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SCHOOL OF BIO AND CHEMICAL ENGINEERING
DEPARTMENT OF CHEMICAL ENGINEERING

## UNIT - I- FLUID MECHANICS- SCHA1303

## UNIT-I <br> FLUID FLOW PHENOMENA

The branch of engineering science that has to do with the behaviour of fluids is called fluid mechanics. Fluid mechanics has two branches: fluid statics which treats fluids in the equilibrium state of no shear stress, and fluid dynamics, which treats fluids when portion of fluid are in motion relative to other parts.

This force, which tends to make one surface slide parallel to an adjacent surface, is called a shear force, and the stress in the glue, the force divided by the area of the glue joint, is called shear stress.

In the attempt to differentiate between fluids and solids, solids are substances, which can permanently resist very large shear forces. When subjected to a shear force solid move a short distance (elastic deformation), thereby setting up internal shear stresses, which resist the external forces, and then they stop moving.

Materials that obviously are fluids cannot permanently resist a shear force, no matter how small. When subjected to a shear force, fluids start to move and keep on moving as long as the force is applied. A fluid is a substance that deforms continuously when subjected to a shear stress, no matter how small that shear stress may be.

## IDEAL AND REAL FLUIDS:

An ideal fluid is one that is non-viscous and with constant density. It also should not have any surface tension and should not vaporize. Then it follows that the flow of an ideal fluid must be frictionless and reversible. Though such a fluid does not exist in reality, the idealization greatly simplifies the analysis of flow. However, simplifications do not have a value if they cannot assist in solving problems.

In situations of extensive flows like motion around airplanes and submarines, the ideal fluid flow analyses give partial solutions. They also help us in obtaining the structure of relationships leaving the coefficients to be determined or modified. The term ideal fluid should not be confused with a perfect gas which may possess viscosity.

A fluid which possesses viscosity is known as real fluid. All the fluids, in actual practice are real fluids.

## PHYSICAL PROPERTIES OF FLUIDS:

## Mass density or Density:

Density is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ' $\rho$ '.

Mathematically, Density $(\rho)=$ Mass of fluid/ volume of fluid
The unit of density in SI unit is $\mathrm{kg} / \mathrm{m}^{3}$
The density of water is $1 \mathrm{~g} / \mathrm{cm}^{3}$ (or) $1000 \mathrm{~kg} / \mathrm{m}^{3}$
For any given substance at a fixed temperature, a graph of mass versus volume produces a linear relationship. The slope yields the density of the substance. Specific weight or Weight density:

Specific weight of a fluid is the ratio between weight of a fluid and its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by ' $\boldsymbol{w}$ '.

$$
\begin{aligned}
& w=\frac{\text { Weightof fluid }}{\text { Volumeof fluid }} \\
& =\frac{\text { Mass of fluid xacceleration dueto gravity }}{\text { Volume of fluid }} \\
& =\frac{\text { Mass of fluid } x \mathrm{~g}}{\text { Volumeof fluid }}=\rho \times g
\end{aligned}
$$

## Specific volume:

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of the fluid is called specific volume.

SpecificVdume $=\frac{\text { Volumeof fluid }}{\text { Mass of fluid }}=\frac{1}{\frac{\text { Massof fluid }}{\text { Volume }}}=\frac{1}{\rho}$

Thus, specific volume is the reciprocal of mass density. The specific volume is the volume occupied by unit mass of fluid. It is expressed as $\mathbf{m}^{3} / \mathrm{kg}$.

Specific gravity or Relative density:
Relative density or Specific gravity = Density of the substance

## Density of water

Experimentally the specific gravity of a substance (liquid) can be obtained by following the steps given below:

1. Weigh empty specific gravity bottle (pycnometer), let the weight be $\mathrm{w}_{1}$.
2. Weigh pycnometer filled with the liquid, let the weight be $\mathrm{w}_{2}$.
3. Weigh pycnometer filled with water, let the weight be $w_{3}$.
4. Find the specific gravity of the liquid as $\left(w_{2}-w_{1}\right) /\left(w_{3}-w_{1}\right)$ i.e., ratio of weight of liquid to weight of water for same volume. (Refer Figure 1.2)


Figure 1.1 Determination of specific gravity of liquid

## Viscosity:

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid. When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say ' $u$ ' and ' $u+d u$ ' as shown in the Figure 1.2.


Figure 1.2 Velocity variations near a solid boundary
The top layer causes a shear stress on the adjacent layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to ' $y$ '. It is denoted by symbol ' $\tau$ '.

Mathematically,

$$
\begin{gathered}
\tau \alpha\left(\frac{d u}{d y}\right) \\
\tau=\mu\left(\frac{d u}{d y}\right)
\end{gathered}
$$

where, $\mu=$ proportionality constant known as absolute viscosity (or) dynamic viscosity,

$$
\left(\frac{d u}{d y}\right)=\text { rate of shear strain or velocity gradient }
$$

Now dynamic viscosity can be written as follows, $\mu=\frac{\tau}{\left(\frac{d u}{d y}\right)}$
Thus viscosity can be defined as the shear stress required producing unit rate of shear strain. The unit of viscosity is $\mathrm{g} / \mathrm{cm} . \mathrm{s}$. Also, 1 Poise $=1 \mathrm{~g} / \mathrm{cm} . \mathrm{s}=0.1 \mathrm{~Pa} . \mathrm{s}$

## Kinematic Viscosity:

The ratio of the absolute viscosity to the density of a fluid is called kinematic viscosity. It is denoted by $v$. In the SI system the unit of $v$ is $\mathrm{m}^{2} / \mathrm{s}$. In the CGS system the unit of $v$ is stoke ( St ) which is equal to $1 \mathrm{~cm}^{2} / \mathrm{s}$.

## Surface tension:

The force required to break a film of specific length is called surface tension. The magnitude of this force per unit length of the free surface has the same value as the surface energy per unit area. In MKS units it is expressed as $\mathrm{kgf} / \mathrm{m}$ while in SI unit as $\mathrm{N} / \mathrm{m}$ (or) $\mathrm{J} / \mathrm{m}^{2}$

## Pressure:

Imagine a closed container with air inside. Air, as a gas, is composed of molecules that can be imagined as round elastic balls. Molecules move in straight lines until they collide with neighbouring molecules or the container wall. Molecules of gas hitting the wall impose a force on the wall. The amount of this impact force per area of the container inner walls is called pressure. The mathematical definition of pressure can be written as

$$
\mathrm{P}=\mathrm{F} / \mathrm{A}
$$

where F is the force of impact of molecules on the walls and A is the area of the walls. The unit for pressure in the SI system is Pa (Pascal). However, there are other units for pressure still in use: bar, atm, and Torr. The ratios between different units are given in Table 1.1.

Table 1.1 Ratios between different pressure units

| Unit | Pa | Bar | atm | Torr |
| :---: | :---: | :---: | :---: | :---: |
| Pa | 1 | 0.00001 | $9.869 \times 10^{-6}$ | $7.501 \times 10^{-3}$ |
| bar | 100000 | 1 | $9.869 \times 10^{-1}$ | 750.1 |
| atm | 101325 | 1.01325 | 1 | 760 |
| Torr | $\mathbf{1 3 3 . 3 2}$ | $\mathbf{0 . 0 0 1 3 3 3}$ | $\mathbf{1 . 3 1 6 \times 1 0} \mathbf{0}^{-3}$ | $\mathbf{1}$ |

## Hydrostatic Pressure:

It is important to understand that pressure defined above is a property of gas. In the case of liquids, the pressure at the certain position in liquid is created by the weight of the fluid column. The pressure imposed by the height of water is called hydrostatic pressure and is directly proportional to the height of the fluid above and its density. The mathematical definition of
hydrostatic pressure is $\Delta \mathrm{P}=\rho \mathrm{gh}$ where ' $h$ ' is the height of the fluid column, ' $\rho$ ' is fluid density and ' $g$ ' is acceleration due to gravity.

## Vapour Pressure:

The vapour molecules exert a partial pressure in the space, known as vapour pressure. The vapour pressure of a given fluid depends upon temperature and increases with it. When the pressure above a liquid equals the vapour pressure of the liquid, boiling occurs.

## COMPRESSIBLE AND INCOMPRESSIBLE FLUIDS:

If the density is little affected by moderate changes in temperature and pressure, the fluid is said to be incompressible, and if the density is sensitive to changes in these variables, the fluid is said to be compressible. Liquids are considered to be incompressible and gases compressible.

## NEWTONIAN AND NON-NEWTONIAN FLUIDS

Newton's law of viscosity:
Newton's law of viscosity states that the shear stress is directly proportional to velocity gradient or shear rate. $\tau=\mu\left(\frac{d u}{d y}\right)$
where, the proportionality factor $\mu$ is called the dynamic viscosity or coefficient of viscosity.
Fluids are classified based on Newton's law of viscosity as
i. Newtonian - fluids which obey Newton's law of viscosity
ii. Non-Newtonian - fluids which does not obey Newton's law of viscosity.


Figure 1.3 Rheological behaviour of non-Newtonian fluids

## Non-Newtonian fluids:

Non-Newtonian fluids can be classified as follows
a. Time independent
$\Rightarrow$ Bingham plastic fluids
$\Rightarrow$ Pseudoplastic fluids
$\Rightarrow$ Dilatant fluids
b. Time dependent
$\Rightarrow$ Thixotropic fluids
$\Rightarrow$ Rheopectic fluids

Time independent fluids
Bingham plastic fluids resist a small shear stress indefinitely, but flow easily under large shear stresses. So, at low stresses, the viscosity in infinite, and at higher stresses, the viscosity decreases with increasing velocity gradient. Examples are bread dough, toothpaste, jellies, and some slurry. Good toothpaste should be a Bingham fluid, so that it can be easily squeezed out of the tube but will not drip off the toothbrush as water.

Pseudoplastic fluids show a viscosity that decreases with increase in velocity gradient. Examples are most slurries, mud, polymer solutions, solutions of natural gums, and blood. Good motor oil should be pseudoplastic, so that in the bearing, where the value of du/dy is high, it will offer little frictional resistance and so that at all the gaskets and joints, where the value of du/dy is low, it will be viscous and not leak through.

Dilatant fluids show a viscosity that increases with increase in velocity gradient. They are uncommon, but starch suspensions, etc., behave in this way.

## Time dependent fluids

The viscosity can remain constant with time, in which case the fluid is called time independent. The viscosity can decrease with time, in which case the fluid is called thixotropic. The viscosity can increase with time, in which case the fluid is called rheopectic.

A good paint should be thixotropic, so that in the container it is very viscous and the pigment
will not settle to the bottom, but when it is stirred, it will become less viscous and can be easily brushed onto a surface. In addition, the brushing should temporarily reduce the viscosity, so that the paint will flow sideways and fill in the brush marks (called levelling in the paint industry); then, as it stands, its viscosity should increase, so that it will not form drops and run down the wall.

A cream should be rheopectic, so that in the container it is less viscous. But, when stirred, it becomes more viscous.

PASCAL'S LAW:
Pascal's law states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. Pascal's law finds numerous examples in our daily life such as:
$>$ ydraulic brake system
> Hydraulic jack
$>$ Hydraulic press
$>$ Hydraulic machines.

## MEASUREMENT OF FLUID PRESSURE (MANOMETRY)

## INTRODUCTION:

Pressure can be measured by allowing it to act on known cross-sectional area. Such method of measuring pressure is called manometry. Manometers are the devices used for measuring pressure at a point in a fluid by balancing the column of fluid by the same or another column of fluid. They are classified as

- Simple manometers
- Differential manometers


## SIMPLE MANOMETERS:

A simple manometer consists of a glass tube, having one of its end connected to a point, where the pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are piezometer or pressure tube and U- tube manometer.

## Piezometer:



Figure 1.4 Piezometer

This is the simplest form of manometer used for measuring gauge pressure. One end of this is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Figure 2.1. The rise of liquid, gives the pressure head at that point. If at a point a, the height of liquid say water is ' $h$ ' in piezometer tube, then the pressure at $A$ is $P_{a}=\rho_{a} g h$. The piezometer will not work for negative gauge pressure, because air will flow into the container through the tube. It is also impractical for measuring large pressure, since the vertical tube would need to be very long.

## Simple U-tube Manometer:

It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in the Figure 2.2. The tube contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

Let ' $B$ ' = Point at which pressure is to be measured, whose value is ' p ',
$\mathrm{A}-\mathrm{A}=$ Datum line
$\mathrm{h}_{1}=$ Height of the light liquid above the datum line
$\mathrm{h}_{2}=$ Height of the heavy liquid above the datum line
$\mathrm{S}_{1}=$ Specific gravity of light liquid
$\rho_{1}=$ Density of light liquid $=1000 \times S_{1}$
$\mathrm{S}_{2}=$ Specific gravity of heavy liquid
$\rho_{2}=$ Density of heavy liquid $=1000 \times S_{2}$


Figure 1.5 Simple U-tube manometer to measure gauge pressure

As the pressure is the same for the horizontal surface, pressure above the horizontal datum lines A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $=P+\rho_{1} g h_{1}$
Pressure above A-A in the right column $=\rho_{2} g h_{2}$

Hence equating the two pressures, $P+\rho_{1} g h_{1}=\rho_{2} g h_{2}$
Therefore, $\quad P=\rho_{2} g h_{2}-\rho_{1} g h_{1}$

To measure negative pressure
Similar to the one solved above, we shall solve for measuring the negative gauge pressure (vacuum) as shown in the Figure 1.6


Figure 1.6 Simple U-tube manometer to measure vacuum pressure

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum lines A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $=P_{a}+\rho_{a} g h_{1}+\rho_{b} g h_{2}$

Pressure above $\mathrm{A}-\mathrm{A}$ in the right column $=0$

Hence equating the two pressures,
$P_{a}+\rho_{a} g h_{1}+\rho_{b} g h_{2}=0$, because point 4 is exposed to atmosphere.
Solving for $\mathrm{P}_{\mathrm{a}}$ will give $\mathrm{P}_{\mathrm{a}}=-\left(\rho_{\mathrm{b}} \mathrm{g} \mathrm{h}_{2}+\rho \mathrm{ag} \mathrm{h}_{1}\right)$

## DIFFERENTIAL MANOMETER:

Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U - tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most common types of differential manometers are U - tube differential manometer and Inverted U-tube differential manometer.

