case of emergency. The schematic arrangements of these two actions are shown in Fig. Valve A are air to close type, indicating, if the air fails, the valve will be fully open. Opposite is the case for valve B.

(b) Number of plugs: Control valves can also be characterized in terms of the number of plugs present, as *single-seated valve* and *double-seated valve*. The difference in construction between a single seated and double-seated valve are illustrated in Fig.

Referring Fig. only one plug is present in the control valve, so it is single seated valve. The advantage of this type of valve is that, it can be fully closed and flow variation from 0 to 100% can be achieved. But looking at its construction, due to the pressure drop across the orifice a large upward force is present in the orifice area, and as a result, the force required to move the valve against this upward thrust is also large. Thus this type of valves is more suitable for small flow rates. On the other hand, there are two plugs in a double- seated valve; flow moves upward in one orifice area, and downward in the other orifice. The resultant upward or downward thrust is almost zero. As a result, the force required to move a double-seated valve is comparatively much less.

But the double-seated valve suffers from one disadvantage. The flow cannot be shut off completely, because of the differential temperature expansion of the stem and the valve seat. If one plug is tightly closed, there is usually a small gap between the other plug and its seat. Thus, single-seated valves are recommended for when the valves are required to be shut off completely. But there are many processes, where the valve used is not expected to operate near shut off position. For this condition, double-seated valves are recommended.

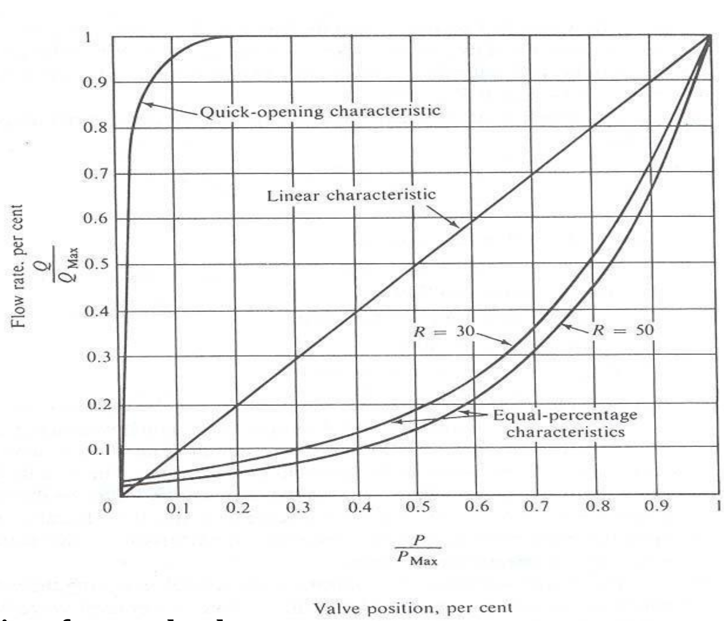


(c) Flow Characteristics: It describes how the flow rate changes with the movement or lift of the stem. The shape of the plug primarily decides the flow characteristics. However, the design of the shape of a control valve and its shape requires further discussions. The flow characteristic of a valve is normally defined in terms of (a) inherent characteristics and (b) effective characteristics. An inherent characteristic is the ideal flow characteristics of a control valve and is decided by the shape and size of the plug. On the other hand, when the

The valve is connected to a pipeline, its overall performance is decided by its effective characteristic.

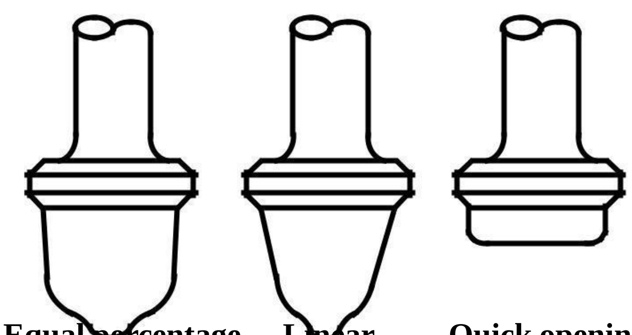
- Quick opening
- Linear
- Equal Percentage.

The characteristics of these control valves are shown in Fig. It has to be kept in mind that all the characteristics are to be determined after maintaining constant pressure difference across the valve as shown in Fig.



Flow characteristics of control valves

Different flow characteristics can be obtained by properly shaping the plugs. Typical shapes of the three types of valves are shown in Fig. 5



Equal percentage Linear Quick opening

Rangeability of a control valve is defined as the ratio of the maximum controllable flow and the minimum controllable flow. Thus:

$$\text{Rangeability} = \frac{\text{maximum controllable flow}}{\text{minimum controllable flow}}$$

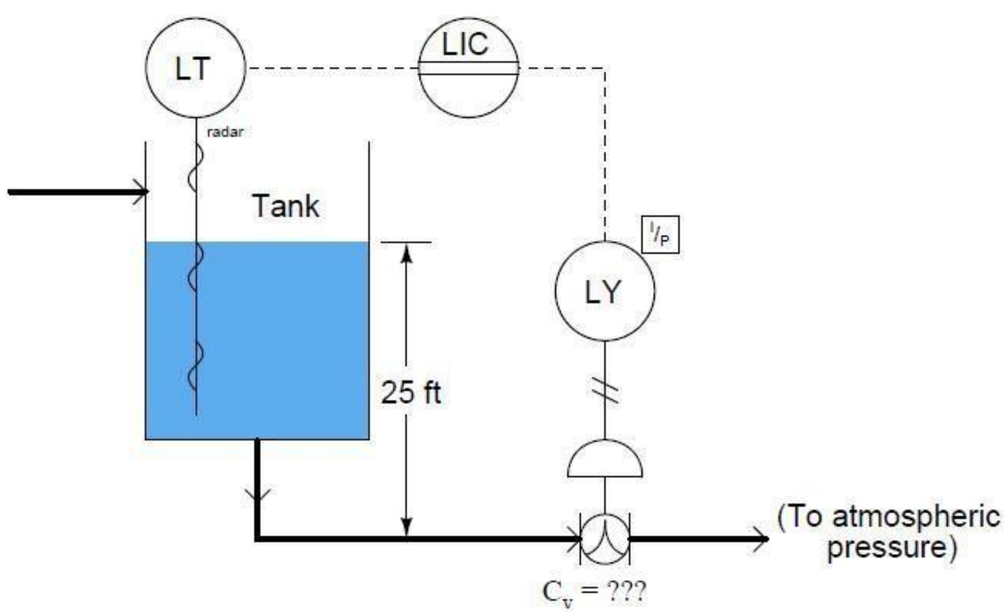
Rangeability of a control valve is normally in between 20 and 70

Importance of proper valve sizing

The flow coefficient of a control valve is a numerical value usually expressing maximum flow capacity. For example, a control valve with a Cv rating of 45 should flow 45 gallons per minute of water through it with a 1 PSI pressure drop when wide open. The flow coefficient value for this same control valve will be less than 45 when the valve position is anything less than fully open. When the control valve is in the fully shut position, its Cv value will be zero. Thus, it should be understood that Cv is truly a variable – not a constant – for any control valve, even though control valves are often specified simply by their maximum flow capacity.

