SCHOOL OF MANAGEMENT STUDIES

UNIT – I - ELEMENTS OF OPERATION RESEARCH - SBAA1305

INTRODUCTION

- The subject OPERATIONS RESEARCH is a branch of mathematics - specially applied mathematics, used to provide a scientific base for management to take timely and effective decisions to their problems.

- Operation Research is an analytical method of problem solving and decision making that is useful in the Management of organizations.

- Operations research is the body of knowledge, which uses mathematical techniques to solve management problems and make timely optimal decisions.

- Operations Research is concerned with helping managers and executives to make better decisions.

- The general approach is to analyse the problem in economic terms and then implement the solution if it does not aggressive or violent to other aspects like human, social and political constraints.

In OR problems are broken into basic components and then solved in defined steps by mathematical analysis.

HISTORY OF OPERATIONS RESEARCH

- Operations Research is a 'war baby'. It is because, the first problem attempted to solve in a systematic way was concerned with how to set the time fuse bomb to be dropped from an aircraft on to a submarine.

- In fact the main origin of Operations Research was during the Second World War (1939-1945) in order to make the best use of limited military resources and the win the war.

- At the time of Second World War, the military management in England invited a team of scientists to study the strategic and tactical problems related to air and land defense of the country.

- The problem attained importance because at that time the resources available with England was very limited and the objective was to win the war with available meager resources.

- The resources such as food, medicines, manpower etc., were required to manage war and for the use of the population of the country.

- It was necessary to decide upon the most effective utilization of the available resources to achieve the objective. It was also necessary to utilize the military resources cautiously. Hence, the Generals of military, invited a team of experts in various walks of life such as scientists, doctors, mathematicians, business people, professors, engineers etc., and the problem of resource utilization is given to them to discuss and come out with a feasible solution. These specialists had a brain storming session and came out with a method of solving the problem, which they coined the name “Linear Programming”. This method worked out well in solving the war problem.

- As the name indicates, the word Operations is used to refer to the “problems of military” and the word Research is use for “inventing new method”. As this method of solving the problem was invented during the war period, the subject is given the name ‘OPERATIONS RESEARCH’ and abbreviated as ‘O.R.’

YEAR	EVENTS
1940	Term OR was coined by Mc.Closky and Trefthen in U.K
1949	• OR unit was set up in India in Hyderabad. (The Regional Research Lab) • OR unit was set up at defence science lab.
1951	• The National Research Council (NRC) in US formed a committee on OR. • The first book was published called “Methods on OR” by Morse and Kimball.
1952	• OR Society of America was formed.
1953	• OR unit was set up in Calcutta in the “Indian Statistical Institute”.
1995	OR society of India was established.

OBJECTIVE OF OPERATIONS RESEARCH

“The objective of Operations Research is to provide a scientific basis to the decision maker for solving the problems involving the interaction of various components of an organization by employing a team of scientists from various disciplines, all working together for finding a solution which is in the best interest of the organization as a whole. The best solution thus obtained is known as optimal decision”.

DEFINITION OF OPERATIONS RESEARCH

- Operations Research is the art of winning wars without actually fighting. – Aurther Clarke.

- perations Research is Research into Operations. - J. Steinhardt.

- Operations Research is defined as Scientific method for providing executive departments a quantitative basis for decisions regarding the operations under their control. - P.M. Morse and G.E. Kimbal

- Operations Research is the application of scientific methods, techniques and tools to operation of a system with optimum solution to the problem. - Churchman, Ackoff and Arnoff.

- OR is the combination of management principles and mathematical concepts (Quantitative techniques) for managerial decision-making purpose.

SCOPE OF OPERATIONS RESEARCH

The scope of quantitative methods is very broad. They are applied in defining the problems and getting solutions of various organizations like, business, Government organizations, profit making units and non-profit units and service units.

They can be applied to variety of problems like deciding plant location, Inventory control, Replacement problems, Production scheduling, Return On Investment analysis (ROI), Portfolio selection, marketing research and so on.

Let us now discuss some of the fields where Operations Research techniques can be applied to understand how the techniques are useful to solve the problems.

In general we can state that whenever there is a problem, simple or complicated, we can use operations research techniques to get best solution.

- **In Defense Operations**
  In fact, the subject Operations research is the baby of World War II. To solve war problems, they have applied team approach, and come out with various models such as resource allocation model, transportation model etc. In any war field two or more parties are involved, each having different resources (manpower, ammunition, etc.), different courses of actions (strategies) for application.

- **In Industry**
  After the II World War, the, Industrial world faced a depression and to solve the various industrial problems, industrialist tried the models, which were successful in solving their problems. Industrialist learnt that the techniques of operations research can conveniently applied to solve industrial problems. Then onwards, various models have been developed to solve industrial problems.

- **In Planning For Economic Growth**
  In India we have five year planning for steady economic growth. Every state government has to prepare plans for balanced growth of the state. Various secretaries belonging to different departments has to co-ordinate and plan for steady economic growth. For this all departments can use Operations research techniques for planning purpose. The question like how many engineers, doctors, software people etc. are required in future and what should be their quality to face the then problems etc. can be easily solved.

- **In Agriculture**

  The demand for food products is increasing day by day due to population explosion. But the land available for agriculture is limited. We must find newer ways of increasing agriculture yield. So the selection of land area for agriculture and the seed of food grains for sowing Operations Research must be meticulously done so that the farmer will not get loss at the same time the users will get what they desire at the desired time and desired cost.

- Linear programming model
- Transportation
- Sequencing and scheduling
- Assignment of jobs to minimize cost or maximize profit
- Game theory
- Inventory model
- Maintenance and Replacement
- Shortest route problems like traveling sales person problem
- Resource allocation problems [BRIEF APPLICATIONS OF ABOVE]

CHARACTERISTICS OF OR

- Aims to find solutions for problems of organized systems.
- Aims to provide optimum solution. Optimization means the best minimum or maximum for the criteria under consideration.
- It is the application of scientific methods, tools and techniques.
- Interdisciplinary team approach is used to solve the problems.
- The solutions that serve best to the organization as a whole is taken into consideration.

MEANING AND NECESSITY OF OPERATIONS RESEARCH MODELS

Classification of Models

The models we use in operations research may broadly classified as: (*i*) Mathematical and Descriptive models, and (*ii*) Static and Dynamic Models.

Mathematical and Descriptive Models

(*i*) Descriptive Model

A descriptive model explains or gives a description of the system giving various variables, constraints and objective of the system or problem. The drawback of this model is as we go on reading and proceed; it is very difficult to remember about the variables and constraints, in case the problem or description of the system is lengthy one. It is practically impossible to keep on reading, as the manager has to decide the course of action to be taken timely.
Hence these models, though necessary to understand the system, have limited use as far as operations research is concerned.

(*ii*) Mathematical Model

we have identified the variables and constraints and objective in the problem statement and given them mathematical symbols x and y and a model is built in the form of an inequality of $\leq$ type. Objective function is also given. This is exactly a mathematical model, which explains the entire system in mathematical language, and enables the operations research person to proceed towards solution.

Linear Programming Model

This model is used for resource allocation when the resources are limited and there are number of competing candidates for the use of resources. The model may be used to maximize the returns or minimize the costs.

Consider the following two situations:

(*a*) A company which is manufacturing variety of products by using available resources, want to use resources optimally and manufacture different quantities of each type of product which yield different returns, so as to maximize the returns.

(*b*) A company manufactures different types of alloys by purchasing the three basic materials and it want to maintain a definite percentage of basic materials in each alloy. The basic materials are to be purchased from the sellers and mix them to produce the desired alloy. This is to be done at minimum cost.
Both of them are resource allocation models, the case (*a*) is maximization problem and the Case (*b*) is minimization problem.

(*c*) Number of factories are manufacturing the same commodities in different capacities and the commodity is sent to various markets for meeting the demands of the consumers, when the cost of transportation is known, the linear programming helps us to formulate a programme to distribute the commodity from factories to markets at minimum cost. The model used is transportation model.

(*d*) When a company has number of orders on its schedule, which are to be processed on same machines and the processing time, is known, then we have to allocate the jobs or orders to the machines, so as to complete all the jobs in minimum time. This we can solve by using Assignment model.

MODELS IN OR

Model is a reasonably simplified representation of reality. It is an abstraction of reality. It helps to arrive at a well-structured view of reality.

MODELS

ICONICMODELS:

It is a pictorial representation or a physical representation of a system. A look alike correspondence is present.
Eg: miniature of a building, toys, globe etc.

- Iconic Models are either scaled up or scaled down.

Scaled up - eg: Atom. Scaled down – eg: globe.

- Iconic models are either two-dimensional or three-dimensional.

ANALOGUEMODEL OR SCHEMATIC MODEL
This model uses one set of properties to describe another set of properties.
There is no look alike correspondence. It is more abstract.
Eg: a set of water pipes that are used to describe current flow.
Eg: Maps, (different colors are used to depict water, land etc
Eg: Organizational chart.

MATHEMATICALMODEL
This uses a set of mathematical symbols (letters and numbers) to represent a system.
V = I * R (Resistance)
(Voltage) (Current)

QUANTITATIVEMODELS
Quantitative models are those, which can measure the observation.
Eg: Models that measure temperature.
APPLICATION/SCOPE OF OR

1. <u>Production:</u>
- Production scheduling
- Project scheduling
- Allocation of resources
- Equipment replacement
- Inventory policy
- Factory size and location

2. <u>Marketing:</u>
- Product introduction with timing
- Product mix selection
- Competitive strategies
- Advertising strategies
- Pricing strategies.

3. <u>Accounts:</u>
- Cash flow analysis (optimum cash balance)
- Credit policies (optimum receivables)

4. <u>Finance:</u>
- Optimum dividend policy
- Portfolio analysis

5. <u>Personnel Management:</u>
- Recruitment and selection
- Assignment of jobs
- Scheduling of training programs

6. <u>Purchasing:</u>
- Rules for purchasing
- EOQ-Economic Order Quantity (how much to order)
- Timing of purchase (when to purchase)

7. <u>Distribution:</u>
- Deciding number of warehouses.

- Location of warehouses
- Size of warehouses
- Transportation strategies

8. <u>Defence:</u>
- Budget allocation
- Allocation of resources

9. <u>Government Departments:</u>
- Transportation
- Budget fixation
- Fiscal policies.

10. <u>R & D (Research and Development):</u>
- Project introduction
- Project control
- Budget allocation for projects

THE MAIN PHASES OF OR

- Formulation of the problem
- Construction of a model (Mathematical model)
- Solve the model
- Control and update the model
- Test the model and validate it
- Implement the model

<u>LIMITATIONS OF OR</u>

<u>1. Magnitude of computation:</u> In order to arrive at an optimum solutions OR takes into account all the variables that affect the system. Hence the magnitude of computation is very large.

<u>2. Non-Quantifiable variables:</u> OR can give an optimum solution to a problem only if all the variables are quantified. Practically all variables in a system cannot be quantified.

<u>3. Time and Cost:</u> To implement OR in an organization, it consumes more time and cost. If the basic decision variables change, OR becomes too costly for an organization to handle it.

<u>4. Implementation of OR:</u> Implementation of OR may lead to HR problems. The psychology of employees should be considered and the success of OR depends on cooperation of the employees.

<u>5. Distance between Manager and OR Specialist:</u> Managers may not be having a complete overview of OR techniques and has to depend upon an OR Specialist. Only if good link is established OR can be a success.

LINEAR PROGRAMMING

DEFINITION

Samuelson, Dorfman, and Solow define LP as "the analysis of problems in which linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of constraints in the form of linear inequalities".

BASIC ASSUMPTIONS OF LINEAR PROGRAMMING:

The following four basic assumptions are necessary for all linear programming models:

1. LINEARITY:

The basic requirements of a LP problem are that both the objectives and constraints must be expressed in terms of linear equations or inequalities. It is well known that if the number of machines in a plant is increased, the production in the plant also proportionately increases. Such a relationship, giving corresponding increment in one variable for every increment in other, is called linear and can be graphically represented in the form of a straight line.

2. DETERMINISTIC (OR CERTAINTY):

In all LP models, it is assumed that all model parameters such as availability of resources, profit (or cost) contribution of a unit of decision variable and consumption of resources by a unit decisions variable must be known and fixed. In other words, this assumptions means that all the **coefficients** in the objectives function and constraints are completely known with certainty and do not change during the period being studied.

3. ADDITIVITY:

The value of the objective function for the given values of decision variables and the total sum of resources used, must be equal to the sum of the contributions (profit or cost) earned from each decision variable and the sum of the resources used by each decision respectively. For example, the total profit earned by the sale of three products A, B and C must be equal to the profits earned separately from A, B and C and similarly, the amount of resources consumed by A, B, and C individually.

4. DIVISIBILITY:

This implies that solution values of decision variables and resources can take any non-negative values, i.e., **fractional values** of the decision variables are **permitted**. This, however, is not always desirable. For example, it is impossible to produce one-fourth of a bus. When it is necessary to have integer variables, a technique known as integer programming could be used.

APPLICATIONS/SCOPE OF LINEAR PROGRAMMING:

(i) **MANUFACTURING PROBLEMS:** to find the number of items of each type that should be manufactured so as to maximize the profit subject to production restrictions imposed by limitations on the use of machinery and labour.

(ii) **ASSEMBLING PROBLEMS:** To have the best combinations of basic components to produce goods according to certain specifications.

(iii) **TRANSPORTATION PROBLEMS**: to find the least costly way of transporting shipments from the warehouses to customers.

(iv) **BLENDING PROBLEM:** To determine the optimal amount of several constitutes to use in producing a set of products which determining the optimal quantity of each product to produce.