## U- tube Differential Manometer:



Figure 1.7 U-tube differential manometer

It consists of a simple U-tube. Both ends of this manometer are connected to two different pipe lines, where the pressure difference is to be measured as shown in Figure 1.7. Let the differential manometer be connected to two different pipe lines, A and B which is not in the same level and whose pressure difference is to be measured as shown in the above figure. Let the liquid in pipe A have more pressure than the liquid in the pipe $B$. Due to high pressure in the pipe $A$, the gauging liquid in the left limb will move down. Consequently, the gauging liquid will rise in the right limb.

In this case the gauging liquid level in the left is taken as datum level Z-Z

Let Z-Z - datum level.
$\mathrm{P}_{\mathrm{a}}$ - pressure of liquid in the pipe A
$\mathrm{P}_{\mathrm{b}}$ - pressure of liquid in the pipe B
$\mathrm{h}_{1}$ - height of liquid in pipe A in the left limb above datum line $\mathrm{Z}-\mathrm{Z}$ $\mathrm{h}_{2}$ - height of gauging liquid in the right limb above datum line $\mathrm{Z}-\mathrm{Z}$
$h_{3}$ - height of liquid of pipe $B$ in the right limb above gauging liquid
$\rho_{1}$ - Density of liquid in pipe A
$\rho_{2}$ - Density of gauging liquid (mercury)
$\rho_{3}$ - Density of liquid in pipe B
Pressure head at $\mathrm{C}=\boldsymbol{P}_{\boldsymbol{a}}+\boldsymbol{h}_{1} \boldsymbol{\rho}_{1} g$

Pressure head at $\mathrm{D}=\boldsymbol{P}_{b}+\boldsymbol{h}_{2} \rho_{2} g+h_{3} \rho_{3} g$ ' m ' of water

We know that,
Pressure head at $\mathrm{C}=$ Pressure head at D

$$
P_{a}+h_{1} \rho_{1} g=P_{b}+h_{2} \rho_{2} g+h_{3} \rho_{3} g
$$

Then pressure difference is given by,

$$
P_{a}-P_{b}=h_{2} \rho_{2} g+h_{3} \rho_{3} g-h_{1} \rho_{1} g
$$

Note - (i)

If both the pipes are at the same level and contains different liquids. In this case, $h_{1}=h_{2}+h_{3}$

Then, the pressure difference is,

$$
\begin{aligned}
& P_{a}-P_{b}=h_{2} \rho_{2} g+h_{3} \rho_{3} g-h_{1} \rho_{1} g \\
& P_{a}-P_{b}=h_{2} \rho_{2} g+h_{3} \rho_{3} g-\left(h_{2}+h_{3}\right) \rho_{1} g \\
& P_{a}-P_{b}=h_{2} \rho_{2} g+h_{3} \rho_{3} g-h_{2} \rho_{1} g-h_{3} \rho_{1} g \\
& P_{a}-P_{b}=h_{2} g\left(\rho_{2}-\rho_{1}\right)+h_{3} g\left(\rho_{3}-\rho_{1}\right)
\end{aligned}
$$



Figure 1.8 Inverted U-tube differential manometer

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured as shown in Figure 2.5. It is used for measuring difference of low pressures. The following shows an inverted U-tube differential manometer connected to the two points A and B . Let the pressure at A is more than the pressure at B.

Let $\quad h_{1}=$ height of liquid in the right limb below the datum line $\mathrm{X}-\mathrm{X}$
$\mathrm{h}_{2}=$ height of liquid in the left limb
$h=$ Height of manometric liquid in left limb
$\rho_{1}=$ Density of liquid at A
$\rho_{2}=$ Density of liquid at B
$\rho_{\mathrm{s}}=$ Density of manometric liquid
$\mathrm{p}_{\mathrm{A}}=$ pressure at A
$\mathrm{p}_{\mathrm{B}}=$ Pressure at B

Taking $\mathrm{X}-\mathrm{X}$ as datum line, then,
Pressure in the right limb below the $\mathrm{X}-\mathrm{X}=p_{A}-\rho_{1} g h_{1}$

Pressure in the left limb below the $\mathrm{X}-\mathrm{X}=p_{B}-\rho_{2} g h_{2}-\rho_{S} g h$

Equating the two pressures,

$$
\begin{aligned}
& p_{A}-\rho_{1} g h_{1}=p_{B}-\rho_{2} g h_{2}-\rho_{S} g h \\
& p_{B}-p_{B}=\rho_{1} g h_{1}-\rho_{2} g h_{2}-\rho_{S} g h
\end{aligned}
$$

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## UNIT - II- FLUID MECHANICS- SCHA1303

# UNIT-II <br> KINEMATICS OF FLUID FLOW <br> <br> TYPES OF FLOW 

 <br> <br> TYPES OF FLOW}

## INTRODUCTION

Fluid dynamics is the study of fluids in motion relative to other parts. The flow patterns are classified in the following different categories.

## Steady and unsteady flow:

The flow of fluid is said to be steady when the flow conditions like velocity, pressure, density and other similar characteristics at a point do not change with respect to time.

$$
\left(\frac{\delta \rho}{\delta t}\right)=0 \quad\left(\frac{\delta \mathrm{P}}{\delta \mathrm{t}}\right)=0 \quad\left(\frac{\delta \mathrm{v}}{\delta \mathrm{t}}\right)=0
$$

The flow of fluid is said to be unsteady, when the flow characteristics at a point change with time.

$$
\left(\frac{\delta \rho}{\delta t}\right) \neq 0 \quad\left(\frac{\delta \mathrm{P}}{\delta \mathrm{t}}\right) \neq 0 \quad\left(\frac{\delta \mathrm{v}}{\delta \mathrm{t}}\right) \neq 0
$$

## Uniform and non- uniform flow:

The flow is said to be uniform, when there is no variation in the magnitude and direction of the velocity from one point to another along the path of the flow (both velocity and area of flow must be same at every cross section) or the velocity does not change along the length of the flow. A flow of a fluid is said to be non-uniform, when the velocity of flow doesn't remain constant at all the points in space during a given interval.

## Rotational and irrotational flow:

Rotational flow is that type of flow in which the fluid particles while flowing along stream lines also rotate about their own mass axis. If the fluid particles while flowing along stream lines do not rotate about their own mass axis, then it is called irrotational flow.

## Compressible and incompressible flow:

Compressible flow is that type of flow in which the density of the fluid changes from point to point. Incompressible flow is one in which the density of the flowing fluid is constant.

## One-, two- and three-dimensional flow:

One-dimensional flow is the flow in which flow parameter such as velocity is a function of time and one space co-ordinate. Two-dimensional flow is one in which flow parameter such as velocity is a function of time and two rectangular space co-ordinates. Three-dimensional flow is one in which the flow parameter, velocity is a function of time and three mutually perpendicular space coordinates.

## Potential flow

A moving fluid uninfluenced by stationary solid walls is not subjected to shear, and shear stresses do not exist within it. The flow of incompressible fluid with no shear is called potential flow. Potential flow has two characteristics,
i. Neither circulation nor eddies can form with in the stream, so that potential flow is also called irrotational flow.
ii. Friction cannot develop, so that there is no dissipation of mechanical energy into heat.

## REYNOLDS NUMBER

Reynolds number is a dimensionless number, which is defined as the ratio between inertia force to viscous force. It is denoted by $\mathrm{N}_{\mathrm{Re}}$

Reynolds Number $\left(\mathrm{N}_{\mathrm{Re}}\right)=\frac{\text { Inertiafore }}{\text { Viscousforce }}$

Reynolds number of a fluid flowing through a pipe is calculated as follows;
Reynolds Number $\left(\mathrm{N}_{\mathrm{Re}}\right)=\frac{D u \rho}{\mu}$
where, $\mathrm{D}=$ Diameter of the pipe, m

$$
\begin{aligned}
& \mathrm{u}=\text { Velocity of flowing fluid, } \mathrm{m} / \mathrm{sec}, \\
& \rho=\text { Density of the flowing fluid, } \mathrm{kg} / \mathrm{m}^{3} \\
& \mu=\text { Viscosity of fluid, } \mathrm{kg} / \mathrm{m} . \mathrm{sec} .
\end{aligned}
$$

Reynolds number is used to decide whether the flow is laminar flow or turbulent flow.
If $\mathbf{N R e}<\mathbf{2 1 0 0}$, then the flow is laminar flow.
If $\mathbf{N}_{\mathrm{Re}}>4000$, then the flow is turbulent flow.
If $N_{R e}$ is between 2100 and 4000, then the flow is in transition region.

## Laminar flow

Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream- lines are straight and parallel. Thus, the particles move in laminas or layers sliding smoothly over the adjacent layer (Figure 2.1). This type of flow is called stream-line or Laminar flow or viscous flow.


Figure 2.1 Laminar flow

## Characteristics of Laminar flow:

i. The fluid particles move in a definite and observable path parallel to the side of the pipe as shown in the above figure.
ii. In any layer, the velocity of fluid particle is low.
iii. Viscosity of the fluid is responsible for this type of flow. High viscosity fluids develop laminar flow.
iv. Reynolds number for laminar flow is $\mathbf{<} \mathbf{2 1 0 0}$.
v. Velocity profile for laminar flow is as shown in Figure 2.3.

## Turbulent flow:

Turbulent flow is defined as that type of flow in which the fluid particles move in a zigzag way. Due to the movement of fluid particles in zigzag way, eddies formation take place which are responsible for high energy loss (Figure 3.2).


Figure 2.2 Turbulent flow

## Characteristics of turbulent flow:

i. The fluid particles do not follow a definite path and move at random as shown in the above figure.
ii. Fluid particles in one layer collide with that of another layer.
iii. In any layer, the velocity of fluid particle is high.
iv. The magnitude and direction of fluid particles varies with time.
v. Reynolds number for turbulent flow is $>4000$.
vi. Velocity profile for turbulent flow is as shown in Figure 2.3.


Figure 2.3 Velocity profile in laminar and turbulent flow

## Transitional flow:

It is the type of flow when there is a change over from the laminar flow to turbulent flow or vice versa.

## APPLICATION OF CONTROL VOLUME ANALYSIS

Basic equations of flowing fluid
The rate of mass entering the flowing system equals that leaving as mass can neither be accumulated nor depleted with in a flow system under steady conditions.

## Continuity equation

Consider a stream tube as shown in the Figure 1.4,


Velocity,
$u_{b}$

Figure 1.4 Stream tube

Let the fluid enter at a point where the area of cross section of the tube is $S_{a}$ and leaves where the area of cross section is $S_{b}$. Let the velocity and density at the entrance be $u_{a}$ and $\rho_{\mathrm{a}}$ respectively and the corresponding quantities at the exit be $\mathrm{u}_{\mathrm{b}}$ and $\rho_{\mathrm{b}}$. Assume density in a single cross section is constant. Also assume that the flow through the tube is potential flow. Then the velocity $u_{a}$ is constant across the area $\mathrm{S}_{\mathrm{a}}$ and velocity $\mathrm{u}_{\mathrm{b}}$ is constant across area $\mathrm{S}_{\mathrm{b}}$.

The mass of fluid entering and leaving the tube per unit time is $\dot{\mathrm{m}}=\rho_{a} \mathrm{u}_{\mathrm{a}} \mathrm{S}_{\mathrm{a}}=\rho_{\mathrm{b}} \mathrm{u}_{\mathrm{b}} \mathrm{S}_{\mathrm{b}}$ where m is the rate of flow in mass per unit time.

For a stream tube $\dot{\mathrm{m}}=\rho \mathrm{u} \mathrm{S}=$ constant

This equation is called the equation of continuity. It applies to both compressible and incompressible flows

For incompressible flow, the above equation reduces to

$$
\mathrm{Q}=\mathrm{u}_{\mathrm{a}} \mathrm{~S}_{\mathrm{a}}=\mathrm{u}_{\mathrm{b}} \mathrm{~S}_{\mathrm{b}}
$$

## Average velocity:

Average velocity equals the total volumetric flow rate of the fluid divided by the cross-sectional area of the conduit. The average velocity $\bar{u}$ can be described as the volume flux of the fluid.

Average velocity $\bar{u}=\frac{\mathrm{Q}}{\mathrm{S}}$
where Q is the volumetric flow rate, $\mathrm{m}^{3} / \mathrm{s}$, and
$S$ is the cross sectional area of the conduit.

## Mass Velocity:

Mass velocity G, is calculated by dividing the mass flow rate by the cross sectional area of channel, [unit: $\mathrm{kg} / \mathrm{m}^{2} . \mathrm{s}$ ]. The mass velocity ' $G$ ' can also be described as the mass current density or mass flux, where flux is defined generally as any quantity passing through an unit area in unit time.

