## SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF AUTOMOBILE

## I. INTRODUCTION TO THERMODYNAMICS

Thermodynamics may be defined as follows:

- Thermodynamics is an axiomatic science which deals with the relations among heat, work and properties of system which are in equilibrium. It describes state and changes in state of physical systems.

Or

- Thermodynamics is the science of the regularities governing processes of energy conversion.

Or

- Thermodynamics is the science that deals with the interaction between energy and material systems.

Thermodynamics, basically entails four laws or axioms known as Zeroth, First, Second and Third law of thermodynamics.

- The First law throws light on concept of internal energy.
- The Zeroth law deals with thermal equilibrium and establishes a concept of temperature.
- The Second law indicates the limit of converting heat into work and introduces the
- principle of increase of entropy.
- The Third law defines the absolute zero of entropy.

Thermodynamic System: A system is a finite quantity of matter or a prescribed region of space (Refer Fig. 2.2)

Boundary. The actual or hypothetical envelope enclosing the system is the boundary of the system. The boundary may be fixed or it may move, as and when a system containing a gas is compressed or expanded. The boundary may be real or imaginary.


Fig. 1. Description of a system


Fig. 2. Real and Imaginary Boundary

## Closed System

If the boundary of the system is impervious to the flow of matter, it is called a closed system. An example of this system is mass of gas or vapour contained in an engine cylinder, the boundary of which is drawn by the cylinder walls, the cylinder head and piston crown. Here the boundary is continuous and no matter may enter or leave.


Fig. 3. Closed system


Fig. 4. Open system

## Open System

An open system is one in which matter flows into or out of the system. Most of the engineering systems are open.

## Isolated System

An isolated system is that system which exchanges neither energy nor matter with any other system or with environment.

Phase. A phase is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.

## Homogeneous System

A system which consists of a single phase is termed as homogeneous system. Examples: Mixture of air and water vapour, water plus nitric acid and octane plus heptane.

## Heterogeneous System

A system which consists of two or more phases is called a heterogeneous system. Examples: Water plus steam, ice plus water and water plus oil.

Macroscopic and microscopic points of view

| S.No | Macroscopic approach | Microscopic approach |
| :---: | :--- | :--- |
| 1 | In this approach a certain quantity of <br> matter is considered without taking <br> into account the events occurring at <br> molecular level. In other words, this <br> approach to thermodynamics is <br> concerned with gross or overall <br> behaviour. This is known as classical <br> thermodynamics. | The approach considers that the system is <br> made up of a very large number of discrete <br> particles known as molecules. These <br> molecules have different velocities and <br> energies. The values of these energies are <br> constantly changing with time. This <br> approach to thermodynamics which is <br> concerned directly with the structure of the <br> matter is known as statistical |


|  |  | thermodynamics. |
| :---: | :--- | :--- |
| 2 | The analysis of macroscopic system <br> requires simple mathematical <br> formulae. | The behaviour of the system is found by <br> using statistical methods as the number of <br> molecules is very large. So advanced <br> statistical and mathematical methods are <br> needed to explain the changes in the <br> system. |
| 3 | The values of the properties of the <br> system are their average values. <br> These properties like pressure and <br> temperature can be measured very <br> easily. The changes in properties can <br> be felt by our senses | The properties like velocity, momentum, <br> impulse, kinetic energy, force of impact <br> etc. which describe the molecule cannot be <br> easily measured by instruments. Our <br> senses cannot feel them. |
| 4 | In order to describe a system only a <br> few properties are needed. | Large number of variables are needed to <br> describe a system. So, the approach is <br> complicated. |

## Pure substance

A pure substance is one that has a homogeneous and invariable chemical composition even though there is a change of phase. In other words, it is a system which is (a) homogeneous in composition, (b) homogeneous in chemical aggregation. Examples: Liquid, water, mixture of liquid water and steam, mixture of ice and water. The mixture of liquid air and gaseous air is not a pure substance.

## Thermodynamic equilibrium

A system is in thermodynamic equilibrium if the temperature and pressure at all points are same; there should be no velocity gradient; the chemical equilibrium is also necessary.

Thus, for attaining a state of thermodynamic equilibrium the following three types of equilibrium states must be achieved:

1. Thermal equilibrium. The temperature of the system does not change with time and has same value at all points of the system.
2. Mechanical equilibrium. There are no unbalanced forces within the system or between the surroundings. The pressure in the system is same at all points and does not change with respect to time.
3. Chemical equilibrium. No chemical reaction takes place in the system and the chemical composition which is same throughout the system does not vary with time.

## Properties of systems

A property of a system is a characteristic of the system which depends upon its state, but not upon how the state is reached. There are two sorts of property:

1. Intensive properties. These properties do not depend on the mass of the system. Examples: Temperature and pressure.
2. Extensive properties. These properties depend on the mass of the system. Example: Volume. Extensive properties are often divided by mass associated with them to obtain the intensive properties. For example, if the volume of a system of mass $m$ is $V$, then the specific volume of matter within the system is $\frac{V}{m}=v$ which is an intensive property.

## State

State is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of a system is called a state.

## Process

A process occurs when the system undergoes a change in a state or an energy transfer at a steady state.

## Non-flow process

A process may be non-flow in which a fixed mass within the defined boundary is undergoing a change of state. Example: A substance which is being heated in a closed cylinder undergoes a non-flow process. Closed systems undergo non-flow processes.

## Flow process

A process may be a flow process in which mass is entering and leaving through the boundary of an open system. In a steady flow process mass is crossing the boundary from surroundings
at entry, and an equal mass is crossing the boundary at the exit so that the total mass of the system remains constant.

Quasi-static process. Quasi means 'almost'. A quasi-static process is also called a reversible process. This process is a succession of equilibrium states and infinite slowness is its characteristic feature.

## Cycle

Any process or series of processes whose end states are identical is termed a cycle.. Fig. 5 shows such a cycle in which a system commencing at condition ' 1 ' changes in pressure and volume through a path 123 and returns to its initial condition ' 1 '.


Fig.5. Cycle

## Point function

When two properties locate a point on the graph (co-ordinate axes) then those properties are called as point function. Examples. Pressure, temperature, volume etc.

$$
\int_{1}^{2} d V=V_{2}-V_{1} \text { (an exact differential). }
$$

## Path function

There are certain quantities which cannot be located on a graph by a point but are given by the area or so, on that graph. In that case, the area on the graph, pertaining to the particular process, is a function of the path of the process. Such quantities are called path functions. Examples. Heat, work etc. Heat and work are inexact differentials. Their change cannot be written as difference between their end states.

$$
\begin{aligned}
& \text { Thus } \int_{1}^{2} \delta Q \neq Q_{2}-Q_{1} \text { and is shown as }{ }_{1} Q_{2} \text { or } Q_{1-2} \\
& \text { Similarly } \int_{1}^{2} \delta W \neq W_{2}-W_{1} \text {, and is shown as }{ }_{1} W_{2} \text { or } W_{1-2}
\end{aligned}
$$

## Zeroth law of thermodynamics

Zeroth law of thermodynamics states that if two systems are each equal in temperature to a third, they are equal in temperature to each other.

## Energy

Energy is a general term embracing energy in transition and stored energy. The stored energy of a substance may be in the forms of mechanical energy and internal energy (other forms of stored energy may be chemical energy and electrical energy). Part of the stored energy may take the form of either potential energy (which is the gravitational energy due to height above a chosen datum line) or kinetic energy due to velocity. The balance part of the energy is known as internal energy. In a non-flow process usually, there is no change of potential or kinetic energy and hence change of mechanical energy will not enter the calculations. In a flow process, however, there may be changes in both potential and kinetic energy and these must be taken into account while considering the changes of stored energy. Heat and work are the forms of energy in transition. These are the only forms in which energy can cross the boundaries of a system. Neither heat nor work can exist as stored energy.

## Work

Work is said to be done when a force moves through a distance. If a part of the boundary of a system undergoes a displacement under the action of a pressure, the work done W is the product of the force (pressure $\times$ area), and the distance it moves in the direction of the force. Fig. 6 (a) illustrates this with the conventional piston and cylinder arrangement, the heavy line defining the boundary of the system. Fig. 6 (b) illustrates another way in which work might be applied to a system. A force is exerted by the paddle as it changes the momentum of the fluid, and since this force moves during rotation of the paddle work is done.


Fig. 6. Work Transfer

Internal energy ( $\Delta \mathbf{U}$ ).

It is the heat energy stored in a gas. If a certain amount of heat is supplied to a gas the result is that temperature of gas may increase or volume of gas may increase thereby doing some external work or both temperature and volume may increase; but it will be decided by the conditions under which the gas is supplied heat. If during heating of the gas the temperature increases its internal energy will also increase.

Joule's law of internal energy states that internal energy of a perfect gas is a function of temperature only. In other words, internal energy of a gas is dependent on the temperature change only and is not affected by the change in pressure and volume.

## First law of thermodynamics

When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to net work done by the system on its surroundings.

$$
\oint d Q=\oint d W
$$

## Application of first law to a process

When a process is executed by a system, the change in stored energy of the system is numerically equal to the net heat interactions minus the network interaction during the process.

$$
\begin{aligned}
\therefore & E_{2}-E_{1} & =Q-W \\
\therefore & \Delta E & =Q-W \quad[\text { or } Q=\Delta E+W] \\
& \int_{1}^{2} d(Q-W) & =\Delta E=E_{2}-E_{1}
\end{aligned}
$$

where E represents the total internal energy

## Application of first law of thermodynamics to non-flow or Closed system

Reversible Constant Volume (or Isochoric) Process (v = constant) :

In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle-wheel work input. It will be assumed that 'constant volume' implies
zero work unless stated otherwise. Fig. 7 shows the system and states before and after the heat addition at constant volume.


Fig. 7 Reversible constant volume process.
Considering mass of the working substance unity and applying first law of thermodynamics to the process,

$$
\begin{array}{ll} 
& Q=\left(u_{2}-u_{1}\right)+W \\
\text { The work done } & W=\int_{1}^{2} p d v=0 \text { as } d v=0 . \\
\therefore & Q=\left(u_{2}-u_{1}\right)=c_{v}\left(T_{2}-T_{1}\right)
\end{array}
$$

where $c_{v}=$ Specific heat at constant volume.
For mass, $m$, of working substance

$$
Q=U_{2}-U_{1}=m c_{v}\left(T_{2}-T_{1}\right)
$$

Reversible Constant Pressure (or Isobaric) Process ( $p=$ constant).

It can be seen from Fig. 7 (b) that when the boundary of the system is inflexible as in a constant volume process, then the pressure rises when heat is supplied. Hence for a constant pressure process, the boundary must move against an external resistance as heat is supplied; for instance, a gas [Fig. 8 (a)] in a cylinder behind a piston can be made to undergo a constant pressure process. Since the piston is pushed through a certain distance by the force exerted by the gas, then the work is done by the gas on its surroundings. Fig. 8 shows the system and states before and after the heat addition at constant pressure.


Fig. 8 Reversible constant pressure process.

Considering unit mass of working substance and applying first law of thermodynamics to the process,

$$
Q=\left(u_{2}-u_{1}\right)+W
$$

The work done, $\quad W=\int_{1}^{2} p d v=p\left(v_{2}-v_{1}\right)$
$\therefore \quad Q=\left(u_{2}-u_{1}\right)+p\left(v_{2}-v_{1}\right)=u_{2}-u_{1}+p v_{2}-p v_{1}$
or

$$
\begin{aligned}
& =\left(u_{2}+p v_{2}\right)-\left(u_{1}+p v_{1}\right)=h_{2}-h_{1} \\
Q & =h_{2}-h_{1}=c_{p}\left(T_{2}-T_{1}\right)
\end{aligned}
$$

where $h=$ Enthalpy (specific), and
$c_{p}=$ Specific heat at constant pressure.
For mass, $m$, of working substance

$$
Q=H_{2}-H_{1}=m c_{p}\left(T_{2}-T_{1}\right)
$$

Reversible Temperature (or Isothermal) Process ( $p v=$ constant, $T=$ constant) :
A process at a constant temperature is called an isothermal process. When a working substance in a cylinder behind a piston expands from a high pressure to a low pressure there is a tendency for the temperature to fall. In an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly, in an isothermal compression heat must be removed from the working substance continuously during the process. Fig. 9 shows the system and states before and after the heat addition at constant temperature.


Fig. 9. Reversible isothermal process.
Considering unit mass of working substance and applying first law to the process,

$$
\begin{aligned}
Q & =\left(u_{2}-u_{1}\right)+W \\
& =c_{v}\left(T_{2}-T_{1}\right)+W
\end{aligned}
$$

$$
=0+W \quad\left[\because T_{2}=T_{1}\right]
$$

The work done, $\quad W=\int_{1}^{2} p d v$
In this case $p v=$ constant or $p=\frac{C}{v}$ (where $C=$ constant)

$$
\therefore \quad W=\int_{v_{1}}^{v_{2}} C \frac{d v}{v}=C\left[\log _{e} v\right]_{v_{1}}^{v_{2}}=C \log _{e} \frac{v_{2}}{v_{1}}
$$

The constant $C$ can either be written as $p_{1} v_{1}$ or as $p_{2} v_{2}$, since

$$
p_{1} v_{1}=p_{2} v_{2}=\text { constant }, C
$$

i.e.,

$$
W=p_{1} v_{1} \log _{e} \frac{v_{2}}{v_{1}} \text { per unit mass of working substance }
$$

or

$$
W=p_{2} v_{2} \log _{e} \frac{v_{2}}{v_{1}} \text { per unit mass of working substance }
$$

$\therefore \quad Q=W=p_{1} v_{1} \log _{e} \frac{v_{2}}{v_{1}}$
For mass, $m$, of the working substance

$$
\begin{aligned}
& Q=p_{1} V_{1} \log _{e} \frac{V_{2}}{V_{1}} \\
& Q=p_{1} V_{1} \log _{e} \frac{p_{1}}{p_{2}} \quad\left[\because \frac{V_{2}}{V_{1}}=\frac{p_{1}}{p_{2}}\right]
\end{aligned}
$$

or

Reversible Adiabatic Process ( $p v^{\nu}=$ constant) :

An adiabatic process is one in which no heat is transferred to or from the fluid during the process. Such a process can be reversible or irreversible.
Considering unit mass of working substance and applying first law to the process,

$$
\begin{aligned}
& Q=\left(u_{2}-u_{1}\right)+W \\
& O=\left(u_{2}-u_{1}\right)+W \\
& W=\left(u_{1}-u_{2}\right) \text { for any adiabatic process }
\end{aligned}
$$

In an adiabatic expansion, the work done W by the fluid is at the expense of a reduction in the internal energy of the fluid. Similarly, in an adiabatic compression process all the work done on the fluid goes to increase the internal energy of the fluid. For an adiabatic process to take place, perfect thermal insulation for the system must be available.


Fig. 9. Reversible adiabatic process

Polytropic Reversible Process ( $\boldsymbol{p}^{v^{n}}=$ constant) :

It is found that many processes in practice approximate to a reversible law of form $\mathrm{pv}^{\mathrm{n}}=$ constant, where n is a constant. Both vapours and perfect gases obey this type of law closely in many non-flow processes. Such processes are internally reversible.

$$
\text { Work done, } \begin{aligned}
W & =\frac{p_{1} v_{1}-p_{2} v_{2}}{n-1} \\
W & =\frac{R\left(T_{1}-T_{2}\right)}{n-1}
\end{aligned}
$$

Summary of Processes for Perfect Gas (Unit mass)

| Process | Index <br> n | Heat added | $\int_{1}^{2} p d v$ | $p, v, T$ <br> relations | Specific heat, c |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Constant pressure | $n=0$ | $c_{p}\left(T_{2}-T_{1}\right)$ | $p\left(v_{2}-v_{1}\right)$ | $\frac{T_{2}}{T_{1}}=\frac{v_{2}}{v_{1}}$ | $c_{p}$ |
| Constant volume | $n=\infty$ | $c_{v}\left(T_{2}-T_{1}\right)$ | 0 | $\frac{T_{1}}{T_{2}}=\frac{p_{1}}{p_{2}}$ | $c_{v}$ |
| Constant temperature | $n=1$ | $p_{1} v_{1} \log _{e} \frac{v_{2}}{v_{1}}$ | $p_{1} v_{1} \log _{e} \frac{v_{2}}{v_{1}}$ | $p_{1} v_{1}=p_{2} v_{2}$ | $\infty$ |
| Reversible adiabatic | $n=\gamma$ | 0 | $\frac{p_{1} v_{1}-p_{2} v_{2}}{\gamma-1}$ | $\begin{aligned} & p_{1} v_{1}^{\gamma}=p_{2} v_{2}^{\gamma} \\ & \frac{T_{2}}{T_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma-1} \\ & =\left(\frac{p_{2}}{P_{1}}\right)^{\frac{\gamma-1}{\gamma}} \end{aligned}$ | 0 |
| Polytropic | $n=n$ | $\begin{gathered} c_{n}\left(T_{2}-T_{1}\right) \\ =c_{v}\left(\frac{\gamma-n}{1-n}\right) \\ \quad \times\left(T_{2}-T_{1}\right) \\ =\frac{\gamma-n}{\gamma-1} \times \text { work } \\ \text { done }(\text { non }- \text { flow }) \end{gathered}$ | $\frac{p_{1} v_{1}-p_{2} v_{2}}{n-1}$ | $\begin{aligned} & p_{1} v_{1}^{n}=p_{2} v_{2}^{n} \\ & \frac{T_{2}}{T_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{n-1} \\ & =\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}} \end{aligned}$ | $c_{n}=c_{v}\left(\frac{\gamma-n}{1-n}\right)$ |

## Problems:

Problem 1: In an internal combustion engine, during the compression stroke the heat rejected to the cooling water is $50 \mathrm{~kJ} / \mathrm{kg}$ and the work input is $100 \mathrm{~kJ} / \mathrm{kg}$. Calculate the change in internal energy of the working fluid stating whether it is a gain or loss.

Solution. Heat rejected to the cooling water, $Q=-50 \mathrm{~kJ} / \mathrm{kg}$
(-ve sign since heat is rejected)
Work input,

$$
W=-100 \mathrm{~kJ} / \mathrm{kg}
$$

(-ve sign since work is supplied to the system)
Using the relation,

$$
\begin{aligned}
Q & =\left(u_{2}-u_{1}\right)+W \\
-50 & =\left(u_{2}-u_{1}\right)-100 \\
u_{2}-u_{1} & =-50+100=50 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

Hence, gain in internal energy $=50 \mathrm{~kJ} / \mathrm{kg}$. (Ans.)

Problem 2: Air enters a compressor at 105 Pa and $25^{\circ} \mathrm{C}$ having volume of $1.8 \mathrm{~m} 3 / \mathrm{kg}$ and is compressed to $5 \times 105 \mathrm{~Pa}$ isothermally. Determine: (i) Work done; (ii) Change in internal energy ; and (iii) Heat transferred.