(v) **PRODUCTION PROBLEMS:** To decide the production schedule to satisfy demand and minimize cost in face of fluctuating rates and storage expenses.

(vi) **DIET PROBLEMS**: To determine the minimum requirement of nutrients subject to availability of foods and their prices.

(vii) **JOB ASSIGNING PROBLEMS:** To assign job to workers for maximum effectiveness and optimal results subject to restrictions of wages and other costs.

(viii) **TRIM-LOSS PROBLEMS:** To determine the best way to obtain a variety of smaller rolls of paper from a standard width of roll that it kept its stock and at the same time minimize wastage.

(ix) **STAFFING PROBLEM**: To find optimal staff in hotels, police stations and hospitals to maximize the efficiency.

(x) **TELEPHONE EXCHANGE PROBLEMS:** To determine optimal facilities in telephone exchange to have minimum breakdowns.

KEY TERMS

<u>Artificial variables:</u> A variable that has no meaning in a physical sense, but acts as tool to help generate an initial LP solution.

<u>Basic variables:</u> The set of variables that are in the solution (i.e., have positive, on-zero values) are listed in the product mix column. The variables that normally take non-zero values to obtain a solution.

<u>Basic solution:</u> A solution to m simultaneous linear equations in n unknowns, m<n, with the property that n-m of the variables have the value zero and the values of the remaining m variables are unique determined; obtained when a set of non-basic variables are assigned the value zero.

<u>Basic feasible solution:</u> A basic solution, for which the values of all variables are non-negative, corresponds to a corner of the LP feasible region.

<u>Degeneracy :</u> A condition that arises when there is a tie in the values used to determine which variables indicated will enter the solution next. It can lead to cycling back and forth between two non-optimal solutions.

Degenerate solution: The number of variables in the standard equality form (counting decision variables, surpluses, and slacks) with positive optimal value is less than the number of constraints.

Optimal solution: A solution that is optimal for the given solution.

Pivot column: The column with the largest positive number in the Ci-Zj row of a maximization problem, or the largest negative Cj-Zj value in a minimization problem. It indicates which variable will enter the solution next.

Pivot row: The corresponding to the variable that will leave the basis in order to make room for the variable entering (as indicated by the new pivot column). This is the smallest positive ratio found by dividing the quantity column values by the pivot column values for each row.

Slack variable: A variable added to less than or equal to constraints in order to create an equality for a simplex method. It represents a quantity of unused resources.

Surplus variable: A variable inserted in a greater than or equal to constraint to create equality. If represents the amount of resources usage above the minimum required usage.

Unboundedness: A condition describing LP maximization problems having solutions that can become infinitely large without violating any stated constraints.

ADVATNTAGES OF LPP:

- It provides an insight and perspective into the problem environment. This generally results in clear picture of the true problem.
- It makes a scientific and mathematical analysis of the problem situations.
- It gives an opportunity to the decision-maker to formulate his strategies consistent with the constraints and the objectives.
- It deals with changing situations. Once a plan is arrived through the LP it can also be revaluated for changing conditions.
- By using LP, the decision maker makes sure that he is considering the best solution.

LIMITATIONS OF LPP:

- The major limitation of LP is that it treats all relationships as linear but it is not true in many real life situations.
- The decision variables in some LPP would be meaningful only if they have integer values. But sometimes we get fractional values to the optimal solution, where only integer values are meaningful.
- All the parameters in the LP model are assumed to be known constants. But in real life they may not be known completely or they may be probabilistic and they may be liable for changes from time to time.
- The problems are complex if the number of variables and constraints are quite large.
- It deals with only single objective problems, whereas in real life situations, there may be more than one objective.

FORMULATION OF LPP:

- Identify the objective function
- Identify the decision variables
- Express the objective function in terms of decision variables
- Identify the constraints and express them
- Value of decision variables is ≥ 0 (always non-negativity)

EXAMPLE PROBLEM:

An organization wants to produce Tables and Chairs. Profit of one table is Rs.100 and profit of one Chair is Rs.50.

Particulars	Tables	Chairs	Maximum hours available
Cutting hours	4	1	300
Painting hours	1	5	100

Solution :

Max Z =100x+50 y

Subjected to :
$4x+y \leq 300$
$x+0.5y \leq 100$
$x,y \geq 0$

STEPS IN GRAPHICAL SOLUTION METHOD:

- Formulate the objective and constraint functions.
- Draw a graph with one variable on the horizontal axis and one on the vertical axis.
- Plot each of the constraints as if they are inequalities.
- Outline the solution area.
- Circle the potential solutions points. These are the intersections of the constraints on the perimeter of the solution area. (vertices of the solution space)
- Substitute each of the potential extreme point values of the two decision variables into the objective function and solve for Z.
- Select the solution that optimizes Z.

PROCEEDURE FOR SOLVING LPP PROBLEM USING SIMPLEX METHOD

STEP:1
Convert all the inequality functions into equality:
For converting all the inequalities into equalities, we should use slack and surplus variables.
In case of ≤ inequalities, we should add Slack variable so as to convert that inequality into equality. For example, $3x + 2y \leq 6$ will become $3x + 2y + S_1 = 6$, where S1 is the slack variable.
In case of ≥ inequalities, we should deduct Surplus variable so as to convert that inequality into equation. For example, $5x + 6y \geq 10$ will become $5x + 6y - S_2 = 10$, where S_2 is the surplus variable.
In case if the given constraint is an equation category, we should not use

either slack variable or surplus variable.

STEP 2:
Find out the basic and non basic variables: Non Basic variable is the variable whose value is zero. Basic variable is the variable which will have either positive or negative value.

After converting all the inequality into equality, we should assume some variables as Non basic variables and find out the values of the other (Basic) variables. This solution is called as initial solution. If all the basic variable values are positive, then that initial solution is called as BASIC FEASIBLE SOLUTION.

STEP:3
Preparation of simplex table: The format of the simplex table is as follows:

Coefficients of Basic variables	Basic Variables	Variables	Solution	Ratio
EVALUATION ROW				

STEP 4:
Calculation of values in Evaluation row: To calculate the values in the evaluation row, we should use the following formula for each variable column:
Evaluation row values = (Variable coefficients x coefficients of basic variables) – Coefficients of the variables in the objective function.

All the values in the Evaluation row should be either positive or zero. Then it indicates that we have reached the optimum stage and thereby we can derive the optimum solution.

If any negative persists, we should proceed further by doing the following steps.

STEP 5:
IDENTIFICATION OF KEY COLUMN: The column that represents least value in the evaluation row is known as KEY COLYMN. The variable in that column is known as ENTERING VARIABLE.

STEP 6:
IDENTIFICATION OF KEY ROW: To find out the Key row, we should calculate the ratio. **Ratio = solution column values / Key column values.** The least ratio row is treated as KEY ROW and the value in that row is known as LEAVING VARIABLE. **THE VARIABLE THAT PREVAILS IN BOTH KEY ROW AND KEY COLUMN IS**

KNOWN AS KEY ELEMENT. After finding the key element, we should prepare next simplex table. In that table, should bring the entering variable and should write the new values of the entering variable.

New values of the Left out row= Old values of the left out row – (New values of the entering variable X value in the key column of the Old left out row)

After calculating the new values for all the rows, we should proceed to STEP 3.

Maximize z = $3x_1 + 2x_2$

subject to

$-x_1+2x_2\leq4$
$3x_1+2x_2\leq14$
$x_1 - x_2 \leq 3$

$x_1, x_2 \geq 0$

Solution:

First, convert every inequality constraints in the LPP into an equality constraint, so that the problem can be written in a standard from. This can be accomplished by adding a slack variable to each constraint. Slack variables are always added to the less than type constraints.

Converting inequalities to equalities

$-x_1+2x_2+x_3=$

$2x_2+x_4=14$
$x_1-x_2+x_5=3$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

Where x_3, x_4 and x_5 are slack variables.

Since slack variables represent unused resources, their contribution in the objective function is zero. Including these slack variables in the objective function, we get

Maximize z = $3x_1 + 2x_2 + 0x_3 + 0x_4 + 0x_5$

Initial basic feasible solution

Now we assume that nothing can be produced. Therefore, the values of the decision variables arezero.
$x_1 = 0$, $x_2 = 0$, $z = 0$

When we are not producing anything, obviously we are left with unused capacity
$x_3 = 4$, $x_4 = 14$, $x_5 = 3$

Simplex Method: Table 1

	c_j	3	2	0	0	0	
c_B	Basic variables B	x_1	x_2	x_3	x_4	x_5	Solution values b (=X_B)
0	x_3	-1	2	1	0	0	4
0	x_4	3	2	0	1	0	14
0	x_5	1	-1	0	0	1	3
z_j-c_j		-3	-2	0	0	0	

$a_{11} = -1$, $a_{12} = 2$, $a_{13} = 1$, $a_{14} = 0$, $a_{15} = 0$, $b_1 = 4$
$a_{21} = 3$, $a_{22} = 2$, $a_{23} = 0$, $a_{24} = 1$, $a_{25} = 0$, $b_2 = 14$
$a_{31} = 1$, $a_{32} = -1$, $a_{33} = 0$, $a_{34} = 0$, $a_{35} = 1$, $b_3 = 3$

Calculating values for the index row ($z_j - c_j$)

$z_1 - c_1 = (0 X (-1) + 0 X 3 + 0 X 1) - 3 = -3$
$z_2 - c_2 = (0 X 2 + 0 X 2 + 0 X (-1)) - 2 = -2$
$z_3 - c_3 = (0 X 1 + 0 X 0 + 0 X 0) - 0 = 0$
$z_4 - c_4 = (0 X 0 + 0 X 1 + 0 X 0) - 0 = 0$
$z_5 - c_5 = (0 X 0 + 0 X 0 + 0 X 1) - 0 = 0$

Choose the smallest negative value from $z_j - c_j$ (i.e., – 3). So column under x_1 is the key column.
Now find out the minimum positive value
Minimum (14/3, 3/1) = 3
So row x_5 is the key row.
Here, the pivot (key) element = 1 (the value at the point of intersection).
Therefore, x_5 departs and x_1 enters.

We obtain the elements of the next table using the following rules:

1. If the values of $z_j – c_j$ are positive, the inclusion of any basic variable will not increase the value of the objective function. Hence, the present solution maximizes the objective function. If there are more than one negative values, we choose the variable as a basic variable corresponding to which the value of $z_j – c_j$ is least (most negative) as this will maximize the profit.

2. The numbers in the replacing row may be obtained by dividing the key row elements by the pivot element and the numbers in the other two rows may be calculated by using the formula:

New number=	old number-	(corresponding no. of key row) X (corresponding no. of key column)
		pivot element

Calculating values for table 2

x_3 row

a_{11}= -1 – 1 X ((-1)/1) = 0
a_{12} = 2 – (-1) X ((-1)/1) = 1
a_{13} = 1 – 0 X ((-1)/1) = 1
a_{14} = 0 – 0 X ((-1)/1) = 0
a_{15} = 0 – 1 X ((-1)/1) = 1
b_1 = 4 – 3 X ((-1)/1) = 7

x_4 row

a_{21} = 3 – 1 X (3/1) = 0
a_{22} = 2 – (-1) X (3/1) = 5
a_{23} = 0 – 0 X (3/1) = 0
a_{24} = 1 – 0 X (3/1) = 1
a_{25} = 0 – 1 X (3/1) = -3
b_2 = 14 – 3 X (3/1) = 5

x_1 row

a_{31} = 1/1 = 1
a_{32} = -1/1 = -1
a_{33} = 0/1 = 0
a_{34} = 0/1 = 0
a_{35} = 1/1 = 1
b_3 = 3/1 = 3

Table 2

	c_j	3	2	0	0	0	
c_B	**Basic variables B**	x_1	x_2	x_3	x_4	x_5	**Solution values b (= X_B)**
0	x_3	0	1	1	0	1	7
0	x_4	0	5	0	1	-3	5
3	x_1	1	-1	0	0	1	3
z_j-c_j		0	-5	0	0	3	

Calculating values for the index row ($z_j - c_j$)

$z_1 - c_1 = (0 \text{ X } 0 + 0 \text{ X } 0 + 3 \text{ X } 1) - 3 = 0$
$z_2 - c_2 = (0 \text{ X } 1 + 0 \text{ X } 5 + 3 \text{ X } (-1)) - 2 = -5$
$z_3 - c_3 = (0 \text{ X } 1 + 0 \text{ X } 0 + 3 \text{ X } 0) - 0 = 0$
$z_4 - c_4 = (0 \text{ X } 0 + 0 \text{ X } 1 + 3 \text{ X } 0) - 0 = 0$
$z_5 - c_5 = (0 \text{ X } 1 + 0 \text{ X } (-3) + 3 \text{ X } 1) - 0 = 3$

Key column = x_2 column
Minimum (7/1, 5/5) = 1
Key row = x_4 row
Pivot element = 5
x_4 departs and x_2 enters.

Calculating values for table 3

x_3 row

$a_{11} = 0 - 0 \text{ X } (1/5) = 0$
$a_{12} = 1 - 5 \text{ X } (1/5) = 0$
$a_{13} = 1 - 0 \text{ X } (1/5) = 1$
$a_{14} = 0 - 1 \text{ X } (1/5) = -1/5$
$a_{15} = 1 - (-3) \text{ X } (1/5) = 8/5$
$b_1 = 7 - 5 \text{ X } (1/5) = 6$

x_2 row

$a_{21} = 0/5 = 0$
$a_{22} = 5/5 = 1$
$a_{23} = 0/5 = 0$
$a_{24} = 1/5$
$a_{25} = -3/5$
$b_2 = 5/5 = 1$

x_1 row

$a_{31} = 1 - 0 \text{ X } (-1/5) = 1$
$a_{32} = -1 - 5 \text{ X } (-1/5) = 0$
$a_{33} = 0 - 0 \text{ X } (-1/5) = 0$

$a_{34} = 0 – 1 X (-1/5) = 1/5$
$a_{35} = 1 – (-3) X (-1/5) = 2/5$
$b_3 = 3 – 5 X (-1/5) =$

Simplex Method: Final Optimal Table

	c_j	**3**	**2**	**0**	**0**	**0**	
c_B	**Basic variables B**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**Solution values b (= X_B)**
0	x_3	0	0	1	-1/5	8/5	6
2	x_2	0	1	0	1/5	-3/5	1
3	x_1	1	0	0	1/5	2/5	4
z_j-c_j		0	0	0	1	0	

Since all the values of zj – c_j are positive, this is the optimal solution.
$x_1 = 4$, $x_2 = 1$
$z = 3 x_1 + 2 x_2 = 14$.

