



**SATHYABAMA**

INSTITUTE OF SCIENCE AND TECHNOLOGY  
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**SCHOOL OF MECHANICAL ENGINEERING**

**DEPARTMENT OF AUTOMOBILE ENGINEERING**

**SAUA1501 AUTOMOTIVE ENGINE COMPONENT DESIGN**

## **UNIT I INTRODUCTION-SAUA1501**

## **UNIT I INTRODUCTION**

### **Introduction**

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. In this chapter, we shall discuss the commonly used engineering materials and their properties in Machine Design.

### **Classification of Engineering Materials**

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminium, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

- (a) Ferrous metals, and
- (b) Non-ferrous metals.

The ferrous metals are those which have the iron as their main constituent, such as cast iron, wrought iron and steel. The non-ferrous metals are those which have a metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc, etc

### **Selection of Materials for Engineering Purposes**

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serve the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

### **Mechanical Properties of Metals**

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include

strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. **Strength.** It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.
2. **Stiffness.** It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.
3. **Elasticity.** It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.
4. **Plasticity.** It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in Ornamental work.
5. **Ductility.** It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.
6. **Brittleness.** It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads, snap off without giving any sensible elongation. Cast iron is a brittle material.
7. **Malleability.** It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.
8. **Toughness.** It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed up to the point of fracture. This property is desirable in parts subjected to shock and impact loads.
9. **Machinability.** It is the property of a material which refers to a relative ease with which a material can be cut. The Machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. **Resilience.** It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. **Creep.** When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.

### **Completely Reversed or Cyclic Stresses**

Consider a rotating beam of circular cross-section and carrying a load  $W$ , This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (i.e. at point A) are under compressive stress and the lower fibres (i.e. at point B) are under tensile stress. After half a revolution, the point B occupies the position of point A and the point A occupies the position of point B. Thus the point B is now under compressive stress and the point A under tensile stress. The speed of variation of these stresses depends upon the speed of the beam. From above it states that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or vice versa, are known as completely reversed or cyclic stresses.

#### **Notes:**

1. The stresses which vary from a minimum value to a maximum value of the same nature, (i.e. tensile or compressive) are called fluctuating stresses.
2. The stresses which vary from zero to a certain maximum value are called repeated stresses.
3. The stresses which vary from a minimum value to a maximum value of the opposite nature (i.e. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called alternating stresses.

### **Fatigue and Endurance Limit**

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

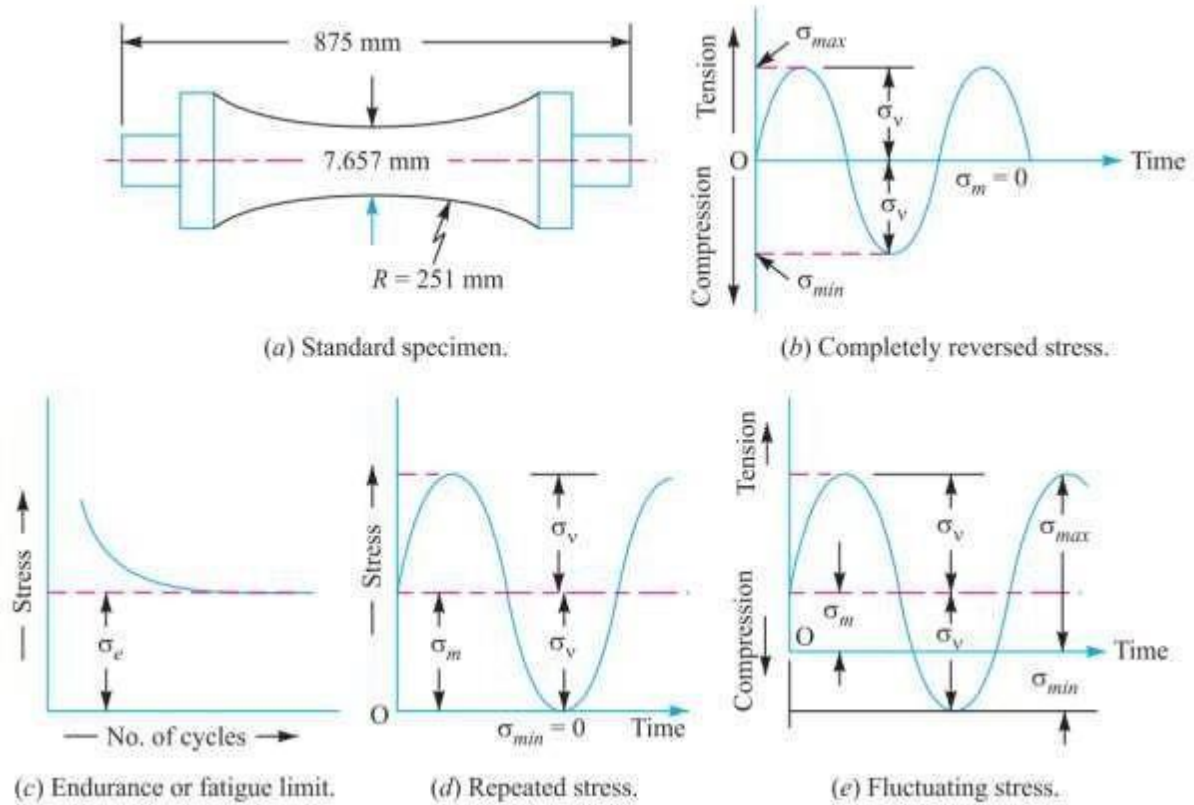


Figure.1.1: Fatigue and Endurance Limit

### Effect of Loading on Endurance Limit—Load Factor

The endurance limit ( $e$ ) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads.

Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

- Let  $K_b$  = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.
- $K_a$  = Load correction factor for the reversed axial load. Its value may be taken as 0.8.
- $K_s$  = Load correction factor for the reversed torsional or shear load.

Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

### Important Terms used in Limit System

The following terms used in limit system (or interchangeable system) is important from the subject point of view:

1. **Nominal size.** It is the size of a part specified in the drawing as a matter of convenience.
2. **Basic size.** It is the size of a part to which all limits of variation (i.e. tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.
3. **Actual size.** It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit, otherwise it will interfere with the interchange ability of the mating parts.
4. **Limits of sizes.** There are two extreme permissible sizes for a dimension of the part as shown in Fig. The largest permissible size for a dimension of the part is called upper or high or maximum limit, whereas the smallest size of the part is known as lower or minimum limit.
5. **Allowance.** It is the difference between the basic dimensions of the mating parts. The allowance may be positive or negative. When the shaft size is less than the hole size, then the allowance is positive and when the shaft size is greater than the hole size, then the allowance is negative.
6. **Tolerance.** It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be unilateral or bilateral. When all the tolerance is allowed on one side of the nominal size, e.g.  $0.000\ 20-0.004$ , then it is said to be unilateral system of tolerance. The unilateral system is mostly used in industries as it permits changing the tolerance value while still retaining the same allowance or type of fit. When the tolerance is allowed on both sides of the nominal size, e.g.  $20^{+0.002}$  then it is said to be bilateral system of tolerance. In this case  $+0.002$  is the upper limit and  $-0.002$  is the lower limit. The method of assigning unilateral and bilateral tolerance is shown in Fig. (a) and (b) respectively.
7. **Tolerance zone.** It is the zone between the maximum and minimum limit size Tolerance zone.
8. **Zero line.** It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.
9. **Upper deviation.** It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol ES (Ecart Superior) and of a shaft, it is represented by es.

10. **Lower deviation.** It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol  $EI$  (Ecart Inferior) and of a shaft, it is represented by  $ei$ .

11. **Actual deviation.** It is the algebraic difference between an actual size and the corresponding basic size.

12. **Mean deviation.** It is the arithmetical mean between the upper and lower deviations.

13. **Fundamental deviation.** It is one of the two deviations which is conventionally chosen to define the position of the tolerance zone in relation to zero line, as shown in fig

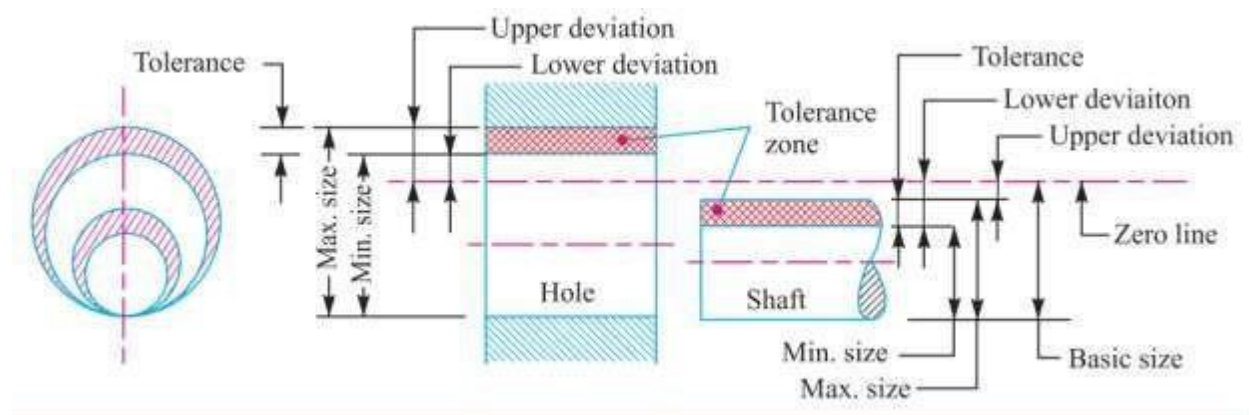


Figure.1.2: Limits System

## NOTCH SENSITIVITY FACTOR

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term notch sensitivity is applied to this behavior. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

When the notch sensitivity factor  $q$  is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:



$$q = \frac{K_f - 1}{K_t - 1}$$

$$K_f = 1 + q (K_t - 1)$$

$$K_{fs} = 1 + q (K_{ts} - 1)$$

$K_t$  = Theoretical stress concentration factor for axial or bending loading, and

$K_{ts}$  = Theoretical stress concentration factor for torsional or shear loading

## FACTOR OF SAFETY

The ratio of the maximum stress to the working stress. In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress.

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore the factor of safety for brittle materials is based on ultimate stress.

## SELECTION OF FACTOR OF SAFETY

The selection of a proper factor of safety to be used in designing any machine components depends upon a number of considerations

- Materials
- Mode of manufacturing
- Type of stress
- General Service condition
- Shape of parts

Selecting a proper factor of safety, a design engineer should consider the following points,

1. Reliability of the properties of the material and change of these properties during service.
2. Reliability of test results and accuracy of application of these results to actual machine parts,
3. Reliability of applied load,

4. To exact mode of failure
5. Extent of simplifying assumptions,
6. Extent of initial stresses set up during manufacture,
7. Extent of loss of life if failure occurs and,
8. Extent of loss of property if failure occurs,

## Fits

The degree of tightness or looseness between the two mating parts is known as a fit of the parts. The nature of fit is characterized by the presence and size of clearance and interference. The clearance is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly. In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be positive.

The interference is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly. In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be negative.

## Types of Fits

According to Indian standards, the fits are classified into the following three groups:

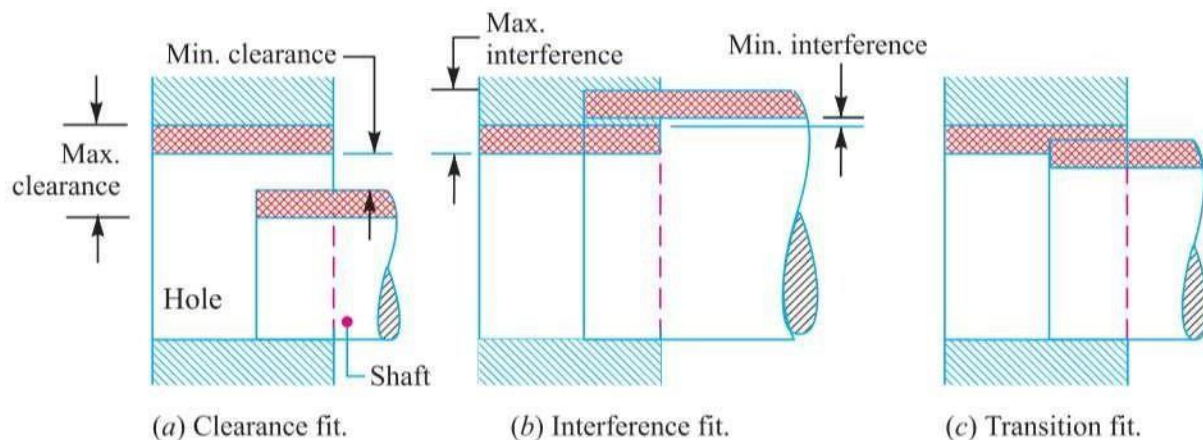


Figure.1.3: Types of Fits

1. **Clearance fit.** In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, It may be noted that in a clearance fit, the tolerance zone of the hole

is entirely above the tolerance zone of the shaft. In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as minimum clearance whereas the difference between the maximum size of the hole and minimum size of the shaft is called maximum clearance

2. **Interference fit.** In this type of fit, the size limits for the mating parts are so selected that interference between them always occur. It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft. In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as minimum interference, whereas the difference between the minimum size of the hole and the maximum size of the shaft is called maximum interference. The interference fits may be shrink fit, heavy drive fit and light drive fit.

3. **Transition fit.** In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts. It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap. The transition fits may be force fit, tight fit and push fit.

## **UNIT II CYLINDER AND PISTON**

## 2. DESIGN OF PISTON AND CYLINDER

### PRINCIPAL PARTS OF AN ENGINE

The principal parts of an IC engine, as shown in fig below are as follows.

Cylinder and cylinder liner, Piston, piston rings and piston pin or gudgeon pin Connecting rod with small and big end bearing. Crank, crank shaft and crank pin, and Valve gear mechanism. The design of the above mentioned principal parts are discussed, below

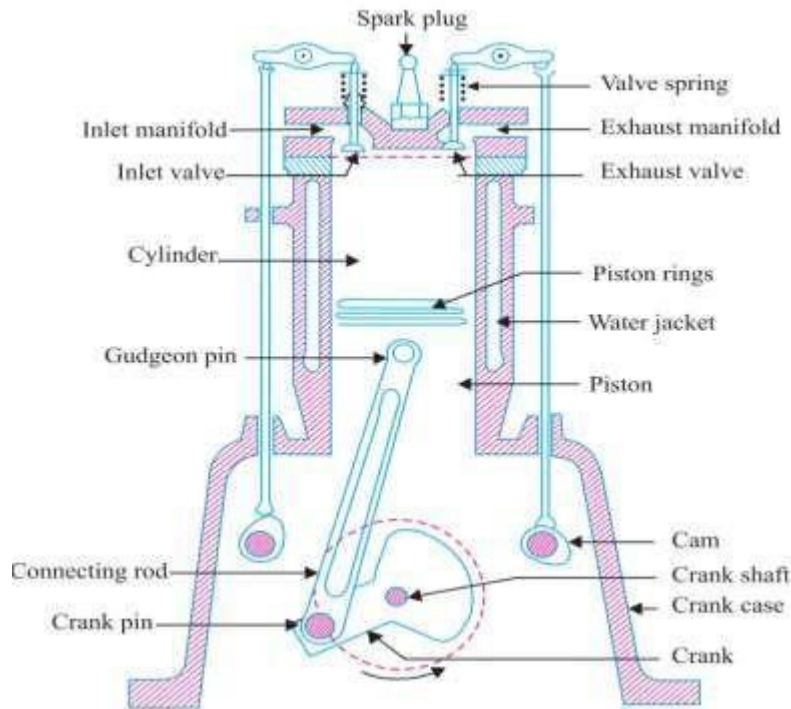


Figure.2.1: Engine Parts

### CYLINDER AND CYLINDER LINER

The function of a cylinder is to retain the working fluid and to guide the piston. The cylinders are usually made of cast iron or cast steel. Since the cylinder has to withstand high temperature due to the combustion of fuel, therefore, some arrangement must be provided to cool the cylinder. The single cylinder engines (such as scooters and motorcycles) are generally air cooled. They are provided with fins around the cylinder. The multi-cylinder engines (such as of cars) are provided with water jackets around the cylinders to cool it. In smaller engines. The cylinder, water jacket and the frame are made as one piece, but for all the larger engines, these parts are manufactured separately. The cylinders are provided with cylinder liners so that in case of wear, they can be easily replaced.

**The cylinder liners are of the following two types:**

1. Dry liner, and
2. Wet liner

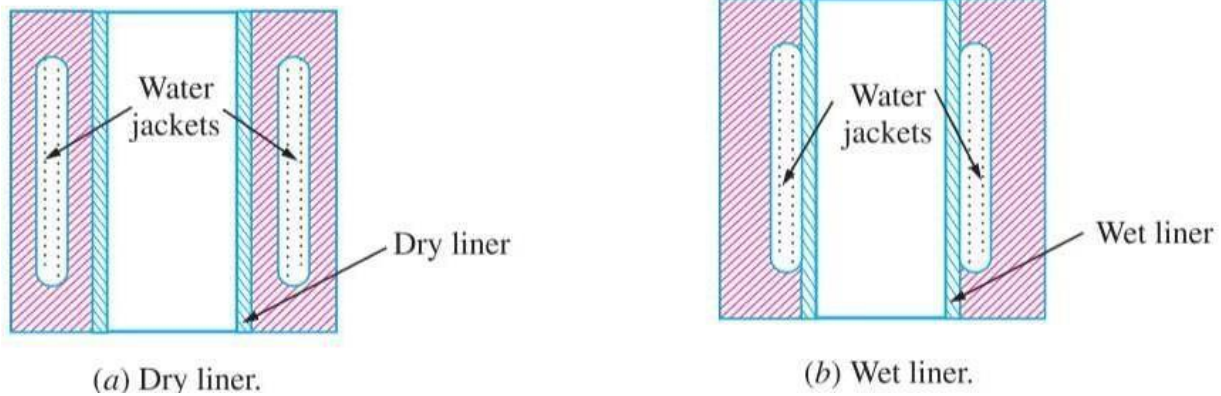


Figure.2.2: Types of Liners

A cylinder liner which does not have any direct contact with the engine cooling water, is known as dry liner, as shown in Fig. (a). A cylinder liner which have its outer surface in direct contact with the engine cooling water, is known as wet liner, as shown in Fig. (b).

The cylinder liners are made from good quality close grained cast iron (i.e. pearlitic cast iron), nickel cast iron, nickel chromium cast iron. In some cases, nickel chromium cast steel with molybdenum may be used. The inner surface of the liner should be properly heat-treated in order to obtain a hard surface to reduce wear.

### **Design of a Cylinder**

In designing a cylinder for an I. C. engine, it is required to determine the following values:

1. **Thickness of the cylinder wall.** The cylinder wall is subjected to gas pressure and the piston side thrust. The gas pressure produces the following two types of stresses:

- (a) Longitudinal stress, and
- (b) Circumferential stress

Since these two stresses act at right angles to each other, therefore, the net stress in each direction is reduced. The piston side thrust tends to bend the cylinder wall, but the stress in the wall due to side thrust is very small and hence it may be neglected

Let  $D_0$  = Outside diameter of the cylinder in mm

$D$  = Inside diameter of the cylinder in mm

p= Maximum pressure inside the engine cylinder in N/mm<sup>2</sup>

t = Thickness of the cylinder wall in mm,

1/m = Poisson's ratio. It is usually taken as 0.25.

The apparent longitudinal stress is given by

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$$\sigma_l = \frac{\text{Force}}{\text{Area}} = \frac{\frac{\pi}{4} \times D^2 \times p}{\frac{\pi}{4} [(D_0)^2 - D^2]} = \frac{D^2 \cdot p}{(D_0)^2 - D^2}$$

and the apparent circumferential stress is given by

$$\sigma_c = \frac{\text{Force}}{\text{Area}} = \frac{D \times l \times p}{2t \times l} = \frac{D \times p}{2t}$$

... (where  $l$  is the length of the cylinder and area is the projected area)

$$\therefore \text{Net longitudinal stress} = \sigma_l - \frac{\sigma_c}{m}$$

$$\text{and net circumferential stress} = \sigma_c - \frac{\sigma_l}{m}$$

The thickness of a cylinder wall ( $t$ ) is usually obtained by using a thin cylindrical formula, *i.e.*,

$$t = \frac{p \times D}{2\sigma_c} + C$$

p = Maximum pressure inside the cylinder in N/mm<sup>2</sup>,

D = Inside diameter of the cylinder or cylinder bore in mm,

$\sigma_c$  = Permissible circumferential or hoop stress for the cylinder material in MPa or N/mm<sup>2</sup>. Its value may be taken from 35 MPa to 100 MPa depending upon the size and material of the cylinder.

C = Allowance for reboring. The allowance for reboring (C) depending upon the cylinder bore (D) for I. C. engines is given in the following table:

Table.2.1: Allowance for Reboring

D (mm)	75	100	150	200	250	300	350	400	450	500
C (mm)	1.5	2.4	4.0	6.3	8.0	9.5	11.0	12.5	12.5	12.5

The thickness of the cylinder wall usually varies from 4.5 mm to 25 mm or more depending upon the size of the cylinder. The thickness of the cylinder wall ( $t$ ) may also be obtained from the following empirical relation, *i.e.*

$$t = 0.045 D + 1.6 \text{ mm}$$

The other empirical relations are as follows: Thickness of the dry liner = 0.03 D to 0.035 D

Thickness of the water jacket wall

$$= 0.032 D + 1.6 \text{ mm or } t / 3 \text{ m for bigger cylinders and } 3t / 4 \text{ for smaller cylinders}$$

Water space between the outer cylinder wall and inner jacket wall

$$= 10 \text{ mm for a 75 mm cylinder to 75 mm for a 750 mm cylinder or } 0.08 D + 6.5 \text{ mm}$$

**2. Bore and length of the cylinder.** The bore (i.e. inner diameter) and length of the cylinder may be determined as discussed below:

Let  $p_m$  = Indicated mean effective pressure in  $\text{N/mm}^2$ ,

$D$  = Cylinder bore in mm,

$A$  = Cross-sectional area of the cylinder in  $\text{mm}^2$ ,

$$= \pi D^2 / 4$$

$l$  = Length of stroke in meters,

$N$  = Speed of the engine in r.p.m., and

$n$  = Number of working strokes per min

=  $N$ , for two stroke engine

=  $N/2$ , for four stroke engine.

We know that the power produced inside the engine cylinder, i.e. indicated power,

$$I.P. = \frac{p_m \times l \times A \times n}{60} \text{ watts}$$

From this expression, the bore ( $D$ ) and length of stroke ( $l$ ) is determined. The length of stroke is generally taken as 1.25  $D$  to 2 $D$ .

Since there is a clearance on both sides of the cylinder, therefore length of the cylinder is taken as 15 percent greater than the length of stroke. In other words,

$$\text{Length of the cylinder, } L = 1.15 \times \text{Length of stroke} = 1.15 l$$

**3. Cylinder flange and studs.**



The cylinders are cast integral with the upper half of the crankcase or they are attached to the crankcase by means of a flange with studs or bolts and nuts. The cylinder flange is integral with the cylinder and should be made thicker than the cylinder wall. The flange thickness should be taken as 1.2 t to 1.4 t, where t is the thickness of cylinder wall.

The diameter of the studs or bolts may be obtained by equating the gas load due to the maximum pressure in the cylinder to the resisting force offered by all the studs or bolts. Mathematically,

$$\frac{\pi}{4} \times D^2 \cdot p = n_s \times \frac{\pi}{4} (d_c)^2 \sigma_t$$

D = Cylinder bore in mm,

p = Maximum pressure in N/mm<sup>2</sup>,

n<sub>s</sub> = Number of studs. It may be taken as 0.01 D + 4 to 0.02 D + 4

d<sub>c</sub> = Core or minor diameter, i.e. diameter at the root of the thread in mm,

σ<sub>t</sub> = Allowable tensile stress for the material of studs or bolts in MPa or N/mm<sup>2</sup>. It may be taken as 35 to 70 MPa. The nominal or major diameter of the stud or bolt (d) usually lies between 0.75 t<sub>f</sub> to t<sub>f</sub>, where t<sub>f</sub> is the thickness of flange. In no case, a stud or bolt less than 16 mm diameter should be used.