It should be obvious that any control valve must be sized large enough (i.e. possess sufficient maximum Cv capacity) to flow the greatest expected flow rate in any given process installation. A valve that is too small for an application will not be able to pass enough process fluid through it when needed.

Given this fact, it may seem safe to choose a valve sized much larger than what is needed, just to avoid the possibility of not having enough flow capacity. For instance, consider this control valve sizing problem, where a characterized ball valve controls the flow rate of water out of a surge tank to maintain a constant water level 25 feet higher than the height of the valve:



According to the process engineers, the maximum expected flow rate for this valve is 470 GPM. What should the maximum Cv rating be for this valve? To begin, we must know the expected pressure drop across the valve. The 25 foot water column height upstream provides us with the means to calculate P1:

$$P = \gamma h$$

$$P_1 = (62.4 \text{ lb/ft}^3)(25 \text{ feet})$$

$$P_1 = 1560 \text{ PSF} = 10.8 \text{ PSI}$$

There is no need to calculate P2, since the P&ID shows us that the downstream side of the valve is vented to atmosphere, and is thus 0 PSI gauge pressure. This gives us a pressure drop of 10.8 PSI across the control valve, with an expected maximum flow rate of 470 GPM. Manipulating our flow capacity equation to solve for Cv:

$$Q = C_v \sqrt{\frac{P_1 - P_2}{G_f}}$$

$$C_v = \frac{Q}{\sqrt{\frac{P_1 - P_2}{G_f}}}$$

$$C_v = \frac{470 \text{ GPM}}{\sqrt{\frac{10.8 \text{ PSI}}{1}}}$$

$$C_v = 143$$

This tells us we need a control valve with a Cv value of at least 143 to meet the specified (maximum) flow rate. A valve with insufficient Cv would not be able to flow the required 470 gallons per minute of water with only 10.8 PSI of pressure drop.

In order to understand how an over-sized control valve leads to unstable control, an exaggerated example is helpful to consider: imagine installing a fire hydrant valve on your kitchen sink faucet. Certainly, a wide-open hydrant valve would allow sufficient water flow into your kitchen sink. However, most of this valve’s usable range of throttling will be limited to the first percent of stem travel. After the valve is opened just a few percent from fully shut, restrictions in the piping of your house’s water system will have limited the flow rate to its maximum, thus rendering the rest of the valve’s stem travel capacity utterly useless. It would be challenging indeed to try filling a drinking cup with water from this hydrant valve: just a little bit too much stem motion and the cup would be subjected to a full-flow stream of water!

Control valve over-sizing is a common problem in industry, often created by future planning for expanded process flow. “If we buy a large valve now,” so the reasoning goes, “we won’t have to replace a smaller valve with a large valve when the time comes to increase our production rate.” In the interim period when that larger valve must serve to control a meager flow rate, however, problems caused by poor control quality may end up costing the enterprise more than the cost of an additional valve.

Gas valve sizing

Sizing a control valve for gas or vapor service is more complicated than for liquid service, due to the compressibility of gases and vapors. As a gas or vapor compresses with changes in pressure, its density changes correspondingly. In previous mathematical analyses of fluid flow restriction, one of our assumptions was that fluid density (ρ) remained constant. This assumption may hold true for some flowing gas conditions as well, provided minimal pressure changes within the path of flow. However, for most control valve applications where the very purpose of the valve is to introduce substantial pressure changes in a fluid stream, the assumption of constant fluid density is unrealistic. Shown here is one of the simpler gas valve sizing equations you will encounter:

$$Q = 963\, C_v \sqrt{\frac{\Delta P (P_1 + P_2)}{G_g T}}$$

Where,

Q = Gas flow rate, in units of Standard Cubic Feet per Hour (SCFH) Cv = Valve capacity coefficient

ΔP = Pressure dropped across valve, pounds per square inch differential (PSID) P1 = Upstream valve pressure, pounds per square inch absolute (PSIA)

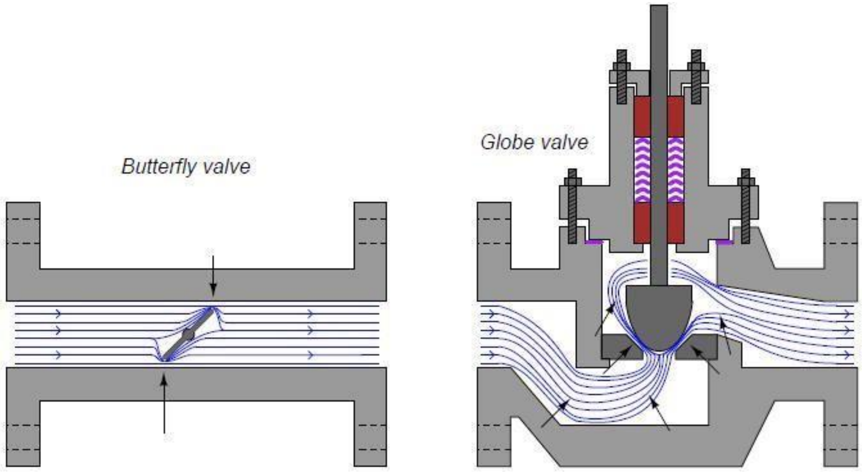
P2 = Downstream valve pressure, pounds per square inch absolute (PSIA) Gg = Specific gravity of gas (Air at standard temperature and pressure = 1.0) T = Absolute temperature of gas in degrees Rankine (oR)

Valve sizing is complicated enough, both for liquid and gas service, that the use of valve sizing computer software is strongly recommended as opposed to hand-calculations. The number of important parameters, nonlinear factors, and alternative equations relevant to control valve sizing are numerous enough to bewilder most technicians (and more than a few engineers). Valve sizing software will also predict noise levels generated by the valve, and in many cases specify actual valve trim styles offered by the manufacturer for mitigating problems such as noise.

Relative flow capacity

The flow capacity of a valve (Cv) is a quantitative rating of its ability to pass a fluid flow for a set of given pressure and density conditions. Cv may be predicted, or empirically measured, for any type of control valve given the proper information.

Not all control valve types exhibit the same Cv coefficients, however, for the same pipe size. A 4 inch butterfly valve, for example, has a much greater full-open Cv rating than a 4 inch globe valve, due to the much more direct path it offers to a moving fluid. A simple comparison of these two valve types clearly shows why this is true (note the “constriction” points labeled with arrows):



A globe valve is simply more efficient at generating fluid turbulence – and therefore dissipating fluid kinetic energy – than a butterfly valve of the same pipe size, because the globe valve design forces the fluid to change direction more often and in different ways.

One way to help quantify a particular valve design's ability to throttle fluid flow is to express this ability as a ratio of flow coefficient (Cv) versus cross-sectional pipe area. The basic principle here is that we should expect the Cv of any particular valve design to be proportional to pipe area (e.g. a ball valve with twice the pipe area should have twice the flow capacity, all other factors being equal), and therefore a ratio of these two quantities should be fairly constant for any valve design.