Graphical Method of Solution of a Linear Programming Problem

So far we have learnt how to construct a mathematical model for a linear programming problem. If we can find the values of the decision variables x_1, x_2, x_3, ... x_n, which can optimize (maximize or minimize) the objective function Z, then we say that these values of x_i are the optimal solution of the Linear Program (LP).

The graphical method is applicable to solve the LPP involving two decision variables x_1, and x_2, we usually take these decision variables as x, y instead of x_1, x_2. To solve an LP, the graphical method includes two major steps.

a) The determination of the solution space that defines the feasible solution. Note that the set of values of the variable x_1, x_2, x_3,....x_n which satisfy all the constraints and also the non-negative conditions is called the feasible solution of the LP.

b) The determination of the optimal solution from the feasible region.

a) To determine the feasible solution of an LP, we have the following steps.

Step 1: Since the two decision variable x and y are non-negative, consider only the first quadrant of xy-coordinate plane

Step 2: Each constraint is of the form $ax + by \leq c$ or $ax + by \geq c$.

Draw the line $ax + by = c$ (1)

For each constraint,

the line (1) divides the first quadrant in to two regions say R_1 and R_2, suppose $(x_1, 0)$ is a point in R_1. If this point satisfies the in equation ax + byc or (c), then shade the region R_1. If $(x_1, 0)$ does not satisfy the inequality, shade the region R_2.

Step 3: Corresponding to each constant, we obtain a shaded region. The intersection of all these shaded regions is the feasible region or feasible solution of the LP.

Let us find the feasible solution for the problem of a decorative item dealer whose LPP is to maximize profit function.

$Z = 50x + 18y$ (1)

Subject to the constraints

$2x + y \leq 100$

$x + y \leq 80$

$x \geq 0, \quad y \geq 0$

Step 1: Since $x \geq 0$, $y \geq 0$, we consider only the first quadrant of the xy - plane

Step 2: We draw straight lines for the equation

$2x + y = 100$ (2)

$x + y = 80$

To determine two points on the straight line $2x + y = 100$

Put $y = 0$, $2x = 100$

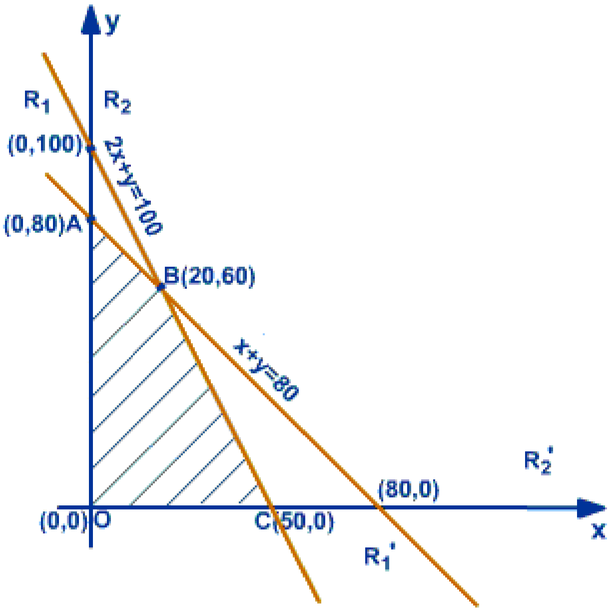
$\Rightarrow x = 50$

$\Rightarrow (50, 0)$ is a point on the line (2)

put $x = 0$ in (2), $y = 100$

$\Rightarrow (0, 100)$ is the other point on the line (2)

Plotting these two points on the graph paper draw the line which represent the line $2x + y = 100$.



This line divides the 1st quadrant into two regions, say R_1 and R_2. Choose a point say (1, 0) in R_1. (1, 0) satisfy the inequality $2x + y \leq 100$. Therefore R_1 is the required region for the constraint $2x + y \leq 100$.

Similarly draw the straight line $x + y = 80$ by joining the point (0, 80) and (80, 0). Find the required region say R_1', for the constraint $x + y \leq 80$.

The intersection of both the region R_1 and R_1' is the feasible solution of the LPP. Therefore every point in the shaded region OABC is a feasible solution of the LPP, since this point satisfies all the constraints including the non-negative constraints.

b) There are two techniques to find the optimal solution of an LPP.

Corner Point Method

The optimal solution to a LPP, if it exists, occurs at the corners of the feasible region.

The method includes the following steps

Step 1: Find the feasible region of the LLP.

Step 2: Find the co-ordinates of each vertex of the feasible region.

These co-ordinates can be obtained from the graph or by solving the equation of the lines.

Step 3: At each vertex (corner point) compute the value of the objective function.

Step 4: Identify the corner point at which the value of the objective function is maximum (or minimum depending on the LP)

The co-ordinates of this vertex is the optimal solution and the value of Z is the optimal value

Example: Find the optimal solution in the above problem of decorative item dealer whose objective function is $Z = 50x + 18y$.

In the graph, the corners of the feasible region are

O (0, 0), A (0, 80), B(20, 60), C(50, 0)

At (0, 0) $Z = 0$

At (0, 80) $Z = 50(0) + 18(80)$

$= 1440$

At (20, 60), Z = 50 (20) +18 (60)

= 1000 + 1080 = Rs.2080

At (50, 0) Z = 50 (50)+ 18 (0)

= 2500.

Since our object is to maximize Z and Z has maximum at (50, 0) the optimal solution is x = 50 and y = 0.

The optimal value is 2500.

If an LPP has many constraints, then it may be long and tedious to find all the corners of the feasible region. There is another alternate and more general method to find the optimal solution of an LP, known as 'ISO profit or ISO cost method'

ISO- PROFIT (OR ISO-COST)

Method of Solving Linear Programming Problems

Suppose the LPP is to

Optimize Z = ax + by subject to the constraints

$a_1x + b_1y \leq (\text{or} \geq) c_1$

$a_2x + b_2y \leq (\text{or} \geq) c_2$

$x \geq 0, \quad y \geq 0.$

This method of optimization involves the following method.

Step 1: Draw the half planes of all the constraints

Step 2: Shade the intersection of all the half planes which is the feasible region.

Step 3: Since the objective function is Z = ax + by, draw a dotted line for the equation ax + by = k, where k is any constant. Sometimes it is convenient to take k as the LCM of a and b.

Step 4: To maximise Z draw a line parallel to ax + by = k and farthest from the origin. This line should contain at least one point of the feasible region. Find the coordinates of this point by solving the equations of the lines on which it lies.

To minimise Z draw a line parallel to ax + by = k and nearest to the origin. This line should contain at least one point of the feasible region. Find the co-ordinates of this point by solving the equation of the line on which it lies.

Step 5: If (x_1, y_1) is the point found in step 4, then

$x = x_1$, $y = y_1$, is the optimal solution of the LPP and

$Z = ax_1 + by_1$ is the optimal value.

The above method of solving an LPP is more clear with the following example.

Example: Solve the following LPP graphically using ISO- profit method.

maximize Z =100 + 100y.

Subject to the constraints

$10x + 5y \leq 80$

$6x + 6y \leq 66$

$4x + 8y \geq 24$

$5x + 6y \leq 90$

$x \geq 0, \quad y \geq 0$

Suggested answer:

since $x \geq 0$, $y \geq 0$, consider only the first quadrant of the plane graph the following straight lines on a graph paper

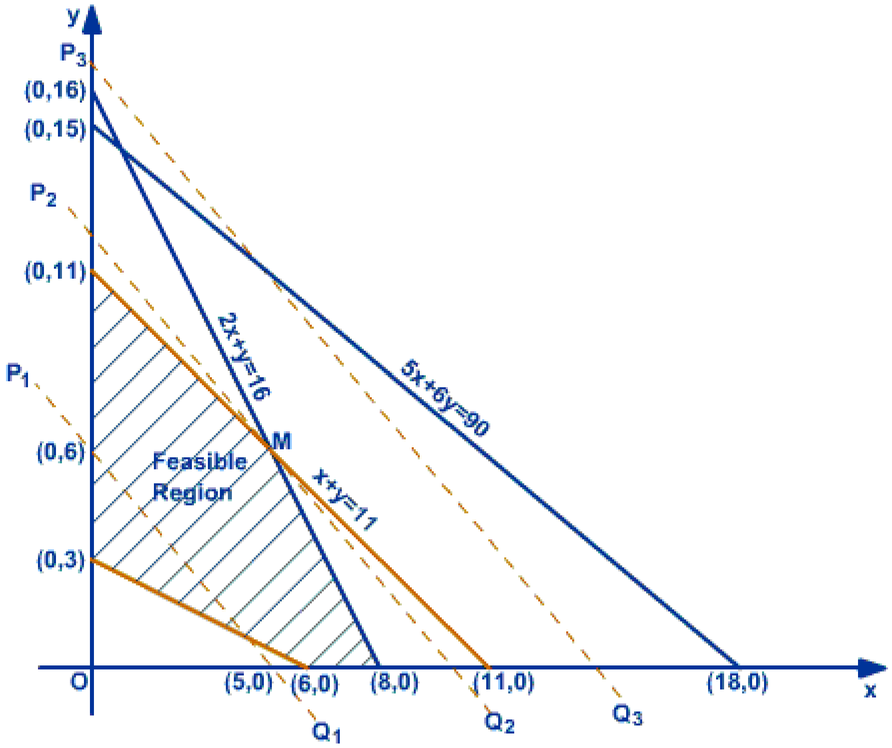
$10x + 5y = 80$ or $2x+y =16$

$6x + 6y = 66$ or $x +y =11$

$4x+ 8y = 24$ or $x+ 2y = 6$

$5x + 6y = 90$

Identify all the half planes of the constraints. The intersection of all these half planes is the feasible region as shown in the figure.



Give a constant value 600 to Z in the objective function, then we have an equation of the line

$120x + 100y = 600$ (1)

or $6x + 5y = 30$ (Dividing both sides by 20)

P_1Q_1 is the line corresponding to the equation $6x + 5y = 30$. We give a constant 1200 to Z then the P_2Q_2 represents the line.

$120x + 100y = 1200$

$6x + 5y = 60$

P_2Q_2 is a line parallel to P_1Q_1 and has one point 'M' which belongs to feasible region and farthest from the origin. If we take any line P_3Q_3 parallel to P_2Q_2 away from the origin, it does not touch any point of the feasible region.

The co-ordinates of the point M can be obtained by solving the equation $2x + y = 16$

$x + y =11$ which give

$x = 5$ and $y = 6$

$\Rightarrow$The optimal solution for the objective function is $x = 5$ and $y = 6$

The optimal value of Z

$120 (5) + 100 (6) = 600 + 600$

$= 1200$

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SCHOOL OF MANAGEMENT STUDIES

UNIT – II - ELEMENTS OF OPERATION RESEARCH - SBAA1305

INTRODUCTION

The transportation problem is a special type of LPP in which the objective is to determine the quantities to be shifted from each source to destination, so that the total transportation cost is minimum.

Suppose a factory owns ware houses in 3 different locations in a city and has to despatch the monthly requirement of the product manufactured by them to 5 different wholesale markets located in the same city. The cost of transporting one unit of the product from the i-th warehouse to the j-th market is known and is cij. It is assumed that the total cost is a linear function so that the total transportation cost of transporting xij, units of the product from the i-th warehouse to the j-th market is given by $\sum c_{ij}x_{ij.}$

It is clear that the factory management will be interested in obtaining a solution that minimizes the total cost of transportation. During the process of transportation they will also face the constraints that from a warehouse they cannot transport more than what is stored or available in the warehouse (supply) and that they need to transport to a market the total monthly requirement of the market (demand).

ASSUMPTIONS

- Quantity of supply at each source is known.
- Quantity demanded at each destination is known.
- The cost of transportation of a commodity from each source to destination is known.

PROCEDURE TO SOLVE TRANSPORTATION
PROBLEM Step I : Deriving the initial basic feasible solution.
Step II : Deriving the final optimal solution.

DERIVING THE INITIAL BASIC FEASIBLE SOLUTION

- North West corner method.
- Matrix minimum method.

Vogel's approximation method (VAM Method) penalty method.

DERIVING THE FINAL SOLUTION

Modified distribution method / Modi method / UV method.

- If total demand = total supply, then it is a balanced transportation problem.
- If the total supply not equal to total demand, then the transportation problem is unbalanced transportation problem.

I. NORTH WEST CORNER METHOD

1. Check if Demand=Supply. If not add dummy row or column.
2. Select the North west (upper left hand) corner cell.
3. Allocate as large as possible in the north west corner cell.
4. If demand is satisfied , strike off the respective column and deduct supply accordingly If supply is exhausted ,strike off the respective row and deduct demand accordingly
5. From the resultant array, locate the north west corner cell and repeat the procedure Note : The assignment done is not taking cost into consideration.