The distance of the flange from the centre of the hole for the stud or bolt should not be less than d + 6 mm and not more than 1.5 d, where d is the nominal diameter of the stud or bolt. In order to make a leak proof joint, the pitch of the studs or bolts should lie between 19 d to 28.5 d, where d is in mm.

4. Cylinder head. Usually, a separate cylinder head or cover is provided with most of the engines. It is, usually, made of box type section of considerable depth to accommodate ports for air and gas passages, inlet valve, exhaust valve and spark plug (in case of petrol engines) or atomizer at the centre of the cover (in case of diesel engines).

The cylinder head may be approximately taken as a flat circular plate whose thickness (t<sub>h</sub>) may be determined from the following relation:

$$t_h = D \sqrt{\frac{C \cdot p}{\sigma_c}}$$

D = Cylinder bore in mm,

p = Maximum pressure inside the cylinder in N/mm<sup>2</sup>,

σ<sub>c</sub> = Allowable circumferential stress in MPa or N/mm<sup>2</sup>. It may be taken as 30 to 50 MPa, and

$C$  = Constant whose value is taken as 0.1.

The studs or bolts are screwed up tightly along with a metal gasket or asbestos packing to provide a leak proof joint between the cylinder and cylinder head. The tightness of the joint also depends upon the pitch of the bolts or studs, which should lie between  $19d$  to  $28.5d$ . The pitch circle diameter ( $D_p$ ) is usually taken as  $D + 3d$ . The studs or bolts are designed in the same way as discussed above.

Problem 1. A four stroke diesel engine has the following specifications:

Brake power = 5 kW; Speed = 1200 r.p.m. ; Indicated mean effective pressure =  $0.35 \text{ N/mm}^2$  ; Mechanical efficiency = 80 %.

Determine: 1. bore and length of the cylinder; 2. thickness of the cylinder head; and 3. size of studs for the cylinder head.

Solution. Given: B.P. = 5kW = 5000 W;  $N = 1200 \text{ r.p.m.}$  or  $n = N / 2 = 600$ ;

$p_m = 0.35 \text{ N/mm}^2$ ;  $\eta_m = 80\% = 0.8$

### 1. Bore and length of cylinder

We know that the indicated power,

$$\text{I.P.} = \text{B.P.} / \eta_m = 5000 / 0.8 = 6250 \text{ W}$$

We also know that the indicated power (I.P.),

$$6250 = \frac{p_m \cdot l \cdot A \cdot n}{60} = \frac{0.35 \times 1.5D \times \pi D^2 \times 600}{60 \times 1000 \times 4} = 4.12 \times 10^{-3} D^3$$

...( $\because$  For four stroke engine,  $n = N/2$ )

$$D^3 = 6250 / 4.12 \times 10^{-3} = 1517 \times 10^3 \text{ or } D = 115 \text{ mm Ans. and } l = 1.5 D = 1.5 \times 115 = 172.5 \text{ mm}$$

Taking a clearance on both sides of the cylinder equal to 15% of the stroke, therefore length of the cylinder,

$$L = 1.15 l = 1.15 \times 172.5 = 198 \text{ say } 200 \text{ mm Ans.}$$

### 2. Thickness of the cylinder head

Since the maximum pressure ( $p$ ) in the engine cylinder is taken as 9 to 10 times the mean effective pressure ( $p_m$ ), therefore let us take

$$p = 9 p_m = 9 \times 0.35 = 3.15 \text{ N/mm}^2$$

We know that thickness of the cylinder head,

$$t_h = D \sqrt{\frac{C \cdot p}{\sigma_t}} = 115 \sqrt{\frac{0.1 \times 3.15}{42}} = 9.96 \text{ say } 10 \text{ mm } \text{Ans.}$$

...(Taking  $C = 0.1$  and  $\sigma_t = 42 \text{ MPa} = 42 \text{ N/mm}^2$ )

### 3. Size of studs for the cylinder head

Let  $d$  = Nominal diameter of the stud in mm,

$d_c$  = Core diameter of the stud in mm. It is usually taken as  $0.84 d$ .

$\sigma_t$  = Tensile stress for the material of the stud which is usually nickel steel.

$n_s$  = Number of studs.

We know that the force acting on the cylinder head (or on the studs)

$$= \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (115)^2 3.15 = 32\,702 \text{ N} \quad \dots(i)$$

The number of studs ( $n_s$ ) are usually taken between  $0.01 D + 4$  (i.e.  $0.01 \times 115 + 4 = 5.15$ ) and  $0.02 D + 4$  (i.e.  $0.02 \times 115 + 4 = 6.3$ ). Let us take  $n_s = 6$ .

We know that resisting force offered by all the studs

$$= n_s \times \frac{\pi}{4} (d_c)^2 \sigma_t = 6 \times \frac{\pi}{4} (0.84d)^2 65 = 216 d^2 \text{ N} \quad \dots(ii)$$

...(Taking  $\sigma_t = 65 \text{ MPa} = 65 \text{ N/mm}^2$ )

From equations (i) and (ii),

$$d^2 = 32\,702 / 216 = 151 \text{ or } d = 12.3 \text{ say } 14 \text{ mm}$$

The pitch circle diameter of the studs ( $D_p$ ) is taken  $D + 3d$ .

$$\therefore D_p = 115 + 3 \times 14 = 157 \text{ mm}$$

We know that pitch of the studs

$$= \frac{\pi \times D_p}{n_s} = \frac{\pi \times 157}{6} = 82.2 \text{ mm}$$

We know that for a leak-proof joint, the pitch of the studs should lie between  $19\sqrt{d}$  to  $28.5\sqrt{d}$ , where  $d$  is the nominal diameter of the stud.

$\therefore$  Minimum pitch of the studs

$$= 19\sqrt{d} = 19\sqrt{14} = 71.1 \text{ mm}$$

and maximum pitch of the studs

$$= 28.5\sqrt{d} = 28.5\sqrt{14} = 106.6 \text{ mm}$$

Since the pitch of the studs obtained above (i.e. 82.2 mm) lies within 71.1 mm and 106.6 mm, therefore, size of the stud ( $d$ ) calculated above is satisfactory.

$$\therefore d = 14 \text{ mm Ans.}$$

## Piston

The piston is a disc which reciprocates within a cylinder. It is either moved by the fluid or it moves the fluid which enters the cylinder. The main function of the piston of an internal combustion engine is to receive the impulse from the expanding gas and to transmit the energy to the crankshaft through the connecting rod. The piston must also disperse a large amount of heat from the combustion chamber to the cylinder walls.

The pistons of internal combustion engines are usually of trunk type as shown in Fig. pistons are open at one end and consist of the following parts:

1. **Head or crown.** The piston head or crown may be flat, convex or concave depending upon the design of combustion chamber. It withstands the pressure of gas in the cylinder.
2. **Piston rings.** The piston rings are used to seal the cylinder in order to prevent leakage of the gas past the piston.
3. **Skirt.** The skirt acts as a bearing for the side thrust of the connecting rod on the walls of cylinder.
4. **Piston pin.** It is also called gudgeon pin or wrist pin. It is used to connect the piston to the connecting rod.

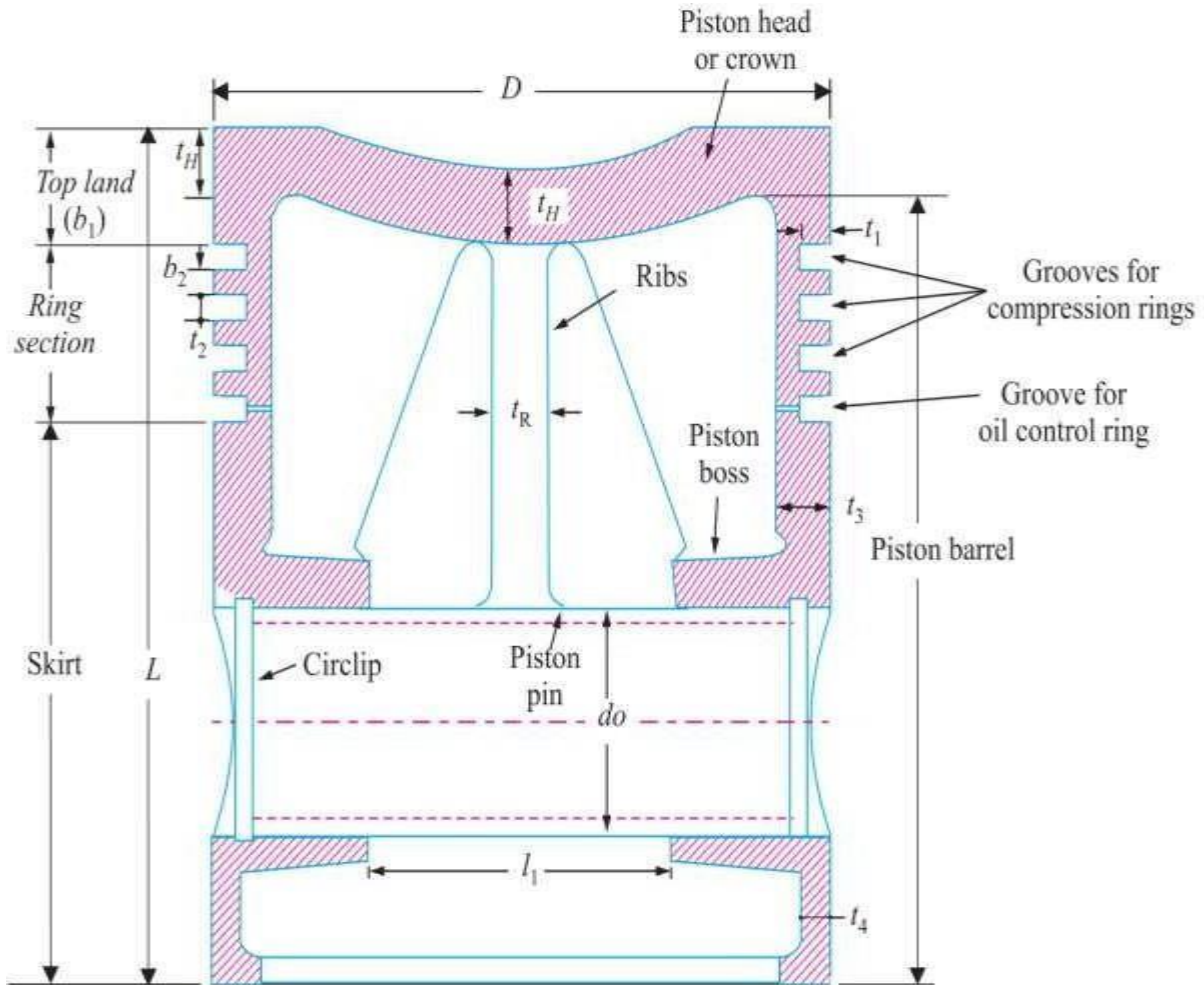


Figure.2.3: Piston

### Design Considerations for a Piston

In designing a piston for I.C. engine, the following points should be taken into consideration:

1. It should have enormous strength to withstand the high gas pressure and inertia forces.
2. It should have minimum mass to minimize the inertia forces.
3. It should form an effective gas and oil sealing of the cylinder.
4. It should provide sufficient bearing area to prevent undue wear.
5. It should disperse the heat of combustion quickly to the cylinder walls.
6. It should have high speed reciprocation without noise.
7. It should be of sufficient rigid construction to withstand thermal and mechanical distortion.

8. It should have sufficient support for the piston pin.

### Material for Pistons

The most commonly used materials for pistons of I.C. engines are cast iron, cast aluminium, forged aluminium, cast steel and forged steel. The cast iron pistons are used for moderately rated engines with piston speeds below 6 m/s and aluminium alloy pistons are used for highly rated engines running at higher piston speeds. It may be noted that

1. Since the coefficient of thermal expansion for aluminium is about 2.5 times that of cast iron, therefore, a greater clearance must be provided between the piston and the cylinder wall (than with cast iron piston) in order to prevent siezing of the piston when engine runs continuously under heavy loads. But if excessive clearance is allowed, then the piston will develop ‘piston slap’ while it is cold and this tendency increases with wear. The less clearance between the piston and the cylinder wall will lead to siezing of piston.

2. Since the aluminium alloys used for pistons have high heat conductivity (nearly four times that of cast iron), therefore, these pistons ensure high rate of heat transfer and thus keeps down the maximum temperature difference between the centre and edges of the piston head or crown.

### Piston Head or Crown

The piston head or crown is designed keeping in view the following two main considerations, i.e.

1. It should have adequate strength to withstand the straining action due to pressure of explosion inside the engine cylinder, and

2. It should dissipate the heat of combustion to the cylinder walls as quickly as possible. On the basis of first consideration of straining action, the thickness of the piston head is determined by treating it as a flat circular plate of uniform thickness, fixed at the outer edges and subjected to a uniformly distributed load due to the gas pressure over the entire cross-section.

The thickness of the piston head ( $t_H$ ), according to Grashoff’s formula is given by

$$t_H = \sqrt{\frac{3p \cdot D^2}{16\sigma_t}} \text{ (in mm)} \quad \dots(i)$$

$p$  = Maximum gas pressure or explosion pressure in  $\text{N/mm}^2$ ,

$D$  = Cylinder bore or outside diameter of the piston in mm, and

$\sigma_t$  = Permissible bending (tensile) stress for the material of the piston in  $\text{MPa}$  or  $\text{N/mm}^2$ . It may be taken as 35 to 40  $\text{MPa}$  for grey cast iron, 50 to 90  $\text{MPa}$  for nickel cast iron and aluminium alloy and 60 to 100  $\text{MPa}$  for forged steel.

On the basis of second consideration of heat transfer, the thickness of the piston head should be such that the heat absorbed by the piston due combustion of fuel is quickly transferred to the cylinder walls. Treating the piston head as a flat circular plate, its thickness is given by

$$t_H = \frac{H}{12.56k(T_C - T_E)} \text{ (in mm)} \quad \dots(ii)$$

H = Heat flowing through the piston head in kJ/s or watts,

k =Heat conductivity factor in W/m/°C. Its value is 46.6 W/m/°C for grey cast iron, 51.25 W/m/°C for steel and 174.75 W/m/°C for aluminium alloys.

T<sub>C</sub> = Temperature at the centre of the piston head in °C, and

T<sub>E</sub> = Temperature at the edges of the piston head in °C.

The temperature difference (T<sub>C</sub> – T<sub>E</sub>) may be taken as 220°C for cast iron and 75°C for aluminium.

The heat flowing through the piston head (H) may be determined by the following expression, i.e.,

$$H = C \times \text{HCV} \times m \times \text{B.P. (in kW)}$$

Where C = Constant representing that portion of the heat supplied to the engine which is absorbed by the piston. Its value is usually taken as 0.05.

HCV = Higher calorific value of the fuel in kJ/kg. It may be taken as 45 × 103 kJ/kg for diesel and 47 × 103 kJ/ kg for petrol,

m = Mass of the fuel used in kg per brake power per second, and

B.P. = Brake power of the engine per cylinder

### **Piston Rings**

The piston rings are used to impart the necessary radial pressure to maintain the seal between the piston and the cylinder bore. These are usually made of grey cast iron or alloy cast iron because of their good wearing properties and also they retain spring characteristics even at high temperatures.

The piston rings are of the following two types:

**1. Compression rings or pressure rings, and**

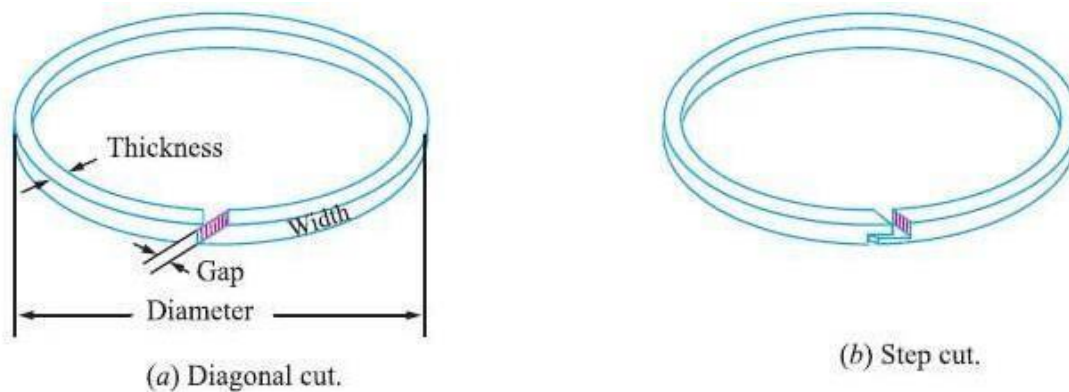
**2. Oil control rings or oil scraper.**



The compression rings or pressure rings are inserted in the grooves at the top portion of the piston and may be three to seven in number. These rings also transfer heat from the piston to the cylinder liner and absorb some part of the piston fluctuation due to the side thrust.

The oil control rings or oil scrapers are provided below the compression rings. These rings provide proper lubrication to the liner by allowing sufficient oil to move up during upward stroke and at the same time scrap the lubricating oil from the surface of the liner in order to minimize the flow of the oil to the combustion chamber.

The compression rings are usually made of rectangular cross-section and the diameter of the ring is slightly larger than the cylinder bore. A part of the ring is cut-off in order to permit it to go into the cylinder against the liner wall. The diagonal cut or step cut ends, as shown in Fig. (a) and (b) respectively, may be used. The gap between the ends should be sufficiently large when the ring is put cold so that even at the highest temperature, the ends do not touch each other when the ring expands, otherwise there might be buckling of the ring.



The radial thickness ( $t_1$ ) of the ring may be obtained by considering the radial pressure between the cylinder wall and the ring. From bending stress consideration in the ring, the radial thickness is given by

$$t_1 = D \sqrt{\frac{3p_w}{\sigma_t}}$$

$D$  = Cylinder bore in mm,

$p_w$  = Pressure of gas on the cylinder wall in N/mm<sup>2</sup>. Its value is limited from 0.025 N/mm<sup>2</sup> to 0.042 N/mm<sup>2</sup>, and

$\sigma_t$  = Allowable bending (tensile) stress in MPa. Its value may be taken from 85 MPa to 110 MPa for cast iron rings.



The axial thickness ( $t_2$ ) of the rings may be taken as 0.7  $t_1$  to  $t_1$ .

The minimum axial thickness ( $t_2$ ) may also be obtained from the following empirical relation:

$$t_2 = \frac{D}{10n_R}$$

$n_R$  = Number of rings.

The width of the top land (i.e. the distance from the top of the piston to the first ring groove) is made larger than other ring lands to protect the top ring from high temperature conditions existing at the top of the piston,

∴ Width of top land,

$$b_1 = t_H \text{ to } 1.2 t_H$$

The width of other ring lands (i.e. the distance between the ring grooves) in the piston may be made equal to or slightly less than the axial thickness of the ring ( $t_2$ ).

∴ Width of other ring lands,

$$b_2 = 0.75 t_2 \text{ to } t_2$$

The depth of the ring grooves should be more than the depth of the ring so that the ring does not take any piston side thrust.

The gap between the free ends of the ring is given by 3.5  $t_1$  to 4  $t_1$ . The gap, when the ring is in the cylinder, should be 0.002  $D$  to 0.004  $D$ .

### **Piston Barrel**

It is a cylindrical portion of the piston. The maximum thickness ( $t_3$ ) of the piston barrel may be obtained from the following empirical relation:

$$t_3 = 0.03 D + b + 4.5 \text{ mm}$$

$b$  = Radial depth of piston ring groove which is taken as 0.4 mm larger than the radial thickness of the piston ring ( $t_1$ ) =  $t_1 + 0.4$  mm

Thus, the above relation may be written as

$$t_3 = 0.03 D + t_1 + 4.9 \text{ mm}$$

The piston wall thickness ( $t_4$ ) towards the open end is decreased and should be taken as 0.25  $t_3$  to 0.35  $t_3$ .

## Piston Skirt

The portion of the piston below the ring section is known as piston skirt. It acts as a bearing for the side thrust of the connecting rod. The length of the piston skirt should be such that the bearing pressure on the piston barrel due to the side thrust does not exceed 0.25 N/mm<sup>2</sup> of the projected area for low speed engines and 0.5 N/mm<sup>2</sup> for high speed engines. It may be noted that the maximum thrust will be during the expansion stroke. The side thrust (R) on the cylinder liner is usually taken as 1/10 of the maximum gas load on the piston

We know that maximum gas load on the piston,

$$P = p \times \frac{\pi D^2}{4}$$

∴ Maximum side thrust on the cylinder,

$$R = P/10 = 0.1 p \times \frac{\pi D^2}{4} \quad \dots(i)$$

p = Maximum gas pressure in N/mm<sup>2</sup>, and

D = Cylinder bore in mm.

The side thrust (R) is also given by

R = Bearing pressure × Projected bearing area of the piston skirt =  $p_b \times D \times l$

Where l = Length of the piston skirt in mm.

From equations (i) and (ii), the length of the piston skirt (l) is determined. In actual practice, the length of the piston skirt is taken as 0.65 to 0.8 times the cylinder bore. Now the total length of the piston (L) is given by

L = Length of skirt + Length of ring section + Top land

The length of the piston usually varies between D and 1.5 D. It may be noted that a longer piston provides better bearing surface for quiet running of the engine, but it should not be made unnecessarily long as it will increase its own mass and thus the inertia forces.

## Piston Pin

The piston pin (also called gudgeon pin or wrist pin) is used to connect the piston and the connecting rod. It is usually made hollow and tapered on the inside, the smallest inside diameter being at the centre of the pin, as shown in Fig. The piston pin passes through the bosses provided on the inside of the piston skirt and the bush of the small end of the connecting rod.

The centre of piston pin should be 0.02 D to 0.04 D above the centre of the skirt, in order to offset the turning effect of the friction and to obtain uniform distribution of pressure between the piston and the cylinder liner.

The material used for the piston pin is usually case hardened steel alloy containing nickel, chromium, molybdenum or vanadium having tensile strength from 710 MPa to 910 MPa

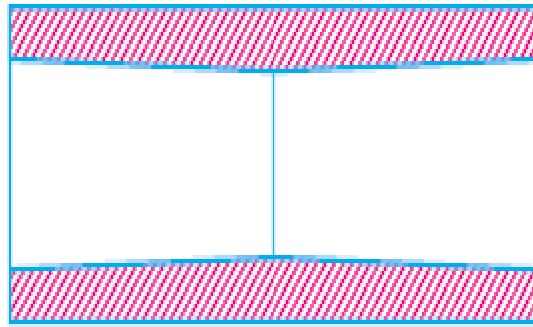


Figure.2.4: Piston Pin

$$Z = \frac{\pi}{32} \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right]$$

We know that maximum bending moment,

$$M = Z \times \sigma_b = \frac{\pi}{32} \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right] \sigma_b$$

$\sigma_b$  = Allowable bending stress for the material of the piston pin. It is usually taken as 84 MPa for case hardened carbon steel and 140 MPa for heat treated alloy steel.

Assuming  $d_i = 0.6 d_o$ , the induced bending stress in the piston pin may be checked.

**Problem 2.** Design a cast iron piston for a single acting four stroke engine for the following data: Cylinder bore = 100 mm; Stroke = 125 mm; Maximum gas pressure = 5 N/mm<sup>2</sup>; Indicated mean effective pressure = 0.75 N/mm<sup>2</sup>; Mechanical efficiency = 80%; Fuel consumption = 0.15 kg per brake power per hour; Higher calorific value of fuel = 42 × 10<sup>3</sup> kJ/kg; Speed = 2000 r.p.m. Any other data required for the design may be assumed.