Mass velocity $\mathrm{G}=\frac{\dot{m}}{s}$
where $\dot{m}$ is the mass flow rate, $\mathrm{kg} / \mathrm{s}$.
$S$ is the cross-sectional area of the conduit.

## BERNOULLI'S EQUATION (OR) THEOREM

## BERNOULLI'S EQUATION

For potential flow, the total head at any cross section is constant. The total head consists of pressure head, velocity head (kinetic head) and datum head (potential head).

$$
\begin{gathered}
\frac{\mathrm{P}}{\rho \mathrm{~g}}+\frac{\mathrm{u}^{2}}{2 \mathrm{~g}}+\mathrm{z}=\text { constant } \\
\frac{\mathrm{P}_{1}}{\rho \mathrm{~g}}+\frac{\mathrm{u}_{1}{ }^{2}}{2 \mathrm{~g}}+\mathrm{z}_{1}=\frac{\mathrm{P}_{2}}{\rho \mathrm{~g}}+\frac{\mathrm{u}_{2}{ }^{2}}{2 \mathrm{~g}}+\mathrm{z}_{2}
\end{gathered}
$$

## MODIFIED BERNOULLI'S EQUATION

For real fluids that are passing through pipe are influenced by the solid boundaries. To extend the Bernoulli's equation to cover these practical situations two modifications are required. They are
i. Fluid friction correction factor
ii. Kinetic energy correction factor

## Fluid friction correction factor:

Fluid friction correction factor can be defined as conversion of mechanical energy into heat in the flowing stream. So, in frictional fluid the total head is not constant along a stream line and always decrease in the direction of flow. In accordance with the principle of conservation of energy an amount of heat generated is equivalent to the loss in mechanical energy. So, for incompressible fluids, the Bernoulli's equation is corrected for friction by adding a term to the right hand side of the equation.

$$
\frac{\mathrm{P}_{\mathrm{a}}}{\rho}+\mathrm{gz}_{\mathrm{a}}+\frac{\mathrm{u}_{\mathrm{a}}^{2}}{2}=\frac{\mathrm{P}_{\mathrm{b}}}{\rho}+\mathrm{gz}_{b}+\frac{\mathrm{u}_{\mathrm{b}}^{2}}{2}+\mathrm{h}_{\mathrm{f}}
$$

where $\mathrm{h}_{\mathrm{f}}$ represents all the friction generated per unit fluid that occurs in the fluid between stations $a$ and $b$.

Friction appears in boundary layers because the work done by shear forces in maintaining the velocity gradients in both laminar and turbulent flow is eventually converted into heat by viscous action. Friction generated in unseparated boundary layers is called skin friction. When the boundary layers separate and form wakes, additional energy dissipation appears within the wake and friction of this type is called form friction since it is a function of the position and shape of the solid. In a given situation both skin friction and form friction may be active in varying degrees. The total friction $\mathrm{h}_{\mathrm{f}}$ in the above equation includes both types of frictional loss.

Kinetic energy correction factor: Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section. It is denoted by $\alpha$. Hence mathematically,

$$
\alpha=\frac{\text { Kinetic energy of the flow per second based on actual velocity }}{\text { Kinetic energy of the flow per second based on average velocity }}
$$

Kinetic energy of the flow per second based on actual velocity across a section: Consider an element of cross-sectional area S . The mass flow rate through this is $\rho \mathrm{u}$ ds. Each kg of fluid flowing through area ds carries kinetic energy in amount $u^{2} / 2$ and the energy flow rate through area ds is therefore, $\mathrm{dE}_{\mathrm{k}}=(\rho \mathrm{uds}) \frac{\mathrm{u}^{2}}{2}=\frac{\rho \mathrm{u}^{3}}{2} \mathrm{ds}$, where $\mathrm{E}_{\mathrm{k}}$ represents the time rate of flow of kinetic energy.
$\therefore$ the total rate of flow of kinetic energy through the entire cross section $\mathrm{s}, \mathrm{E}=\frac{\rho}{2} \int_{0}^{\mathrm{s}} \mathrm{u}^{3} \mathrm{ds}$.

But total rate of mass flow, $\dot{\mathrm{m}}=\frac{\frac{\rho}{2} \int_{0}^{\mathrm{s}} \mathrm{u}^{3} \mathrm{ds}}{\rho \int_{0}^{\mathrm{s}} \mathrm{uds}}=\frac{\frac{1}{2} \int_{0}^{\mathrm{s}} \mathrm{u}^{3} \mathrm{ds}}{\overline{\mathrm{u} d s}}$,
so $\mathrm{K} . \mathrm{E} /$ sec based on average velocity $=\frac{\overline{\mathrm{u}}^{2}}{2}$
$\therefore$ K.E. correction factor $\alpha=\frac{\frac{\frac{1}{2} \int_{0}^{\mathrm{s}} \mathrm{u}^{3} \mathrm{ds}}{\frac{\overline{\mathrm{u}} \mathrm{ds}}{2}}}{\frac{\overline{\mathrm{u}}^{2}}{2}} \quad \alpha=\frac{\int_{0}^{\mathrm{s}} \mathrm{u}^{3} \mathrm{ds}}{\overline{\mathrm{u}}^{3} \mathrm{~S}}$

## PUMP PROBLEMS:

Work supplied to the pump from shaft work $=\mathrm{w}_{\mathrm{s}}$
Pump work $=w_{p}$
Total friction in the pump $/ \mathrm{kg}$ of fluid $=\mathrm{h}_{\mathrm{fp}}$
Net mechanical energy available to the flowing fluid $=\mathrm{w}_{\mathrm{p}}-\mathrm{h}_{\mathrm{fp}}$
Pump efficiency $\eta=\mathrm{w}_{\mathrm{p}}-\mathrm{h}_{\mathrm{fp}} / \mathrm{w}_{\mathrm{p}}$
$\therefore$ The mechanical energy delivered to the flowing fluid $=\eta \omega_{p} \dot{\mathrm{~m}}$
$\therefore$ Power, $\mathrm{P}=\eta \mathrm{w}_{\mathrm{p}} \dot{\mathrm{m}}$, watts or $\eta \mathrm{w}_{\mathrm{p}} \dot{\mathrm{m}} / 746$, hp
Applying the bernoulli's equation around the pump, the pressure drop developed by the pump is calculated as follows
$\frac{P_{1}}{\rho}+g z_{1}+\frac{u_{1}^{2}}{2}+\eta w_{p}=\frac{P_{2}}{\rho}+g z_{2}+\frac{u_{2}^{2}}{2}$, here $z_{1}=z_{2}$ (as pump is parallel to datum line)
$\overline{\mathrm{u}}_{1}=\overline{\mathrm{u}}_{2}$ (rate of flow is steady and dia of the pipe on suction side and delivery side are same )
$\therefore \frac{\mathrm{P}_{2}}{\rho}-\frac{\mathrm{P}_{1}}{\rho}=\eta \mathrm{w}_{\mathrm{p}}$
if $\overline{\mathbf{u}}_{1} \neq \overline{\mathbf{u}}_{2} \frac{\mathrm{P}_{2}}{\rho}-\frac{\mathrm{P}_{1}}{\rho}=\eta \mathrm{w}_{\mathrm{p}}\left(\frac{\overline{\mathrm{u}}_{1}^{2}-\overline{\mathrm{u}}_{2}^{2}}{2}\right)$

## PROBLEMS ON PUMPS:

4.11. In the figure shown a pump draws a solution of specific gravity $\mathbf{1 . 8 4}$ from a storage tank through a 75 mm pipe. The efficiency of the pump is $\mathbf{6 0 \%}$. The velocity in the suction line is $0.914 \mathrm{~m} / \mathrm{s}$. The pump discharges through a 50 mm pipe to an overhead tank. The end of the discharge pipe is 15 m above the level of the solution in the feed tank. A frictional loss in the entire piping system is $30 \mathrm{~J} / \mathrm{kg}$. What pressure must the pump develop and what is the power of the pump?


Let ' $a$ ' be the surface of the solution in the feed tank. ' $b$ ' be the discharge point.
From the figure $\mathrm{z}_{\mathrm{a}}=0$ and $\mathrm{z}_{\mathrm{b}}=15 \mathrm{~m}$.
$\mathrm{v}_{\mathrm{a}}$ is very small compared to $\mathrm{v}_{\mathrm{b}}$ (due to large cross sectional area of the feed tank)
$\therefore \frac{\mathrm{v}_{\mathrm{a}}^{2}}{2}$ can be neglected
Let the datum line passes through the surface of the liquid in the tank.

Pressures at both a and $b$ will same, because both are opened to atmosphere.
Writing Bernoulli's equation in energy form (because the frictional loss is given in the form of energy)
$\frac{P_{a}}{\rho}+\frac{v_{a}^{2}}{2}+g z_{a}+\eta W_{p}=\frac{P_{b}}{\rho}+\frac{v_{b}^{2}}{2}+g z_{b}+h_{f}$, this equation reduces to the following equation after incorporating the above assumptions
$\eta \mathrm{W}_{\mathrm{p}}=\frac{\mathrm{v}_{\mathrm{b}}^{2}}{2}+\mathrm{gz}_{\mathrm{b}}+\mathrm{h}_{\mathrm{f}} \rightarrow(1)$

Diameter of suction pipe, $\mathrm{D}_{\mathrm{a}}=0.075 \mathrm{~m}$
Diameter of delivery pipe, $\mathrm{D}_{\mathrm{b}}=0.075 \mathrm{~m}$
Velocity in suction pipe, $\mathrm{v}_{\mathrm{a}}=0.914 \mathrm{~m} / \mathrm{s}$
Area of suction pipe, $\mathrm{S}_{\mathrm{a}}=(\pi / 4) * \mathrm{D}_{\mathrm{a}}{ }^{2}=4.417 * 10^{-3} \mathrm{~m}^{2}$.
Area of delivery pipe, $\mathrm{S}_{\mathrm{b}}=(\pi / 4)^{*} \mathrm{D}_{\mathrm{b}}{ }^{2}=1.963 * 10^{-3} \mathrm{~m}^{2}$.
We know from continuity equation that $\mathrm{S}_{\mathrm{a}} \mathrm{V}_{\mathrm{a}}=\mathrm{S}_{\mathrm{b}} \mathrm{V}_{\mathrm{b}}$
$\mathrm{v}_{\mathrm{b}}=\frac{\mathrm{S}_{\mathrm{a}} \mathrm{v}_{\mathrm{a}}}{\mathrm{S}_{\mathrm{b}}}=\frac{4.417 * 10^{-3} * 0.075^{2}}{1.963 * 10^{-3}}=2.057 \mathrm{~m} / \mathrm{s}$
Substituting $\mathrm{v}_{\mathrm{b}}$ in equation (1), and solving
$0.6 \mathrm{~W}_{\mathrm{p}}=\frac{2.057^{2}}{2}+9.81 * 15+30 \Rightarrow \mathbf{W}_{\mathbf{p}}=\mathbf{2 9 8 . 7 7 6 J} / \mathbf{k g}$.
Power, $\mathrm{P}=\dot{\mathrm{m}} \mathrm{w}_{\mathrm{p}}=\rho * \mathrm{va}_{\mathrm{a}} * \mathrm{~S}_{\mathrm{a}} * \mathrm{~W}_{\mathrm{p}}$
$\mathrm{P}=1840 * 0.914 * 4.417 * 10^{-3} * 298.776$
$\mathrm{P}=2219.408 \mathrm{~J} / \mathrm{s}=\mathbf{2 2 1 9 . 4 0 8 W}$
$P=2.219 k W$
To find the pressure drop developed across the pump, let
1 be input to the pump and 2 be output from the pump
from the figure $\mathrm{z}_{1} \approx \mathrm{z}_{2}$.
Applying Bernoulli's equation across points 1 and 2,
$\frac{P_{1}}{\rho}+\frac{v_{1}^{2}}{2}+g z_{1}+\eta W_{p}=\frac{P_{2}}{\rho}+\frac{v_{2}^{2}}{2}+g z_{2}+h_{f}$,
here $\mathrm{h}_{\mathrm{f}}$, because there is no piping system for the loss of head/energy in the pipe.
$\therefore$ Bernoulli's equation reduces to
$\frac{\mathrm{P}_{1}}{1840}+\frac{0.914^{2}}{2}+0.6 * 298.779=\frac{\mathrm{P}_{2}}{1840}+\frac{2.057^{2}}{2}$
solving for $\mathrm{P}_{2}-\mathrm{P}_{1}$, we get
$P_{2}-P_{1}=3.267 * 10{ }^{5} \mathrm{~N} / \mathrm{m}^{2}$

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## UNIT - III FLUID MECHANICS SCHA1303

# UNIT-III <br> FLOW PAST IMMERSED BODIES AND FLUIDIZATION <br> FLOW OVER SPHERE AND NON-SPHERICAL OBJECTS: 



Figure 3.1 Shows flow over sphere and non-spherical objects
Flow of fluids outside immersed bodies appears in many chemical engineering applications and other processing applications. These occur, for example, in flow past spheres in settling, flow through packed beds in drying and filtration, flow past tubes in heat exchangers, and so on. It is useful to be able to predict the frictional losses and /or the force on the submerged objects in these various applications.

Consider the flow of fluid inside of a conduit, the transfer of momentum perpendicular to the surface resulted in a tangential shear stress or drag on the smooth surface parallel to the direction of flow. This force exerted by the fluid on the solid in the direction of flow is called skin or wall drag. For any surface in contact with a flowing fluid, skin friction will exist. In addition to skin friction, if the fluid is not flowing parallel to the surface but change directions to pass around a solid body such as sphere, significant additional losses will occur and this is called form drag. The flow of fluid over different type of objects is as shown in the following figure.

