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SCHOOL OF BIO AND CHEMICAL ENGINEERING
DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – I – MASS TRANSFER – II – SCHA1501

UNIT 1

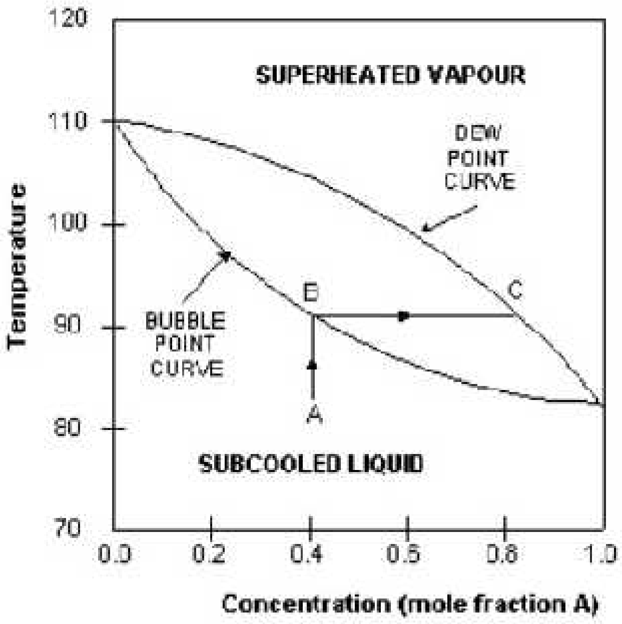
VAPOR PRESSURE AND BOILING

The vapor pressure of a liquid at a particular temperature is the equilibrium pressure exerted by molecules leaving and entering the liquid surface.

Here are some important points regarding vapor pressure:

- Energy input raises vapor pressure
- Vapor pressure is related to boiling
- A liquid is said to ‘boil’ when its vapor pressure equals the surrounding pressure
- The ease with which a liquid boils depends on its volatility
- Liquids with high vapor pressures (volatile liquids) will boil at lower temperatures
- The vapor pressure and hence the boiling point of a liquid mixture depends on the relative amounts of the components in the mixture
- Distillation occurs because of the differences in the volatility of the components in the liquid mixture.

THE BOILING POINT DIAGRAM



- The Boiling Point Diagram The boiling point diagram shows how the equilibrium compositions of the components in a liquid mixture vary with temperature at a fixed pressure.
- Consider an example of a liquid mixture containing 2 components (A and B) - a binary mixture..

The boiling point of A is that at which the mole fraction of A is 1. The boiling point of B is that at which the mole fraction of A is 0. In this example, A is the more volatile component and therefore has a lower boiling point than B.

The upper curve in the diagram is called the dew-point curve while the lower one is called the bubble-point curve.

- Dew-point: is the temperature at which the saturated vapor starts to condense.
- Bubble-point: is the temperature at which the liquid starts to boil.
- The region above the dew-point curve shows the equilibrium composition of the superheated vapor while the region below the bubble-point curve shows the equilibrium composition of the sub-cooled liquid.

- For example, when a subcooled liquid with mole fraction of A=0.4 (point A) is heated, its concentration remains constant until it reaches the bubble-point (point B), when it starts to boil. The vapors evolved during the boiling has the equilibrium composition given by point C, approximately 0.8 mole fraction A. This is approximately 50% richer in A than the original liquid.
- This difference between liquid and vapor compositions is the basis for distillation operations.

Relative Volatility

- Relative volatility: is a measure of the differences in volatility between 2 components, and hence their boiling points. It indicates how easy or difficult a particular separation will be.
- The relative volatility of component ‘i’ with respect to component ‘j’ is defined as

$\alpha_{ij} =$

yi = mole fraction of component ‘i’ in the vapour

xi = mole fraction of component ‘i’ in the liquid

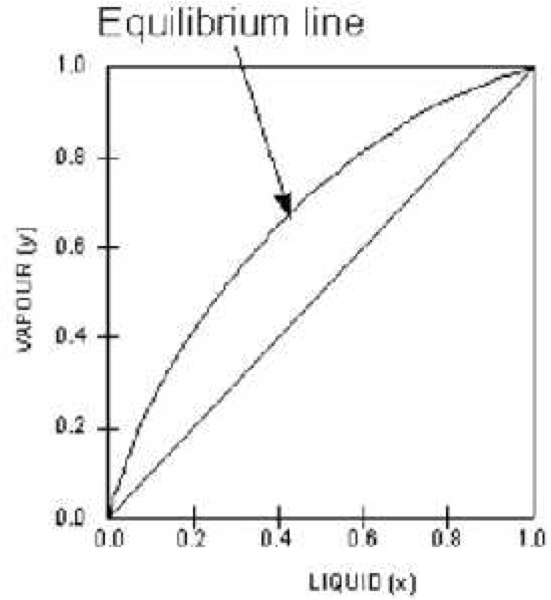
Thus if the relative volatility between 2 components is very close to one, it is an indication that they have very similar vapor pressure characteristics. This means that they have very similar boiling points and therefore, it will be difficult to separate the two components via distillation.

VAPOUR LIQUID EQUILIBRIA

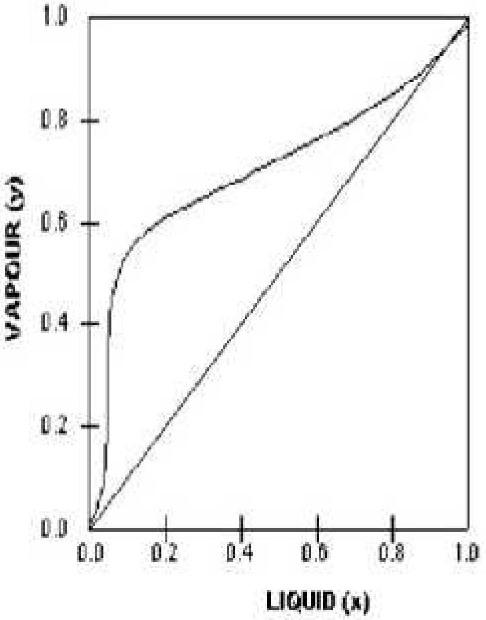
Distillation columns are designed based on the boiling point properties of the components in the mixtures being separated. Thus the sizes, particularly the height, of distillation columns are determined by the vapor liquid equilibrium (VLE) data for the mixtures.

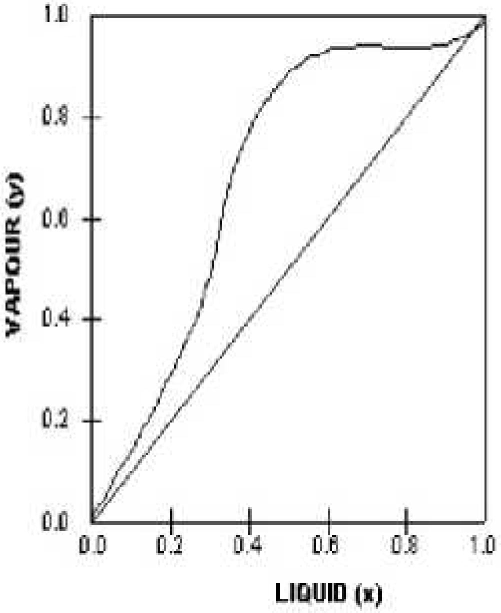
Vapor-Liquid-Equilibrium (VLE) Curves

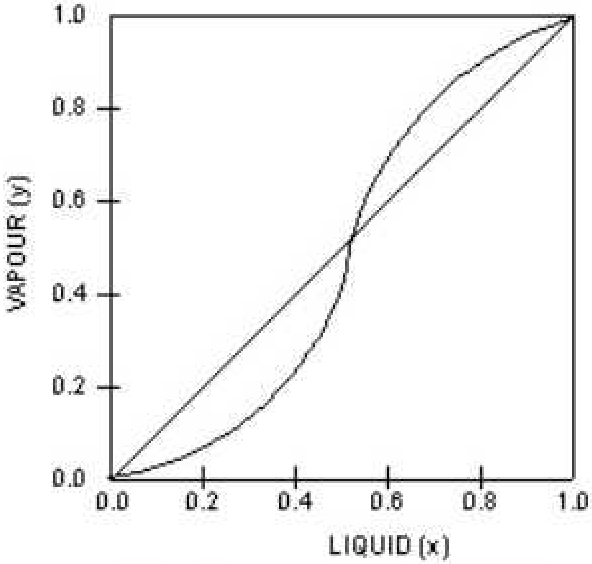
Constant pressure VLE data is obtained from boiling point diagrams. VLE data of binary mixtures is often presented as a plot, as shown in the figure. The VLE plot expresses the bubble-point and the dew-point of a binary mixture at constant pressure. The curved line is called the equilibrium line and describes the compositions of the liquid and vapour in equilibrium at some fixed pressure



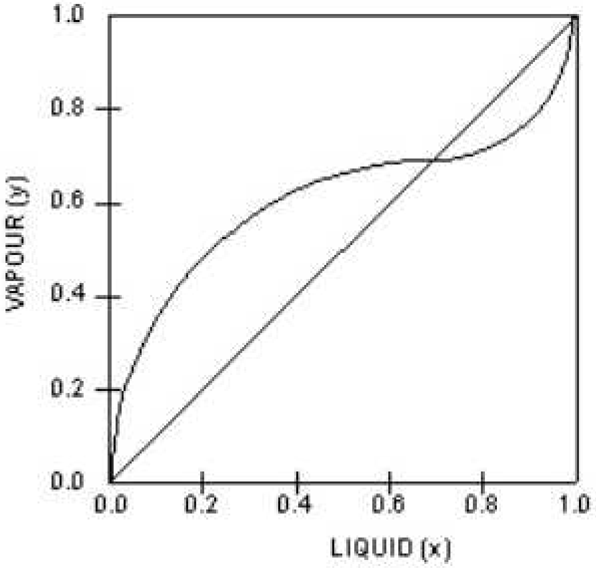
The previous particular VLE plot shows a binary mixture that has a uniform vapor-liquid equilibrium that is relatively easy to separate. The next two VLE plots below on the other hand, shows non-ideal systems which will present more difficult separations.







Maximum Boiling Point



Minimum Boiling Point

The most intriguing VLE curves are generated by azeotropic systems. An azeotrope is a liquid mixture which when vaporised, produces the same composition as the liquid. The two VLE plots below, show two different azeotropic systems, one with a minimum boiling point and one with a maximum boiling point. In both plots, the equilibrium curves cross the diagonal lines, and this are azeotropic points where the azeotropes occur. In other words azeotropic systems give rise to VLE plots where the equilibrium curves crosses the diagonals

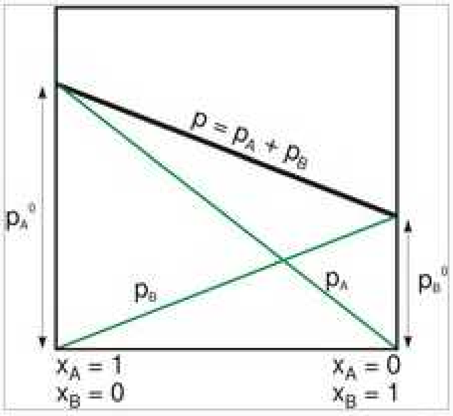
Raoult's law

It states that the partial vapor pressure of each component of an ideal mixture of liquids is equal to the vapor pressure of the pure component multiplied by its mole fraction in the mixture.

Mathematically, Raoult's law for a single component in an ideal solution is stated as

$$p_i = p_i^\star x_i ,$$

where p_i is the partial vapor pressure of the component i in the gaseous mixture (above the solution), $p_i^\star$ is the vapor pressure of the pure component i, and x_i is the mole fraction of the component i in the mixture (in the solution).



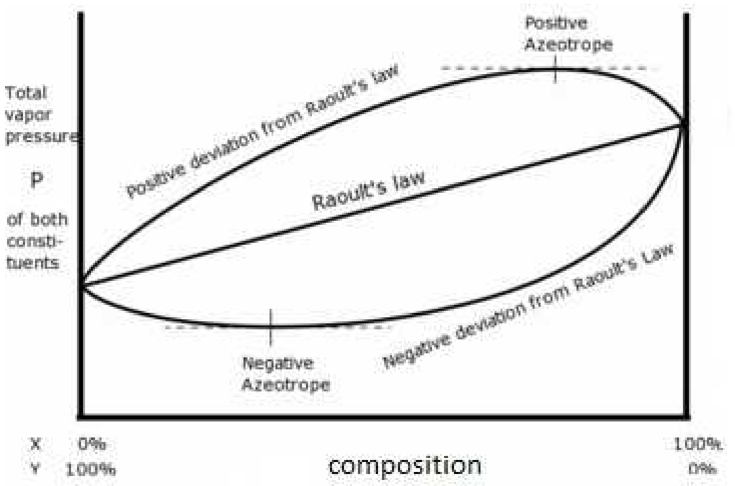
Once the components in the solution have reached equilibrium, the total vapor pressure of the solution can be determined by combining Raoult's law with Dalton's law of partial pressures to give

$$p = p_A^\star x_A + p_B^\star x_B + \cdots .$$

If a non-volatile solute (zero vapor pressure, does not evaporate) is dissolved into a solvent to form an ideal solution, the vapor pressure of the final solution will be lower than that of the pure solvent.

Ideal solution: An ideal solution will obey Raoult's Law,

Real solutions: Solutions which deviate from Raoults law are called as real solutions. Many pairs of liquids are present in which there is no uniformity of attractive forces, i.e., the adhesive and cohesive forces of attraction are not uniform between the two liquids, so that they deviate from the Raoult's law.



Negative deviation:

If the vapor pressure of a mixture is lower than expected from Raoult's law, there is said to be a negative deviation. This is evidence that the adhesive forces between different components are stronger than the average cohesive forces between like components. In consequence each component is retained in the liquid phase by attractive forces that are stronger than in the pure liquid so that its partial vapor pressure is lower.

For example, the system of chloroform ($CHCl_3$) and acetone(CH_3COCH_3) has a negative deviation from Raoult's law

Positive deviation

When the cohesive forces between like molecules are greater than the adhesive forces between dissimilar molecules, the dissimilarities of polarity leads both components to escape solution more easily. Therefore, the vapor pressure is greater than expected from the Raoult's law, showing positive deviation. If the deviation is large, then the vapor pressure curve shows a maximum at a particular composition and form a positive azeotrope. Some mixtures in which this happens are (1)benzene and methanol, (2) carbon disulfide and acetone, and (3) chloroform and ethanol

TYPES OF DISTILLATION COLUMNS

There are many types of distillation columns, each designed to perform specific types of separations, and each design differs in terms of complexity.

Batch and Continuous Columns

One way of classifying distillation column type is to look at how they are operated. Thus we have: batch and continuous columns.

Batch Columns In batch operation, the feed to the column is introduced batch-wise. That is, the column is charged with a 'batch' and then the distillation process is carried out. When the desired task is achieved, a next batch of feed is introduced.

Continuous columns

They process a continuous feed stream. No interruptions occur unless there is a problem with the column or surrounding process units. They are capable of handling high throughputs and they are most common of the two types.

Types of Continuous Columns

Continuous columns can be further classified according to:

- The nature of the feed that they are processing,
    - binary column - feed contains only two components
    - multi-component column - feed contains more than two components
- The number of product streams they have
    - Multi-product column - column has more than two product streams

  - Extractive distillation - where the extra feed appears in the bottom product stream

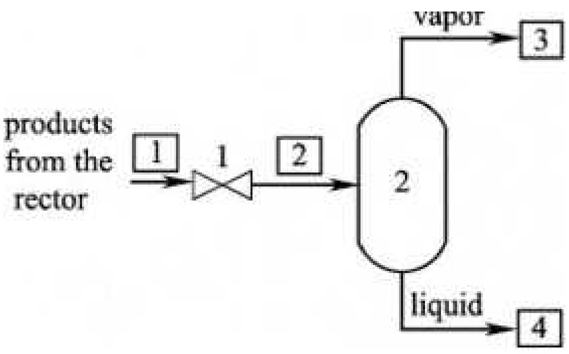
- Where the extra feed exits when it is used to help with the separation,
  - azeotropic distillation - where the extra feed appears at the top product stream

- The type of column internals
  - tray column - where trays of various designs are used to hold up the liquid to provide better contact between vapor and liquid, hence better separation
  - packed column - where instead of trays, 'packings' are used to enhance contact between vapor and liquid

Equilibrium or Flash Distillation

This is type of distillation that occurs in a single-stage in which a liquid mixture is partially vaporized. The vapor is allowed to come to equilibrium with the residual liquid and the resulting vapor and liquid phases are separated and removed from the apparatus. Consider a binary mixture of A and B flowing at a rate of F mol/h into a heater partial vaporization.

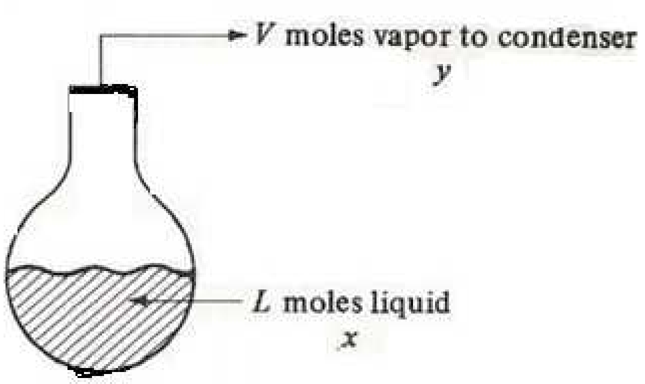
Defining f as the molar fraction of the feed that is vaporized (i.e. f = V/F), the material balance for the flash drum gives:



$y = -[(1-f)/f]x + x_F/f$

Equilibrium relation: $y = f(x)$ Solved simultaneously for x and y

Simple batch or Differential Distillation



liquid is first charged to a heated kettle - The liquid charge is boiled slowly and the vapors are withdrawn as rapidly as possible to a condenser, where the condensed vapor (distillate) is collected - The first portion of vapor condensed is richest in the more volatile component A. - As vaporization proceeds, the vaporized product becomes leaner in A. Following is the design equation for the batch distillation (material balance):

ln =

Where L_1 is the original moles charged

L_2: the moles left in the still

x_1: the original composition of liquid

x_2: the final composition of liquid

$y = f(x)$ is needed From equilibrium relation

DISTILLATION COLUMN DESIGN

Distillation columns are designed using VLE data for the mixtures to be separated. The vapor-liquid equilibrium characteristics (indicated by the shape of the equilibrium curve) of the mixture will determine the number of stages, and hence the number of trays, required for the separation. This is illustrated clearly by applying the McCabe-Thiele method to design a binary column.