**Solution.** Given : D = 100 mm ; L = 125 mm = 0.125 m ; p = 5 N/mm<sup>2</sup> ; p<sub>m</sub> = 0.75 N/mm<sup>2</sup>;  $\eta_m$  = 80% = 0.8 ; m = 0.15 kg / BP / h = 41.7 × 10<sup>-6</sup> kg / BP / s; HCV = 42 × 10<sup>3</sup> kJ / kg ; N = 2000 r.p.m.

The dimensions for various components of the piston are determined as follows:

### 1. Piston head or crown

The thickness of the piston head or crown is determined on the basis of strength as well as on the basis of heat dissipation and the larger of the two values is adopted.

We know that the thickness of piston head on the basis of strength,

$$t_H = \sqrt{\frac{3p \cdot D^2}{16 \sigma_t}} = \sqrt{\frac{3 \times 5(100)^2}{16 \times 38}} = 15.7 \text{ say } 16 \text{ mm}$$

...(Taking  $\sigma_t$  for cast iron = 38 MPa = 38 N/mm<sup>2</sup>)

Since the engine is a four stroke engine, therefore, the number of working strokes per minute,

$$n = N / 2 = 2000 / 2 = 1000$$

and cross-sectional area of the cylinder,

$$A = \frac{\pi D^2}{4} = \frac{\pi (100)^2}{4} = 7855 \text{ mm}^2$$

We know that indicated power,

$$IP = \frac{p_m \cdot L \cdot A \cdot n}{60} = \frac{0.75 \times 0.125 \times 7855 \times 1000}{60} = 12\,270 \text{ W} \\ = 12.27 \text{ kW}$$

$$\therefore \text{ Brake power, } BP = IP \times \eta_m = 12.27 \times 0.8 = 9.8 \text{ kW} \quad \dots (\because \eta_m = BP / IP)$$

We know that the heat flowing through the piston head,

$$H = C \times HCV \times m \times BP \\ = 0.05 \times 42 \times 10^3 \times 41.7 \times 10^{-6} \times 9.8 = 0.86 \text{ kW} = 860 \text{ W} \\ \dots (\text{Taking } C = 0.05)$$

$\therefore$  Thickness of the piston head on the basis of heat dissipation,

$$t_H = \frac{H}{12.56 k (T_C - T_E)} = \frac{860}{12.56 \times 46.6 \times 220} = 0.0067 \text{ m} = 6.7 \text{ mm} \\ \dots (\because \text{ For cast iron, } k = 46.6 \text{ W/m}^\circ\text{C, and } T_C - T_E = 220^\circ\text{C})$$

Taking the larger of the two values, we shall adopt

$$t_H = 16 \text{ mm } \text{Ans.}$$

Since the ratio of  $L / D$  is 1.25, therefore a cup in the top of the piston head with a radius equal to  $0.7 D$  (i.e. 70 mm) is provided.

### 2. Radial ribs

The radial ribs may be four in number. The thickness of the ribs varies from  $t_H / 3$  to  $t_H / 2$ .

$$\therefore \text{ Thickness of the ribs, } t_R = 16 / 3 \text{ to } 16 / 2 = 5.33 \text{ to } 8 \text{ mm}$$

$$\text{Let us adopt } t_R = 7 \text{ mm } \text{Ans.}$$

### 3. Piston rings

Let us assume that there are total four rings (i.e.  $n_r = 4$ ) out of which three are compression rings and one is an oil ring.

We know that the radial thickness of the piston rings,

$$t_1 = D \sqrt{\frac{3p_w}{\sigma_t}} = 100 \sqrt{\frac{3 \times 0.035}{90}} = 3.4 \text{ mm} \\ \dots (\text{Taking } p_w = 0.035 \text{ N/mm}^2, \text{ and } \sigma_t = 90 \text{ MPa})$$

and axial thickness of the piston rings

$$t_2 = 0.7 t_1 \text{ to } t_1 = 0.7 \times 3.4 \text{ to } 3.4 \text{ mm} = 2.38 \text{ to } 3.4 \text{ mm}$$

$$\text{Let us adopt } t_2 = 3 \text{ mm}$$

We also know that the minimum axial thickness of the piston ring,

$$t_2 = \frac{D}{10 n_r} = \frac{100}{10 \times 4} = 2.5 \text{ mm}$$

Thus the axial thickness of the piston ring as already calculated (i.e.  $t_2 = 3 \text{ mm}$ ) is satisfactory.  
Ans.

The distance from the top of the piston to the first ring groove, i.e. the width of the top land,

$$b_1 = t_H \text{ to } 1.2 t_H = 16 \text{ to } 1.2 \times 16 \text{ mm} = 16 \text{ to } 19.2 \text{ mm and width of other ring lands,}$$

$$b_2 = 0.75 t_2 \text{ to } t_2 = 0.75 \times 3 \text{ to } 3 \text{ mm} = 2.25 \text{ to } 3 \text{ mm}$$

Let us adopt  $b_1 = 18 \text{ mm}$ ; and  $b_2 = 2.5 \text{ mm}$  Ans.

We know that the gap between the free ends of the ring,

$$G_1 = 3.5 t_1 \text{ to } 4 t_1 = 3.5 \times 3.4 \text{ to } 4 \times 3.4 \text{ mm} = 11.9 \text{ to } 13.6 \text{ mm and the gap when the ring is in the cylinder,}$$

$$G_2 = 0.002 D \text{ to } 0.004 D = 0.002 \times 100 \text{ to } 0.004 \times 100 \text{ mm} = 0.2 \text{ to } 0.4 \text{ mm}$$

Let us adopt  $G_1 = 12.8 \text{ mm}$ ; and  $G_2 = 0.3 \text{ mm}$

#### 4. Piston barrel

Since the radial depth of the piston ring grooves ( $b$ ) is about  $0.4 \text{ mm}$  more than the radial thickness of the piston rings ( $t_1$ ), therefore,

$$b = t_1 + 0.4 = 3.4 + 0.4 = 3.8 \text{ mm}$$

We know that the maximum thickness of barrel,

$$t_3 = 0.03 D + b + 4.5 \text{ mm} = 0.03 \times 100 + 3.8 + 4.5 = 11.3 \text{ mm and piston wall thickness towards the open end,}$$

$$t_4 = 0.25 t_3 \text{ to } 0.35 t_3 = 0.25 \times 11.3 \text{ to } 0.35 \times 11.3 = 2.8 \text{ to } 3.9 \text{ mm}$$

Let us adopt  $t_4 = 3.4 \text{ mm}$

#### 5. Piston skirt

Let  $l$  = Length of the skirt in mm.

We know that the maximum side thrust on the cylinder due to gas pressure ( $p$ ),

$$R = \mu \times \frac{\pi D^2}{4} \times p = 0.1 \times \frac{\pi (100)^2}{4} \times 5 = 3928 \text{ N}$$

We also know that the side thrust due to bearing pressure on the piston barrel ( $p_b$ ),

$$R = p_b \times D \times l = 0.45 \times 100 \times l = 45 l \text{ N} \quad \dots (\text{Taking } p_b = 0.45 \text{ N/mm}^2)$$

From above, we find that

$$45 l = 3928 \text{ or } l = 3928 / 45 = 87.3 \text{ say } 90 \text{ mm Ans.}$$

$\therefore$  Total length of the piston,

$$L = \text{Length of the skirt} + \text{Length of the ring section} + \text{Top land}$$

$$= l + (4 t_2 + 3 b_2) + b_1$$

$$= 90 + (4 \times 3 + 3 \times 3) + 18 = 129 \text{ say } 130 \text{ mm Ans.}$$

## 6. Piston pin

Let  $d_0$  = Outside diameter of the pin in mm,

$l_1$  = Length of pin in the bush of the small end of the connecting rod in mm, and

$p_{b1}$  = Bearing pressure at the small end of the connecting rod bushing in  $\text{N/mm}^2$ . Its value for bronze bushing is taken as  $25 \text{ N/mm}^2$ .

We know that load on the pin due to bearing pressure

$$= \text{Bearing pressure} \times \text{Bearing area} = p_{b1} \times d_0 \times l_1$$

$$= 25 \times d_0 \times 0.45 \times 100 = 1125 d_0 \text{ N} \quad \dots (\text{Taking } l_1 = 0.45 D)$$

We also know that maximum load on the piston due to gas pressure or maximum gas load

$$= \frac{\pi D^2}{4} \times p = \frac{\pi (100)^2}{4} \times 5 = 392875 \text{ N}$$

From above, we find that

$$1125 d_0 = 392875 \text{ or } d_0 = 392875 / 1125 = 34.9 \text{ say } 35 \text{ mm Ans.}$$

The inside diameter of the pin ( $d_i$ ) is usually taken as  $0.6 d_0$ .

$$\therefore d_i = 0.6 \times 35 = 21 \text{ mm Ans.}$$

Let the piston pin be made of heat treated alloy steel for which the bending stress ( $\sigma_b$ ) may be taken as 140 MPa. Now let us check the induced bending stress in the pin.

We know that maximum bending moment at the centre of the pin,

$$M = \frac{P.D}{8} = \frac{39\,275 \times 100}{8} = 491 \times 10^3 \text{ N-mm}$$

We also know that maximum bending moment ( $M$ ),

$$491 \times 10^3 = \frac{\pi}{32} \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right] \sigma_b = \frac{\pi}{32} \left[ \frac{(35)^4 - (21)^4}{35} \right] \sigma_b = 3664 \sigma_b$$

$$\therefore \sigma_b = 491 \times 10^3 / 3664 = 134 \text{ N/mm}^2 \text{ or MPa}$$

Since the induced bending stress in the pin is less than the permissible value of 140 MPa (i.e. 140 N/mm<sup>2</sup>), therefore, the dimensions for the pin as calculated above (i.e.  $d_o = 35$  mm and  $d_i = 21$  mm) are satisfactory.

### Ceramic coated engine

Research for decreasing costs and consumed fuel in internal combustion engines and technological innovation studies have been continuing. Engine efficiency improvement efforts via constructional modifications are increased today; for instance, parallel to development of advanced technology ceramics, ceramic coating applications in internal combustion engines grow rapidly. To improve engine performance, fuel energy must be converted to mechanical energy at the most possible rate. Coating combustion chamber with low heat conducting ceramic materials leads to increasing temperature and pressure in internal combustion engine cylinders. Hence, an increase in engine efficiency should be observed.

Ceramic coatings applied to diesel engine combustion chambers are aimed to reduce heat which passes from in-cylinder to engine cooling system. Engine cooling systems are planned to be removed from internal combustion engines by the development of advanced technology ceramics. One can expect that engine power can be increased and engine weight and cost can be decreased by removing cooling system elements (coolant pump, ventilator, water jackets and radiators etc.)



Initiation of the engine can be easier like shortened ignition delay in ceramic coated diesel engines due to increased temperature after compression because of low heat rejection. More silent engine operation can be obtained considering less detonation and noise causing from uncontrolled combustion. Engine can be operated at lower compression ratios due to shortened ignition delay. Thus better mechanical efficiency can be obtained and fuel economy can be improved

Another important topic from the view point of internal combustion engines is exhaust emissions. Increased combustion chamber temperature of ceramic coated internal combustion engines causes a decrease in soot and carbon monoxide emissions. When increased exhaust gas temperature is considered, it is obvious that turbocharging and consequently total thermal efficiency of the engine is increased.

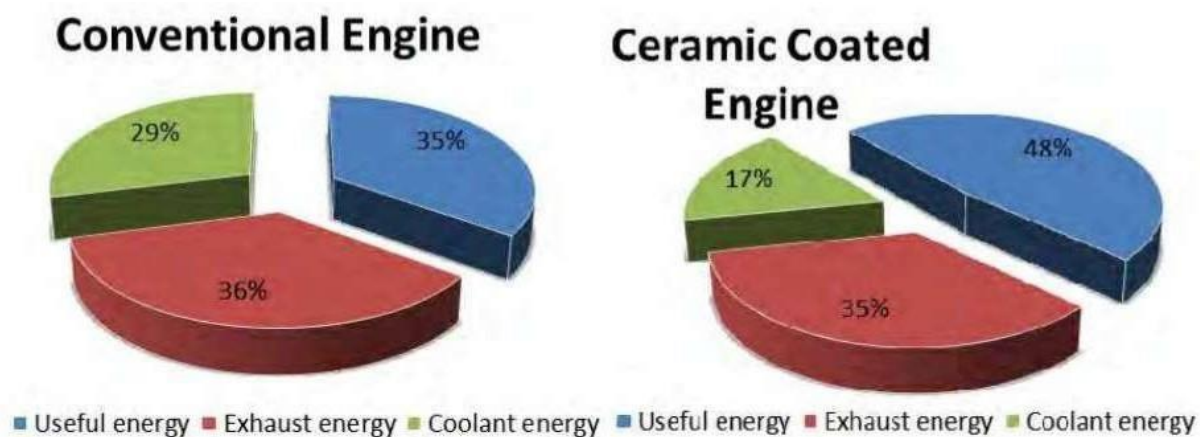


Figure.2.5: Energy Consumption Convention Vs Ceramic Coated

Engine Advantages of advanced technology ceramics can be listed as below;

- Resistant to high temperatures
- High chemical stability
- High hardness values
- Low densities
- Can be found as raw material form in environment
- Resistant to wear
- Low heat conduction coefficient
- High compression strength

### Engine emissions

Diesel engine, like other internal combustion engines, converts chemical energy contained in the fuel into mechanical power. Diesel fuel is a mixture of hydrocarbons which during an ideal combustion process would produce only carbon dioxide (CO<sub>2</sub>) and water vapor (H<sub>2</sub>O). Indeed,



diesel exhaust gases are primarily composed of CO<sub>2</sub>, H<sub>2</sub>O and the unused portion of engine charge air. The volumetric concentrations of these gases in diesel exhaust are typically in the following ranges:

CO<sub>2</sub> - 2 ... 12%

H<sub>2</sub>O - 2 ... 12%

O<sub>2</sub> - 3 ... 17

N<sub>2</sub> - balance.

The concentrations depend on the engine load, with the content of CO<sub>2</sub> and H<sub>2</sub>O increasing and that of O<sub>2</sub> decreasing with increasing engine load. None of these principal diesel emissions (with the exception of CO<sub>2</sub> for its greenhouse gas properties) have adverse health or environmental effects.

Diesel emissions include also pollutants that can have adverse health and/or environmental effects. Most of these pollutants originate from various non-ideal processes during combustion, such as incomplete combustion of fuel, reactions between mixture components under high temperature and pressure, combustion of engine lubricating oil and oil additives as well as combustion of non-hydrocarbon components of diesel fuel, such as sulfur compounds and fuel additives. Common pollutants include unburned hydrocarbons (HC), carbon monoxide (CO), nitrogen oxides (NO<sub>x</sub>) or particulate matter (PM). Total concentration of pollutants in diesel exhaust gases typically amounts to some tenths of one percent—this is schematically illustrated in Figure 1. Much lower, “near-zero” levels of pollutants are emitted from modern diesel engines equipped with emission aftertreatment devices such as NO<sub>x</sub> reduction catalysts and particulate filters.

## **UNIT III DESIGN OF CONNECTING ROD AND CRANK SHAFT**

### 3. DESIGN OF CONNECTING ROD, CRANKSHAFT

#### Connecting Rod

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crankpin and thus convert the reciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, I-section or H-section. Generally circular section is used for low speed engines while I-section is preferred for high speed engines.

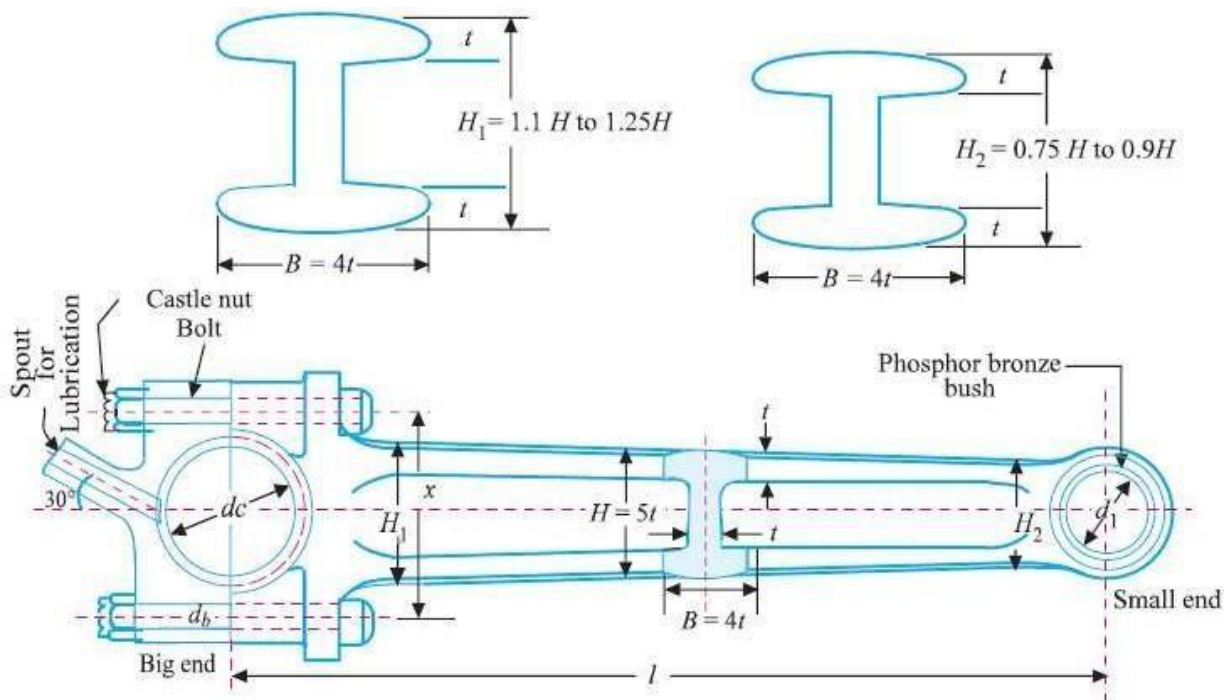


Figure.3.1: Connecting Rod

The length of the connecting rod ( $l$ ) depends upon the ratio of  $l/r$ , where  $r$  is the radius of crank. It may be noted that the smaller length will decrease the ratio  $l/r$ . This increases the angularity of the connecting rod which increases the side thrust of the piston against the cylinder liner which in turn increases the wear of the liner. The larger length of the connecting rod will increase the ratio  $l/r$ . This decreases the angularity of the connecting rod and thus decreases the

side thrust and the resulting wear of the cylinder. But the larger length of the connecting rod increases the overall height of the engine. Hence, a compromise is made and the ratio  $l / r$  is generally kept as 4 to 5.

The small end of the connecting rod is usually made in the form of an eye and is provided with a bush of phosphor bronze. It is connected to the piston by means of a piston pin. The big end of the connecting rod is usually made split (in two halves) so that it can be mounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two cap bolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about 0.75 mm) of white metal or babbitt metal. The wear of the big end bearing is allowed for by inserting thin metallic strips (known as shims) about 0.04 mm thick between the cap and the fixed half of the connecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

The connecting rods are usually manufactured by drop forging process and it should have adequate strength, stiffness and minimum weight. The material mostly used for connecting rods varies from mild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome- nickel or chromemoly steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength of about 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimate tensile strength of 750 MPa. These steels are used for connecting rods of industrial engines.

The alloy steels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods of aeroengines and automobile engines. The bearings at the two ends of the connecting rod are either splash lubricated or pressure lubricated. The big end bearing is usually splash lubricated while the small end bearing is pressure lubricated. In the splash lubrication system, the cap at the big end is provided with a dipper or spout and set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holes provided on the upper surface of the small end. In the pressure lubricating system, the lubricating oil is fed under pressure to the big end bearing through the holes drilled in crankshaft, crank webs and crank pin. From the big end bearing, the oil is fed to small end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the small end bearing is lubricated by the oil scrapped from the walls of the cylinder liner by the oil scraper rings.

### **Forces Acting on the Connecting Rod**

The various forces acting on the connecting rod are as follows:

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,

2. Force due to inertia of the connecting rod or inertia bending forces,
3. Force due to friction of the piston rings and of the piston, and
4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below

### **Design of Connecting Rod**

In designing a connecting rod, the following dimensions are required to be determined:

1. Dimensions of cross-section of the connecting rod,
2. Dimensions of the crankpin at the big end and the piston pin at the small end,
3. Size of bolts for securing the big end cap, and
4. Thickness of the big end cap.

The procedure adopted in determining the above mentioned dimensions is discussed as below:

#### **1. Dimensions of cross-section of the connecting rod**

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore, the cross-section of the connecting rod is designed as a strut and the Rankine's formula is used.

A connecting rod, as shown in Fig. Subjected to an axial load  $W$  may buckle with X-axis as neutral axis (i.e. in the plane of motion of the connecting rod) or Y-axis as neutral axis (i.e. in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis.

A connecting rod should be equally strong in buckling about both the axes.

Let  $A$  = Cross-sectional area of the connecting rod,

$l$  = Length of the connecting rod,

$\sigma_c$  = Compressive yield stress,

$W_B$  = Buckling load,

$I_{xx}$  and  $I_{yy}$  = Moment of inertia of the section about X-axis and Y-axis respectively, and  $k_{xx}$  and  $k_{yy}$  = Radius of gyration of the section about X-axis and Y-axis respectively.

According to Rankine's formula,

$$W_B \text{ about } X\text{-axis} = \frac{\sigma_c \cdot A}{1 + a \left( \frac{L}{k_{xx}} \right)^2} = \frac{\sigma_c \cdot A}{1 + a \left( \frac{l}{k_{xx}} \right)^2} \quad \dots (\because \text{For both ends hinged, } L = l)$$

$$W_B \text{ about } Y\text{-axis} = \frac{\sigma_c \cdot A}{1 + a \left( \frac{L}{k_{yy}} \right)^2} = \frac{\sigma_c \cdot A}{1 + a \left( \frac{l}{2 k_{yy}} \right)^2} \quad \dots [\because \text{For both ends fixed, } L = \frac{l}{2}]$$

$L$  = Equivalent length of the connecting rod, and

$a$  = Constant

= 1 / 7500, for mild steel

= 1 / 9000, for wrought iron

= 1 / 1600, for cast iron

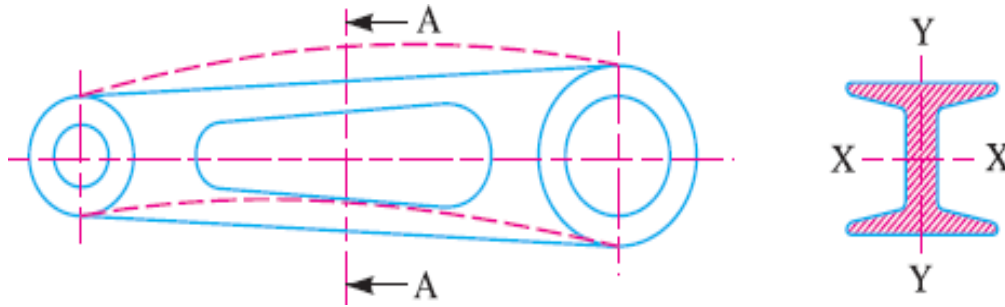


Figure.3.2: Connecting Rod I - Section

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, i.e.

$$\frac{\sigma_c \cdot A}{1 + a \left( \frac{l}{k_{xx}} \right)^2} = \frac{\sigma_c \cdot A}{1 + a \left( \frac{l}{2 k_{yy}} \right)^2} \quad \text{or} \quad \left( \frac{l}{k_{xx}} \right)^2 = \left( \frac{l}{2 k_{yy}} \right)^2$$

$$k_{xx}^2 = 4 k_{yy}^2 \quad \text{or} \quad I_{xx} = 4 I_{yy}$$

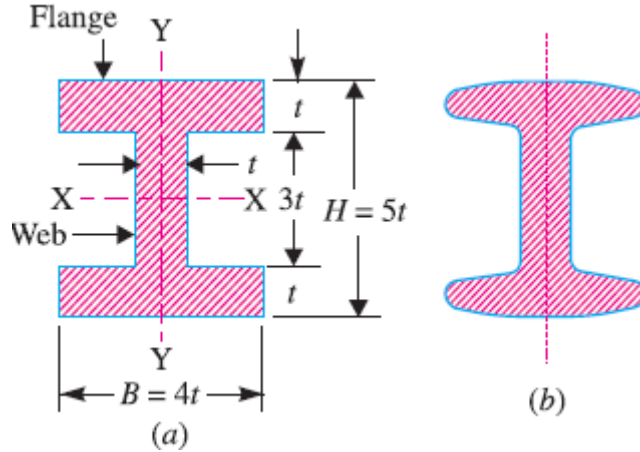


Figure.3.3: Cross Section Area

This shows that the connecting rod is four times strong in buckling about Y-axis than about X-axis. If  $I_{xx} > 4 I_{yy}$ , then buckling will occur about Y-axis and if  $I_{xx} < 4 I_{yy}$ , buckling will occur about X-axis. In actual practice,  $I_{xx}$  is kept slightly less than  $4 I_{yy}$ . It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis. The design will always be satisfactory for buckling about Y-axis. The most suitable section for the connecting rod is I-section with the proportions as shown in Fig.