## Drag coefficient:

From the above discussions it is evident that the geometry of the immersed solid is a main factor in determining the amount of total drag force exerted on the body. Correlations of the geometry and flow characteristics for solid objects suspended or held in a free stream (immersed objects) are similar in concept and form to the friction factor- Reynolds number correlation given for flow inside conduits. In flow through conduits, the friction factor was defined as the ratio of the drag force per unit area (shear stress) to the product of density times velocity head.

In similar manner for flow past immersed objects, the drag coefficient ' $\mathrm{C}_{\mathrm{D}}$ ' is defined as the ratio of total drag force per unit area to $\frac{\rho v_{o}^{2}}{2}$.
$C_{D}=\frac{\left(F_{D} / A_{P}\right)}{\left(\rho v_{o}^{2} / 2\right)}$
where, $\mathrm{F}_{\mathrm{D}}$ - total drag force

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{P}}-\text { Area } \\
& v_{0-\text { free }- \text { stream velocity. }} \\
& \rho \text { - density of fluid }
\end{aligned}
$$

$\mathrm{A}_{\mathrm{P}}$ is area obtained by projecting the body on a plane perpendicular to the line of flow.
For a sphere, $\mathrm{A}_{P}=\frac{\pi D_{P}^{2}}{4}$, where $\mathrm{D}_{\mathrm{p}}-$ diameter of sphere.

For cylinder, $\mathrm{A}_{\mathrm{P}}=\mathrm{LD} \mathrm{D}_{\mathrm{P}}$, where $\mathrm{L}-$ Length of cylinder
From the above equation the total drag force is,
$\mathrm{F}_{\mathrm{D}}=C_{D} \frac{v_{0}^{2}}{2} \rho A_{P}$

Reynolds number for a given solid immersed in a flowing liquid is,
$\mathrm{N}_{\mathrm{Re}}=\frac{D_{P} v_{0} \rho}{\mu}$
$\mathrm{NRe}=\frac{D_{P} G_{0}}{\mu}$ Where $\mathrm{G}_{0}=v_{0} \rho$

## FLOW THROUGH PACKED BED:

A porous medium is a continuous solid phase with many pores, or void spaces in it. Sponges, plaster walls, filters, and the packed beds used for adsorption, distillation etc. are some examples of porous media. In many porous solid, such as foamed polystyrene drinking cups or ice boxes, the void spaces are not connected, so there is no possibility of a fluid flowing through them. A bed of granular solids or sand has fewer pores than a foamed plastic cup, but the pores are all connected, so a fluid can easily flow through it. Porous media which do not have interconnected pores are permeable. The flow of fluids through permeable porous media is of great practical signification in ground-water hydrology, oil and gas production, filters, packed absorption and distillation columns and fixed bed catalytic reactors.

Consider the flow of a single-phase fluid through a bed of uniformly sized spherical particles. If the average velocity at any cross-section perpendicular to flow is based on the entire crosssectional area of the pipe (bed), it is called the superficial velocity. It is given by

$$
\begin{equation*}
u_{0}=\frac{Q}{A_{p i p e}} \tag{1}
\end{equation*}
$$

It may, on the other hand, be based on the area actually open to the flowing fluid, in which case it is called the interstitial velocity, given by
$\mathrm{u}=\frac{u_{0}}{e}=\frac{Q}{e A_{p i p e}}$
where, e is the porosity, or void fraction, of the bed.
$e=\frac{\text { total volume of bed }- \text { volume of solidsin bed }}{\text { total volume of bed }}$

$$
\begin{equation*}
=\frac{\text { freevolume available for fluid flow }}{\text { total volumeof fixedbed }} \tag{3}
\end{equation*}
$$

From the theoretical standpoint interstitial velocity is more important; it determines the kinetic energy, the fluid forces and whether the flow is streamline or turbulent. From the practical standpoint the superficial velocity is generally more useful.

For a packed bed the hydraulic radius varies from point to point. It is given by
$\mathrm{r}_{\mathrm{H}}=\frac{\text { wetted crosssec } \text { tion normal to flow }}{\text { wetted perimeter }}$
$\mathrm{r}_{\mathrm{H}}=\frac{\text { volumeopento flow }}{\text { total wetted surfaceareaof packing }}$
For a porous medium made of equally sized spherical particles,
$\mathrm{r}_{\mathrm{H}}=\frac{\text { total volume of bed } \times e}{\text { numberof particles } \times \text { surfaceareaof one particle }}$
But, number of particles $=\frac{\text { volumeof bed } \times(1-e)}{\text { volumeof one particle }}$

$$
\begin{align*}
& \text { Therefore, } \mathrm{r}_{\mathrm{H}}=\frac{e}{(1-e) \times\left[\frac{\text { surfacearea }}{\text { volume }}\right] \text { of a particle }} \\
& =\frac{e}{(1-e) \times\left[\frac{\pi d_{p}^{2}}{(\pi / 6) d_{p}^{3}}\right]} \\
& =\frac{d_{p}}{6} \times \frac{e}{1-e} \tag{4}
\end{align*}
$$

where, $\mathrm{d}_{\mathrm{p}}$ is the particle diameter.

In this case of flow through packed beds, the Reynolds number and the friction factor become, Re $=\left(4 \mathrm{r}_{\mathrm{H}}\right)\left(\frac{u_{0}}{e}\right) \frac{\rho}{\mu}$

$$
=4 \frac{d_{p}}{6} \times \frac{e}{1-e} \times\left(\frac{u_{0}}{e}\right) \frac{\rho}{\mu}
$$

$$
\begin{equation*}
=\frac{2 d_{p} u_{0} \rho}{3 \mu(1-e)} \tag{5}
\end{equation*}
$$

$$
\begin{align*}
& \mathrm{f}=\left(-\frac{\Delta P_{f}}{\rho L}\right) \frac{\left(4 r_{H}\right)}{2\left(u_{0} / e\right)^{2}} \\
&=\left(-\frac{\Delta P_{f}}{\rho L}\right) 4 \frac{d_{p}}{6} \times \frac{e}{1-e} \times \frac{e^{2}}{2 u_{0}^{2}} \\
&=\frac{1}{3}\left(-\frac{\Delta P_{f}}{\rho L}\right) \frac{d_{p}}{u_{0}^{2}} \times \frac{e^{3}}{1-e} \cdots \cdots-\cdots \tag{6}
\end{align*}
$$

Thus, the modified friction factor and the Reynolds number are defined as

$$
\begin{gather*}
\mathrm{f}_{\mathrm{m}}=\left(-\frac{\Delta P_{f}}{\rho L}\right) \frac{d_{p}}{u_{0}^{2}} \times \frac{e^{2}}{1-e}  \tag{7}\\
\operatorname{Re}_{\mathrm{m}}=\frac{d_{p} u_{0} \rho}{\mu(1-e)}
\end{gather*}
$$

Correlation of experimental data on the pressure drop in the flow of single-phase fluids through packed beds gives,

$$
\begin{equation*}
\mathrm{f}_{\mathrm{m}}=\frac{150}{\mathrm{Re}_{m}}+1.75 \tag{9}
\end{equation*}
$$

Equation (9) is known as the Ergun equation. It fits experimental data well for $\mathrm{Re}_{\mathrm{m}}$ ranging from 1 to over 2000.

For $\operatorname{Re}_{\mathrm{m}}<10$, when the flow is streamline, the resistance to the flow is due to the viscous drag at the surface of the particles and the losses can be attributed mainly to skin friction. The second term in Eq. (9) becomes negligible compared tom the first and,

$$
\begin{align*}
\mathrm{f}_{\mathrm{m}} & =\frac{150}{\mathrm{Re}_{m}}  \tag{10}\\
\text { or, }-\frac{\Delta P_{f}}{\rho} & =150 \frac{\mu u_{0} L}{d_{p}^{2} \rho} \times \frac{(1-e)^{2}}{e^{3}} \tag{11}
\end{align*}
$$

Equation (11) is known as the Blake-Kozeny, or Kozeny-Carman equation.
It may be noted that the flow of fluids in the earth or in industrial filters is usually streamline.
For $\operatorname{Re}_{\mathrm{m}}>1000$, when the flow is turbulent, resistance to the flow is mainly due to the loss caused by turbulent eddies and the sudden changes in the cross-sections of the flow channels. At high flow rates and in very thin beds, where the enlargement and contraction losses become significant, the effect of viscous force is negligible.
$\mathrm{f}_{\mathrm{m}} \quad=1.75$
or, $-\frac{\Delta P_{f}}{\rho}=1.75 \frac{u_{0}^{2} L}{d_{p}} \times \frac{1-e}{e^{3}}$

This equation is known as the Burke-Plummer equation.
The diameter $d_{v}$ of a nonspherical particle may be defined as the diameter of a sphere with the same volume as that of the particle. A screen analysis approximates $d_{v}$ fro irregular, nearly spherical particles. A screen analysis may underestimate or overestimates $\mathrm{d}_{\mathrm{v}}$ of regular, non spherical particles, depending on the shape of the particles. In general it gives the second largest dimension of the particle.

For non spherical particles, the shape factor or sphericity $\phi_{\mathrm{s}}$ is defined as,

$$
\begin{equation*}
\phi_{\mathrm{s}}=\frac{\text { Surfaceareaof a spherehaving the same volumeas that of the particle }}{\text { Surfaceof a particle }} \tag{14}
\end{equation*}
$$

For spheres $\phi_{\mathrm{s}}=1$, and $0<\phi_{\mathrm{s}}<1$ for all other particle shapes.
The diameter $d_{p}$ in Eqs (4)-(12) is replaced by $\phi{ }_{s} d_{v}$ for beds of non spherical granular solids, so that the Ergun equation becomes,

$$
\begin{equation*}
\frac{-\Delta P_{f}}{\rho}=150 \frac{\mu u_{0}}{\rho\left(\phi_{s} d_{v}\right)^{2}} \times \frac{(1-e)^{2}}{e^{3}}+1.75 \frac{u_{0}^{2} L}{\phi_{s} d_{v}} \times \frac{1-e}{e^{3}} \tag{15}
\end{equation*}
$$

## Motion of solids in fluid:

Many processing steps, especially mechanical separation, involve the movement of solid particles or liquid drops through a fluid. The fluid may be gas or liquid, and it may be flowing or at rest. Examples are the elimination of dust and fumes from air or flue gas, than removal solids from liquid wastes, and the recovery of acid mists from the waste gas of an acid plant.

## Mechanics of particles in motion:

The movement of particle through a fluid requires external force acting on the particle. This force may come from a density difference between the particles and the fluid, or it may be the result of electric or magnetic fields. In this section only gravitational or centrifugal forces, which arise from density differences are considered.

Three forces acting on the particles moving through a fluid are,

1. The external force, gravitational, or centrifugal
2. The buoyant force which acts parallel with the external force but in the opposite direction, and
3. The drag force which appears, whenever there is a relative motion between the particle and fluid.

The drag force acts to oppose the motion and parallel with the direction of movement but in the opposite direction.

Consider a particle of mass ' $m$ ' moving through a fluid under the action of external force ' Fe ' the velocity of particle is ' $u$ '. Let the buoyant force is ' $F_{b}$ ' and the drag force ' $F_{D}$ '. then the resultant force on the particle is $\mathbf{F e}-\mathbf{F}_{\mathbf{b}}-\mathbf{F}_{\mathbf{D}}$. The acceleration of the particle is (du/dt). The force balance is,

$$
m \frac{d u}{d t}=F_{e}-F_{b}-F_{D}
$$

where, m - is constant,

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{e}}=\text { External force }=\mathrm{m} \mathrm{a}_{\mathrm{e}} \\
& \mathrm{~F}_{\mathrm{b}}=\text { Buoyant force }=\frac{m \rho a_{e}}{\rho_{P}} \\
& \mathrm{~F}_{\mathrm{D}}=\text { Drag force }=\frac{C_{D} u_{0}^{2} \rho A_{P}}{2} \\
& \mathrm{a}_{\mathrm{e}}=\text { acceleration }
\end{aligned}
$$

### 4.8 FLUIDIZED BED AND FLUIDIZATION:

Fluidization is an operation by which fine granular solids are transformed into a fluid-like state through contact movement of granular solid particles through a series of processing steps in a continuous fashion.

The chief advantages of fluidization are that it ensures contact of the fluid with all parts of the solid particles, prevents segregation of these particles by thoroughly agitating the bed, minimizes temperature variations even in a large reactor and ensures high heat and mass transfer rates. Fluidization finds application in the catalytic-cracking reactors in the petroleum industry, drying and sizing crystals, transporting solids, coating metal surfaces with plastic materials, roasting ores, and synthesis reactions.

The disadvantages of fluidization include greater power requirement, higher breakage of solid particles, serious erosion of pipelines and containers, and need for bigger reactors.

Consider the upward flow of a fluid through a vertical bed of fine particles. At a low flow rate, the fluid merely percolates through the void spaces between stationary particles in a fixed bed. Neglecting kinetic energy terms, Bernoulli's equation becomes,
$\Delta \mathrm{P}+\rho g \Delta L=-\Delta P_{f}$
where the friction loss is obtained from Kozeny-Carman equation, if the flow is streamline. Since $\rho g \Delta L$ is usually much less compared to $\Delta P_{f}$, the pressure difference across the bed of solid particles $\Delta P$ is found to be linearly proportional to the superficial fluid velocity $\mathrm{u}_{0}$.