Solution. Initial pressure of air, Initial temperature of air, Final pressure of air, Final temperature of air,

$$
\begin{aligned}
p_{1} & =10^{5} \mathrm{~Pa} \\
T_{1} & =25+273=298 \mathrm{~K} \\
p_{2} & =5 \times 10^{5} \mathrm{~Pa} \\
T_{2} & =T_{1}=298 \mathrm{~K} \text { (isothermal process) }
\end{aligned}
$$

Since, it is a closed steady state process, we can write down the first law of thermodynamics

$$
Q=\left(u_{2}-u_{1}\right)+W \ldots . . \text { per } \mathrm{kg}
$$

(i) For isothermal process :

$$
W_{1-2}=\int_{1}^{2} p \cdot d v=p_{1} v_{1} \log _{e}\left(\frac{p_{1}}{p_{2}}\right)
$$



$$
\begin{aligned}
p_{1} v_{1} & =p_{2} v_{2} \text { for isothermal process } \\
W_{1-2} & =-10^{5} \times 1.8 \log _{e}\left(\frac{1 \times 10^{5}}{5 \times 10^{5}}\right) \\
& =-2.897 \times 10^{5}=-289.7 \mathrm{~kJ} / \mathrm{kg} .
\end{aligned}
$$

(- ve sign indicates that the work is supplied to the air)
$\therefore$ Work done on the air $=\mathbf{2 8 9 . 7} \mathbf{k J} / \mathrm{kg}$. (Ans.)
(ii) Since temperature is constant,
$\therefore \quad u_{2}-u_{1}=0$
$\therefore$ Change in internal energy $=$ zero. (Ans.)
(iii) Again, $\quad Q_{1-2}=\left(u_{2}-u_{1}\right)+W$

$$
=0+(-289.7)=-289.7 \mathrm{~kJ}
$$

(- ve sign indicates that heat is lost from the system to the surroundings)
$\therefore$ Heat rejected $=\mathbf{2 8 9 . 7} \mathbf{k J} / \mathrm{kg}$. (Ans.)

Problem 3: A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is $\mathbf{- 3 4 0}$ kJ . The system completes 200 cycles per min. Complete the following table showing the method for each item, and compute the net rate of work output in $k W$

Process
1-2 Q (kJ/min)

0
$2-3 \quad 42000$
$3-4 \quad-4200$
4-1
Solution. Sum of all heat transferred during the cycle $=-340 \mathrm{~kJ}$.
Number of cycles completed by the system $=200$ cycles $/ \mathrm{min}$.
Process 1-2 :

$$
\begin{aligned}
Q & =\Delta E+W \\
\therefore \quad 0 & =\Delta E+4340 \\
\therefore \quad \Delta E & =-4340 \mathrm{~kJ} / \mathrm{min} .
\end{aligned}
$$

Process 2-3:

$$
\begin{aligned}
Q & =\Delta E+W \\
42000 & =\Delta E+0 \\
\Delta E & =42000 \mathrm{~kJ} / \mathrm{min} .
\end{aligned}
$$

Process 3-4 :

$$
\begin{array}{rlrl}
Q & =\Delta E+W \\
& & & \\
\therefore & -4200 & =-73200+W \\
& W & =69000 \mathrm{~kJ} / \mathrm{min} .
\end{array}
$$

Process 4-1 :

$$
\underset{\text { cycle }}{\Sigma Q}=-340 \mathrm{~kJ}
$$

The system completes $200 \mathrm{cycles} / \mathrm{min}$

$$
\begin{aligned}
\because \quad Q_{1-2}=Q_{2-3}+Q_{3-4}+Q_{4-1}=-340 \times 200 & =-68000 \mathrm{~kJ} / \mathrm{min} \\
0+42000+(-4200)+Q_{4-1} & =-68000 \\
Q_{4-1} & =-105800 \mathrm{~kJ} / \mathrm{min}
\end{aligned}
$$

Now, $\int d E=0$, since cyclic integral of any property is zero.

$$
\begin{array}{rlrl} 
& \Delta E_{1-2}+\Delta E_{2-3}+\Delta E_{3-4}+\Delta E_{4-1}=0 \\
& & -4340 & +42000+(-73200)+\Delta E_{4-1}=0 \\
\therefore & \Delta E_{4-1} & =35540 \mathrm{~kJ} / \mathrm{min} \\
\therefore & W_{4-1} & =Q_{4-1}-\Delta E_{4-1} \\
& & =-105800-35540=-141340 \mathrm{~kJ} / \mathrm{min}
\end{array}
$$

The completed table is given below :

| Process | $Q(k J / m i n)$ | W(kJ/min $)$ | $\Delta E(\mathrm{~kJ} / \mathrm{min})$ |
| :--- | :---: | :---: | :---: |
| $1-2$ | 0 | 4340 | -4340 |
| $2-3$ | 42000 | 0 | 42000 |
| $3-4$ | -4200 | 69000 | -73200 |
| $4-1$ | -105800 | -141340 | 35540 |

Since

$$
\underset{\text { cycle }}{\Sigma Q}=\underset{\text { cycle }}{\Sigma W}
$$

Rate of work output $=-68000 \mathrm{~kJ} / \mathrm{min}=-\frac{68000}{60} \mathrm{~kJ} / \mathrm{s}$ or kW

$$
=1133.33 \mathrm{~kW} . \text { (Ans.) }
$$


$\qquad$
$-73200$

Problem 4: 0.44 kg of air at $180^{\circ} \mathrm{C}$ expands adiabatically to three times its original volume and during the process, there is a fall in temperature to $15^{\circ} \mathrm{C}$. The work done during the process is 52.5 kJ . Calculate cp and cv

## Solution:

Mass of air,
Initial temperature,

$$
m=0.44 \mathrm{~kg}
$$

$$
T_{1}=180+273=453 \mathrm{~K}
$$

$$
\text { Ratio }=\frac{V_{2}}{V_{1}}=3
$$

Final temperature, $\quad T_{2}=15+273=288 \mathrm{~K}$
Work done during the process, $W_{1-2}=52.5 \mathrm{~kJ}$

$$
c_{p}=?, c_{v}=?
$$

For adiabatic process, we have

$$
\frac{T_{2}}{T_{1}}=\left(\frac{V_{1}}{V_{2}}\right)^{\gamma-1}
$$

$$
\frac{288}{453}=\left(\frac{1}{3}\right)^{\gamma-1} \quad \text { or } \quad 0.6357=(0.333)^{r-1}
$$

or
Taking $\log$ on both sides, we get

$$
\begin{aligned}
\log _{e}(0.6357) & =(\gamma-1) \log _{e}(0.333) \\
-0.453 & =(\gamma-1) \times(-1.0996) \\
\therefore \quad \gamma & =\frac{0.453}{1.0996}+1=1.41
\end{aligned}
$$

Also,

$$
\frac{c_{p}}{c_{v}}=\gamma=1.41
$$

Work done during adiabatic process,

$$
\begin{array}{ll} 
& W_{1-2}=\frac{m R\left(T_{1}-T_{2}\right)}{\gamma-1} \\
\therefore & 52.5=\frac{0.44 R(453-288)}{(1.41-1)} \\
\therefore & R=\frac{52.5(1.41-1)}{0.44(453-288)}=0.296 \\
\therefore & c_{p}-c_{v}=0.296
\end{array} \quad\left[\because R=c_{p}-c_{v}\right]
$$

$$
\text { Also } \quad \frac{c_{p}}{c_{v}}=1.41 \quad \text { or } \quad c_{p}=1.41 c_{v}
$$

$$
\therefore \quad 1.41 c_{v}-c_{p}=0.296
$$

$$
c_{v}=0.722 \mathbf{k J} / \mathbf{k g ~ K} . \quad \text { (Ans.) }
$$

$$
c_{p}=1.018 \mathrm{kJJ} / \mathrm{kg} \mathrm{K.} \text {. (Ans.) }
$$

Problem 5: 0.1 m 3 of an ideal gas at 300 K and 1 bar is compressed adiabatically to 8
bar. It is then cooled at constant volume and further expanded isothermally so as to
reach the condition from where it started. Calculate : (i) Pressure at the end of constant volume cooling. (ii) Change in internal energy during constant volume process. (iii) Net work done and heat transferred during the cycle. Assume $\mathbf{c p}=14.3 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ and $\mathrm{cv}=$ $10.2 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$

Solution. Given : $V_{1}=0.1 \mathrm{~m}^{3} ; T_{1}=300 \mathrm{~K} ; p_{1}=1 \mathrm{bar} ; c_{p}=14.3 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$;

$$
c_{v}=10.2 \mathrm{~kJ} / \mathrm{kg} \mathrm{~K}
$$

Refer to Fig. 4.24.
(i) Pressure at the end of constant volume cooling, $\mathbf{p}_{3}$ :

$$
\gamma=\frac{c_{p}}{c_{v}}=\frac{14.3}{10.2}=1.402
$$

Characteristic gas constant,

$$
R=c_{p}-c_{v}=14.3-10.2=4.1 \mathrm{~kJ} / \mathrm{kg} \mathrm{~K}
$$

Considering process 1-2, we have :

$$
\begin{aligned}
& p_{1} V_{1}^{\gamma}=p_{2} V_{2}^{\gamma} \\
& V_{2}=V_{1} \times\left(\frac{p_{1}}{p_{2}}\right)^{\frac{1}{\gamma}}=0.1 \times\left(\frac{1}{8}\right)^{\frac{1}{1.402}}=0.0227 \mathrm{~m}^{3}
\end{aligned}
$$

Also, $\frac{T_{2}}{T_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{8}{1}\right)^{\frac{1402-1}{1402}}=1.815$

$$
T_{2}=T_{1} \times 1.815=300 \times 1.815=544.5 \mathrm{~K}
$$



Fig. 4.24

Considering process $3-1$, we have

$$
\begin{aligned}
& p_{3} V_{3}=p_{1} V_{1} \\
\therefore \quad & p_{3}=\frac{p_{1} V_{1}}{V_{3}}=\frac{1 \times 0.1}{0.0227}=4.4 \text { bar. (Ans. } \quad\left(\because \quad V_{3}=V_{2}\right)
\end{aligned}
$$

(ii) Change in internal energy during constant volume process, $\left(U_{3}-U_{2}\right)$ :

Mass of gas, $\quad m=\frac{p_{1} V_{1}}{R T_{1}}=\frac{\left(1 \times 10^{5}\right) \times 0.1}{(4.1 \times 1000) \times 300}=0.00813 \mathrm{~kg}$
$\therefore$ Change in internal energy during constant volume process 2-3,

$$
\begin{aligned}
U_{3}-U_{2} & =m c_{v}\left(T_{3}-T_{2}\right) \\
& =0.00813 \times 10.2(300- \\
& =-20.27 \mathrm{~kJ} . \quad \text { (Ans.) }
\end{aligned}
$$

$$
=0.00813 \times 10.2(300-544.5) \quad\left(\because T_{3}=T_{1}\right)
$$

(- ve sign means decrease in internal energy)

- During constant volume cooling process, temperature and hence internal energy is reduced. This decrease in internal energy equals to heat flow to surroundings since work done is zero.
(iii) Net work done and heat transferred during the cycle :

$$
\begin{aligned}
W_{1-2} & =\frac{p_{1} V_{1}-p_{2} V_{2}}{\gamma-1}=\frac{m R\left(T_{1}-T_{2}\right)}{\gamma-1} \\
& =\frac{0.00813 \times 4.1(300-544.5)}{1.402-1}=-20.27 \mathrm{kJJ}
\end{aligned}
$$

$$
W_{2-3}=0 \quad \text {... since volume remains constant }
$$

$$
W_{3-1}=p_{3} V_{3} \log _{e}\left(\frac{V_{1}}{V_{3}}\right)=p_{1} V_{1} \log _{e}\left(\frac{p_{3}}{p_{1}}\right) \quad\left(\because p_{3} V_{3}=p_{1} V_{1}\right)
$$

$$
=\left(1 \times 10^{5}\right) \times 0.1 \times \log _{e}\left(\frac{4.4}{1}\right)
$$

$$
=14816 \mathrm{Nm}(\text { or J) or } 14.82 \mathrm{~kJ}
$$

$\therefore$ Net work done $=W_{1-2}+W_{2-3}+W_{3-1}$

$$
=(-20.27)+0+14.82=-5.45 \mathrm{kJJ}
$$

-ve sign indicates that work has been done on the system. (Ans.)
For a cyclic process : $\oint \delta Q=\oint \delta W$
$\therefore \quad$ Heat transferred during the complete cycle $=-\mathbf{5 . 4 5} \mathbf{~ k J}$
-ve sign means heat has been rejected i.e., lost from the system. (Ans.)

## APPLICATION OF FIRST LAW TO STEADY FLOW PROCESS

## Steady Flow Energy Equation (S.F.E.E.)

In many practical problems, the rate at which the fluid flows through a machine or piece of apparatus is constant. This type of flow is called steady flow.

Assumptions : The following assumptions are made in the system analysis : (i) The mass flow through the system remains constant. (ii) Fluid is uniform in composition. (iii) The only interaction between the system and surroundings are work and heat. (iv) The state of fluid at any point remains constant with time. (v) In the analysis only potential, kinetic and flow energies are considered.

Fig. 10. shows a schematic flow process for an open system. An open system is one in which both mass and energy may cross the boundaries. A wide interchange of energy may take place within an open system. Let the system be an automatic engine with the inlet manifold at the first state point and exhaust pipe as the second point. There would be an interchange of chemical energy in the fuel, kinetic energy of moving particles, internal energy of gas and heat transferred and shaft work within the system. From Fig. 4.30 it is obvious that if there is no variation of flow of mass or energy with time across the boundaries of the system the steady flow will prevail. The conditions may pass through the cyclic or non-cyclic changes within the system. As a result, the mass entering the system equals the mass leaving, also energy entering the system equals energy leaving.


Fig.10. Open system
The steady flow equation can be expressed as follows :

$$
\begin{gather*}
u_{1}+\frac{C_{1}^{2}}{2}+Z_{1} g+p_{1} v_{1}+Q=u_{2}+\frac{C_{2}^{2}}{2}+Z_{2} g+p_{2} v_{2}+W \\
\left(u_{1}+p_{1} v_{1}\right)+\frac{C_{1}^{2}}{2}+Z_{1} g+Q=\left(u_{2}+p_{2} v_{2}\right)+\frac{C_{2}^{2}}{2}+Z_{2} g+W \\
h_{1}+\frac{C_{1}^{2}}{2}+Z_{1} g+Q=h_{2}+\frac{C_{2}^{2}}{2}+Z_{2} g+W
\end{gather*}
$$

If $Z_{1}$ and $Z_{2}$ are neglected, we get

$$
h_{1}+\frac{C_{1}^{2}}{2}+Q=h_{2}+\frac{C_{2}^{2}}{2}+W
$$

where, $\quad Q=$ Heat supplied (or entering the boundary) per kg of fluid, $W=$ Work done by (or work coming out of the boundary) 1 kg of fluid,
$\mathrm{C}=$ Velocity of fluid, $\mathrm{Z}=$ Height above datum, $\mathrm{p}=$ Pressure of the fluid, $\mathrm{u}=$ Internal energy per kg of fluid, and $\mathrm{pv}=$ Energy required for 1 kg of fluid.

This equation is applicable to any medium in any steady flow. It is applicable not only to rotary machines such as centrifugal fans, pumps and compressors but also to reciprocating machines such as steam engines. In a steady flow the rate of mass flow of fluid at any section is the same as at any other section. Consider any section of cross-sectional area $A$, where the fluid velocity is C, the rate of volume flow past the section is CA. Also, since mass flow is volume flow divided by specific volume,

$$
\therefore \quad \text { Mass flow rate, } \dot{m}=\frac{C A}{v}
$$

(where $v=$ Specific volume at the section)
This equation is known as the continuity of mass equation.
With reference to Fig. 4.30.

$$
\therefore \quad \dot{m}=\frac{C_{1} A_{1}}{v_{1}}=\frac{C_{2} A_{2}}{v_{2}}
$$

Problem 6. In a gas turbine unit, the gases flow through the turbine is $15 \mathrm{~kg} / \mathrm{s}$ and the power developed by the turbine is 12000 kW . The enthalpies of gases at the inlet and outlet are $1260 \mathrm{~kJ} / \mathrm{kg}$ and $400 \mathrm{~kJ} / \mathrm{kg}$ respectively, and the velocity of gases at the inlet and outlet are $50 \mathrm{~m} / \mathrm{s}$ and $110 \mathrm{~m} / \mathrm{s}$ respectively. Calculate : (i) The rate at which heat is rejected to the turbine, and (ii) The area of the inlet pipe given that the specific volume of the gases at the inlet is $0.45 \mathrm{~m} 3 / \mathrm{kg}$

Solution. Rate of flow of gases, $\dot{m}=15 \mathrm{~kg} / \mathrm{s}$
Volume of gases at the inlet, $\quad v=0.45 \mathrm{~m}^{3} / \mathrm{kg}$
Power developed by the turbine, $\quad P=12000 \mathrm{~kW}$
$\therefore$ Work done,
Enthalpy of gases at the inlet,
Enthalpy of gases at the oulet,
Velocity of gases at the inlet,
Velocity of gases at the outlet,

$$
\begin{gathered}
W=\frac{12000}{15}=800 \mathrm{~kJ} / \mathrm{kg} \\
h_{1}=1260 \mathrm{~kJ} / \mathrm{kg} \\
h_{2}=400 \mathrm{~kJ} / \mathrm{kg} \\
C_{1}=50 \mathrm{~m} / \mathrm{s} \\
C_{2}=110 \mathrm{~m} / \mathrm{s} .
\end{gathered}
$$


(i) Heat rejected, Q :

Using the flow equation,

$$
\begin{equation*}
h_{1}+\frac{C_{1}^{2}}{2}+Q=h_{2}+\frac{C_{2}^{2}}{2}+W \tag{i}
\end{equation*}
$$

Kinetic energy at inlet $=\frac{C_{1}^{2}}{2}=\frac{50^{2}}{2} \mathrm{~m}^{2} / \mathrm{s}^{2}=\frac{50^{2} \mathrm{~kg} \mathrm{~m}^{3}}{2 \mathrm{~s}^{2} \mathrm{~kg}}=1250 \mathrm{Nm} / \mathrm{kg}=1.25 \mathrm{~kJ} / \mathrm{kg}$
Kinetic energy at outlet $=\frac{C_{1}^{2}}{2}=\frac{110^{2}}{2 \times 1000}=6.05 \mathrm{~kJ} / \mathrm{kg}$
Substituting these values in eqn. (i), we get

$$
1260+1.25+Q=400+6.05+800
$$

$\therefore$
Heat rejected

$$
\begin{aligned}
Q & =-55.2 \mathrm{~kJ} / \mathrm{kg} \\
& =+55.2 \mathrm{~kJ} / \mathrm{kg}=55.2 \times 15 \mathrm{~kJ} / \mathrm{s}=828 \mathrm{~kW} . \quad \text { (Ans.) }
\end{aligned}
$$