Since we know the area of a pipe is proportional to the square of either radius or diameter ($A = \frac{\pi d^2}{4}$4 or $A = \pi r^2$), we may simplify this ratio by omitting all constants such as π and simply relating Cv factor to the square of pipe diameter (d^2). This ratio is called the relative flow capacity, or Cd:

$$C_d = \frac{C_v}{d^2}$$

Several valve capacity factors (Cd) for different control valve types are shown here, assuming full-area trim and a full- open position:

5.2 Transducers

Basically transducer is defined as a device, which converts energy or information from one form to another. These are widely used in measurement work because not all quantities that need to be measured can be displayed as easily as others. A better measurement of a quantity can usually be made if it may be converted to another form, which is more conveniently or accurately displayed. For example, the common mercury thermometer converts variations in temperature into variations in the length of a column of mercury. Since the variation in the length of the mercury column is rather simple to measure, the mercury thermometer becomes a convenient device for measuring temperature. On the other hand, the actual temperature variation is not as easy to display directly. Another example is manometer, which detects pressure and indicates it directly on a scale calibrated in actual units of pressure.

Thus the transducer is a device, which provides a usable output in response to specific input measurand, which may be physical or mechanical quantity, property or condition. The transducer may be mechanical, electrical, magnetic, optical, chemical, acoustic, thermal nuclear, or a combination of any two or more of these.

Mechanical transducers:

Are simple and rugged in construction, cheaper in cost, accurate and operate without external power supplies but are not advantageous for many of the modern scientific experiments and process control instrumentation owing to their poor frequency response, requirement of large forces to overcome mechanical friction, in compatibility when remote control or indication is required, and a lot of other limitations. All these drawbacks have been overcome with the introduction of electrical transducers.

Electrical Transducers:

Mostly quantities to be measured are non-electrical such as temperature, pressure, displacement, humidity, fluid flow, speed etc., but these quantities cannot be measured directly. Hence such quantities are required to be sensed and changed into some other form for easy measurement.

Electrical quantities such as current, voltage, resistance inductance and capacitance etc. can be conveniently measured, transferred and stored, and therefore, for measurement of non-electrical quantities these are to be converted into electrical quantities first and then measured. The function of converting non-electrical quantity into electrical one is accomplished by a device called the electrical transducer. Basically an electrical transducer is a sensing device by which a physical, mechanical or optical quantity to be measured is transformed directly, with a suitable mechanism, into an electrical signal (current, voltage or frequency). The production of these signals is based upon electrical effects which may be resistive, inductive, capacitive etc in nature. The input versus output energy relationship takes a definite reproducible function. The output to input and the output to time behavior is predictable to a known degree of accuracy, sensitivity and response, within the specified environmental conditions.

Basic Requirements of a Transducer:

The main function of a transducer is to respond only for the measurement under specified limits for which it is designed. It is, therefore, necessary to know the relationship between the input and output quantities and it should be fixed. Transducers should meet the following basic requirements.

1. ***Ruggedness.*** It should be capable of withstanding overload and some safety arrangement should be provided for overload protection.
2. ***Linearity***. Its input-output characteristics should be linear and it should produce these characteristics in symmetrical way.
3. ***Repeatability.*** It should reproduce same output signal when the same input signal is applied again and again under fixed environmental conditions e.g. temperature, pressure, humidity etc.
4. ***High Output Signal Quality.*** The quality of output signal should be good i.e. the ratio of the signal to the noise should be high and the amplitude of the output signal should be enough.
5. ***High Reliability and Stability.*** It should give minimum error in measurement for temperature variations, vibrations and other various changes in surroundings.

6. ***Good Dynamic Response***. Its output should be faithful to input when taken as a function of time. The effect is analyzed as the frequency response.
7. ***No Hysteretic.*** It should not give any hysteretic during measurement while input signal is varied from its low value to high value and vice-versa.
8. ***Residual Deformation.*** It should be no deformation on removal of local after long period of application.

Selection of Transducers:

In a measurement system the transducer (or a combination of transducers) is the input element with the critical function of transforming some physical quantity to a proportional electrical signal. So selection of an appropriate transducer is most important for having accurate results.

The first step in the selection procedure is to clearly define the nature of quantity under measurement (measurand) and know the range of magnitudes and frequencies that the measurand is expected to exhibit. Next step will be to examine the available transducer principles for measurement of desired quantity.

The type of transducer selected must be compatible with the type and range of the quantity to be measured and the output device.

In case one or more transducer principles are capable of generating a satisfactory signal, decision is to be taken whether to employ a commercially available transducer or build a suitable transducer. If the transducers are available in the market at a suitable price, the choice will probably be to purchase one of them, otherwise own transducer will have to be designed, built and calibrated.

The points to be considered in determining a transducer suitable for a specific measurement are as follows:

1. ***Range***. The range of the transducer should be large enough to encompass all the expected magnitudes of the measurand.
2. ***Sensitivity***. The transducer should give a sufficient output signal per unit of measured input in order to yield meaningful data.
3. ***Electrical Output Characteristics***. The electrical characteristics-the output impedance, the frequency response, and the response time of the transducer output signal should be compatible with the recording device and the rest of the measuring system equipment.
4. ***Physical Environment***. The transducer selected should be able to withstand the environmental conditions to which it is likely to be subjected while carrying out measurements and tests.
   Such parameters are temperature, acceleration, shock and vibration, moisture, and corrosive chemicals might damage some transducers but not others.

5. ***Errors***. The errors inherent in the operation of the transducer itself, or those errors caused by environmental conditions of the measurement, should be small enough or controllable enough that they allow meaningful data to be taken.
   However the total measurement error in a transducer-activated system may be reduced to fall within the required accuracy range by adopting the following techniques.
   1. Calibrating the transducer output against some known standards while in use under actual test conditions. This calibration should be performed regularly as the measurement proceeds.

2. Continuous monitoring of variations in the environmental conditions of the transducer and correcting the data accordingly.

Controlling the measurement environment artificially in order to reduce possible transducer errors artificial environmental control includes the enclosing of the transducer in a temperature-controlled housing and isolating the device from external shocks and vibrations.

Classification of Transducers:

The transducers may be classified in various ways such as on the basis of electrical principles involved, methods of application, methods of energy conversion used, nature of output signal etc as shown in table (1.1).

1. **Primary and Secondary Transducers:** Transducers, on the basis of methods of applications, may be classified into primary and secondary transducers. When the input signal is directly sensed by the transducer and physical phenomenon is converted into the electrical form directly then such a transducer is called the primary transducer.
   For example a thermistor used for the measurement of temperature fall in this category the thermistor senses the temperature directly and causes the change in resistance with the change in temperature.

   When the input signal is sensed first by some detector or sensor and then its output being of some form other than input signals is given as input to a transducer for conversion into electrical form, then such a transducer falls in the category of secondary transducers.

   For example, in case of pressure measurement, bourdon tube is a primary sensor which converts pressure first into displacement, and then the displacement is converted into an output voltage by an LVDT. In this case LVDT is secondary transducer.