McCABE-THIELE DESIGN METHOD

The McCabe-Thiele approach is a graphical one, and uses the VLE plot to determine the theoretical number of stages required to effect the separation of a binary mixture. It assumes constant molar overflow and this implies that:

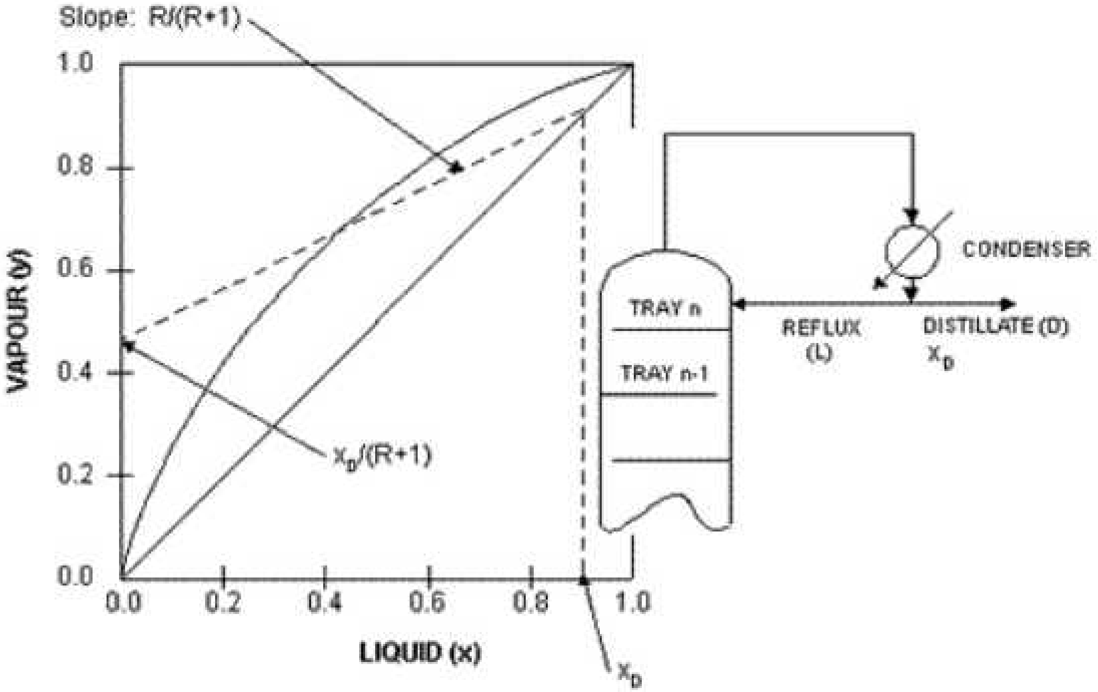
- Molal heats of vaporization of the components are roughly the same
- Heat effects (heats of solution, heat losses to and from column, etc.) are negligible
- for every mole of vapour condensed, 1 mole of liquid is vaporised

The design procedure is simple. Given the VLE diagram of the binary mixture, operating lines are drawn first.

- ➢ Operating lines define the mass balance relationships between the liquid and vapor phases in the column.
- ➢ There is one operating line for the bottom (stripping) section of the column, and on for the top (rectification or enriching) section of the column.
- ➢ Use of the constant molar overflow assumption also ensures the operating lines are straight lines.

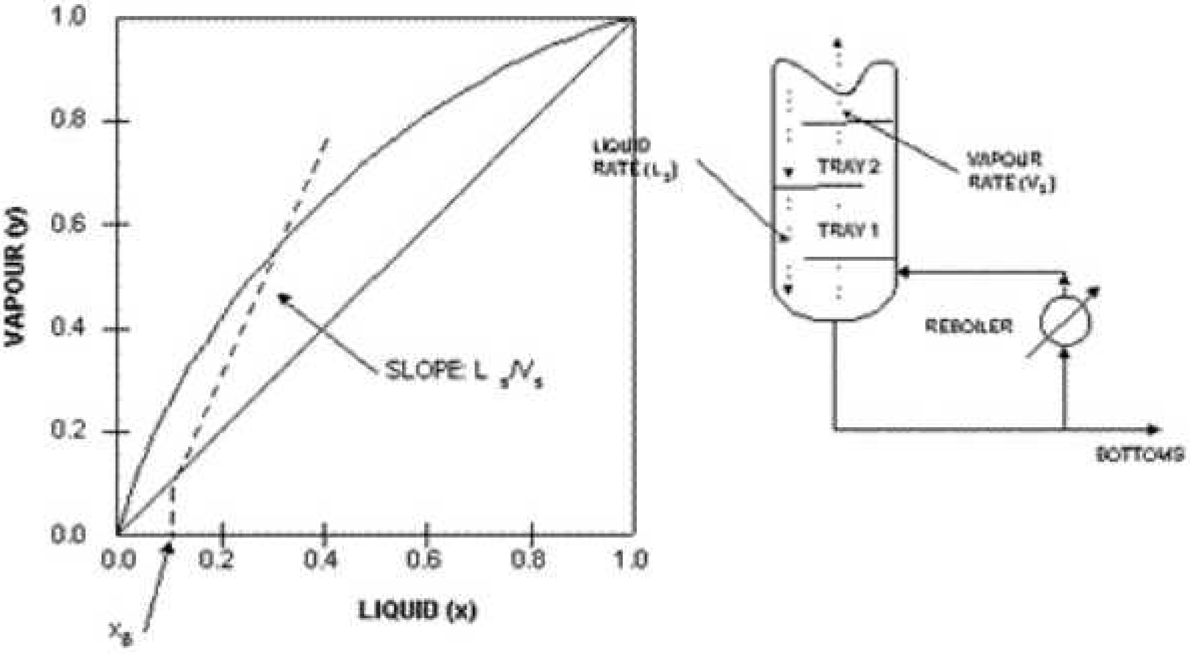
Operating Line for the Rectification Section

The operating line for the rectification section is constructed as follows: ™First the desired top product composition is located on the VLE diagram, and a vertical line produced until it intersects the diagonal line that splits the VLE plot in half. A line with slope R/(R+1) is then drawn from this instersection point as shown in the diagram below. R is the ratio of reflux flow (L) to distillate flow (D) and is called the reflux ratio and is a measure of how much of the material going up the top of the column is returned back to the column as reflux.

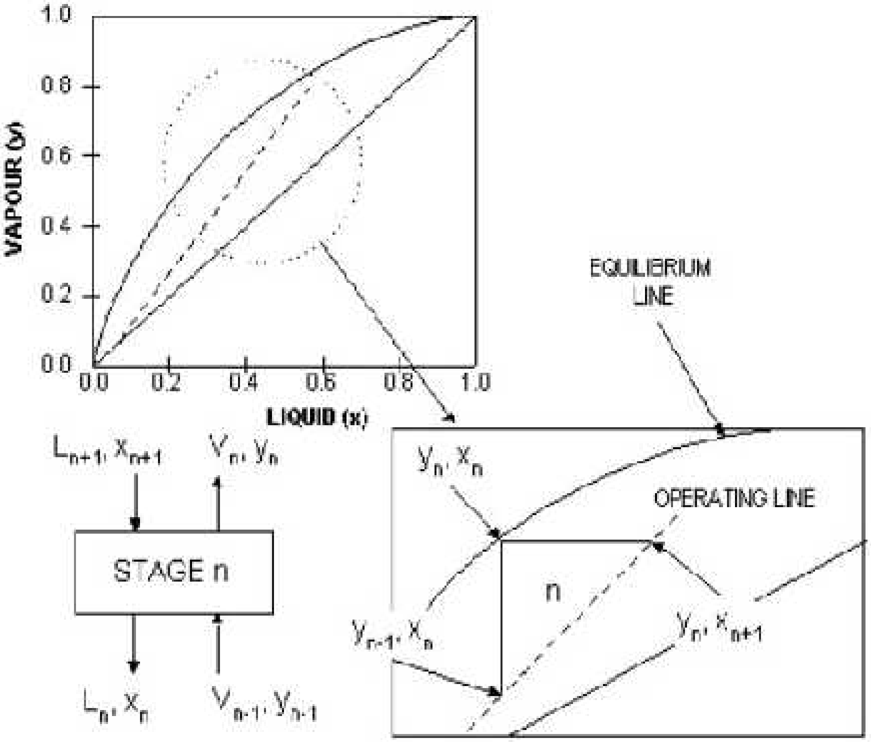


Operating Line for the Stripping Section ™

The operating line for the stripping section is constructed in a similar manner. However, the starting point is the desired bottom product composition. A vertical line is drawn from this point to the diagonal line, and a line of slope Ls/Vs is drawn as illustrated in the diagram below. Ls is the liquid rate down the stripping section of the column, while Vs is the vapor rate up the stripping section of the column. Thus the slope of the operating line for the stripping section is a ratio between the liquid and vapor flows in that part of the column.



Equilibrium and Operating Lines ™The McCabe-Thiele method assumes that the liquid on a tray and the vapor above it are in equilibrium. How this is related to the VLE plot and the operating lines is depicted graphically in the diagram on the right.



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A magnified section of the operating line for the stripping section is shown in relation to the corresponding n'th stage in the column. L's are the liquid flows while V's are the vapor flows. x and y denote liquid and vapor compositions and the subscripts denote the origin of the flows or compositions. That is 'n-1' will mean from the stage below stage 'n' while 'n+1' will mean from the stage above stage 'n'. The liquid in stage 'n' and the vapor above it are in equilibrium, therefore, xn and yn lie on the equilibrium line. Since the vapor is carried to the tray above without changing composition, this is depicted as a horizontal line on the VLE plot. Its intersection with the operating line will give the composition of the liquid on tray 'n+1' as the operating line defines the material balance on the trays. The composition of the vapor above the 'n+1' tray is obtained from the intersection of the vertical line from this point to the equilibrium line

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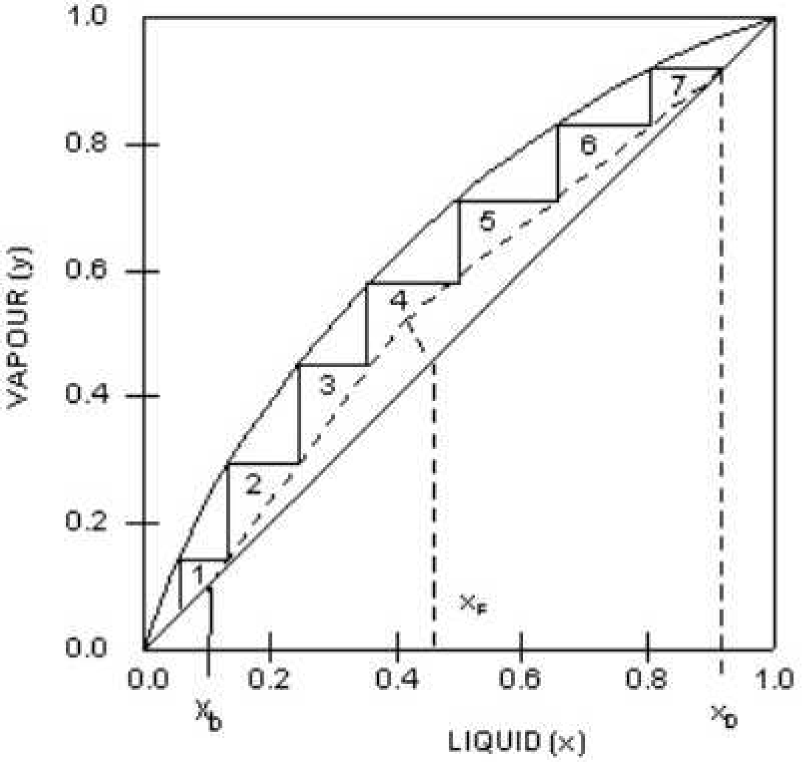
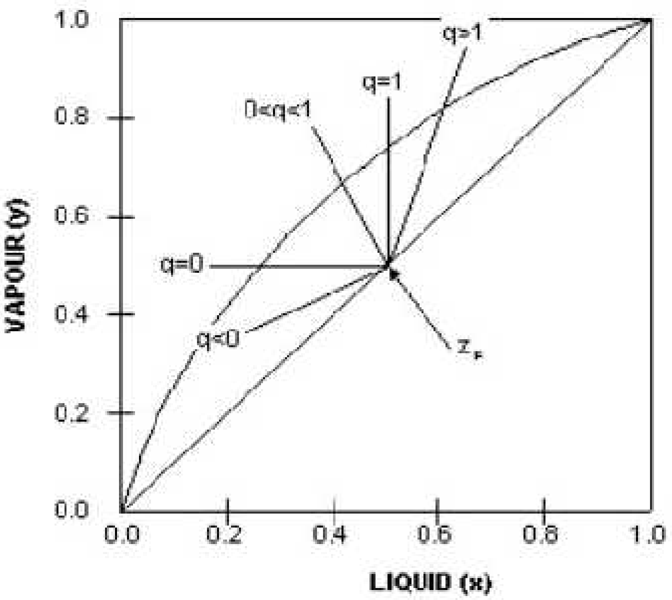


Doing the graphical construction repeatedly will give rise to a number of 'corner' sections, and each section will be equivalent to a stage of the distillation. This is the basis of sizing distillation columns using the McCabeThiele graphical design methodology as shown in the following example. sections, the graphical construction described above was applied. This particular example shows that 7 theoretical stages are required to achieve the desired separationThe required number of trays (as opposed to stages) is one less than the number of stages since the graphical construction includes the contribution of the reboiler in carrying out the separation.

The actual number of trays required is given by the formula: (number of theoretical trays)/(tray efficiency)

Typical values for tray efficiency ranges from 0.5 to 0.7 and depends on a number of factors, such as the type of trays being used, and internal liquid and vapor flow conditions. Sometimes, additional trays are added (up to 10%) to accomodate the possibility that the column may be under-designed.

The Feed Line (q-line)



The diagram above also shows that the binary feed should be introduced at the 4'th stage. However, if the feed composition is such that it does not coincide with the intersection of the operating lines, this means that the feed is not a saturated liquid. The condition of the feed can be deduced by the slope of the feed line or q-line. The q-line is that drawn between the intersection of the operating lines, and where the feed composition lies on the diagonal line. Depending on the state of the feed, the feed lines will have different slopes.

For example:

- q = 0 (saturated vapor)
- q = 1 (saturated liquid)
- 0 < q < 1 (mix of liquid and vapor)
- q > 1 (subcooled liquid)
- q < 0 (superheated vapour)

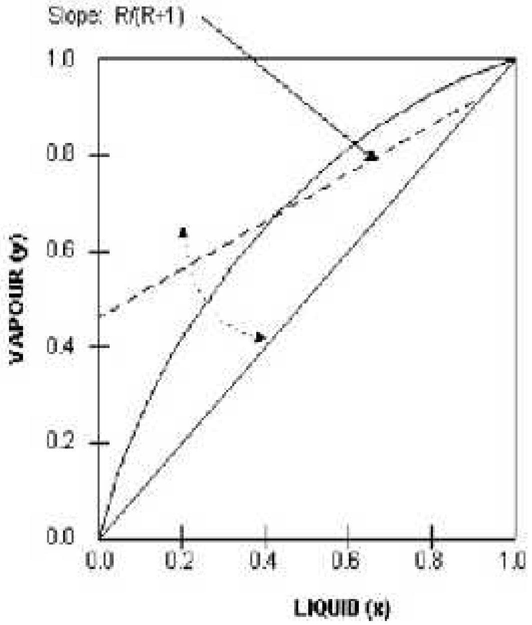
The q-lines for the various feed conditions are shown in the diagram above.

Using Operating Lines and the Feed Line in McCabe-Thiele Design If we have information about the condition of the feed mixture, then we can construct the q-line and use it in the McCabe-Thiele design. However, excluding the equilibrium line, only two other pairs of lines can be used in the McCabe-Thiele procedure. These are:

- feed-line and rectification section operating line
- feed-line and stripping section operating line
- stripping and rectification operating lines

This is because these pairs of lines determine the third.

Reflux Conditions



As the reflux ratio is increased, the gradient of operating line for the rectification section moves towards a maximum value of 1. Physically, what this means is that more and more liquid that is rich in the more volatile components are being recycled back into the column. Separation then becomes better and thus less trays are needed to achieve the same degree of separation. Minimum trays are required under total reflux conditions, i.e. there is no withdrawal of distillate. 9 On the other hand, as reflux is decreased, the operating line for the rectification section moves towards the equilibrium line. The ‘pinch’ between operating and equilibrium lines becomes more pronounced and more and more trays are required.This is easy to verify using the McCabeThiele method.

The limiting condition occurs at minimum reflux ration, when an infinite number of trays will be required to effect separation. Most columns are designed to operate between 1.2 to 1.5 times the minimum reflux ratio because this is approximately the region of minimum operating costs (more reflux means higher reboiler duty).

Vapor Flow Conditions

Adverse vapor flow conditions can cause:

- Foaming
- Entrainment
- Weeping/dumping
- Flooding

Foaming

Foaming refers to the expansion of liquid due to passage of vapour or gas. Although it provides high interfacial liquid-vapour contact, excessive foaming often leads to liquid buildup on trays. In some cases, foaming may be so bad that the foam mixes with liquid on the tray above. Whether foaming will occur depends primarily on physical properties of the liquid mixtures, but is sometimes due to tray designs and condition. Whatever the cause, separation efficiency is always reduced.

Entrainment

Entrainment refers to the liquid carried by vapor up to the tray above and is again caused by high vapor flow rates. It is detrimental because tray efficiency is reduced: lower volatile material is carried to a plate holding liquid of higher volatility. It could also contaminate high purity distillate. Excessive entrainment can lead to flooding. Weeping/Dumping This phenomenon is caused by low vapor flow. The pressure exerted by the vapour is insufficient to hold up the liquid on the tray. Therefore, liquid starts to leak through perforations. Excessive weeping will lead to dumping. That is the liquid on all trays will crash (dump) through to the base of the column (via a domino effect) and the column will have to be re-started. Weeping is indicated by a sharp pressure drop in the column and reduced separation efficiency.

Weeping/Dumping

This phenomenon is caused by low vapor flow. The pressure exerted by the vapour is insufficient to hold up the liquid on the tray. Therefore, liquid starts to leak through perforations. Excessive weeping will lead to dumping. That is the liquid on all trays will crash (dump) through to the base of the column (via a domino effect) and the column will have to be re-started. Weeping is indicated by a sharp pressure drop in the column and reduced separation efficiency.

Flooding:

Flooding is brought about by excessive vapor flow, causing liquid to be entrained in the vapor up the column. The increased pressure from excessive vapor also backs up the liquid in the downcomer, causing an increase in liquid holdup on the plate above. Depending on the degree of flooding, the maximum capacity of the column may be severely reduced. Flooding is detected by sharp increases in column differential pressure and significant decrease in separation efficiency.

Steam distillation:

At atmospheric pressure, high-boiling liquids cannot be purified by distillation, since components of the liquid may decompose at the high temperatures required. - Often the high-boiling substances are essentially insoluble in water, so a separation at lower temperatures can be obtained by simple steam distillation. Steam distillation is often used to separate a high-boiling component from small amounts of non volatile.

If a layer of liquid water (A) and an immiscible high-boiling component (B) such as hydrocarbon are boiled at 101.3 kPa, then the phase rule, for the three phases (2 L and 1 V) and two components,

$F = C – P + 2 = 2 – 3 + 2 = 1$ degree of freedom Hence, if the total pressure is fixed, the system is fixed • Since there are two liquid phases, each will exert its own vapor pressure at the prevailing temperature, and cannot be influenced by the presence of the other.

When the sum of the separate vapor pressures equals the total pressure, the mixture boils and

$p_A + p_B = P$ where p_A is vapor pressure of pure water

p_B is vapor pressure of pure B

Vapor composition: $y_A = p_A / P$ $y_B = p_B / P$

As long as the two liquid phases are present, the mixture will boil at the same temperature, giving a vapor of constant composition. ™Note: as long as liquid water is present, the high-boiling component B vaporizes at a temperature well below its normal boiling point without vacuum. ™The vapors of water (A) and high-boiling component (B) are usually condensed in a condenser and the resulting two immiscible liquid phases are separated. ™

Disadvantage: large amounts of heat must be used to simultaneously evaporate the water with the high-boiling compound ™Used in the food industry for the removal of volatile taints and flavors from edible fats and oils.

Azeotropic, Extractive and Molecular distillation

An azeotrope is a liquid mixture which when vaporised, produces the same composition as the liquid. An azeotrope that contains one liquid phase in contact with vapour is called a homogenous azeotrope. A homogenous azeotrope cannot be separated by conventional distillation. However, vacumn distillation may be used as the lower pressures can shift the azeotropic point. Alternatively, an additional substance may added to shift the azeotropic point to a more ‘favourable’ position.

Azeotropic distillation: When this additional component appears in appreciable amounts at the top of the column, the operation is called azeotropic distillation.