6. Continue allocation until all demand is satisfied and all supply is exhausted.

7. Multiply the allocated quantity *cost of transportation for each occupied cell and add it to find the total cost.

II. LEAST COST METHOD

1. Check if Demand=Supply. If not add a dummy row /column.
2. The lowest cost cell in the matrix is allocated as much as possible based on demand and supply requirement.
   - If there are more than one least cost cell , select the one where maximum units can be allocated.
   - If the tie exist, follow the serial order.
3. If demand is satisfied, strike off the respective column and deduct supply accordingly. If supply is exhausted, strike off the respective row and deduct demand accordingly.
4. From the resultant array, locate the least cost cell and repeat the procedure.
5. Continue allocation until all demand is satisfied and all supply is exhausted.
6. Find total cost.

VOGEL'S APPROXIMATION METHOD (VAM)

This method gives better initial solution in terms of less transportation cost through the concept of ‘penalty numbers’ which indicate the possible cost penalty associated with not assigning an allocation to given cell.

1. Check if demand = supply, if not add a dummy row or column.
2. Calculate penalty of each row & column by taking the difference between the lowest unit transportation cost. This difference indicates the penalty or extra cost which has to be paid for not assigning an allocation to the cell with the minimum transportation cost.
3. Select the row or column which has got the largest penalty number.(If there is a tie it can be broken by selecting the cell where the maximum allocation can be made.)
4. In that row or column choose the minimum cost cell and allocate accordingly.
   - If there are more than one minimum cost cell, select the one where maximum units can be allocated.
   - If the tie exists, follow the serial order.
5. If demand is satisfied, strike off the respective column and deduct supply accordingly. If supply is exhausted, strike off the respective row and deduct demand accordingly.
6. From the resultant array, calculate penalty and repeat the procedure.
7. Continue allocation until all demand is satisfied and all supply is exhausted.
8. Find the total cost.

OPTIMAL SOLUTION

Work out the basic feasible solution using by any one method

a) Northwest corner method

b) Least cost method
c) VAM/Penalty method. (preferably VAM)

<u>STEP 1</u>:

Check if the number of occupied cells is m+n-1 (i.e., number of rows +number of columns-1) Note : Rows & columns include dummy rows & columns.

- If number of occupied cells = m+n-1, then the solution to the transportation problem is **basic feasible solution**.
- If number of occupied cells < m+n-1, then the solution is **degenerate solution**. Degeneracy may occur either at the initial stage or at an intermediate stage at some subsequent iteration.

- In case of degeneracy, we allocate an extremely small amount, close to zero [(ξ) epsilon] to one or more empty cells of the transportation table (unoccupied least cost cell). So that total no of occupied cells equals to m+n-1.

<u>STEP 2</u>:

If the basic feasible solution is achieved then MODI method is used to obtain final optimal solution

- ❖ Defining the occupied cells.

  $c_{ij} = u_i + v_j$ where, c_{ij} →cost.

  u_i →row.

  V_j →column.

- ❖ Assume any one u_i or v_j is to be zero such that max. no of allocations are done in that row(i) or column (j) & find value of all other u_i's & v_j's

<u>STEP 3</u>:

- ❖ Evaluate the unoccupied cells. $d_{ij} = u_i + v_j - c_{ij}$
- ❖ If all evaluation values are either negative or zero,then the initial solution is optimal solution.
- ❖ If any positive value exist, initial solution is not an optimal solution.

Feasible Solution: A feasible solution to a transportation problem is a set of non-negative values x_{ij}(i=1,2,..,m, j=1,2,…n) that satisfies the constraints.

Basic Feasible Solution: A feasible solution is called a basic feasible solution if it contains not more than m+n–1 allocations, where m is the number of rows and n is the number of columns in a transportation problem.

Optimal Solution: Optimal Solution is a feasible solution (not necessarily basic) which optimizes(minimize) the total transportation cost.

Non degenerate basic feasible Solution: If a basic feasible solution to a transportation problem contains exactly m+n–1 allocations in independent positions, it is called a Non degenerate basic feasible solution. Here m is the number of rows and n is the number of columns in a transportation problem.

Degeneracy : If a basic feasible solution to a transportation problem contains less than *m*+*n*–1 allocations , it is called a degenerate basic feasible solution. Here m is the number of rows and n is the number of columns in a transportation problem.

Definition

The transportation problem is a special type of linear programming problem, where the objective is to minimize the cost of distributing a product from a number of sources to a number of destinations**.**

For a feasible solution to exist, it is necessary that total capacity equals total to the requirements. If $\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j$ i.e. If total supply = total demand then it is a balanced transportation problem otherwise it is called unbalanced Transportation problem. There will be (m + n - 1) basic independent variables out of (m x n) variables

What are the understanding assumptions?

1. Only a single type of commodity is being shipped from an origin to a destination.
2. Total supply is equal to the total demand.
   $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, a_i (supply) and b_j (demand) are all positive integers.
3. The unit transportation cost of the item from all sources to destinations is certainly and preciously known.
4. The objective is to minimize the total cost.

Example 1: The ICARE Company has three plants located throughout a state with production capacity 50, 75 and 25 gallons. Each day the firm must furnish its four retail shops R_1, R_2, R_3, & R_4 with at least 20, 20, 50, and 60 gallons respectively. The transportation costs (in Rs.) are given below.

Company	Retail				Supply
	R1	R2	R3	R4	
P1	3	5	7	6	50
P2	2	5	8	2	75
P3	3	6	9	2	25
Demand	20	20	50	60	

The economic problem is to distribute the available product to different retail shops in such a way so that the total transportation cost is minimum?

Solution: Starting from the North West corner, we allocate min (50, 20) to P_1R_1, i.e., 20 units to cell P_1R_1. The demand for the first column is satisfied. The allocation is shown in the following table.

Table 1

Company	Retail				Supply
	R1	R2	R3	R4	
P1	3 (20)	5 (20)	7 (10)	6	50
P2	2	5	8 (40)	2 (35)	75
P3	3	6	9	2 (25)	25
Demand	20	20	50	60	

Now we move horizontally to the second column in the first row and allocate 20 units to cell P_1R_2. The demand for the second column is also satisfied.

Proceeding in this way, we observe that $P_1R_3 = 10$, $P_2R_3 = 40$, $P_2R_4 = 35$, $P_3R_4 = 25$.
The resulting feasible solution is shown in the following table.

Here, number of retail shops (n) = 4, and
Number of plants (m) = 3.Number of basic variables = m + n – 1 = 3 + 4 – 1 = 6.

Initial basic feasible solution

The initial basic feasible solution is x_{11}=20, x_{12}=5, x_{13}=20, x_{23}=40, x_{24}=35, x_{34}=25 and minimum cost of transportation = 670

Matrix Minimum Method

Example 2: Consider the transportation problem presented in the following table**:**

Factory	Retail shop				Supply
	1	2	3	4	
1	3	5	7	6	50
2	2	5	8	2	75
3	3	6	9	2	25
Demand	20	20	50	60	

Solution:

Factory	Retail shop				Supply
	1	2	3	4	
1	3	5 20	7 30	6	50
2	2 20	5	8	2 55	75
3	3	6	9 20	2 5	25
Demand	20	20	50	60	

Number of basic variables = m + n –1 = 3 + 4 – 1 = 6.

Initial basic feasible solution

The initial basic feasible solution is x_{12}=20, x_{13}=30, x_{21}=20, x_{24}=55, x_{233}=20, x_{34}=5 and minimum cost of transportation=20 X 2 + 20 X 5 + 30 X 7 + 55 X 2 + 20 X 9 + 5 X 2 = 650.

Vogel Approximation Method (VAM)

The Vogel approximation (Unit penalty) method is an iterative procedure for computing a basic feasible solution of a transportation problem. This method is preferred over the two methods discussed in the previous sections, because the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution.

Example 3: Obtain an Initial BFS to the following Transportation problem using VAM method?

Origin	Destination				Supply
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

Solution:

$$\sum_{=}^{4} a_i = \sum_{=}^{3} b_j$$, the given problem is balanced TP., Therefore there exists a

Step -1: Select the lowest and next to lowest cost for each row and each column, then the difference between them for each row and column displayed them with in first bracket against respective rows and columns. Here all the differences have been shown within first compartment. Maximum difference is 15 which is occurs at the second column. Allocate min (40,120) in the minimum cost cell (1,2).

Step -2: Appling the same techniques we obtained the initial BFS. Where all capacities and demand have been exhausted

Table Initial

	Destination										
Origin	**1**	**2**	**3**	**4**	**Supply**	**Penalty**					
1	20	**22**	17	**4**	~~123~~	13	13	-	-	-	-
2	**24**	37	**9**	**7**	~~70~~	2	2	2	17	24	24
3	**32**	37	20	15	~~50~~	5	5	5	17	32	-
Demand	~~60~~	~~40~~	~~30~~	~~110~~	240						
Penalty	4	15	8	3							
	4	-	8	3							
	8	-	11	8							
	8	-	-	8							
	8	-	-	-							
	24	-	-	-							

Feasible solution

The initial basic feasible solution is x_{12}=40, x_{14}=40, x_{21}=10, x_{23}=30, x_{24}=30, x_{31}=50.and minimum cost of transportation=3520.

Optimality Test for Transportation problem

There are basically two methods

a) Modified Distribution Method (MODI)
b) Stepping Stone Method.

Modified Distribution Method (MODI)

The modified distribution method, also known as MODI method or (u - v) method provides a minimum cost solution to the transportation problem. In the stepping stone method, we have to draw as many closed paths as equal to the unoccupied cells for their evaluation. To the contrary, in MODI method, only closed path for the unoccupied cell with highest opportunity cost is drawn.

Steps

1. Determine an initial basic feasible solution using any one of the three methods given below:

    a) North West Corner Rule
    b) Matrix Minimum Method
    c) Vogel Approximation Method

2. Determine the values of dual variables, u_i and v_j, using $u_i + v_j = c_{ij}$

3. Compute the opportunity cost using $\Delta_{ij} = c_{ij} - (u_i + v_j)$.

4. Check the sign of each opportunity cost.

    a) If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimal solution. On the other hand,
    b) if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimal solution and further savings in transportation cost are possible.

5. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.

6. Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at

the original unoccupied cell.

7. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.

8. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, **add** this quantity to all the cells on the corner points of the closed path marked with plus signs, and **subtract** it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

9. Repeat the whole procedure until an optimal solution is obtained.

Degeneracy in Transportation Problem:

If the basic feasible solution of a transportation problem with m origins and n destinations has fewer than (m + n – 1) positive x_{ij} (occupied cells), the problem is said to be a degenerate transportation problem.

Degeneracy can occur at two stages:

1. At the initial solution
2. During the testing of the optimal solution

To resolve degeneracy, we make use of an artificial quantity (*d*). The quantity *d* is assigned to that unoccupied cell, which has the minimum transportation cost.

Example01: Find the optimum transportation schedule and minimum total cost of transportation.

	D 1	D 2	D 3	ai
O1	10	7	8	40
O2	15	12	9	15
O3	7	8	12	40
bj	25	55	20	

Solution: Since $\sum^{4} a_i = \sum^{3} b_i$ = 100. So the given transportation problem is balanced. To find initial basic feasible solution, we apply VAM method. In this method we are to find the difference between two least elements in each row and column.

4 10	40 7	5 8
uuu 8 15	4 12	15 9
25 7	15 8	3 12

We first take the least element cell (3,1) lies on the highest difference column where the demand is 25 units and capacity is 40 units. We choose x_{31}=25, the min of these two, to convert into it basic cell, and demand is exhausted, Neglecting that column. We again find the difference and applying the same method to get all initial basic feasible solution. The solutions are x_{12}=40, x_{13}=5, x_{23}=15, x_{31}=25, x_{32}=15.Here the number of solutions =5= (m+n-1) = (3+3-1). This implies, the solution is non-degenerate solution.

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SCHOOL OF MANAGEMENT STUDIES

UNIT – III ELEMENTS OF OPERATION RESEARCH - SBAA1305

INTRODUCTION

In a printing press there is one machine and one operator is there to operate. How would you employ the worker? Your immediate answer will be, the available operator will operate the machine. Again suppose there are two machines in the press and two operators are engaged at different rates to operate them. Which operator should operate which machine for maximizing profit?

Similarly, if there are n machines available and n persons are engaged at different rates to operate them. Which operator should be assigned to which machine to ensure maximum efficiency? While answering the above questions we have to think about the interest of the press, so we have to find such an assignment by which the press gets maximum profit on minimum investment. Such problems are known as "assignment problems".

Assignment problem is a particular case of the transportation problem in which objective is to assign number of task to equal number of facilities at minimum cost and maximum profit. Suppose there are 'm' facilities and 'n' jobs and the effectiveness of each facility for each job are given, the objective is to assign one facility to one job so that the given measure of effectiveness is optimized.
If the matrix contains the cost involved in assignment the aim is to **minimize the cost**. If the matrix contains revenue or profit the aim is to **maximize the revenue or profit**.

Example.:

	JOBS a	b	c	d
1	C1a	C1b	C1c	C1d
2	C2a	C2b	C2c	C2d
FACILITIES 3	C3a	C3b	C3c	C3d
4	C4a	C4b	C4c	C4d

1,2,3,4 indicates the facilities and a, b, c, d indicates the jobs. The matrix entries are the cost associated with the assignment of facilities with the jobs. The objective is to assign one facility to one job, so that the total cost is minimum.
Ex.:

Facility Job

1 $\longrightarrow$ d

2 $\longrightarrow$ c

3 $\longrightarrow$ a

4 $\longrightarrow$ b

Total cost = C1d + C2c +C3a +C4b

Hungarian assignment method

The Hungarian method of assignment provides us with an efficient means of finding the optimal solution. The Hungarian method is based upon the following principles:

(i) If a constant is added to every element of a row and/or column of the cost matrix of an assignment problem the resulting assignment problem has the same optimum solution as the original problem or vice versa.