(ii) Inlet area, A :

Using the relation,

$$
\begin{aligned}
\dot{m} & =\frac{C A}{v} \\
\therefore \quad A & =\frac{v \dot{m}}{C}=\frac{0.45 \times 15}{50}=0.135 \mathrm{~m}^{2} .
\end{aligned}
$$

Problem 7. In an air compressor air flows steadily at the rate of $0.5 \mathrm{~kg} / \mathrm{s}$ through an air compressor. It enters the compressor at $6 \mathrm{~m} / \mathrm{s}$ with a pressure of 1 bar and a specific volume of $0.85 \mathrm{~m} 3 / \mathrm{kg}$ and leaves at $5 \mathrm{~m} / \mathrm{s}$ with a pressure of 7 bar and a specific volume of $0.16 \mathrm{~m} 3 / \mathrm{kg}$. The internal energy of the air leaving is $90 \mathrm{~kJ} / \mathrm{kg}$ greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of $60 \mathrm{~kJ} / \mathrm{s}$. Calculate : (i) The power required to drive the compressor ; (ii) The inlet and output pipe cross-sectional areas. Solution. Air flow rate through the compressor, $m=0.5 \mathrm{~kg} / \mathrm{s}$ Velocity of air at the inlet to compressor, $\mathrm{C} 1=6 \mathrm{~m} / \mathrm{s}$ Velocity of air at the outlet of compressor, $C 2=5 \mathrm{~m} / \mathrm{s}$ Pressure of air at the inlet to the compressor, p1 = 1 bar


Pressure of air at outlet to the compressor, $\quad p_{2}=7 \mathrm{bar}$
Specific volume of air at inlet to the compressor, $v_{1}=0.85 \mathrm{~m}^{3} / \mathrm{kg}$
Specific volume of air at outlet to the compressor, $v_{2}=0.16 \mathrm{~m}^{3} / \mathrm{kg}$
Difference of internal energy at the outlet and inlet of the compressor,

$$
\left(u_{2}-u_{1}\right)=90 \mathrm{~kJ} / \mathrm{kg}
$$

Heat rejected by air (to cooling water),

$$
Q=-\frac{60}{0.5}=-120 \mathrm{~kJ} / \mathrm{kg} .
$$

(i) Power required to drive the compressor :

Using the steady flow energy equation,

$$
\begin{aligned}
& u_{1}+\frac{C_{1}^{2}}{2}+p_{1} v_{1}+Q=u_{2}+\frac{C_{2}^{2}}{2}+p_{2} v_{2}+W \\
\therefore \quad W & =\left(u_{1}-u_{2}\right)+\left(\frac{C_{1}^{2}}{2}-\frac{C_{2}^{2}}{2}\right)+\left(p_{1} v_{1}-p_{2} v_{2}\right)+Q \\
& =-90+\frac{1}{1000}\left(\frac{6^{2}}{2}-\frac{5^{2}}{2}\right)+\frac{10^{5}}{1000}(1 \times 0.85-7 \times 0.16)+(-120) \\
& =-90+0.0055-27-120=-237 \mathrm{~kJ} / \mathrm{kg}(\mathrm{app} .) .
\end{aligned}
$$

(Note that the change in kinetic energy is negligibly small in comparison with the other terms).
i.e., Work input required $=237 \mathrm{~kJ} / \mathrm{kg}=237 \times 0.5 \mathrm{~kJ} / \mathrm{s}=118.5 \mathrm{~kW}$

Hence, power required to drive the compressor $=118.5 \mathrm{~kW}$. (Ans.)
(ii) Inlet and outlet pipe cross-sectional areas, $A_{1}$ and $A_{2}$ :

Using the relation,

$$
\begin{aligned}
& \dot{m}
\end{aligned}=\frac{C A}{v}, ~\left(A_{1}=\frac{\dot{m} v_{1}}{C_{1}}=\frac{0.5 \times 0.85}{6} \mathrm{~m}^{2}=0.0708 \mathrm{~m}^{2}\right.
$$

i.e., Inlet pipe cross-sectional area, $A_{1}=0.0708 \mathrm{~m}^{2}$. (Ans.)

Again,

$$
A_{2}=\frac{\dot{m} v_{2}}{C_{2}}=\frac{0.5 \times 0.16}{5} \mathrm{~m}^{2}=0.016 \mathrm{~m}^{2}
$$

i.e., Outlet pipe cross-sectional area, $A_{2}=\mathbf{0 . 0 1 6} \mathbf{~ m}^{2}$. (Ans.)

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## SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF AUTOMOBILE

## UNIT - II - Applied Thermodynamics - SAUA1301

## Limitations of first law of thermodynamics and introduction to second law

It has been observed that energy can flow from a system in the form of heat or work. The first law of thermodynamics sets no limit to the amount of the total energy of a system which can be caused to flow out as work. A limit is imposed, however, as a result of the principle enunciated in the second law of thermodynamics which states that heat will flow naturally from one energy reservoir to another at a lower temperature, but not in opposite direction without assistance. This is very important because a heat engine operates between two energy reservoirs at different temperatures. Further the first law of thermodynamics establishes equivalence between the quantity of heat used and the mechanical work but does not specify the conditions under which conversion of heat into work is possible, neither the direction in which heat transfer can take place. This gap has been bridged by the second law of thermodynamics.

## Performance of heat engines and reversed heat engines

A heat engine is used to produce the maximum work transfer from a given positive heat transfer. The measure of success is called the thermal efficiency of the engine and is defined by the ratio :

$$
\text { Thermal efficiency, } \eta_{t h}=\frac{W}{Q_{1}}
$$

where, $\mathrm{W}=$ Net work transfer from the engine, and $\mathrm{Q}_{1}=$ Heat transfer to engine.

For a reversed heat engine acting as a refrigerator when the purpose is to achieve the maximum heat transfer from the cold reservoir, the measure of success is called the coefficient of performance (C.O.P.). It is defined by the ratio :

Co-efficient of performance, (C.O.P. $)_{\text {ref. }}=\frac{Q_{2}}{W}$
where, $\mathrm{Q}_{1}=$ Heat transfer to hot reservoir, and $\mathrm{W}=$ Net work transfer to the heat pump.


Fig. 1

In all the above three cases application of the first law gives the relation $\mathrm{Q}_{1}-\mathrm{Q}_{2}=\mathrm{W}$, and this can be used to rewrite the expressions for thermal efficiency and co-efficient of performance solely in terms of the heat transfers.

$$
\begin{aligned}
\eta_{k h} & =\frac{Q_{1}-Q_{2}}{Q_{1}} \\
\text { (C.O.P. })_{\text {ref }} & =\frac{Q_{2}}{Q_{1}-Q_{2}} \\
\text { (C.O.P. })_{\text {heat pump }} & =\frac{Q_{1}}{Q_{1}-Q_{2}}
\end{aligned}
$$

It may be seen that $\eta_{\text {th }}$ is always less than unity and (C.O.P.) heat pump is always greater than unity.

## Reversible processes:

A reversible process should fulfill the following conditions :

1. The process should not involve friction of any kind.
2. Heat transfer should not take place with finite temperature difference.
3. The energy transfer as heat and work during the forward process should be identically equal to energy transfer as heat and work during the reversal of the process.
4. There should be no free or unrestricted expansion.
5. There should be no mixing of the fluids. 6. The process must proceed in a series of equilibrium states.

Some examples of ideal reversible processes are :
(i) Frictionless adiabatic expansion or compression ;
(ii) Frictionless isothermal expansion or compression ;
(iii) Condensation and boiling of liquids.

Some examples of irreversible processes are :
(i) Combustion process;
(ii) Mixing of two fluids ;
(iii) All processes involving friction;
(iv) Flow of electric current through a resistance ;
(v) Heat flow from a higher temperature to lower temperature.

Reversible processes are preferred because the devices which produce work such as engines and turbines, reversible process of the working fluid delivers more work than the corresponding irreversible processes. Also in case of fans, compressors, refrigerators and pumps less power input is required when reversible processes are used in place of corresponding irreversible ones. In thermodynamic analysis concept of reversibility, though hypothetical, is very important because a reversible process is the most efficient process. Only reversible processes can be truely represented on property diagrams. Thermodynamic reversibility can only be approached but can never be achieved. Thus the main task of the engineer is to design the system which will evolve approximate reversible processes.

## Statements of second law of thermodynamics:

The second law of thermodynamics has been enunciated meticulously by Clausius, Kelvin and Planck in slightly different words although both statements are basically identical. Each statement is based on an irreversible process. The first considers transformation of heat between two thermal reservoirs while the second considers the transformation of heat into work.

## Clausius Statement:

"It is impossible for a self-acting machine working in a cyclic process unaided by any external agency, to convey heat from a body at a lower temperature to a body at a higher temperature"

## Kelvin-Planck Statement:

"It is impossible to construct an engine, which while operating in a cycle produces no other effect except to extract heat from a single reservoir and do equivalent amount of work".

## Equivalence of Clausius Statement to the Kelvin-Planck Statement:

Consider a higher temperature reservoir $\mathrm{T}_{1}$ and low temperature reservoir $\mathrm{T}_{2}$. Fig. shows a heat pump which requires no work and transfers an amount of $\mathrm{Q}_{2}$ from a low temperature to a higher temperature reservoir (in violation of the Clausius statement). Let an amount of heat $\mathrm{Q}_{1}$ (greater than $\mathrm{Q}_{2}$ ) be transferred from high temperature reservoir to heat engine which devolops a net work, $\mathrm{W}=\mathrm{Q}_{1}-\mathrm{Q}_{2}$ and rejects $\mathrm{Q}_{2}$ to the low temperature reservoir. Since there is no heat interaction with the low temperature, it can be eliminated. The combined system of the heat engine and heat pump acts then like a heat engine exchanging heat with a single reservoir, which is the violation of the Kelvin-Planck statement.


Fig. 2 Equivalence of Clausius statement to Kelvin-Planck statement

## Clausius inequality:

When a reversible engine uses more than two reservoirs the third or higher numbered reservoirs will not be equal in temperature to the original two. Consideration of expression for efficiency of the engine indicates that for maximum efficiency, all the heat transfer should
take place at maximum or minimum reservoir temperatures. Any intermediate reservoir used will, therefore, lower the efficiency of the heat engine. Practical engine cycles often involve continuous changes of temperature during heat transfer. A relationship among processes in which these sort of changes occur is necessary. The ideal approach to a cycle in which temperature continually changes is to consider the system to be in communication with a large number of reservoirs in procession. Each reservoir is considered to have a temperature differing by a small amount from the previous one. In such a model it is possible to imagine that each reservoir is replaced by a reversible heat engine in communication with standard reservoirs at same temperature $\mathrm{T}_{0}$. Fig. shows one example to this substitution.

(a)

(b)

Fig. 3

The system to which the heat transfer is effected is neither concerned with the source of energy it receives nor with the method of transfer, save that it must be reversible. Associated with the small heat transfer dQ to the original system is a small work transfer dW and for this system the first law gives,

$$
\sum_{\text {cycle }}(\delta Q-\delta W)=0
$$

Now consider the engine replacing the reservoirs and apply the second law to the new system in Fig. (b). If the new system is not a perpetual motion machine of second kind, no positive work transfer is possible with a single reservoir.

$$
\text { Therefore, } \quad \sum_{\text {cycle }}\left(\delta W-\delta W_{R}\right) \leq 0
$$

But by the definition of thermodynamic temperature in equation

$$
\begin{aligned}
& \frac{\delta W_{R}}{\delta Q}=\frac{\delta Q_{0}-\delta Q}{\delta Q}=\frac{T_{0}-T}{T} \\
& T_{0} \sum_{\text {cycle }}\left(\frac{\delta Q}{T}\right) \leq 0 \text { but } T_{0} \neq 0 \text { and therefore ; } \\
& \sum_{\text {cvcle }}\left(\frac{\delta Q}{T}\right) \leq 0
\end{aligned}
$$

This is known as Clausius inequality.

## Carnot cycle:

The cycle was first suggested by a French engineer Sadi Carnot in 1824 which works on reversible cycle and is known as Carnot cycle. Any fluid may be used to operate the Carnot cycle which is performed in an engine cylinder the head of which is supposed alternatively to be perfect conductor or a perfect insulator of a heat. Heat is caused to flow into the cylinder by the application of high temperature energy source to the cylinder head during expansion, and to flow from the cylinder by the application of a lower temperature energy source to the head during compression.


Fig. 4


Fig. 6
The assumptions made for describing the working of the Carnot engine are as follows :
(i) The piston moving in a cylinder does not develop any friction during motion.
(ii) The walls of piston and cylinder are considered as perfect insulators of heat.
(iii)The cylinder head is so arranged that it can be a perfect heat conductor or perfect heat insulator.
(iv) The transfer of heat does not affect the temperature of source or sink.
(v) Working medium is a perfect gas and has constant specific heat.
(vi) Compression and expansion are reversible.

Following are the four stages of Carnot cycle :
Stage 1. (Process 1-2). Hot energy source is applied. Heat $Q_{1}$ is taken in whilst the fluid expands isothermally and reversibly at constant high temperature $\mathrm{T}_{1}$.
Stage 2. (Process 2-3). The cylinder becomes a perfect insulator so that no heat flow takes place. The fluid expands adiabatically and reversibly whilst temperature falls from $\mathrm{T}_{1}$ to $\mathrm{T}_{2}$.

Stage 3. (Process 3-4). Cold energy source is applied. Heat $\mathrm{Q}_{2}$ flows from the fluid whilst it is compressed isothermally and reversibly at constant lower temperature $\mathrm{T}_{2}$.
Stage 4. (Process 4-1). Cylinder head becomes a perfect insulator so that no heat flow occurs. The compression is continued adiabatically and reversibly during which temperature is raised from $T_{2}$ to $T_{1}$. The work delivered from the system during the cycle is represented by the enclosed area of the cycle. Again for a closed cycle, according to first law of the thermodynamics the work obtained is equal to the difference between the heat supplied by the source $\left(\mathrm{Q}_{1}\right)$ and the heat rejected to the $\operatorname{sink}\left(\mathrm{Q}_{2}\right)$.

$$
\begin{aligned}
& \therefore \quad W=Q_{1}-Q_{2} \\
& \text { Also, thermal efficiency, } \eta_{t h}=\frac{\text { Work done }}{\text { Heat supplied by the source }}=\frac{Q_{1}-Q_{2}}{Q_{1}} \\
& =1-\frac{Q_{2}}{Q_{1}}\left(=1-\frac{T_{2}}{T_{1}}\right) \\
& {\left[\begin{array}{rl}
\because \quad Q_{1} & =m c_{p} T_{1} \\
Q_{2} & =m c_{p} T_{2} \\
\text { where, } m & =\text { mass of fluid. }
\end{array}\right]}
\end{aligned}
$$

Such an engine since it consists entirely of reversible processes, can operate in the reverse direction so that it follows the cycle shown in Fig. (b) and operates as a heat pump. Q2 is being taken in at the lower temperature T2 during the isothermal expansion (process 4-3) and heat Q1 is being rejected at the upper temperature T1 (process 2-1). Work W will be needed to drive the pump. Again, the enclosed area represents this work which is exactly equal to that flowing from it when used as engine.

The Carnot cycle cannot be performed in practice because of the following reasons :

1. It is impossible to perform a frictionless process.
2. It is impossible to transfer the heat without temperature potential.
3. Isothermal process can be achieved only if the piston moves very slowly to allow heat transfer so that the temperature remains contant. Adiabatic process can be achieved only if the piston moves as fast as possible so that the heat transfer is negligible due to very short time available. The isothermal and adiabatic processes take place during the same stroke therefore the piston has to move very slowly for part of the stroke and it has to move very fast during remaining stroke. This variation of motion of the piston during the same stroke is not possible.

## Carnot's theorem:

"It states that of all engines operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine".


Fig. 7Two cyclic heat engines $\mathrm{HE}_{A}$ and $\mathrm{HE}_{\mathrm{B}}$ operating between the same source and sink, of which $\mathrm{HE}_{\text {в }}$ is reversible.
$\mathrm{HE}_{\mathrm{A}}$ and $\mathrm{HE}_{\mathrm{B}}$ are the two engines operating between the given source at temperature $\mathrm{T}_{1}$ and the given sink at temperature $\mathrm{T}_{2}$.

Let $\mathrm{HE}_{\mathrm{A}}$ be any heat engine and $\mathrm{HE}_{\mathrm{B}}$ be any reversible heat engine. We have to prove that efficiency of $\mathrm{HE}_{\mathrm{B}}$ is more than that of $\mathrm{HE}_{\mathrm{A}}$. Let us assume that $\eta_{\mathrm{A}}>\eta_{\mathrm{B}}$. Let the rates of working of the engines be such that,

$$
\begin{aligned}
& Q_{1 A}=Q_{1 B}=Q_{1} \\
& \eta_{A}>\eta_{B} \\
& \frac{W_{A}}{Q_{1 A}}>W_{B} \\
& Q_{1 B} \\
& W_{A}>W_{B}
\end{aligned}
$$

Now, let $\mathrm{HE}_{\mathrm{B}}$ be reversed. Since $\mathrm{HE}_{\mathrm{B}}$ is a reversible heat engine, the magnitudes of heat and work transfer quantities will remain the same, but their directions will be reversed, as shown in Fig. Since $W_{A}>W_{B}$, some part of $W_{A}$ (equal to $W_{B}$ ) may be fed to drive the reversed heat engine $\exists H_{B}$. Since $Q_{1 A}=Q_{1 B}=Q_{1}$, the heat discharged by $\exists H_{B}$ may be supplied to $\mathrm{HE}_{A}$. The source may, therefore, be eliminated. The net result is that $\mathrm{HE}_{\mathrm{A}}$ and $\exists \mathrm{H}_{\mathrm{B}}$ together constitute a heat engine which, operating in a cycle produces network $W_{A}-W_{B}$ while exchanging heat with a single reservoir at $\mathrm{T}_{2}$. This violates the Kelvin-Planck statement of the second law. Hence the assumption that $\eta_{A}>\eta_{B}$ is wrong.


Fig. HEв $_{B}$ is reversed


Fig. $8 \mathrm{HE}_{\mathrm{A}}$ and $\exists \mathrm{H}_{\mathrm{B}}$ together violate the Kelvin-Planck statement.
Therefore,

$$
\eta_{\mathrm{B}} \geq \eta_{\mathrm{A}} .
$$

## Corollary of carnot's theorem:

"The efficiency of all reversible heat engines operating between the same temperature levels is the same".