Let thickness of the flange and web of the section =  $t$

Width of the section,  $B = 4t$  and depth or height of the section,

$$H = 5t$$

From Fig.(a), we find that area of the section,

$$A = 2(4t \times t) + 3t \times t = 11t^2$$

Moment of inertia of the section about X-axis,

$$I_{xx} = \frac{1}{12} [4t(5t)^3 - 3t(3t)^3] = \frac{419}{12} t^4$$

and moment of inertia of the section about Y-axis,

$$I_{yy} = \left[ 2 \times \frac{1}{12} t \times (4t)^3 + \frac{1}{12} (3t) t^3 \right] = \frac{131}{12} t^4$$

$$\therefore \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

After deciding the proportions for I-section of the connecting rod, its dimensions are determined by considering the buckling of the rod about X-axis (assuming both ends hinged) and applying the Rankine's formula. We know that buckling load,

$$W_B = \frac{\sigma_c \cdot A}{1 + a \left( \frac{L}{k_{xx}} \right)^2}$$

The buckling load ( $W_B$ ) may be calculated by using the following relation, i.e.

$W_B = \text{Max. gas force} \times \text{Factor of safety}$  The factor of safety may be taken as 5 to 6.

The dimensions  $B = 4t$  and  $H = 5t$ , as obtained above by applying the Rankine's formula, are at the middle of the connecting rod. The width of the section ( $B$ ) is kept constant throughout the length of the connecting rod, but the depth or height varies. The depth near the small end (or piston end) is taken as  $H_1 = 0.75H$  to  $0.9H$  and the depth near the big end (or crank end) is taken

$$H_2 = 1.1H \text{ to } 1.25H.$$

## **2. Dimensions of the crankpin at the big end and the piston pin at the small end**

Since the dimensions of the crankpin at the big end and the piston pin (also known as gudgeon pin or wrist pin) at the small end are limited, therefore, fairly high bearing pressures have to be allowed at the bearings of these two pins.

The crankpin at the big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1 mm or less) of bearing metal (such as tin, lead, babbitt, copper, lead) on the inner surface of the shell. The allowable bearing pressure on the crankpin depends upon many factors such as material of the bearing,

Viscosity of the lubricating oil, method of lubrication and the space limitations. The value of bearing pressure may be taken as  $7 \text{ N/mm}^2$  to  $12.5 \text{ N/mm}^2$  depending upon the material and method of lubrication used.

The piston pin bearing is usually a phosphor bronze bush of about 3 mm thickness and the allowable bearing pressure may be taken as  $10.5 \text{ N/mm}^2$  to  $15 \text{ N/mm}^2$ .

Since the maximum load to be carried by the crankpin and piston pin bearings is the maximum force in the connecting rod ( $F_C$ ), therefore the dimensions for these two pins are determined for the maximum force in the connecting rod ( $F_C$ ) which is taken equal to the maximum force on the piston due to gas pressure ( $F_L$ ) neglecting the inertia forces.

We know that maximum gas force,



$$F_L = \frac{\pi D^2}{4} \times p \quad \dots(i)$$

D = Cylinder bore or piston diameter in mm, and

p = Maximum gas pressure in N/mm<sup>2</sup>

Now the dimensions of the crankpin and piston pin are determined as discussed below :

Let  $d_c$  = Diameter of the crank pin in mm,

$l_c$  = Length of the crank pin in mm,

$p_{bc}$  = Allowable bearing pressure in N/mm<sup>2</sup>, and

$d_p$ ,  $l_p$  and  $p_{bp}$  = Corresponding values for the piston pin,

We know that load on the crank pin

= Projected area  $\times$  Bearing pressure

$$= d_c \cdot l_c \cdot p_{bc} \dots(ii)$$

Similarly, load on the piston pin

$$= d_p \cdot l_p \cdot p_{bp} \dots(iii)$$

Equating equations (i) and (ii), we have

$$F_L = d_c \cdot l_c \cdot p_{bc}$$

Taking  $l_c = 1.25 d_c$  to  $1.5 d_c$ , the value of  $d_c$  and  $l_c$  are determined from the above expression.

Again, equating equations (i) and (iii), we have

$$F_L = d_p \cdot l_p \cdot p_{bp}$$

Taking  $l_p = 1.5 d_p$  to  $2 d_p$ , the value of  $d_p$  and  $l_p$  are determined from the above expression.

### 3. Size of bolts for securing the big end cap

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force of the reciprocating parts,

$$\therefore F_I = m_R \cdot \omega^2 \cdot r \left( \cos \theta + \frac{\cos 2\theta}{l/r} \right)$$

We also know that at the top dead centre, the angle of inclination of the crank with the line of stroke,  $\theta = 0$

$$\therefore F_I = m_R \cdot \omega^2 \cdot r \left( 1 + \frac{r}{l} \right)$$

$m_R$  = Mass of the reciprocating parts in kg,

$\omega$  = Angular speed of the engine in rad / s,

$r$  = Radius of the crank in metres, and

$l$  = Length of the connecting rod in metres.

The bolts may be made of high carbon steel or nickel alloy steel. Since the bolts are under repeated stresses but not alternating stresses, therefore, a factor of safety may be taken as 6.

Let  $d_{cb}$  = Core diameter of the bolt in mm,

$\sigma_t$  = Allowable tensile stress for the material of the bolts in MPa, and

$n_b$  = Number of bolts. Generally two bolts are used.

$\therefore$  Force on the bolts

$$= \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b$$

Equating the inertia force to the force on the bolts, we have

$$F_I = \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b$$

From this expression,  $d_{cb}$  is obtained. The nominal or major diameter ( $d_b$ ) of the bolt is given by

$$d_b = \frac{d_{cb}}{0.84}$$

#### 4. Thickness of the big end cap

The thickness of the big end cap ( $t_c$ ) may be determined by treating the cap as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (i.e.  $F_I$  when  $\theta = 0$ ). This load is assumed to act in between the uniformly distributed load and the centrally concentrated load. Therefore, the maximum bending moment acting on the cap will be taken as

$$M_C = \frac{F_1 \times x}{6}$$

$x$  = Distance between the bolt centres.

= Dia. of crankpin or big end bearing ( $d_c$ ) + 2 × Thickness of bearing liner (3 mm) + Clearance (3 mm)

Let  $b_c$  = Width of the cap in mm. It is equal to the length of the crankpin or big end bearing ( $l_c$ ), and

$\sigma_b$  = Allowable bending stress for the material of the cap in MPa.

We know that section modulus for the cap,

$$Z_C = \frac{b_c (t_c)^2}{6}$$

$$\therefore \text{Bending stress, } \sigma_b = \frac{M_C}{Z_C} = \frac{F_1 \times x}{6} \times \frac{6}{b_c (t_c)^2} = \frac{F_1 \times x}{b_c (t_c)^2}$$

From this expression, the value of  $t_c$  is obtained.

**Problem 1.** Design a connecting rod for an I.C. engine running at 1800 r.p.m. and developing a maximum pressure of 3.15 N/mm<sup>2</sup>. The diameter of the piston is 100 mm; mass of the reciprocating parts per cylinder 2.25 kg; length of connecting rod 380 mm; stroke of piston 190 mm and compression ratio 6: 1. Take a factor of safety of 6 for the design. Take length to diameter ratio for big end bearing as 1.3 and small end bearing as 2 and the corresponding bearing pressures as 10 N/mm<sup>2</sup> and 15 N/mm<sup>2</sup>. The density of material of the rod may be taken as 8000 kg/m<sup>3</sup> and the allowable stress in the bolts as 60 N/mm<sup>2</sup> and in cap as 80 N/mm<sup>2</sup>. The rod is to be of I-section for which you can choose your own proportions.

Draw a neat dimensioned sketch showing provision for lubrication. Use Rankine formula for which the numerator constant may be taken as 320 N/mm<sup>2</sup> and the denominator constant 1 / 7500

**Solution.** Given :  $N = 1800$  r.p.m. ;  $p = 3.15$  N/mm<sup>2</sup> ;  $D = 100$  mm ;  $m_R = 2.25$  kg ;  $l = 380$  mm = 0.38 m ; Stroke = 190 mm ; Compression ratio = 6 : 1 ; F. S. = 6.

The connecting rod is designed as discussed below:

1. **Dimension of I- section of the connecting rod** Let us consider an I-section of the connecting rod, as shown in Fig. (a), with the following proportions:

Flange and web thickness of the section =  $t$

Width of the section,  $B = 4t$  and depth or height of the section,

$$H = 5t$$

First of all, let us find whether the section chosen is satisfactory or not.

We have already discussed that the connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis. The connecting rod should be equally strong in buckling about both the axes. We know that in order to have a connecting rod equally strong about both the axes,

$$I_{xx} = 4 I_{yy}$$

where  $I_{xx}$  = Moment of inertia of the section about X-axis, and

$I_{yy}$  = Moment of inertia of the section about Y-axis.

In actual practice,  $I_{xx}$  is kept slightly less than  $4 I_{yy}$ . It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis.

Now, for the section as shown in Fig. (a), area of the section,

$$A = 2 (4 t \times t) + 3t \times t = 11 t^2$$

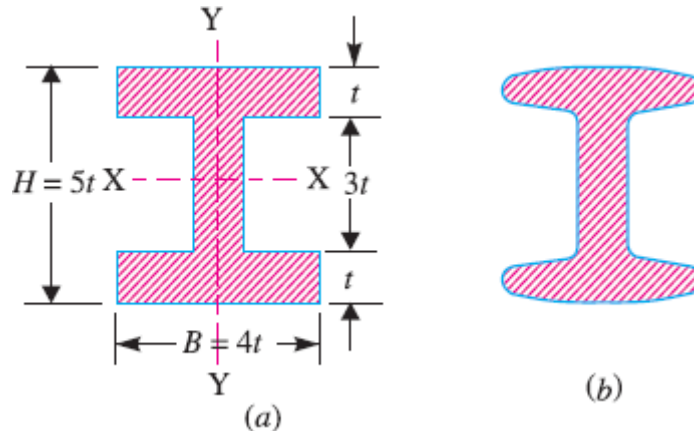


Figure.3.4: I section

$$A = 2(4t \times t) + 3t \times t = 11t^2$$

$$I_{xx} = \frac{1}{12} [4t(5t)^3 - 3t \times (3t)^3] = \frac{419}{12} t^4$$

and

$$I_{yy} = 2 \times \frac{1}{12} \times t(4t)^3 + \frac{1}{12} \times 3t \times t^3 = \frac{131}{12} t^4$$

$$\therefore \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since  $\frac{I_{xx}}{I_{yy}} = 3.2$ , therefore the section chosen is quite satisfactory.

Now let us find the dimensions of this *I*-section. Since the connecting rod is designed by taking the force on the connecting rod ( $F_C$ ) equal to the maximum force on the piston ( $F_L$ ) due to gas pressure, therefore,

$$F_C = F_L = \frac{\pi D^2}{4} \times p = \frac{\pi(100)^2}{4} \times 3.15 = 24\,740 \text{ N}$$

We know that the connecting rod is designed for buckling about X-axis (i.e. in the plane of motion of the connecting rod) assuming both ends hinged. Since a factor of safety is given as 6, therefore the buckling load,

$$W_B = F_C \times F. S. = 24\,740 \times 6 = 148\,440 \text{ N}$$

We know that radius of gyration of the section about  $X$ -axis,

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419t^4}{12} \times \frac{1}{11t^2}} = 1.78 t$$

Length of crank,

$$r = \frac{\text{Stroke of piston}}{2} = \frac{190}{2} = 95 \text{ mm}$$

Length of the connecting rod,

$$l = 380 \text{ mm}$$

...(Given)

$\therefore$  Equivalent length of the connecting rod for both ends hinged,

$$L = l = 380 \text{ mm}$$

Now according to Rankine's formula, we know that buckling load ( $W_B$ ),

$$148\,440 = \frac{\sigma_c A}{1 + a \left( \frac{L}{k_{xx}} \right)^2} = \frac{320 \times 11t^2}{1 + \frac{1}{7500} \left( \frac{380}{1.78t} \right)^2}$$

... (It is given that  $\sigma_c = 320 \text{ MPa}$  or  $\text{N/mm}^2$  and  $a = 1/7500$ )

$$\frac{148\,440}{320} = \frac{11t^2}{1 + \frac{6.1}{t^2}} = \frac{11t^4}{t^2 + 6.1}$$

$$\text{or } t^4 - 42.2t^2 - 257.3 = 0$$

$$\therefore t^2 = \frac{42.2 \pm \sqrt{(42.2)^2 + 4 \times 257.3}}{2} = \frac{42.2 \pm 53}{2} = 47.6$$

... (Taking +ve sign)

$$\text{or } t = 6.9 \text{ say } 7 \text{ mm}$$

Thus, the dimensions of  $I$ -section of the connecting rod are :

Thickness of flange and web of the section

$$= t = 7 \text{ mm Ans.}$$

Width of the section,  $B = 4t = 4 \times 7 = 28 \text{ mm Ans.}$

and depth or height of the section,

$$H = 5t = 5 \times 7 = 35 \text{ mm Ans.}$$

These dimensions are at the middle of the connecting rod. The width (B) is kept constant throughout the length of the rod, but the depth (H) varies. The depth near the big end or crank end is kept as 1.1H to 1.25H and the depth near the small end or piston end is kept as 0.75H to 0.9H. Let us take

Depth near the big end,

$$H_1 = 1.2H = 1.2 \times 35 = 42 \text{ mm and depth near the small end,}$$

$$H_2 = 0.85H = 0.85 \times 35 = 29.75 \text{ say } 30 \text{ mm}$$

$\therefore$  Dimensions of the section near the big end

= 42 mm × 28 mm Ans. and dimensions of the section near the small end

= 30 mm × 28 mm Ans.

Since the connecting rod is manufactured by forging, therefore the sharp corners of I-section are rounded off, as shown in Fig. (b), for easy removal of the section from the dies.

## 2. Dimensions of the crankpin or the big end bearing and piston pin or small end bearing

Let  $d_c$  = Diameter of the crankpin or big end bearing,

$l_c$  = length of the crankpin or big end bearing =  $1.3 d_c$  ...(Given)

$p_{bc}$  = Bearing pressure = 10 N/mm<sup>2</sup> ...(Given)

We know that load on the crankpin or big end bearing

= Projected area × Bearing pressure

$$= d_c \cdot l_c \cdot p_{bc} = d_c \times 1.3 d_c \times 10 = 13 (d_c)^2$$

Since the crankpin or the big end bearing is designed for the maximum gas force ( $F_L$ ), therefore, equating the load on the crankpin or big end bearing to the maximum gas force, i.e.

$$13 (d_c)^2 = F_L = 24\,740 \text{ N}$$

$$\therefore (d_c)^2 = 24\,740 / 13 = 1903 \text{ or } d_c = 43.6 \text{ say } 44 \text{ mm Ans.}$$

$$\text{and } l_c = 1.3 d_c = 1.3 \times 44 = 57.2 \text{ say } 58 \text{ mm Ans.}$$

The big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1mm or less) of bearing metal such as babbit.

Again, let  $d_p$  = Diameter of the piston pin or small end bearing,

$l_p$  = Length of the piston pin or small end bearing =  $2d_p$  ...(Given)

$p_{bp}$  = Bearing pressure = 15 N/mm<sup>2</sup> ..(Given)

We know that the load on the piston pin or small end bearing

= Project area × Bearing pressure

$$= d_p \cdot l_p \cdot p_{bp} = d_p \times 2 d_p \times 15 = 30 (d_p)^2$$

Since the piston pin or the small end bearing is designed for the maximum gas force ( $F_L$ ), therefore, equating the load on the piston pin or the small end bearing to the maximum gas force,

$$30 (d_p)^2 = 24\,740 \text{ N}$$

$$\therefore (d_p)^2 = 24\,740 / 30 = 825 \text{ or } d_p = 28.7 \text{ say } 29 \text{ mm Ans.}$$

$$\text{and } l_p = 2 d_p = 2 \times 29 = 58 \text{ mm Ans.}$$

The small end bearing is usually a phosphor bronze bush of about 3 mm thickness.

### 3. Size of bolts for securing the big end cap

Let  $d_{cb}$  = Core diameter of the bolts,

$\sigma_t$  = Allowable tensile stress for the material of the bolts

$$= 60 \text{ N/mm}^2 \dots (\text{Given})$$

and  $n_b$  = Number of bolts. Generally two bolts are used.

We know that force on the bolts

$$= \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b = \frac{\pi}{4} (d_{cb})^2 60 \times 2 = 94.26 (d_{cb})^2$$

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force of the reciprocating parts,

$$F_I = m_R \cdot \omega^2 \cdot r \left( \cos \theta + \frac{\cos 2\theta}{l/r} \right)$$

We also know that at top dead centre on the exhaust stroke,  $\theta = 0$ .

$$\therefore F_I = m_R \cdot \omega^2 \cdot r \left( 1 + \frac{r}{l} \right) = 2.25 \left( \frac{2\pi \times 1800}{60} \right)^2 0.095 \left( 1 + \frac{0.095}{0.38} \right) \text{ N}$$

$$= 9490 \text{ N}$$

Equating the inertia force to the force on the bolts, we have

$$9490 = 94.26 (d_{cb})^2 \text{ or } (d_{cb})^2 = 9490 / 94.26 = 100.7$$

$$\therefore d_{cb} = 10.03 \text{ mm}$$

and nominal diameter of the bolt,

$$d_b = \frac{d_{cb}}{0.84} = \frac{10.03}{0.84} = 11.94$$

### 4. Thickness of the big end cap

Let  $t_c$  = Thickness of the big end cap,

$b_c$  = Width of the big end cap.



t is taken equal to the length of the crankpin or big end bearing ( $l_c$ )

= 58 mm (calculated above)

$\sigma_b$  = Allowable bending stress for the material of the cap

= 80 N/mm<sup>2</sup> ...(Given)

The big end cap is designed as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (i.e. FI when  $\theta = 0$ ). Since the load is assumed to act in between the uniformly distributed load and the centrally concentrated load, therefore, maximum bending moment is taken as

$$M_C = \frac{F_I \times x}{6}$$

= Dia. of crank pin or big end bearing + 2 × Thickness of bearing liner + Nominal dia. of bolt + Clearance

= ( $d_c + 2 \times 3 + d_b + 3$ ) mm = 44 + 6 + 12 + 3 = 65 mm

∴ Maximum bending moment acting on the cap,

$$\begin{aligned}
&= \text{Dia. of crank pin or big end bearing} + 2 \times \text{Thickness of bearing liner} + \text{Nominal dia. of bolt} + \text{Clearance} \\
&= (d_c + 2 \times 3 + d_b + 3) \text{ mm} = 44 + 6 + 12 + 3 = 65 \text{ mm}
\end{aligned}$$

∴ Maximum bending moment acting on the cap,

$$M_C = \frac{F_I \times x}{6} = \frac{9490 \times 65}{6} = 102\,810 \text{ N-mm}$$

Section modulus for the cap

$$Z_C = \frac{b_c (t_c)^2}{6} = \frac{58 (t_c)^2}{6} = 9.7 (t_c)^2$$

We know that bending stress ( $\sigma_b$ ),

$$80 = \frac{M_C}{Z_C} = \frac{102\,810}{9.7 (t_c)^2} = \frac{10\,600}{(t_c)^2}$$

$$\therefore (t_c)^2 = 10\,600 / 80 = 132.5 \quad \text{or} \quad t_c = 11.5 \text{ mm} \quad \text{Ans.}$$

Let us now check the design for the induced bending stress due to inertia bending forces on the connecting rod (*i.e.* whipping stress).

We know that mass of the connecting rod per metre length,

$$\begin{aligned}
m_1 &= \text{Volume} \times \text{density} = \text{Area} \times \text{length} \times \text{density} \\
&= A \times l \times \rho = 11t^2 \times l \times \rho \quad \dots (\because A = 11t^2) \\
&= 11(0.007)^2 (0.38) 8000 = 1.64 \text{ kg} \quad \dots [\because \rho = 8\,000 \text{ kg/m}^3 \text{ (given)}]
\end{aligned}$$

∴ Maximum bending moment,

$$\begin{aligned}
M_{\max} &= m \cdot \omega^2 \cdot r \times \frac{l}{9\sqrt{3}} = m_1 \cdot \omega^2 \cdot r \times \frac{l^2}{9\sqrt{3}} \quad \dots (\because m = m_1 \cdot l) \\
&= 1.64 \left( \frac{2\pi \times 1800}{60} \right)^2 (0.095) \frac{(0.38)^2}{9\sqrt{3}} = 51.3 \text{ N-m} \\
&= 51\,300 \text{ N-mm}
\end{aligned}$$

and section modulus, 
$$Z_{xx} = \frac{I_{xx}}{5t/2} = \frac{419 t^4}{12} \times \frac{2}{5t} = 13.97 t^3 = 13.97 \times 7^3 = 4792 \text{ mm}^3$$

∴ Maximum bending stress (induced) due to inertia bending forces or whipping stress,

$$\sigma_{b(\max)} = \frac{M_{\max}}{Z_{xx}} = \frac{51\,300}{4792} = 10.7 \text{ N/mm}^2$$

Since the maximum bending stress induced is less than the allowable bending stress of 80 N/mm<sup>2</sup>, therefore the design is safe.

## CRANKSHAFT

A crankshaft (i.e. a shaft with a crank) is used to convert reciprocating motion of the piston into rotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the main bearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms or webs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, depending Upon the position of crank, may be divided into the following two types:

1. Side crankshaft or overhung crankshaft, as shown in Fig. (a), and
2. Centre crankshaft, as shown in Fig. 32.(b).

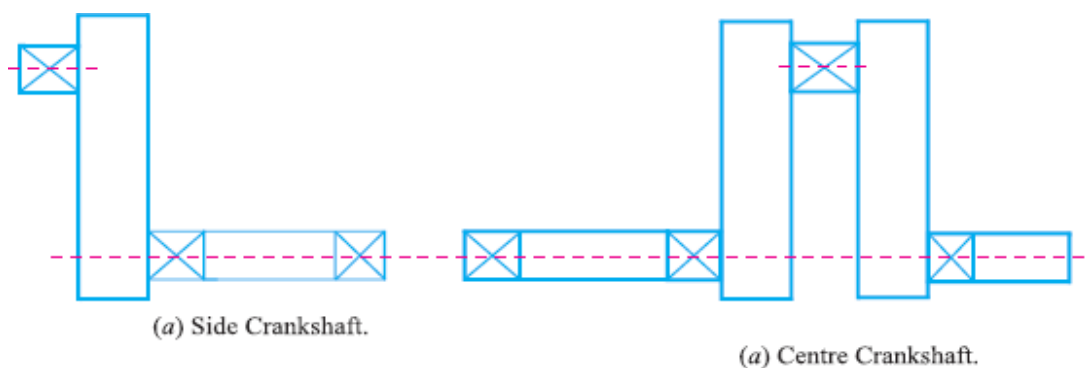


Figure.3.5: Types of Crank Shaft

The crankshaft, depending upon the number of cranks in the shaft, may also be classified as single throw or multi-throw crankshafts. A crankshaft with only one side crank or center crank is called a single throw crankshaft whereas the crankshaft with two side cranks, one on each end or with two or more center cranks is known as multi-throw crankshaft. The side crankshafts are used for medium and large size horizontal engines.