(ii) The solution having zero total cost is considered as optimum solution.

Hungarian method of assignment problem (minimization case) can be summarized in the following steps:

Step I: Subtract the minimum cost of each row of the cost (effectiveness) matrix from all the elements of the respective row so as to get first reduced matrix.

Step II: Similarly subtract the minimum cost of each column of the cost matrix from all the elements of the respective column of the first reduced matrix. This is first modified matrix.

Step III: Starting with row 1 of the first modified matrix, examine the rows one by one until a row containing exactly single zero elements is found. Make any assignment by making that zero in or enclose the zero inside a. Then cross (X) all other zeros in the column in which the assignment was made. This eliminates the possibility of making further assignments in that column.

Step IV: When the set of rows have been completely examined, an identical procedure is applied successively to columns that is examine columns one by one until a column containing exactly single zero element is found. Then make an experimental assignment in that position and cross other zeros in the row in which the assignment has been made.

Step V: Continue these successive operations on rows and columns until all zeros have been either assigned or crossed out and there is exactly one assignment in each row and in each column. In such case optimal assignment for the given problem is obtained.

Step VI: There may be some rows (or columns) without assignment i.e. the total number of marked zeros is less than the order of the matrix. In such case proceed to step VII.

Step VII: Draw the least possible number of horizontal and vertical lines to cover all zeros of the starting table. This can be done as follows:

1. Mark (√) in the rows in which assignments has not been made.
2. Mark column with (√) which have zeros in the marked rows.
3. Mark rows with (√) which contains assignment in the marked column.
4. Repeat 2 and 3 until the chain of marking is completed.
5. Draw straight lines through marked columns.
6. Draw straight lines through unmarked rows.

By this way we draw the minimum number of horizontal and vertical lines necessary to cover all zeros at least once. It should, however, be observed that in all n x n matrices less than n lines will cover the zeros only when there is no solution among them. Conversely, if the minimum number of lines is n, there is a solution.

Step VIII: In this step, we

1. Select the smallest element, say X, among all the not covered by any of the lines of the table; and
2. Subtract this value X from all of the elements in the matrix not covered by lines and add X to all those elements that lie at the intersection of the horizontal and vertical lines, thus obtaining the second modified cost matrix.

Step IX: Repeat Steps IV, V and VI until we get the number of lines equal to the order of matrix I, till an optimum solution is attained.

Step X: We now have exactly one encircled zero in each row and each column of the cost matrix. The assignment schedule corresponding to these zeros is the optimum assignment. The above technique is explained by taking the following examples

Example 1

A plant manager has four subordinates, and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. This estimate of the times each man would take to perform each task is given in the effectiveness matrix below.

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man hours?

Solution

Step I : Subtracting the smallest element in each row from every element in that row, we get the first reduced matrix.

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Step II: Next, we subtract the smallest element in each column from every element in that column; we get the second reduced matrix.

Step III: Now we test whether it is possible to make an assignment using only zero distances.

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

(a) Starting with row 1 of the matrix, we examine rows one by one until a row containing exactly single zero elements are found. We make an experimental assignment (indicated by) to that cell. Then we cross all other zeros in the column in which the assignment was made.

(b) When the set of rows has been completely examined an identical procedure is applied successively to columns. Starting with Column 1, we examine columns until a column containing exactly one remaining zero is found. We make an experimental assignment in that position and cross other zeros in the row in which the assignment was made. It is found that no additional assignments are possible. Thus, we have the complete Zero assignment,

A $\rightarrow$ I, B $\rightarrow$ III, C $\rightarrow$ II, D $\rightarrow$ IV

The minimum total man hours are computed as

Optimal assignment	Man hours
A $\rightarrow$ I	8
B $\rightarrow$ III	4
C $\rightarrow$ II	19
D $\rightarrow$ IV	10
Total	**41 hours**

Example 2

A dairy plant has five milk tankers I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D, and E. The distances (in kms) between dairy plant and the delivery routes are given in the following distance matrix

	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How the milk tankers should be assigned to the chilling centers so as to minimize the distance travelled?

Solution

Step I: Subtracting minimum element in each row we get the first reduced matrix as

30	0	45	60	70
15	0	10	40	55
30	0	45	60	75
0	0	30	30	60
20	0	35	45	70

Step II: Subtracting minimum element in each column we get the second reduced matrix as

30	0	35	30	15
15	0	0	10	0
30	0	35	30	20

0	0	20	0	5
20	0	25	15	15

Step III: Row 1 has a single zero in column 2. We make an assignment by putting around it and delete other zeros in column 2 by marking X. Now column1 has a single zero in column 4 we make an assignment by putting and cross the other zero which is not yet crossed. Column 3 has a single zero in row 2; we make an assignment and delete the other zero which is uncrossed. Now we see that there are no remaining zeros; and row 3, row 5 and column 4 has no assignment. Therefore, we cannot get our desired solution at this stage.

30	$\boxed{0}$	35	30	15		√
15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$	L_2	
30	$\cancel{0}$	35	30	20		√
$\boxed{0}$	$\cancel{0}$	20	$\cancel{0}$	5	L_3	
20	$\cancel{0}$	25	15	15		√
	√ L_1					

Step IV: Draw the minimum number of horizontal and vertical lines necessary to cover all zeros at least once by using the following procedure

1. Mark (√) row 3 and row 5 as having no assignments and column 2 as having zeros in rows 3 and 5.

2. Next we mark (√) row 2 because this row contains assignment in marked column 2. No further rows or columns will be required to mark during this procedure.

3. Draw line L_1 through marked col.2.

4. Draw lines L_2 & L_3 through unmarked rows.

Step V: Select the smallest element say X among all uncovered elements which is X = 15. Subtract this value X=15 from all of the values in the matrix not covered by lines and add X to all those values that lie at the intersections of the lines L_1, L_2 & L_3.

Applying these two rules, we get a new matrix

15	0	20	15	0
15	15	0	10	0
15	0	20	15	5
0	15	20	0	5
5	0	10	0	0

Step VI: Now reapply the test of Step III to obtain the desired solution.

15	$\cancel{0}$	20	15	$\boxed{0}$
15	15	$\boxed{0}$	10	$\cancel{0}$
15	$\boxed{0}$	20	15	5
$\boxed{0}$	15	20	$\cancel{0}$	5
5	$\cancel{0}$	10	$\boxed{0}$	$\cancel{0}$

The assignments are

A → V   B → III   C → II   D → I   E → I

Total Distance 200 + 130 + 110 + 50 + 80 = 570

column-wise) choose arbitrarily one zero for assignment and cancel all zeros in the corresponding rows and columns.

- Repeat the procedure by choosing another zero for assignment till all such zeroes are considered.
- Each assignment by this procedure will provide different set of assignments keeping the total minimum cost as constant. This implies multiple optimal solutions with the same optimal assignment cost.

SOLVING MAXIMISATION PROBLEMS IN ASSIGNMENT USING HUNGARIAN METHOD

- The maximization problem can be converted in to a minimization problem by subtracting all the elements of the matrix from the highest value.
- Follow the steps 1 to 9 of Hungarian Algorithm.

Note: While calculating the total profits take corresponding values from initial assignment problem (data before conversion of the problem)

RESTRICTED ASSIGNMENT PROBLEMS

The assignment technique assumes that the problem is free from practical restrictions and any task could be assigned to any facility. But in some cases, it may not be possible to assign a particular task to a particular facility due to space, size of the task, process capability of the facility, technical difficulties or other restrictions. This can be overcome by assigning a very high processing time of cost element (□ infinity) to the corresponding cell.

- Use Hungarian method for assignment steps 1 to 9.

NOTE:

- For maximization problems in restricted assignments, convert the problem in to a minimization problems given in the procedure above.
- Substitute (infinity) in the matrix for the restricted assignments.
- Use Hungarian method for assignment steps 1 to 9.

TRAVELLING SALESMAN PROBLEM

A salesman normally visits numbers of cities starting from high head quarters. The distance (or time or cost) between every pair of cities are assumed to be known. If a salesman has to visit 'n' cities, then he will have a total of (n-1)! Possible round trips. The problem of finding the shortest distance (or minimum time or minimum cost) if the salesman starts from his headquarters and passes through each city under his jurisdiction exactly once and returns to the headquarters is called the Travelling salesman problem or A Travelling Salesperson problem.

A travelling salesman problem is very similar to the assignment problem with the additional constraints.

a) **Route Conditions**:

The salesman should go through every city exactly once except the starting city (headquarters).

Example:

A travelling salesman, named Rolling Stone plans to visit five cities 1, 2, 3, 4 & 5. The travel time (in hours) between these cities is shown below:

	To				
From	**1**	**2**	**3**	**4**	**5**
1	∞	5	8	4	5
2	5	∞	7	4	5
3	8	7	∞	8	6
4	4	4	8	∞	8
5	5	5	6	8	∞

How should Mr. Rolling Stone schedule his touring plan in order to **minimize** the total travel **time**, if he visits each city once a week?

Solution

After applying steps 1 to 3 of the Hungarian method, we get the following assignments.

Table

	To				
From	**1**	**2**	**3**	**4**	**5**
1	∞	1	3	$\boxed{0}$	1
2	1	∞	2	$\cancel{0}$	1
3	2	1	∞	2	$\boxed{0}$
4	$\boxed{0}$	$\cancel{0}$	3	∞	4
5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	3	∞

Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced matrix.

Table

	To				
From	1	2	3	4	5
1	∞	1	3	$\boxed{0}$	1
2	1	∞	2	$\cancel{0}$	1
3	2	1	∞	2	$\boxed{0}$
4	$\boxed{0}$	$\cancel{0}$	3	∞	4
5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	3	∞

Select the smallest element from all the uncovered elements. Subtract this smallest element from all the uncovered elements and add it to the elements, which lie at the intersection of two lines. Thus, we obtain another reduced matrix for fresh assignment. Repeating step 3 on the reduced matrix, we get the following assignments.

Table

	To				
From	**1**	**2**	**3**	**4**	**5**
1	∞	$\cancel{0}$	2	$\boxed{0}$	1
2	$\boxed{0}$	∞	1	$\cancel{0}$	1
3	1	$\cancel{0}$	∞	2	$\boxed{0}$
4	$\cancel{0}$	$\boxed{0}$	3	∞	5
5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	4	∞

The above solution suggests that the salesman should go from city 1 to city 4, city 4 to city 2, and then city 2 to 1 (original starting point). The above solution is not a solution to the travelling salesman problem as he visits city 1 twice.

The next best solution can be obtained by bringing the minimum non-zero element, i.e., 1 into the solution. Please note that the value 1 occurs at four places. We will consider all the cases separately until the acceptable solution is obtained. To make the assignment in the cell (2, 3), delete the row & the column containing this cell so that no other assignment can be made in the second row and third column.

Now, make the assignments in the usual manner as shown in the following table.

Table

	To				
From	1	2	3	4	5
1	∝	~~0~~	2	$\boxed{0}$	1
2	~~0~~	∝	$\boxed{1}$	~~0~~	1
3	1	~~0~~	∝	2	$\boxed{0}$
4	~~0~~	$\boxed{0}$	3	∝	5
5	$\boxed{0}$	~~0~~	~~0~~	4	∝

He starts from city 1 and goes to city 4; from city 4 to city 2; from city 2 to city 3; from city 3 to city 5; from city 5 to city 1.

Substituting values from original table:
4 + 7 + 6+ 4 + 5 = 26 hours.

- The salesman starts from one city (headquarters) and comes back to that city (headquarters).

b) Obviously going from any city to the same city directly is not allowed (i.e., no assignments should be made along the diagonal line).

Steps to solve travelling salesman problem:

i. Assigning an infinitely large element in each of the squares along the diagonal line in the cost matrix.
ii. Solving the problem as a routine assignment problem.
iii. Scrutinizing the solution obtained under (ii) to see if the 'route' conditions are satisfied.
iv. If not, making adjustments in assignments to satisfy the condition with minimum increase in total cost (i.e. to satisfy route condition, 'next best solution' may require to be considered).

SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

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SCHOOL OF MANAGEMENT STUDIES

UNIT – IV ELEMENTS OF OPERATION RESEARCH - SBAA 1305

When a number of jobs are given to be done and they require processing on two or more machines, the main concern of a manager is to find the order or sequence to perform these jobs. We shall consider the sequencing problems in respect of the jobs to be performed in a factory and study the method of their solution. Such sequencing problems can be broadly divided in two groups. In the first one, there are n jobs to be done, each of which requires processing on some or all of the k different machines. We can determine the effectiveness of each of the sequences that the technologically feasible (that is to say, those satisfying the restrictions on the order in which each job must be processed through the machines) and choose a sequence which optimizes the effectiveness. To illustrate, the timings of processing of each of the n jobs on each of the **k** machines, in a certain given order, may be given and the time for performing the jobs may be the measure of effectiveness. We shall select the sequences for which the total time taken in processing all the jobs on the machines would be the minimum.

In this unit we will look into solution of a sequencing problem. In this lesson the solutions of following cases will be discussed:

a) n jobs and two machines A and B, all jobs processed in the order AB.

b) n jobs and three machines A, B and C all jobs processed in the order ABC

c) Problems with n jobs and m machines. b) Each job is processed in the order AB.

c) The exact or expected processing times A_1, A_2, A_3, --- , A_n ; B_1, B_2, B_3, --- , B_n are known and are provided in the following table

Machine	Job(s)								
	1	2	3	--	-	i	- -	-	n
A	A_1	A_2	A_3	--	-	A_i	--	-	A_n
B	B_1	B_2	B_3	--	-	B_i	--	-	B_n

The problem is to find the sequence (or order) of jobs so as to minimize the total elapsed time T. The solution of the above problem is also known as Johnsons procedure which involves the following steps:

Step 1. Select the smallest processing time occurring in the list A_1, A_2, A_3, --- , A_n ; B_1, B_2, B_3, --- , B_n if there is a tie, either of the smallest processing times can be selected.