Let both the heat engines $\mathrm{HE}_{\mathrm{A}}$ and $\mathrm{HE}_{\mathrm{B}}$ be reversible. Let us assume $\eta_{\mathrm{A}}>\eta_{\mathrm{B}}$. if $\mathrm{HE}_{\mathrm{B}}$ is reversed to run say, as a heat pump using some part of the work output $\left(\mathrm{W}_{\mathrm{A}}\right)$ of engine $\mathrm{HE}_{\mathrm{A}}$, we see that the combined system of heat pump $\mathrm{HE}_{\mathrm{B}}$ and engine $\mathrm{HE}_{\mathrm{A}}$, becomes a PMM2. So $\eta_{A}$ cannot be greater than $\eta_{B}$. Similary, if we assume $\eta_{B}>\eta_{A}$ and reverse the engine $H E_{A}$, we observe that $\eta_{B}$ cannot be greater than $\eta_{A} \therefore \eta_{A}=\eta_{B}$. Since the efficiencies of all reversible engines operating between the same heat reservoirs are the same, the efficiency of a
reversible engine is independent of the nature or amount of the working substance undergoing the cycle.

## Efficiency of the reversible heat engine:

The efficiency of a reversible heat engine in which heat is received solely at $T_{1}$ is found to be,

$$
\begin{aligned}
& \eta_{\text {rev. }}=\eta_{\max }=1-\left(\frac{Q_{2}}{Q_{1}}\right)_{\text {rev. }}=1-\frac{T_{2}}{T_{1}} \\
& \eta_{\text {rev. }}=\frac{T_{1}-T_{2}}{T_{1}}
\end{aligned}
$$

From the above expression, it may be noted that as $T_{2}$ decreases and $T_{1}$ increases, the efficiency of the reversible cycle increases.

Since $\eta$ is always less than unity, $T_{2}$ is always greater than zero and $+v e$. The C.O.P. of a refrigerator is given by,

$$
(\text { C.O.P. })_{\text {ref. }}=\frac{Q_{2}}{Q_{1}-Q_{2}}=\frac{1}{\frac{Q_{1}}{Q_{2}}-1}
$$

For a reversible refrigerator, using

$$
\begin{aligned}
\frac{Q_{1}}{Q_{2}} & =\frac{T_{1}}{T_{2}} \\
\therefore \quad \text { (C.O.P.) })_{\text {rev. }} & =\frac{1}{\frac{T_{1}}{T_{2}}-1} \\
\left.\therefore \quad[\text { (C.O.P. })_{\text {ref. }}\right]_{\text {rev. }} & =\frac{T_{2}}{T_{1}-T_{2}}
\end{aligned}
$$

Similarly, for a reversible heat pump

$$
\text { [(C.O.P. } \left.)_{\text {heat pump }}\right]_{\text {ree. }}=\frac{T_{1}}{T_{1}-T_{2}}
$$

Problem 1: A heat engine receives heat at the rate of $1500 \mathrm{~kJ} / \mathrm{min}$ and gives an output of 8.2 kW. Determine : (i) The thermal efficiency ; (ii) The rate of heat rejection.

Solution. Heat received by the heat engine,

$$
\begin{aligned}
Q_{1} & =1500 \mathrm{~kJ} / \mathrm{min} \\
& =\frac{1500}{60}=25 \mathrm{~kJ} / \mathrm{s}
\end{aligned}
$$

Work output, $W=8.2 \mathrm{~kW}=8.2 \mathrm{~kJ} / \mathrm{s}$.
(i) Thermal efficiency, $\eta_{t h}=\frac{W}{Q_{1}}$

$$
=\frac{8.2}{25}=0.328=32.8 \%
$$

Hence, thermal efficiency $=32.8 \%$. (Ans.)
(ii) Rate of heat rejection,

$$
\begin{aligned}
Q_{2} & =Q_{1}-W=25-8.2 \\
& =16.8 \mathrm{~kJ} / \mathrm{s}
\end{aligned}
$$



Hence, the rate of heat rejection $=16.8 \mathrm{~kJ} / \mathrm{s}$.

Problem 2: Find the co-efficient of performance and heat transfer rate in the condenser of a refrigerator in $\mathrm{kJ} / \mathrm{h}$ which has a refrigeration capacity of $12000 \mathrm{~kJ} / \mathrm{h}$ when power input is 0.75 kW.

## Solution:

## Solution.

Refrigeration capacity, $Q_{2}=12000 \mathrm{~kJ} / \mathrm{h}$
Power input, $W=0.75 \mathrm{~kW}(=0.75 \times 60 \times 60 \mathrm{~kJ} / \mathrm{h})$
Co-efficient of performance, C.O.P. :

## Heat transfer rate :

$(\text { C.O.P. })_{\text {refrigerator }}=\frac{\text { Heat absorbed at lower temperature }}{\text { Work input }}$
$\therefore \quad$ C.O.P. $=\frac{Q_{2}}{W}=\frac{12000}{0.75 \times 60 \times 60}=4.44$
Hence C.O.P. = 4.44. (Ans.)
Hence transfer rate in condenser $=Q_{1}$


According to the first law
$Q_{1}=Q_{2}+W=12000+0.75 \times 60 \times 60=14700 \mathrm{~kJ} / \mathrm{h}$
Hence, heat transfer rate $=14700 \mathrm{~kJ} / \mathrm{h}$. (Ans.)

Problem 3: A domestic food refrigerator maintains a temperature of $-12^{\circ} \mathrm{C}$. The ambient air temperature is $35^{\circ} \mathrm{C}$. If heat leaks into the freezer at the continuous rate of $2 \mathrm{~kJ} / \mathrm{s}$ determine the least power necessary to pump this heat out continuously.

Solution. Freezer temperature,

$$
T_{2}=-12+273=261 \mathrm{~K}
$$

Ambient air temperature,

$$
T_{1}=35+273=308 \mathrm{~K}
$$

Rate of heat leakage into the freezer $=2 \mathrm{~kJ} / \mathrm{s}$
Least power required to pump the heat :
The refrigerator cycle removes heat from the freezer at the same rate at which heat leaks into it (Fig. 5.12).

For minimum power requirement

$$
\begin{aligned}
& \frac{Q_{2}}{T_{2}} & =\frac{Q_{1}}{T_{1}} \\
\therefore & Q_{1} & =\frac{Q_{2}}{T_{2}} \times T_{1}=\frac{2}{261} \times 308=2.36 \mathrm{~kJ} / \mathrm{s} \\
\therefore & W & =Q_{1}-Q_{2} \\
& & =2.36-2=0.36 \mathrm{~kJ} / \mathrm{s}=0.36 \mathrm{~kW}
\end{aligned}
$$



Hence, least power required to pump the heat continuously

$$
=0.36 \mathrm{~kW} . \quad \text { (Ans.) }
$$

Problem 4: An inventor claims that his engine has the following specifications:
Temperature limits ...... $750^{\circ} \mathrm{C}$ and $25^{\circ} \mathrm{C}$
Power developed ...... 75 kW
Fuel burned per hour ...... 3.9 kg
Heating value of the fuel ...... $74500 \mathrm{~kJ} / \mathrm{kg}$
State whether his claim is valid or not

Solution. Temperature of source, $T_{1}=750+273=1023 \mathrm{~K}$
Temperature of sink, $T_{2}=25+273=298 \mathrm{~K}$
We know that the thermal efficiency of Carnot cycle is the maximum between the specified temperature limits.

Now,

$$
\eta_{\text {carnot }}=1-\frac{T_{2}}{T_{1}}=1-\frac{298}{1023}=0.7086 \text { or } 70.86 \%
$$

The actual thermal efficiency claimed,

$$
\eta_{\text {thermal }}=\frac{\text { Work done }}{\text { Heat supplied }}=\frac{75 \times 1000 \times 60 \times 60}{3.9 \times 74500 \times 1000}=0.9292 \text { or } 92.92 \% \text {. }
$$

Since $\eta_{\text {thermal }}>\eta_{\text {carnot }}$, therefore claim of the inventor is not valid (or possible). (Ans.)

Problem 5: A cyclic heat engine operates between a source temperature of $1000^{\circ} \mathrm{C}$ and a sink temperature of $40^{\circ} \mathrm{C}$. Find the least rate of heat rejection per kW net output of the engine ?

Solution. Temperature of source,

$$
T_{1}=1000+273=1273 \mathrm{~K}
$$

Temperature of sink,

$$
T_{2}=40+273=313 \mathrm{~K}
$$

Least rate of heat rejection per kW net output :

For a reversible heat engine, the rate of heat rejection will be minimum (Fig. 5.13)

$$
\begin{aligned}
\eta_{\max } & =\eta_{\text {rev. }}=1-\frac{T_{2}}{T_{1}} \\
& =1-\frac{313}{1273}=0.754
\end{aligned}
$$

Now $\quad \frac{W_{\text {net }}}{Q_{1}}=\eta_{\max }=0.754$

$\therefore \quad Q_{1}=\frac{W_{\text {net }}}{0.754}=\frac{1}{0.754}=1.326 \mathrm{~kW}$
Now $\quad Q_{2}=Q_{1}-W_{\text {net }}=1.326-1=0.326 \mathrm{~kW}$
Hence, the least rate of heat rejection $=0.326 \mathrm{~kW}$. (Ans.)

Problem 6: A reversible heat engine operates between two reservoirs at temperatures $700^{\circ} \mathrm{C}$ and $50^{\circ} \mathrm{C}$. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of $50^{\circ} \mathrm{C}$ and $-25^{\circ} \mathrm{C}$. The heat transfer to the engine is 2500 kJ and the net work output of the combined engine refrigerator plant is 400 kJ . (i) Determine the heat transfer to the refrigerant and the net heat transfer to the reservoir at $50^{\circ} \mathrm{C}$; (ii) Reconsider (i) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 45 per cent of their maximum possible values.

## Solution:



Temperature,
Temperature,
Temperature,
$T_{1}=700+273=973 \mathrm{~K}$
$T_{2}=50+273=323 \mathrm{~K}$
$T_{3}=-25+273=248 \mathrm{~K}$
The heat transfer to the heat engine, $Q_{1}=2500 \mathrm{~kJ}$
The network output of the combined engine refrigerator plant,

$$
W=W_{1}-W_{2}=400 \mathrm{~kJ} .
$$

(i) Maximum efficiency of the heat engine cycle is given by

$$
\begin{array}{ll} 
& \begin{aligned}
\eta_{\max } & =1-\frac{T_{2}}{T_{1}}=1-\frac{323}{973}=0.668 \\
& \text { Again, } \\
\therefore \quad & \frac{W_{1}}{Q_{1}}
\end{aligned}=0.668 \\
& W_{1}
\end{array}=0.668 \times 2500=1670 \mathrm{~kJ}, ~(\text { C.O.P. })_{\max }=\frac{T_{3}}{T_{2}-T_{3}}=\frac{248}{323-248}=3.306
$$

$$
\text { Also, } \quad \text { C.O.P. }=\frac{Q_{4}}{W_{2}}=3.306
$$

Since,

$$
\begin{aligned}
& \text { Since, } \quad W_{1}-W_{2}=W=400 \mathrm{~kJ} \\
& W_{2}=W_{1}-W=1670-400=1270 \mathrm{~kJ} \\
& \therefore \quad Q_{4}=3.306 \times 1270=4198.6 \mathrm{~kJ} \\
& Q_{3}=Q_{4}+W_{2}=4198.6+1270=5468.6 \mathrm{~kJ} \\
& Q_{2}=Q_{1}-W_{1}=2500-1670=830 \mathrm{~kJ} .
\end{aligned}
$$

## Heat rejection to the $50^{\circ} \mathrm{C}$ reservoir

$$
=Q_{2}+Q_{3}=830+5468.6=6298.6 \mathbf{k J J} . \quad(\text { Ans. })
$$

(ii) Efficiency of actual heat engine cycle,

$$
\begin{array}{lrl} 
& \eta & =0.45 \eta_{\max }=0.45 \times 0.668=0.3 \\
\therefore & W_{1} & =\eta \times Q_{1}=0.3 \times 2500=750 \mathrm{~kJ} \\
\therefore & W_{2} & =750-400=350 \mathrm{~kJ}
\end{array}
$$

C.O.P. of the actual refrigerator cycle,

$$
\begin{aligned}
\text { C.O.P. } & =\frac{Q_{4}}{W_{2}}=0.45 \times 3.306=1.48 \\
\therefore \quad Q_{4} & =350 \times 1.48=518 \mathrm{~kJ} . \quad \text { (Ans.) } \\
Q_{3} & =518+350=868 \mathrm{~kJ} \\
Q_{2} & =2500-750=1750 \mathrm{~kJ}
\end{aligned}
$$

Heat rejected to $50^{\circ} \mathrm{C}$ reservoir

$$
=Q_{2}+Q_{3}=1750+868=2618 \text { kJ. (Ans.) }
$$

Problem 7: A reversible heat pump is used to maintain a temperature of $0^{\circ} \mathrm{C}$ in a refrigerator when it rejects the heat to the surroundings at $25^{\circ} \mathrm{C}$. If the heat removal rate from the refrigerator is $1440 \mathrm{~kJ} / \mathrm{min}$, determine the C.O.P. of the machine and work input required. (ii) If the required input to run the pump is developed by a reversible engine which receives heat at $380^{\circ} \mathrm{C}$ and rejects heat to atmosphere, then determine the overall C.O.P. of the system.

Solution. (i) Temperature, $\mathrm{T} 1=25+273=298 \mathrm{~K}$ Temperature, $\mathrm{T} 2=0+273=273 \mathrm{~K}$

(a) Single system

(b) Combined system

Heat removal rate from the refrigerator,

$$
Q_{1}=1440 \mathrm{~kJ} / \mathrm{min}=24 \mathrm{~kJ} / \mathrm{s}
$$

Now, co-efficient of performance, for reversible heat pump,

$$
\begin{array}{ll}
\text { C.O.P. }=\frac{T_{1}}{T_{1}-T_{2}}=\frac{298}{(298-273)}=11.92 . \quad \text { (Ans.) } \\
\therefore & \text { (C.O.P. })_{\text {ref. }}=\frac{T_{2}}{T_{1}-T_{2}}=\frac{273}{298-273}=10.92
\end{array}
$$

Now,

$$
10.92=\frac{Q_{1}}{W}=\frac{24}{W}
$$

$\therefore \quad W=2.2 \mathrm{~kW}$
i.e.,

Work input required

$$
W=2.2 \mathrm{~kW}
$$

$$
Q_{2}=Q_{1}+W=24+2.2=26.2 \mathrm{~kJ} / \mathrm{s}
$$

(ii) Refer Fig. 5.15 (b).

The overall C.O.P. is given by,

$$
\begin{aligned}
\text { C.O.P. } & =\frac{\text { Heat removed from the refrigerator }}{\text { Heat supplied from the source }} \\
& =\frac{Q_{1}}{Q_{3}}
\end{aligned}
$$

For the reversible engine, we can write
or
or
or
or
or
or

$$
\begin{aligned}
\frac{Q_{3}}{T_{3}} & =\frac{Q_{4}}{T_{4}} \\
\frac{Q_{4}+W}{T_{3}} & =\frac{Q_{4}}{T_{4}} \\
\frac{Q_{4}+2.2}{(380+273)} & =\frac{Q_{4}}{(25+273)} \\
\frac{Q_{4}+2.2}{653} & =\frac{Q_{4}}{298} \\
298\left(Q_{4}+2.2\right) & =653 Q_{4} \\
Q_{4}(653-298) & =298 \times 2.2 \\
Q_{4} & =\frac{298 \times 2.2}{(653-298)}=1.847 \mathrm{~kJ} / \mathrm{s} \\
Q_{3} & =Q_{4}+W=1.847+2.2=4.047 \mathrm{~kJ} / \mathrm{s}
\end{aligned}
$$

Substituting this value in eqn. (i), we get

$$
\text { C.O.P. }=\frac{24}{4.047}=5.93 . \quad \text { (Ans.) }
$$

If the purpose of the system is to supply the heat to the $\operatorname{sink}$ at $25^{\circ} \mathrm{C}$, then

$$
\text { Overall C.O.P. }=\frac{Q_{2}+Q_{4}}{Q_{3}}=\frac{26.2+1.847}{4.047}=6.93 . \quad \text { (Ans.) }
$$

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## SCHOOL OF MECHANICAL ENGINEERING DEPARTMENT OF AUTOMOBILE ENGINEERING

UNIT - 3 - Applied Thermodynamics - SAUA1301

## UNIT 3 GAS POWER CYCLES AND INTERNAL COMBUSTION ENGINES

Air standard cycles - Otto, Diesel and Dual cycles. Derivation of expression for air standard efficiency and mean effective pressure .IC Engines- Introduction-Classification, Comparison between four stroke and two stroke, petrol \& diesel engines-Performance testing on internal combustion engines, Performance curves..

## DEFINITION OF A CYCLE

A cycle is defined as a repeated series of operations occurring in a certain order. It may be repeated by repeating the processes in the same order. The cycle may be of imaginary perfect engine or actual engine. The former is called ideal cycle and the latter actual cycle. In ideal cycle all accidental heat losses are prevented and the working substance is assumed to behave like a perfect working substance.

## AIR STANDARD EFFICIENCY

To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed to behave as a perfect gas) as the working substance in the engine cylinder. The efficiency of engine using air as the working medium is known as an "Air standard efficiency". This efficiency is often called ideal efficiency. The actual efficiency of a cycle is always less than the air-standard efficiency of that cycle under ideal conditions. This is taken into account by introducing a new term "Relative efficiency" which is defined as the ratio of Actual thermal efficiency to Air standard efficiency.
The analysis of all air standard cycles is based upon the following assumptions:

1. The gas in the engine cylinder is a perfect gas i.e., it obeys the gas laws and has constant specific heats.
2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures i.e., the molecular weight of cylinder gas is $29 . \mathrm{Cp}=1.005 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}, \mathrm{Cv}=0.718$ kJ/kg-K.
3. The compression and expansion processes are adiabatic and they take place without internal friction, i.e., these processes are isentropic.
4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot bodyor a cold body in contact with cylinder at appropriate points during the process.
5. The cycle is considered closed with the same 'air' always remaining in the cylinder to repeat the cycle.

## CONSTANT VOLUME OR OTTO CYCLE

This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines.

Figs. 1 (a) and (b) shows the theoretical p-V diagram and T-s diagrams of this cycle respectively.