2. **Active and Passive Transducers:** Transducers, on the basis of methods of energy conversion used, may be classified into active and passive transducers. Self-generating type transducers i.e. the transducers, which develop their output the form of electrical voltage or current without any auxiliary source, are called the active transducers. Such transducers draw energy from the system under measurement. Normal such transducers give very small output and, therefore, use of amplifier becomes essential.
   Transducers, in which electrical parameters i.e. resistance, inductance or capacitance changes with the change in input signal, are called the passive transducers. These transducers require external power source for energy conversion. In such transducer electrical parameters i.e. resistance, inductance or capacitance causes a change in voltages current or frequency of the external power source. These transducers may draw sour energy from the system under measurement. Resistive, inductive and capacitive transducer falls in this category.

3. **Analog and Digital Transducers:** Transducers, on the basis of nature of output signal, may be classified into analog and digital transducers. Analog transducer converts input signal into output signal, which is a continuous function of time such as thermistor, strain gauge, LVDT, thermo-couple etc. Digital transducer converts input signal into the output signal of the form of pulse e.g. it gives discrete output. These transducers are becoming more and more popular now-a-days because of advantages associated with digital measuring instruments and also due to the effect that digital signals can be transmitted over a long distance without causing much distortion due to amplitude variation and phase shift. Sometimes an analog transducer combined with an ADC (analog-digital convector) is called a digital transducer.

4. **Transducers and Inverse Transducers:** Transducer, as already defined, is a device that converts a non-electrical quantity into an electrical quantity. Normally a transducer and associated circuit has a non-electrical input and an electrical output, for example a thermo- couple, photoconductive cell, pressure gauge, strain gauge etc. An inverse transducer is a device that converts an electrical quantity into a non-electrical quantity. It is a precision actuator having an electrical input and a low-power non-electrical output. For examples a piezoelectric crystal and transnational and angular moving-coil elements can be employed as inverse transducers. Many data-indicating and recording devices are basically inverse transducers. An ammeter or voltmeter converts electric current into mechanical movement and the characteristics of such an instrument placed at the output of a measuring system are important. A most useful application of inverse transducers is in feedback measuring systems.

Table

Classification of Electrical Transducers

Class	Electrical Parameters	Types of Transducers	Principle of Operation	Typical Applications
Passive Transducers	Resistance	Potentiometer	Variation of resistance in a potentiometer or a bridge circuit due to positioning of a slide contact by an external force.	Pressure, displace ment, position
		Resistance strain gauge	Variation of resistance of a wire or a semi-conductor b y elongation or compression due to externally applied stress.	Force, torque, displacement.
		Pirani gauge or hot-wire meter	Variation of resistance of a heating element by convection cooling of a stream of gas.	Gas flow, gas pressure.
		Resistance thermo-meter or pyrometer	Variation of resistance of pure metal wire with the variation in temperature	Temperature, radiant heat.
		Thermistor	Variation of resistance of certain metal oxides having negative temperature coefficient of resistance with the variation in temperature.	Temperature
		Resistance hygro-meter	Variation of resistance of a conductive strip with moisture content.	Relative humidity.

		Photoconductive cell	Variation of resistance of a cell as a circuit element with incident light.	Photosensitive relay.

Class	Electrical Parameters	Types of Transducers	Principle of Operation	Typical Applications
Passive Transducers	Inductance	Magnetic circuit breaker	Variation of self or mutual inductance of an ac- excited coil by changes in the magnetic circuit	Pressure, displace ment
		Reluctance pick up	Variation of reluctance of the magnetic circuit by changing the position of the iron core of a coil.	Pressure, displace-ment, vibration position.
		Differential trans-former	Variation of differential voltage of two secondary windings of a transformer by varying the position of the magnetic core by an externally applied force.	Force, pressure, position, displacement
		Eddy current gauge	Variation of coil inductance by the proximity of an eddy current plate	Displacement, thick ness
		Magnetostriction gauge	Variation of magnetic properties by pressure and stress.	Force, pressure, sound.
	capacitan ce	Variable capaci-tance pressure gauge	Variation in capacitance due to variation of distance between two parallel plates by an externally applied force.	Pressure, displace ment.
		Capacitor micro - phone	Variation of capacitance between a fixed plate and a movable diaphragm due to sound pressure.	Speech, music, noise.
		Dielectric gauge	Variation in capacitance because of changes in dielectric.	Liquid level, thickness.

Class	Electrical Parameters	Types of Transducers	Principle of Operation	Typical Applications
Passive Transducers	Voltage and current	Hall effect pickup	Generation of a potential difference across a semiconductor(germanium) plate due to interaction of magnetic flux with an applied current.	Magnetic flux, current.
		Ionization chamber	Induced electron flow by gas ionization due to radioactive radiation.	Particle counting.
		Photoemissive cell	Electron emission due to incident radiation on photoemissive surface	Light and radiations.
		Photomultiplier tube	Secondary electron emission due to incident radiation on photosensitive cathode	Light and radiation, photosensitive relays.
Active Transducers	Voltage and current	Thermocouple and thermopile	Development of an emf across the junction of two dissimilar metals or semi conductors when that junction is heated.	Temperature, heat flow, radiation
		Moving-coil gene- rator	Generation of an emf due to motion of a coil in a magnetic field.	Velocity, vibration
		Piezoelectric pickup	Generation of an emf on applying an external force to a certain crystalline material such as quartz.	Sound, vibration, acceleration, pressure variations.
		Photovoltaic cell	Generation of a voltage in a semi-conductor junction device when radiant energy stimulates the cell	Light meter, solar cell

Class	Electrical Parameters	Types of Transducers	Principle of Operation	Typical Applications

Digital transducers	Train of pulses	Encoders	Translation of the shaft angular position into a digital number.	Angular position
		Even counting	Conversion of angular and translational motions into train of pulses by using either electromagnetic, capacitive or photo-electric method.	Motion.
		Frequency output	Conversion of analog signals into frequency. For example inductive or capacitive transducers can be incorporated in the tuned resonant circuit of an LC oscillator.	Displacement, force, pressure, vibration