If the flow rate of the fluid be increased steadily, a point is reached when the particles are all just suspended in the upward flowing fluid. At this point the friction force between a particle and fluid counterbalances the weight of the particle, and the pressure drop through any section of the bed nearly equals the weight of the particles in that any section. The bed is considered to be just fluidized and is referred to as an incipiently bed. Thus, for bed at minimum fluidization

$$
\begin{equation*}
\Delta P A=W=\left(A L_{m f}\right)\left(1-e_{m f}\right)\left(\rho_{p}-\rho\right) g \tag{2}
\end{equation*}
$$

where, A is the cross-sectional area of the tube, and $\rho_{p}$ is the density of the particle.

Therefore, $\frac{\Delta P}{L_{m f}}=\left(1-e_{m f}\right)\left(\rho_{p}-\rho\right) g$

In many practical applications of fluidization, the particles are very small and the fluid velocity is low, so that the flow is streamline, and from Kozeny-Carman equation.
$\frac{-\Delta P}{L_{m f}}=150 \frac{\mu u_{m f}}{\phi_{s}{ }^{2} d_{p}{ }^{2}} \times \frac{\left(1-e_{m f}\right)^{2}}{e^{3}{ }_{m f}}$

Equating right hand sides of Eqs (3) and (4), the superficial fluid velocity at minimum fluidization conditions given by
$u_{m f}=\frac{d_{p}^{2}}{150} \times \frac{\rho_{p}-\rho}{\mu} g\left[\frac{e_{m f}^{3}}{1-e_{m f}}\right]$

For fluid velocities less than the minimum fluidization velocity $u_{m f}$, the bed behaves as a packed bed. However, as the velocity increases past $u_{\mathrm{mf}}$, not only does the bed expand, but the particles move about relative to each other, as e increases, the particles move apart and are able to slide past each other, and the entire particle-fluid mass behaves as a fluid which can be poured from one vessel to another an be pumped, etc. as the velocity increases further, the bed becomes more expanded, and the solid content becomes more and more dilute. Finally, as the velocity becomes
equal to the terminal settling velocity of the individual particles, the last particles are swept out of the system. Thus the velocity range for which a fluidized bed can exist is from $u_{m f}$ to the free settling velocity of the particles.

For a given fluid-solid system, Eqs (3) and (4) predict that

$$
\begin{aligned}
& \frac{\Delta P}{L(1-e)}=k_{1} \\
& =\mathrm{k}_{2} \mathrm{u}_{\mathrm{o}} \frac{1-e}{e^{3}}
\end{aligned}
$$

where, $\mathrm{k}_{1}$ and $\mathrm{k}_{2}$ are constants.
Thus the variation of the bed porosity e with superficial fluid velocity $u_{0}$ in a fluidized bed is given by
$\mathrm{u}_{0}=\mathrm{k} k \frac{e^{3}}{1-e}$
where, k is constant for the system.
Equation (6) has been found to hold good for a liquid-solid system for $\mathrm{e}<0.80$. The relationship between bed heights and bed voidages is defined by

$$
\begin{equation*}
\frac{L}{L_{m f}}=\frac{1-e_{m f}}{1-e} \tag{7}
\end{equation*}
$$

The free settling velocity $u_{t}$ of a spherical particle in a fluid medium is given by stokes' law

$$
\begin{equation*}
u_{t}=\frac{d_{p}^{2}\left(\rho_{p}-\rho\right) g}{18 \mu} \tag{8}
\end{equation*}
$$

Provided that,
$\operatorname{Re}_{\mathrm{t}}=\frac{d_{p} u_{t} \rho}{\mu}<0.3$

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# UNIT-IV <br> TRANSPORTATION OF FLUIDS 

## TRANSPORTATION OF FLUIIDS

## TRANSPORTATION OF LIQUIDS - PUMPS:

A pump may be defined as a mechanical device which converts mechanical energy supplied to it (by an electrical motor or oil engine) into hydraulic energy.

Pump may also be defined as a devices used for lifting liquids from a lower level to a higher level. Pumps are broadly classified as follows:

1. Positive displacement pumps
(i). Reciprocating pumps
a).Piston pumps
b).Plunger pumps
c).Diaphragm pumps
(ii). Rotary pumps
a). Gear pumps (Internal and External)
b). Lobe pumps
c). Screw pumps
d). Vane pumps
2. Centrifugal pumps (kinematics pumps)
(i). Single suction pumps
(ii). Double suction pumps

## POSITIVE DISPLACEMENT PUMPS:

In the positive displacement pump a definite volume of liquid is trapped in a chamber, which is alternatively filled from the inlet and emptied at a high pressure through the discharge.

Positive displacement pumps apply pressure directly to the liquid by reciprocating piston or by rotating members which forms chambers alternatively filled by and emptied of the liquid.

## Reciprocating pump:

In reciprocating pumps, the liquid is sucked and displaced due to the thrust exerted on it by a moving piston or plunger. Hence they are also known as positive displacement pumps. The maximum discharge pressure of piston pump is 50 atm . The reciprocating pumps can be classified as single acting pump and double acting pump according to the contact of liquid with the sides of piston or plunger.

## Single acting reciprocating pump:

In a single acting pump, the liquid is in contact with one side of the piston or plunger. It has one suction pipe and one delivery pipe. Delivery of liquid takes place only in delivery stroke.


Figure 4.1 Single acting reciprocating pump

Figure 5.1 shows a single acting reciprocating pump. It consists of a pump cylinder, piston or plunger, piston rod, crank, connecting rod, suction pipe, delivery pipe, suction valve, delivery valve. Suction and delivery valves are one way (non return) valves. The crank is driven by the prime mover. The rotation of crank moves the piston to and fro inside the cylinder.

## Working:

During suction stroke, piston moves to the right and vacuum is created inside the cylinder. Due to this, suction valve opens and the liquid from the sump enters (sucked into) the cylinder through the suction pipe and suction valve. Delivery valve remains closed during this stroke.

During delivery stroke, piston moves to the left and forces liquid out of the cylinder. Due to this, delivery valve opens and the liquid is delivered to the required height through the delivery valve and delivery pipe. Suction valve will remain closed during this stroke. Crank rotates one revolution for completing one suction and one delivery stroke. The same cycle is repeated as the crank revolves.

## Applications:

The speed of this pump is limited and hence they are suitable for small capacity and high heads. It is generally used for

1. Marines (for pumping water),
2. Pneumatic pressure system,
3. Pumping feed water to small boilers,
4. Pumping light oil.

## Double acting reciprocating pump:

In a double acting pump, the liquid is in contact with both sides of the piston or plunger. It has two suction pipes and two delivery pipes. Liquid is delivered during each stroke.

## Construction:

A double acting reciprocating pump is as shown in the following figure. It has two suction pipes, two delivery pipes, two suction valves and two delivery valves. The valves are non return valves (Figure 4.2).


Figure 4.2 Double acting reciprocating pump

## Working:

When the piston moves to the right, a partial vacuum is created in the left side of the piston. The liquid from the sump is forced into the cylinder through the left side suction valve. When the piston returns (moves to the left) vacuum is created in the right side of the piston. The liquid from the sump is sucked in to the cylinder through the right side suction valve. At the same time, the liquid in the left side of the piston is forced out through the left side delivery valve and delivery pipe. The operations are repeated. During each stroke, suction takes place on one side of the piston and delivery takes place on the other side of the piston. Thus the liquid is delivered continuously.

## Plunger pump:

## Construction:

The plunger pump consists of heavy walled cylinder inside of which a movable plunger is connected to motor through a shaft. A stationary packed seal is provided around the plunger. The
diameter of plunger is small when compared to piston diameter. Plunger pumps also available with more than one cylinder. Inlet and outlet paths are provided with check valves (Figure 7.3).


Figure 4.3 Plunger pump

## Working:

During suction stroke, plunger moves to the right and vacuum is created inside the cylinder. Due to this, suction valve opens and the liquid from the sump enters (sucked into) the cylinder through the suction pipe and suction valve. Delivery valve remains closed during this stroke.

During delivery stroke, plunger moves to the left and forces liquid out of the cylinder. Due to this, delivery valve opens and the liquid is delivered to the required height through the delivery valve and delivery pipe. Suction valve will remain closed during this stroke. Crank rotates one revolution for completing one suction and one delivery stroke. The same cycle is repeated as the crank revolves.

These pumps build up very high pressures up to 1500 atm . They are suitable for rough works such as pumping of water containing sand. The plunger pump may be single acting or double acting.

## Diaphragm pump:

## Construction:

The diaphragm pump consists of a flexible diaphragm made up of metal, plastic or rubber as a reciprocating member. This eliminates the need for packing or seals exposed to the liquid being pumped. The diaphragm pump also consists of pumping chamber, crank, connecting rod, suction pipe, delivery pipe, suction valve(inlet), delivery valve(outlet). Suction and delivery valves are ball valves. The crank is driven by the electric motor. The rotation of crank moves the piston to and fro inside the cylinder (Figure 7.4).


Figure 4.4 Diaphragm pump

## Working:

During suction stroke, diaphragm moves up and vacuum is created inside the chamber. Due to this, suction valve opens and the liquid from the sump enters (sucked into) the chamber through the suction pipe and suction valve. Delivery valve remains closed during this stroke.

During delivery stroke, diaphragm moves down and forces liquid out of the chamber. Due to this, delivery valve opens and the liquid is delivered to the required height through the delivery valve and delivery pipe. Suction valve will remain closed during this stroke.

Crank rotates one revolution for completing one suction and one delivery stroke. The same cycle is repeated as the crank revolves.

## Applications:

These pumps build up very high pressures in excess of 100 atm . They are suitable for pumping of toxic or corrosive liquids. The diaphragm pump handle small to moderate amounts of liquid, up to about $100 \mathrm{gal} / \mathrm{min}$.

## Differences between piston and plunger pumps:

| Piston pump |  | Plunger pump |
| :---: | :--- | :--- |
| 1. | The reciprocating component is a piston <br> with a piston rod. | The reciprocating component is a plunger. |
| 2. | Piston requires piston rings (packing rings) | The plunger has no piston rings. Packing <br> rings are held in position with stationary <br> cylinder by packing glands. |
| 3. | It is used for moderate pressure up to 50 <br> atm. | It can develop very high pressure up to <br> 1500 atm. |
| 4. | It is used for pumping clean water as the <br> clearance between the cylinder and the <br> piston is very small. | It can be used for pumping water <br> containing sand and similar material. |
| 5. | More wear and tear. | Less wear and tear. |
| 6. | Liners are provided in the cylinder which <br> requires periodical replacement. | No liners are used. |

## Discharge, work done and power required to drive reciprocating pump:

Let,
$D=$ Diameter of the cylinder
$A=$ Area of cross section of the cylinder or piston
$=\frac{\pi}{4} D^{2}$
$r=$ Radius of crank
$N=$ Speed of the crank in $r p m$.
$L=$ Length of the stroke ( $=2 r$ )
$h_{s}=$ Height of the axis of the cylinder from water surface in sump
$h_{d}=$ Height of the delivery outlet above the cylinder axis
Volume of water delivered in one revolution

$$
=\text { Area } \times \text { Length of stroke }=A \times L
$$

Number of revolution per second $=\frac{\mathrm{N}}{60}$
$\therefore$ Discharge of the pump per second,

$$
Q=A \times L \times \frac{N}{60}=\frac{A L N}{60}
$$

Weight of water delivered per second

$$
W=w Q=\frac{w A L N}{60}
$$

Work done per second $=$ Weight of water lifted $/ \sec \times$ total height through whith liquid is lifted

$$
\begin{aligned}
& =W\left(h_{s}+h_{d}\right) \\
& =\frac{w A L N}{60}\left(h_{s}+h_{d}\right) \text { (Where, } w=\text { weight density of liquid }
\end{aligned}
$$

Case (ii): Double acting pump:
For double acting pumps, all the values are doubled since it gives twice the output that of a single acting pump. Therefore,

Discharge, $Q=\frac{2 A L N}{60}$

Work done per second $=\frac{2 w A L N}{60}\left(h_{s}+h_{d}\right)$
Power required to drive the pump

$$
P=\frac{2 w A L N}{60}\left(h_{s}+h_{d}\right)
$$

## Slip:

The difference between the theoretical discharge and actual discharge is called slip of the pump.

$$
\text { Slip }=Q_{t h}-Q_{a c t}
$$

Generally, the slip is expressed in percentage which is given by

$$
\begin{aligned}
& \% \text { slip }=\frac{Q_{t h}-Q_{a c t}}{Q_{t h}} \times 100=1-\frac{Q_{a c t}}{Q_{t h}} \times 100 \\
& \% \text { slip }=\left(1-C_{d}\right) \times 100
\end{aligned}
$$

where, $\mathrm{C}_{\mathrm{d}}$ is coefficient of discharge

## Problem 4.1

A single acting reciprocating pump, running at 60 rpm delivers $0.53 \mathrm{~m}^{-}$of water per minute. The diameter of the piston is 200 mm and stroke length 300 mm . The suction and delivery heads are $4 m$ and $12 m$ respectively. Determine
(i) Theoretical discharge.
(ii) Co-efficient of discharge.
(iii) Percentage slip of the pump, and
(iv) Power required to run the pump.