### Material and manufacture of Crankshafts

The crankshafts are subjected to shock and fatigue loads. Thus material of the crankshaft should be tough and fatigue resistant. The crankshafts are generally made of carbon steel, special steel or special cast iron.

In industrial engines, the crankshafts are commonly made from carbon steel such as 40 C 8, 55 C 8 and 60 C 4. In transport engines, manganese steel such as 20 Mn 2, 27 Mn 2 and 37 Mn 2 are generally used for the making of crankshaft. In aero engines, nickel chromium steel such as 35 Ni 1 Cr 60 and 40 Ni 2 Cr 1 Mo 28 are extensively used for the crankshaft.

The crankshafts are made by drop forging or casting process but the former method is more common. The surface of the crankpin is hardened by case carburizing, nitriding or induction hardening.

### **Bearing Pressures and Stresses in Crankshaft**

The bearing pressures are very important in the design of crankshafts. The maximum permissible bearing pressure depends upon the maximum gas pressure, journal velocity, amount and method of lubrication and change of direction of bearing pressure. The following two types of stresses are induced in the crankshaft.

1. Bending stress ; and 2. Shear stress due to torsional moment on the shaft

Most crankshaft failures are caused by a progressive fracture due to repeated bending or reversed torsional stresses. Thus the crankshaft is under fatigue loading and, therefore, its design should be based upon endurance limit. Since the failure of a crankshaft is likely to cause a serious engine destruction and neither all the forces nor all the stresses acting on the crankshaft can be determined accurately, therefore a high factor of safety from 3 to 4, based on the endurance limit, is used.

### **Design Procedure for Crankshaft**

The following procedure may be adopted for designing a crankshaft.

1. First of all, find the magnitude of the various loads on the crankshaft.
2. Determine the distances between the supports and their position with respect to the loads.
3. For the sake of simplicity and also for safety, the shaft is considered to be supported at the centres of the bearings and all the forces and reactions to be acting at these points. The distances between the supports depend on the length of the bearings, which in turn depend on the diameter of the shaft because of the allowable bearing pressures.
4. The thickness of the cheeks or webs is assumed to be from  $0.4 d_s$  to  $0.6 d_s$ , where  $d_s$  is the diameter of the shaft. It may also be taken as  $0.22D$  to  $0.32 D$ , where  $D$  is the bore of cylinder in mm.
5. Now calculate the distances between the supports.
6. Assuming the allowable bending and shear stresses, determine the main dimensions of the crankshaft.

### **Design of Centre Crankshaft**

We shall design the centre crankshaft by considering the two crank positions, i.e. when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and

when the crank is at angle at which the twisting moment is maximum. These two cases are discussed in detail as below :

1. **When the crank is at dead centre.** At this position of the crank, the maximum gas pressure on the piston will transmit maximum force on the crankpin in the plane of the crank causing only bending of the shaft. The crankpin as well as ends of the crankshaft will be only subjected to bending moment. Thus, when the crank is at the dead centre, the bending moment on the shaft is maximum and the twisting moment is zero.

$D$  = Piston diameter or cylinder bore in mm,

$p$  = Maximum intensity of pressure on the piston in  $\text{N/mm}^2$ ,

$W$  = Weight of the flywheel acting downwards in N, and  $T_1 + T_2$  = Resultant belt tension or pull acting horizontally in N. The thrust in the connecting rod will be equal to the gas load on the piston (FP). We know that gas load on the piston,

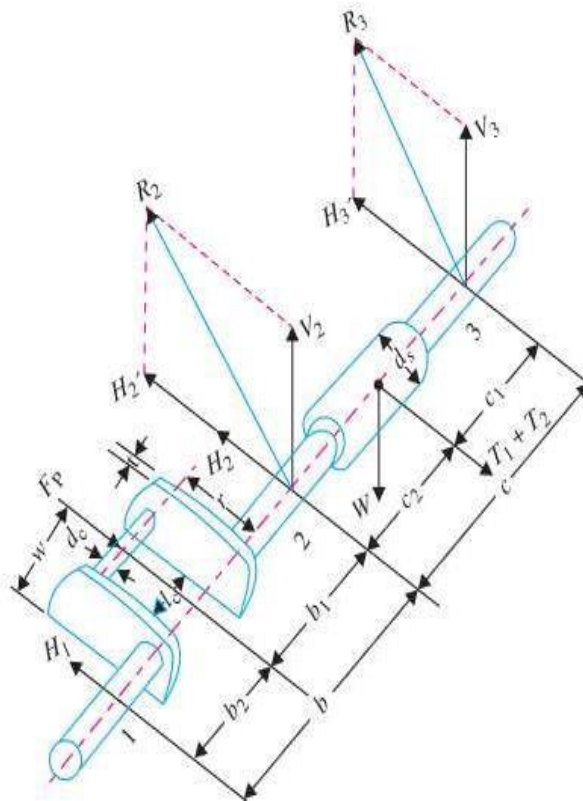


Figure.3.6: Crank Shaft

$$F_p = \frac{\pi}{4} \times D^2 \times p$$

Due to this piston gas load ( $F_p$ ) acting horizontally, there will be two horizontal reactions  $H_1$  and  $H_2$  at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_p \times b_1}{b}; \quad \text{and} \quad H_2 = \frac{F_p \times b_2}{b}$$

Due to the weight of the flywheel ( $W$ ) acting downwards, there will be two vertical reactions  $V_2$  and  $V_3$  at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c}; \quad \text{and} \quad V_3 = \frac{W \times c_2}{c}$$

Now due to the resultant belt tension ( $T_1 + T_2$ ), acting horizontally, there will be two horizontal reactions  $H_2'$  and  $H_3'$  at bearings 2 and 3 respectively, such that

$$H_2' = \frac{(T_1 + T_2) c_1}{c}; \quad \text{and} \quad H_3' = \frac{(T_1 + T_2) c_2}{c}$$

The resultant force at bearing 2 is given by

$$R_2 = \sqrt{(H_2 + H_2')^2 + (V_2)^2}$$

and the resultant force at bearing 3 is given by

$$R_3 = \sqrt{(H_3)^2 + (V_3)^2}$$

Now the various parts of the centre crankshaft are designed for bending only, as discussed below:

**(a) Design of crankpin**

Let  $d_c$  = Diameter of the crankpin in mm,  
 $l_c$  = Length of the crankpin in mm,  
 $\sigma_b$  = Allowable bending stress for the crankpin in N/mm<sup>2</sup>.

We know that bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 \quad \dots(i)$$

We also know that

$$M_C = \frac{\pi}{32} (d_c)^3 \sigma_b \quad \dots(ii)$$

From equations (i) and (ii), diameter of the crankpin is determined. The length of the crankpin is given by

$$l_c = \frac{F_p}{d_c \cdot p_b}$$

where

$p_b$  = Permissible bearing pressure in N/mm<sup>2</sup>.

**(b) Design of left hand crank web**

The crank web is designed for eccentric loading. There will be two stresses acting on the crank web, one is direct compressive stress and the other is bending stress due to piston gas load ( $F_p$ ).

The thickness (t) of the crank web is given empirically as  $t = 0.4 d_s$  to  $0.6$

$$d_s = 0.22D \text{ to } 0.32D = 0.65 d_c + 6.35 \text{ mm}$$

where  $d_s$  = Shaft diameter in mm,

$D$  = Bore diameter in mm, and  $d_c$  = Crankpin diameter in mm,

The width of crank web ( $w$ ) is taken as  $w = 1.125 d_c + 12.7 \text{ mm}$

We know that maximum bending moment on the crank web,

$$M = H_1 \left( b_2 - \frac{l_c}{2} - \frac{t}{2} \right)$$

and section modulus,  $Z = \frac{1}{6} \times w \cdot t^2$

$$\therefore \text{Bending stress, } \sigma_b = \frac{M}{Z} = \frac{6H_1 \left( b_2 - \frac{l_c}{2} - \frac{t}{2} \right)}{w \cdot t^2}$$

and direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w \cdot t}$$

$\therefore$  Total stress on the crank web

$$= \text{Bending stress} + \text{Direct stress} = \sigma_b + \sigma_c$$

$$= \frac{6H_1 \left( b_2 - \frac{l_c}{2} - \frac{t}{2} \right)}{w \cdot t^2} + \frac{H_1}{w \cdot t}$$

This total stress should be less than the permissible bending stress.

**(c) Design of right hand crank web**

The dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to left hand crank web from the balancing point of view.

**(d) Design of shaft under the flywheel**

Let  $d_s$  = Diameter of shaft in mm.

We know that bending moment due to the weight of flywheel,

$$M_W = V_3 \cdot c_1$$

and bending moment due to belt tension,

$$M_T = H_3' \cdot c_1$$

These two bending moments act at right angles to each other. Therefore, the resultant bending moment at the flywheel location,

$$M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(V_3 \cdot c_1)^2 + (H_3' \cdot c_1)^2} \quad \dots (i)$$

We also know that the bending moment at the shaft,

$$M_S = \frac{\pi}{32} (d_s)^3 \sigma_b \quad \dots (ii)$$

where  $\sigma_b$  = Allowable bending stress in N/mm<sup>2</sup>.

From equations (i) and (ii), we may determine the shaft diameter ( $d_s$ ).

### When the crank is at an angle of maximum twisting moment

The twisting moment on the crankshaft will be maximum when the tangential force on the crank ( $F_T$ ) is maximum. The maximum value of tangential force lies when the crank is at angle of 25° to 30° from the dead centre for a constant volume combustion engines (*i.e.*, petrol engines) and 30° to 40° for constant pressure combustion engines (*i.e.*, diesel engines). Consider a position of the crank at an angle of maximum twisting moment as shown in Fig. If  $p'$  is the intensity of pressure on the piston at this instant, then the piston gas load at this position of crank



and thrust on the connecting rod,

$$F_P = \frac{\pi}{4} \times D^2 \times p'$$

$$F_Q = \frac{F_P}{\cos \phi}$$

where

$\phi$  = Angle of inclination of the connecting rod with the line of stroke  $PO$ .

The thrust in the connecting rod ( $F_Q$ ) may be divided into two components, one perpendicular to the crank and the other along the crank. The component of  $F_Q$  perpendicular to the crank is the tangential force ( $F_T$ ) and the component of  $F_Q$  along the crank is the radial force ( $F_R$ ) which produces thrust on the crankshaft bearings. From Fig, we find that

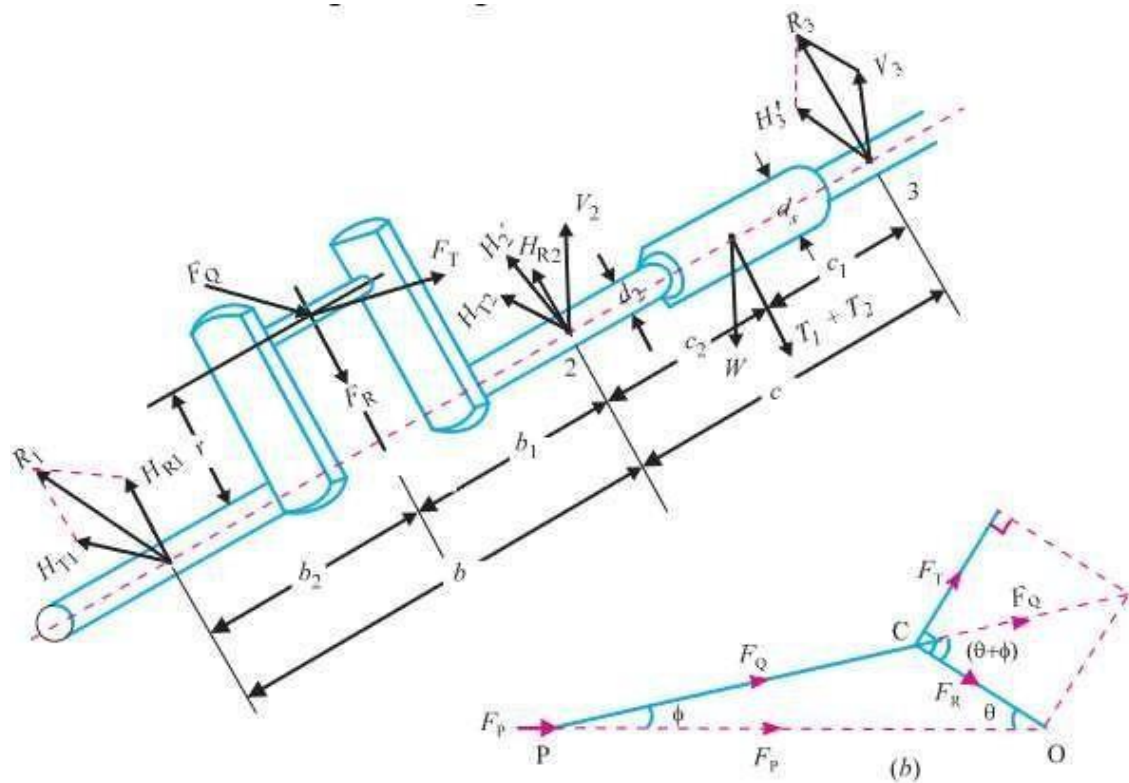


Figure.3.7: Crank Shaft

$$F_T = F_Q \sin (\theta + \phi)$$

and

$$F_R = F_Q \cos (\theta + \phi)$$

It may be noted that the tangential force will cause twisting of the crankpin and shaft while the radial force will cause bending of the shaft.

Due to the tangential force ( $F_T$ ), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b}; \quad \text{and} \quad H_{T2} = \frac{F_T \times b_2}{b}$$

Due to the radial force ( $F_R$ ), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b}; \quad \text{and} \quad H_{R2} = \frac{F_R \times b_2}{b}$$

The reactions at the bearings 2 and 3, due to the flywheel weight ( $W$ ) and resultant belt pull ( $T_1 + T_2$ ) will be same as discussed earlier.

Now the various parts of the crankshaft are designed as discussed below :

#### (a) Design of crankpin

Let  $d_c$  = Diameter of the crankpin in mm.

We know that bending moment at the centre of the crankpin,

$$M_C = H_{R1} \times b_2$$

and twisting moment on the crankpin,

$$T_C = H_{T1} \times r$$

∴ Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(H_{R1} \times b_2)^2 + (H_{T1} \times r)^2} \quad \dots(i)$$

We also know that twisting moment on the crankpin,

$$T_e = \frac{\pi}{16} (d_c)^3 \tau \quad \dots(ii)$$

where

$\tau$  = Allowable shear stress in the crankpin.

From equations (i) and (ii), the diameter of the crankpin is determined.

**(b) Design of shaft under the flywheel**

Let  $d_s$  = Diameter of the shaft in mm.

We know that bending moment on the shaft,

$$M_s = R_3 \times c_1$$

and twisting moment on the shaft,

$$T_s = F_T \times r$$

∴ Equivalent twisting moment on the shaft,

$$T_e = \sqrt{(M_s)^2 + (T_s)^2} = \sqrt{(R_3 \times c_1)^2 + (F_T \times r)^2} \quad \dots (i)$$

We also know that equivalent twisting moment on the shaft,

$$T_e = \frac{\pi}{16} (d_s)^3 \tau \quad \dots (ii)$$

where  $\tau$  = Allowable shear stress in the shaft.

From equations (i) and (ii), the diameter of the shaft is determined.

**(c) Design of shaft at the juncture of right hand crank arm**

Let  $d_{s1}$  = Diameter of the shaft at the juncture of right hand crank arm.

We know that bending moment at the juncture of the right hand crank arm,

$$M_{s1} = R_1 \left( b_2 + \frac{l_c}{2} + \frac{t}{2} \right) - F_Q \left( \frac{l_c}{2} + \frac{t}{2} \right)$$

and the twisting moment at the juncture of the right hand crank arm,

$$T_{s1} = F_T \times r$$

∴ Equivalent twisting moment at the juncture of the right hand crank arm,

$$T_e = \sqrt{(M_{s1})^2 + (T_{s1})^2} \quad \dots (i)$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16} (d_{s1})^3 \tau \quad \dots (ii)$$

where  $\tau$  = Allowable shear stress in the shaft.

From equations (i) and (ii), the diameter of the shaft at the juncture of the right hand crank arm is determined.

**(d) Design of right hand crank web**

The right hand crank web is subjected to the following stresses:

- (i) Bending stresses in two planes normal to each other, due to the radial and tangential components of  $F_Q$ .
- (ii) Direct compressive stress due to  $F_R$ , and
- (iii) Torsional stress.

The bending moment due to the radial component of  $F_Q$  is given by,

$$M_R = H_{R2} \left( b_1 - \frac{l_c}{2} - \frac{t}{2} \right) \quad \dots (i)$$

We also know that  $M_R = \sigma_{bR} \times Z = \sigma_{bR} \times \frac{1}{6} \times w \cdot t^2 \quad \dots (ii)$

where

$\sigma_{bR}$  = Bending stress in the radial direction, and

$$Z = \text{Section modulus} = \frac{1}{6} \times w \cdot t^2$$

From equations (i) and (ii), the value of bending stress  $\sigma_{bR}$  is determined.

The bending moment due to the tangential component of  $F_Q$  is maximum at the juncture of crank and shaft. It is given by

$$M_T = F_T \left[ r - \frac{d_{s1}}{2} \right] \quad \dots (iii)$$

where

$d_{s1}$  = Shaft diameter at juncture of right hand crank arm, i.e. at bearing 2.

$$\text{We also know that } M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t \cdot w^2 \quad \dots (iv)$$

where

$\sigma_{bT}$  = Bending stress in tangential direction.

From equations (iii) and (iv), the value of bending stress  $\sigma_{bT}$  is determined.

The direct compressive stress is given by,

$$\sigma_d = \frac{F_R}{2w \cdot t}$$

The maximum compressive stress ( $\sigma_c$ ) will occur at the upper left corner of the cross-section of the crank.

$$\therefore \sigma_c = \sigma_{bR} + \sigma_{bT} + \sigma_d$$

Now, the twisting moment on the arm,

$$T = H_{T1} \left( b_2 + \frac{l_c}{2} \right) - F_T \times \frac{l_c}{2} = H_{T2} \left( b_1 - \frac{l_c}{2} \right)$$

We know that shear stress on the arm,

$$\tau = \frac{T}{Z_p} = \frac{4.5 T}{w \cdot t^2}$$

where

$$Z_p = \text{Polar section modulus} = \frac{w \cdot t^2}{4.5}$$

$\therefore$  Maximum or total combined stress,

$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2} \sqrt{(\sigma_c)^2 + 4\tau^2}$$

The value of  $(\sigma_c)_{max}$  should be within safe limits. If it exceeds the safe value, then the dimension  $w$  may be increased because it does not affect other dimensions.

#### (e) Design of left hand crank web

Since the left hand crank web is not stressed to the extent as the right hand crank web, therefore, the dimensions for the left hand crank web may be made same as for right hand crank web.

#### (f) Design of crankshaft bearings

The bearing 2 is the most heavily loaded and should be checked for the safe bearing pressure.

We know that the total reaction at the bearing 2,

$$R_2 = \frac{F_p}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}$$

$$\therefore \text{Total bearing pressure} = \frac{R_2}{l_2 \cdot d_{s1}}$$

where

$l_2$  = Length of bearing 2.

**Problem 1.** Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data:

Bore = 400 mm ; Stroke = 600 mm ; Engine speed = 200 r.p.m. ; Mean effective pressure = 0.5 N/mm<sup>2</sup>; Maximum combustion pressure = 2.5 N/mm<sup>2</sup>; Weight of flywheel used as a pulley = 50 kN; Total belt pull = 6.5 kN. When the crank has turned through 35° from the top dead centre, the pressure on the piston is 1N/mm<sup>2</sup> and the torque on the crank is maximum. The ratio of the connecting rod length to the crank radius is 5. Assume any other data required for the design.

**Solution.** Given : D = 400 mm ; L = 600 mm or r = 300 mm ; p<sub>m</sub> = 0.5 N/mm<sup>2</sup> ; p = 2.5 N/mm<sup>2</sup>; W = 50 kN ; T<sub>1</sub> + T<sub>2</sub> = 6.5 kN ; θ = 35° ; p' = 1N/mm<sup>2</sup> ; l / r = 5 We shall design the crankshaft for the two positions of the crank, i.e. firstly when the crank is at the dead centre ; and secondly when the crank is at an angle of maximum twisting moment.

1. Design of the crankshaft when the crank is at the dead centre We know that the piston gas load,

$$F_p = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (400)^2 2.5 = 314\,200 \text{ N} = 314.2 \text{ kN}$$

Assume that the distance (b) between the bearings 1 and 2 is equal to twice the piston diameter (D).

$$\therefore b = 2D = 2 \times 400 = 800 \text{ mm}$$

and 
$$b_1 = b_2 = \frac{b}{2} = \frac{800}{2} = 400 \text{ mm}$$

We know that due to the piston gas load, there will be two horizontal reactions  $H_1$  and  $H_2$  at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_p \times b_1}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}$$

and 
$$H_2 = \frac{F_p \times b_2}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}$$

Assume that the length of the main bearings to be equal, i.e.,  $c_1 = c_2 = c / 2$ .

We know that due to the weight of the flywheel acting downwards, there will be two vertical reactions  $V_2$  and  $V_3$  at bearings 2 and 3 respectively, such that



$$V_2 = \frac{W \times c_1}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}$$

and

$$V_3 = \frac{W \times c_2}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}$$

Due to the resultant belt tension  $(T_1 + T_2)$  acting horizontally, there will be two horizontal reactions  $H_2'$  and  $H_3'$  respectively, such that

$$H_2' = \frac{(T_1 + T_2) c_1}{c} = \frac{(T_1 + T_2) c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25 \text{ kN}$$

and

$$H_3' = \frac{(T_1 + T_2) c_2}{c} = \frac{(T_1 + T_2) c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25 \text{ kN}$$

Now the various parts of the crankshaft are designed as discussed below:

**(a) Design of crankpin**

Let

$d_c$  = Diameter of the crankpin in mm ;

$l_c$  = Length of the crankpin in mm ; and

$\sigma_b$  = Allowable bending stress for the crankpin. It may be assumed as 75 MPa or N/mm<sup>2</sup>.