Extractive distillation: When the additional component appears mostly at the bottom of the column, the operation is called extractive distillation.

Molecular distillation: Distillation that is carried out under a high vacuum in an apparatus so designed as to permit molecules escaping from the warm liquid to reach the cooled surface of the condenser before colliding with other molecules and consequently returning to the liquid and that is used in the purification of substances of low volatility (as in the separation of vitamin A and vitamin E from fish-liver oils)

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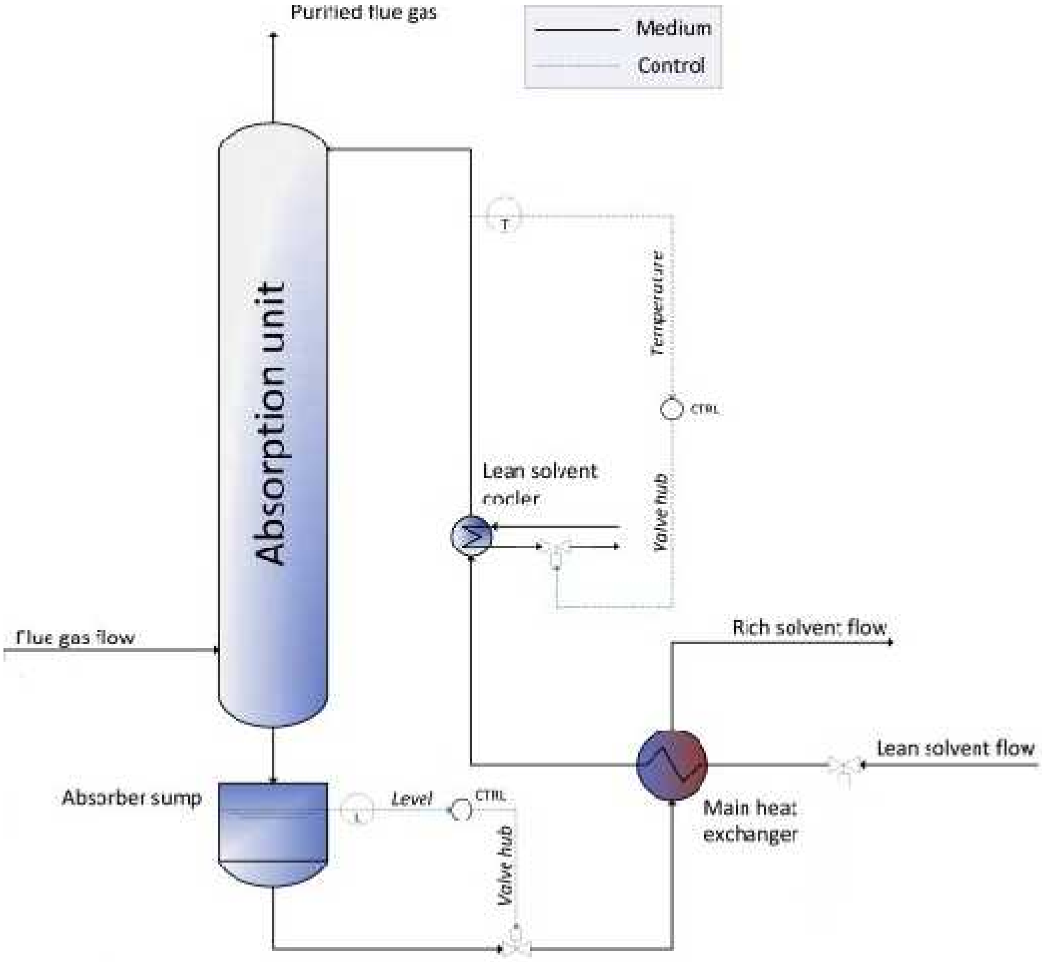
UNIT – II – MASS TRANSFER – II – SCHA1501

ABSORPTION

- Absorption refers to an operation in which the transfer of material is from a gas phase to a liquid phase. A gas is absorbed by means of liquid in which the solute gas is more or less soluble from its mixture with an inert gas as well as more or less insoluble gas. The liquid is essentially immiscible in the gas phase.
- Removal of solute from the gas mixture by using suitable liquid solvent.
- The separation of ammonia from an air-ammonia mixture by means of water is a typical example of absorption.



Selectionofsolventforabsorptionandstripping

(A) Gas Solubility: High solubility of a gas in the solvent is preferred, utilizing low quantity of solvent. Absorbent should not dissolve carrier gas. Similar chemical nature of solute and absorbent (solvent) gives a good solubility. If chemical reaction takes place between solute and solvent, rate of absorption is extremely high. But the reaction should be reversible to recover solvent during desorption.

(B) Volatility: Low volatility or low vapor pressure of the solvent enhances the adsorption operation as solvent loss with carrier gas is very small. Sometimes, a second less volatile solvent is used to recover the first solvent.

(C) Viscosity: For better absorption, a solvent of low viscosity is required. In mechanically agitated absorber, greater amount of power is required for high viscous solvent and flooding is also caused at lower liquid and gas flow rates.

(D) Corrosiveness: Non-corrosive or less corrosive solvent reduces equipment construction cost as well as maintenance cost.

(E) Cost: The solvent should be cheap so that losses will be insignificant and should be easily available.

(F) Toxicity and Hazard: The solvent should be non-toxic, non-flammable, non-hazardous and should be chemically stable.

TypesofAbsorbers

1. Tray column
2. Packed column

PackingMaterials

Packing materials are utilized to provide large interfacial area of contact between two phases. These are made from either of ceramics, metals or plastics. A number of packing materials with various size, shape and performance are available. These are classified into three types, namely, **dumped or random**, **structured** and **grid**.

The packing materials have following characteristics:

(a) Cost: The cost of the packing materials should be very low.

(b) Surface area: A large interfacial area of contact is always recommended. In that case, pressure drop will be more.

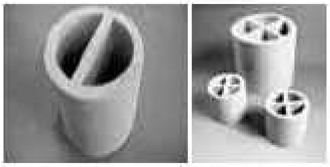
(c) Void volume: A high void volume is needed to maintain low pressure drop.

(d) Fouling resistance: Packing materials should not trap suspended solids present in liquid. Bigger packing materials generally give low fouling resistance.

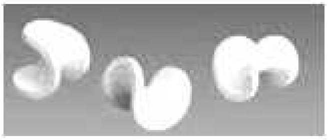
(e) Mechanical strength: Good mechanical strength is desired for choosing packing materials as this will not break or deform during filling or operation.

(f) Uniform flow of streams: Stack of packing materials should have uniform void spaces through which both the streams (gas and liquid) can flow uniformly. Nonuniform flow of streams leads to stagnant liquid pool which in turn gives low mass transfer.

(a) Raschig rings;



(b) Lessing rings and modified Raschig rings (Cross-partition rings)



(c) Berl saddle

(a) Intalox saddle and modification

(b) Pall ring and modification

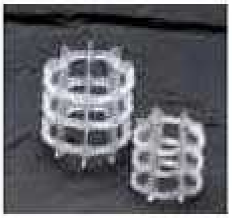
(a) Intalox Metal Tower Packing (IMTP)

(b) Nutter ring

(c) Cascade Mini-Ring (CMR)

(d) Jaeger Tripac

(e) Koch Flexisaddle



(f) Nor-Pac

(g) Hiflow ring

StructuredPackings

These materials are used widely as packing materials in packed tower due to low gas pressure drop and improved efficiency. Corrugated metal sheet structured packing and Wire mesh structured packing materials are widely used in the industries.

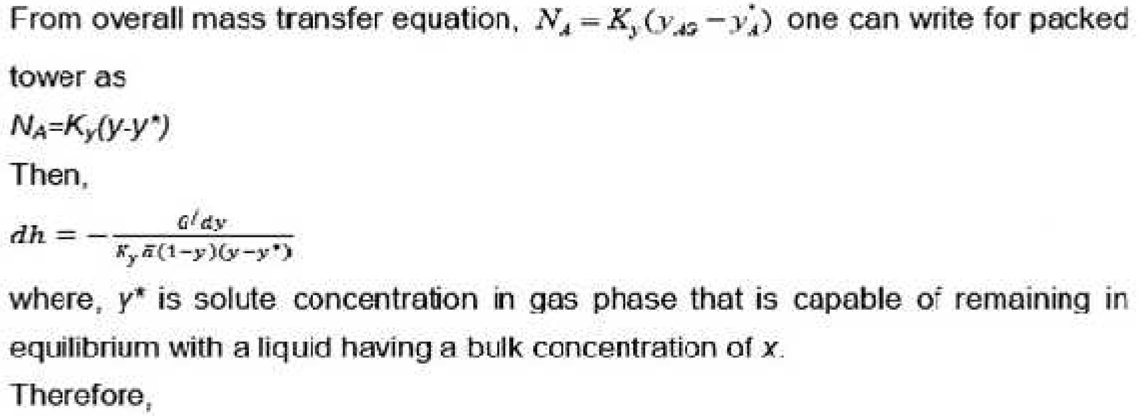
Mellapak

Flexipak

Montez
Corrugated metal
sheet

Wire mesh packing

Design of packed tower based on overall mass transfer Coefficient



From overall mass transfer equation, $N_A = K_y(y_{AG} - y_A^*)$ one can write for packed tower as

$N_A = K_y(y - y^*)$

Then,

$$dh = -\frac{G' dy}{K_y \bar{a}(1-y)(y-y^*)}$$

where, y^* is solute concentration in gas phase that is capable of remaining in equilibrium with a liquid having a bulk concentration of x.

Therefore,

where, y^* is solute concentration in gas phase that is capable of remaining in equilibrium with a liquid having a bulk concentration of x.

Therefore,

$$h_T = \int_0^{h_T} dh = \int_{y2}^{y1} \frac{G^{/} dy}{K_y\,\overline{a}(1-y)(y-y^*)}$$

$$= \int_{y2}^{y1} \frac{G^{/} dy}{k_G\,\overline{a}P(1-y)(y-y_i)}$$

$$= \int_{x2}^{x1} \frac{L^{/} dx}{k_L\,\overline{a}(C_{av})(1-x)(x_i-x)}$$

Graphical integration of right hand side of Equation (4.11):

Operating line AB is drawn in xy plane. Any point (x,y) is taken in operating line. A vertical line is drawn upto equilibrium line to get y^*.

$$h_T = \int_{x2}^{x1} \frac{L^{/} dx}{K_x\,\overline{a}(1-x)(x^*-x)} = \int_{y2}^{y1} \frac{G^{/} dy}{K_G\,\overline{a}P(1-y)(y-y^*)} = \int_{x2}^{x1} \frac{L^{/} dx}{k_L\,\overline{a}(C_{av})(1-x)(x^*-x)}$$

Equation 4.7 can be written as

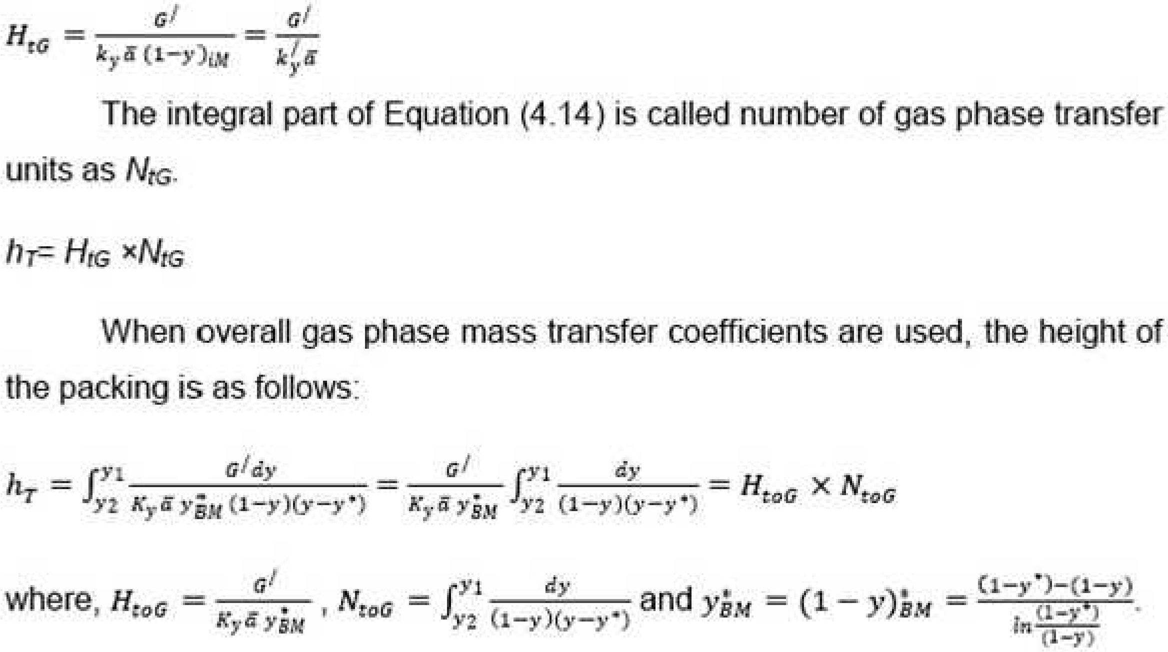
$$h_T = \int_0^{h_T} dh = \int_{y2}^{y1} \frac{G^{/} y_{iBM}\, dy}{k_y \bar{a}\, y_{iBM}\,(1-y)(y-y_i)} = \int_{y2}^{y1} \frac{G^{/} (1-y)_{iM}\, dy}{k_y \bar{a}\,(1-y)_{iM}\,(1-y)(y-y_i)}$$

where, $y_{iBM} = (1-y)_{iM} = \frac{(1-y_i)-(1-y)}{ln\frac{(1-y_i)}{(1-y)}}$

$$h_T = \frac{G^{/}}{k_y \bar{a}\,(1-y)_{iM}} \int_{y2}^{y_1} \frac{(1-y)_{iM}\, dy}{(1-y)(y-y_i)}$$

As, $\frac{G^{/}}{k_y \bar{a}\,(1-y)_{iM}}$ remains constant at the packing section though G/ varies. This quantity is called 'height if transfer units' (HTU) and designated as H_{tG}. It is important to measure the separation effectiveness of the particular packings for a particular separation process. It also describes the mass transfer coefficient. Larger mass transfer coefficient leads to the smaller value of HTU.

Hence,



$$H_{tG} = \frac{G^/}{k_y \bar{a}\,(1-y)_{iM}} = \frac{G^/}{k_y^/ \bar{a}}$$

The integral part of Equation (4.14) is called number of gas phase transfer units as N_{tG}.

$h_T = H_{tG} \times N_{tG}$

When overall gas phase mass transfer coefficients are used, the height of the packing is as follows:

$$h_T = \int_{y2}^{y1} \frac{G^/ dy}{K_y \bar{a}\, y_{BM}^{*} (1-y)(y-y^*)} = \frac{G^/}{K_y \bar{a}\, y_{BM}^{*}} \int_{y2}^{y1} \frac{dy}{(1-y)(y-y^*)} = H_{toG} \times N_{toG}$$

where, $H_{toG} = \frac{G^/}{K_y \bar{a}\, y_{BM}^{*}}$, $N_{toG} = \int_{y2}^{y1} \frac{dy}{(1-y)(y-y^*)}$ and $y_{BM}^{*} = (1-y)_{BM}^{*} = \frac{(1-y^*)-(1-y)}{ln\frac{(1-y^*)}{(1-y)}}$.

DefineNTU

Height of Packing (z) = NTU x HTU

Where,

NTU = number of transfer units - dimensionless
HTU = height of transfer units - dimension of length

The number of transfer units (NTU) required is a measure of the difficulty of the separation. A single transfer unit gives the change of composition of one of the phases equal to the average driving force producing the change. The NTU is similar to the number of theoretical trays required for tray column. Hence, a larger number of transfer units will be required for a very high purity product.

HeightofTransferUnits(HTU)

$\frac{G/}{k_y a\,(1-y)_{iM}}$ remains constant at the packing section though G/ varies. This quantity is called 'height if transfer units' (HTU) and designated as H_{tG}. It is important to measure the separation effectiveness of the particular packings for a particular separation process. It also describes the mass transfer coefficient. Larger mass transfer coefficient leads to the smaller value of HTU.

HeightEquivalenttoTheoreticalPlate(HETP)

For a specified separation job, in packed tower, the height of packing is to be determined and in tray tower, numbers of ideal trays are determined. The ratio between packing height to number of trays required for the same separation is called height equivalent to theoretical plate (HETP).

$$HETP = \frac{h_T}{N_T} = \frac{Height\ of\ packing}{Number\ of\ ideal\ trays}$$

Height of packing (Z) = HETP x Number of trays

HETP varies with size and type of packing, flow rate of gas and liquid, concentration of solute, physical and transport properties as well as equilibrium relationship and uniformity of liquid and gas distribution. HETP is used to characterize the packing. A good packing has small HETP.

AbsorptionFactor

Where *A* is absorption factor and is defined as $A = L/(mG)$.

Absorptionwithchemicalreaction

In ammonia removal from ammonia air mixture by using water has solvent the chemical reaction will not play an appreciable role.

In the following examples

- removal of NO_x and SO_x from the dust laden gas by using water,

- Removal of CO_2 from industrial flue gas by using sodium hydroxide as solvent, chemical reaction play a vital role.

Henry'sLaw

A law stating that the mass of a dissolved gas in a given volume of solvent at equilibrium is proportional to the partial pressure of the gas

Where, H is the Henry's constant, is the partial pressure of A in gas phase, is the concentration of component A in the liquid phase.

StrippingorDesorption

- Mass transfer occurs from liquid to gas phase.
- The solute is removed from the liquid solution by contacting with gas.