Electro-Pneumatic Transducers Information

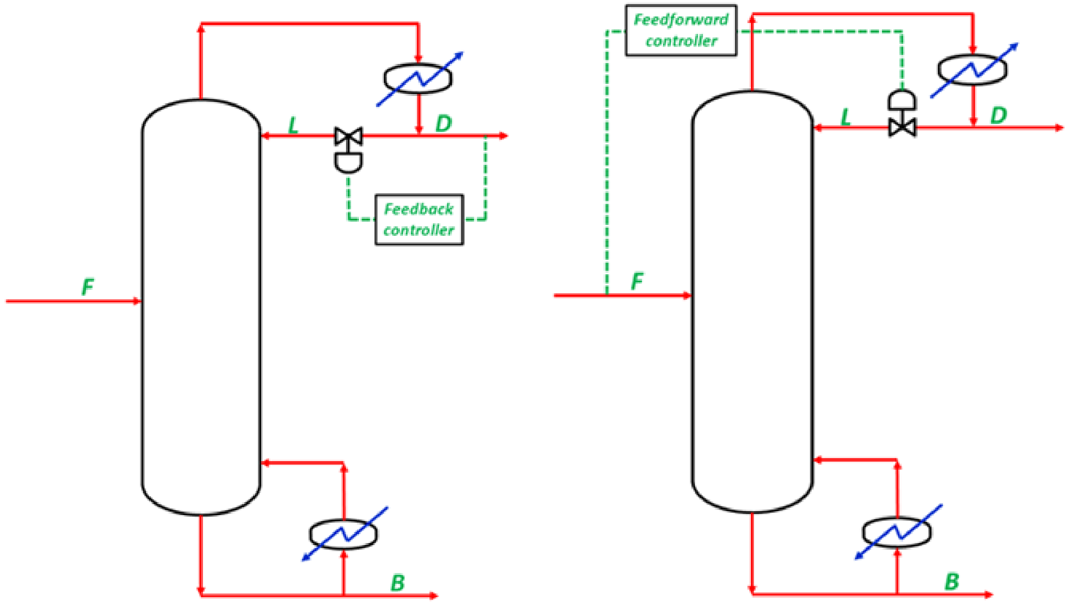
Electro-pneumatic transducers convert a current or voltage input into proportional output pressure. They are often paired with valves, pneumatic relays, and flow regulators in process control applications.

Electro-pneumatic (also known as E/P or I/P) transducers typically accept a standard current loop, often 4-20 mA, or a 0-5V or 0-10V voltage signal. As in all transducers, the device's output values must be calibrated with the input range to ensure accurate output pressure. Important calibration specifications include **zero**, the lowest possible pressure matched to the lowest input value, and **span**, the numerical value between the minimum and maximum output. Adding the span to the zero value yields the maximum output pressure for a calibrated device.

5.3 Feedforward control

A feedback controller responds only after it detects a deviation in the value of the controlled output from its desired set point. On the other hand, a feedforward controller detects the disturbance directly and takes an appropriate control action in order to eliminate its effect on the process output.

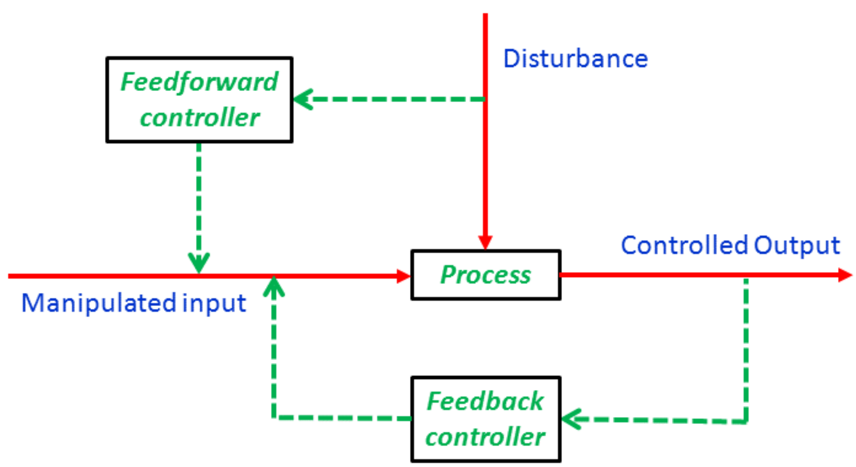
Consider the distillation column shown in Fig. The control objective is to keep the distillate concentration at a desired set point despite any changes in the inlet feed stream.



(a) Feedback control configuration (b) Feedforward control configuration

Feedback and Feedforward control configuration of a distillation column

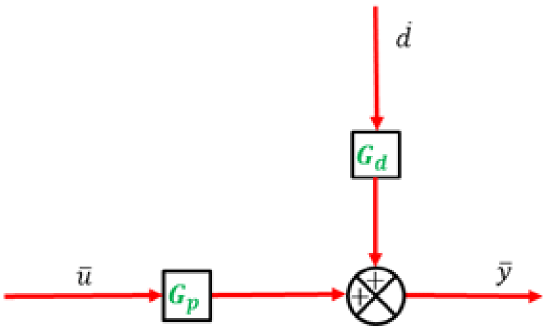
Fig. shows the conventional feedback loop, which measures the distillate concentration and after comparing it with the desired setpoint, increases or decreases the reflux ratio. A feedforward control system uses a different approach. It measures the changes in the inlet feed stream (disturbance) and adjusts the reflux ratio appropriately. Fig shows the feedforward control configuration.



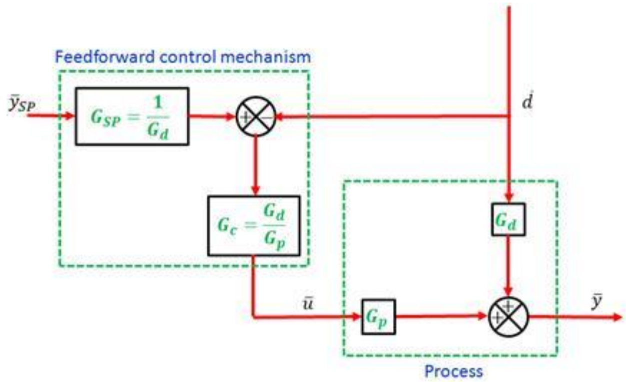
The comparative schematic of feedback and feedforward control structure

Fig shows the general form of a feedforward control system. It directly measures the disturbance to the process and anticipates its effect on the process output. Eventually it alters the manipulated input in such a way that the impact of the disturbance on the process output gets eliminated. In other words, where the feedback control action starts after the disturbance is "felt" through the changes in process output, the feedforward control action starts immediately after the disturbance is "measured" directly. Hence, feedback controller acts in a *compensatory* manner whereas the feedforward controller acts in an *anticipatory* manner.

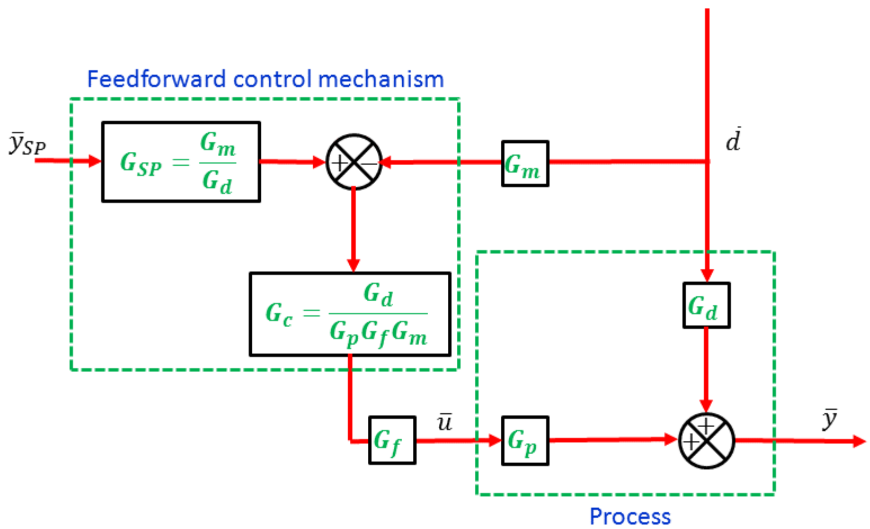
Design of feedforward controller::Let us consider the block diagram of a process shown in Fig. The Fig. presents the open-loop diagram of the process. The process and disturbance transfer functions are represented by G_p and G_d respectively. The controlled output, manipulated input and the disturbance variable are indicated as $\bar{y}, \bar{u}$ and $\bar{d}$ respectively.