Step 2. If the least processing time is A_r , select the r^{th} job first. If it is B_s, do the s^{th} job last as the given order is AB

Step 3. There are now (n-1) jobs left to be ordered. Repeat steps I and II for the remaining set of processing times obtained by deleting the processing time for both the machines corresponding to the job already assigned.

Step 4. Continue in the same manner till the entire jobs have been ordered. The resulting ordering will minimize the total elapsed time T and is called the optimal sequence.

Step 5. After finding the optimal sequence as stated above find the total elapsed time and idle times on machines A and B as under:

Total elapsed time =	The time between starting the first job in the optimal sequence on machine A and completing the last job in the optimal machine B.
Idle time on machine A =	(Time when the last job in the optimal sequence on sequences is completed on machine B)- (Time when the last job in the optimal sequences is completed on machine A)
Idle time on machine B =	(Time when the first job in the optimal sequences is completed on machine A)

Example 1

There are nine jobs, each of which must go through two machines P and Q in the order PQ, the processing times (in hours) are given below:

Machine	Job(s)								
	A	B	C	D	E	F	G	H	I

P	2	5	4	9	6	8	7	5	4
Q	6	8	7	4	3	9	3	8	11

Find the sequence that minimizes the total elapsed time T. Also calculate the total idle time for the machines in this period.

Solution

The minimum processing time on two machines is 2 which correspond to task A on machine P. This shows that task A will be preceding first. After assigning task A, we are left with 8 tasks on two machines

Machine	B	C	D	E	F	G	H	I
P	5	4	9	6	8	7	5	4
Q	8	7	4	3	9	3	8	11

Minimum processing time in this reduced problem is 3 which correspond to jobs E and G (both on machine Q). Now since the corresponding processing time of task E on machine P is less than the corresponding processing time of task G on machine Q therefore task E will be processed in the last and task G next to last. The situation will be dealt as

A							G	E

The problem now reduces to following 6 tasks on two machines with processing time as follows:

Machine	B	C	D	F	H	I
P	5	4	9	8	5	4
Q	8	7	4	9	8	11

Here since the minimum processing time is 4 which occurs for tasks C and I on machine P and task D on machine Q. Therefore, the task C which has less processing time on P will be processed first and then task I and task D will be placed at the last i.e., 7^{th} sequence cell.

The sequence will appear as follows:

A	C	I				D	E	G

The problem now reduces to the following 3 tasks on two machines

Machine	B	F	H
P	5	8	5
Q	8	9	8

n this reduced table the minimum processing time is 5 which occurs for tasks B and H both on machine P. Now since the corresponding time of tasks B and H on machine Q are same i.e. 8. Tasks B or H may be placed arbitrarily in the 4^{th} and 5^{th} sequence cells. The remaining task F can then be placed in the 6^{th} sequence cell. Thus the optimal sequences are represented as

A	I	C	B	H	F	D	E	G

or

A	1	C	H	B	F	D	E	G

Further, it is also possible to calculate the minimum elapsed time corresponding to the optimal sequencing $A \rightarrow I \rightarrow C \rightarrow B \rightarrow H \rightarrow F \rightarrow D \rightarrow E \rightarrow G$.

Job Sequence	Machine A		Machine B	
	Time In	Time Out	Time In	Time Out
A	0	2	2	8
I	2	6	8	19
C	6	10	19	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
E	37	43	55	58
G	43	50	58	61

Hence the total elapsed time for this proposed sequence staring from job A to completion of job G is 61 hours .During this time machine P remains idle for 11 hours (from 50 hours to 61 hours)and the machine Q remains idle for 2 hours only (from 0 hour to 2 hour).

Processing of n Jobs through Three Machines

The type of sequencing problem can be described as follows:

a) Only three machines A, B and C are involved;

b) Each job is processed in the prescribed order ABC

c) No passing of jobs is permitted i.e. the same order over each machine is maintained.

d) The exact or expected processing times A_1, A_2, A_3, --- , A_n ; B_1, B_2, B_3, --- , B_n and C_1, C_2, C_3, --- , C_n are known and are denoted by the following table

Machine	Job(s)								
	1	2	3	--	-	i	- -	-	n

A	A_1	A_2	A_3	--	-	A_i	--	-	A_n
B	B_1	B_2	B_3	--	-	B_i	--	-	B_n
C	C_1	C_2	C_3			C_i			C_n

Our objective will be to find the optimal sequence of jobs which minimizes the total elapsed time. No general procedure is available so far for obtaining an optimal sequence in such case. However, the Johnsons procedure can be extended to cover the special cases where either one or both of the following conditions hold:

a) The minimum processing time on machine A ≥ the maximum processing time on machine B.

b) The minimum processing time on machine C ≥ the maximum processing time on machine B.

The method is to replace the problem by an equivalent problem involving n jobs and two machines. These two fictitious machines are denoted by G and H and the corresponding time G_i and H_i are defined by

$G_i = A_i + B$ and $B_i + C_i$

Now this problem with prescribed ordering GH is solved by the method with n jobs through two machines, the resulting sequence will also be optimal for the original problem. The above methodology is illustrated by following example:

Example 2

There are five jobs (namely 1,2,3,4 and 5), each of which must go through machines A, B and C in the order ABC. Processing Time (in hours) are given below:

Jobs	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Find the sequence that minimum the total elapsed time required to complete the jobs.

Solution

Here Min A_i = 5; B_i = 5 and C_i =3 since the condition of Min. $A_i \geq$ Max. B_i is satisfied the given problem can be converted into five jobs and two machines problem.

Jobs	$G_i = A_i + B_i$	$H_i = B_i + C_i$
1	7	5
2	8	8
3	10	9
4	14	11
5	8	10

The Optimal Sequence will be

2 5 4 3 1

Total elapsed Time will be

Jobs	Machine A		Machine B		Machine C	
	In	Out	In	Out	In	Out
2	0	7	7	8	8	15
5	7	12	12	15	15	22
4	12	21	21	26	26	32
3	21	27	27	31	32	37
1	27	32	32	34	37	40

Min. total elapsed time is 40 hours.

Idle time for Machine A is 8 hrs. (32-40)

Idle time for Machine B is 25 hours (0-7, 8-12, 15-21, 26-27, 31-32 and 34-40)

Idle time for Machine C is 12 hours (0-8, 22-26.)

Problems with n Jobs and m Machines

Let there be n jobs, each of which is to be processed through m machines, say M_1, M_2, --- , M_m in the order M_1, M_2, M_3, --- , M_m. Let T_{ij} be the time taken by the i^{th} machine to complete the j^{th} job.

The iterative procedure of obtaining an optimal sequence is as follows:

Step I: Find (i) $\min_j (T_{1j})$ ii) $\min_j (T_{mj})$ iii) $\max_j (T_{2j}, T_{3j}, T_{4j}, --- , T_{(m-1)j})$ for j=1,2,---, n

Step II: Check whether

a. $\min_j(T_{1j}) \geq \max_j (T_{ij})$ for i=2,3,----,m-1

Or

b. $\min_j(T_{mj}) \geq \max_j (T_{ij})$ for i=2,3,---,m-1

Step III: If the inequalities in Step II are not satisfied, method fails, otherwise, go to next step.

Step IV: Convert the m machine problem into two machine problem by introducing two fictitious machines G and H, such that

$T_{Gj} = T_{1j} + T_{2j} + --- + T_{(m-1)j}$ and $T_{Hj} = T_{2j} + T_{3j} + --- + T_{mj}$

Determine the optimal sequence of n jobs through 2 machines by using optimal sequence algorithm.

Step V: In addition to condition given in Step IV, if $T_{ij} = T_{2j} + T_{3j} + --- + T_{mj} = C$ is a fixed positive constant for all i = 1, 2, 3, , n then determine the optimal sequence of n jobs and two machines M_1 and M_m in the order M_1M_m by using the optimal sequence algorithm.

Example 3

Find an optimal sequence for the following sequencing problem of four jobs and five machines when passing is not allowed, of which processing time (in hours) is given below:

Job	Machine				
	A	B	C	D	E
1	7	5	2	3	9
2	6	6	4	5	10
3	5	4	5	6	8
4	8	3	3	2	6

Also find the total elapsed time.

Solution:

Here Min. $A_i = 5$, Min. $E_i = 6$

Max. $(B_i, C_i, D_i) = 6, 5, 6$ respectively

Since Min. E_i = Max. (B_i, D_i) and Min. A_i = Max. C_i satisfied therefore the problem can be converted into 4 jobs and 2 fictitious machines G and H as follows:

Job	Fictitious Machine	
	$G_i = A_i + B_i + C_i + D_i$	$H_i = B_i + C_i + D_i + E_i$
1	17	19
2	21	25
3	20	23
4	16	14

The above sequence will be:

1 3 2 4

Total Elapsed Time Corresponding to Optimal Sequence can be obtained as follows:

	Machine A		Machine B		Machine C		Machine D		Machine E	
Job	In	Out	In	Out	In	Out	In	Out	In	Out
1	0	7	7	12	12	14	14	17	17	26
3	7	12	12	16	16	21	21	27	27	35
2	12	18	18	24	24	28	28	33	35	45
4	18	26	26	29	29	32	33	35	45	51

Thus the minimum elapsed time is 51 hours.

Idle time for machine A = 25 hours(26-51)

Idle time for machine B = 33 hours(0-7,16-18,24-26,29-51)

Idle time for machine C = 37 hours(0-12,14-16,21-24,28-29,32-51)

Idle time for machine D = 35 hours (0-14,17-21,27-28,35-51)

Idle time for machine E = 18 hours (0-17,26-27)

GAME THEORY

INTRODUCTION

A competitive situation in business can be treated similar to a **game**. There are two or more players and each player uses a strategy to out play the opponent.

A strategy is an action plan adopted by a player in-order to counter the other player.In our of game theory we have two players namely Player A and Player B.

The basic objective would be that

Player A – plays to **Maximize profit** (offensive) - Maxi (min) criteria

Player B – plays to **Minimize losses** (defensive) - Mini (max) criteria

The Maxi (Min) criteria is that – Maximum profit out of minimum possibilities The Mini (max) criteria is that – Minimze losses out of maximum possibilities.

Game theory helps in finding out the best course of action for a firm in view of the anticipated counter-moves from the competing organizations.

Characteristics of a game

A competitive situation is a competitive game if the following properties hold good

1. The number of competitors is finite, say N.
2. A finite set of possible courses of action is available to each of the N competitors.
3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that all the players select their courses of action simultaneously. As a result,no competitor will be in a position to know the choices of his competitors.
4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

TERMINOLOGIES

Zero Sum game because the Gain of A – Loss of B = 0. In other words, the gain of Player A is the Loss of Player B.

Pure strategy If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to minimize the gain Therefore the pure strategy is a decision rule always to select a particular course of action.

Mixed strategy If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus, the mixed strategy is a selection among pure strategies with fixed probabilities.

Optimal strategy The strategy that puts the player in the most preferred position irrespective of the strategy of his opponents is called an optimal strategy Any deviation from this strategy would reduce his payoff

Saddle Point : If the Maxi (min) of A = Mini (max) of B then it is known as the Saddle Point Saddle point is the number, which is lowest in its row and highest in its column.When minimax value is equal to maximin value , the game is said to have saddle point. It is the cell in the payoff matrix which satisfies minimax to maximin value

Value of the Game: It is the average wining per play over a long no. of plays.It is the expected pay off when all the players adopt their optimum strategies .If the value of game is zero it is said to be a fair game, If the value of game is not zero it is said to be a unfair game .

In all problems relating to game theory, first look for saddle point, then check out for rule of dominance and see if you can reduce the matrix.

Rule of Dominance:

The dominance and modified dominance principles and their applications for reducing the size of a game with or without a saddle point. If every value of one strategy of A is lesser than that o the other strategy of A,Then A will play the strategy with greater values and remove the strategy with the lesser payoff values.

If every value of one strategy of B is greater than that of other strategy of B , B will play the lesser value strategy and remove the strategy with higher payoff values.

Dominance rule for the row

If all the elements in a particular row is lower than or equal to all the elements in another row, then the row with the lower items are said to be dominated by row with higher ones, Then the row with lower elements will be eliminated.

Dominance rule for the column

If all the elements in a particular column is higher than or equal to all the elements in another column, then the column with the higher items are said to be dominated by column with lower ones, Then the column with higher elements will be eliminated.

Modified Dominance Rule

In few cases, if the given strategy is inferior to the average of two or more pure strategies, then the inferior strategy is deleted from the pay-off matrix and the size of the matrix is reduced considerably. In other words , if a given row has lower elements than the elements of average of two rows then particular row can be eliminated. Similarly if a given column has higher elements than the elements of average of two columns then particular column can be eliminated. Average row/column cannot be eliminated under any circumstances.. This type of dominance property is known as the modified dominance property

Graphical Method

If one of the players, play only two strategies or if the game can be reduced such that one of the players play only two strategies. Then the game can be solved by the graphical method.

In case the pay-off matrix is of higher order (say m x n), then we try to reduce as much as possible using dominance and modified dominance ,f we get a pay-off matrix of order 2 x n or n x 2 we try to reduce the size of the pay-off matrix to that of order 2 x 2 with the graphical method so that the value of game could be obtained

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Managerial Applications of the Theory of Games

The techniques of game theory can be effectively applied to various managerial problems as detailed below:

1. Analysis of the market strategies of a business organization in the long run.
2. Evaluation of the responses of the consumers to a newproduct.
3. Resolving the conflict between two groups in a business organization.
4. Decision making on the techniques to increase market share.
5. Material procurement process.
6. Decision making for transportation problem.
7. Evaluation of the distribution system.
8. Evaluation of the location of the facilities.
9. Examination of new business ventures and
10. Competitive economic environment.