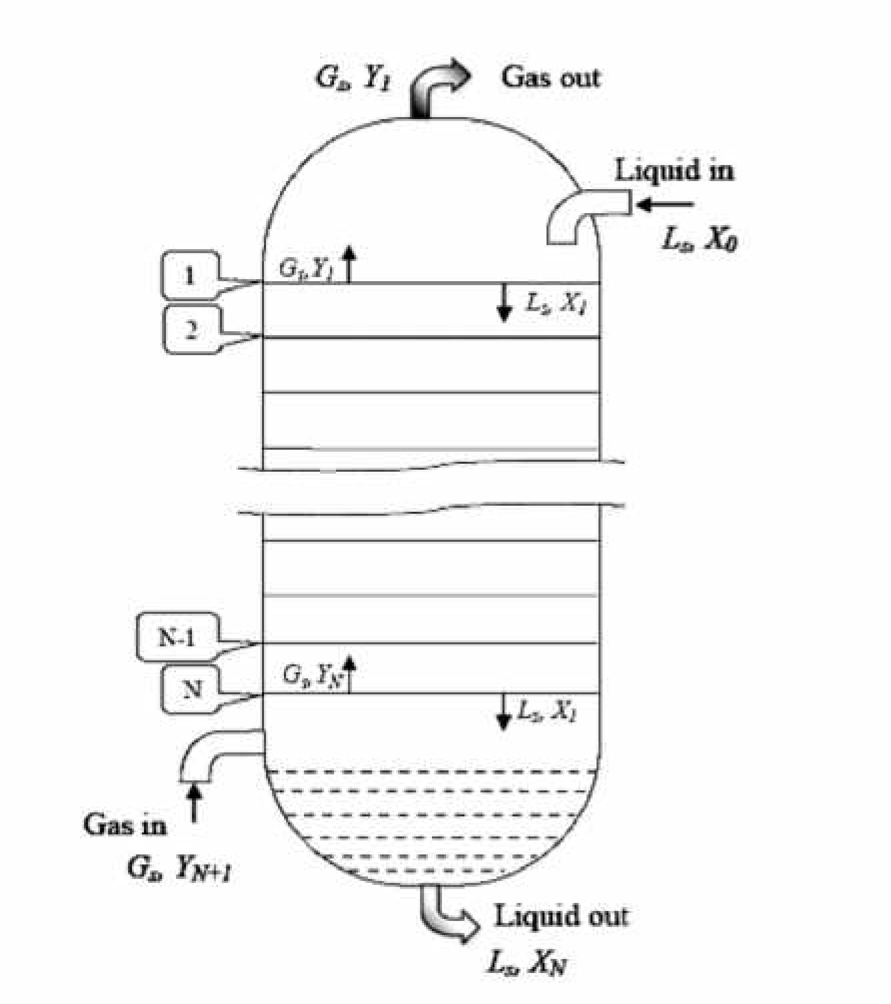
SinglestagecountercurrentUnit

G_1, G_2 : molar flow rates of entering and leaving gas

G_S: molar flow rate of inert gas

L_1, L_2 are the molar flow rates of leaving and entering liquid

L_S is the molar flow rate of pure solvent



Where X = Y =

X,Y are the mole ratios of solute to **inert component** in liquid and gas phase respectively

x,y are the mole ratio of solute in liquid and gas phases

Making Material Balance

$$G_s(Y_1 - Y_2) = L_s(X_1 - X_2)$$

STEP-BY-STEP PROCEDURE

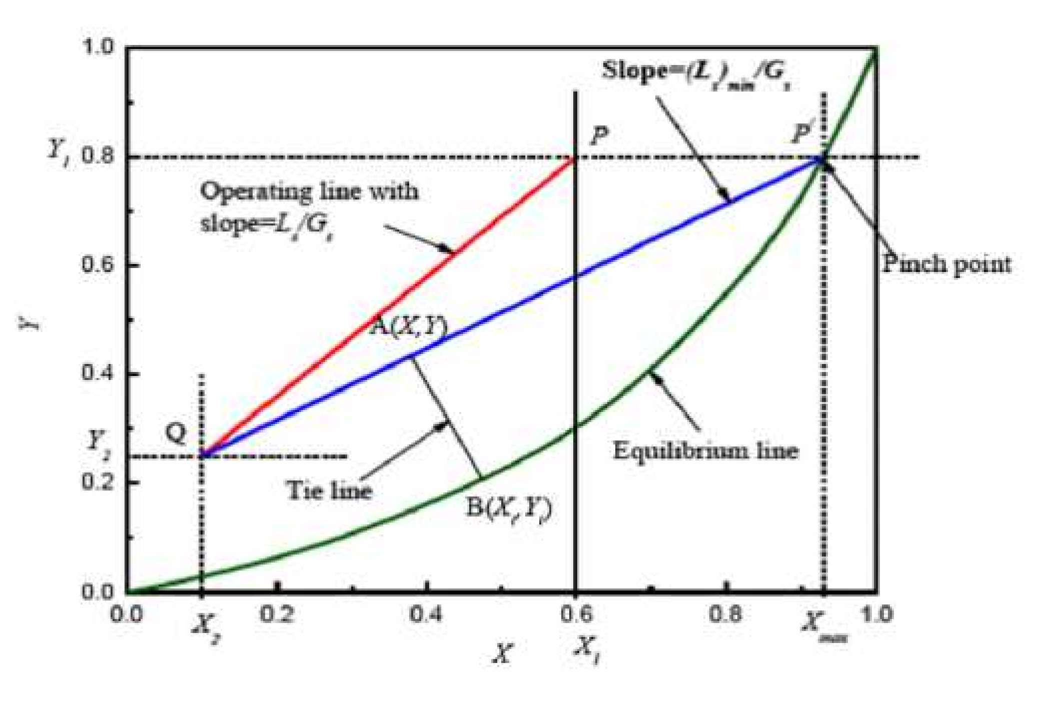
(1) For a particular gas-liquid system, draw equilibrium curve on *X*-*Y* plane.

(2) Draw operating line in *X*-*Y* plane (*PQ*) using material balance Equation. Lower terminal *Q* (*X2, Y2*) and upper terminal *P* (*X1, Y1*) are placed in *x*-*y* plane.

Making Material Balance

$$G_s(Y_1 - Y_2) = L_s(X_1 - X_2)$$

If liquid mass flow rate, *Ls* is not known, minimum liquid mass flow rate *(Ls)min* is to be determined. *Ls* is generally 1.2 to 2 times the *(Ls)min*.



Graphical determination of (L_s)$_{min}$ for absorption.

In the above figure, lower terminal of absorption tower is represented by *Q* (*X2, Y2*); i.e., bottom of the tower. Operating line is *PQ*. If liquid rate is decreased, slope of operating line (*Ls/Gs*) also decreases and operating line shifts from *PQ* to *P'Q* when touches equilibrium line. This perating line is tangent to equilibrium line.

$$\text{Slope of } P^{/}Q = \frac{(L_s)_{min}}{G_s}$$

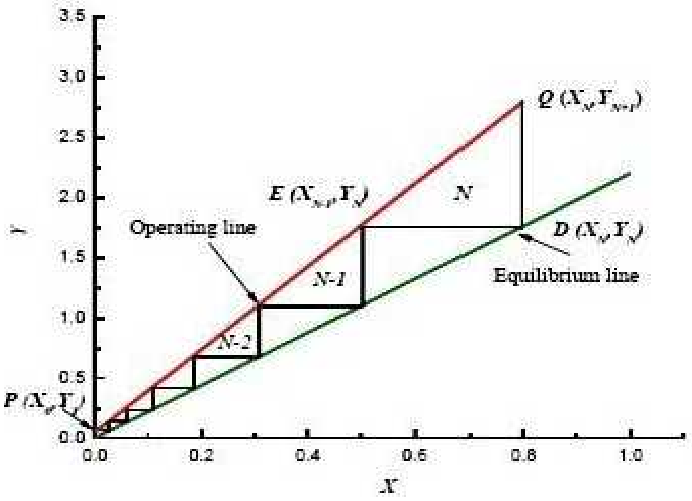
The driving force for absorption is zero at $P^{/}$ and is called "PINCH POINT".

(3) A point *A* (*x, y*) is taken on the operating line. From the known value of k_x and k_y or $k_x\bar{a}$ and $k_y\bar{a}$, a line is drawn with slope of k_x/k_y to equilibrium line, $B(x_i,y_i)$. Line *AB* is called "TIE LINE" and x_i and y_i are known for a set of values of *x* and *y*.

(4) Step (3) is repeated for other points in the operating line to get several (x_i,y_i) sets for $y_1 \geq y \geq y_2$.

(5) Calculate flow rate of gas *G* (kg/h) at each point as $G=G_s(1+y)$.

The following parameters should be known for the determination of “number of stages” (1) Gas feed rate, (2) Concentration of gas at inlet and outlet of the tower, (3) Minimum liquid rate; actual liquid rate is 1.2 to 2 times the minimum liquid rate, (4) Equilibrium data for construction of equilibrium curve now,



Packed Bed Absorber

-
- **HTU =**
-
-
- **Z = No of trays x HETP**

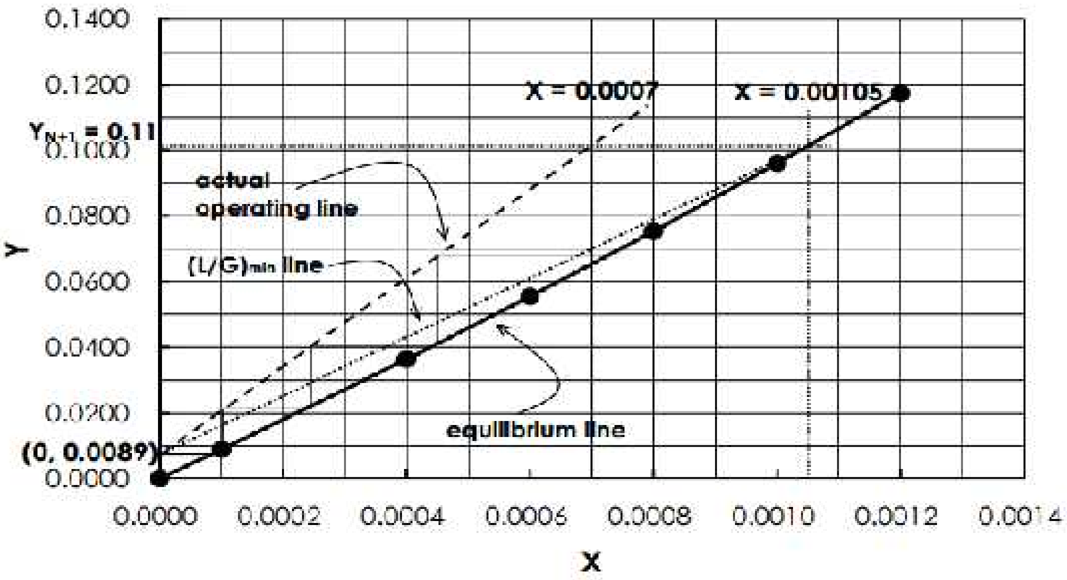
Example

A gas stream containing 90 mol% N_2 and 10% CO_2 is passed through an absorber, in which pure and cool water at 5°C is used as a solvent The operation is assumed to be isothermal at 5 o C and isobaric at 10 atm The liquid flow rate is 1.5 times the minimum liquid flow rate Determine the number of equilibrium stages required to absorb 92 mol% of CO_2 Given Henry's constant of CO_2 in water at 5ºC of 876 atm/mole fraction.

=

the equilibrium or the y-x data can be computed

x	**0**	**0.0001**	**0.0004**	**0.0006**	**0.0008**	**0.0010**	**0.0012**
y	**0**	**0.00876**	**0.0350**	**0.0526**	**0.0701**	**0.0876**	**0.1051**
X	**0**	**0.0001**	**0.0004**	**0.0006**	**0.0008**	**0.0010**	**0.0012**
Y	**0**	**0.00884**	**0.0363**	**0.0555**	**0.0754**	**0.0960**	**0.1175**



The slope of the min L/G minimum is found to be 97.2

Hence, the slope of the actual operating line is 1.5 times of minimum = 1.5 x 97.2 = 145. 8

X_1 = 0.0007

Yields the number of equilibrium stages of ~3.8 = 4

Exercise problems-

1. NH_3 – air mixture containing 5% NH_3 is scrubbed with water to remove NH_3, 5000 kg/hr of gas mixture is to be processed with 2,00,000 kg/hr of water to reduce NH_3 in the exit gas to 0.15%.Calculate the height of packing required assuming dilute solutions are involved. Equilibrium relation is given by Y = 20 X where X,Y = mole fraction of NH_3 in liquid and vapor. HTU = 2m.
2. SO_2–air mixture containing 8% SO_2 at 20°C and 1 atm is to be scrubbed with water in a sieve plate column. The air and water flow rate is 32.6kg/m²s. If 95% of SO_2 is to be recovered. Calculate the number of theoretical trays required.

   Equilibrium Data at 20°C and 1 atm:

p_A, partial pressure in mmHg of SO_2 gas	12	18.2	31.7	50	69.6	106
C_A, gm of SO_2/100gmof water	2	3.5	5.0	7.5	10.0	20

3. Ammonia is absorbed from gas by using water in a scrubber under atm. Pressure the initial ammonia content is 0.04 kmol / kmol of inert gas. The recovery of ammonia by absorption is 90%. The water enters the tower free from solute. Estimate the concentration of ammonia in the exit stream and number of theoretical stages required?

X	0.005	0.01	0.0125	0.015	0.02	0.027
Y	0.0045	0.0102	0.0138	0.018	0.027	0.039

4. It is desired to absorb 95% ammonia from a feed mixture contain 10% ammonia and rest air. The gas enters at rate of 500kmol/hr. if water is used as solvent at the rate of 1.5 times of the minimum calculate (a) NTU (b) Ls actual (c) height of the tower if HTU = 1m the equilibrium relationship is given by Y=20X

5. A packed tower is designed to recover 98% CO_2 from gas mixture contain 10% CO_2 and 90% air using water. The equilibrium relation is Y=14X where Y is kg CO_2 / kg dry air. And X is kg CO_2 / kg water. The water to gas ratio is kept at 30% more than the minimum value. Calculate height of tower if HTU is 1.25m.

6. A packed tower is used to remove 96 % SO_2 from a stream of air containing 0.0291 mole fraction SO_2. The SO_2 is removed by absorbing it in water which enters the tower with a mass velocity of 0.51 k mol/m²s and the gas stream enters the tower counter currently at the mass velocity of 9.75 X 10^{-3}kmol/m²s. Determine the packed tower height. The following data are available.
   K_x a = 0.94 kmol/m³s, K_ya = 0.0496 kmol/m³s. Slope of the equilibrium line = 28.7

7. A mixture of acetone vapour and air containing 5% by volume of acetone is to be freed of its acetone content by scrubbing it with water in a packed bed absorber. The flow rate of the gas mixture is 700 m³/hr of acetone-free air measured at NTP and that of water is 1500 kg/hr. The absorber operates at an average temperature of 20°C and a pressure of 101 kPa. The scrubber absorbs 98% acetone. The equilibriumrelation for the acetone vapour-water system is given by: $Y^* = 1.68X$

Where, Y^* = kmol acetone/kmol dry air

X = kmol acetone/kmol water

Calculate: (a) mean driving force for absorption

(b) mass transfer if the overall mass transfer coefficient is $k_G = 0.4$ kmol of acetone/m^{2}hr(kmol acetone/kmol dry air)

8. One kilo-mole per unit time of gas consisting of 75% methane and 25% n-pentane vapor at 27 deg C 1atm, is to be scrubbed with 2kmol/unit time of non volatile paraffin oil, molecular weight is 200 entering the absorber free of pentane at 35 deg C. Compute the number of ideal trays for 98% recovery of pentane. The equilibrium data

X	0	0.13	0.18	0.2	0.28	0.33
Y	0	0.1	0.125	0.15	0.2	0.25

9. Ammonia air mixture contain 2% by volume is to be scrubbed with water at 20deg C in a tower packed with 1.27cm Rasching Rings. The water and gas rate are 1170 kg/hr each based on the empty tower cross section. Estimate the height of the tower required if 98% of the ammonia is entering gas is to be absorbed. The tower operates at 1atm. The equilibrium relationship is given by $Y = 0.746\ X$

10. Acetone is to be recovered from a 5 % acetone air mixture by scrubbing with water in a packed tower the liquid rate is 0.85kg/s m^2 and the gas rate is 0.5 kg/s m^2. The overall mass transfer coefficient is 1.5×10^{-4} what should be the height of the tower for 98% recovery?

X	0.0099	0.0196	0.0360	0.04
Y	0.0076	0.0156	0.0306	0.0333

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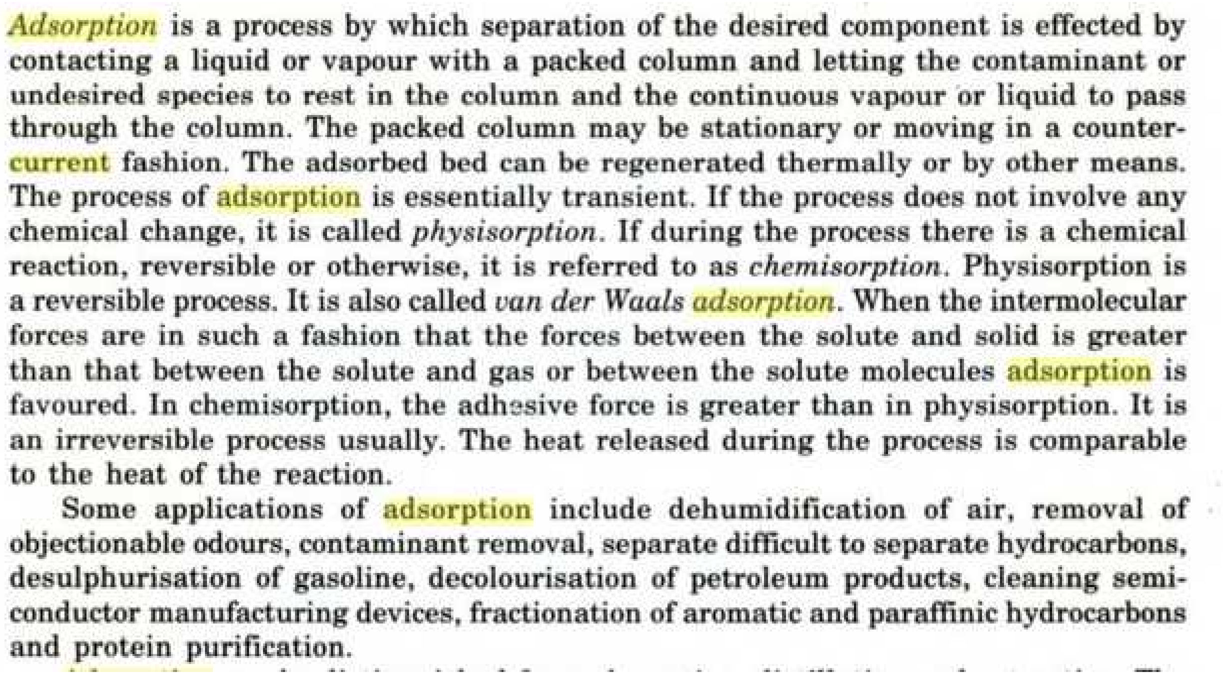
UNIT – III – MASS TRANSFER – II – SCHA1501

UNIT4ADSORPTION

- Removal of solute from the fluids by using suitable solid.
- The separation of colour from effluent by using charcoal.



Adsorption is a process by which separation of the desired component is effected by contacting a liquid or vapour with a packed column and letting the contaminant or undesired species to rest in the column and the continuous vapour or liquid to pass through the column. The packed column may be stationary or moving in a counter-current fashion. The adsorbed bed can be regenerated thermally or by other means. The process of adsorption is essentially transient. If the process does not involve any chemical change, it is called *physisorption*. If during the process there is a chemical reaction, reversible or otherwise, it is referred to as *chemisorption*. Physisorption is a reversible process. It is also called *van der Waals* adsorption. When the intermolecular forces are in such a fashion that the forces between the solute and solid is greater than that between the solute and gas or between the solute molecules adsorption is favoured. In chemisorption, the adhesive force is greater than in physisorption. It is an irreversible process usually. The heat released during the process is comparable to the heat of the reaction.

Some applications of adsorption include dehumidification of air, removal of objectionable odours, contaminant removal, separate difficult to separate hydrocarbons, desulphurisation of gasoline, decolourisation of petroleum products, cleaning semi-conductor manufacturing devices, fractionation of aromatic and paraffinic hydrocarbons and protein purification.

DifferencesbetweenPhysisorptionandChemisorption

PHYSICAL ADSORPTION	CHEMISORPTIONS
The forces operating in these are weak vander Waal's forces.	The forces operating in these cases are similar to those of a chemical bond.
The heat of adsorption are low i.e. about 20 – 40 kJ mol^{-1}	The heat of adsorption are high i.e. about 40 – 400 kJ mol^{-1}
No compound formation takes place in these cases.	Surface compounds are formed.
The process is reversible i.e. desorption of the gas occurs by increasing the temperature or decreasing the pressure.	The process is irreversible. Efforts to free the adsorbed gas give some definite compound.
It does not require any activation energy.	It requires any activation energy.
This type of adsorption decreases with increase of	This type of adsorption first increases with

temperature.	increase of temperature. The effect is called activated adsorption.
It is not specific in nature i.e. all gases are adsorbed on all solids to some extent.	It is specific in nature and occurs only when there is some possibility of compound formation between the gas being adsorbed and the solid adsorbent.
The amount of the gas adsorbed is related to the ease of liquefaction of the gas.	There is no such correlation exists.
It forms multimolecular layer.	It forms unimolecular layer.

AdsorptionIsotherm

At constant temperature the relation between the amount of gas adsorbed on the solid and the concentration of fluid is termed as adsorption isotherm.