We know that the bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 = 157.1 \times 400 = 62\,840 \text{ kN-mm} \quad \dots(i)$$

We also know that

$$\begin{aligned} M_C &= \frac{\pi}{32} (d_c)^3 \sigma_b = \frac{\pi}{32} (d_c)^3 75 = 7.364 (d_c)^3 \text{ N-mm} \\ &= 7.364 \times 10^{-3} (d_c)^3 \text{ kN-mm} \end{aligned} \quad \dots(ii)$$

Equating equations (i) and (ii), we have

$$(d_c)^3 = 62\,840 / 7.364 \times 10^{-3} = 8.53 \times 10^6$$

or

$$d_c = 204.35 \text{ say } 205 \text{ mm Ans.}$$

We know that length of the crankpin,

$$l_c = \frac{F_p}{d_c \cdot p_b} = \frac{314.2 \times 10^3}{205 \times 10} = 153.3 \text{ say } 155 \text{ mm Ans.}$$

...(Taking  $p_b = 10 \text{ N/mm}^2$ )

**(b) Design of left hand crank web**

We know that thickness of the crank web,

$$\begin{aligned} t &= 0.65 d_c + 6.35 \text{ mm} \\ &= 0.65 \times 205 + 6.35 = 139.6 \text{ say } 140 \text{ mm Ans.} \end{aligned}$$

and width of the crank web,  $w = 1.125 d_c + 12.7 \text{ mm}$

$$= 1.125 \times 205 + 12.7 = 243.3 \text{ say } 245 \text{ mm Ans.}$$

We know that maximum bending moment on the crank web,

$$\begin{aligned} M &= H_1 \left( b_2 - \frac{l_c}{2} - \frac{t}{2} \right) \\ &= 157.1 \left( 400 - \frac{155}{2} - \frac{140}{2} \right) = 39\,668 \text{ kN-mm} \end{aligned}$$

$$\text{Section modulus, } Z = \frac{1}{6} \times w \cdot t^2 = \frac{1}{6} \times 245 (140)^2 = 800 \times 10^3 \text{ mm}^3$$

$$\therefore \text{ Bending stress, } \sigma_b = \frac{M}{Z} = \frac{39\,668}{800 \times 10^3} = 49.6 \times 10^{-3} \text{ kN/mm}^2 = 49.6 \text{ N/mm}^2$$

We know that direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w \cdot t} = \frac{157.1}{245 \times 140} = 4.58 \times 10^{-3} \text{ kN/mm}^2 = 4.58 \text{ N/mm}^2$$

$\therefore$  Total stress on the crank web

$$= \sigma_b + \sigma_c = 49.6 + 4.58 = 54.18 \text{ N/mm}^2 \text{ or MPa}$$

Since the total stress on the crank web is less than the allowable bending stress of 75 MPa, therefore, the design of the left hand crank web is safe.

**(c) Design of right hand crank web**

From the balancing point of view, the dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to the dimensions of the left hand crank web.

**(d) Design of shaft under the flywheel**

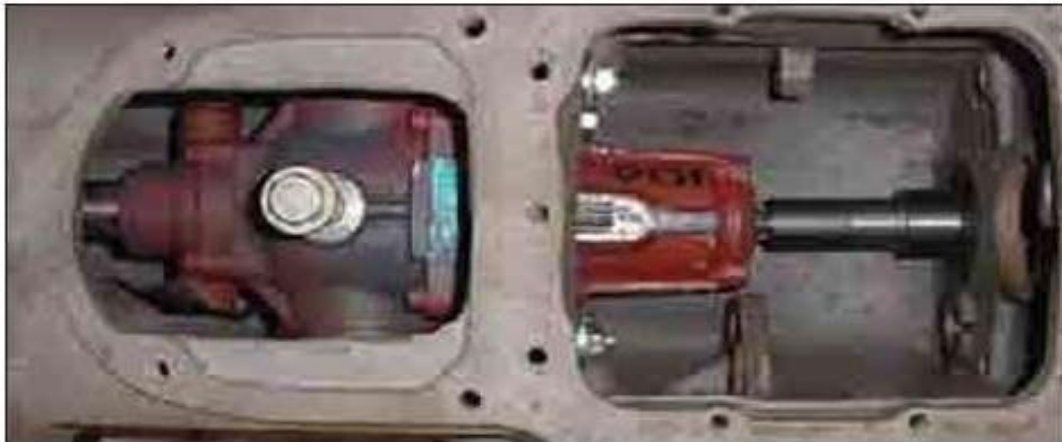
Let  $d_s$  = Diameter of the shaft in mm.

Since the lengths of the main bearings are equal, therefore

$$l_1 = l_2 = l_3 = 2 \left( \frac{b}{2} - \frac{l_c}{2} - t \right) = 2 \left( 400 - \frac{155}{2} - 140 \right) = 365 \text{ mm}$$

Assuming width of the flywheel as 300 mm, we have

$$c = 365 + 300 = 665 \text{ mm}$$



*Hydrostatic transmission inside a tractor engine*

Figure.3.8: Hydrostatic Transmission

Allowing space for gearing and clearance, let us take  $c = 800$  mm.

$$\therefore c_1 = c_2 = \frac{c}{2} = \frac{800}{2} = 400 \text{ mm}$$

We know that bending moment due to the weight of flywheel,

$$M_W = V_3 \cdot c_1 = 25 \times 400 = 10\,000 \text{ kN-mm} = 10 \times 10^6 \text{ N-mm}$$

and bending moment due to the belt pull,

$$M_T = H_3' \cdot c_1 = 3.25 \times 400 = 1300 \text{ kN-mm} = 1.3 \times 10^6 \text{ N-mm}$$

$\therefore$  Resultant bending moment on the shaft,

$$\begin{aligned} M_S &= \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(10 \times 10^6)^2 + (1.3 \times 10^6)^2} \\ &= 10.08 \times 10^6 \text{ N-mm} \end{aligned}$$

We also know that bending moment on the shaft ( $M_S$ ),

$$10.08 \times 10^6 = \frac{\pi}{32} (d_s)^3 \sigma_b = \frac{\pi}{32} (d_s)^3 42 = 4.12 (d_s)^3$$

$$\therefore (d_s)^3 = 10.08 \times 10^6 / 4.12 = 2.45 \times 10^6 \text{ or } d_s = 134.7 \text{ say } 135 \text{ mm Ans.}$$

## 2. Design of the crankshaft when the crank is at an angle of maximum twisting moment

We know that piston gas load,

$$F_p = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4} (400)^2 1 = 125\,680 \text{ N} = 125.68 \text{ kN}$$

In order to find the thrust in the connecting rod ( $F_Q$ ), we should first find out the angle of inclination of the connecting rod with the line of stroke (i.e. angle  $\phi$ ). We know that

$$\sin \phi = \frac{\sin \theta}{l/r} = \frac{\sin 35^\circ}{5} = 0.1147$$

$$\therefore \phi = \sin^{-1} (0.1147) = 6.58^\circ$$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_p}{\cos \phi} = \frac{125.68}{\cos 6.58^\circ} = \frac{125.68}{0.9934} = 126.5 \text{ kN}$$

Tangential force acting on the crankshaft,

$$F_T = F_Q \sin (\theta + \phi) = 126.5 \sin (35^\circ + 6.58^\circ) = 84 \text{ kN}$$

and radial force,  $F_R = F_Q \cos (\theta + \phi) = 126.5 \cos (35^\circ + 6.58^\circ) = 94.6 \text{ kN}$

Due to the tangential force ( $F_T$ ), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$$

and  $H_{T2} = \frac{F_T \times b_2}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$

Due to the radial force ( $F_R$ ), there will be two reactions at bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

$$H_{R2} = \frac{F_R \times b_2}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

Now the various parts of the crankshaft are designed as discussed below:

### (a) Design of crankpin

Let  $d_c$  = Diameter of crankpin in mm.



We know that the bending moment at the centre of the crankpin,

$$M_C = H_{R1} \times b_2 = 47.3 \times 400 = 18\,920 \text{ kN-mm}$$

and twisting moment on the crankpin,

$$T_C = H_{T1} \times r = 42 \times 300 = 12\,600 \text{ kN-mm}$$

∴ Equivalent twisting moment on the crankpin,

$$\begin{aligned} T_e &= \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(18\,920)^2 + (12\,600)^2} \\ &= 22\,740 \text{ kN-mm} = 22.74 \times 10^6 \text{ N-mm} \end{aligned}$$

We know that equivalent twisting moment ( $T_e$ ),

$$22.74 \times 10^6 = \frac{\pi}{16} (d_c)^3 \tau = \frac{\pi}{16} (d_c)^3 35 = 6.873 (d_c)^3 \quad \dots (\text{Taking } \tau = 35 \text{ MPa or N/mm}^2)$$

$$\therefore (d_c)^3 = 22.74 \times 10^6 / 6.873 = 3.3 \times 10^6 \text{ or } d_c = 149 \text{ mm}$$

Since this value of crankpin diameter (*i.e.*  $d_c = 149 \text{ mm}$ ) is less than the already calculated value of  $d_c = 205 \text{ mm}$ , therefore, we shall take  $d_c = 205 \text{ mm}$ . **Ans.**

**(b) Design of shaft under the flywheel**

Let  $d_s$  = Diameter of the shaft in mm.

The resulting bending moment on the shaft will be same as calculated earlier, *i.e.*

$$M_s = 10.08 \times 10^6 \text{ N-mm}$$

and twisting moment on the shaft,

$$T_s = F_T \times r = 84 \times 300 = 25\,200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment on shaft,

$$\begin{aligned} T_e &= \sqrt{(M_s)^2 + (T_s)^2} \\ &= \sqrt{(10.08 \times 10^6)^2 + (25.2 \times 10^6)^2} = 27.14 \times 10^6 \text{ N-mm} \end{aligned}$$

We know that equivalent twisting moment ( $T_e$ ),

$$27.14 \times 10^6 = \frac{\pi}{16} (d_s)^3 \tau = \frac{\pi}{16} (135)^3 \tau = 483\,156 \tau$$

$$\therefore \tau = 27.14 \times 10^6 / 483\,156 = 56.17 \text{ N/mm}^2$$

From above, we see that by taking the already calculated value of  $d_s = 135 \text{ mm}$ , the induced shear stress is more than the allowable shear stress of 31 to 42 MPa. Hence, the value of  $d_s$  is calculated by taking  $\tau = 35 \text{ MPa or N/mm}^2$  in the above equation, *i.e.*

$$27.14 \times 10^6 = \frac{\pi}{16} (d_s)^3 35 = 6.873 (d_s)^3$$

$$\therefore (d_s)^3 = 27.14 \times 10^6 / 6.873 = 3.95 \times 10^6 \text{ or } d_s = 158 \text{ say } 160 \text{ mm} \text{ **Ans.**}$$

**(c) Design of shaft at the juncture of right hand crank arm**

Let  $d_{s1}$  = Diameter of the shaft at the juncture of the right hand crank arm.

We know that the resultant force at the bearing 1,

$$R_1 = \sqrt{(H_{T1})^2 + (H_{R1})^2} = \sqrt{(42)^2 + (47.3)^2} = 63.3 \text{ kN}$$

∴ Bending moment at the juncture of the right hand crank arm,

$$M_{s1} = R_1 \left( b_2 + \frac{l_c}{2} + \frac{t}{2} \right) - F_Q \left( \frac{l_c}{2} + \frac{t}{2} \right)$$

$$\begin{aligned}
&= 63.3 \left( 400 + \frac{155}{2} + \frac{140}{2} \right) - 126.5 \left( \frac{155}{2} + \frac{140}{2} \right) \\
&= 34.7 \times 10^3 - 18.7 \times 10^3 = 16 \times 10^3 \text{ kN-mm} = 16 \times 10^6 \text{ N-mm}
\end{aligned}$$

and twisting moment at the juncture of the right hand crank arm,

$$T_{S1} = F_T \times r = 84 \times 300 = 25\,200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

$\therefore$  Equivalent twisting moment at the juncture of the right hand crank arm,

$$\begin{aligned}
T_e &= \sqrt{(M_{S1})^2 + (T_{S1})^2} \\
&= \sqrt{(16 \times 10^6)^2 + (25.2 \times 10^6)^2} = 29.85 \times 10^6 \text{ N-mm}
\end{aligned}$$

We know that equivalent twisting moment ( $T_e$ ),

$$29.85 \times 10^6 = \frac{\pi}{16} (d_{J1})^3 \tau = \frac{\pi}{16} (d_{J1})^3 42 = 8.25 (d_{J1})^3 \quad \dots (\text{Taking } \tau = 42 \text{ MPa or N/mm}^2)$$

$$\therefore (d_{J1})^3 = 29.85 \times 10^6 / 8.25 = 3.62 \times 10^6 \text{ or } d_{J1} = 153.5 \text{ say } 155 \text{ mm Ans.}$$

**(d) Design of right hand crank web**

Let  $\sigma_{bR}$  = Bending stress in the radial direction ; and  
 $\sigma_{bT}$  = Bending stress in the tangential direction.

We also know that bending moment due to the radial component of  $F_Q$ ,

$$\begin{aligned}
M_R &= H_{R2} \left( b_1 - \frac{l_c}{2} - \frac{t}{2} \right) = 47.3 \left( 400 - \frac{155}{2} - \frac{140}{2} \right) \text{ kN-mm} \\
&= 11.94 \times 10^3 \text{ kN-mm} = 11.94 \times 10^6 \text{ N-mm} \quad \dots (i)
\end{aligned}$$

We also know that bending moment,

$$M_R = \sigma_{bR} \times Z = \sigma_{bR} \times \frac{1}{6} \times w \cdot t^2 \quad \dots (\because Z = \frac{1}{6} \times w \cdot t^2)$$

$$11.94 \times 10^6 = \sigma_{bR} \times \frac{1}{6} \times 245 (140)^2 = 800 \times 10^3 \sigma_{bR}$$

$$\therefore \sigma_{bR} = 11.94 \times 10^6 / 800 \times 10^3 = 14.9 \text{ N/mm}^2 \text{ or MPa}$$

We know that bending moment due to the tangential component of  $F_Q$ ,

$$\begin{aligned}
M_T &= F_T \left( r - \frac{d_{J1}}{2} \right) = 84 \left( 300 - \frac{155}{2} \right) = 18\,690 \text{ kN-mm} \\
&= 18.69 \times 10^6 \text{ N-mm}
\end{aligned}$$

We also know that bending moment,

$$M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t \cdot w^2 \quad \dots (\because Z = \frac{1}{6} \times t \cdot w^2)$$

$$18.69 \times 10^6 = \sigma_{bT} \times \frac{1}{6} \times 140 (245)^2 = 1.4 \times 10^6 \sigma_{bT}$$

$$\therefore \sigma_{bT} = 18.69 \times 10^6 / 1.4 \times 10^6 = 13.35 \text{ N/mm}^2 \text{ or MPa}$$

Direct compressive stress,

$$\sigma_b = \frac{F_R}{2w \cdot t} = \frac{94.6}{2 \times 245 \times 140} = 1.38 \times 10^{-3} \text{ kN/mm}^2 = 1.38 \text{ N/mm}^2$$

and total compressive stress,

$$\begin{aligned}\sigma_c &= \sigma_{br} + \sigma_{bt} + \sigma_d \\ &= 14.9 + 13.35 + 1.38 = 29.63 \text{ N/mm}^2 \text{ or MPa}\end{aligned}$$

We know that twisting moment on the arm,

$$\begin{aligned}T &= H_{T2} \left( b_1 - \frac{l_c}{2} \right) = 42 \left( 400 - \frac{155}{2} \right) = 13\,545 \text{ kN-mm} \\ &= 13.545 \times 10^6 \text{ N-mm}\end{aligned}$$



*Piston and piston rod*

and shear stress on the arm,

$$\tau = \frac{T}{Z_p} = \frac{4.5T}{w \cdot l^2} = \frac{4.5 \times 13.545 \times 10^6}{245 (140)^2} = 12.7 \text{ N/mm}^2 \text{ or MPa}$$

We know that total or maximum combined stress,

$$\begin{aligned}(\sigma_c)_{max} &= \frac{\sigma_c}{2} + \frac{1}{2} \sqrt{(\sigma_c)^2 + 4\tau^2} \\ &= \frac{29.63}{2} + \frac{1}{2} \sqrt{(29.63)^2 + 4(12.7)^2} = 14.815 + 19.5 = 34.315 \text{ MPa}\end{aligned}$$

Since the maximum combined stress is within the safe limits, therefore, the dimension  $w = 245 \text{ mm}$  is accepted.

#### **(e) Design of left hand crank web**

The dimensions for the left hand crank web may be made same as for right hand crank web.

#### **(f) Design of crankshaft bearings**

Since the bearing 2 is the most heavily loaded, therefore, only this bearing should be checked for bearing pressure.

We know that the total reaction at bearing 2,

$$R_2 = \frac{F_p}{2} + \frac{W}{2} + \frac{I_1 + I_2}{2} = \frac{314.2}{2} + \frac{50}{2} + \frac{6.5}{2} = 185.35 \text{ kN} = 185\,350 \text{ N}$$

## Significant of Firing Order

Every engine cylinder must fire once in every cycle. This requires that for a four-stroke four cylinder engine the ignition system must fire for every 180 degrees of crank rotation. For a six cylinder engine the time available is only 120 degrees of crank rotation.

The order in which various cylinders of a multi cylinder engine fire is called the firing order. The number of possibilities of firing order depends upon the number of cylinders and throws of the crankshaft. It is desirable to have the power impulses equally spaced and from the point of view of balancing this has led to certain conventional arrangements of crankshaft throws. Further, there are three factors which must be considered before deciding the optimum firing order of an engine. These are:

- (i) Engine vibrations
- (ii) Engine cooling and
- (iii) Development of back pressure

It should be noted that to some extent all the above three requirements are conflicting and therefore a trade-off is necessary.

For 4-Cylinder engines the possible firing orders are: 1-3-4-2 or 1-2-4-3

The former is more commonly used in the vertical configuration of cylinders.

For a 6-Cylinder engine firing orders can be: 1-5-3-6-2-4 or 1-5-4-6-2-3 or 1-2-4-6-5-3 or 1-2-3-6-5-4

The first one is more commonly used.

### Other Firing Orders

For 3 Cylinder engine 1-3-2

8 Cylinder in-line engine 1-6-2-5-8-3-7-4

8 Cylinder V shape engine 1-5-4-8-6-3-7-2, 1-8-4-3-6-5-7-2, 1-6-2-5-8-3-7-4,

1-8-7-3-6-5-4-2, 1-5-4-2-6-3-7-8

Cylinder No. 1 is taken from front of the in-line engines whereas in V shape front cylinder on right side-bank is considered cylinder No.1 for fixing H.T. leads according to engine firing order.

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**UNIT IV DESIGN OF VALVES AND  
FLYWHEEL-SAUA1501**

## **4. DESIGN OF VALVES AND FLYWHEEL**

### **Design aspects of intake and exhaust manifold.**

An intake manifold's job is to guide the air into the cylinder head. In a fuel injected car a throttle plate or throttle body is attached to one end and is used to control the air flow entering the manifold. Many race engines use a separate throttle plate for each cylinder opposed to a street driver car which normally uses one.

### **Parts to be manifold.**

#### **Plenum**

The plenum is the big usually circular part of the manifold. All of the runners are fed by the plenum.

Plenum size should be 50-70% of the actual engine displacement.

#### **Runner**

The runners stem from the plenum and are connected to the cylinder head.

They have a tapered shape starting large at the plenum and gradually get smaller near the cylinder head.

Variable runner length effects the power band of your car.

Short runners and wide are optimal for higher engine function and long and narrow runners are optimal for low-mid rpm function.

#### **Throttle body**

Controls the air flow into the intake plenum.

Size of the throttle body effects the speed at which the air enters.

The air velocity should be kept at approximately at 300 ft/sec for smooth throttle response.

$$V = (\text{Airflow rate} / \text{Area of section})$$

#### **Fuel injector location**

Two main guidelines to follow

Aim directly down the center of the port, located on each runner.

Discharge at a point where velocity is greatest and at an angle of less than 20 with respect to the runner.

High velocity helps to atomize the fuel with the air. Also decreases the chance of fuel to puddle inside the manifold.

A secondary injector can also be added and it can be placed so it discharged upstream, but the airflow must be large. This can also help in atomization.

## **Ram Air Theory**

This theory is used to help explain the boost at a certain RPM that is noticed when varying runner lengths.

To describe how this works we have to take into account that mass air flow can be explained if you characterize it as a sound wave and its corresponding frequencies.

When the piston drops in a naturally aspirated engine, it creates a low pressure area inside the cylinder. This allows the atmospheric pressure to enter the valve once it opens. But the air does not just stop once the valve shuts. The air (as a sound wave) gets reflected back up the intake manifold runner which in return hits the plenum and is reflected back down the runner.

The plenum acts as a resonance chamber. Each reflection from the resonance chamber adds more (energy, tone, amplitude) to the sound wave.

The idea is to get these maximized waves to the valves so they enter the motor with increased energy, which in some cases can be over atmospheric pressure.

## **Exhaust manifold**

Exhaust manifolds collect the exhaust gases from the engine cylinders and discharge to the atmosphere through the exhaust system. The engine efficiency, combustion characteristics would depend upon how the exhaust gases were removed from the cylinder. The following are some of the critical aspects related to the design of exhaust manifold for the internal combustion engine.

The exhaust manifold design should result in maintaining high temperature in the exhaust pipe. This is necessary because the catalyst, placed near the end of the exhaust pipe, would absorb more pollutant in high temperature conditions.

Also, the design should ensure that exhaust manifold natural frequencies are not in the range of the excitation frequency range of engine vibration to prevent any damages.

The mass of the exhaust manifold should be as low as possible.

## **Valve Gear Mechanism**

The valve gear mechanism of an I.C. engine consists of those parts which actuate the inlet and exhaust valves at the required time with respect to the position of piston and crankshaft. Fig (a) shows the valve gear arrangement for vertical engines. The main components of the mechanism are valves, rocker arm, valve springs, push rod, cam and camshaft.



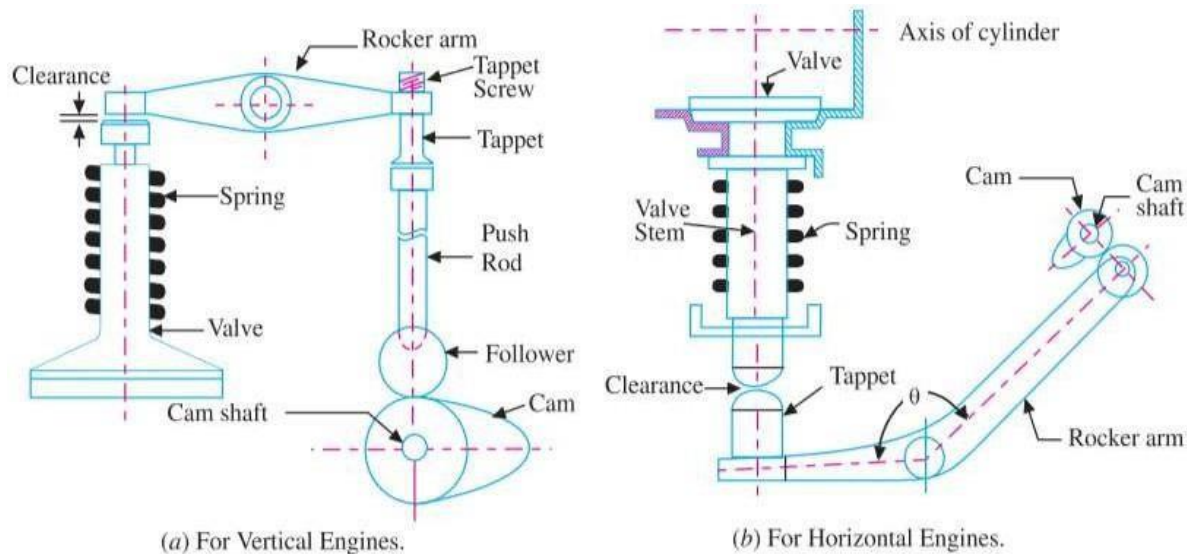


Figure.4.1.Valve Mechanism

The fuel is admitted to the engine by the inlet valve and the burnt gases are escaped through the exhaust valve. In vertical engines, the cam moving on the rotating camshaft pushes the cam follower and push rod upwards, thereby transmitting the cam action to rocker arm. The camshaft is rotated by the toothed belt from the crankshaft. The rocker arm is pivoted at its centre by a fulcrum pin. When one end of the rocker arm is pushed up by the push rod, the other end moves downward. This pushes down the valve stem causing the valve to move down, thereby opening the port. When the cam follower moves over the circular portion of cam, the pushing action of the rocker arm on the valve is released and the valve returns to its seat and closes it by the action of the valve spring. In some of the modern engines, the camshaft is located at cylinder head level. In such cases, the push rod is eliminated and the roller type camfollower is made part of the rocker arm. Such an arrangement for the horizontal engines is shown in Fig.