- The point 1 represents that cylinder is full of air with volume V 1 , pressure P 1 and absolute temperature T 1 .
- Line 1-2 represents the adiabatic compression of air due to which P1, V1 and T1 change to $\mathrm{P} 2, \mathrm{~V} 2$ and T 2 respectively.
- Line 2-3 shows the supply of heat to the air at constant volume so that P2 and T2 change to P 3 and T 3 ( V 3 being the same as V 2 ).
- Line 3-4 represents the adiabatic expansion of the air. During expansion P3, V3 and T 3 change to a final value of $\mathrm{P} 4, \mathrm{~V} 4$ or V 1 and T 4 , respectively.
- Line 4-1 shows the rejection of heat by air at constant volume till original state (point 1) reaches. Consider 1 kg of air (working substance):


Fig. 1

Heat supplied at constant volume $=c_{v}\left(T_{3}-T_{2}\right)$.
Heat rejected at constant volume $=c_{v}\left(T_{4}-T_{1}\right)$.
But, work done $\quad=$ Heat supplied - Heat rejected

$$
=c_{v}\left(T_{3}-T_{2}\right)-c_{v}\left(T_{4}-T_{1}\right)
$$

$\therefore \quad$ Efficiency $=\frac{\text { Work done }}{\text { Heat supplied }}=\frac{c_{v}\left(T_{3}-T_{2}\right)-c_{v}\left(T_{4}-T_{1}\right)}{c_{v}\left(T_{3}-T_{2}\right)}$
$=1-\frac{T_{4}-T_{1}}{T_{3}-T_{2}}$
Let compression ratio, $r_{c}(=r)=\frac{v_{1}}{v_{2}}$
and expansion ratio, $\quad r_{e}(=r)=\frac{v_{4}}{v_{3}}$
(These two ratios are same in this cycle)

As

$$
\frac{T_{2}}{T_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma-1}
$$

Then,

$$
\begin{aligned}
& T_{2}=T_{1} \cdot(r)^{\gamma-1} \\
& \frac{T_{3}}{T_{4}}=\left(\frac{v_{4}}{v_{3}}\right)^{\gamma-1}
\end{aligned}
$$

Similarly,
or

$$
T_{3}=T_{4} \cdot(r)^{\gamma-1}
$$

Inserting the values of $T_{2}$ and $T_{3}$ in equation ( $i$ ), we get

$$
\begin{aligned}
\eta_{\text {otto }} & =1-\frac{T_{4}-T_{1}}{T_{4} \cdot(r)^{\gamma-1}-T_{1} \cdot(r)^{\gamma-1}}=1-\frac{T_{4}-T_{1}}{r^{\gamma-1}\left(T_{4}-T_{1}\right)} \\
& =1-\frac{1}{(r)^{\gamma-1}}
\end{aligned}
$$

This expression is known as the air standard efficiency of the Otto cycle. It is clear from the above expression that efficiency increases with the increase in the value of r , which means we can have maximum efficiency by increasing $r$ to a considerable extent, but due to practical difficulties its value is limited to about 8 . The net work done per kg in the Otto cycle can also be expressed in terms of $p$, $v$. If $p$ is expressed in bar i.e., $10^{5} \mathrm{~N} / \mathrm{m}^{2}$, then work done

$$
\begin{aligned}
& W=\left(\frac{p_{3} v_{3}-p_{4} v_{4}}{\gamma-1}-\frac{p_{2} v_{2}-p_{1} v_{1}}{\gamma-1}\right) \times 10^{2} \mathrm{~kJ} \\
& \frac{p_{3}}{p_{4}}=r^{\gamma}=\frac{p_{2}}{p_{1}} \\
& \frac{p_{3}}{p_{2}}=\frac{p_{4}}{p_{1}}=r_{p}
\end{aligned}
$$

where $r_{p}$ stands for pressure ratio.
and

$$
\begin{aligned}
v_{1} & =r v_{2}=v_{4}=r v_{3} \\
W & =\frac{1}{\gamma-1}\left[p_{4} v_{4}\left(\frac{p_{3} v_{3}}{p_{4} v_{4}}-1\right)-p_{1} v_{1}\left(\frac{p_{2} v_{2}}{p_{1} v_{1}}-1\right)\right] \\
& =\frac{1}{\gamma-1}\left[p_{4} v_{4}\left(\frac{p_{3}}{p_{4} r}-1\right)-p_{1} v_{1}\left(\frac{p_{2}}{p_{1} r}-1\right)\right] \\
& =\frac{v_{1}}{\gamma-1}\left[p_{4}\left(r^{\gamma-1}-1\right)-p_{1}\left(r^{\gamma-1}-1\right)\right] \\
& =\frac{v_{1}}{\gamma-1}\left[\left(r^{\gamma-1}-1\right)\left(p_{4}-p_{1}\right)\right] \\
& =\frac{p_{1} v_{1}}{\gamma-1}\left[\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right)\right]
\end{aligned}
$$

Mean effective pressure ( $\mathbf{p}_{\mathrm{m}}$ ) is given by :

$$
\begin{aligned}
& p_{m}=\left[\left(\frac{p_{3} v_{3}-p_{4} v_{4}}{\gamma-1}-\frac{p_{2} v_{2}-p_{1} v_{1}}{\gamma-1}\right) \div\left(v_{1}-v_{2}\right)\right] \text { bar } \\
& p_{m}= \frac{\left[\frac{p_{1} v_{1}}{\gamma-1}\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right)\right]}{\left(v_{1}-v_{2}\right)} \\
&= \frac{\frac{p_{1} v_{1}}{\gamma-1}\left[\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right)\right.}{v_{1}-\frac{v_{1}}{r}}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{\frac{p_{1} v_{1}}{\gamma-1}\left[\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right)\right]}{v_{1}\left(\frac{r-1}{r}\right)} \\
& \mathbf{p}_{\mathrm{m}}=\frac{p_{1} r\left[\left(r^{\gamma-1}-1\right)\left(r_{p}-1\right)\right]}{(\gamma-1)(r-1)}
\end{aligned}
$$

MEP may be thought of as the average pressure acting on a piston during different portions of its cycle.It is the ratio of the work done to stoke volume of the cycle

## CONSTANT PRESSURE OR DIESEL CYCLE

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle in that heat is supplied at constant pressure instead of at constant volume. Fig. 2 shows the p-v and T-s diagrams of this cycle respectively.

This cycle comprises of the following operations:


Fig. 2 P-V and T-S diagrams of Ideal Diesel Cycle
(i) 1-2 ...Adiabatic compression.
(ii) 2-3... Addition of heat at constant pressure.
(iii) 3-4... Adiabatic expansion.
(iv) 4-1 ... Rejection of heat at constant volume.

Point 1 represents that the cylinder is full of air. Let $\mathrm{P} 1, \mathrm{~V}_{1}$ and T 1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (i.e., $\mathrm{pV}^{\mathrm{r}}=$ constant) till the values become P2, V2 and T2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let volume increases from V 2 to V 3 and temperature T 2 to T 3 , corresponding to point 3. This point (3) is
called the point of cut-off. The air then expands adiabatically to the conditions P4, V4 and T4 respectively corresponding to point 4 . Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state
Consider 1 kg of air.
Heat supplied at constant pressure $=c_{p}\left(T_{3}-T_{2}\right)$
Heat rejected at constant volume $=c_{v}\left(T_{4}-T_{1}\right)$
Work done $\quad=$ Heat supplied - heat rejected

$$
\begin{aligned}
& =c_{p}\left(T_{3}-T_{2}\right)-c_{v}\left(T_{4}-T_{1}\right) \\
\therefore \quad \eta_{\text {diesel }} & =\frac{\text { Work done }}{\text { Heat supplied }} \\
& =\frac{c_{p}\left(T_{3}-T_{2}\right)-c_{v}\left(T_{4}-T_{1}\right)}{c_{p}\left(T_{3}-T_{2}\right)} \\
& =1-\frac{\left(T_{4}-T_{1}\right)}{\gamma\left(T_{3}-T_{2}\right)}
\end{aligned}
$$

$$
\ldots(i)\left[\because \frac{c_{p}}{c_{v}}=\gamma\right]
$$

Let compression ratio, $\quad r=\frac{v_{1}}{v_{2}}$, and cut-off ratio, $\rho=\frac{v_{3}}{v_{2}}$ i.e., $\frac{\text { Volume at cut-off }}{\text { Clearance volume }}$ Now, during adiabatic compression 1-2,

$$
\frac{T_{2}}{T_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma-1}=(r)^{\gamma-1} \quad \text { or } \quad T_{2}=T_{1} \cdot(r)^{\gamma-1}
$$

During constant pressure process 2-3,

$$
\frac{T_{3}}{T_{2}}=\frac{v_{3}}{v_{2}}=\rho \quad \text { or } \quad T_{3}=\rho \cdot T_{2}=\rho \cdot T_{1} \cdot(r)^{\gamma-1}
$$

During adiabatic expansion 3-4

$$
\begin{aligned}
\frac{T_{3}}{T_{4}} & =\left(\frac{v_{4}}{v_{3}}\right)^{\gamma-1} \\
& =\left(\frac{r}{\rho}\right)^{\gamma-1} \quad\left(\because \frac{v_{4}}{v_{3}}=\frac{v_{1}}{v_{3}}=\frac{v_{1}}{v_{2}} \times \frac{v_{2}}{v_{3}}=\frac{r}{\rho}\right) \\
\therefore \quad T_{4} & =\frac{T_{3}}{\left(\frac{r}{\rho}\right)^{\gamma-1}}=\frac{\rho \cdot T_{1}(r)^{\gamma-1}}{\left(\frac{r}{\rho}\right)^{\gamma-1}}=T_{1} \cdot \rho^{\gamma}
\end{aligned}
$$

By inserting values of $T_{2}, T_{3}$ and $T_{4}$ in eqn. (i), we get

$$
\eta_{\text {diesel }}=1-\frac{\left(T_{1} \cdot \rho^{\gamma}-T_{1}\right)}{\gamma\left(\rho \cdot T_{1} \cdot(r)^{\gamma-1}-T_{1} \cdot(r)^{\gamma-1}\right)}=1-\frac{\left(\rho^{\gamma}-1\right)}{\gamma(r)^{\gamma-1}(\rho-1)}
$$

It may be observed that eqn. for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $r>1$. Hence for a given compression ratio, the Otto cycle is more efficient.

Mean effective pressure $\mathbf{p}_{\mathrm{m}}$ is given by :

$$
\begin{aligned}
& p_{m}=\frac{p_{1} v_{1} r^{\gamma-1}\left[\gamma(\rho-1)-r^{1-\gamma}\left(\rho^{\gamma}-1\right)\right]}{(\gamma-1) v_{1}\left(\frac{r-1}{r}\right)} \\
& \mathbf{p}_{\mathrm{m}}=\frac{p_{1} r^{\gamma}\left[\gamma(\rho-1)-r^{1-\gamma}\left(\rho^{\gamma}-1\right)\right]}{(\gamma-1)(r-1)} .
\end{aligned}
$$

## DUAL COMBUSTION CYCLE

This cycle (also called the limited pressure cycle or mixed cycle) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines. The dual combustion cycle (Fig 3) consists of the following operations :
(i) 1-2-Adiabatic compression
(ii) 2-3-Addition of heat at constant volume
(iii) 3-4-Addition of heat at constant pressure
(iv) 4-5-Adiabatic expansion
(v) 5-1-Rejection of heat at constant volume.


Fig. 3

Consider 1 kg of air.
Total heat supplied $\quad=$ Heat supplied during the operation 2-3 + heat supplied during the operation 3-4

$$
=c_{v}\left(T_{3}-T_{2}\right)+c_{p}\left(T_{4}-T_{3}\right)
$$

Heat rejected during operation $5-1=c_{v}\left(T_{5}-T_{1}\right)$
Work done

$$
\begin{aligned}
& =\text { Heat supplied }- \text { heat rejected } \\
& =c_{v}\left(T_{3}-T_{2}\right)+c_{p}\left(T_{4}-T_{3}\right)-c_{v}\left(T_{5}-T_{1}\right) \\
\eta_{\text {dual }} & =\frac{\text { Work done }}{\text { Heat supplied }}=\frac{c_{v}\left(T_{3}-T_{2}\right)+c_{p}\left(T_{4}-T_{3}\right)-c_{v}\left(T_{5}-T_{1}\right)}{c_{v}\left(T_{3}-T_{2}\right)+c_{p}\left(T_{4}-T_{3}\right)} \\
& =1-\frac{c_{v}\left(T_{5}-T_{1}\right)}{c_{v}\left(T_{3}-T_{2}\right)+c_{p}\left(T_{4}-T_{3}\right)} \\
& =1-\frac{c_{v}\left(T_{5}-T_{1}\right)}{\left(T_{3}-T_{2}\right)+\gamma\left(T_{4}-T_{3}\right)} \quad \ldots(i) \quad\left(\because \gamma=\frac{c_{p}}{c_{v}}\right)
\end{aligned}
$$

Compression ratio,

$$
r=\frac{v_{1}}{v_{2}}
$$

During adiabatic compression process 1-2,

$$
\begin{equation*}
\frac{T_{2}}{T_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma-1}=(r)^{\gamma-1} \tag{ii}
\end{equation*}
$$

During constant volume heating process,

$$
\begin{align*}
& \frac{p_{3}}{T_{3}}=\frac{p_{2}}{T_{2}} \\
& \frac{T_{3}}{T_{2}}=\frac{p_{3}}{p_{2}}=\beta, \text { where } \beta \text { is known as pressure or explosion ratio. } \\
& T_{2}=\frac{T_{3}}{\beta} \tag{iii}
\end{align*}
$$

During adiabatic expansion process,

$$
\begin{aligned}
& \frac{T_{4}}{T_{5}}=\left(\frac{v_{5}}{v_{4}}\right)^{\gamma-1} \\
&=\left(\frac{r}{\rho}\right)^{\gamma-1} \\
&\left(\because \frac{v_{5}}{v_{4}}=\frac{v_{1}}{v_{4}}=\frac{v_{1}}{v_{2}} \times \frac{v_{2}}{v_{4}}=\frac{v_{1}}{v_{2}} \times \frac{v_{3}}{v_{4}}=\frac{r}{\rho}, \rho \text { being the cut-off ratio }\right)
\end{aligned}
$$

During constant pressure heating process,

$$
\begin{align*}
& \frac{v_{3}}{T_{3}}=\frac{v_{4}}{T_{4}} \\
& T_{4}=T_{3} \frac{v_{4}}{v_{3}}=\rho T_{3} \tag{v}
\end{align*}
$$

Putting the value of $T_{4}$ in the eqn. (iv), we get

$$
\frac{\rho T_{3}}{T_{5}}=\left(\frac{r}{\rho}\right)^{\gamma-1} \quad \text { or } \quad T_{5}=\rho \cdot T_{3} \cdot\left(\frac{\rho}{r}\right)^{\gamma-1}
$$

Putting the value of $T_{2}$ in eqn. (ii), we get

$$
\begin{aligned}
& \frac{\frac{T_{3}}{\beta}}{T_{1}}=(r)^{\gamma-1} \\
& T_{1}=\frac{T_{3}}{\beta} \cdot \frac{1}{(r)^{\gamma-1}}
\end{aligned}
$$

Now inserting the values of $T_{1}, T_{2}, T_{4}$ and $T_{5}$ in eqn. (i), we get

$$
\begin{gathered}
\eta_{\text {dual }}=1-\frac{\left[\rho \cdot T_{3}\left(\frac{\rho}{r}\right)^{\gamma-1}-\frac{T_{3}}{\beta} \cdot \frac{1}{(r)^{\gamma-1}}\right]}{\left[\left(T_{3}-\frac{T_{3}}{\beta}\right)+\gamma\left(\rho T_{3}-T_{3}\right)\right]}=1-\frac{\frac{1}{(r)^{\gamma-1}}\left(\rho^{\gamma}-\frac{1}{\beta}\right)}{\left(1-\frac{1}{\beta}\right)+\gamma(\rho-1)} \\
\eta_{\text {dual }}=1-\frac{1}{(r)^{\gamma-1}} \cdot \frac{\left(\beta \cdot \rho^{\gamma}-1\right)}{[(\beta-1)+\beta \gamma(\rho-1)]} \\
\mathbf{p}_{\mathbf{m}}=\frac{p_{1}(r)^{\gamma}\left[\beta(\rho-1)+(\beta-1)-r^{1-\gamma}\left(\beta \rho^{\gamma}-1\right)\right]}{(\gamma-1)(r-1)}
\end{gathered}
$$

## COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

Following are the important variable factors which are used as a basis for comparison of the cycles:

- Compression ratio.
- Maximum pressure
- Heat supplied
- Heat rejected
- Net work

Some of the above mentioned variables are fixed when the performance of Otto, Diesel and dual combustion cycles is to be compared.

## Efficiency Versus Compression Ratio

Fig shows the comparison for the air standard efficiencies of the Otto, Diesel and Dual combustion cycles at various compression ratios and with given cut-off ratio for the Diesel and Dual combustion cycles. It is evident from the Fig that the air standard efficiencies increase with the increase in the compression ratio. For a given compression ratio Otto cycle is the most efficient while the Diesel cycle is the least efficient.

Note. The maximum compression ratio for the petrol engine is limited by detonation. In their respective ratio ranges, the Diesel cycle is more efficient than the Otto cycle.

## For the Same Compression Ratio and the Same Heat Input

A comparison of the cycles (Otto, Diesel and Dual) on the p-v and T-s diagrams for the same compression ratio and heat supplied is shown in the Fig. 4


Fig. 4

(a)

(b)

Fig. 5

We know that,

$$
\begin{equation*}
\eta=1-\frac{\text { Heat rejected }}{\text { Heat supplied }} \tag{13.13}
\end{equation*}
$$

Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1 , the quantity of heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram. As is evident from the eqn. the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.

$$
\eta o t t o>\eta d u a l>\eta d i e s e l
$$

## For Constant Maximum Pressure and Heat Supplied

Fig. 6 shows the Otto and Diesel cycles on p -v and T-s diagrams for constant maximum pressure and heat input respectively.