(a) Open-loop process diagram



(b) Process diagram with feedforward controller



(c) Process diagram with feedforward controller, sensor and valve
The schematic of a feedforward controller mechanism

The process output is represented by

$$\bar{y} = G_p\bar{u} + G_d\bar{d} \qquad \text{(V.1)}$$

The control objective is to maintain $\bar{y}$ at the desired setpoint $\bar{y}_{SP}$. Hence the eq (V.1) can be rewritten as

$$\bar{y}_{SP} = G_p\bar{u} + G_d\bar{d} \qquad \text{(V.2)}$$

The eq. (V.2) can be rearranged in the following manner:

$$\bar{y}_{SP} - G_d\bar{d} = G_p\bar{u}$$

or

$$\bar{u} = \frac{1}{G_p}\left(\bar{y}_{SP} - G_d\bar{d}\right) = \frac{G_d}{G_p}\left(\frac{1}{G_d}\bar{y}_{SP} - \bar{d}\right) = G_c\left(G_{SP}\bar{y}_{SP} - \bar{d}\right) \qquad (V.3)$$

The eq. (V.3) can be schematically represented by Fig V.3(b).

For the sake of simplicity, measuring element and final control element were not considered as parts of the feedforward control configuration as shown in Fig V.3(b). In a more generalized case, when such elements are added in the controller configuration, the resulting control structure takes the form of Fig V.3(c). A generalized form of controller equation can be written as

$$\bar{u} = G_c G_f\left(G_{SP}\bar{y}_{SP} - G_m\bar{d}\right) \qquad (V.4)$$

And

$$\bar{y} = G_p\bar{u} + G_d\bar{d} = G_p\{G_c G_f\left(G_{SP}\bar{y}_{SP} - G_m\bar{d}\right)\} + G_d\bar{d} = \{G_p G_c G_f G_{SP}\}\bar{y}_{SP} + \{G_d - G_p G_c G_f G_m\}\bar{d} \qquad (V.5)$$

In case of regulatory problem (disturbance rejection) i.e. when $\bar{y}_{SP} = 0$, the controller should be able to reject the effect of disturbance and ensure no deviation in the output, i.e. $\bar{y} = 0$. In other words,

$$G_d - G_p G_c G_f G_m = 0 \qquad (V.6)$$

or

$$G_c = \frac{G_d}{G_m G_f G_p} \qquad (V.7)$$

In case of servo problem (setpoint tracking), i.e. when $\bar{d} = 0$, the controller should be able to ensure that output tracks the setpoint, i.e. $\bar{y} = \bar{y}_{SP}$. In other words,

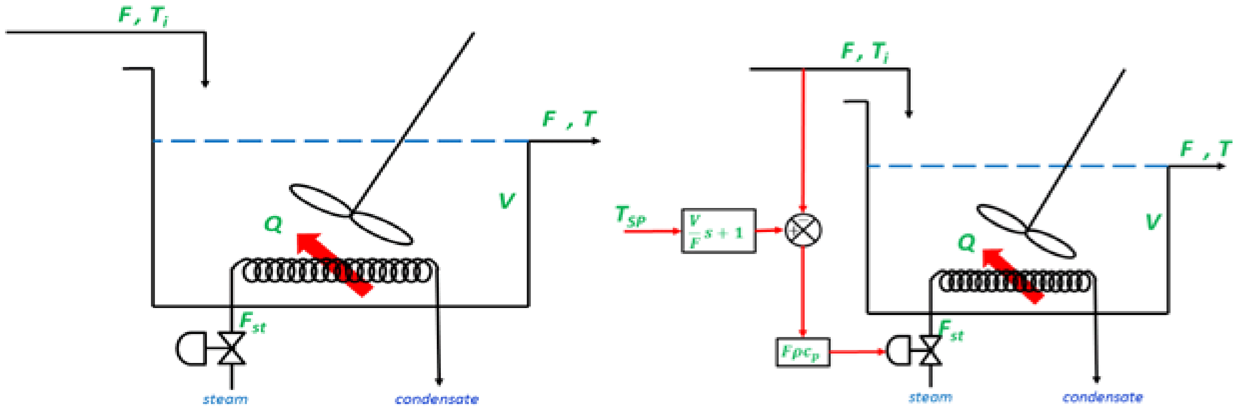
$$G_p G_c G_f G_{SP} = 1 \qquad (V.8)$$

or

$$G_{SP} = \frac{1}{G_p G_c G_f} = \frac{1}{G_p\left(\frac{G_d}{G_m G_f G_p}\right)G_f} = \frac{G_m}{G_d}$$

Example of design of feedforward controller:

Consider an overflow type continuous stirred tank heater shown in Fig V.4. The fluid inside the tank is heated with steam whose flow rate is $\boldsymbol{F_{st}}$ and supplying heat at a rate of $\boldsymbol{Q}$ to the fluid. Temperatures of the inlet and outlet streams are $\boldsymbol{T_i}$ and T respectively. V is the volume of liquid which is practically constant in an overflow type reactor. A control valve in the steam line indicates that the steam flow rate can be manipulated in order to keep the liquid temperature at a desired setpoint. Temperature of the inlet stream flow is the source of disturbance (change in $\boldsymbol{T_i}$) to the process.



(a) Process without a controller (b) Process with feedforward controller

Feedforward control configuration of an overflow type continuous stirred tank heater

A simple energy balance exercise will yield the model equation of the above process as:

$$V\frac{dT}{dt} = F(T_i - T) + \frac{Q}{\rho c_p} \qquad \text{(V.10)}$$

All the variables are assumed to be in the deviation form. Hence, taking Laplace transform on both sides we obtain:

$$VsT(s) = F\{T_i(s) - T(s)\} + \frac{Q(s)}{\rho c_p} \qquad \text{(V.11)}$$

or,

$$VsT(s) + FT(s) = FT_i(s) + \frac{Q(s)}{\rho c_p} \qquad \text{(V.12)}$$

$$\frac{V}{F}sT(s) + T(s) = T_i(s) + \frac{Q(s)}{F\rho c_p} \qquad \text{(V.13)}$$

or,

$$T(s) = \left\{\frac{1}{\left(\frac{V}{F}s + 1\right)}\right\}T_i(s) + \left[\left\{\frac{1}{F\rho c_p}\right\}\left\{\frac{1}{\left(\frac{V}{F}s + 1\right)}\right\}\right]Q(s) \qquad \text{(V.14)}$$

The feedforward controller is meant for ensuring $T = T_{SP}$. Hence,

$$\left(\frac{V}{F}s + 1\right)T_{sp}(s) = T_i(s) + \left[\left\{\frac{1}{F\rho c_p}\right\}\right]Q(s) \qquad \text{(V.15)}$$

or

$$Q(s) = F\rho c_p\left\{\left(\frac{V}{F}s + 1\right)T_{sp}(s) - T_i(s)\right\} \qquad \text{(V.16)}$$

Hence, one needs to set $\boldsymbol{F_{st}}$ in such a way that $\boldsymbol{Q}$ amount of heat as given in eq.(V.16) is transferred to the process. Fig (b) represents the feedforward structure of the controller.