## Valves

The valves used in internal combustion engines are of the following three types;

1. Poppet or mushroom valve
2. Sleeve valve
3. Rotary valve.

Out of these three valves, poppet valve, as shown in Fig. is very frequently used. It consists of head, face and stem. The head and face of the valve is separated by a small margin, to avoid sharp edge of the valve and also to provide provision for the regrinding of the face. The face angle generally varies from  $30^\circ$  to  $45^\circ$ . The lower part of the stem is provided with a groove in which spring retainer lock is installed. Since both the inlet and exhaust valves are subjected to high temperatures of  $1930^\circ\text{C}$  to  $2200^\circ\text{C}$  during the power stroke, therefore, it is necessary that the material of the valves should withstand these temperatures. Thus the material of the valves must have good heat conduction, heat resistance, corrosion resistance, wear resistance and shock resistance. It may be noted that the temperature at the inlet valve is less



as compared to exhaust valve. Thus, the inlet valve is generally made of nickel chromium alloy steel and the exhaust valve (which is subjected to very high temperature of exhaust gases) is made from silchrome steel which is a special alloy of silicon and chromium. In designing a valve, it is required to determine the following dimensions

**(a) Size of the valve port**

Let  $a_p$  = Area of the port,

$v_p$  = Mean velocity of gas flowing through the port,

$a$  = Area of the piston, and

$v$  = Mean velocity of the piston.

We know that  $a_p \cdot v_p = a \cdot v$

$$a_p = \frac{a \cdot v}{v_p}$$

**(b) Thickness of the valve disc** The thickness of the valve disc ( $t$ ), as shown in Fig. may be determined empirically from the following relation

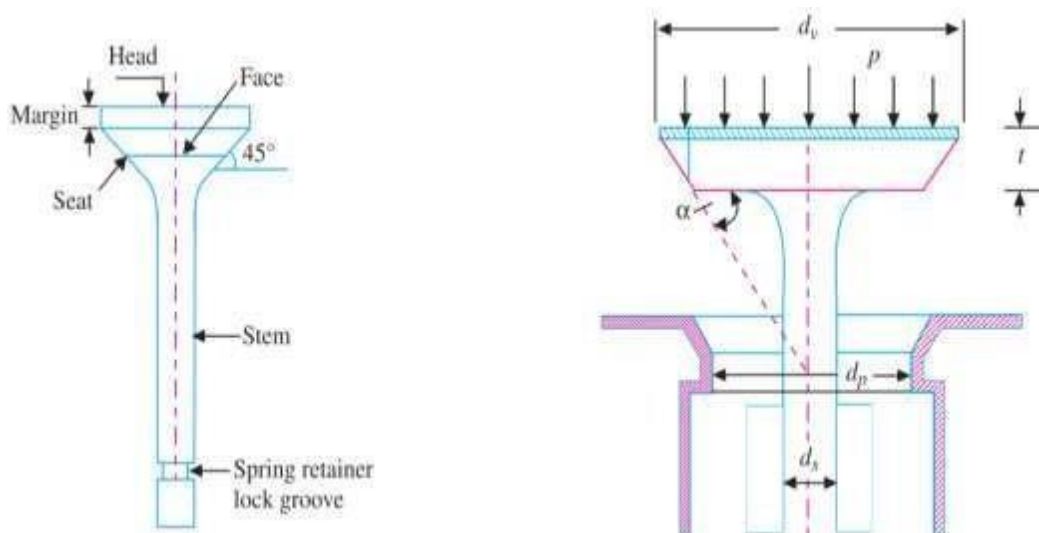


Figure.4.2: Valve

$$t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}}$$

$k$  = Constant = 0.42 for steel and 0.54 for cast iron,

$d_p$  = Diameter of the port in mm,

$p$  = Maximum gas pressure in  $\text{N/mm}^2$ , and

$\sigma_b$  = Permissible bending stress in MPa or  $\text{N/mm}^2$

= 50 to 60 MPa for carbon steel and 100 to 120 MPa for alloy steel

**Maximum lift of the valve**  $h$  = Lift of the valve. The lift of the valve may be obtained by equating the area across the valve seat to the area of the port. For a conical valve, as shown in Fig.

$\alpha$  = Angle at which the valve seat is tapered =  $30^\circ$  to  $45^\circ$

$$\pi d_p \cdot h \cos \alpha = \frac{\pi}{4} (d_p)^2 \quad \text{or} \quad h = \frac{d_p}{4 \cos \alpha}$$

In case of flat headed valve, the lift of valve is given by

$$h = \frac{d_p}{4}$$

The valve seats usually have the same angle as the valve seating surface. But it is preferable to make the angle of valve seat  $1/2^\circ$  to  $1^\circ$  larger than the valve angle as shown in Fig. This results in more effective seat.

#### (d) Valve stem diameter

The valve stem diameter ( $d_s$ ) is given by

$$d_s = \frac{d_p}{8} + 6.35 \text{ mm to } \frac{d_p}{8} + 11 \text{ mm}$$

**Problem 1.** The conical valve of an I.C. engine is 60 mm in diameter and is subjected to a maximum gas pressure of  $4 \text{ N/mm}^2$ . The safe stress in bending for the valve material is 46 MPa. The valve is made of steel for which  $k = 0.42$ . The angle at which the valve disc seat is tapered is  $30^\circ$ . Determine : 1. thickness of the valve head ; 2. stem diameter ; and 3. maximum lift of the valve.

**Solution.** Given :  $d_p = 60 \text{ mm}$  ;  $p = 4 \text{ N/mm}^2$  ;  $\sigma_b = 46 \text{ MPa} = 46 \text{ N/mm}^2$  ;  $k = 0.42$  ;  $\alpha = 30^\circ$

1. Thickness of the valve head We know that thickness of the valve head,

$$t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}} = 0.42 \times 60 \sqrt{\frac{4}{46}} = 7.43 \text{ say } 7.5 \text{ mm}$$

2. Stem diameter We know that stem diameter,

$$d_s = \frac{d_p}{8} + 6.35 = \frac{60}{8} + 6.35 = 13.85 \text{ say } 14 \text{ mm}$$

3. Maximum lift of the valve We know that maximum lift of the valve,

$$h = \frac{d_p}{4 \cos \alpha} = \frac{60}{4 \cos 30^\circ} = \frac{60}{4 \times 0.866} = 17.32 \text{ say } 17.4 \text{ mm}$$

## Rocker Arm

The rocker arm is used to actuate the inlet and exhaust valves motion as directed by the cam and follower. It may be made of cast iron, cast steel, or malleable iron. In order to reduce inertia of the rocker arm, an I section is used for the high speed engines and it may be rectangular section for low speed engines. In four stroke engines, the rocker arms for the exhaust valve is the most heavily loaded. Though the force required to operate the inlet valve is relatively small, yet it is usual practice to make the rocker arm for the inlet valve of the same dimensions as that for exhaust valve. A typical rocker arm for operating the exhaust valve is shown in Fig. The lever ratio  $a / b$  is generally decided by considering the space available for rocker arm. For moderate and low speed engines,  $a / b$  is equal to one. For high speed engines, the ratio  $a / b$  is taken as  $1 / 1.3$ . The various forces acting on the rocker arm of exhaust valve are the gas load, spring force and force due to valve acceleration.

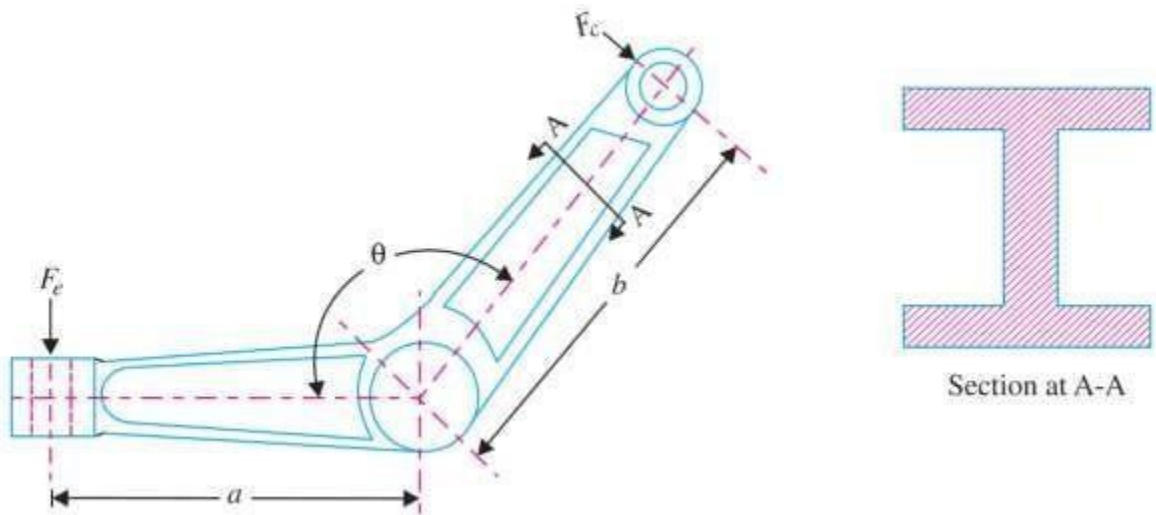


Figure.4.3: Rocker Arm

$m_v$  = Mass of the valve,

$d_v$  = Diameter of the valve head,

$h$  = Lift of the valve,

$a$  = Acceleration of the valve,

$p_c$  = Cylinder pressure or back pressure when the exhaust valve opens, and

$p_s$  = Maximum suction pressure.

We know that gas load,  $P$  = Area of valve  $\times$  Cylinder pressure when the exhaust valve opens

$$= \frac{\pi}{4} (d_v)^2 p_c$$

Spring force,

$$F_s = \text{Area of valve} \times \text{Maximum suction pressure}$$

$$= \frac{\pi}{4} (d_v)^2 p_s$$

$$F_{va} = \text{Mass of valve} \times \text{Acceleration of valve} = m_v \times a$$

∴ Maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_{va}$$

It may be noted that maximum load on the rocker arm for inlet valve is

$$F_i = F_s + F_{va}$$

Since the maximum load on the rocker arm for exhaust valve is more than that of inlet valve, therefore, the rocker arm must be designed on the basis of maximum load on the rocker arm for exhaust valve.

**Design for fulcrum pin.** The load acting on the fulcrum pin is the total reaction ( $R_F$ ) at the fulcrum point.

$d_1$  = Diameter of the fulcrum pin, and

$l_1$  = Length of the fulcrum pin.

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin,

$R_F = d_1 \cdot l_1 \cdot p_b$  The ratio of  $l_1 / d_1$  is taken as 1.25 and the bearing pressure ( $p_b$ ) for ordinary lubrication is taken from 3.5 to 6 N / mm<sup>2</sup> and it may go up to 10.5 N/mm<sup>2</sup> for forced lubrication. The pin should be checked for the induced shear stress.

The thickness of the phosphor bronze bush may be taken from 2 to 4 mm. The outside diameter of the boss at the fulcrum is usually taken twice the diameter of the fulcrum pin.

**Design for forked end.**

The forked end of the rocker arm carries a roller by means of a pin. For uniform wear, the roller should revolve in the eyes. The load acting on the roller pin is  $F_c$ .

Let  $d_2$  = Diameter of the roller pin, and

$l_2$  = Length of the roller pin.

Considering the bearing of the roller pin. We know that load on the roller pin,

$$F_c = d_2 \cdot l_2 \cdot p_b$$

The ratio of  $l_2 / d_2$  may be taken as 1.25. The roller pin should be checked for induced shear stress.

The roller pin is fixed in eye and the thickness of each eye is taken as half the length of the roller pin.

∴ Thickness of each eye =  $l_2 / 2$  The radial thickness of eye ( $t_3$ ) is taken as  $d_1 / 2$ . Therefore overall diameter of the eye,  $D_1 = 2 d_1$

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye.

A clearance of 1.5 mm between the roller and the fork on either side of the roller is provided.

**Design for rocker arm cross-section.**

The rocker arm may be treated as a simply supported beam and loaded at the fulcrum point. We have already discussed that the rocker arm is generally of I-section but for low speed engines, it can be of rectangular section. Due to the load on the valve, the rocker arm is subjected to bending moment.

Let  $l$  = Effective length of each rocker arm, and

$\sigma_b$  = Permissible bending stress.

We know that bending moment on the rocker arm,

$$M = F_e \times l \dots (i)$$

We also know that bending moment,  $M = \sigma_b \times Z \dots (ii)$

where  $Z$  = Section modulus.

From equations (i) and (ii), the value of  $Z$  is obtained and thus the dimensions of the section are determined.

### **Design for tappet.**

The tappet end of the rocker arm is made circular to receive the tappet which is a stud with a lock nut. The compressive load acting on the tappet is the maximum load on the rocker arm for the exhaust valve ( $F_e$ ).

Let  $d_c$  = Core diameter of the tappet, and

$\sigma_c$  = Permissible compressive stress for the material of the tappet which is made of mild steel.

It may be taken as 50 MPa. We know that load on the tappet,

$$F_e = \frac{\pi}{4} (d_c)^2 \sigma_c$$

From this expression, the core diameter of the tappet is determined. The outer or nominal diameter of the tappet ( $d_n$ ) is given as

$$d_n = d_c / 0.84$$

The diameter of the circular end of the rocker arm ( $D_3$ ) and its depth ( $t_4$ ) is taken as twice the nominal diameter of the tappet ( $d_n$ ), i.e.

$$D_3 = 2 d_n ; \text{ and } t_4 = 2 d_n$$

### **Design for valve spring.**

The valve spring is used to provide sufficient force during the valve lifting process in order to overcome the inertia of valve gear and to keep it with the cam without bouncing. The spring is generally made from plain carbon spring steel. The total load for which the spring is designed is equal to the sum of initial load and load at full lift.

Let  $W_1$  = Initial load on the spring = Force on the valve tending to draw it into the cylinder on suction stroke,

$W_2$  = Load at full lift = Full lift  $\times$  Stiffness of spring

∴ Total load on the spring,  $W = W_1 + W_2$

### **Flywheel**

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply. In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In machines where the operation is intermittent like punching machines, shearing machines, riveting machines, crushers etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

### **Coefficient of Fluctuation of Speed**

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called coefficient of fluctuation of speed.

Let  $N_1$  = Maximum speed in r.p.m. during the cycle,

$N_2$  = Minimum speed in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

∴ Coefficient of fluctuation of speed,

$$\begin{aligned} C_s &= \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2} \\ &= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \\ &= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \end{aligned}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed.

## Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. The horizontal line AG represents the mean torque line. Let  $a_1, a_3, a_5$  be the areas above the mean torque line and  $a_2, a_4$  and  $a_6$  be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

Let the energy in the flywheel at A = E, then from Fig.

we have Energy at B =  $E + a_1$

Energy at C =  $E + a_1 - a_2$

Energy at D =  $E + a_1 - a_2 + a_3$

Energy at E =  $E + a_1 - a_2 + a_3 - a_4$

Energy at F =  $E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at G =  $E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6 = \text{Energy at A}$

Let us now suppose that the maximum of these energies is at B and minimum at E.

$\therefore$  Maximum energy in the flywheel =  $E + a_1$  and minimum energy in the flywheel =  $E + a_1 - a_2 + a_3 - a_4$

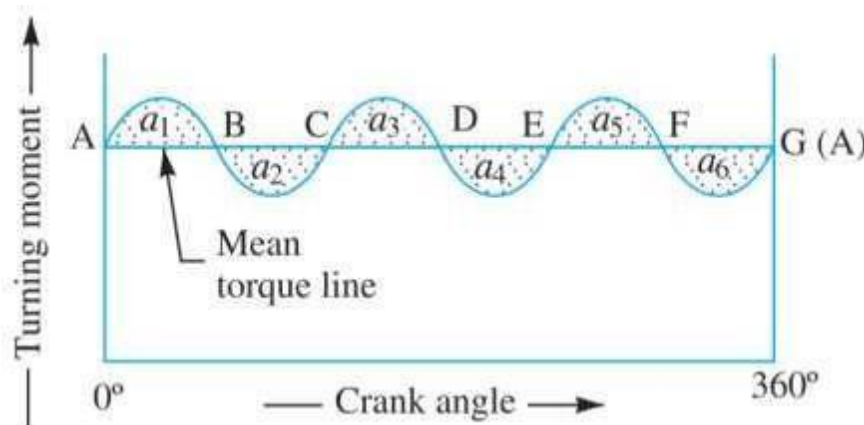


Figure.4.4: Turning Moment Diagram

$\therefore$  Maximum fluctuation of energy,

$\Delta E = \text{Maximum energy} - \text{Minimum energy}$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

**Problem 2.** The turning moment diagram for a petrol engine is drawn to the following scales: Turning moment, 1 mm = 5 N-m; Crank angle, 1 mm =  $1^\circ$ . The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line, taken in order are 295, 685, 40, 340, 960, 270 mm<sup>2</sup>. Determine the mass of 300 mm diameter flywheel rim when the coefficient of fluctuation of speed is 0.3% and the engine runs at 1800 r.p.m. Also determine the cross-section of the rim when the width of the rim is twice of thickness. Assume density of rim material as 7250 kg / m<sup>3</sup>.

Solution. Given :  $D = 300 \text{ mm}$  or  $R = 150 \text{ mm} = 0.15 \text{ m}$  ;  $CS = 0.3\% = 0.003$  ;  $N = 1800 \text{ r.p.m.}$   
 or  $\omega = 2\pi \times 1800 / 60 = 188.5 \text{ rad/s}$  ;  $\rho = 7250 \text{ kg / m}^3$

Mass of the flywheel Let  $m = \text{Mass of the flywheel in kg.}$

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig.

Since the scale of turning moment is  $1 \text{ mm} = 5 \text{ N-m}$ , and scale of the crank angle is  $1 \text{ mm} = 1^\circ = \pi/180 \text{ rad}$ , therefore  $1 \text{ mm}^2$  on the turning moment diagram.

$$= 5 \times \pi/180 = 0.087 \text{ N-m}$$

Let the total energy at  $A = E$ . Therefore from Fig. we find that

$$\text{Energy at B} = E + 295$$

$$\text{Energy at C} = E + 295 - 685 = E - 390$$

$$\text{Energy at D} = E - 390 + 40 = E - 350$$

$$\text{Energy at E} = E - 350 - 340 = E - 690$$

$$\text{Energy at F} = E - 690 + 960 = E + 270$$

Energy at  $G = E + 270 - 270 = E = \text{Energy at A}$  From above we see that the energy is maximum at B and minimum at E.

$$\therefore \text{Maximum energy} = E + 295 \text{ and minimum energy} = E - 690$$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 295) - (E - 690) = 985 \text{ mm}^2 = 985 \times 0.087 = 86 \text{ N-m}$$

We also know that maximum fluctuation of energy ( $\Delta E$ ),

$$86 = m.R^2.\omega^2.CS = m (0.15)^2 (188.5)^2 (0.003) = 2.4 m$$

$$\therefore m = 86 / 2.4 = 35.8 \text{ kg}$$

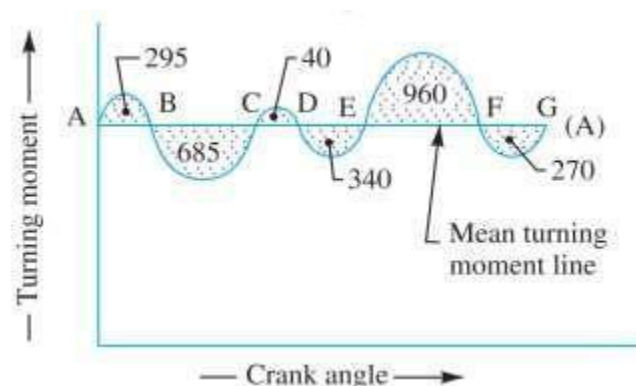


Figure.4.5: Turning Moment Diagram



Cross-section of the flywheel rim Let  $t$  = Thickness of rim in metres, and

$b$  = Width of rim in metres =  $2t$  ...(Given)

$\therefore$  Cross-sectional area of rim,  $A = b \times t = 2t \times t = 2t^2$

We know that mass of the flywheel rim ( $m$ ),

$$35.8 = A \times 2\pi R \times \rho = 2t^2 \times 2\pi \times 0.15 \times 7250 = 13\,668\,t^2$$

$$\therefore t^2 = 35.8 / 13\,668 = 0.0026 \quad \text{or}$$

$$t = 0.051 \text{ m} = 51 \text{ mm.}$$

$$\text{and } b = 2t = 2 \times 51 = 102 \text{ mm}$$

**UNIT V DESIGN OF TRANSMISSION SHAFT AND BEARING-  
SAUA1501**

## **5. DESIGN OF TRANSMISSION SHAFTS, BEARINGS AND SPRINGS**

### **Shaft**

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending. In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

### **Material Used for Shafts**

The material used for shafts should have the following properties:

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12. The mechanical properties of these grades of carbon steel are given in the following table.

### **Manufacturing of Shafts**

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

### **Types of Shafts**

The following two types of shafts are important from the subject point of view :

#### **1. Transmission shafts.**

These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, overhead shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

#### **2. Machine shafts.**

These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft

## Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are: 25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps; 110 mm to 140 mm with 15 mm steps; and 140 mm to 500 mm with 20 mm steps. The standard length of the shafts are 5 m, 6 m and 7 m.

## Stresses in Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (i.e. due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

## Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(a) 112 MPa for shafts without allowance for keyways.

(b) 84 MPa for shafts with allowance for keyways. For shafts purchased under definite physical specifications, the permissible tensile stress ( $\sigma_t$ ) may be taken as 60 per cent of the elastic limit in tension ( $\sigma_{el}$ ), but not more than 36 per cent of the ultimate tensile strength ( $\sigma_u$ ). In other words, the permissible tensile stress,

$$\sigma_t = 0.6 \sigma_{el} \text{ or } 0.36 \sigma_u,$$

Whichever is less, the maximum permissible shear stress may be taken as

(a) 56 MPa for shafts without allowance for key ways.

(b) 42 MPa for shafts with allowance for keyways. For shafts purchased under definite physical specifications, the permissible shear stress ( $\tau$ ) may be taken as 30 per cent of the elastic limit in tension ( $\sigma_{el}$ ) but not more than 18 per cent of the ultimate tensile strength ( $\sigma_u$ ). In other words, the permissible shear stress,  $\tau = 0.3 \sigma_{el}$  or  $0.18 \sigma_u$ , whichever is less.

## Design of Shafts

The shafts may be designed on the basis of

1. Strength, and
2. Rigidity and stiffness. In designing shafts on the basis of strength, the following cases may be considered:
  - (a) Shafts subjected to twisting moment or torque only,
  - (b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads. We shall now discuss the above cases, in detail, in the following pages.

### Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that  $T/J = r/\tau$  ... (i)

where  $T$  = Twisting moment (or torque) acting upon the shaft,

$J$  = Polar moment of inertia of the shaft about the axis of rotation,

$\tau$  = Torsional shear stress, and

$$r = \text{Distance from neutral axis to the outer most fibre} \\ = d/2; \text{ where } d \text{ is the diameter of the shaft.}$$

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (i) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \quad \dots(ii)$$

From this equation, we may determine the diameter of round solid shaft ( $d$ ).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32} [(d_o)^4 - (d_i)^4]$$

where  $d_o$  and  $d_i$  = Outside and inside diameter of the shaft, and  $r = d_o/2$ .

Substituting these values in equation (i), we have

$$\frac{T}{\frac{\pi}{32} [(d_o)^4 - (d_i)^4]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right] \quad \dots(iii)$$

Let  $k = \text{Ratio of inside diameter and outside diameter of the shaft}$   
 $= d_i/d_o$

Now the equation (iii) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o} \left[ 1 - \left( \frac{d_i}{d_o} \right)^4 \right] = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) \quad \dots(iv)$$

From the equations (iii) or (iv), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment ( $T$ ) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where

$T$  = Twisting moment in N-m, and

$N$  = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment ( $T$ ) is given by

$$T = (T_1 - T_2) R$$

where

$T_1$  and  $T_2$  = Tensions in the tight side and slack side of the belt respectively, and

$R$  = Radius of the pulley.