Fig. 6

For the maximum pressure the points 3 and $3^{\prime}$ must lie on a constant pressure line.
On T-s diagram the heat rejected from the Diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve $4^{\prime}$ to 1 ; hence the Diesel cycle is more efficient than the Otto cycle for the condition of maximum pressure and heat supplied

1. An engine working on the Otto cycle is supplied with air at $0.1 \mathrm{MPa}, 350 \mathrm{C}$. The compression ratio is 8 . Heat supplied is $2100 \mathrm{~kJ} / \mathrm{kg}$. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency, and the mean effective pressure. (For air, $\mathrm{cp}=1.005 \mathrm{~kJ} / \mathrm{kgK}, \mathrm{cv}=0.718 \mathrm{~kJ} / \mathrm{kgK}$, and $\mathrm{R}=0.287 \mathrm{~kJ} / \mathrm{kgK}$ )

## Data given:

- Otto Cycle
- $\mathrm{p}_{1}=0.1 \mathrm{MPa}=0.1 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}$
- $\mathrm{T}_{1}=35^{\circ} \mathrm{C}=35+273=308 \mathrm{~K}$
- $r_{c}=8$
- $q_{\text {in }}=q_{2-3}=2100 \mathrm{~kJ} / \mathrm{kg}=2100 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $\mathrm{c}_{\mathrm{p}}=2100 \mathrm{~kJ} / \mathrm{kg}=2100 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $\mathrm{c}_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kgK}=1.005 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $\mathrm{c}_{\mathrm{v}}=0.718 \mathrm{~kJ} / \mathrm{kgK}=0.718 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $\mathrm{R}=0.287 \mathrm{~kJ} / \mathrm{kgK}=0.287 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$


## To calculate:

(i) Maximum pressure $\left(p_{3}\right)$
(ii) Maximum temperature $\left(T_{3}\right)$
(iii) Cycle efficiency $(\eta)$
(iv) $\operatorname{MEP}\left(p_{\mathrm{m}}\right)$

## Solution:

Given that: $r_{c}=\frac{v_{1}}{v_{2}}=8$

p -V diagram

## Cycle efficiency:

$\eta=1-\frac{1}{r_{c}^{\gamma-1}}=1-\frac{1}{8^{1.4-1}}=0.5647(56.47 \%)$

## Also, Cycle efficiency:

$$
\begin{aligned}
& \eta=\frac{w_{\text {net }}}{q_{2-3}} \Rightarrow w_{\text {net }}=\eta \times q_{2-3} \\
& \therefore w_{\text {net }}=0.5647 \times 2100 \times 10^{3}=1185.87 \times 10^{3} \mathrm{~J} / \mathrm{kg}
\end{aligned}
$$

## At State 1:

$p_{1} v_{1}=R T_{1} \Rightarrow v_{1}=\frac{R T_{1}}{p_{1}}=\frac{0.287 \times 10^{3} \times 308}{0.1 \times 10^{6}}$
$\therefore v_{1}=0.88 \mathrm{~m}^{3} / \mathrm{kg}$
Given that:
$r_{c}=\frac{v_{1}}{v_{2}}=8 \Rightarrow v_{2}=\frac{v_{1}}{8}=\frac{0.88}{8}$
$\therefore v_{2}=0.11 \mathrm{~m}^{3} / \mathrm{kg}$

## Mean effective pressure:

$$
p_{m}=\frac{w_{\text {net }}}{v_{1}-v_{2}}=\frac{1185.87 \times 10^{3}}{0.88-0.11}=1540.09 \times 10^{3} \mathrm{~N} / \mathrm{m}^{2}
$$

Given that: $c_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kgK}, c_{\mathrm{v}}=0.718 \mathrm{~kJ} / \mathrm{kgK}$

$$
\gamma=\frac{c_{p}}{c_{v}}=\frac{1.005}{0.718}=1.4
$$

## For isentropic process 1-2:

$$
\begin{aligned}
& \frac{T_{2}}{T_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma-1}=\left(r_{c}\right)^{\gamma-1} \Rightarrow T_{2}=T_{1} r_{c}^{\gamma-1} \\
& \therefore T_{2}=308 \times 8^{1.4-1}=707.6 K\left(434.6^{\circ} \mathrm{C}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{p_{2}}{p_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma}=\left(r_{c}\right)^{\gamma} \Rightarrow p_{2}=p_{1} r_{c}^{\gamma} \\
& \therefore p_{2}=0.1 \times 10^{6} \times 8^{1.4}=1.84 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

Given that: $q_{2-3}=2100 \times 10^{3} \mathrm{~J} / \mathrm{kg}$

$$
\begin{aligned}
& q_{2-3}=c_{v}\left(T_{3}-T_{2}\right) \Rightarrow 2100 \times 10^{3}=0.718 \times 10^{3}\left(T_{3}-707.6\right) \\
& \therefore T_{3}=3632.39 K\left(3359.39^{\circ} \mathrm{C}\right)
\end{aligned}
$$

## For constant volume process 2-3:

$$
\begin{aligned}
& \frac{p_{3}}{p_{2}}=\frac{T_{3}}{T_{2}} \Rightarrow p_{3}=p_{2}\left(\frac{T_{3}}{T_{2}}\right)=1.84 \times 10^{6}\left(\frac{3632.39}{707.6}\right) \\
& \therefore p_{3}=9.46 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

2. An engine of 250 mm bore and 375 mm stroke works on constant volume cycle. The clearance volume is 0.00263 m 3 . The initial pressure and temperature are 1 bar and 500 C . If the maximum pressure is 25 bar, determine (i) the air standard efficiency of the cycle and (ii) the mean effective pressure.

## Data given:

Constant Volume Cycle (Otto Cycle)

- $d=250 \mathrm{~mm}=250 \times 10^{-3} \mathrm{~m}$
- $\mathrm{L}=375 \mathrm{~mm}=375 \times 10^{-3} \mathrm{~m}$
- $v_{2}=v_{3}=0.00263 \mathrm{~m}^{3}$
- $p_{1}=1 \mathrm{bar}=1 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$
- $T_{1}=50^{\circ} \mathrm{C}=50+273=323 \mathrm{~K}$
- $p_{3}=25 \mathrm{bar}=25 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$


## For air, assume that:

- $\mathrm{c}_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kgK}=1.005 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $\mathrm{c}_{\mathrm{v}}=0.718 \mathrm{~kJ} / \mathrm{kgK}=0.718 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $\mathrm{R}=0.287 \mathrm{~kJ} / \mathrm{kgK}=0.287 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$

- $\gamma=1.4$


## To determine:

(i) Air standard efficiency ( $\eta_{\text {Otto }}$ )
(ii) Mean effective pressure ( $p_{\mathrm{m}}$ )

## Solution:

Stroke volume, $\mathrm{V}_{\mathrm{s}}$ :
$V_{s}=\frac{\pi d^{2} L}{4}=\frac{\pi \times\left(250 \times 10^{-3}\right)^{2} \times 375 \times 10^{-3}}{4}=0.0184 \mathrm{~m}^{3}$


## Total volume, V :

$$
V=V_{c}+V_{s}=0.00263+0.0184=0.02103 \mathrm{~m}^{3}
$$

## Compression ratio, $\mathbf{r}_{\mathrm{c}}$ :

$$
r_{c}=\frac{V_{1}}{V_{2}}=\frac{0.02103}{0.00263}=7.996
$$

## Air standard efficiency, $\eta$ :

$$
\eta=1-\frac{1}{r_{c}^{\gamma-1}}=1-\frac{1}{7.996^{1.4-1}}=0.5646(56.46 \%)
$$

## For isentropic process 1-2:

$$
\frac{p_{2}}{p_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma}=\left(r_{c}\right)^{\gamma} \Rightarrow p_{2}=p_{1} r_{c}^{\gamma}
$$

$$
\therefore p_{2}=1 \times 10^{5} \times 7.996^{1.4}=18.37 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
$$

Given that: $p_{3}=25 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$

## Explosion ratio, $\alpha$ :

$\alpha=\frac{p_{3}}{p_{2}}=\frac{25}{18.37}=1.36$

## Mean effective pressure, $\mathrm{p}_{\mathrm{m}}$ :

$$
p_{m}=\frac{p_{1} r_{c}(\alpha-1)\left(r_{c}^{-1}-1\right)}{(\gamma-1)\left(r_{c}-1\right)}
$$

$p_{m}=\frac{1 \times 10^{5} \times 7.996(1.36-1)\left(7.996^{1+1}-1\right)}{(1.4-1)(7.996-1)}$
$\therefore p_{m}=1.334 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$
3. The compression ratio of an air standard diesel cycle is 16 . The temperature and pressure at the beginning of isentropic compression are 150 C and 0.1 MPa respectively. During the constant pressure process, the heat is added until the temperature reaches 14800 C . Determine (i) the cutoff ratio (ii) the heat supplied per kg of air, (iii) the cycle efficiency, and (iv) the m.e.p

## Data given:

- Diesel Cycle
- $r_{c}=\left(v_{1} / v_{2}\right)=16$
- $T_{1}=15^{0} \mathrm{C}=15+273=288 \mathrm{~K}$
- $p_{1}=0.1 \mathrm{MPa}=0.1 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}$
- $T_{3}=1480^{\circ} \mathrm{C}=1480+273=1753 \mathrm{~K}$


## For air, assume that

- $c_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kgK}=1.005 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $c_{\mathrm{v}}=0.718 \mathrm{~kJ} / \mathrm{kgK}=0.718 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $R=0.287 \mathrm{~kJ} / \mathrm{kgK}=0.287 \times 10^{3} \mathrm{~J} / \mathrm{kgK}$
- $\gamma=1.4$

To calculate:
(i) Cut-off ratio $\left(v_{3} / v_{2}\right)$
(ii) Heat supplied per kg of air $\left(q_{2-3}\right)$
(iii) Cycle efficiency $(\eta)$
(iv) $\operatorname{MEP}\left(p_{\mathrm{m}}\right)$

Solution:
For isentropic process 1-2:
$\frac{T_{2}}{T_{1}}=\left(\frac{v_{1}}{v_{2}}\right)^{\gamma-1}=\left(r_{c}\right)^{\gamma-1} \Rightarrow T_{2}=T_{1} r_{c}^{\gamma-1}$
$\therefore T_{2}=288 \times 16^{1.4-1}=873.05 K\left(600.05^{0} \mathrm{C}\right)$

For constant pressure process 2-3:
Cut-off ratio: $\rho=\frac{V_{3}}{V_{2}}=\frac{T_{3}}{T_{2}}=\frac{1753}{873.05}=2.01$
Heat Supplied, $\mathbf{q}_{2-3}$ :
$q_{2-3}=c_{p}\left(T_{3}-T_{2}\right)=1.005 \times 10^{3} \times(1753-873.05)$
$\therefore q_{2-3}=884.35 \times 10^{3} \mathrm{~J} / \mathrm{kg}$

## For Isentropic expansion process 3-4:

$\frac{T_{4}}{T_{3}}=\left(\frac{\rho}{r_{c}}\right)^{\gamma-1} \Rightarrow T_{4}=T_{3}\left(\frac{\rho}{r_{c}}\right)^{\gamma-1}=1753 \times\left(\frac{2.01}{16}\right)^{1.4-1}$
$\therefore T_{4}=764.56 \mathrm{~K}\left(491.56^{\circ} \mathrm{C}\right)$

Heat rejected, $\mathbf{q}_{4-1}$ :
$q_{4-1}=c_{v}\left(T_{4}-T_{1}\right)=0.718 \times 10^{3} \times(764.56-288)$
$\therefore q_{4-1}=342.17 \times 10^{3} \mathrm{~J} / \mathrm{kg}$

## Cycle efficiency, $\boldsymbol{\eta}$ :

$$
\eta=1-\frac{q_{4-1}}{q_{2-3}}=1-\frac{342.17}{884.35}=0.6131(61.31 \%)
$$

## At State 1:

$$
p_{1} v_{1}=R T_{1} \Rightarrow v_{1}=\frac{R T_{1}}{p_{1}}=\frac{0.287 \times 10^{3} \times 288}{0.1 \times 10^{6}}=0.827 \mathrm{~m}^{3} / \mathrm{kg}
$$

## Mean effective pressure, $\mathbf{p}_{\mathrm{m}}$ :

$$
\begin{aligned}
& p_{m}=\frac{w_{n e t}}{v_{s}}=\frac{q_{2-3}-q_{4-1}}{v_{1}-v_{2}}=\frac{q_{2-3}-q_{4-1}}{v_{1}\left(1-\frac{1}{r_{c}}\right)}=\frac{884.35-342.17}{0.827\left(1-\frac{1}{16}\right)} \\
& \therefore p_{m}=6.993 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

## INTERNAL COMBUSTION ENGINES

## INTRODUCTION

As the name implies or suggests, the internal combustion engines (briefly written as IC engines) are those engines in which the combustion of fuel takes place inside the engine cylinder. These are petrol, diesel, and gas engines.

## CLASSIFICATION OF IC ENGINES

The internal combustion engines may be classified in many ways, but the following are important from the subject point of view

1. According to the type of fuel used
(a) Petrol engines. (b) Diesel engines or oil engines, and (c) Gas engines.
2. According to the method of igniting the fuel
(a) Spark ignition engines (briefly written as S.1. engines), (b) Compression ignition engines (briefly written as C.I. engines), and (c) Hot spot ignition engines
3. According to the number of strokes per cycle
(a) Four stroke cycle engines, and (b) Two stroke cycle engines.
4. According to the cycle of operation
(a) Otto. cycle (also known as constant volume cycle) engines, (b) Diesel cycle (also known as constant pressure cycle) engines, and (c) Dual combustion cycle (also known as semidiesel cycle) engines.
5. According to the speed of the engine
(a) Slow speed engines, (b) Medium speed engines, (c) High speed engines.
6. According to the cooling system
(a) Air-cooled engines. (b) Water-cooled engines. (c) Evaporative cooling engines.
7. According to the method of fuel injection
(a) Carburetor engines, (b) Air injection engines, (c) Airless or solid injection engines.
8. According to the number of cylinders
(a) Single cylinder engines (b) Multi-cylinder engines.
9. According to the arrangement of cylinders
(a) Vertical engines, (b) Horizontal engines, (c) Radial engines, (d) In-line multi-cylinder engines, (e)V-type multi-cylinder engines, (j) Opposite-cylinder engines, (g) Oppositepiston engines

## MAIN COMPONENTS OF IC ENGINES



Fig. 7
As a matter of fact, an IC engine consists of hundreds of different parts, which are important for its proper working. The description of all these parts is beyond the scope. However, the main components, which are important from academic point of view, are shown and are discussed below:

1. Cylinder. It is one of the most important part of the engine, in which the piston moves
to and fro in order to develop power. Generally, the engine cylinder has to withstand a high pressure (more than 50 bar) and temperature (more than $2000^{\circ} \mathrm{C}$ ). Thus the materials for an engine cylinder should be such that it can retain sufficient strength at such a high pressure and temperature. For ordinary engines, the cylinder is made of ordinary cast iron. But for heavy duty engines, it is made of steel alloys or aluminium alloys. In case of multiple cylinder engines, the cylinders are cast in one block known as cylinder block.Sometimes, a liner or sleeve is inserted into the cylinder, which can be replaced when worn out. As the material required for liner is comparatively small, it can be made of alloy cast iron having long life and sufficient resistance to rapid wear and tear to the fast moving reciprocating parts.
2. Cylinder head: It is fitted on one end of the cylinder, and acts as a cover to close the cylinder bore. Generally, the cylinder head contains inlet and exit valves for admitting fresh charge and exhausting the burnt gases. In petrol engines, the cylinder head also contains a spark plug for igniting the fuel-air mixture, towards the end of compression stroke. But in diesel engines, the cylinder head contains nozzle (i.e. fuel valve) for injecting the fuel into the cylinder. The cylinder head is, usually, cast as one piece and bolted to one end of the cylinder. Generally, the cylinder block and cylinder head are made from the same material. A copper or asbestos gasket is provided between the engine cylinder and cylinder head to make an air-tight joint.
3. Piston: It is considered as the heart of an I.C. engine, whose main function is to transmit the force exeIled by the burning of charge to the connecting rod. The pistons are generally made of aluminium alloys which are light in weight. They have good heat conducting property and also greater strength at higher temperatures.
4. Piston rings: These are circular rings and made of special steel alloys which retain elastic properties even at high temperatures. The piston rings are housed in the circumferential grooves provided on the outer surface of the piston. Generally, there are two sets of rings mounted for the piston. The function of the upper rings is to provide air tight seal to prevent leakage of the burnt gases into the lower portion. Similarly, the function of the lower rings is to provide effective seal to prevent leakage of the oil into the engine cylinder.
5. Connecting rod: It is a link between the piston and crankshaft, whose main function is to transmit force from the piston to the crankshaft. Moreover, it converts reciprocating motion of the piston into circular motion of the crankshaft, in the working stroke. The upper (i.e. smaller) end of the connecting rod is fitted to the piston and the lower (i.e. bigger) end to the crank. The special steel alloys or aluminium alloys are used for the manufacture of connecting rods. A special care is required for the design and manufacture of connecting rod, as it is subjected to alternatively compressive and tensile stresses as well as bending stresses.
6. Crankshaft: It is considered as the backbone of an l.c. engine whose function is to convert the reciprocating motion of the piston into the rotary motion with the help of connecting rod. This shaft contains one or more eccentric portions called cranks. That part of the crank, to which bigger end of the connecting rod is fitted, is called crank pin.It has been experienced that too many main bearings create difficulty of correct alignment. Special steel alloys are used for the manufacture of crankshaft. A special care is required for the design and manufacture of crankshaft.
7. Crank case: It is a cast iron case, which holds the cylinder and crankshaft of an I.c. engine. It also serves as a sump for the lubricating oil. The lower portion of the crank case is known as bed plate, which is fixed with the help of bolts.
8. Flywheel: It is a big wheel, mounted on the crankshaft, whose function is to maintain its speed constant. It is done by storing excess energy during the power stroke, which is returned during other strokes.

## FOUR STROKE CYCLE PETROL ENGINE

It is also known as Otto cycle. It requires four strokes of the piston to complete one cycle of operation in the engine cylinder. The four strokes of a petrol engine sucking fuel-air mixture (petrol mixed with proportionate quantity of air in the carburetor known as charge) are described below:

1. Suction or charging stroke: In this stroke, the inlet valve opens and charge is sucked into the cylinder as the piston moves downward from top dead centre (T.D.C.). It continues till the piston reaches its bottom dead centre (B.D. C.) as shown in (a).
2. Compression stroke: In this stroke, both the inlet and exhaust valves are closed and the charge is compressed as the piston moves upwards from B.D. C. to TD. C. As a result of compression, the pressure and temperature of the charge increases considerably (the actual values depend upon the compression ratio). This completes one revolution of the crank shaft. The compression stroke is shown in (b).


Four-stroke cycle petrol engine.

Fig. 8
3. Expansion or working stroke Shortly before the piston reaches T.D.C. (during compression stroke), the charge is ignited with the help of a spark plug. It suddenly increases the pressure and temperature of the products of combustion but the volume, practically, remains constant. Due to the rise in pressure, the piston is pushed down with a great force. The hot burnt gases expand due to high speed of the piston. During this expansion, some of the heat energy produced is transformed into mechanical work. It may be noted that during this working stroke, as shown in (c), both the valves are closed and piston moves from T.D.C. to B.D.C
4. Exhaust stroke: In this stroke, the exhaust valve is open as piston moves from B.D.C. to T.D.C. This movement of the piston pushes out the products of combustion, from the engine cylinder and are exhausted through the exhaust valve into the atmosphere, as shown in (d). This completes the cycle, and the engine cylinder is ready to suck the charge again.