Remarks:

• The feedforward controller ideally does not get any feedback from the process output. Hence, it solely works on the merit of the model(s). The better a model represents the behavior of a process,the better would be the performance of a feedforward controller designed on the basis of that model. Perfect control necessitates perfect knowledge of process and disturbance models and this is practically impossible. This inturn is the main drawback of a feedforward controller.

• The feedforward control configuration can be developed for more than one disturbance in multi-controller configuration. Any controller in that configuration would act according to the disturbance for which it is designed.

• External characteristics of a feedforward loop are same as that of a feedback loop. The primary measurement (disturbance in case of feedforward control and process output in case of feedback control) is compared to a setpoint and the result of the comparison is used as the actuating signal for the controller. Except the controller, all other hardware elements of the feedforward control configuration such as sensor, transducer, transmitter, valves are same as that of an equivalent feedback control configuration.

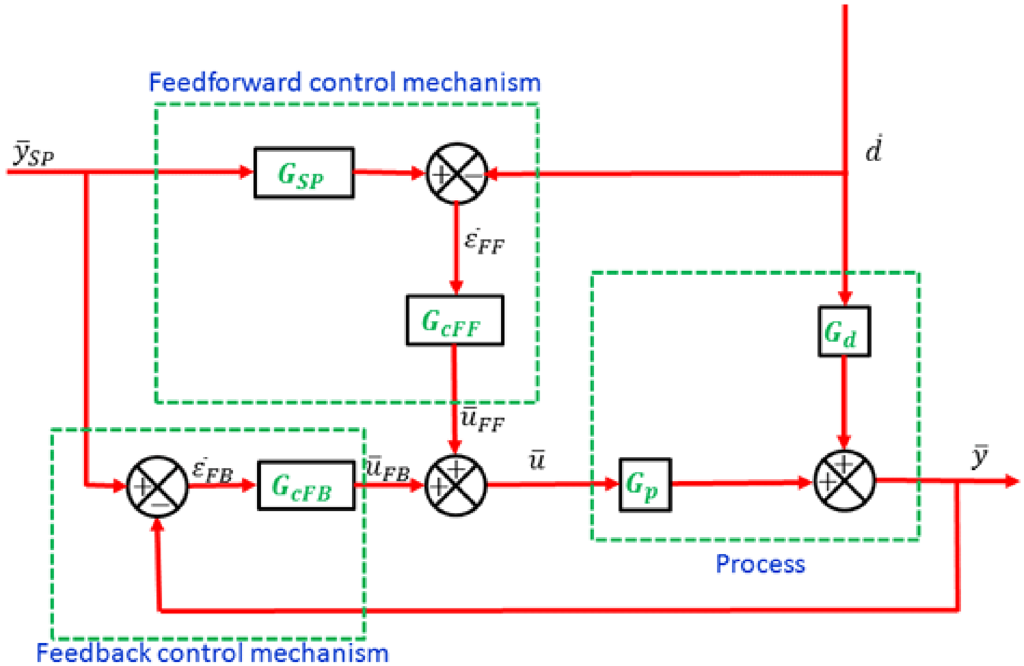
• Feedforward controller cannot be expressed in the feedback form such as P, PI and PID controllers. It is regarded as a special purpose computing machine .

Combination of Feedforward-Feedback Controller:

Table: Merits and demerits of feedforward and feedback controllers

Merits	**Demerits**
Feedforward controllers	
Takes corrective action before the process "feels" the disturbance	Requires measurement of all disturbances affecting the system
Good for sluggish systems and/or system with large deadtime	Sensitive to variation in process parameters
Does not affect the stability of the process	Requires a "near perfect" model of the process
Feedback controllers	
Does not require disturbance measurement	Acts to take corrective action after the process "feels" the disturbance
Insensitive to mild errors in modeling	Bad for sluggish systems and/or system with large deadtime
Insensitive to mild changes in process parameters	May affect the stability of the process

Let us now explore how a combination of feedforward and feedback controller would perform when they are designed to act simultaneously. The schematic of a feedforward-feedback controller is shown in Fig



The schematic of a feedforward-feedback controller

Without losing the generality we shall ignore the transfer functions of the measuring element and the final control element.

Now the closed loop transfer function of feedforward-feedback controller can be derived in the following manner:

$$\begin{aligned}\bar{y} = G_p\bar{u} + G_d\bar{d} &= G_p(\bar{u}_{FF} + \bar{u}_{FB}) + G_d\bar{d} = G_p(G_{cFF}\bar{\varepsilon}_{FF} + G_{cFB}\bar{\varepsilon}_{FB}) + G_d\bar{d} \\ &= G_p\left(G_{cFF}\{G_{SP}\bar{y}_{SP} - \bar{d}\} + G_{cFB}\{\bar{y}_{SP} - \bar{y}\}\right) + G_d\bar{d}\end{aligned} \quad \text{(V.17)}$$

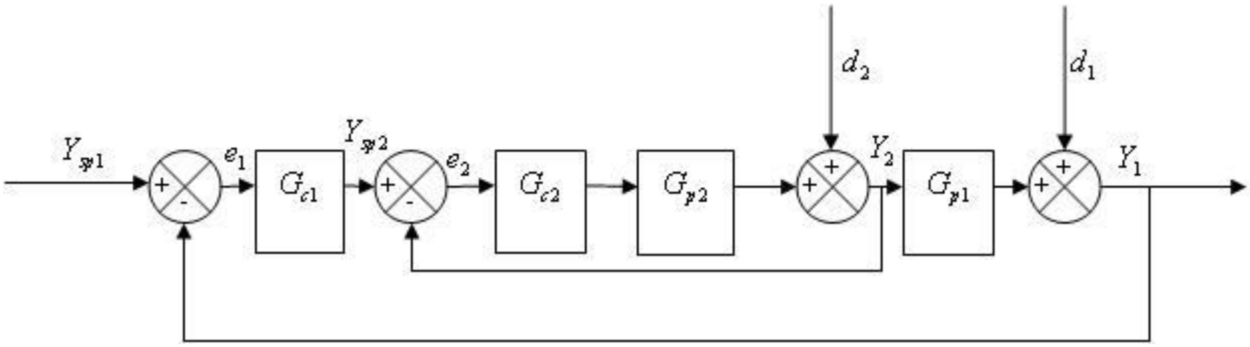
Rearranging the above we get,

$$\bar{y} = \frac{G_p(G_{cFB} + G_{SP}G_{cFF})}{1 + G_pG_{cFB}}\bar{y}_{SP} + \frac{G_d - G_pG_{cFF}}{1 + G_pG_{cFB}}\bar{d} \quad \text{(V.18)}$$

It is observed that the stability of the closed loop response is determined by the roots of the characteristic equation: $1 + G_pG_{cFB} = 0$. Hence, the stability characteristics of a process does not change with the addition of a feedforward loop.