**Problem1.** Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

**Solution.**

**Given :**

$P = 20 \text{ kW} = 20 \times 10^3 \text{ W}$ ;  $N = 200 \text{ r.p.m.}$ ;  $\tau_u = 360 \text{ MPa} = 360 \text{ N/mm}^2$ ; F.S. = 8 ;  $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

### **Diameter of the solid shaft**

Let  $d$  = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft ( $T$ ),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84 d^3$$

$$\therefore d^3 = 955 \times 10^3 / 8.84 = 108\,032 \text{ or } d = 47.6 \text{ say } 50 \text{ mm Ans.}$$

### **Diameter of hollow shaft**

Let  $d_i$  = Inside diameter, and

$d_o$  = Outside diameter.

We know that the torque transmitted by the hollow shaft ( $T$ ),

$$\begin{aligned} 955 \times 10^3 &= \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) \\ &= \frac{\pi}{16} \times 45 (d_o)^3 [1 - (0.5)^4] = 8.3 (d_o)^3 \end{aligned}$$

$$\therefore (d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060 \text{ or } d_o = 48.6 \text{ say } 50 \text{ mm Ans.}$$

and  $d_i = 0.5 d_o = 0.5 \times 50 = 25 \text{ mm Ans.}$

## **Bearings**

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc.

## Classification of Bearings

Though the bearings may be classified in many ways, yet the following are important from the subject point of view:

**Depending upon the direction of load to be supported.** The bearings under this group are classified as:

(a) Radial bearings, and

(b) Thrust bearings. In radial bearings, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. In thrust bearings, the load acts along the axis of rotation as shown in Fig.

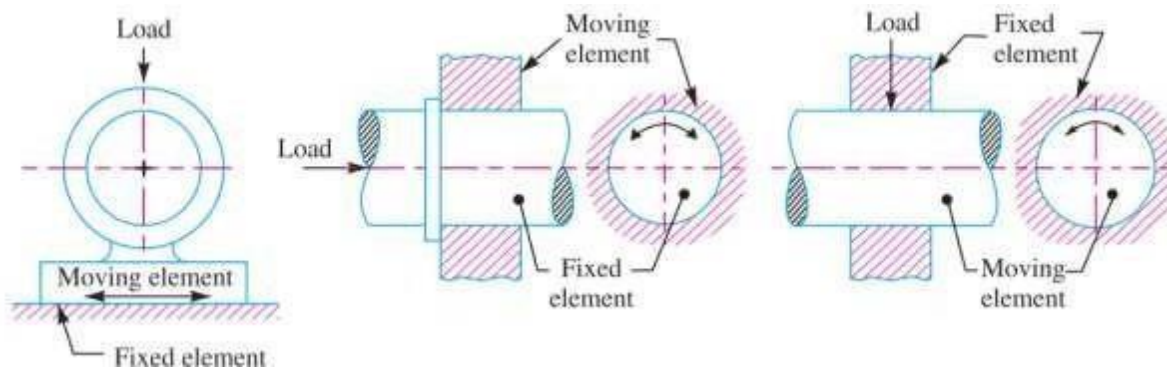


Figure.5.1: Radial and Thrust Bearing

**Depending upon the nature of contact.** The bearings under this group are classified as:

(a) Sliding contact bearings, and

(b) Rolling contact bearings. In sliding contact bearings, as shown in Fig. the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

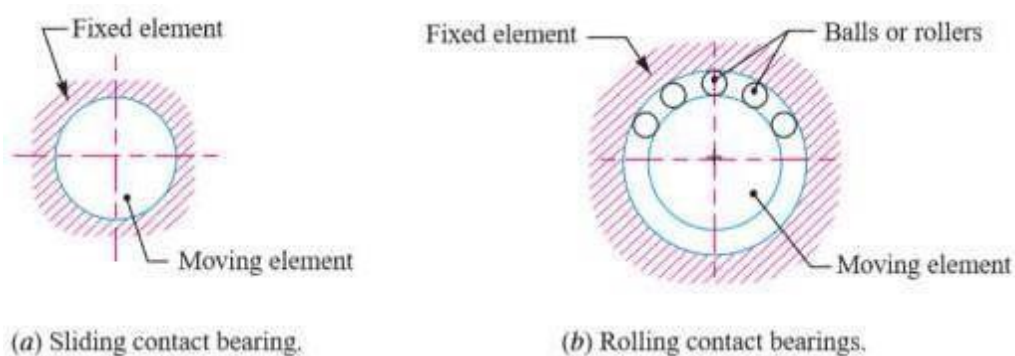


Figure.5.2: Sliding Contact and Rolling Contact Bearing

## Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. may be called slipper or guide bearings. Such type of bearings are usually found in cross-head of steam engines.



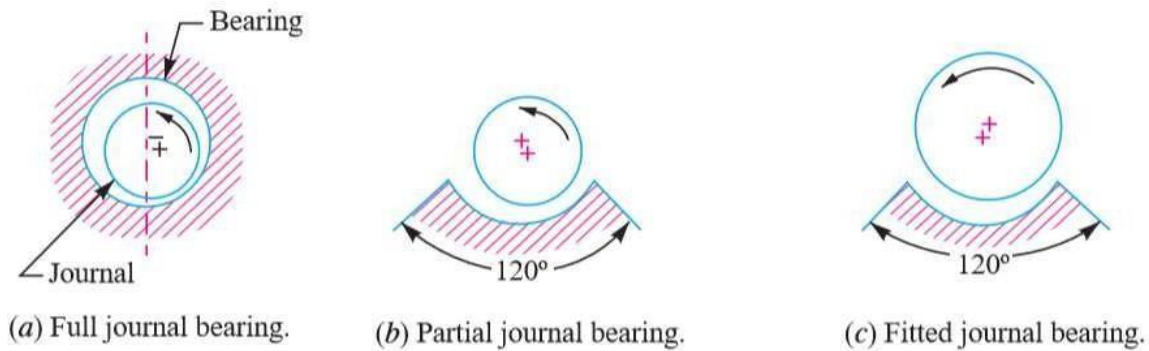


Figure.5.3: Journal Bearing

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as journal or sleeve bearings. When the angle of contact of the bearing with the journal is  $360^\circ$  as shown in Fig. then the bearing is called a full journal bearing. This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction. When the angle of contact of the bearing with the journal is  $120^\circ$ , as shown in Fig. then the bearing is said to be partial journal bearing. This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as clearance bearings because the diameter of the journal is less than that of bearing.

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows :

**1. Thick film bearings.** The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as hydrodynamic lubricated bearings.

**2. Thin film bearings.** The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other at least part of the time. Such type of bearings are also called boundary lubricated bearings.

**3. Zero film bearings.** The zero film bearings are those which operate without any lubricant present.

**4. Hydrostatic or externally pressurized lubricated bearings.** The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members

#### Assumptions in Hydrodynamic Lubricated

Bearings The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.

4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, i.e. the side leakage is neglected.

### **Important Factors for the Formation of Thick Oil Film in Hydrodynamic Lubricated Bearings**

According to Reynolds, the following factors are essential for the formation of a thick film of

1. A continuous supply of oil.
2. A relative motion between the two surfaces in a direction approximately tangential to the surfaces.
3. The ability of one of the surfaces to take up a small inclination to the other surface in the direction of the relative motion.
4. The line of action of resultant oil pressure must coincide with the line of action of the external load between the surfaces

### **Lubricants**

The lubricants are used in bearings to reduce friction between the rubbing surfaces and to carry away the heat generated by friction. It also protects the bearing against corrosion. All lubricants are classified into the following three groups:

1. Liquid,
2. Semi-liquid, and
3. Solid.

The liquid lubricants usually used in bearings are mineral oils and synthetic oils. The mineral oils are most commonly used because of their cheapness and stability. The liquid lubricants are usually preferred where they may be retained. A grease is a semi-liquid lubricant having higher viscosity than oils. The greases are employed where slow speed and heavy pressure exist and where oil drip from the bearing is undesirable. The solid lubricants are useful in reducing friction where oil films cannot be maintained because of pressures or temperatures. They should be softer than materials being lubricated. A graphite is the most common of the solid lubricants either alone or mixed with oil or grease

### **Viscosity of lubricant.**

It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film-under heat and pressure. The greater the heat and pressure, the greater viscosity is required of a lubricant to prevent thinning and squeezing out of the film.

The fundamental meaning of viscosity may be understood by considering a flat plate moving under a force  $P$  parallel to a stationary plate, the two plates being separated by a thin film of a fluid lubricant of thickness  $h$ , as shown in Fig. The particles of the lubricant adhere strongly to the moving and stationary plates. The motion is accompanied by a linear slip or shear between the particles

throughout the entire height ( $h$ ) of the film thickness. If  $A$  is the area of the plate in contact with the lubricant, then the unit shear stress is given by  $\tau = P/A$

According to Newton's law of viscous flow, the magnitude of this shear stress varies directly with the velocity gradient ( $dv / dy$ ). It is assumed that

- (a) the lubricant completely fills the space between the two surfaces,
- (b) the velocity of the lubricant at each surface is same as that of the surface, and
- (c) any flow of the lubricant perpendicular to the velocity of the plate is negligible

$$\therefore \tau = \frac{P}{A} \propto \frac{dV}{dy} \quad \text{or} \quad \tau = Z \times \frac{dV}{dy}$$

where  $Z$  is a constant of proportionality and is known as *absolute viscosity* (or simply viscosity) of the lubricant.

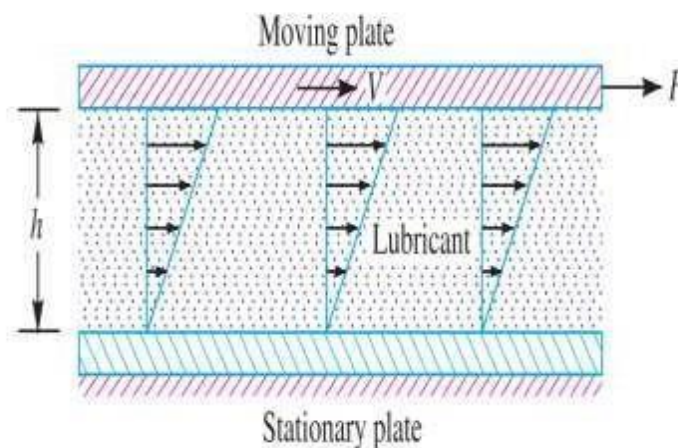


Figure.5.4: Absolute Viscosity of the Lubricant

When the thickness of the fluid lubricant is small which is the case for bearings, then the velocity gradient is very nearly constant as shown in Fig. 26.6, so that

$$\frac{dV}{dy} = \frac{V}{y} = \frac{V}{h}$$

$$\therefore \tau = Z \times \frac{V}{h} \quad \text{or} \quad Z = \tau \times \frac{h}{V}$$

When  $\tau$  is in  $\text{N/m}^2$ ,  $h$  is in metres and  $V$  is in  $\text{m/s}$ , then the unit of absolute viscosity is given by

$$Z = \tau \times \frac{h}{V} = \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}}{\text{m/s}} = \text{N-s/m}^2$$

However, the common practice is to express the absolute viscosity in mass units, such that

$$1 \text{ N-s/m}^2 = \frac{1 \text{ kg-m}}{\text{s}^2} \times \frac{\text{s}}{\text{m}^2} = 1 \text{ kg/m-s} \quad \dots (\because 1 \text{ N} = 1 \text{ kg-m/s}^2)$$

Thus the unit of absolute viscosity in S.I. units is  $\text{kg/m-s}$ .

The viscosity of the lubricant is measured by Saybolt universal viscometer. It determines the time required for a standard volume of oil at a certain temperature to flow under a certain head through a tube of standard diameter and length. The time so determined in seconds is the Saybolt universal viscosity. In order to convert Saybolt universal viscosity in seconds to absolute viscosity (in  $\text{kg/m-s}$ ), the following formula may be used:

$$Z = \text{Sp. gr. of oil} \left( 0.00022S - \frac{0.18}{S} \right) \text{ kg/m-s} \quad \dots (i)$$

**Problem 1.** Design a journal bearing for a centrifugal pump from the following data : Load on the journal = 20 000 N; Speed of the journal = 900 r.p.m.; Type of oil is SAE 10, for which the absolute viscosity at  $55^\circ\text{C}$  =  $0.017 \text{ kg/m-s}$ ; Ambient temperature of oil =  $15.5^\circ\text{C}$  ; Maximum bearing pressure for the pump =  $1.5 \text{ N/mm}^2$ . Calculate also mass of the lubricating oil required for artificial cooling, if rise of temperature of oil be limited to  $10^\circ\text{C}$ . Heat dissipation coefficient =  $1232 \text{ W/m}^2/^\circ\text{C}$ .

**Solution.**

Given :  $W = 20\,000 \text{ N}$  ;  $N = 900 \text{ r.p.m.}$  ;  $t_0 = 55^\circ\text{C}$  ;  $Z = 0.017 \text{ kg/m-s}$  ;  $t_a = 15.5^\circ\text{C}$  ;  $p = 1.5 \text{ N/mm}^2$  ;  $t = 10^\circ\text{C}$  ;  $C = 1232 \text{ W/m}^2/^\circ\text{C}$  The journal bearing is designed as discussed in the following steps :

First of all, let us find the length of the journal ( $l$ ). Assume the diameter of the journal ( $d$ ) as 100 mm. From Table 26.3, we find that the ratio of  $l/d$  for centrifugal pumps varies from 1 to 2. Let us take  $l/d = 1.6$ .

$$\therefore l = 1.6 d = 1.6 \times 100 = 160 \text{ mm Ans.}$$

2. We know that bearing pressure,

$$p = \frac{W}{l.d} = \frac{20\,000}{160 \times 100} = 1.25$$

Since the given bearing pressure for the pump is  $1.5 \text{ N/mm}^2$ , therefore the above value of  $p$  is safe and hence the dimensions of  $l$  and  $d$  are safe.

$$3. \quad \frac{Z.N}{p} = \frac{0.017 \times 900}{1.25} = 12.24$$

From Table 26.3, we find that the operating value of

$$\frac{Z.N}{p} = 28$$

We have discussed in Art. 26.14, that the minimum value of the bearing modulus at which the oil film will break is given by

$$3 K = \frac{Z.N}{p}$$

$\therefore$  Bearing modulus at the minimum point of friction,

$$K = \frac{1}{3} \left( \frac{Z.N}{p} \right) = \frac{1}{3} \times 28 = 9.33$$

Since the calculated value of bearing characteristic number  $\left( \frac{Z.N}{p} = 12.24 \right)$  is more than 9.33, therefore the bearing will operate under hydrodynamic conditions.

4. From Table 26.3, we find that for centrifugal pumps, the clearance ratio ( $c/d$ ) = 0.0013

5. We know that coefficient of friction,

$$\mu = \frac{33}{10^8} \left( \frac{Z.N}{p} \right) \left( \frac{d}{c} \right) + k = \frac{33}{10^8} \times 12.24 \times \frac{1}{0.0013} + 0.002$$

$$= 0.0031 + 0.002 = 0.0051 \quad \dots [\text{From Art. 26.13, } k = 0.002]$$

6. Heat generated,

$$Q_g = \mu W V = \mu W \left( \frac{\pi d.N}{60} \right) \text{ W} \quad \dots \left( \because V = \frac{\pi d.N}{60} \right)$$

$$= 0.0051 \times 20\,000 \left( \frac{\pi \times 0.1 \times 900}{60} \right) = 480.7 \text{ W}$$

$\dots (d \text{ is taken in metres})$

7. Heat dissipated,

$$Q_d = C.A (t_b - t_a) = C.l.d (t_b - t_a) \text{ W} \quad \dots (\because A = l \times d)$$

We know that

$$(t_b - t_a) = \frac{1}{2} (t_o - t_a) = \frac{1}{2} (55^\circ - 15.5^\circ) = 19.75^\circ \text{C}$$

$$\therefore Q_d = 1232 \times 0.16 \times 0.1 \times 19.75 = 389.3 \text{ W}$$

$\dots (l \text{ and } d \text{ are taken in metres})$

We see that the heat generated is greater than the heat dissipated which indicates that the bearing is warming up. Therefore, either the bearing should be redesigned by taking  $t_o = 63^\circ \text{C}$  or the bearing should be cooled artificially.

We know that the amount of artificial cooling required

$$= \text{Heat generated} - \text{Heat dissipated} = Q_g - Q_d$$

$$= 480.7 - 389.3 = 91.4 \text{ W}$$

**Problem 3.** A wall bracket supports a plummer block for 80 mm diameter shaft. The length of bearing is 120 mm. The cap of bearing is fastened by means of four bolts, two on each side of the shaft. The cap is to withstand a load of 16.5 kN. The distance between the centre lines of

the bolts is 150 mm. Determine the thickness of the bearing cap and the diameter of the bolts. Assume safe stresses in tension for the material of the cap, which is cast iron, as 15 MPa and for bolts as 35 MPa. Also check the deflection of the bearing cap taking  $E = 110 \text{ kN/mm}^2$ .

Solution : Given :  $d = 80 \text{ mm}$  ;  $l = 120 \text{ mm}$  ;  $n = 4$  ;  $W = 16.5 \text{ kN} = 16.5 \times 10^3 \text{ N}$  ;  $a = 150 \text{ mm}$  ;  $\sigma_b = 15 \text{ MPa} = 15 \text{ N/mm}^2$  ;  $\sigma_t = 35 \text{ MPa} = 35 \text{ N/mm}^2$  ;  $E = 110 \text{ kN/mm}^2 = 110 \times 10^3 \text{ N/mm}^2$

### Thickness of the bearing cap

We know that thickness of the bearing cap,

$$t = \sqrt{\frac{3 W . a}{2 \sigma_b l}} = \sqrt{\frac{3 \times 16.5 \times 10^3 \times 150}{2 \times 15 \times 120}} = \sqrt{2062.5}$$

$$= 45.4 \text{ say } 46 \text{ mm Ans.}$$

### Diameter of the bolts

Let  $d_c$  = Core diameter of the bolts.

We know that

$$\frac{\pi}{4} (d_c)^2 \sigma_t = \frac{4}{3} \times \frac{W}{n}$$

or 
$$\frac{\pi}{4} (d_c)^2 35 = \frac{4}{3} \times \frac{16.5 \times 10^3}{4} = 5.5 \times 10^3$$

$$\therefore (d_c)^2 = \frac{5.5 \times 10^3 \times 4}{\pi \times 35} = 200 \quad \text{or} \quad d_c = 14.2 \text{ mm Ans.}$$

### Deflection of the cap

We know that deflection of the cap,

$$\delta = \frac{W . a^3}{4 E l t^3} = \frac{16.5 \times 10^3 (150)^3}{4 \times 110 \times 10^3 \times 120 (46)^3} = 0.0108 \text{ mm Ans.}$$

Since the limited value of the deflection is 0.025 mm, therefore the above value of deflection is within limits.

## Springs

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

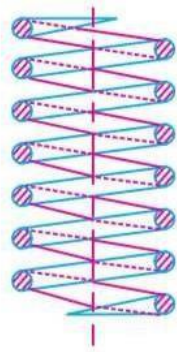


2. To apply forces, as in brakes, clutches and spring loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

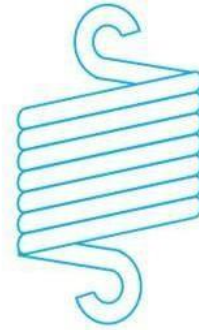
### Types of Springs

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

1. **Helical springs.** The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring as shown in Fig. and tension helical spring as shown in Fig



(a) Compression helical spring.



(b) Tension helical spring.

Figure.5.5: Types of Springs

The helical springs are said to be closely coiled when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than  $10^\circ$ . The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring. In open coiled helical springs, the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only. The helical springs have the following advantages:

- (a) These are easy to manufacture.
- (b) These are available in wide range.
- (c) These are reliable.
- (d) These have constant spring rate.
- (e) Their performance can be predicted more accurately.
- (f) Their characteristics can be varied by changing dimensions.

## Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used i.e. severe service, average service or light service. Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs. Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs. The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

Table shows the values of allowable shear stress, modulus of rigidity and modulus of elasticity for various materials used for springs. The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

### End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in Fig.

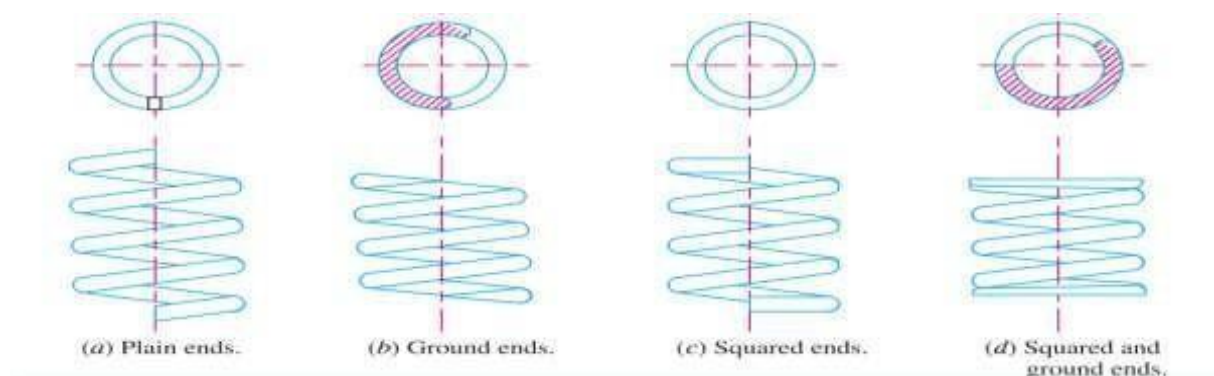


Figure.5.6: Helical Springs

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, this effect must be taken into account. The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and hence are termed as inactive coils. The turns which impart spring action are known as active turns. As the load increases, the number of inactive coils also increases due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table shows the total number of turns, solid length and free length for different types of end connections.



Problem 4. A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm<sup>2</sup>, find the axial load which the spring can carry and the deflection per active turn.

Solution. Given :  $d = 6 \text{ mm}$  ;  $D_o = 75 \text{ mm}$  ;  $\tau = 350 \text{ MPa} = 350 \text{ N/mm}^2$ ;  $G = 84 \text{ kN/mm}^2 = 84 \times 10^3 \text{ N/mm}^2$

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$$\therefore \text{Spring index, } C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let  $W = \text{Axial load, and}$

$\delta / n = \text{Deflection per active turn.}$

### 1. Neglecting the effect of curvature

We know that the shear stress factor,

$$K_s = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire ( $\tau$ ),

$$350 = K_s \times \frac{8 W \cdot D}{\pi d^3} = 1.043 \times \frac{8 W \times 69}{\pi \times 6^3} = 0.848 W$$

$$\therefore W = 350 / 0.848 = 412.7 \text{ N Ans.}$$

We know that deflection of the spring,

$$\delta = \frac{8 W \cdot D^3 \cdot n}{G \cdot d^4}$$

$\therefore$  Deflection per active turn,

$$\frac{\delta}{n} = \frac{8 W \cdot D^3}{G \cdot d^4} = \frac{8 \times 412.7 (69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm Ans.}$$

### 2. Considering the effect of curvature

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire ( $\tau$ ),

$$350 = K \times \frac{8 W \cdot C}{\pi d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913 W$$

$$\therefore W = 350 / 0.913 = 383.4 \text{ N Ans.}$$

and deflection of the spring,

$$\delta = \frac{8 W \cdot D^3 \cdot n}{G \cdot d^4}$$

$\therefore$  Deflection per active turn,

$$\frac{\delta}{n} = \frac{8 W \cdot D^3}{G \cdot d^4} = \frac{8 \times 383.4 (69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm Ans.}$$