## FOUR-STROKE CYCLE DIESEL ENGINE

It is also known as compression ignition engine because the ignition takes place due to the heat produced in the engine cylinder at the end of compression stroke. The four strokes of a diesel engine sucking pure air are described below:

1. Suction or charging stroke: In this stroke, the inlet valve opens and pure air is sucked into the cylinder as the piston moves downwards from the top dead centre (TDC). It continues till the piston reaches its bottom dead centre ( $B D C$ ) as shown ( $a$ ).
2. Compression stroke: In this stroke, both the valves are closed and the air is compressed as the piston moves upwards from $B D C$ to $T D C$. As a result of compression, pressure and temperature of the air increases considerably (the actual value depends upon the compression ratio). This completes one revolution of the crank shaft. The compression stroke is shown in (b).
 charging stroke.

(c) Expansion or working stroke.

(d) Exhaust stroke.

Fig. 9
3. Expansion or working stroke: Shortly before the piston reaches the $T D C$ (during the compression stroke), fuel oil is injected in the form of very fine spray into the engine cylinder, through the nozzle, known as fuel injection valve. At this moment temperature of the compressed
air is sufficiently high to ignite the fuel. It suddenly increases the pressure and temperature of the products of combustion. The fuel oil is continuously injected for a fraction of the revolution. The fuel oil is assumed to be burnt at constant pressure. Due to increased pressure, the piston is pushed down with a great force. The hot burnt gases expand due to high speed of the piston. During this expansion, some of the heat energy is transformed into mechanical work. It may be noted that during this working stroke, both the valves are closed and the piston moves from $T D C$ to $B D C$.
4. Exhaust stroke: In this stroke, the exhaust valve is open as the piston moves from $B D C$ to $T D C$. This movement of the piston pushes out the products of combustion from the engine cylinder through the exhaust valve into the atmosphere. This completes the cycle and the engine cylinder is ready to suck the fresh air again.

## TWO-STROKE CYCLE PETROL ENGINE

A two-stroke cycle petrol engine was devised by Duglad Clerk in I RHO. In this cycle, the suction, compression, expansion and exhaust takes place during two strokes of the piston. It means that there is one working stroke after every revolution of the crank shaft. A two stroke engine has ports instead of valves. All the four stages of a two stroke petrol engine are described below:

1. Suction stage: In this stage, the piston, while going down towards $B D C$, uncovers both the transfer port and the exhaust port The fresh fuel-air mixture flows into the engine cylinder from the crank case, as shown (a).
2. Compression stage: In this stage, the piston, while moving up, first covers the transfer port and then exhaust port. After that the fuel is compressed as the piston moves upwards as shown (b). In this stage, the inlet port opens and fresh fuel-air mixture enters into the crank case.
3. Expansion stage: Shortly before this piston reaches the TDC (during compression stroke), the charge is ignited with the help of a spark plug. It suddenly increases the pressure and temperature of the products of combustion. But the volume, practically, remains constant. Due to rise in the pressure, the piston is pushed downwards with a great force as shown in (c). The hot burnt gases expand due to high speed of the piston. During this expansion, some of the heat energy produced is transformed into mechanical work.


Fig. 10

1. Exhaust stage: In this stage, the exhaust port is opened as the piston moves downwards. The products of combustion, from the engine cylinder are exhausted through the exhaust port into the atmosphere, as shown $(d)$. This completes the cycle and the engine cylinder is ready to suck the charge again

## TWO STROKE AND FOUR STROKE CYCLE ENGINE

In a two-stroke engine, the working cycle is completed in two strokes of the piston or one revolution of the crankshaft. This is achieved by carrying out the suction and compression processes in one stroke (or more precisely in inward stroke), expansion and exhaust processes in the second stroke (or more precisely in outward stroke). In a four-stroke engine, the working cycle is completed in four-strokes of the piston or two-revolutions of the crankshaft. This is achieved by carrying out suction, compression, expansion and exhaust processes in each stroke. It will be interesting to know that from the thermodynamic point of view, there is no difference between twostroke and four-stroke cycle engines. The difference is purely mechanical.

## Advantages and Disadvantage of Two-stroke over Four-stroke Cycle Engines

## Advantages

1. A two stroke cycle engine gives twice the number of power strokes than the four stroke cycle engine at the same engine speed. Theoretically, a two-stroke cycle engine should develop twice the power as that of a four-stroke cycle engine. But in actual practice, a twostroke cycle engine develops 1.7 to 1.8 times greater value for slow speed engines the power developed by four-stroke cycle engine of the same dimensions and speed. This is due to lower compression ratio and effective stroke being less than the theoretical stroke.
2. For the same power developed, a two-stroke cycle engine is lighter, less bulky and occupies less floor area. Thus it makes, a two-stroke cycle engine suitable for marine engines and other light vehicles.
3. As the number of working strokes in a two-stroke cycle engine are twice than the
four-stroke cycle engine, so the turning moment of a two-stroke cycle engine is more uniform. Thus it makes a two-stroke cycle engine to have a lighter flywheel and foundations. This also leads to a higher mechanical efficiency of a two-stroke cycle engine.
4. The initial cost of a two-stroke cycle engine is considerably less than a four-stroke cycle engine.
5. The mechanism of a two-stroke cycle engine is much simpler than a four-stroke cycle engine.
6. The two-stroke cycle engines are much easier to start.

## Disadvantages

1. Thermal efficiency of a two-stroke cycle engine is less than that a four-stroke cycle engine, because a two-stroke cycle engine has less compression ratio than that of a four-stroke cycle engine.
2. Overall efficiency of a two stroke cycle engine is also less than that of a four-stroke cycle engine because in a two-stroke cycle, inlet and exhaust ports remain open simultaneously for some time. In spite of careful design, a small quantity of charge is lost from the engine cylinder.
3. In case of a two-stroke cycle engine, the number of power strokes is twice as those of a four-stroke cycle engine. Thus the capacity of the cooling system must be higher. Beyond a certain limit, the cooling capacity offers a considerable difficulty. Moreover, there is a greater wear and tear in a two-stroke cycle engine.
4. The consumption of lubricating oil is large in a two-stroke cycle engine because of high operating temperature.
5. The exhaust gases in a two-stroke cycle engine create noise, because of short time available for their exhaust.

## COMPARISON OF PETROL AND DIESEL ENGINES (SI and CI Engines)

Following points are important for the comparison of petrol engines and diesel engines:
Table. 1

| Petrol Engines | Diesel Engines |
| :---: | :---: |
| $\bullet$ A petrol engine draws a mixture of | $\bullet$A diesel engine draws only air <br> petrol and air during suction stroke. |
| during suction stroke |  |

engine during suction stroke

- Pressure at the end of compression is about 10 bar
- The charge (i.e. petrol and air mixture) is ignited with the help of spark plug
- The combustion of fuel takes place approximately at constant volume. In other words, it works on Otto cycle
- A petrol engine has compression ratio approximately from 6 to 10 .
- The starting' is easy due to low compression ratio.
- As the compression ratio is low, the petrol engines are lighter and cheaper.
- The running cost of a petrol engine is high because of the higher cost of petrol.
- The maintenance cost is less.
- The thermal efficiency is up to about $26 \%$.
- Overheating trouble is more due to low thermal efficiency.
- These are high speed engines.
- The petrol engines arc generally employed in light duty vehicles such as scooters, motorcycles, cars. These are also used in aero planes
- Pressure at the end of compression is about 35 bar.
- The fuel is injected in the form of fine spray. The temperature of the compressed air (about 600 "C at a pressure of about 35bar) is sufficiently high to ignite the fuel.
- The combustion of fuel takes place approximately at constant pressure. In other words. It works on Diesel cycle.
- A diesel engine has compression ratio approximately from 15 to 25 .
- The starting is little difficult due. to high compression ratio.
- As the compression ratio is high. the diesel engine;; are heavier and costlier.
- The running cost of diesel engine is low because of the lower cost of diesel.
- The maintenance cost is more.
- The thermal efficiency is up to about $40 \%$
- Overheating trouble is less due to high thermal efficiency
- These are relatively low speed engines.
- The diesel engines are generally employed in heavy duty vehicles like buses. trucks, and earth moving machines etc.


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SCHOOL OF MECHANICAL ENGINEERING DEPARTMENT OF AUTOMOBILE ENGINEERING

UNIT - 4 - Applied Thermodynamics - SAUA1301

## Unit 4: Air Compressors

### 4.1 Introduction:

Compression of air and vapour plays an important role in engineering fields. Compression of air is mostly used since it is easy to transmit air compared with vapour.

### 4.2 Uses of compressed air:

The applications of compressed air are listed below:

1) It is used in gas turbines and propulsion units.
2) It is used in striking type pneumatic tools for concrete breaking, clay or rock drilling, chipping, caulking, riveting etc.
3) It is used in rotary type pneumatic tools for drilling, grinding, hammering etc.
4) Pneumatic lifts and elevators work by compressed air.
5) It is used for cleaning purposes
6) It is used as an atomiser in paint spray and insecticides spray guns.
7) Pile drivers, extractors, concrete vibrators require compressed air.
8) Air-operated brakes are used in railways and heavy vehicles such as buses and lorries.
9) Sand blasting operation for cleaning of iron castings needs compressed air.
10) It is used for blast furnaces and air-operated chucks.
11) Compressed air is used for starting I.C.engines and also super charging them.

### 4.3 Working principle of a compressor:



Fig:4.1 Air Compressor
A line diagram of a compressor unit is shown in fig:4.1. The compression process requires work input. Hence a compressor is driven by a prime mover. Generally, an electric motor is used as prime mover. Air from atmosphere enters into the compressor It is compressed to a high pressure. Then, this high pressure air is delivered to a storage vessel (reservoir). From the reservoir, it can be conveyed to the desired place through pipe lines.
Some of the energy supplied by the prime mover is absorbed in work done against friction. Some portion of energy is lost due to radiation and coolant. The rest of the energy is maintained within the high pressure air delivered.

### 4.4 Classification of compressors:

Air compressors may be classified as follows:
According to design and principle of operation:
(a) Reciprocating compressors in which a piston reciprocates inside the cylinder.
(b) Rotary compressors in which a rotor is rotated.

According to number of stages:
(a) Single stage compressors in which compression of air takes place in one cylinder only.
(b) Multi stage compressors in which compression of air takes place in more than one cylinder.

According to pressure limit:
(a) Low pressure compressors in which the final delivery pressure is less than 10 bar,
(b) Medium pressure compressor in which the final delivery pressure is 10 bar to 80 bar and
(c) High pressure compressors in which the final delivery pressure is 80 to 100 bar.

According to capacity:
(a) Low capacity compressor (delivers $0.15 \mathrm{~m}^{3} / \mathrm{s}$ of compressed air),
(b) Medium capacity compressor (delivers $5 \mathrm{~m}^{3} / \mathrm{s}$ of compressed air) and
(c) High capacity compressor (delivers more than $5 \mathrm{~m}^{3} / \mathrm{s}$ of compressed air).

According to method of cooling:
(a) Air cooled compressor (Air is the cooling medium) and
(b)Water cooled compressor (Water is the cooling medium).

According to the nature of installation:
(a) Portable compressors (can be moved from one place to another).
(b) Semi-fixed compressors and
(c) Fixed compressors (They are permanently installed in one place).

According to applications:
(a) Rock drill compressors (used for drilling rocks),
(b) Quarrying compressors (used in quarries),
(c) Sandblasting compressors (used for cleaning of cast iron) and
(d) Spray painting compressors (used for spray painting).

According to number of air cylinders
(a) Simplex - contains one air cylinder
(b) Duplex - contains two air cylinders
(c) Triplex - contains three air cylinders
4.4.1 Reciprocating compressors may be classified as follows:
(a) Single acting compressors in which suction, compression and delivery of air (or gas) take place on one side of the piston.
(b) Double acting compressors in which suction, compression and delivery of air (or gas) take place on both sides of the piston.

### 4.5 Single stage reciprocating air compressor:

In a single stage compressor, the compression of air (or gas) takes place in a single cylinder. A schematic diagram of a single stage, single acting compressor is shown in fig:4.2.

Construction: It consists of a piston which reciprocates inside a cylinder. The piston is connected to the crankshaft by means of a connecting rod and a crank. Thus, the rotary movement of the crankshaft is converted into the reciprocating motion of the piston. Inlet and outlet valves (suction and delivery valves) are provided at the top of the cylinder.


Fig :4.2 Single stage reciprocating Air
Compressor

Working: When the piston moves down, the pressure inside the cylinder is reduced. When the cylinder pressure is reduced below atmospheric pressure, the inlet valve opens. Atmospheric air is drawn into the cylinder till the piston reaches the bottom dead centre. The delivery valve remains closed during this period. When the piston moves up, the pressure inside the cylinder increases. The inlet valve is closed, since the pressure inside the cylinder is above atmospheric. The pressure of air
inside the cylinder is increased steadily. The outlet valve is then opened and the high pressure air is delivered through the outlet valve in to the delivery pipe line.

At the top dead centre of the piston, a small volume of high pressure air is left in the clearance space. When the piston moves down again, this air is expanded and pressure reduces, Again the inlet valve opens and thus the cycle is repeated.

## Disadvantages

1. Handling of high pressure air results in leakage through the piston.
2. Cooling of the gas is not effective.
3. Requires a stronger cylinder to withstand high delivery pressure.

Applications: It is used in places where the required pressure ratio is small.

### 4.6 Compression processes:

The air may be compressed by the following processes.
(a) Isentropic or adiabatic compression,
(b) Polytropic compression and
(c) Isothermal compression
(a)Isentropic(or)adiabatic compression:

In internal combustion engines, the air (or air fuel mixture) is compressed isentropically. By isentropic compression, maximum available energy in the gas is obtained.
(b)Polytropic compression:


Fig: 4.3 Compression processes
A-B": Isothermal; A-B: Polvtropic; A-B': Isentropic
The compression follows the law $p V^{n}=$ Constant. This type of compression may be used in Bell-Coleman cycle of refrigeration.

## (c)Isothermal compression:

When compressed air (or gas) is stored in a tank, it loses its heat to the surroundings. It attains the temperature of surroundings after some time. Hence, the overall effect of this compression process is to increase the pressure of the gas keeping the temperature constant. Thus isothermal compression is suitable if the compressed air (or gas) is to be stored.

### 4.7 Power required for driving the compressor:

The following assumptions are made in deriving the power required to drive the compressor.

1. There is no pressure drop through suction and delivery valves.
2. Complete compression process takes place in one cylinder.
3. There is no clearance volume in the compressor cylinder.
4. Pressure in the suction line remains constant. Similarly, pressure in the delivery line remains constant.
5. The working fluid behaves as a perfect gas.

6 . There is no frictional losses.
The cycle can be analysed for the three different case of compression. Work required can be obtained from the $\mathrm{p}-\mathrm{V}$ diagram.

Let,
$p_{l}=$ Pressure of the air $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$, before compression
$\mathrm{V}_{1}=$ Volume of the air $\left(\mathrm{m}^{3}\right)$, before compression
$\mathrm{T}_{1}=$ Temperature of the air $(K)$, before compression
$\mathrm{p}_{2}, \mathrm{~V}_{2}$ and $\mathrm{T}_{2}$ be the corresponding values after compression.
$m$ - Mass of air induced or delivered by the cycle (kg).
$N$ - Speed in RPM.

### 4.7.1 Polytropic Compression



Fig:4.4 Polytropic compression
(Compression follows $\mathrm{pV}^{\mathrm{n}}=$ Constant)
Let $\mathrm{n}=$ Index of polytropic compression
Net work done on air/cycle is given by
$\mathrm{W}=$ Area 1-2-3-4-1
$=$ Work done during compression (1-2) + Work done during air delivery (2-3) - Work done during suction (4-1).

$$
\begin{aligned}
& \mathrm{W}=\frac{p_{2} v_{2}-p_{1} v_{1}}{n-1}+p_{2} v_{2}-p_{1} v_{1} \\
& \mathrm{~W}=\frac{p_{2} v_{2-} p_{1}+(n-1) p_{2} v_{2}-(n-1) p_{1} v_{1}}{n-1}
\end{aligned}
$$

$$
=\frac{n p_{2} v_{2}-n p_{1} v_{1}}{n-1}=\left(\frac{n}{n-1}\right) p_{2} v_{2}-p_{1} v_{1}
$$

We know that, $\mathrm{p}_{1} \mathrm{~V}_{1}=\mathrm{mR} T_{1} \& \mathrm{p}_{2} \mathrm{~V}_{2}=\mathrm{mR} \mathrm{T}_{2}$

$$
\text { Therefore, } \mathrm{W}=\frac{n}{n-1} \mathrm{~m} \mathrm{R}\left(\mathrm{~T}_{2}-\mathrm{T}_{1}\right)
$$

$$
\mathrm{W}=\frac{n}{n-1} \mathrm{mR} \mathrm{~T}_{1}\left[\frac{T_{2}}{T_{1}}-1\right]
$$

For polytropic process, $\frac{T_{2}}{T_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}$
Therefore, $\mathrm{W}=\frac{n}{n-1} \mathrm{mR} \mathrm{T}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right] \mathrm{kJ} / \mathrm{cycle}$

$$
\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right] \mathrm{kJ} / \text { cycle }
$$

Indicated power (or) Power required, $\mathrm{P}=\mathrm{W} \times \mathrm{N}, \mathrm{kW}$ for single acting reciprocating compressor;

$$
=\mathrm{W} \times 2 \mathrm{~N}, \mathrm{~kW} \text { for double acting reciprocating compressor. }
$$

### 4.7.2 Isentropic compression

## Compression follows, $\mathrm{p}^{\boldsymbol{V}}=$ Constant

Let $\gamma=$ Index of isentropic compression
Net work done on air/cycle is given by

$$
\mathrm{W}=\text { Area } 1-2-3-4-1
$$

$=$ Work done during compression (1-2) + Work done during air delivery (2-3) - Work done during suction (4-1).

$$
\begin{aligned}
\mathrm{W} & =\frac{p_{2} v_{2}-p_{1} v_{1}}{\gamma-1}+p_{2} v_{2}-p_{1} v_{1} \\
\mathrm{~W} & =\frac{p_{2} v_{2-} p_{1}+(\gamma-1) p_{2} v_{2}-(\gamma-1) p_{1} v_{1}}{\gamma-1} \\
& =\frac{\gamma p_{2} v_{2}-\gamma p_{1} v_{1}}{\gamma-1}=\left(\frac{\gamma}{\gamma-1}\right) p_{2} v_{2}-p_{1} v_{1}
\end{aligned}
$$

We know that, $\mathrm{p}_{1} \mathrm{~V}_{1}=\mathrm{mR} \mathrm{T}_{1} \& \mathrm{p}_{2} \mathrm{~V}_{2}=\mathrm{mR} \mathrm{T}_{2}$

$$
\begin{aligned}
& \mathrm{W}=\frac{\gamma}{\gamma-1} \mathrm{~m} \mathbf{R}\left(\mathrm{~T}_{2}-\mathrm{T}_{1}\right) \\
& \mathrm{W}=\frac{\gamma}{\gamma-1} \mathrm{~m} \mathbf{R} \mathbf{T}_{1}\left[\frac{T_{2}}{T_{1}}-\mathbf{1}\right]
\end{aligned}
$$

For isentropic process, $\frac{T_{2}}{T_{1}}=\left(\frac{p_{2}}{p_{1}}\right)^{\frac{\gamma-1}{\gamma}}$

Therefore, $\mathrm{W}=\frac{\gamma}{\gamma-1} \mathrm{mR} \mathrm{T}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{\gamma-1}{\gamma}}-1\right] \mathrm{kJ} /$ cycle

$$
\mathrm{W}=\frac{\gamma}{\gamma-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{\gamma-1}{\gamma}}-1\right] \quad \mathrm{kJ} / \text { cycle }
$$

### 4.7.3 Isothermal Compression

Compression follows, $\mathrm{pV}=$ Constant


Fig: 4.5 Isothermal Compression
Isothermal Work input, $\mathrm{W}=$ Area $1-2-3-4-1=$ area under 1-2 + area under 2-3 - area under 4-1
$\mathrm{W}=p_{1} V_{1} \ln \left(\frac{V_{1}}{V_{2}}\right)+p_{2} V_{2}-p_{1} V_{1}$
But $p_{1} V_{1}=p_{2} V_{2}$
$\mathrm{W}=p_{1} V_{1} \ln \left(\frac{V_{1}}{V_{2}}\right) \quad$ and $\frac{V_{1}}{V_{2}}=\frac{p_{2}}{p_{1}}$
Therefore, $\mathrm{W}=p_{2} V_{2} \ln \left(\frac{p_{2}}{p_{1}}\right) \mathrm{kJ} /$ cycle
4.8 Isothermal efficiency: Isothermal efficiency is defined as the ratio of isothermal work input to the actual work input. This is used for comparing the compressors.