## Given data:

Speed, $N=60 \mathrm{rpm}$
Discharge, $Q_{a c t}=0.53 \mathrm{~m}^{3} / \mathrm{min}=\frac{0.53}{60}=8.83 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}$
Diameter of piston, $D=200 \mathrm{~mm}=0.2 \mathrm{~m}$
Stroke length, $L=300 \mathrm{~mm}=0.3 \mathrm{~m}$
Suction head, $h_{s}=4 m$
Delivery head, $h_{d}=12 \mathrm{~m}$

$$
\text { Theoretical discharge, } \begin{aligned}
Q_{t h}=\frac{A L N}{60} & =\frac{4^{\pi} \times 0.2^{2} \times 0.3 \times 60}{60} \\
& =9.42 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

Co-efficient of discharge, $C_{d}=\frac{Q_{a c t}}{Q_{t h}}=\frac{8.83 \times 10^{-3}}{9.42 \times 10^{-3}}=0.937$
Ans.

$$
\begin{aligned}
\% \text { slip of the pump } & =\frac{Q_{t h}-Q_{a c t}}{Q_{t h}} \times 100 \\
& =\frac{9.42 \times 10^{-3}-8.83 \times 10^{-3}}{8.83 \times 10^{-3}} \times 100 \\
& =6.68 \%
\end{aligned}
$$

Dower required to run the pump,
Ans.

Ans.

$$
P=\frac{w A L N}{60}\left(h_{s}+h_{d}\right)
$$

## Problem 4.2

A double acting reciprocating pump, running at 50 rpm is discharging 900 liters of of the pump and power required to drive the pump. Given data:

Speed, $N=50 \mathrm{rpm}$
Discharge, $Q=900 \mathrm{lit} / \mathrm{min}=\frac{900}{1000 \times 60}=0.015 \mathrm{~m}^{3} / \mathrm{s}$

$$
\text { Strokes, } L=400 \mathrm{~mm}=0.4 \mathrm{~m}
$$

Diameter of piston, $D=250 \mathrm{~mm}=0.25 \mathrm{~m}$
Delivery head, $h_{d}=25 \mathrm{~m}$
Suction head, $h_{s}=4 \mathrm{~m}$
© Solution:
Solution:
Theoretical discharge, $Q_{t h}=\frac{2 A L N}{60}=\frac{2 \times \frac{\pi}{4} \times 0.25^{2} \times 0.4 \times 50}{60}=0.03272 \mathrm{~m}^{3} / \mathrm{s}$

$$
\text { Slip }=Q_{t h}-Q_{a c t}=0.03272-0.015=0.01772 \mathrm{~m}^{3} / \mathrm{s} \quad \text { Ans. }
$$

$$
\begin{aligned}
& \text { Power required to drive the double acting pump, } \quad P=\frac{2 w A L N}{60}\left(h_{s}+h_{d}\right) \\
& =\frac{2 \times 9.81 \times \frac{\pi}{4} \times 0.25^{2} \times 0.4 \times 50}{60}(25+4) \\
& =9.3 \mathrm{~kW}
\end{aligned}
$$

### 4.2 Rotary pump:

In rotary pump the liquid is carried from the suction side to the discharge side through rotation of a body. The rotating body may be a gear, lobe, vane etc. Rotary pumps are classified as follows:
i. External gear pump
ii. Internal gear pump
iii. Vane pump
iv. Lobe pump
v. Screw pump.

## External Gear Pump:

## Construction:



### 4.5 External gear pump

In external gear pump gear meshing arrangement is actually outside of the gear wheel, so it is called external gear pump. Gear pump consists of two gears in a casing. One gear is rotating in clockwise direction and another gear is rotating in anticlockwise direction. Only one gear is connected to a driving shaft and another gear is rotates freely. In special circumstances both the gears are connected to driving shaft (Figure 4.5).

## Working:

When the gears rotate, it creates partial vacuum inside the casing which causes the suction of liquid. The entered liquid is carried to the discharge line by the rotation of gears, since the space between the casing and gears are so small the pressure of the liquid increases whiles it carrying from suction to discharge. Due to the meshing of gear the liquid cannot return back to the suction. The liquid leaves the gear pump at moderate pressure.

## Internal Gear Pump:

## Construction:



### 4.6 Internal gear pump

In internal gear pump two gears are meshed internally inside a casing, where the inside is driven gear and other one is stationary one. The driven gear runs eccentrically to the stationary gear. The spaces between the gears are sealed with a crescent shaped projection which is actually acts as partition to prevent the liquid entering from discharge side to suction side (Figure 4.6).

## Working:

When the pump starts, the teeth in the gear wheels intermesh completely. Due to eccentric movement, when the teeth come out of the stationary gear, and hence partial vacuum is created which causes suction of liquid inside the casing. When the pump runs continuously the trapped liquid between the driven gear and crescent is discharged through the outlet at high pressure.

## Vane Pump:

## Construction:



### 4.7 Vane pump

A Vane pump consists of a circular casing consists of a rotor disk which is connected to the motor shaft. The rotor disk consists of radial rectangular slots. Inside the rectangular slot, rectangular vanes are present which can freely moves inside the slot. They all there one put inside a casing (Figure 4.7).

## Working:

When the pump runs in clockwise direction, due to the centrifugal force the vanes moves outward radially and makes air tight contact with the casing, due to which a partial vacuum is created between the vanes and casing which causes suction of liquid. When the pump runs continuously the trapped liquid discharges at high pressure. And the pumping of liquid continues.

## Lobe Pump:

## Construction:


4.8 Lobe pump

A lobe pump actually resembles gear pump. It consists of two lobes are rotates inside a casing. One lobe is connected to driven motor shaft which rotates clockwise direction and another lobe rotates freely in anticlockwise direction. Numerous designs of lobes are available; in every design functions are same (Figure 4.8).

## Working:

When the lobes are rotates, it creates a partial vacuum inside the casing and causes suction of liquid. When the lobes run continuously the trapped liquid is discharged at high pressure in the discharge side. The main disadvantage of lobe pump is discharge of liquid is not constant when compared with gear pump.

## Screw Pump:

## Construction:



### 4.9 Screw pump

Screw pumps are suitable for the liquids having high viscosity. Screw pumps are modified form of external gear pump. There are two types of screw pump:
i. Single screw pump- where only one rotor rotates.
ii. Double screw pump- where two rotors rotate.

In screw pump, a rotating screw (rotor) is present in a casing. The rotor is connected to a motor through a shaft. The rotors are meshed with casing internally. The space between rotor and casing is so small (Figure 4.9).

## Working:

When the shaft rotates, the liquid enters in the suction side. It divides into two streams. The liquid flows into two extreme sides of rotor. The pressure of the fluid increases when it moves axially when the shaft rotates. The increase in pressure due to the meshing of the rotor screws with casing. The liquid discharges through the opening present in the centre of the rotor.

### 4.2 CENTRIFUGAL PUMPS:

A hydraulic machine which converts mechanical energy of liquid into pressure energy by means of centrifugal force acting on the liquid is called centrifugal pump.

### 4.2.1 Principle:

The centrifugal pump acts as a reversed of an inward radial flow reaction turbine. This means that the flow in centrifugal pumps is in the radial outward directions. The centrifugal pump works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of rotating liquid takes place. The rise in pressure head at any point of rotating liquid is proportional to the square of tangential velocity of the liquid at that point [i.e. rise in pressure head $=\left(\mathrm{V}^{2} / 2 \mathrm{~g}\right)$ or $\left(\mathrm{w}^{2} \mathrm{r}^{2} / 2 \mathrm{~g}\right)$ ]. Thus at the outlet of the impeller where radius is more, rise in pressure head will be more and the liquid will be discharged at the outlet with high pressure head. Due to this high-pressure head, the liquid can be lifted to a high level.

### 4.1.2.2 Construction:

The main components of centrifugal pump are

1. Impeller,
2. Casing,
3. Suction pipe,
4. Delivery pipe,
5. Delivery valve and
6. A prime mover.


Figure 4.9 Centrifugal pump

## Impeller:

Impeller is a rotor, provided with a series of curved vanes or blades. It is mounted on a shaft. This shaft is rotated by a prime mover such as electric motor or oil engine.

## Casing:

The casing surrounds the impeller. It is an air -tight and water-tight casing. The casing is designed with a gradually increasing area. Hence, when water flows through the casing, the kinetic energy of water is converted into pressure energy, before the water leaves the casing.

## Suction pipe:

The upper end of the suction pipe is connected to the inlet of the pump. The lower end is submerged into suction well or sump from which water is to be pumped. The lower end of suction pipe is fitted with a foot valve and strainer.

## Delivery pipe:

The lower end of delivery pipe is connected to the outlet of the pump. The other end delivers water at the required level.

## Delivery valve:

A delivery valve is provided in the delivery pipe just near the outlet of the pump. It is provided to control (to regulate) the flow from the pump into the delivery pipe.

## Prime mover:

It is used to drive (rotate) the impeller of the pump. Usually, an electric motor is provided for this purpose

### 4.1.2.3 Working:

The first step in the operation of a centrifugal pump is 'priming'. Priming is the operation of filling up water in the suction pipe, casing and a portion of delivery valve. It is done to remove the air present inside. If any air is present, the pressure developed across the impeller will not be sufficient to suck the water from the sump. The delivery valve is kept closed during priming.

After priming, the impeller is rotated by a prime mover keeping the delivery valve still closed. The rotating vanes give centrifugal head to the pump. When the pump attains a constant speed, the delivery valve is gradually opened. The water flows in a radially outward direction. Then, it leaves the vanes at the outer circumference with high velocity and pressure.

As the liquid flows along the spiral casing, its kinetic energy gradually converted into pressure energy. The high pressure water is discharged through the delivery pipe to the required length.

When the water is forced away from the centre of the impeller by the centrifugal force, partial vacuum is created at the centre of the impeller known as 'eye'. Due to this, water from the sump enters into the eye of the impeller through the suction pipe. Thus the water enters and leaves the impeller continuously to maintain a continuous discharge to the required height. The suction head is generally limited to 7.90 ' m ' of water to avoid cavitation.

### 4.1.2.4 Applications:

Centrifugal pump is most commonly used in water works, sewage works, irrigation, water pressure schemes, drainage, oil refineries, etc.

### 4.1.2.5 Terminology:

Suction Head ( $\boldsymbol{h}_{s}$ ):
Suction head is defined as the vertical height of the centre line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted. This height is also called suction lift and is denoted by ' $h_{s}$ '.

## Discharge HeaD ( $h_{d}$ ):

Discharge head is defined as the vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by ' $h_{d}$ '.

## Developed Head $\left(H_{d}\right)$ :

Developed head is defined as the sum of suction head and discharge head. This is denoted by ' $\mathrm{H}_{\mathrm{d}}$ '. It is also known as static head $\mathbf{H}_{\mathbf{d}}=\mathbf{h}_{\mathrm{s}}+\mathbf{h}_{\mathbf{d}}$

Suction limitations of a pump: Whenever the pressure in a liquid drops below the vapour pressure corresponding to its temperature, the liquid will vaporize. When this happens within an operating pump, vapor bubbles will be carried along to a point of higher pressure, where they suddenly collapse. This phenomenon is known as cavitation.

Cavitation in a pump should be avoided, as it is accompanied by metal removal, vibration, reduced flow, loss of efficiency, and noise. When the absolute suction pressure is low, cavitation may occur in the pump inlet and damage result in the pump suction and one of the impeller vanes near the inlet edges. To avoid the phenomena, it is necessary to maintain a required net positive suction head (NPSH) $\mathbf{R}$ which is the equivalent total head of liquid at the pump centre line less the vapor pressure 'p'. Each pump manufacturer publishes curves relating (NPSH) $)_{R}$ to capacity and speed for each pump.

When a pump installation is being designed, the available net positive suction head must be equal to or greater than the $(\mathrm{NPSH})_{\mathrm{R}}$ for the desired capacity. The (NPSH) $)_{\mathrm{A}}$ can be calculated as follows; $(\mathbf{N P S H})_{A}=h_{s s}-h_{f s}-\mathbf{p}$.

If (NPSH) $)_{A}$ is to be checked on a exiting installation, it can be determined as follows; $(\mathbf{N P S H})_{A}=$ $\mathbf{a t m}+\mathbf{h g s}_{\mathrm{gs}}-\mathbf{p}+\mathbf{h}_{\mathrm{vs}}$

Practically, the NPSH required for operation without cavitation and vibration in the pump is somewhat greater than the theoretical. The actual (NPSH) ${ }_{R}$ depends on the characteristics of the liquid, the total head, the pump speed, the capacity, and impeller design. Any suction condition which reduce $(\mathrm{NPSH})_{\mathrm{A}}$ below that required to prevent cavitation at the desired capacity will produce an unsatisfactory installation and can led to mechanical difficulty.

### 4.1.2.6 Cavitation:

When the suction lift is high for a centrifugal pump, a large vacuum is created ate the pump inlet. This vacuum pressure falls below the vapour pressure of the water corresponding to that temperature. At this condition, water will vaporize and cavities are formed. This phenomenon is known as cavitation.

When cavitation occurs, vapour bubbles are formed. They move from low pressure side (entry to the impeller) to the high pressure (exit of the impeller) and collapse suddenly. When they collapse, they hit the vanes of impeller and impeller may worn out. This damaging of impeller is known as 'pitting'.

## Effects of cavitation:

1. The metallic surfaces are damaged and cavities are formed on the surfaces.
2. Due sudden collapses of vapour bubble, considerable noise and vibrations are produced.
3. Due to pitting the efficiency of the pump is decreases.

## Prevention of cavitation:

1. The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure
2. The special materials or coatings such as aluminium - bronze and stainless steel, which are cavitation resistance material.
3. Suction lift is kept between 5 m to 6 m ,
4. The velocity in the suction pipe should be low,
5. The pump speed should be reasonably low.
6. Sharp bends in the suction pipe should be avoided to reduce frictional losses.
7. Impeller is provided with more number of vanes to reduce turbulence.