5.4 Cascade Control

The primary disadvantage of conventional feedback control is that the corrective action for disturbances does not begin until after the controlled variable deviates from the setpoint. In other words, the disturbance must be “felt” by the process before the control system responds. Feedforward control offers large improvements over feedback control for processes that have large time constant and/or delay. However, feedforward control requires that the disturbances be measured explicitly and that a model be available to calculate the controller output. Cascade control is an alternative approach that can significantly improve the dynamic response to disturbances by employing a secondary measurement and a secondary feedback controller. The secondary measurement point is located so that it recognizes the upset condition sooner than the controlled variable, but the disturbance is not necessarily measured.



Cascade Control Structure

Let us consider the following block diagram of cascade control structure.The outer loop and its controller are called master loop and master controller whereas the inner loop and its controller are called slave loop and slave controller respectively.

$$Y_2 = G_{p2}G_{c2}e_2 + d_2 = G_{p2}G_{c2}\left(Y_{sp2} - Y_2\right) + d_2 \qquad \text{V.20}$$

Simplifying

$$Y_2 = \frac{G_{p2}G_{c2}}{\left(1+G_{p2}G_{c2}\right)}Y_{sp2} + \frac{1}{\left(1+G_{p2}G_{c2}\right)}d_2 \qquad \text{V.21}$$

Similarly

$$\begin{aligned}
Y_1 &= G_{p1}Y_2 + d_1 \\
&= G_{p1}\left[\frac{G_{p2}G_{c2}}{\left(1+G_{p2}G_{c2}\right)}Y_{sp2} + \frac{1}{\left(1+G_{p2}G_{c2}\right)}d_2\right] + d_1 \\
&= G_{p1}\left[\frac{G_{p2}G_{c2}}{\left(1+G_{p2}G_{c2}\right)}G_{c1}e_1 + \frac{1}{\left(1+G_{p2}G_{c2}\right)}d_2\right] + d_1 \\
&= G_{p1}\left[\frac{G_{p2}G_{c2}}{\left(1+G_{p2}G_{c2}\right)}G_{c1}\left(Y_{sp1} - Y_1\right) + \frac{1}{\left(1+G_{p2}G_{c2}\right)}d_2\right] + d_1
\end{aligned} \qquad \text{V.22}$$

Again simplifying the above eqn:

$$Y_1 = \frac{G_{p1}G_{p2}G_{c2}G_{c1}}{1+G_{p2}G_{c2}+G_{p1}G_{p2}G_{c2}G_{c1}}Y_{sp1} + \frac{G_{p1}}{1+G_{p2}G_{c2}+G_{p1}G_{p2}G_{c2}G_{c1}}d_2 + \frac{1+G_{p2}G_{c2}}{1+G_{p2}G_{c2}+G_{p1}G_{p2}G_{c2}G_{c1}}d_1 \qquad \text{V.23}$$

Now see what happens if the secondary loop is absent. In that case:

$$Y_2 = G_{p2}Y_{sp2} \qquad \text{V.24}$$

And

$$\begin{aligned} Y_1 &= G_{p1}Y_2 + d_1 \\ &= G_{p1}\left[G_{p2}Y_{sp2} + d_2\right] + d_1 \\ &= G_{p1}\left[G_{p2}G_{c1}e_1 + d_2\right] + d_1 \\ &= G_{p1}\left[G_{p2}G_{c1}\left(Y_{sp1} - Y_1\right) + d_2\right] + d_1 \end{aligned} \qquad \text{V.25}$$

Simplifying

$$Y_1 = \frac{G_{p1}G_{p2}G_{c1}}{\left(1+G_{p1}G_{p2}G_{c1}\right)}Y_{sp1} + \frac{G_{p1}}{\left(1+G_{p1}G_{p2}G_{c1}\right)}d_2 + \frac{1}{\left(1+G_{p1}G_{p2}G_{c1}\right)}d_1 \qquad \text{V.26}$$

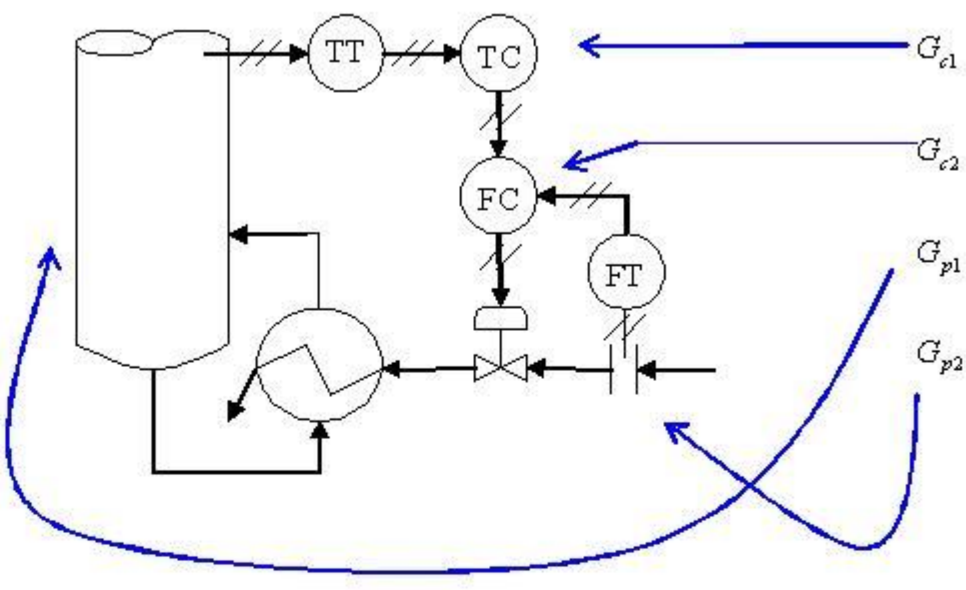
Few points to remember on Cascade controller:

- The slave loop should be tuned before the master loop. After the slave loop is tuned and closed, the master loop should be designed based on the dynamics of inner loop.

- There is little or no advantage to using cascade control if the secondary process is not significantly faster than the primary process dynamics. In particular, if there is long dead time in the secondary process, it is unlikely that the cascade controller will be better than the standard feedback control.

- The most common cascade control loop involves flow controller (eg. TC/FC example in distillation column) as the inner loop. This loop easily rejects the disturbances in fluid steam pressure, either upstream or downstream of the valve.

The Fig presents the process and Instrumentation diagram of a distillation column on which a Cascade controller structure has been employed. Only the bottom part of the column has been shown in the figure.



Partial P & ID of a distillation column with cascade structure of temperature a flow control.

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