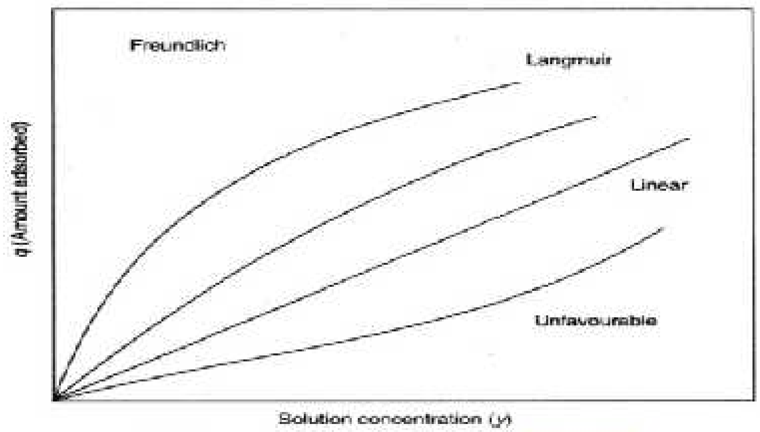


FIGURE Typical isotherms during adsorption.

AdsorptionIsobar

At constant pressure the relation between temperature and amount of gas adsorbed is termed as adsorption isobar

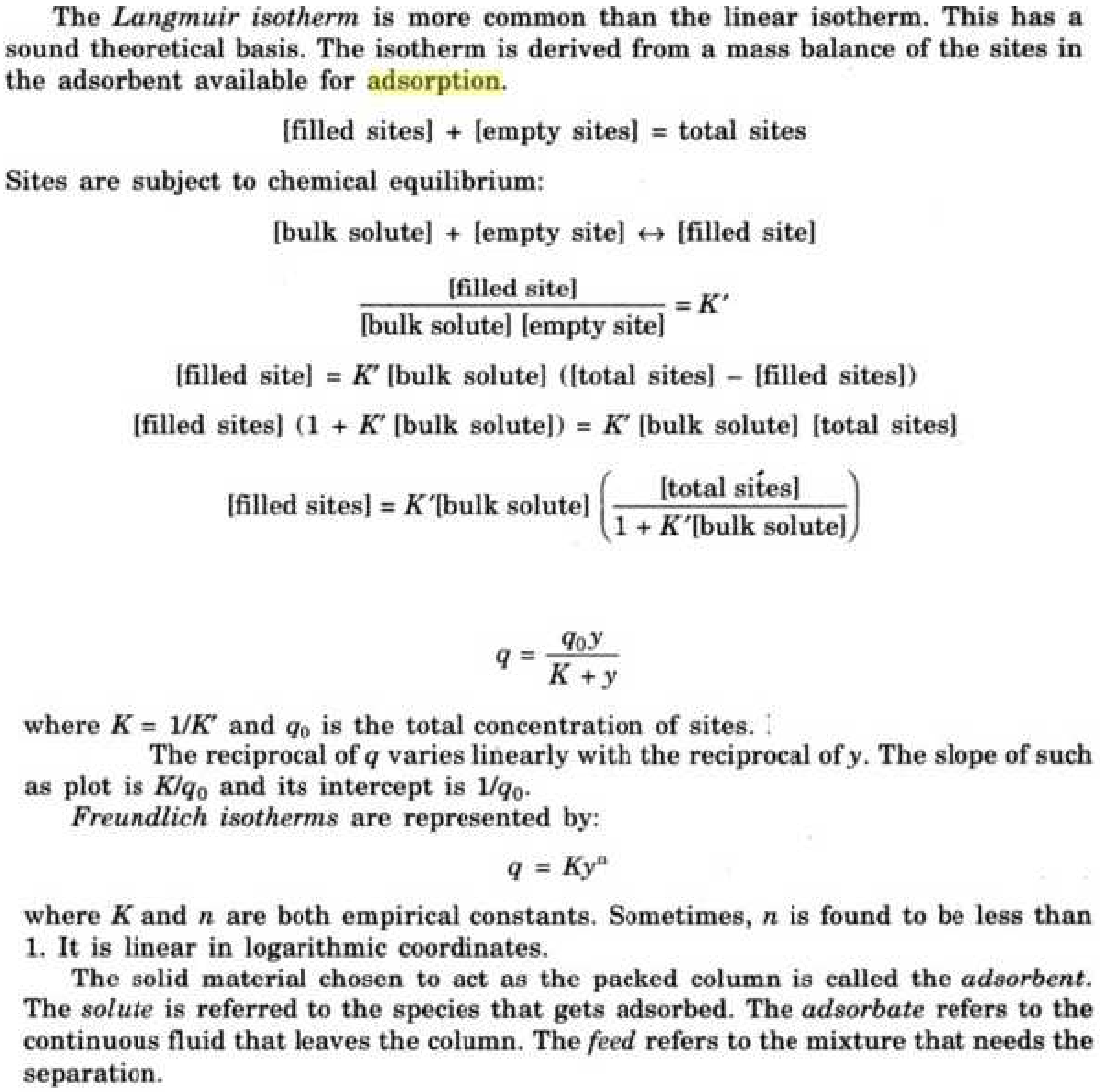
AdsorptionIsostere

At constant amount of gas adsorption the relation between temperature and pressure is termed as adsorption isobar.

Types of Adsorption Isotherms

1. Langmuir
2. BET
3. Freundlich

Langmuir Isotherm



The *Langmuir isotherm* is more common than the linear isotherm. This has a sound theoretical basis. The isotherm is derived from a mass balance of the sites in the adsorbent available for adsorption.

$$[\text{filled sites}] + [\text{empty sites}] = \text{total sites}$$

Sites are subject to chemical equilibrium:

$$[\text{bulk solute}] + [\text{empty site}] \leftrightarrow [\text{filled site}]$$

$$\frac{[\text{filled site}]}{[\text{bulk solute}]\,[\text{empty site}]} = K'$$

$$[\text{filled site}] = K'\,[\text{bulk solute}]\,([\text{total sites}] - [\text{filled sites}])$$

$$[\text{filled sites}]\,(1 + K'\,[\text{bulk solute}]) = K'\,[\text{bulk solute}]\,[\text{total sites}]$$

$$[\text{filled sites}] = K'[\text{bulk solute}]\left(\frac{[\text{total sites}]}{1 + K'[\text{bulk solute}]}\right)$$

$$q = \frac{q_0 y}{K + y}$$

where $K = 1/K'$ and q_0 is the total concentration of sites.

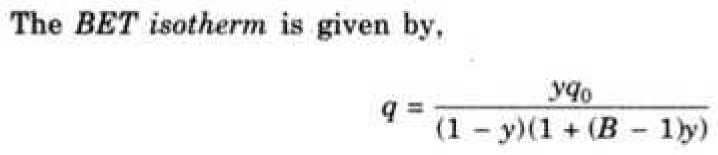
The reciprocal of q varies linearly with the reciprocal of y. The slope of such as plot is K/q_0 and its intercept is $1/q_0$.

Freundlich isotherms are represented by:

$$q = Ky^n$$

where K and n are both empirical constants. Sometimes, n is found to be less than 1. It is linear in logarithmic coordinates.

The solid material chosen to act as the packed column is called the *adsorbent*. The *solute* is referred to the species that gets adsorbed. The *adsorbate* refers to the continuous fluid that leaves the column. The *feed* refers to the mixture that needs the separation.



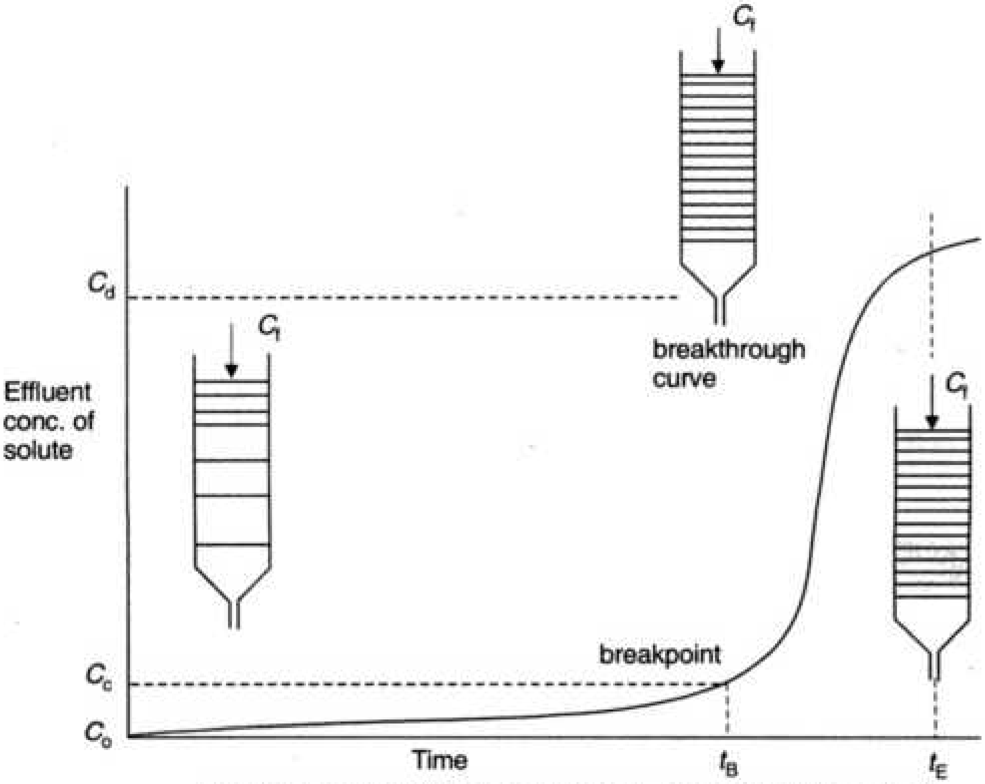
The *BET isotherm* is given by,

$$q = \frac{yq_0}{(1 - y)(1 + (B - 1)y)}$$

IndustrialAdsorbents

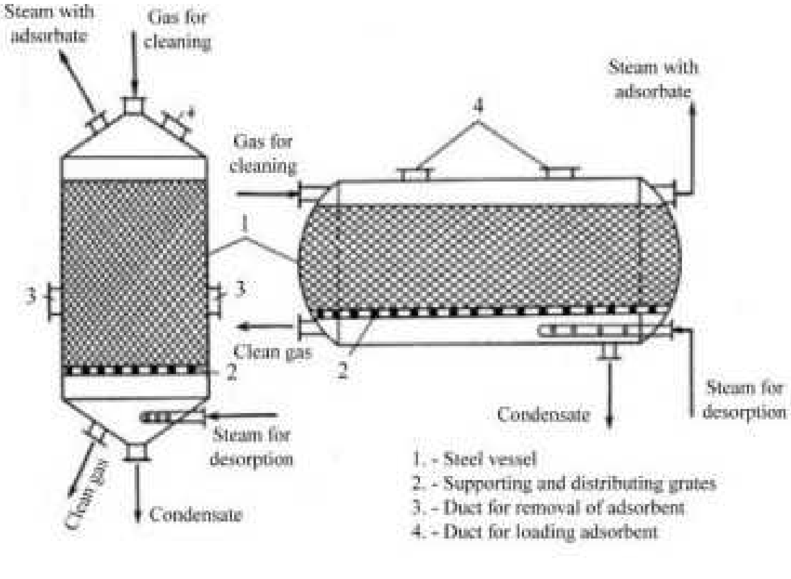
1. Silica gel
2. Activated carbon
3. Alumina
4. Bauxite
5. Molecular sieves
6. Bone char

BreakthroughCurve

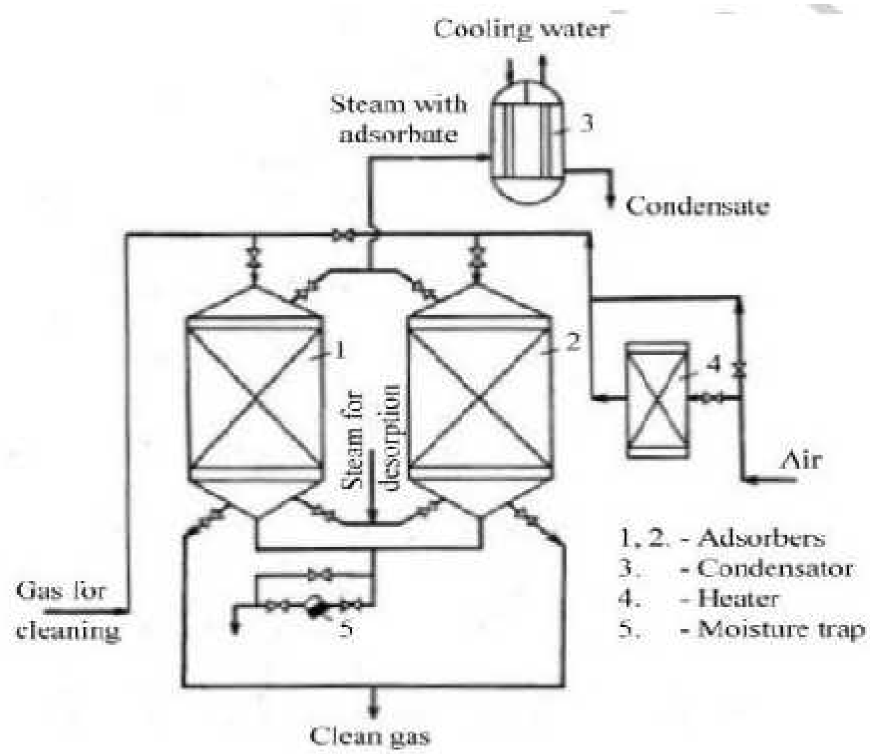


Breakthrough curve, breakthrough time, exhaustion time.

FixedBedAdsorption



Continuous Adsorbers



Cross Current Adsorption

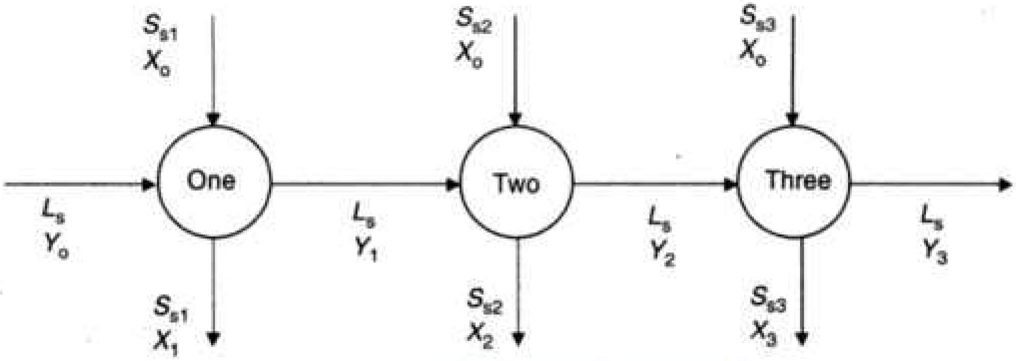


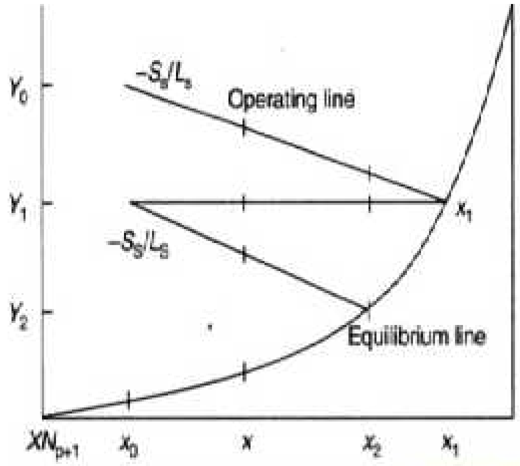
FIGURE 5.6 Multi-stage cross-current adsorption operation.

When the costs are tallied, the total cost including the filtration costs in between stages and the actual carbon used as a adsorbent, it is found that usually two stages are economical. From the mass balances as is shown in Eqs. (5.15 and 5.16),

$$-\frac{S_{s1}}{L_s}=\frac{(Y_0-Y_1)}{(X_0-X_1)}$$

$$-\frac{S_{s2}}{L_s}=\frac{(Y_1-Y_2)}{(X_0-X_2)}$$

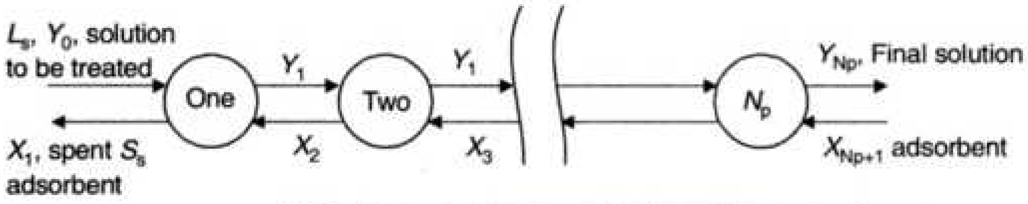
$$-\frac{S_{s3}}{L_s}=\frac{(Y_2-Y_3)}{(X_0-X_3)}$$



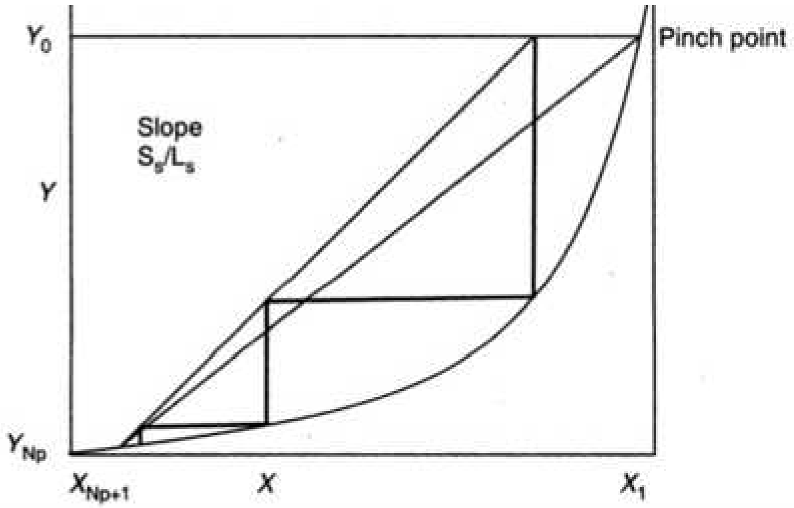
Operating line, equilibrium for staged adsorption operations.

Multistage Counter Current Adsorption

$$L_s(Y_0 - Y_{Np}) = S_s(X_1 - X_{Np+1})$$



Multi-stage counter-current adsorption operations.



Number of ideal stages required for counter-current multi-stage adsorption.