THEORY QUESTIONS &PROBLEMS

1. What is meant Game theory?
2. What do you mean by pay-off matrix?
3. What do you mean by i) Saddle point ii) Zero-Sumgame?
4. Write short notes on i) pure strategy ii) Mixed strategy.
5. What are the methods available to solve gametheory?
6. What do you mean by Dominance principle?
7. Write the rules of dominance principle?
8. What is meant by i) Maximinprinciple ii) Minimax principle?

9. Find the value of game

		Player B	
		B1	B2
Player A	A1	6	-3
	A2	-3	7

SCHOOL OF MANAGEMENT STUDIES

UNIT –V - ELEMENTS OF OPERATION RESEARCH - SBAA1305

INTRODUCTION

If any equipment or machine is used for a long period of time, due to wear and tear, the item tends to worsen. A remedial action to bring the item or equipment to the original level is desired. Then the need for replacement becomes necessary. This may be due physical impairment, due to normal wear and tear, obsolescence etc. The resale value of the item goes on diminishing with the passage of time.

The depreciation of the original equipment is a factor, which is responsible not to favor replacement because the capital is being spread over a long time leading to a lower average cost. Thus there exists an economic trade-off between increasing and decreasing cost functions. We strike a balance between the two opposing costs with the aim of obtaining a minimum cost.

Replacement model aims at identifying the **time** at which the assets must be replaced in order to minimize the cost.

REASONS FOR REPLACEMENT OF EQUIPMENT:

1. Physical impairment or malfunctioningof various parts refers to
   - The physical condition of the equipment itself
   - Leads to a decline in the value of service rendered by the equipment
   - Increasing operating cost of the equipment
   - Increased maintenance cost of the equipment
   - Or a combination of the above.
2. Obsolescence of the equipment, caused due to improvement in the existing tools and machinery mainly when the technology becomes advanced.
3. When there is sudden failure or breakdown.

REPLACEMENT MODELS:

- **Assets that fails Gradually:**

Certain assets wear and tear as they are used. The efficiency of the assets decline with time. The maintenance cost keeps increasing as the years pass by eg. Machinery, automobiles, etc.

1. Gradual failure without taking time value of money into consideration
2. Gradual failure taking time value of money into consideration

- **Assets which fail suddenly**

  Certain assets fail suddenly and have to be replaced from time to time eg. bulbs.

  1. Individual Replacement policy (IRP)
  2. Group Replacement policy (GRP)

I. Gradual failure without taking time value of money into consideration

As mentioned earlier the equipments, machineries and vehicles undergo wear and tear with the passage of time. The cost of operation and the maintenance are bound to increase year by year. A stage may be reached that the maintenance cost amounts prohibitively large that it is better and economical to replace the equipment with a new one. We also take into account the salvage value of the items in assessing the appropriate or opportune time to replace the item. We assume that the details regarding the costs of operation, maintenance and the salvage value of the item are already known

Procedure for replacement of an asset that fails gradually (without considering Time value of money):

a) Note down the years
b) Note down the running cost 'R' (Running cost or operating cost or Maintenance cost or other expenses)
c) Calculate Cumulative the running cost 'R'
d) Note down the capital cost 'C'
e) Note down the scrap or resale value 'S'
f) Calculate Depreciation = Capital Cost – Resale value
g) Find the Total Cost

   Total Cost = Cumulative Running cost + Depreciation
h) Find the average cost

   Average cost = Total cost/No. of corresponding year
i) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

year	Running cost	Cumulative running cost	Capital cost	Salvage value or Resale value	Depn. = Capital cost – salvage value	Total cost= Cumulative running cost + Depreciation	Average annual cost P_n = Total cost / no. of corresponding year
n	R_n	□ R_n	C	S_n	C - S_n	□R_n+C -S_n	(□R_n + C – S_n) /n
1	2	3	4	5	6 (4-5)	7 (3+6)	8 (7/1)

II. Gradual failure taking time value of money into consideration

In the previous section we did not take the interest for the money invested, the running costs and resale value. If the effect of time value of money is to be taken into account, the analysis must be based on an equivalent cost. This is done with the present value or present worth analysis.

For example, suppose the interest rate is given as 10% and Rs. 100 today would amount to Rs. 110 after a year's time. In other words the expenditure of Rs. 110 in year's time is equivalent to Rs. 100 today. Likewise one rupee a year from now is equivalent to (1.1)-1 rupees today and one-rupee in '*n*' years from now is equivalent to (1.1)-n rupees today. This quantity (1.1)-n is called the present value or present worth of one rupee spent 'n' years from now

Procedure for replacement of an asset that fails gradually (with considering Time value of money):

Assumption:

i. Maintenance cost will be calculated at the beginning of the year
ii. Resale value at the end of the year

Procedure:

a) Note down the years
b) Note down the running cost 'R' (Running cost or operating cost or Maintenance cost or other expenses)
c) Write the present value factor at the beginning for running cost
d) Calculate present value for Running cost
e) Calculate Cumulative the running cost R'

f) Note down the capital cost ‘C’

g) Note down the scrap or resale value ‘S’

h) Write the present value factor at the end of the year and also calculate present value for salvage or scrap or resale value.

i) Calculate Depreciation = Capital Cost – Resale value

j) Find the Total Cost = Cumulative Running cost + Depreciation

k) Calculate annuity factor (Cumulative present value factor at the beginning)

l) Find the Average cost = Total cost / Annuity

m) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

Year n	R_n	Pv^{n-1}	R_nPv^{n-1}	□R_nPv^{n-1}	C	S_n	Pv^n	S_nPv^n	C - S_nPv^n	□R_nv^{n-1} + C - S_nPv^n	□PV^{n-1}	W_n
1	2	3	4(2*3)	5	6	7	8	9(7*8)	10	11(5+10)	12	13

ITEMS THAT FAIL COMPLETELY AND SUDDENLY

There is another type of problem where we consider the items that fail completely. The item fails such that the loss is sudden and complete. Common examples are the electric bulbs, transistors and replacement of items, which follow sudden failure mechanism.

I. INDIVIDUAL REPLACEMENT POLICY(IRP):

Under this strategy equipments or facilities break down at various times. Each breakdown canbe remedied as it occurs by replacement or repair of the faulty unit.

Examples: Vacuum tubes, transistors

No. of failures = $\frac{\text{Total no. of items}}{\text{Average life of an item}}$

Total IRP Cost = No. of failures * IRP cost

II. GROUP REPLACEMENT

As per this strategy, an optimal group replacement period '*P*' is determined and common preventive replacement is carried out as follows.

(a) Replacement an item if it fails before the optimum period '*P*'.

(b) Replace all the items every optimum period of '*P*' irrespective of the life of individual item. Examples: Bulbs, Tubes, and Switches.

Among the three strategies that may be adopted, the third one namely the group replacement policy turns out to be economical if items are supplied cheap when purchased in bulk quantities. With this policy, all items are replaced at certain fixed intervals.

Procedure for Group Replacement Policy (GRP):

1. Write down the weeks
2. Write down the individual probability of failure during that week
3. Calculate No. of failures:

   N_0 - No. of items at the beginning

   N_1 - No. of failure during 1st week (N_0P_1)

   N_2 - No. of failure during 2nd week ($N_0P_2 + N_1P_1$)

   N_3 - No. of failure during 3rd week ($N_0P_3 + N_1P_2 + N_2P_1$)
4. Calculate cumulative failures
5. Calculate IRP Cost = Cumulative no. of failures * IRP cost
6. Calculate and write down GRP Cost = Total items * GRP Cost
7. Calculate Total Cost = IRP Cost + GRP Cost Calculate

Average cost = Total cost / no. of corresponding year

The replacement theory provides answer to this question in terms of optimal replacement period. Replacement theory deals with the analysis of materials and machines which deteriorate with time and fix the optimal time of their replacement so that total cost is the minimum.

Types of Replacement Problems

i) Replacement policy for items, efficiency of which declines gradually with time without change in money value.

ii) Replacement policy for items, efficiency of which declines gradually with time but with change in money value.

iii) Replacement policy of items breaking down suddenly

  a) Individual replacement policy

  b) Group replacement policy

iv) Staff replacement

Example 1 A milk plant is considering replacement of a machine whose cost price is Rs. 12,200 and the scrap value Rs. 200. The running (maintenance and operating) costs in Rs. are found from experience to be as follows:

Year:	1	2	3	4	5	6	7	8
Running Cost:	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced?

Solution The computations can be summarized in the following tabular form:

Calculations for average cost of machine

	(In Rupees)				
Year (t) (1)	**Running Cost** $f(t)$ (2)	**Cumulative Running Cost** $\Sigma f(t)$ (3)	**Depreciation Cost** $(C - S)$ (4)	**Total Cost TC** (5) = (3) + (4)	**Average Cost** $A(n)$ (6) = (5)/(1)
1	200	200	12000	12200	12200
2	500	700	12000	12700	6350
3	800	1500	12000	13500	4500

4	1200	2700	12000	14700	3675
5	1800	4500	12000	16500	3300
6	2500	7000	12000	19000	3167
7	3200	10200	12000	22200	3171
8	4000	14200	12000	26200	3275

From the table it is noted that the average total cost per year, A(n) is minimum in the 6th year (Rs. 3167). Also the average cost in 7th year (Rs.3171) is more than the cost in 6th year. Hence the machine should be replaced after every 6 years.

Example 2

A Machine owner finds from his past records that the maintenance costs per year of a machine whose purchase price is Rs. 8000 are as given below:

Year:	1	2	3	4	5	6	7	8
Maintenance Cost:	1000	1300	1700	2200	2900	3800	4800	6000
Resale Price:	4000	2000	1200	600	500	400	400	400

Determine at which time it is profitable to replace the machine.

Solution C = Rs. 8000. Table 13.2 shows the average cost per year during the life of machine. Here, The computations can be summarized in the following tabular form:

Table Calculations for average cost of machine

Year (t)	**f(t)**	**Cumulative maintenance cost** $\sum \mathbf{f(t)}$	Scrap value $s(t)$	Total cost $TC = c - s(t) + \sum \mathbf{f(t)}$	T_A
1	1000	1000	4000	5000	5000
2	1300	2300	2000	8300	4150
3	1700	4000	1200	10800	3600
4	2200	6200	600	13600	3400
5	2900	9100	500	16600	$\boxed{3200}$

6	3800	12900	400	20500	3417
7	4800	17700	400	25300	3614
8	6000	23700	400	31300	3913

The above table shows that the value of T_A during fifth year is minimum. Hence the machine should be replaced after every fifth year.

Example 3

The cost of a machine is Rs. 6100 and its scrap value is only Rs.100. The maintenance costs are found to be

Year:	1	2	3	4	5	6	7	8
Maintenance Cost (in Rs.):	100	250	400	600	900	1250	1600	2000

When should the Machine be replaced?

Solution

C = 6100, s(t) = 100 The computations can be summarized in the following tabular form:

Table Calculations for average cost of machine

Replace at the end of year	f(t)	**Cumulative maintenance cost** $\sum \mathbf{f(t)}$	Total cost $TC = C - s(t) + \sum \mathbf{f(t)}$	T_A
1	100	100	6100	6100
2	250	350	6350	3175
3	400	750	6750	2250
4	600	1350	7350	1737.50
5	900	2250	8250	1650
6	1250	3500	9500	$\boxed{1583.33}$
7	1600	5100	11100	1585.7
8	2000	7100	13100	1637.50

It is now observed that the machine should be replaced at the end of sixth year otherwise the average cost per year will start to increase.

Replacement of items whose maintenance cost increases with time and the money value changes at a constant rate

To understand this let us define the following terms:

Money Value

Since money has a value over time, therefore the explanation of the statement: Money is worth 10% per year can be given in two ways:

(a) In one way, spending Rs.100 today would be equivalent to spend Rs.110 in years time. In other words if we plan to spend Rs.110 after a year from now, we could spend Rs.100 today and an investment which would be worth Rs.110 next year.

(b) Alternatively if we borrow Rs.100 at the interest of 10% per year and spend Rs.100 today, we have to pay Rs.100 after one year (next year).

Thus, we conclude that Rs.100 is equal to Rs.110 a year from now. Consequently Rs. 1 from a year now is equal to $(1+0.1)^{-1}$ rupee today.

Present Worth Factor

As we have seen, a rupee a year from now will be equivalent to $(1+0.1)^{-1}$ rupee today at the discount rate of 10% per year. So, one rupee in **n** years from now will be equal to $(1+0.1)^{-n}$. Therefore, the quantity $(1+0.1)^{-n}$ is called the Present Worth Factor (PWF) or Present Value (PV) of one rupee spent in **n** years from now. In general, if **r** is the rate of interest, then $(1+r)^{-n}$ is called PWF or PV of one rupee spent in **n** years from now onwards. The expression $(1+r)^{-n}$ is known as compound amount factor of one rupee spent in **n** years.

Discount Rate

Let **r** be the rate of interest. Therefore present worth factor of unit amount to be spent after one year is $v = \frac{1}{1+r}$.

Then **v** is known as the discount rate. The optimum replacement policy for replacement of item where maintenance costs increase with time and money value changes with constant rate can be determined by following method:

Suppose that the item (which may be machine or equipment) is available for use over a series of time periods of equal intervals (say one year). Let

C = Purchase price of the item to be replaceds

R_t = Running (or maintenance) cost in the **t^{th}** year

R = Rate of interest

$v = \frac{1}{1+r}$ is the present worth of a rupee to be spent in a year hence.