### 4.1.2.7 Priming:

Priming is a process of filling up water in the casing and suction pipe of a centrifugal pump for the removal of air before starting it. If any air is present inside the casing, discontinuity of flow may be caused. If the pump is started with air in the casing and suction pipe, there will be only a negligible pressure difference across the impeller. This will not be sufficient to create enough vacuum to suck the water into the casing from the sump. Hence, the pump will not work. Therefore priming is very essential before starting the pump.

The pumps can be primed in several ways. They are,

1. Manual priming,
2. Priming by vacuum, and

## 3. Self-priming

In manual priming, water is poured through the priming cock by a funnel and the air vent in the casing is opened. When all the air has been displaced from the suction- pipe and casing, the cock is then closed and the pump can be started.

In large pumps, priming is done by evacuating the casing and suction pipe with the aid of an air pump or ejector. Thus the water is drawn into the suction pipe from the sump. This is called priming by vacuum. In self-priming, the priming is done automatically by having a special reservoir containing water between suction line and pump. Self-priming devices are used with big size pump only, as it requires large expenses.

### 4.2 GAS MOVING MACHINERIES

Gas moving machinery comprises mechanical devices used for compressing and moving gases. They are often classified from the stand point of the pressure heads produced and are fans for low pressures, blowers for intermediate pressures, and compressors for high pressures.

### 4.2.1 FANS:

Fans discharge large volumes of gas into open spaces or large ducts. They are low speed machines that generate very low pressures, on the order of 0.04 atm . Fans may be radial or axial flow type and they come in sizes ranging from small portable model used in automobiles to very large ventilating and industrial fans. Radial flow fans generate centrifugal force to propel the gas while the axial flow fans impel the gas along the path parallel to the axis of the fan.

The volume of gas propelled by a specific fan under specified operating conditions is directly proportional to the fan speed. Similarly, for a specific fan and operating conditions, the static pressure varies as the square of the fan speed while power consumption varies as the cube of the fan speed.

### 4.2.1.1 Centrifugal fans:



Figure 4.10 Straight Blade Fan
They have 5 to 12 radial blades mounted on rotor of comparatively large diameter. These blades resemble paddle wheels. They are best suited for low speed duty. Frequently they are used as exhaust fan where air stream is involved to transport air-borne waste.

### 4.2.1.2 Axial Flow Fans:

In axial flow fans air enters in an axial direction and leaves in an axial direction. The axial flow fans are two types:

1. Disk type, and
2. Propeller type.

The disk type fans have plain or curved blades. They are used for general circulation or exhaust work without duct.

The axial flow fans have blade designs similar to those used in aircrafts. They are usually of two staged type as shown in the following figure.

Ventilating fans are available for large volumetric flow ( $220740 \mathrm{~m}^{3} / \mathrm{h}$ ) and low pressure increase $(1400 \mathrm{~Pa})$. The fans operate in the range of $\mathbf{4 0 \%}$ to $\mathbf{7 0 \%}$ of efficiency.


Figure 4.11 Two stage axial flow fan

### 4.2.2 BLOWERS:

Blowers are high speed rotary devices that can develop a maximum pressure of about 2 atm . They are used to handle large volume of gas gases at a pressure higher than the fans. There are two types of blowers as follows:

1. Rotary or Positive displacement blowers
2. Turbo or Centrifugal blowers.

### 4.2.1 Rotary or Positive displacement blowers:

A rotary or positive displacement blower (lobe type) is as shown in the above figure. These machines operate as gear pumps do except that, because of the special design of the "teeth" the clearance is only a few thousands of an inch. The relative position of the impellers is maintained precisely by heavy external gears. A single stage blower can discharge gas at 0.4 to 1 atm . Gauge, a two-stage blower at 2 atm . The blower shown in the above figure has two lobes. Three - lobe machines are also common.


Figure7.12 Rotary or Positive displacement blowers

### 4.2.3 COMPRESSOR:

Compressors are used to handle large volume of gas and discharge at pressures from 2 atm . to several thousand atm. Compressor can be classified as follows:

1. Continuous flow compressors

- Centrifugal compressors
- Axial flow compressors

2. Positive-displacement compressors

- Rotary compressors
- Reciprocating compressors


### 4.2.3.1 Reciprocating Compressor:

Reciprocating compressors are used mainly when high- pressure head is required at a low flow. Reciprocating compressors are furnished in either single stage or multi stage types. The number stages are determined by the required compression ratio $\mathrm{p}_{2} / \mathrm{p}_{1}$. The compression ratio per stage is generally limited to 4 , although low capacity units are furnished with compression ratios of 8 and
even higher. The maximum compression ratio is determined by the maximum allowable discharge gas temperature.


## Figure 4.13 Reciprocating compressor

Reciprocating compressors operate mechanically in the same way as reciprocating pumps, with the differences that leak prevention is more difficult and the temperature rise is more important. In reciprocating compressors a piston, a cylinder fitted with suitable suction and discharge valves plus a crank shaft with a drive. The cylinder wall and cylinder heads are cored for cooling jackets using water or refrigerant. Reciprocating compressors are usually motor driven and are nearly always double-acting. The reciprocating motion of the piston propels the gas through the successive stages of expansion, suction, compression and discharge. The overall efficiency of the reciprocating compressor is in the range of $65 \%-80 \%$.

Suction: When the piston moves to left the inlet valve opens and gas enters into the cylinder and discharge valve closed.

Compression: When the piston moves to right the gas is compressed to high pressure, and suction valve closed.

Discharge: When the piston moves to left the discharge valve opens and compressed gas is delivered at high pressure. On multi stage compressors, intercoolers are provided between stages. These heat exchangers remove the heat of compression from the gas and reduce its temperature to approximately the temperature existing at the compressor intake. Such cooling reduces the volume
of the gas going to the high pressure cylinders reduces the power required for compression, and keeps the temperature within safe operating limits.

## References

1. R. K. Bansal, A Textbook of Fluid Mechanics, First Edition, Laxmi Publications (P) LTD., New Delhi, (2012).
2. Warren McCabe, Julian Smith and Peter Harriott, Unit Operations of Chemical Engineering, Seventh Edition, Mc-Graw hill International Edition (2017)
3. C. P. Kothandaraman and R. Rudramoorthy, Fluid Mechanics and Machinery, Third Edition, New Academic Science Limited, UK, (2013).
4. K.A. Gavhane, Unit Operation -1, Nirali Prakashan, Twelth Edition Educational Publishers, India, (2010).

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SCHOOL OF BIO AND CHEMICAL ENGINEERING
DEPARTMENT OF CHEMICAL ENGINEERING

# UNIT-V <br> METERING OF FLUIDS <br> FLUID FLOW MEASUREMENT 

### 5.1 VENTURIMETER

Venturimeter is a variable head meter used for finding the discharge of liquid flowing at any point along a pipe line. This is a device also used to find out the rate of discharge of liquid flowing through a pipe. This apparatus follows the principle of Bernoulli's theorem, that total head of a flowing fluid remains constant i.e., when liquid passes through a converging cone, its increase in kinetic head is due to the decrease in pressure head. By measuring the pressure difference with the help of a differential manometer, the velocity of flow and hence the rate of discharge can be evaluated.

A venturimeter consists of the following three parts (Figure 5.1):


Figure 5.1 Venturimeter [2]

## Converging cone:

It is a short tapering pipe whose diameter gradually decreases from the diameter of the pipe to the smaller diameter at the pipe to the smaller diameter at the throat. The inclined angle is about $20^{\circ} \mathrm{C}$. The length of converging cone is 2.5 times the pipe diameter and its slope is 1 in 4 to 1 in 5 .

## Throat:

It is a small portion of constant diameter which is placed in between the converging cone and diverging cone. Its diameter is $1 / 4$ to $3 / 4$ times the diameter of pipe at inlet so that the pressure at the
throat may not fall below 2.5 m of water, otherwise the separation will take place even at atmospheric temperature.

## Diverging cone:

This is also a tapering pipe whose diameter gradually increases from the diameter at throat to the diameter of the pipe. When fluid flows through this position of venturimeter, it is retarded. Due to the retardation, the velocity head of the fluid decreases and consequently its pressure head increases. If the pressure is rapidly increases then there is a every possibility that the fluid may break away from the walls of pipe due to boundary layer effect. For this reason the length of diverging cone is 3 to 4 times the length of converging cone. The inclined angle is about $5^{\circ}$.

The basic equation for venturimeter can be obtained by writing Bernoulli's equation for incompressible fluid between two stations, upstream taping and throat. Friction is neglected, meter is assumed to be horizontal and there is no pump.
$\frac{\mathrm{P}_{1}}{\rho}+\frac{\overline{\mathrm{u}}_{1}^{2}}{2}+\mathrm{gz}_{1}=\frac{\mathrm{P}_{2}}{\rho}+\frac{\overline{\mathrm{u}}_{2}^{2}}{2}+\mathrm{gz}_{2} \rightarrow(1)$
Since the meter is horizontal $\mathrm{z}_{1}=\mathrm{z}_{2}$
$\therefore \frac{\mathrm{P}_{1}}{\rho}+\frac{\overline{\mathrm{u}}_{1}^{2}}{2}=\frac{\mathrm{P}_{2}}{\rho}+\frac{\overline{\mathrm{u}}_{2}^{2}}{2}$
$\overline{\mathrm{u}}_{2}^{2}-\overline{\mathrm{u}}_{1}^{2}=\frac{2\left(\mathrm{P}_{2}-\mathrm{P}_{1}\right)}{\rho} \rightarrow(2)$

According to continuity equation $\mathrm{S}_{1} \overline{\mathrm{u}}_{1}=\mathrm{S}_{2} \overline{\mathrm{u}}_{2} \rightarrow(3)$
$\frac{\pi}{4} D_{1}^{2} \bar{u}_{1}=\frac{\pi}{4} D_{2}^{2} \bar{u}_{2}$
$\overline{\mathrm{u}}_{1}=\left(\frac{\mathrm{D}_{2}}{\mathrm{D}_{1}}\right)^{2} \overline{\mathrm{u}}_{2}$
$\overline{\mathrm{u}}_{1}=\beta^{2} \overline{\mathrm{u}}_{2}$
Eliminating $\bar{u}_{1}$ from equation 2
$\overline{\mathrm{u}}_{2}^{2}-\beta^{4} \overline{\mathrm{u}}_{2}^{2}=\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\rho}$
$\overline{\mathrm{u}}_{2}^{2}\left(1-\beta^{4}\right)=\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\rho}$
$\bar{u}_{2}^{2}=\frac{2\left(P_{1}-P_{2}\right)}{\left(1-\beta^{4}\right) \rho}$
$\overline{\mathrm{u}}_{2}=\sqrt{\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\left(1-\beta^{4}\right) \rho}} \rightarrow(4)$
But we know $\mathrm{P}_{1}-\mathrm{P}_{2}=\rho \mathrm{gh}$
$\frac{P_{1}-P_{2}}{\rho}=$ gh
Substituting in equation 4
$\overline{\mathrm{u}}_{2}=\sqrt{\frac{2 \mathrm{gh}}{\left(1-\beta^{4}\right)}} \rightarrow(5)$

Equations 4 and 5 apply to the frictionless flow of incompressible flow. To account for friction loss between 1 and 2 , equation 4 is corrected by introducing a empirical factor $\mathrm{C}_{\mathrm{v}}$, called venturi coefficient.
$\overline{\mathrm{u}}_{2}=\mathrm{C}_{\mathrm{v}} \sqrt{\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\left(1-\beta^{4}\right) \rho}} \rightarrow(6)$

Venturi coefficient is determined experimentally. When $D_{2} / D_{1}$ is less than 0.25 the term $\beta$ can be neglected. For well-designed meter $\mathrm{C}_{\mathrm{v}}$ is about 0.98 to 0.99 .

The mass flow rate through venturimeter can be calculated from the velocity at throat $\overline{\mathrm{u}}_{2}$ using continuity equation.
$\dot{\mathrm{m}}=\overline{\mathrm{u}}_{2} \mathrm{~S}_{2} \rho=\mathrm{C}_{\mathrm{v}} \mathrm{S}_{2} \sqrt{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right) \rho} \rightarrow(7)$
where $\dot{\mathrm{m}}$ - mass flow rate

Volumetric flow rate is obtained by dividing mass flow rate by the density

$$
\mathrm{q}=\frac{\dot{\mathrm{m}}}{\rho}=\mathrm{C}_{\mathrm{v}} \mathrm{~S}_{2} \sqrt{\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\left(1-\beta^{4}\right) \rho}} \rightarrow(8)
$$

$\mathrm{C}_{\mathrm{v}}$ can be found experimentally.

## Advantages of Venturi meter

1. High accuracy is attainable.
2. Pressure recovery is high, i.e., pressure losses are minimum.
3. Resistance to abrasion is minimum.
4. Ideally suited where the measured fluid contains large amount of suspended solids.

## Disadvantages of Venturimeter

1. Lot of space is required.
2. Initial cost is high
3. The device is bulky and required good support.

## WORKED EXAMPLES:

5.1. A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is $\mathbf{2 0} \mathbf{~ c m}$ of mercury. Determine the rate of flow. Assume $\mathbf{C}_{\mathbf{v}}=\mathbf{0 . 9 8}$.