The number of stages can be calculated using analytically based on the adsorption factor.

$$\frac{\ln\left((A_d - 1)\frac{(X_{Np+1} - X_1)}{(X_1 - X_0)} + A_d\right)}{\ln(A_d)} - 1 = N_p$$

1. A solid adsorbent is used to remove the colour impurities from an aqueous solution. The original value of colour on an arbitrary scale is 48. It is required to reduce this value to 10% of its original value. Using the following data find the quantity of fresh adsorbent used for 1000 kg of a solution for
    (i) Single stage

    (ii) Plot the adsorption isotherm

Equilibrium data:

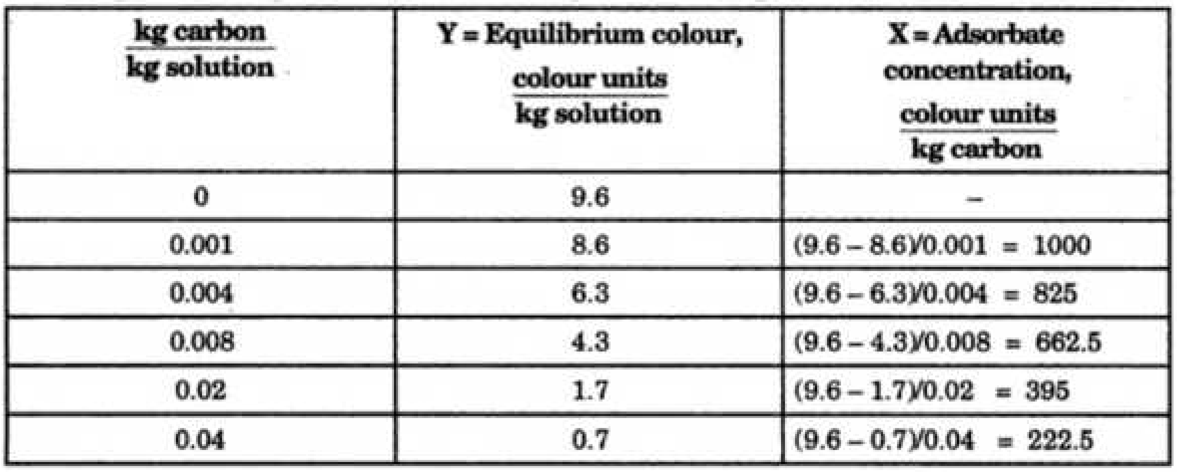
	0	0.001	0.004	0.008	0.02	0.04
Equilibrium colour	9.6	8.6	6.3	4.3	1.7	0.7

Solution

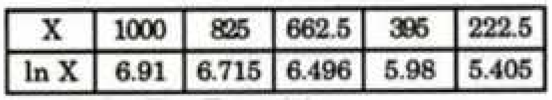
Basis : Ls =1000 kg solution

Unit of Y = unit of colour / kg solution

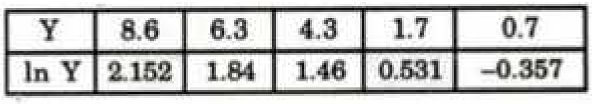
Unit of X = unit of colour / kg adsorbent



$\frac{\text{kg carbon}}{\text{kg solution}}$	Y = Equilibrium colour, $\frac{\text{colour units}}{\text{kg solution}}$	X = Adsorbate concentration, $\frac{\text{colour units}}{\text{kg carbon}}$
0	9.6	–
0.001	8.6	(9.6 – 8.6)/0.001 = 1000
0.004	6.3	(9.6 – 6.3)/0.004 = 825
0.008	4.3	(9.6 – 4.3)/0.008 = 662.5
0.02	1.7	(9.6 – 1.7)/0.02 = 395
0.04	0.7	(9.6 – 0.7)/0.04 = 222.5



X	1000	825	662.5	395	222.5
ln X	6.91	6.715	6.496	5.98	5.405

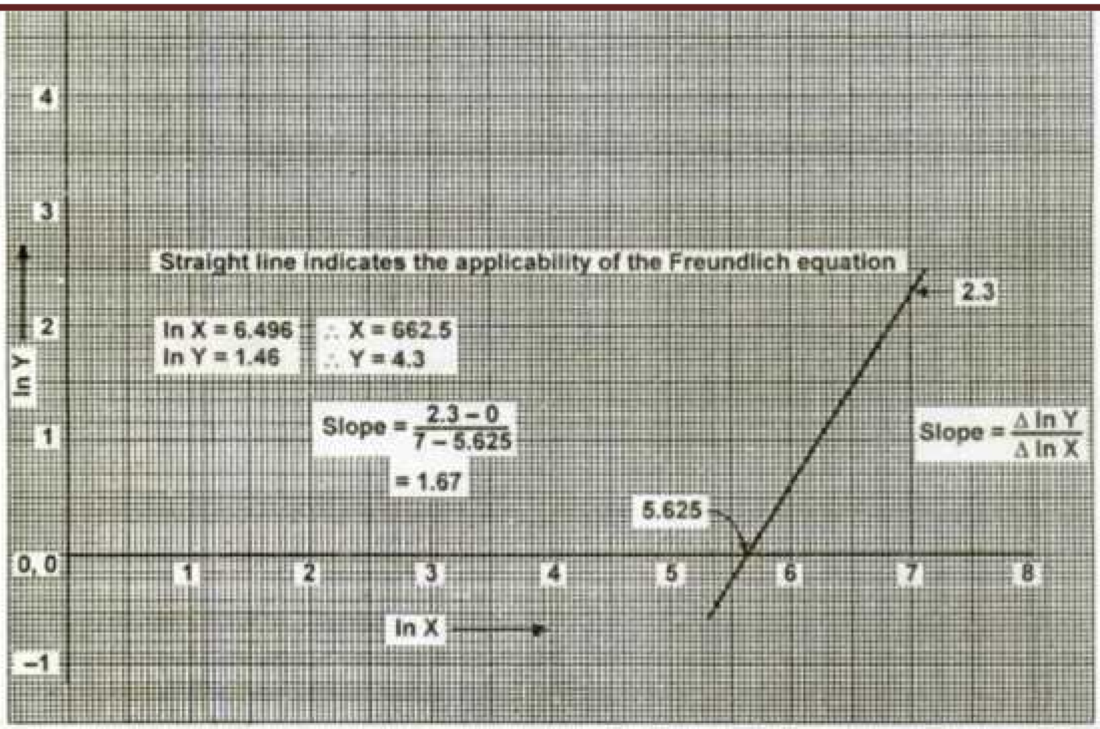


Y	8.6	6.3	4.3	1.7	0.7
ln Y	2.152	1.84	1.46	0.531	–0.357

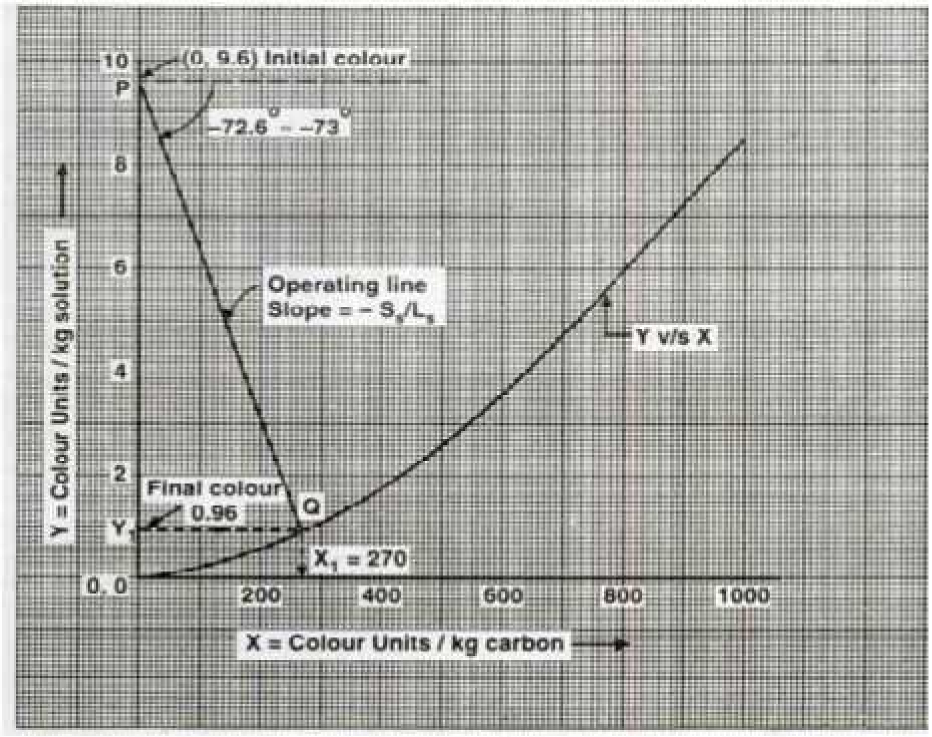
The Freundlich equation

$Y = mX^n$

$Y = 8.36 \times 10^{-5} X^{1.67}$



Log-log plot to determine constants of the Freundlich equation



Fresh solid required = 32kg

Assignment Problems

1. Experiments on decolonization of oil yielded the following equilibrium relationship $y = 0.5x^{0.5}$. where y = grams of colour removed/gm of adsorbent. x = colour in the oil, gm of colour /1 kg of colour free oil. 100kg oil containing 1 part of colour to 3 part of oil is agitated with 25 kg of adsorbent calculate the % of colour removed if
    - (a) All 25 kg absorbent is used in one step
    - (b) 12.5 kg adsorbent is used initially, following by another 12.5 kg of adsorbent.

2. A solid adsorbent is used to remove the colour impurities from an aqueous solution. The original value of colour on an arbitrary scale is 48. It is required to reduce this value to 10% of its original value. Using the following data find the quantity of fresh adsorbent used for 1000 kg of a solution for
    - (i) Single stage
    - (ii) Two stage cross current operation with the intermediate colour value of 24.

Equilibrium data:

	0	0.001	0.004	0.008	0.02	0.04
Equilibrium colour	48	43	31.5	21.5	8.5	3.5

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UNIT – IV – MASS TRANSFER – II – SCHA1501

Liquid Extraction

Liquid extraction produces separation of the constituents of a liquid solution by contact with another insoluble liquid. If the components of the original solution distribute differently between the two liquids, separation will result. The component balances will be essentially identical to those for leaching, but there are two major differences that complicate the calculations:

- the carrier phase is a liquid, not a solid, so the physical separation techniques will change, and
- two distinct phases develop, so the simplicity of uniform solution is lost.

Common applications of liquid extraction include: the separation and purification of lube oils, separation of penicillin from fermentation broth, etc.

Extraction is driven by chemical differences, not by vapor pressure differences, and so can be used in situations when distillation is impractical. For instance, it can be used to separate materials with similar boiling points (so that distillation is impractical) or mixtures containing temperature sensitive compounds.

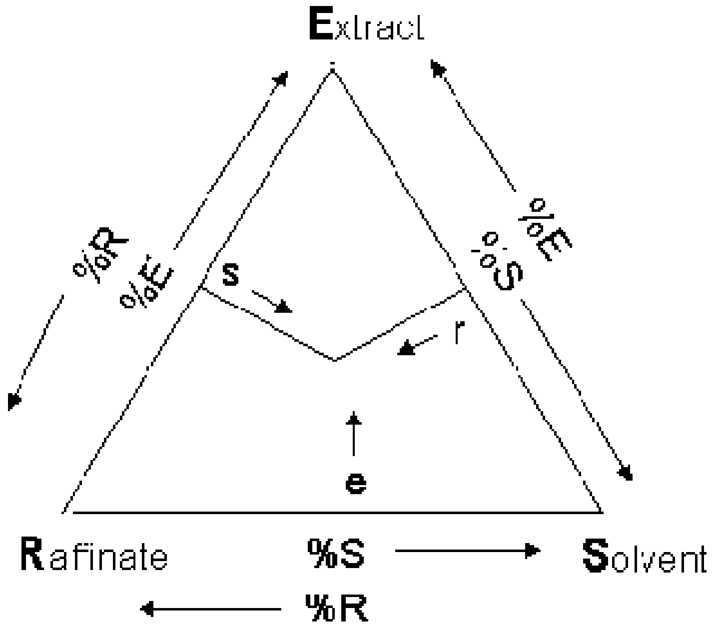
Distillation and evaporation produce finished products; liquid extraction generally does not. The products are still mixtures, although with new compositions, and these must be separated to obtain final products. Secondary separation often requires distillation or evaporation. The overall process cost thus must be considered when choosing extraction.

Extraction may become economical for dilute aqueous solutions when evaporation would require vaporization of very large amounts of water.

Terminology

Certain terms are commonly used when describing extraction processes. The solution to be extracted is called the *feed*, the liquid used in contacting is the *solvent*. The enriched solvent product is the *extract* and the depleted feed is called the *raffinate*.

Extraction processes may be be single stage, multistage crosscurrent, or countercurrent. Cocurrent extraction offers no advantages over a single stage (convince yourself of this!). This class will primarily be concerned with countercurrent systems.

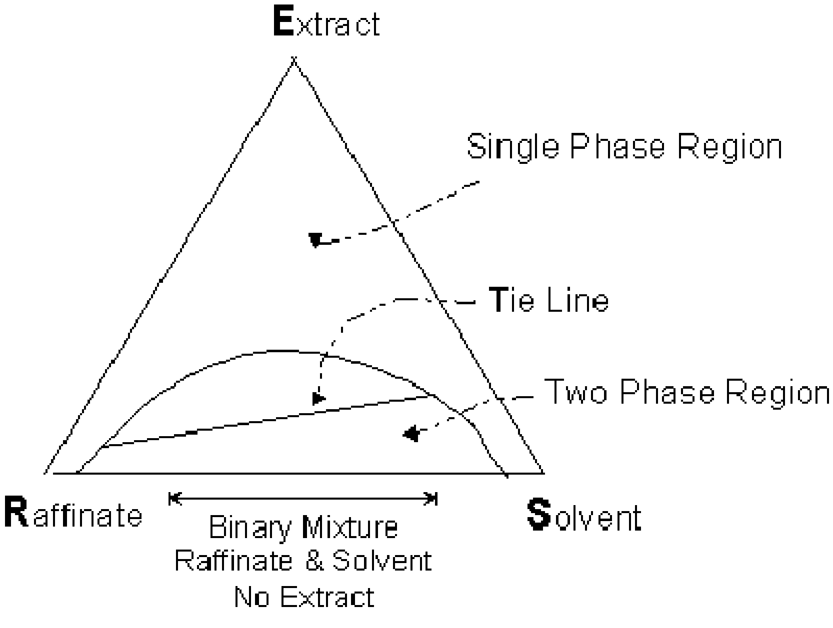


Equilibrium

Extraction calculations require an understanding of ternary equilibrium. You probably should refresh your memory on how ternary diagrams are read and used. I would anticipate that you learned this in your material balance course.

One new term that may not be familiar is the *plait point*. This point is located near the top of the two-phase envelope, at the inflection point. It represents a condition where the 3-component mixture separates into two phases, but the phases have identical compositions. (Compare this with an azeotropic mixture of liquid and vapor.)

There are two main classes of liquid-liquid equilibrium that occur in extraction. A *Class I* system is the one I expect you are familiar with; it has one immiscible pair of compounds and produces the familiar envelope. *Class II* mixtures have two pair of immiscible compounds, and so the two-phase envelope crosses the triangular diagram like a bridge. Class I mixtures are the most common and are preferable -- so if you can pick a solvent to get a Class I, you usually want to do so. Classes can change with temperature, so that is also a concern.



Solvent Selection

One of the key decisions when designing an extraction process is the choice of the solvent to be used. Issues include:

- Selectivity -- compare the equilibrium ratio of solute in each phase
- Distribution Coefficients -- y/x at equilibrium; large values preferable
- Insolubility -- solvent should not be soluble in carrier liquid
- Recoverability -- consider constraints (i.e. azeotropes)
- Density -- must be different so that phases can be separated by settling
- Interfacial Tension -- if too high, liquids will be difficult to mix
- Chemical Reactivity -- solvent should be inert and stable
- Viscosity, Vapor Pressure, Freezing Point -- low values make storage easier
- Safety -- toxicity, flammability
- Cost

Calculations

As with the other separations we discuss, there are two primary calculations:

- the number of stages needed to make a separation (extent)
- the amount of solvent needed to make a separation (rate/capacity)

Since LL equilibrium is seldom available in algebraic form, the calculations tend to be iterative or graphical. You have a choice of graphical approaches depending on the type of equilibrium diagram you have available (or choose to construct):

- A modified McCabe-Thiele approach can be used if *y vs. x* data is available. The coordinates for the diagram are the mass fraction of solute in the extract phase and the mass fraction in the raffinate for the other. The curve is typically concave downward, begins at the origin and ends with the plait point composition.
- When one has a convenient equilateral triangle diagram, construction can be done directly on the triangle. Some authors refer to this as the Hunter-Nash method.
- Rectangular equilibrium diagrams can be constructed. These look a lot like the enthalpy-composition diagrams from distillation calculations or the "solid-free" diagrams used in leaching.

Solvent-to-Feed Ratio

For a given feed mixture, required degree of extraction, operating pressure and temperature, and choice of solvent, there is a minimum solvent-to-feed ratio which corresponds to an infinite number of contact stages.

As with the other separations we have studied, this corresponds to a "pinch" between the equilibrium and operating curves at the feed composition. Algebraically, this corresponds to an extract phase in equilibrium with the entering feed. The pinch can also be found graphically -- on a McCabe-Thiele type construction, minimum solvent ratio corresponds to a pinch (curves intersecting) at the feed composition. During a triangular construction, a feed pinch is represented by the operating line overlapping a tie-line and running through the feed point.

A theoretical upper limit or maximum solvent-to-feed ratio also can be determined. If you visualize the ternary diagrams, you notice that if enough solvent is added, the equilibrium curve is crossed and the single phase region is entered. Once this happens, it is impossible to divide the mixture into different phases, hence no separation is achieved. The maximum solvent-to-feed ratio is thus that which puts the mixture on the phase boundary.

McCabe-Thiele Method

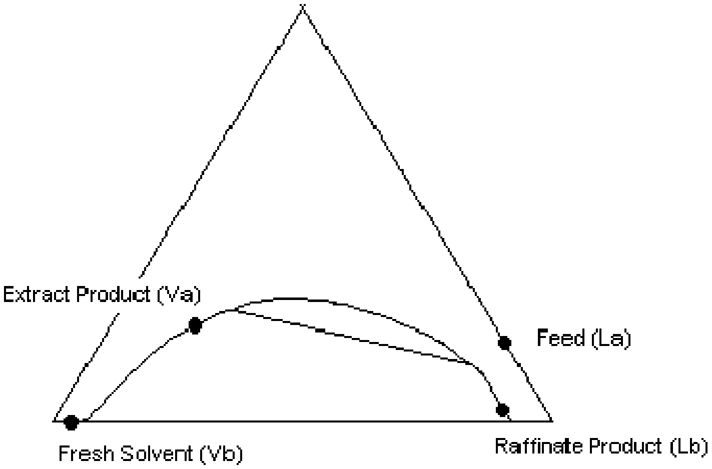
A modified McCabe-Thiele approach is probably the most straightforward graphical technique for solving extraction problems. As always, the main constraint is the equilibrium data. When the data is given in a tabular form, it isn't difficult to construct the needed *y* (solute in the extract phase) vs. *x* (solute in the raffinate phase) diagram; however, it is a bit of a chore to pull the points off of a ternary diagram. In the latter case, it may may sense to construct directly on the triangle.

Once you have the *y vs. x* plot, the component and material balances can be used to set the endpoints of the operating curve. Interior points can be found by selecting an intermediate value of x, and calculating the appropriate y (this typically is an iterative calculation). You want to find enough interior points to be sure of the shape of the curve, but shouldn't have to calculate too many of these points. The operating curve that results will typically be curved. *For extraction calculations, both the equilibrium and the operating equations will be typically be curved.*

Once the curves are available, they can be "stepped off" into triangles, just as one would expect from McCabe-Thiele.

Construction on Ternary Diagram

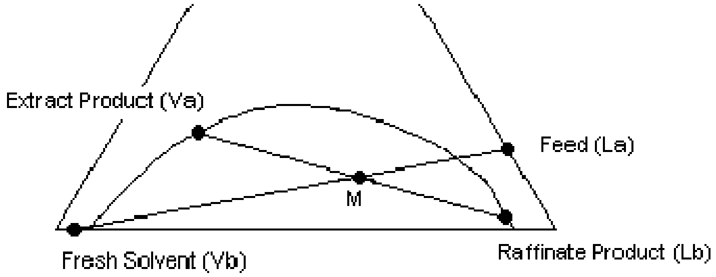
Construction on a ternary diagram is a little messier. The diagrams are typically much more crowded and so counting stages is more complicated. You also typically need substantial extra space on the side of the diagram; often you want to tape a spare sheet of paper in place to get the workspace. Pocket rulers end up being too short, so make sure you



have a longer (~ 2 ft) straightedge around.