Let the item be replaced at the end of every **n^{th}** year. The year wise present worth of expenditure on the item in the successive cycles of **n** years can be calculated as follows:

Year	1	2----	n	n+1	n+2	----	2n	2n+1
Present worth	$C+R_1$	R_2v	R_nv^{n-1}	$(C+R_1)v^n$	R_2v^{n+1}		R_nv^{2n-1}	$(C+R_1)v^{2n}$

Assuming that machines has no resale value at the time of replacement, the present worth of the machine in **n** years will be given by

$$p(n) = [(C + R_1) + R_2v + R_3v^2 + - - - + R_nv^{n-1}] + [(C + R_1)v^n + R_2v^{n+1} + R_3v^{n+2} + - - - + R_nv^{2n-1}] + [(C + R_1)v^{2n} + R_2v^{2n+1} + R_3v^{2n+2} + - - - + R_nv^{3n-1}] + - - \text{and so on}$$

Summing up the right-hand side, column-wise

$$P(n) = (C + R_1)\lfloor 1 + v^n + v^{2n} + --\rfloor + R_2 v\lfloor 1 + v^n + v^{2n} + --\rfloor + --$$
$$+R_n v^{n-1}\lfloor 1 + v^n + v^{2n} + --\rfloor$$

$$= (C + R_1 + R_2 v + --- + R_n v^{n-1})\lfloor 1 + v^n + v^{2n} + --\rfloor$$

$$= (C + R_1 + R_2 v + --- + R_n v^{n-1})\left\lfloor \frac{1}{1-v^n} \right\rfloor,$$

using sum of an infinite G.P.

$$P(n) = \left[\frac{f(n)}{1-v^n}\right] \text{ where } f(n) = (C + R_1 + R_2 v + --- + R_n v^{n-1}),$$

$$(n+1) = \left[\frac{f(n+1)}{1-v^{n+1}}\right]$$

f(n) and f(n+1) given above at n = 0,1,2, are called the weighted average cost of previous **n** years with weights 1,v, v^2, ----v^{n-1}respectively. P (n) is the amount of money required now to pay all the future costs of acquiring and operating the equipment when it is renewed every **n** years. However, if P (n) is less than P (n+1) then replacing the equipment each **n** year is preferable to replacing each **n** years is preferable to replacing each (n+1) years. Further, if the best policy is replacing every **n** years, then the two inequalities P (n+1) > P (n) > 0 and P (n-1) - P (n) < 0 must hold, without giving the proof we shall state the following two inequalities which holds good at **n**, the optimal replacement interval.

$$P(n) < \frac{(C+R_1)+R_2 v+R_3 v^2+\cdots\ldots\ldots+R_{n-1}v^{n-2}}{1+v+v^2+\cdots+v^{n-2}}$$

$$\text{and } P(n+1) > \frac{(C+R_1)+R_2 v+\cdots\ldots\ldots+R_n v^{n-1}}{1+v+v^2+\cdots+v^{n-1}}$$

As a result of these two inequalities, rules for minimizing costs may be stated as follows:

1. Do not replace if the operating cost of next period is less than the weighted average of previous costs.

2. Replace if the operating cost of the next period is greater than the weighted average of the previous costs.

Working Procedure

The step-by-step procedure for solving the problem is stated as under:

1. Write in a column the running/maintenance costs of machine or equipment for different years, R_n.

2. In the next column write the discount factor indicating the present value of a rupee received after (i-1) years, $v^{n-1} = (1/1+r)^{n-1}$

3. The two column values are multiplied to get present value of the maintenance costs, i.e ., $R_n v^{n-1}$.

4. These discounted maintenance costs are then cumulated to the i^{th} year to get $\sum R_n v^{n-1}$.

5. The cost of machine or equipment is added to the values obtained in Step 4 above to

Obtain C+ $\sum R_n v^{n-1}$.

6. The discount factors are then cumulated to get $\sum \mathrm{v}^{n-1}$.

7. The total costs obtained in (Step 5) are divided by the corresponding value of the accumulated discount factor for each of the years.

8. Now compare the column of maintenance costs which is constantly increasing with the last column. Replace the machine in the latest year that the last column exceeds the column of maintenance costs.

Example 4

A milk plant is offered an equipment A which is priced at Rs.60,000 and the costs of operation and maintenance are estimated to be Rs.10,000 for each of the first 5 years, increasing every year by Rs. 3000 per year in the sixth and subsequent years. If money carries the rate of interest 10% per annum what would the optimal replacement period?

Solution

Table Determination of optimal replacement period

At the end of year **(n)**	**Operating & maintenance cost** **R_n**	**Discounted factor** v^{n-1}	**Discounted operation & maintenance cost** $R_n v^{n-1}$	**Cumulative** **Discounted operation & maintenance cost**	**Discounted total cost** $C + \sum R^{n-1}$	**Cumulative discounted factor** $\sum v^{n-1}$	**Weighted average annual cost** $\frac{C + R_n v^{n-1}}{\sum v^{n-1}}$
(1)	(2)	(3)	(4)=(2)x(3)	(5)	(6)=(5)+60000	(7)	(8)=(6)+(7)
1	10000	1.0000	10000.00	10000.00	70000.00	1.00	70000.00
2	10000	0.9091	9091.00	19091.00	79091.00	1.91	41428.42
3	10000	0.8264	8264.00	27355.00	87355.00	2.74	31933.83
4	10000	0.7513	7513.00	34868.00	94868.00	3.49	27207.75
5	10000	0.6830	6830.00	41698.00	101698.00	4.17	24389.18
6	13000	0.6209	8071.70	49769.70	109769.70	4.79	22913.08
7	16000	0.5645	9032.00	58801.70	118801.70	5.36	22184.36
8	19000	0.5132	9750.80	68552.50	128552.50	5.87	21905.89
9	22000	0.4665	10263.00	78815.50	138815.50	6.33	21912.82
10	25000	0.4241	10602.50	89418.00	149418.00	6.76	22106.52

From Table we find the weighted cost is minimum at the end of 8th year, hence the equipment should be replaced at the end of 8th year.

Example 5

A Manufacturer is offered two machines A and B. Machine A is priced at Rs. 5000 and running cost is estimated at Rs. 800 for each of the first five years, increasing by Rs. 200 per year in the sixth and subsequent years. Machine B, with the same capacity as A, costs Rs. 2500, but has running cost of Rs. 1200 per year for six years, thereafter increasing by Rs. 200 per year. If money is worth 10% per year, which machine should be purchased? (Assume that the machines will eventually be sold for scrap at a negligible price).

Solution

Since money is worth 10% per year, therefore discount rate is $v = \frac{1}{(1+0.10)} = 0.9091$

Computation of weighted average cost for machine A

At the end of year **(n)**	**Operating & maintenance cost** **R_n**	**Discounted factor** v^{n-1}	**Discounted operation & maintenance cost** $R_n v^{n-1}$	**Cumulative** **Discounted operation & maintenance cost**	**Discounted total cost** $C + \sum R^{n-1}$	**Cumulative discounted factor** $\sum v^{n-1}$	**Weighted average annual cost** $\frac{C + R_n v^{n-1}}{\sum v^{n-1}}$
(1)	(2)	(3)	(4)=(2)x(3)	(5)	(6)=(5)+ 5000	(7)	(8)=(6)+(7)
1	800	1.0000	800	800	5800	1	5800
2	800	0.9091	727	1527	6527	1.9091	3419.035
3	800	0.8264	661	2188	7188	2.7355	2627.819
4	800	0.7513	601	2789	7789	3.4868	2233.98
5	800	0.6830	546	3336	8336	4.1698	1999.098
6	1000	0.6209	621	3957	8957	4.7907	1869.61
7	1200	0.5645	677	4634	9634	5.3552	1799.025
8	1400	0.5132	718	5353	10353	5.8684	1764.13
9	1600	0.4665	746	6099	11099	6.3349	1752.043
10	1800	0.4241	763	6862	11862	6.759	1755.053

From table 13.5 we conclude that for machine A 1600<1752.043<1800. Since the running cost of 9th year is 1600and that of 10th year is 1800 and 1800>1752.043, it would be economical to replace machine A at the end of nine years.

Table Computation of weighted average cost for machine B

At the end of year (n)	**Operating & maintenance cost R_n**	**Discounted factor v^{n-1}**	**Discounted operation & maintenance cost $R_n v^{n-1}$**	**Cumulative Discounted operation & maintenance cost**	**Discounted total cost $C + \sum R^{n-1}$**	**Cumulative discounted factor $\sum v^{n-1}$**	**Weighted average annual cost $\frac{C + R_n v^{n-1}}{\sum v^{n-1}}$**
(1)	(2)	(3)	(4)=(2)x(3)	(5)	(6)=(5)+ 2500	(7)	(8)=(6)+(7)
1	1200	1.0000	1200.00	1200.00	3700.00	1.00	3700.00
2	1200	0.9091	1090.92	2290.92	4790.92	1.91	2509.52
3	1200	0.8264	991.68	3282.60	5782.60	2.74	2113.91
4	1200	0.7513	901.56	4184.16	6684.16	3.49	1916.99
5	1200	0.6830	819.60	5003.76	7503.76	4.17	1799.55
6	1200	0.6209	745.08	5748.84	8248.84	4.79	1721.84
7	1400	0.5645	790.30	6539.14	9039.14	5.36	1687.92
8	1600	0.5132	821.12	7360.26	9860.26	5.87	1680.23
9	1800	0.4665	839.70	8199.96	10699.96	6.33	1689.05
10	2000	0.4241	848.20	9048.16	11548.16	6.76	1708.56

In table13.6 we find that 1800<1689<2300 so it is better to replace the machine B after 8th year. The equivalent yearly average discounted value of future costs is Rs. 1748.60 for machine A and it is 1680.23for machine B. Hence, it is more economical to buy machine B rather than machine A.

STEP 4:

- Identify the **entering variable.**

The highest **positive** evaluated value(dij) cell is treated as entering variable cell.

STEP 5:

- Identify leaving variable.
    - To identify the leaving variable, construct a closed loop.
    - Loop starts at the entering variable cell.
    - Loop can go clockwise or anticlockwise.
    - The turning point should be occupied cell.
    - Loop can cross each other.

STEP 6:

- Start assigning positive & negative .
- Assign positive (+) for the entering variable & negative (-) alternatively.
- Where is the **minimum allocation quantity** among the negative cells.
- Add to the allocated value in the positive cells and deduct to the allocated value in the negative cells.
- Cell/cells which have zero allocation (after deducting) is the leaving variable.

STEP 7:

- Prepare a new transportation table.
- The values in the loop will get changed as per the step 6 and all other allocations not in the loop remains the same.

STEP 8:

- Check for optimality using step 1 to step 3
    - If the solution is optimal calculate the minimum transportation cost from the allocations and the unit costs given.
    - Repeat the procedure from step 4 to step 8, if the solution is not optimal.

1. Give the meaning for transportation.
2. Define transportation model.

3. Specify the objective of transportation model.
4. Write the steps involved in North-west corner method.
5. List the steps involved in Least-Cost method.
5. What is meant by unbalanced problem in transportation?
6. How will you convert unbalanced problem into balanced problem in transportation?
7. What are the methods used to find initial solution in transportation?
8. What is degeneracy in transportation?
9. Find the initial solution for the given transportation problem using North-West corner method.

Destination

		D1	D2	D3	D4	Supply
	O1	6	1	9	3	100
Origins	O2	11	5	2	8	60
	O3	10	12	4	7	40
Demand		40	60	80	20	

10. Find the initial solution for the given transportation problem using Least-Cost method. Destination

		D1	D2	D3	I 4	Supply
	O1	2	4	6	2	100
Origins	O2	8	6	5	2	60

O3	9	10	7	5	40
Demand	40	60	80	20	

EXTRA QUESTIONS :

1. What is meant by transportation? Specify the objectives of transportation tool. Write the procedure for making unbalanced problem into balanced problem with an example.

OR

2. Solve the transportation problem using MODI method and find optimal solution. Destination

Sources	D1	D2	D3	D4	Supply
S1	7	3	8	6	60
S2	4	2	5	10	100
S3	2	6	5	1	40
Demand	20	50	50	80	

3. Write the procedure to solve transportation problem using MODI method.

OR

4. Find the optimal solution for the given transportation problem using MODI method. Destination

Sources	D1	D2	D3	D4	Supply
S1	6	1	9	3	70
S2	11	5	2	8	55
S3	10	12	4	7	70
Demand	85	35	50	45	

5. Write the procedure for a) North-West corner method b) Least-Cost c) method

Vogel's approximation method.

OR

6. For the given transportation problem, find the optimal solution using MODI method.

Destination

Sources	D1	D2	D3	Supply
S1	16	20	12	50
S2	14	8	18	50
S3	26	24	16	50
Demand	50	50	50	

7. Using U-V method, solve the given transportation problem. Destination

Sources	D1	D2	D3	Supply
S1	5	1	7	10
S2	6	4	6	80
S3	3	2	5	15
S4	5	3	2	40
Demand	75	20	50	

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8. Find the initial solution using VAM and optimal solution using MODI method for the below transportation problem.

Destination

Sources D1 D2 D3 D4 Supply

S1	13	25	12	21	18
S2	18	23	14	9	27
S3	23	15	12	16	21
Demand	14	12	23	27	

I. Introduction

(Title of chapter -- Times New Roman- Bold – Font Size 14 andCentred)

Contents - Course content begins here as per the styling given……………………….

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Insert page number at the bottom of every page.

Mention Figure number, Table Number (If any)

Margins – ONE INCH (Left, Right, Top and Bottom)