Solution:
$\mathrm{D}_{1}=30 \mathrm{~cm}=0.30 \mathrm{~m} ; \quad \mathrm{D}_{2}=15 \mathrm{~cm}=0.15 \mathrm{~m}$
$\Rightarrow \beta=D_{2} / D_{1}=0.15 / 0.30=0.5$
$\mathrm{S}_{2}=\pi \mathrm{D}_{2}{ }^{2} / 4=\pi^{*} 0.15^{2} / 4=0.0177 \mathrm{~m}^{2}$
$\mathrm{R}_{\mathrm{m}}=20 \mathrm{~cm}=0.20 \mathrm{~m}$
$\Delta \mathrm{P}=\mathrm{R}_{\mathrm{m}} * \mathrm{~g} *\left(\rho_{\mathrm{m}}-\rho\right)=0.20 * 9.81 *(13600-1000)=24721 \mathrm{~N} / \mathrm{m}^{2}$.

$$
\begin{aligned}
& \mathrm{Q}=\mathrm{C}_{\mathrm{v}} \mathrm{~S}_{2} \sqrt{\frac{2 \Delta \mathrm{P}}{\rho\left(1-\beta^{4}\right)}} \\
& Q=0.98 * 0.0177 \sqrt{\frac{2 * 24721}{1000 *\left(1-0.5^{4}\right)}}=\mathbf{0 . 1 2 6} \mathbf{~ m}^{3} / \mathrm{s}
\end{aligned}
$$

5.2. A venturimeter has a pipe of 1.2 m diameter and throat of 0.6 m diameter. Find the discharge through the venturimeter and velocity of water at the throat, if the pressure is 4 cm of Hg , venturi coefficient is 0.98 .

Solution: Given: $\mathrm{D}_{1}=1.2 \mathrm{~m}$
$\mathrm{D}_{2}=0.6 \mathrm{~m}$
$\mathrm{R}_{\mathrm{m}}=4 \mathrm{~cm}=0.04 \mathrm{~m}$.
$\mathrm{C}_{\mathrm{v}}=0.98$
$\Rightarrow \beta=D_{2} / D_{1}=0.6 / 1.2=0.5$
$\mathrm{S}_{2}=\pi \mathrm{D}_{2}{ }^{2} / 4=\pi^{*} 0.6^{2} / 4=0.283 \mathrm{~m}^{2}$
$\Delta \mathrm{P}=\mathrm{R}_{\mathrm{m}} * \mathrm{~g}^{*}\left(\rho_{\mathrm{m}}-\rho\right)=0.04 * 9.81 *(13600-1000)=4944.24 \mathrm{~N} / \mathrm{m}^{2}$.
$\mathrm{Q}=\mathrm{C}_{\mathrm{v}} \mathrm{S}_{2} \sqrt{\frac{2 \Delta \mathrm{P}}{\rho\left(1-\beta^{4}\right)}}$
$\mathrm{Q}=0.98 * 0.283 \sqrt{\frac{2 * 4944.24}{1000 *\left(1-0.5^{4}\right)}}=\mathbf{0 . 9} \mathbf{m}^{3} / \mathbf{s}$.

Average velocity $\overline{\mathrm{u}}_{2}=\frac{\mathrm{Q}}{\mathrm{S}_{2}}=\frac{0.9}{0.283}=\mathbf{3 . 1 8} \mathbf{~ m} / \mathbf{s}$.

### 5.2 ORIFICEMETER

It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice which is concentric with the pipe (Figure 5.2). The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

$$
\begin{aligned}
& D_{1}=\text { diameter of pipe } \\
& D_{2}=\text { diameter of throat } \\
& \frac{D_{2}}{D_{1}}=\beta=\text { ratio of diameter of throat to the diameter of pipe. }
\end{aligned}
$$



Figure 5.2 Orificemeter

A differential manometer is connected at section, which is at a distance of about 1.5 to 2 times the pipe diameter upstream from the orifice plate and at section 2 , which is at a distance about half the diameter of the pipe on the downstream side from the orifice plate.

Let $P_{1}, \bar{u}_{1}$ and $S_{1}$ are pressure, average velocity and cross sectional area respectively at section 1 and $P_{2}, \bar{u}_{2}$ and $S_{2}$ are corresponding values section 2.

Applying Bernoulli's equation between 1 and 2
$\frac{\mathrm{P}_{1}}{\rho}+\frac{\overline{\mathrm{u}}_{1}^{2}}{2}+\mathrm{gz}_{1}=\frac{\mathrm{P}_{2}}{\rho}+\frac{\overline{\mathrm{u}}_{2}^{2}}{2}+\mathrm{gz}_{2} \rightarrow(1)$
Since the meter is horizontal $\mathrm{z}_{1}=\mathrm{Z}_{2}$
$\therefore \frac{\mathrm{P}_{1}}{\rho}+\frac{\overline{\mathrm{u}}_{1}^{2}}{2}=\frac{\mathrm{P}_{2}}{\rho}+\frac{\overline{\mathrm{u}}_{2}^{2}}{2}$
$\overline{\mathrm{u}}_{2}^{2}-\overline{\mathrm{u}}_{1}^{2}=\frac{2\left(\mathrm{P}_{2}-\mathrm{P}_{1}\right)}{\rho} \rightarrow(2)$

According to continuity equation $\mathrm{S}_{1} \overline{\mathrm{u}}_{1}=\mathrm{S}_{2} \overline{\mathrm{u}}_{2} \rightarrow$ (3) $\frac{\pi}{4} D_{1}^{2} \overline{\mathrm{u}}_{1}=\frac{\pi}{4} D_{2}^{2} \overline{\mathrm{u}}_{2}$
$\overline{\mathrm{u}}_{1}=\left(\frac{\mathrm{D}_{2}}{\mathrm{D}_{1}}\right)^{2} \overline{\mathrm{u}}_{2}$
$\overline{\mathrm{u}}_{1}=\beta^{2} \overline{\mathrm{u}}_{2}$
Eliminating $\overline{\mathrm{u}}_{1}$ from equation 2
$\overline{\mathrm{u}}_{2}^{2}-\beta^{4} \overline{\mathrm{u}}_{2}^{2}=\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\rho}$
$\overline{\mathrm{u}}_{2}^{2}\left(1-\beta^{4}\right)=\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\rho}$
$\overline{\mathrm{u}}_{2}^{2}=\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\left(1-\beta^{4}\right) \rho}$
$\overline{\mathrm{u}}_{2}=\sqrt{\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\left(1-\beta^{4}\right) \rho}} \rightarrow(4)$

But, $\mathrm{P}_{1}-\mathrm{P}_{2}=\rho \mathrm{gh}$
$\frac{P_{1}-P_{2}}{\rho}=$ gh
Substituting in equation 4
$\overline{\mathrm{u}}_{2}=\sqrt{\frac{2 \mathrm{gh}}{\left(1-\beta^{4}\right)}} \rightarrow(5)$

Equations 4 and 5 apply to the frictionless flow of incompressible flow. In practice the diameter of the stream at vena contracta is not known, but the orifice diameter is known. Hence the above equation may be written in terms of velocity through the orifice. If a constant is inserted in the equation to correct the difference between this velocity and the velocity at the vena contracta, there is some loss by friction and this also may be included in the constant. Then the above equation becomes
$\overline{\mathrm{u}}_{2}=\mathrm{C}_{\mathrm{o}} \sqrt{\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\left(1-\beta^{4}\right) \rho}} \rightarrow(6)$
where $\mathrm{C}_{\mathrm{o}}$ is the orifice coefficient.

The mass flow rate through orifice meter can be calculated from the velocity at throat $\overline{\mathrm{u}}_{2}$ using continuity equation.
$\dot{\mathrm{m}}=\overline{\mathrm{u}}_{2} \mathrm{~S}_{2} \rho=\mathrm{C}_{\mathrm{o}} \mathrm{S}_{2} \sqrt{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right) \rho} \rightarrow(7)$
where $\dot{\mathrm{m}}$ - mass flow rate and $\mathrm{S}_{2}$ - Area of throat.

Volumetric flow rate is obtained by dividing mass flow rate by the density
$\mathrm{q}=\frac{\dot{\mathrm{m}}}{\rho}=\mathrm{C}_{0} \mathrm{~S}_{2} \sqrt{\frac{2\left(\mathrm{P}_{1}-\mathrm{P}_{2}\right)}{\left(1-\beta^{4}\right) \rho}} \rightarrow(8)$
$\mathrm{C}_{\mathrm{o}}$ can be found experimentally and is generally 0.61 .

## Advantages

1. It is cheap device
2. It can be installed between existing pipe flanges.

Disadvantages

1. Pressure recovery is very poor, i.e., pressure losses are high.
2. It is susceptible to inaccuracies resulting from wear and abrasion.
3. It may be damaged by pressure transients because of its lower physical strength.

## WORKED EXAMPLES:

5.3. An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauge fitted to the upstream and downstream of the orifice meter give readings of $1.95 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$ and $1 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$ respectively. Coefficient of discharge for the meter is given as 0.6. Find the discharge of water through pipe.

## Solution:

$\mathrm{D}_{2}=10 \mathrm{~cm}=0.1 \mathrm{~m}$.
$S_{2}=\frac{\pi}{4} D_{2}^{2}=\frac{\pi}{4} 0.1^{2}=7.854 * 10^{-3}=0.007854 \mathrm{~m}^{2}$
$\beta=0.1 / 0.2=0.5$
$\Delta \mathrm{P}=\left(1.95 \times 10^{5}-1 \times 10^{5}\right)=95000 \mathrm{~N} / \mathrm{m}^{2}$.
$\mathrm{Q}=\mathrm{C}_{\mathrm{v}} \mathrm{S}_{2} \sqrt{\frac{2 \Delta \mathrm{P}}{\rho\left(1-\beta^{4}\right)}}$
$Q=0.6 * 0.007854 \sqrt{\frac{2 * 95000}{1000^{*}\left(1-0.5^{4}\right)}}=\mathbf{0 . 0 6 9 3} \mathbf{m}^{3} / \mathrm{s}=\mathbf{6 9 . 3} \mathbf{~ l p s}$
5.4. An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of $\mathbf{H g}$. Find the rate of flow of oil of specific gravity 0.9 , when the coefficient of discharge of the meter is 0.61 .

## Solution:

$\mathrm{D}_{2}=15 \mathrm{~cm}=0.15 \mathrm{~m}$.
$S_{2}=\frac{\pi}{4} D_{2}^{2}=\frac{\pi}{4} 0.15^{2}=0.0177 \mathrm{~m}^{2}$
$\beta=0.15 / 0.3=0.5$
$\mathrm{R}_{\mathrm{m}}=50 \mathrm{~cm}=0.50 \mathrm{~m}$
$\Delta P=R_{m} * g^{*}\left(\rho_{\mathrm{m}}-\rho\right)=0.50 * 9.81 *(13600-1000)=61803 \mathrm{~N} / \mathrm{m}^{2}$.
$\mathrm{Q}=\mathrm{C}_{\mathrm{V}} \mathrm{S}_{2} \sqrt{\frac{2 \Delta \mathrm{P}}{\rho\left(1-\beta^{4}\right)}}$
$Q=0.61 * 0.0177 \sqrt{\frac{2 * 61803}{1000 *\left(1-0.5^{4}\right)}} \mathbf{0 . 1 3 0 9} \mathbf{m}^{3} / \mathbf{s}=\mathbf{1 3 1} \mathbf{~ l p s}$

## PITOT TUBE:

Pitot tube is a flow measuring device used to measure the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the Pitot tube consists of a glass tube, bent at right angles.


Figure 5.3 Pitot tube
The lower end, which is bent through $90^{\circ}$ is directed in the upstream direction as shown in the Figure 5.3. The liquid rises up in the tube due to the conversion of kinetic to pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at the same level in such a way that point (2) is just as the inlet of the pitot tube and point (1) is far away from the tube.

Applying Bernoulli's equation between two points (1) and (2),

$$
\frac{P_{1}}{\rho g}+\frac{\bar{u}_{1}^{2}}{2 g}+z_{1}=\frac{P_{2}}{\rho g}+\frac{\bar{u}_{2}^{2}}{2 g}+z_{2} \rightarrow(1)
$$

Since $\mathrm{z}_{1}=\mathrm{z}_{2}$ and $\mathrm{v}_{2}=0$,
Let pressure head at point (1) be H ,
pressure head at point (2) be (H+h),

$$
\begin{gathered}
H+\frac{\bar{u}_{1}^{2}}{2 g}=H+h \rightarrow(2) \\
v_{1}=\sqrt{2 g h}
\end{gathered}
$$

This is theoretical velocity. Actual velocity is given by $\left(v_{1}\right)_{\mathrm{act}}=C_{v} \sqrt{2 g h}$ where $\mathrm{C}_{\mathrm{v}}$ is pitot coefficient.

## EXERCISES:

5.5. A $30 \mathrm{~cm} \times 15 \mathrm{~cm}$ venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. The difference in elevation of throat section and entrance section of throat section and entrance section of venturimeter is 30 cm . A differential mercury manometer is connected to the inlet and throat gives a reading of $\mathbf{2 5} \mathbf{~ c m}$. Find the discharge. Take $C_{v}=0.98$. Find the pressure difference between entrance and throat.
5.6 An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauge fitted upstream and downstream of the orifice meter reads $14.715 \mathrm{~N} / \mathrm{cm}^{2}$ and $9.81 \mathrm{~N} / \mathrm{cm}^{2}$ respectively. Find the rate of flow of water through the pipe in litres /s. take Co $=0.60$
5.7 A horizontal pipe having diameter of 30 cm and the orifice diameter of 15 cm is used to measure the flow of oil of sp.gr.is 0.8 .. The discharge of oil through the orifice is 50 litres per second. Find the reading of oil mercury differential manometer. Take $\mathbf{C o}=\mathbf{0 . 6 1}$