The first step is to locate the known endpoints.

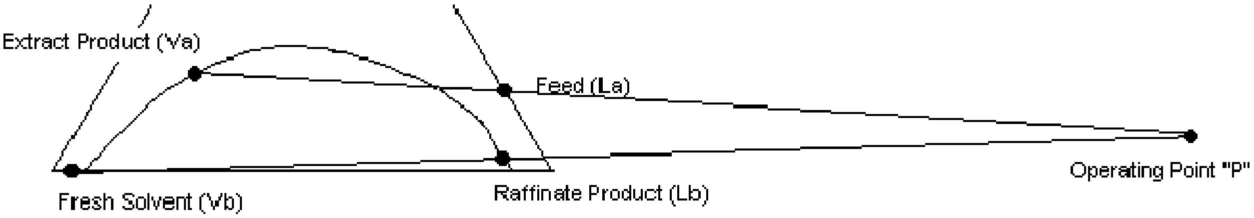
- The fresh solvent point will typically lie on a side of the triangle (it has little or no solute and minimal raffinate). If pure, the fresh solvent point will be at an apex.
- The feed point will lie on the solute-carrier axis; only rarely will the feed contain solvent.
- Products result from equilibrium stage separation, so both will lie on the phase envelope. Often, only one of these is given in the problem statement.



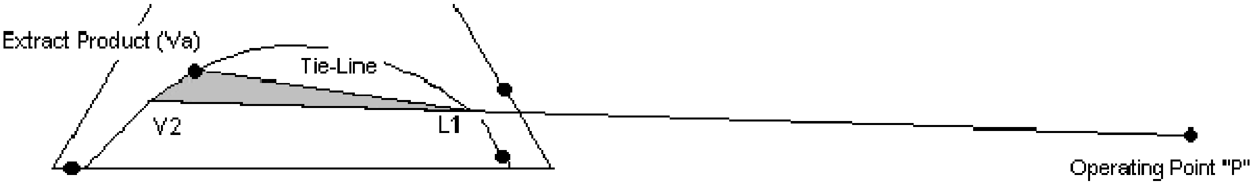
The fundamental idea of all constructions is that a single line connects points made from "mixing" two streams. The endpoints are thus connected two different ways.

First, a segment is drawn connecting the "entering" streams (L_a, V_b) and one between the "leaving" streams (L_b, V_a). These will intersect in the middle of the diagram at a "mixing point", M. Since the two streams leaving an ideal stage, are in equilibrium, this point is related to the equilibrium curve. Lever arm principles apply, so that the M point splits the line segments proportionately to the solvent/feed ratio, so that $\frac{S}{F} = \frac{\overline{MF}}{\overline{SM}}$

An "operating" point, P, is located by connecting the "sides" of the cascade: L_a to V_a and L_b to V_b. The P point can lie on either side of the triangle, depending on the slope of the tie lines. (This is where that extra piece of paper comes in handy!) All possible operating points must pass through the P point.



With the endpoints and the P point, the stages can be stepped off. Begin at the V_a (extract product) point. Trace down a tie- line to the raffinate side of the envelope; the intersection will be at the composition of the stream leaving stage one, so this is the L_1 point. Next, construct a line connecting L_1 with P and extend the line back across to the extract side of the diagram. This is an operating line, and the intersection is the V_2 point. The triangle that has been formed represents one ideal equilibrium stage.



This procedure -- down a tie line, up an operating line -- is repeated until the feed point is reached/passed. The total number of triangles created is the number of stages. Often the construction gets crowded and may be hard to read.

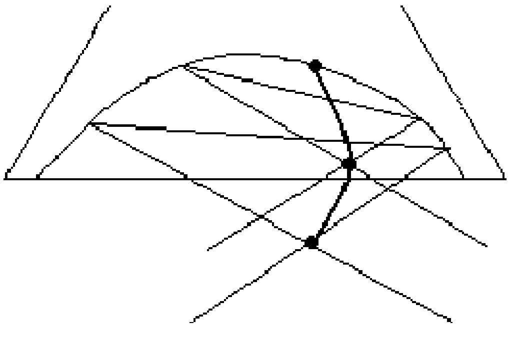
Problems can be varied by changing the given information. You then will need to think through the relationships (solvent/feed ratio, equilibrium tie-lines, operating lines, etc.) and adapt the procedure.

Minimum solvent-to-feed is determined by extending the tie-line that runs through the feed point until it intersects the segment connecting V_b and L_b. This intersection is the point P_{min}, an operating point at minimum S/F. The line is then extended back across the envelope to the extract side. The intersection is the point V_{min}, corresponding to the extract product at minimum S/F. (Remember that a "pinch" on this type of diagram means that the operating and equilibrium (tie-lines) overlap.) A mixing point "M_{min}" can be determined using the feed point, V_b, L_b, and V_{min}, and the lever-arm rule will provide a value for the S/F ratio.

Maximum solvent-to-feed is found by locating a point "M_{max}" on the line connecting the fresh solvent and the feed, at the point where it intersects the extract side of the envelope. S/F is then calculated by lever arm.

Note on Tie Lines

Rarely do you have all the equilibrium tie lines you want. It is thus good to know that there is a fairly easy way of generating additional lines.



To do this, you construct a "conjugate curve" from the existing tie lines. Take each endpoint and draw a line from it downward, perpendicular to base of the triangle. The extensions from the raffinate side will intersect those from the extract side, and each pair forms a point on the conjugate curve. The final point is the plait point. When a new tie line is needed, one composition is chosen, a line is traced down to the conjugate curve, and then back up to the envelope on the other side. This is the other end of the tie line.

Construction on Rectangular Equilibrium Diagram

Graphical solution can be readily done on rectangular equilibrium diagrams, but this method will not be discussed here. This method is usually the first choice only if the equilibrium data is already plotted in rectangular form.

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UNIT – V – MASS TRANSFER – II – SCHA1501

LEACHING

(Liquid-Solid) Leaching

Leaching is a liquid-solid operation. The two phases are in intimate contact, the solute(s) can diffuse from the solid to the liquid phase, which causes a separation of the components originally in the solid. A special leaching process, when an undesirable component is removed from a solid with water, is called washing.

Leaching is widely used in the biological and food processing industries, such as the separation of sugar from sugar beets with hot water, the extraction of oils from peanuts, soybeans, sunflower seeds, cotton seeds, and halibut livers.

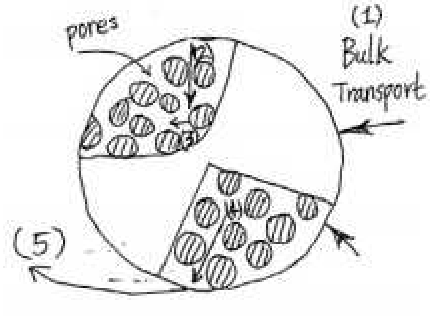
In pharmaceutical industry, many products are obtained by leaching plant roots, leaves, and stems.

In the metals processing industry, leaching is used to remove the metals from their ores, which contains many undesirable constituents, as solute salts. A good example is gold leaching, gold is leached from its ore using an aqueous sodium cyanide solution.

Preparation of Solids for Leaching

This depends on the proportion of the soluble constituent present, its distribution throughout the original solid, the nature of the solid, and the original particle size. If the soluble material is surrounded by a matrix of insoluble matter, the solvent must diffuse inside to contact and dissolve the soluble material and then diffuse out. This is common in leaching metal salts from mineral ores. In these cases crushing and grinding of the ores is used to increase the rate of leaching since the solution portions are made more accessible to the solvent. Biological materials are cellular in structure and the soluble constituents are inside the cells. Because the cell walls provide another resistance to diffusion, the rate of leaching may be slow. In this case the biological materials are cut into thin wedge-shaped slices to reduce the diffusion distance of solvent

Rates of Leaching



Generally there are five rate steps in the leaching process:

1. The solvent is transferred from the bulk solution to the surface of the solid.

2. The solvent penetrates or diffuses into the solid (intraparticle diffusion).

3. The solute dissolves from the solid into the solvent.

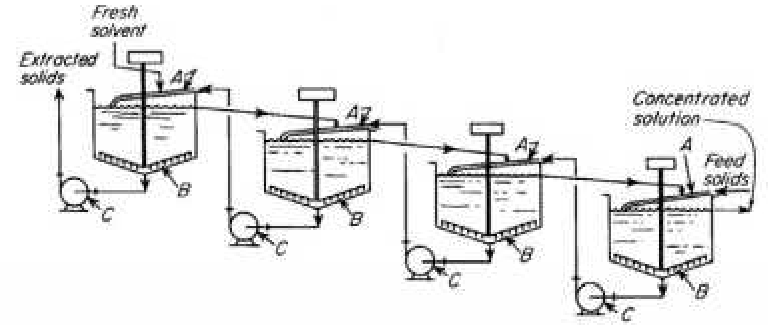
4. The solute diffuses through the mixture to the surface of the solid (intraparticle diffusion).

5. The solute is transferred to the bulk solution.

Step 1 is usually fast. The controlling rate process is generally the intraparticle diffusion or the dissolving step. For the intraparticle diffusion, we will study in more detail in the subject of "Advanced Separation Processes".

Equilibrium Relationship in Leaching

In leaching it is assumed that there is sufficient solvent present so that all the solute in the entering solid can be dissolved into the liquid, equilibrium is reached when the solute is dissolved. Hence, all the solute is completely dissolved in the first stage.



Countercurrent leaching: A, launder; B, rake; C, pump

It is also assumed that the solid is insoluble, and no adsorption will happen for the solute in the solid, meaning that the solution in the liquid phase leaving a stage is the same as the solution remaining with the solid matrix in the settled slurry leaving the same stage. The settled solid leaving a stage always contains some liquid. This solid-liquid stream is called the underflow or slurry stream. The liquid is called the overflow stream. The concentration of oil or solute in the overflow stream is equal to that in the liquid solution accompanying the slurry or underflow stream. Hence, on an xy plot the equilibrium line is on the 45° line.

Equilibrium diagrams for leaching

Since we have three components: solute (A), inert or leached solid (B), and solvent (C), a rectangular diagram is used to show the equilibrium data.

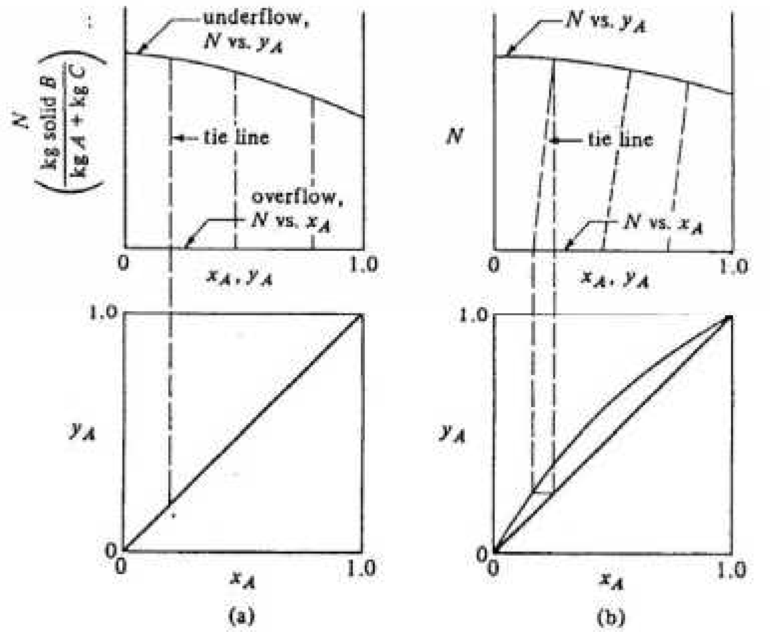
The concentration of inert or insoluble solid B in the solution mixture or the slurry mixture is defined as N= =

For the overflow, N = 0, for the underflow, N depends on the solute concentration in the liquid. The compositions of solute A in the liquid as weight fractions are

$x_A =$ = (overflow liquid)

$y_A =$ = (liquid in slurry)

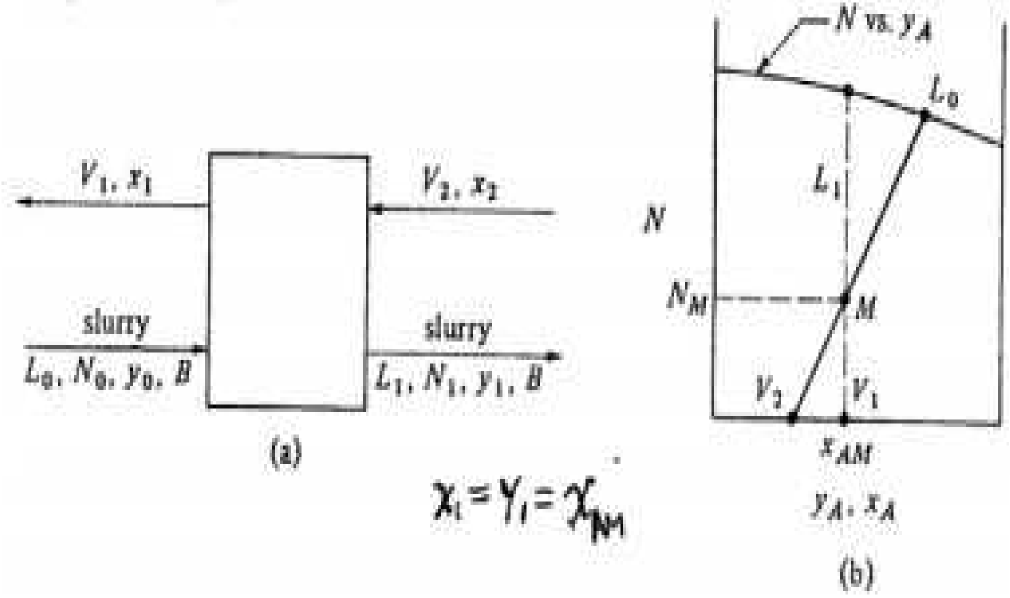
For the entering solid feed to be leached, N is kg inert solid/kg solute A and $y_A = 1.0$. For pure entering solvent N = 0 and $x_A = 0$.



Several typical equilibrium diagrams: (a) case for vertical tie lines and $y_A = x_A$, (b) case where $yA \neq xA$ for tie lines.

Single-Stage Leaching

The following figure shows a single-stage leaching process where V is kg/h of overflow solution with composition x_A and L is the kg/h of liquid in the slurry solution with composition y_A based on a given flow rate B kg/h of dry solute-free solid.



$x_1 = y_1 = x_{AM}$

By doing material balances,

we have total solution (A+C): $L_0 + V_2 = L_1 + V_1 = M$

(A): $L_0 y_{A0} + V_2 X_{A2} = L_1 y_{A1} + V_1 X_{A1} = M X_{AM}$

(B): $L_0 N_0 + 0 = L_1 N_1 + 0 = M N_M$

A balance on C is not needed since $x_A + X_C = 1$

and $y_A + y_C = 1$

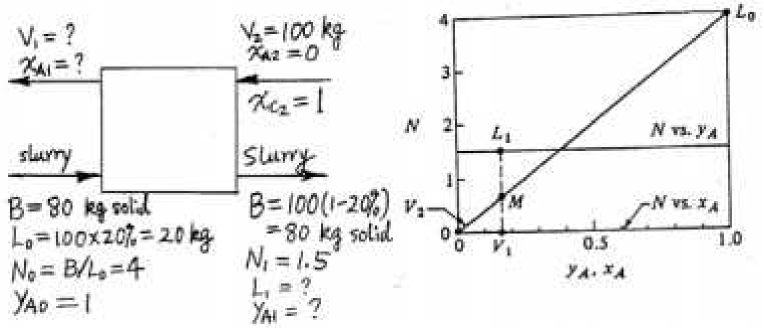
where M is the total flow rate in kg (A+C)/h,

x_{AM} and N_M are the coordinates of this point M.

If L_0 entering is the fresh solid feed to be leached with no solvent C present, it would be located above the N versus y line in the above figure.

Example L1: Single-Stage Leaching of Flaked Soybeans In a Single-Stage Leaching of soybean oil from flaked soybeans with hexane, 100 kg of soybeans containing 20 wt % oil is leached with 100 kg of fresh hexane solvent. The value of N for the slurry underflow is constant at 1.5 kg insoluble solid/kg solution retained. Calculate the amounts and compositions of the overflow V1 and the underflow L1 leaving the stage.

Solution: The process flow diagram and known variables are as follows.



To locate the point M, use the material balances:

$$L_0 + V_2 = 20 + 100 = 120 \text{ kg} = M$$

$$L_0 y_{A0} + V_2 x_{A2} = 20 \times 1 + 100 \times 0 = 20 = M x_{AM} = 120 x_{AM}$$

Hence $x_{AM} = 0.167$.

$$B = L_0 N_0 = 20 \times 4 = M N_M$$

The point M is plotted along with V_2 and L_0. A vertical tie line is drawn to locate L_1 and V_1 in equilibrium with each other. So $N_1 = 1.5$, $y_{A1} = x_{A1} = 0.167$. By using lever-arm rule, we obtain $L_1 = 53.3$ kg and $V_1 = 66.7$ kg.

Countercurrent Multistage Leaching

The ideal stages are numbered in the direction of the solids or underflow stream. The overall balance and the component balance on solute A give

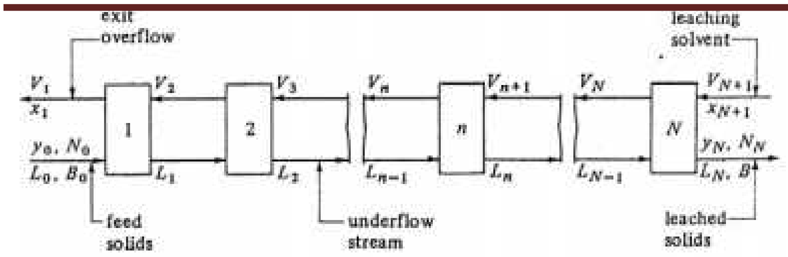
$$L_0 + V_{n+1} = L_n + V_1$$

$$L_0 y_{A0} + V_{n+1} x_{A,n+1} = L_n y_{A,n} + V_1 x_{A1}$$

Solving for xn+1 and eliminating Vn+1, the operating line is obtained as

$$x_{n+1} = \frac{1}{1 + (V_1 - L_0)/L_n} y_n + \frac{V_1 x_1 - L_0 y_0}{L_n + V_1 - L_0}$$

This can be plotted on an xy plot passes through the terminal points (x_1, y_0) and (x_{n+1}, y_n)



If the viscosity and density of the solution change with the solute (A) concentration, the liquid retained in the solid underflow will change and so will the overflow and the slope of the operating line vary. This is called variable underflow. If the amount of solution Ln retained by the solid is constant and independent of concentration, then we will have constant underflow.

Constant Underflow in Countercurrent Leaching

Multistage Leaching In this case the liquid Ln retained in the underflow solids is constant from stage to stage. This means that a plot of N versus yA is a horizontal line and N is constant. The calculation procedure discussed in the previous section for variable underflow can still be used for constant underflow by simply using a horizontal line Δof N versus yA and stepping off the stages with the point. For constant underflow, it is also possible to use the McCabe and Thiele method on the xy diagram, since the operating line is now a straight line. The equilibrium line is usually linear (yA = xA) so we can also use the analytical solution as below. However, special treatment must be given for the first stage, because L0 is generally not equal to Ln , since it contains little or no solvent. A separate material balance must be made on stage 1 to obtain L1 and V_2. Any units may be used in the following equation

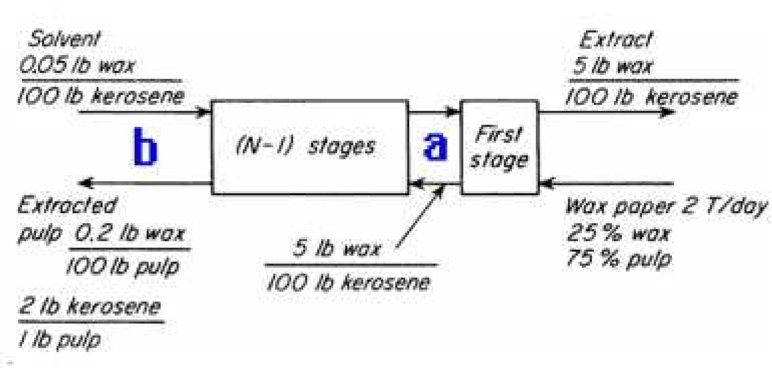
Any units may be used in the following equation

$$N = \frac{\ln\left(\dfrac{x_b - x_b^*}{x_a - x_a^*}\right)}{\ln\left(\dfrac{x_b - x_a}{x_b^* - x_a^*}\right)}$$

as long as the units are consistent and the overflows and underflows are constant.

Example L2. By extraction with kerosene, 2 tons of waxed paper per day is to be dewaxed in a continuous countercurrent extraction system that contains a number of ideal stages. The waxed paper contains, by weight, 25% paraffin wax and 75% paper pulp. The extracted pulp is put through a dryer to evaporate the kerosene. The pulp, which retains the unextracted wax after evaporation, must not contain over 0.2 lb of wax per 100 lb of wax-free pulp. The kerosene used for the extraction contains 0.05 lb of wax per 100 lb of waxfree kerosene. Experiments show that the pulp retains 2.0 lb of kerosene per lb of kerosene- and wax-free pulp as it is transferred from cell to cell. The extract from the battery is to contain 5 lb of wax per 100 lb of wax-free kerosene. How many stages are required?

Solution. Since here the ratio of kerosene to pulp is constant, flow rates should be expressed in pounds of kerosene. All concentrations must be in pounds of wax per pound of wax-free kerosene. The unextracted paper has no kerosene, so the first cell must be treated separately.



The kerosene in with the fresh solvent is found by a wax balance. Take a basis of 100 lb of wax- and kerosene-free pulp, and let s be the pounds of kerosene fed in with the fresh solvent. The wax balance, in lb, is

Wax in with pulp, 100x25/75 = 33.33

Wax in with the solvent, 0.0005s

Total wax input = 33.33 + 0.0005s

Wax out with pulp, 100x0.002 = 0.2

The total kerosene input is S, there are 200 lb of kerosene in the outlet of underflow, so the output of kerosene in the extract = s-200, hence,

Wax out with extract is = (s-200)0.05 = 0.05s – 10

Total wax output = 0.05s – 9.8

Therefore, 33.33 + 0.0005s = 0.05s – 9.8 s = 871 lb. 0.0005s = 0.436

The kerosene in the exhausted pulp is 200 lb, in the strong solution is 871- 200 = 671 lb.

The wax in this solution is 671x0.05 = 33.55 lb.

The concentration in the underflow to the 2nd unit (y_a) = that of the overflow from the 1st stage (x_1) = 33.55/(33.54+671) = 0.0476

The wax in the underflow to unit 2 (W_a) can be found from y_a = 0.0476 = Wa/(Wa+200)

So W_a = 200x0.0476/(1-0.0476) = 9.996 lb.

The wax in the overflow from the 2nd cell to the 1st is, by a wax balance over the 1st unit,

9.996 + 33.55 – 33.33 = 10.22 lb

The concentration of this stream (x_a) = 10.22/(871+10.22) = 0.0116.

$x_a^* = y_a = 0.0476$

$x_b^* = y_b = 0.2/(200+0.2) = 0.001$

$x_b = 0.436/(0.436+871) = 0.0005$

Where x_a, x_b are the overflow concentration of A entering the 1st and final stages, respectively. x_a^* and x_b^* are the overflow concentration of A in equilibrium with y_a an y_b (the underflow concentration of A leaving the 1st and final stages), respectively

Since stage 1 has already been taken into account,

$$N-1=\frac{\ln\left(\frac{x_b-x_b^*}{x_a-x_a^*}\right)}{\ln\left(\frac{x_b-x_a}{x_b^*-x_a^*}\right)}=\frac{ln\left(\frac{0.0005-0.001}{0.0116-0.0476}\right)}{ln\left(\frac{0.0005-0.0116}{0.001-0.0476}\right)}=3$$

The total number of ideal stages is N = 1+3 = 4.

Variable Underflow in Countercurrent Multistage Leaching

The material balances for the whole system are

$$L_0+V_{N+1}=L_N+V_1=M$$
$$L_0y_{A0}+V_{N+1}x_{A,N+1}=L_ny_{A,N}+V_1x_{A1}=Mx_{AM}$$
$$B=L_0N_0=L_NN_N=MN_M$$

L_0MV_{N+1} must lie on a straight line and L_NMV_1 must be another straight line. Usually the flows and compositions of L_0 and V_{N+1} are known and the desired exit concentration $y_{A,N}$ is set. Then the coordinates N_M and x_{AM} can be determined.

In order to go stage by stage, we must derive the operating point equation, as we did for distillation by Ponchon and Savarit method. Making a total balance on stage 1 and then on stage n,

$$L_0+V_2=L_1+V_1$$
$$L_{n-1}+V_{n+1}=L_n+V_n$$

The above equations can be rearranged as

$$L_0-V_1=L_1-V_2=\ \dots\ =L_n-V_{n+1}=L_N-V_{N+1}=\Delta$$

The coordinates of the operating point Δ can be obtained by a balance on solute A and the solid:

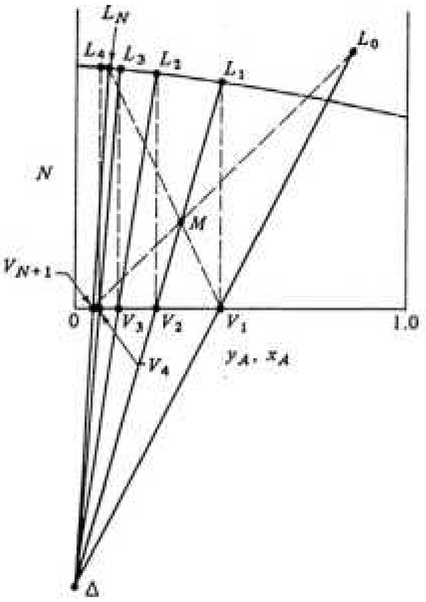
$$L_0 y_{A0} - V_1 x_{A1} = \ \dots\ = L_N y_{AN} - V_{N+1} x_{A,N+1} = \Delta x_{A\Delta}$$

$$B = L_0 N_0 = \ \dots\ = L_N N_N == \Delta N_\Delta$$

Hence,

$$x_{A\Delta} = \frac{L_0 y_{A0} - V_1 x_{A1}}{L_0 - V_1} = \frac{L_N y_{AN} - V_{N+1} x_{A,N+1}}{L_N - V_{N+1}}$$

$$N_\Delta = \frac{B}{L_0 - V_1} = \frac{L_0 N_0}{L_0 - V_1}$$



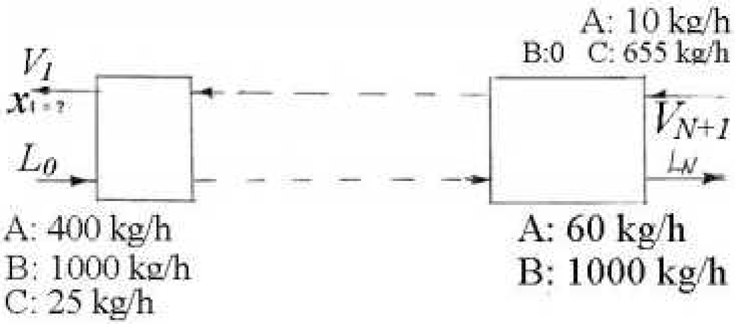
This point Δ can also be located graphically as the intersection of lines L_0V_1 and L_NV_{N+1}.

To graphically determine the number of stages, we start L_0 and draw a line $L_0\Delta$ to locate V_1 . Then an equilibrium tie line through V_1 locates L_1. Line $L_1\ \Delta$ is drawn to give V2. A tie line from V_2 gives L_2. This is continued until the desired L_N is reached.

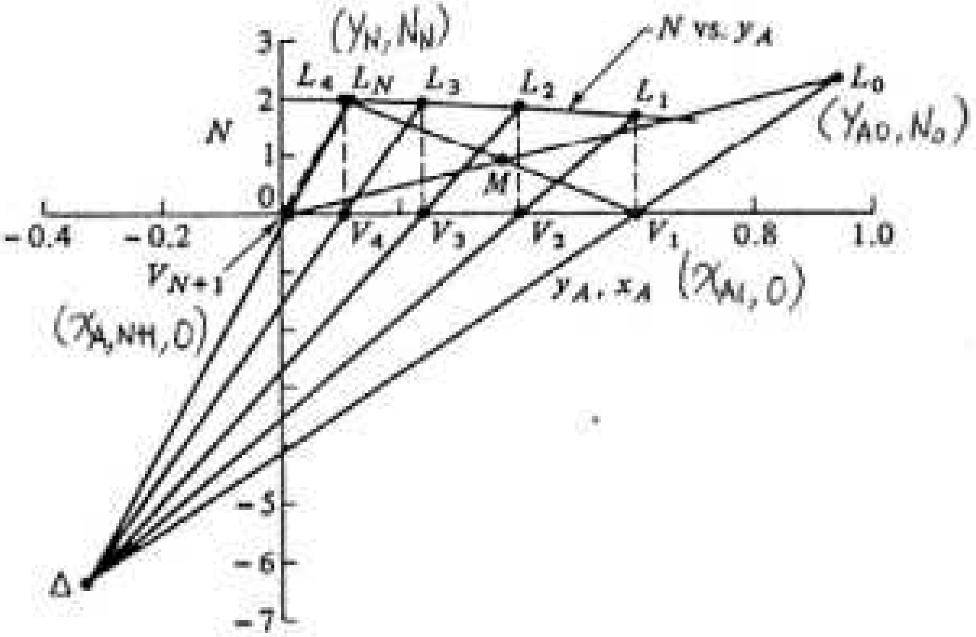
Example L3: Countercurrent Leaching of Oil from Meal A continuous countercurrent multistage system is to be used to leach oil from meal by benzene solvent. The process is to treat 1000 kg/h of inert solid meal (B) containing 400 kg oil (A) and also 25 kg benzene (C). The inlet flow per hour of fresh solvent mixture contains 655 kg benzene and 10 kg oil. The leached solids are to contain 60 kg oil. Settling experiments similar to those in the actual extractor show that the solution retained depends on the concentration of oil in the solution. The data are tabulated below as N kg inert solid B/kg solution and y_A kg oil A/kg solution.

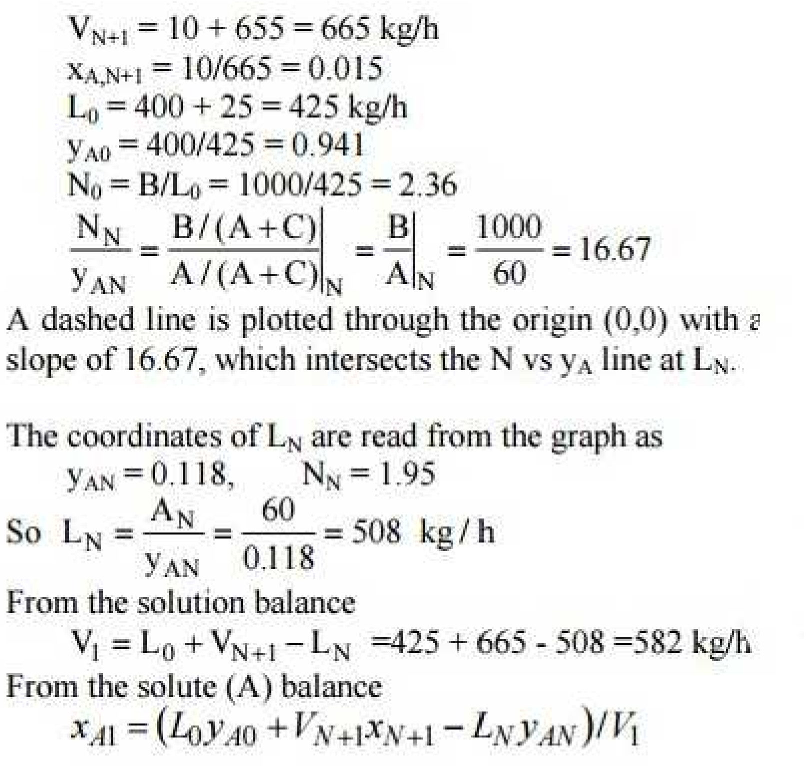
N	2.00	1.98	1.94	1.89	1.82	1.75	1.68	1.61
y_A	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7

Calculate the amounts and concentrations of the stream leaving the process and the number of stages required. Solution:



The underflow data are plotted as N vs y_A.





$V_{N+1} = 10 + 655 = 665$ kg/h

$x_{A,N+1} = 10/665 = 0.015$

$L_0 = 400 + 25 = 425$ kg/h

$y_{A0} = 400/425 = 0.941$

$N_0 = B/L_0 = 1000/425 = 2.36$

$$\frac{N_N}{y_{AN}} = \left.\frac{B/(A+C)}{A/(A+C)}\right|_N = \left.\frac{B}{A}\right|_N = \frac{1000}{60} = 16.67$$

A dashed line is plotted through the origin (0,0) with a slope of 16.67, which intersects the N vs y_A line at L_N.

The coordinates of L_N are read from the graph as

$y_{AN} = 0.118, \qquad N_N = 1.95$

So $L_N = \dfrac{A_N}{y_{AN}} = \dfrac{60}{0.118} = 508$ kg/h

From the solution balance

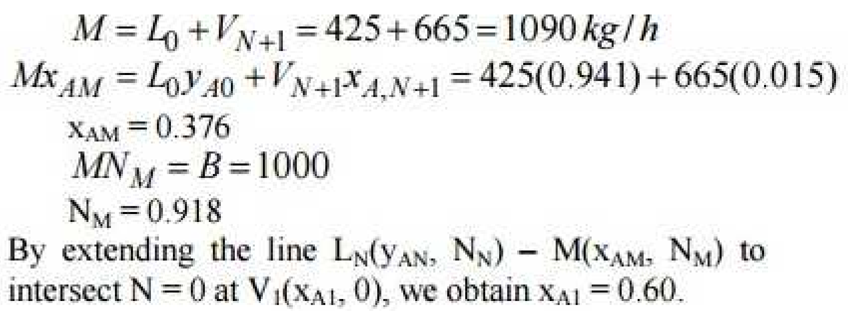
$V_1 = L_0 + V_{N+1} - L_N = 425 + 665 - 508 = 582$ kg/h

From the solute (A) balance

$$x_{A1} = \left(L_0 y_{A0} + V_{N+1} x_{N+1} - L_N y_{AN}\right)/V_1$$

= (425x0.941+665x0.015-508x0.118)/582 = 0.60

Another method to locate V1 is to determine the M point first.



$$M = L_0 + V_{N+1} = 425 + 665 = 1090\, kg/h$$

$$Mx_{AM} = L_0 y_{A0} + V_{N+1} x_{A,N+1} = 425(0.941) + 665(0.015)$$

$x_{AM} = 0.376$

$$MN_M = B = 1000$$

$N_M = 0.918$

By extending the line $L_N(y_{AN}, N_N)$ – $M(x_{AM}, N_M)$ to intersect N = 0 at $V_1(x_{A1}, 0)$, we obtain $x_{A1} = 0.60$.

is obtained as the intersection ofΔThe operating point lines L0V1 and LNVN+1 in the above figure. The stages are stepped off as shown. The fourth stage L4 is slightly past the desired LN. About 3.9 stages are required.

Variable Underflow in Countercurrent Multistage Leaching: **McCabe-Thiele method**

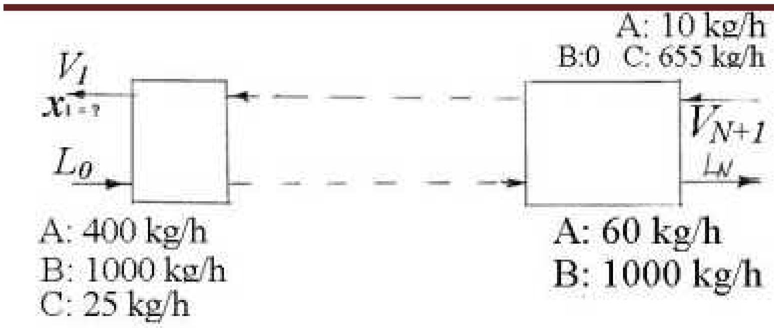
The number of ideal stages for variable underflow can also be determined by the McCabe-Thiele method. The terminal points on the operating line are determined usiny the material balances. Normally only one interdemiate point is reuired to draw the operating line unless there is a large change in L and V or the operating line is very close to the equilibrium line. To locate the intermediate point of the operating line, choose a yn value of about $(y_0+y_N)/2$ to fix L_n, Vn+1 x_{n+1} and are calculated from

$$V_{n+1} = L_n + V_1 - L_0$$

$$x_{n+1} = \frac{L_n}{V_{n+1}} y_n + \frac{V_1 x_1 - L_0 y_0}{V_{n+1}}$$

This point (y_n, x_{n+1}) together with the two terminal points (y_0, x_1) and (y_N, x_N+1) are used to draw the operating curve. The McCabe-Thiele method is used to determine the number of ideal stages by couting the number of triangles between the equilibrium and curved operating lines in the x-y diagram.

Example L4: Use the McCabe-Thiele method to calculate the number of idear stages for the leaching system in Example L3



$V_{N+1} = 10 + 655 = 665$ kg/h

$x_{A,N+1} = 10/665 = 0.015$

$L_0 = 400 + 25 = 425$ kg/h

$y_{A0} = 400/425 = 0.941$

The amount and composition of the solution in the spent solids, L_N and y_N are obtained by trial. Assume $y_{AN} = 0.1$, from the given table,

$N = 1.98 = B/(A+C) = 1000/(60+C)$

$C = 445$ kg/h

$y_{AN} = A/(A+C) = 60/(60+445) = 0.119$

At this new y_{AN}, N = 1.97 by linear interpolation. Hence

$C = 447, y_{AN} = 60/(60+447) = 0.118$ (close enough)

$L_N = A+C = 60+447 = 507$ kg.h

$$V_1 = V_{N+1} + L_0 - L_N = 665 + 425 - 507 = 583\ kg/h$$

Oil balance gives the oil in the extract as

$A_1 = 400+10-60 = 350$ kg/h

$x_{A1} = A_1/V_1 = 350/583 = 0.60$

To determine an intermediate point on the operating curve, choose $y_{An} = 0.5$, $N = 1.75 = B/L_n = 1000/L_n$

$L_n = 571$ kg/h

$$V_{n+1} = L_n + V_1 - L_0 = 571 + 583 - 425 = 729\ kg/h$$

$$x_{A,n+1} = \frac{L_n}{V_{n+1}} y_n + \frac{V_1 x_1 - L_0 y_0}{V_{n+1}}$$

$$= \frac{571}{729}(0.5) + \frac{583 \times 0.60 - 425 \times 0.941}{729} = 0.323$$

The points (y_0, x_1), (y_n, x_{n+1}), and (y_N, x_{N+1}) define a slightly curved operating line. By the McCabe-Thiele method, slightly more than 4 stages are required.