Isothermal efficiency, $\eta_{\text {iso }}=\frac{\text { Isothermal work input }}{\text { Actual work output }}$
4.9 Adiabatic efficiency: Adiabatic efficiency is defined as the ratio of adiabatic work input to the actual work input. This is used for comparing the compressors.

Adiabatic efficiency, $\eta_{\text {adia }}=\frac{\text { Adiabatic work input }}{\text { Actual work output }}$

### 4.10 Mechanical efficiency:

The compressor is driven by a prime mover. The power input to the compressor is the shaft power (brake power) of the prime mover. This is also known as brake power of the compressor.

Mechanical efficiency is defined as the ratio of indicated power of the compressor to the power input to the compressor.

$$
\eta_{\mathrm{m}}=\frac{\text { Indicated power of compressor }}{\text { Power input }}
$$

```
Indicated Power, IP \(=\frac{\boldsymbol{p}_{\boldsymbol{m}} \boldsymbol{l} \boldsymbol{a N k}}{\mathbf{6 0}}\),
    where, \(\mathrm{p}_{\mathrm{m}}=\) mean effective pressure, \(\mathrm{kN} / \mathrm{m}^{2}\)
    \(I=\) length of stroke of piston, m
    \(\mathrm{a}=\) area of cross section of cylinder, \(\mathrm{m}^{2}\)
    \(\mathrm{N}=\) crank speed in rpm, and
    \(K=\) number of cylinders
```


### 4.11 Clearance and clearance volume:

When the piston reaches top dead centre (TDC) in the cylinder, there is a dead space between piston top and the cylinder head. This space is known as clearance space and the volume occupied by this space is known as clearance volume, $\mathrm{V}_{\mathrm{c}}$.

The clearance volume is expressed as percentage of piston displacement. Its value ranges from 5\%-10\% of swept volume or stroke volume $\left(\mathrm{V}_{\mathrm{s}}\right)$. The $\mathrm{p}-\mathrm{V}$ diagram for a single stage compressor, considering clearance volume is shown in fig. . At the end of delivery of high pressure air (at point 3), a small amount of high pressure air at $\mathrm{p}_{2}$ remains in the clearance space. This high pressure air which remains at the clearance space when the piston is at TDC is known as remnant air. It is expanded polytropically till atmospheric pressure ( $\mathrm{p}_{4}=\mathrm{p}_{1}$ ) is reached. The inlet valve is opened and the fresh air is sucked into the cylinder. The suction of air takes place for the rest of stroke (upto point 1). The volume of air sucked is known as effective suction volume $\left(V_{1}-V_{4}\right)$. At point 1 , the air is compressed polytropically till the delivery pressure $\left(p_{2}\right)$ is reached. Then the delivery valve is opened and high pressure air is discharged into the receiver. The delivery of air continues till the piston reaches its top dead centre, then the cycle is repeated.

### 4.11.1 Effect of clearance volume:

The following are the effects of clearance space.

1. Suction volume (volume of air sucked) is reduced.
2. Mass of air is reduced.
3. If clearance volume increases, heavy compression is required.
4. Heavy compression increases mechanical losses


Fig: $4.6 \mathrm{p}-\mathrm{V}$ diagram with clearance volume
4.11.3 Work input considering clearance volume:

Assuming the expansion (3-4) and compression (1-2) follow the law $\mathrm{p} \mathrm{V}^{\mathrm{n}}=\mathrm{C}$,
Work input per cycle is given by,
$\mathrm{W}=$ Area (1-2-3-6-5-4-1)- Area (3-6-5-4-3)
$\mathrm{W}=$ Workdone during compression - Work done during expansion
$\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right]-\frac{n}{n-1} \mathrm{p}_{4} \mathrm{~V}_{4}\left[\left(\frac{p_{3}}{p_{4}}\right)^{\frac{n-1}{n}}-1\right]$
But, $\mathrm{p}_{3}=\mathrm{p}_{2}$ and $\mathrm{p}_{4}=\mathrm{p}_{1}$
therefore
$\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right]-\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{4}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right]$
$\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1}\left(\mathrm{~V}_{1}-\mathrm{V}_{4}\right)\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right] \mathrm{kJ} /$ cycle
$\mathrm{V}_{1}-\mathrm{V}_{4}$ is called as effective suction volume.

### 4.12 Volumetric efficiency:

The clearance volume in a compressor reduces the intake capacity of the cylinder. This leads to a term called volumetric efficiency.
The volumetric efficiency is denned as the volume of free air sucked into the compressor per cycle to the stroke volume of the cylinder, the volume measured at the intake pressure and temperature or at standard atmospheric conditions, $\left(\mathrm{p}_{\mathrm{s}}=101.325 \mathrm{kN} / \mathrm{m}^{2}\right.$ and $\mathrm{T}_{\mathrm{s}}=288 \mathrm{~K}$ )

Volumetric efficiency, $\eta_{\text {vol }}=\frac{\text { Volume of free air taken in per cycle }}{\text { Stroke volume of the cylinder }}$

$$
=\frac{\text { Effective suction volume }}{\text { Swept volume }}=\frac{(V 1-V 4)}{(V 1-V 3)}=\frac{V_{1}-V_{4}}{V_{S}}
$$

Clearance ratio: Clearance ratio is defined as, the ratio of clearance volume to swept volume. It is denoted by the letter C .

Clearance ratio, $\mathrm{C}=\frac{\text { Clearance volume }}{\text { Swept volume }}=\frac{V_{c}}{V_{s}}=\frac{V_{c}}{V_{1-V_{3}}}$
Pressure ratio, $\mathrm{R}_{\mathrm{p}}=\frac{\text { Delivery pressure }}{\text { Suction pressure }}=\frac{p_{2}}{p_{1}}=\frac{p_{3}}{p_{4}}$

### 4.12.1 Expression for Volumetric efficiency

Let the compression and expansion follows the law, $\mathrm{pV}^{\mathrm{n}}=$ Constant.
Clearance ratio, $\mathrm{C}=\frac{\text { Clearance volume }}{\text { Swept volume }}=\frac{V_{c}}{V_{s}}=\frac{V_{3}}{V_{1-V_{3}}}$
$\mathrm{V}_{1}-\mathrm{V}_{3}=\frac{V_{3}}{C}$
$\mathrm{V}_{1}=\frac{V_{3}}{C}+\mathrm{V}_{3}$
$\mathrm{V}_{1}=\mathrm{V}_{3}\left(\frac{1}{c}+1\right)$
We know that, Pressure ratio, $\mathrm{R}_{\mathrm{p}}=\frac{\text { Delivery pressure }}{\text { Suction pressure }}=\frac{p_{2}}{p_{1}}=\frac{p_{3}}{p_{4}}$
By polytropic expansion process 3-4:
$\frac{p_{3}}{p_{4}}=\left(\frac{V_{4}}{V_{3}}\right)^{n}$
$\frac{V_{4}}{V_{3}}=\left(\frac{p_{3}}{p_{4}}\right)^{1 / n}=\left(R_{p}\right)^{\frac{1}{n}}$
Therefore, $V_{4}=\mathrm{V}_{3}\left(R_{p}\right)^{\frac{1}{n}}$
Volumetric efficiency, $\eta_{\text {vol }}=\frac{\text { Effective suction volume }}{\text { Swept volume }}=\frac{\left(V_{1}-V_{4}\right)}{\left(V_{1}-V_{3}\right)}$

Using equations 1,2 and 3 in 4,
$\eta_{\mathrm{vol}}=\frac{V_{3}\left(\frac{1}{C}+1\right)-V_{3}\left[R_{p}\right]^{1 / n}}{\frac{V_{3}}{C}}=\frac{V_{3}\left\{\left(\frac{1}{C}+1\right)-\left[R_{p}\right]^{1 / n}\right\}}{V_{3}\left(\frac{1}{C}\right)}=\frac{\left\{\left(\frac{1}{C}+1\right)-\left[R_{p}\right]^{1 / n}\right\}}{\left(\frac{1}{C}\right)}=\mathrm{C}\left[\left(\frac{1}{c}+1\right)-\left[R_{p}\right]^{1 / n}\right]$
$\eta_{\text {vol }}=1+\mathrm{C}-\mathrm{C}\left[R_{p}\right]^{1 / n}=1+\mathrm{C}-\mathrm{C}\left[\frac{p_{2}}{p_{1}}\right]^{1 / n}$

### 4.13 Multi-stage air compressor:

In a multi stage air compressor, compression of air takes place in more than one cylinder. Multi stage air compressor is used in places where high pressure air is required. Fig. shows the general arrangement of a two-stage air compressor. It consists of a low pressure (L.P) cylinder, an intercooler and a high pressure (H.P) cylinder. Both the pistons (in L.P and H.P cylinders) are driven by a single prime mover through a common shaft.

Atmospheric air at pressure $\mathrm{p}_{1}$ taken into the low pressure cylinder is compressed to a high pressure $\left(\mathrm{p}_{2}\right)$. This pressure is intermediate between intake pressure $\left(\mathrm{p}_{1}\right)$ and delivery pressure $\left.\mathrm{p}_{3}\right)$. Hence this is known as intermediate pressure.

The air from low pressure cylinder is then passed into an intercooler. In the intercooler, the air is cooled at constant pressure by circulating cold water. The cooled air from the intercooler is then taken into the high pressure cylinder. In the high pressure cylinder, air is further compressed to the final delivery pressure $\left(p_{3}\right)$ and supplied to the air receiver tank.


Fig : 4.7 Multistage compressor (Two stage)
stage


Fig:4.8 pV diagram of two compressor

## Advantages:

1. Saving in work input: The air is cooled in an intercooler before entering the high pressure cylinder. Hence less power is required to drive a multistage compressor as compared to a single stage compressor
for delivering same quantity of air at the same delivery pressure.
2. Better balancing: When the air is sucked in one cylinder, there is compression in the other cylinder. This provides more uniform torque. Hence size of the flywheel is reduced.
3. No leakage and better lubrication: The pressure and temperature ranges are kept within desirable limits. This results in a) Minimum air leakage through the piston of the cylinder and b) effective lubrication due to lower temperature.
4. More volumetric efficiency: For small pressure range, effect of expansion of the remnant air (high pressure air in the clearance space) is less. Thus by increasing number of stages, volumetric efficiency is improved.
5. High delivery pressure: The delivery pressure of air is high with reasonable volumetric efficiency.
6. Simple construction of LP cylinder: The maximum pressure in the low pressure cylinder is less. Hence, low pressure cylinder can be made lighter in construction.
7. Cheaper materials: Lower operating temperature permits the use of cheaper materials for construction.

## Disadvantages:

1. More than one cylinder is required.

2 An intercooler is required. This increases initial cost. Also space required is more.
3. Continuous flow of cooling water is required.
4. Complicated in construction.

### 4.14 Intercoolers:

An intercooler is a simple heat exchanger. It exchanges the heat of compressed air from the LP compressor to the circulating water before the air enters the HP compressor. It consists of a number of special metal tubes connected to corrosion resistant plates at both ends. The entire nest of tubes is covered by an outer shell


Working: Cold water enters the bottom of the intercooler through water inlet (1) and flows into the bottom tubes. Then they pass through the top tubes and leaves through the water outlet (2) at the top. Air from LP compressor enters through the air inlet (3) of the intercooler and passes over the tubes. While passing over the tubes, the air is cooled (by the cold water circulated through the tubes). This cold air leaves the intercooler through the air outlet (4). Baffle plates are provided in the intercooler to change the direction of air. This provides a better heat transfer from air to the circulating water.

Fig:4.9 Intercooler

### 4.15 Work input required in multistage compressor:

The following assumptions are made for calculating the work input in multistage compression.

1. Pressure during suction and delivery remains
constant in each stage.
2. Intercooling takes place at constant pressure in each stage.
3. The compression process is same for each stage.
4. The mass of air handled by LP cylinder and HP cylinder is same.
5. There is no clearance volume in each cylinder.

6 There is no pressure drop between the two stages, i.e., exhaust pressure of one stage is equal to the suction pressure of the next stage.


Fig:4.10 Two Stage compression

Work required to drive the multi-stage compressor can be calculated from the area of the $\mathrm{p}-\mathrm{V}$ diagram .
Let, $\mathrm{p}_{1}, \mathrm{~V}_{1}$ and $\mathrm{T}_{1}$ be the condition of air entering the LP cylinder.
$\mathrm{P}_{2}, \mathrm{~V}_{2}$ and $\mathrm{T}_{2}$ be the condition of air entering the HP cylinder.
$p_{3}$ be the final delivery pressure of air.
Then,
Total work input $=$ Work input for LP compressor + Work input for HP compressor.
$\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right]+\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-1\right] \mathrm{kJ} /$ cycle
$\mathrm{W}=\frac{n}{n-1} \mathrm{mR} \mathrm{T}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right]+\frac{n}{n-1} \mathrm{mR} \mathrm{T}_{2}\left[\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-1\right] \mathrm{kJ} /$ cycle

If intercooling is perfect, $\mathrm{T}_{2}=\mathrm{T}_{1}$, therefore,
$\mathrm{W}=\frac{n}{n-1} \mathrm{mR} \mathrm{T}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right]+\frac{n}{n-1} \mathrm{mR} \mathrm{T}_{1}\left[\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-1\right] \mathrm{kJ} /$ cycle
$\mathrm{W}=\frac{n}{n-1} \mathrm{mR} \mathrm{T}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}+\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-2\right]$ kJ/cycle
Or
$\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}+\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-2\right] \quad$ kJ/cycle
4.16 Condition for maximum efficiency (or) Condition for minimum work input (or)

To prove that for minimum work input the intermediate pressure of a two-stage compressor with perfect intercooling is the geometric mean of the intake pressure and delivery pressure (or)
To prove $p_{2}=\sqrt{p_{1} p_{3}}$
Work input for a two-stage air compressor with perfect intercooling is given by,
$\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}+\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-2\right] \mathrm{kJ} /$ cycle

If the initial pressure $\left(\mathbf{p}_{1}\right)$ and final pressure $\left(\mathbf{p}_{3}\right)$ are fixed, the value of intermediate pressure $\left(\mathbf{p}_{2}\right)$ can be determined by differentiating the above equation of work input in terms of $\mathbf{p}_{2}$ and equating it to zero.

Let, $\frac{n}{n-1} p_{1} V_{1}=k($ constant $)$ and $\frac{n-1}{n}=a$
then,

$$
\mathrm{W}=k\left[\left(\frac{p_{2}}{p_{1}}\right)^{a}+\left(\frac{p_{3}}{p_{2}}\right)^{a}-2\right]
$$

or

$$
\begin{equation*}
\mathrm{W}=k\left(p_{2}^{a} p_{1}^{-a}+p_{3}^{a} p_{2}^{-a}-2\right) \tag{1}
\end{equation*}
$$

Differentiating the above equation (1) with respect to $p_{2}$ and equating it to zero,

$$
\begin{aligned}
& \frac{d W}{d p_{2}}=k a p_{2}^{a-1} p_{1}^{-a}+k(-a) p_{3}^{a} p_{2}^{-a-1}=0 \\
& k a \frac{p_{2}^{a}}{p_{2} p_{1}^{a}}-k a p_{3}^{a} \frac{1}{p_{2}^{a} p_{2}}=0
\end{aligned}
$$

or

$$
\begin{aligned}
& \frac{k a p_{2}^{a}}{p_{2} p_{1}^{a}}=\frac{k a p_{3}^{a}}{p_{2} p_{2}^{a}} \\
& \left(\frac{p_{2}}{p_{1}}\right)^{a}=\left(\frac{p_{3}}{p_{2}}\right)^{a}
\end{aligned}
$$

or

$$
\begin{aligned}
& \frac{p_{2}}{p_{1}}=\frac{p_{3}}{p_{2}} \\
=>\quad p_{2}^{2} & =p_{1} p_{3}
\end{aligned}
$$

or
Intermediate pressure, $\mathbf{p}_{2}=\sqrt{\mathbf{p}_{1} \mathbf{p}_{3}}$
Thus for maximum efficiency the intermediate pressure is the geometric mean of the initial and final pressures.

### 4.17 Minimum work input for multistage compression with perfect intercooling:

Work input for a two-stage compressor with perfect intercooling is given by

$$
\begin{equation*}
\mathrm{W}=\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}+\left(\frac{p_{3}}{p_{2}}\right)^{\frac{n-1}{n}}-2\right] \tag{1}
\end{equation*}
$$

Work input will be minimum if $\frac{p_{2}}{p_{1}}=\frac{p_{3}}{p_{2}}$ $\qquad$

$$
p_{2}^{2}=p_{1} p_{3}
$$

Dividing both sides by $p_{1}^{2}$,
$\left(\frac{p_{2}}{p_{1}}\right)^{2}=\frac{p_{3}}{p_{1}} \Rightarrow \frac{p_{2}}{p_{1}}=\left(\frac{p_{3}}{p_{1}}\right)^{1 / 2}$
From (2), $\frac{p_{3}}{p_{2}}=\frac{p_{2}}{p_{1}}=\left(\frac{p_{3}}{p_{1}}\right)^{1 / 2}$
Substituting the equation (4) in equation (1), work input for a two stage compressor,

$$
\begin{aligned}
& \begin{aligned}
W_{\min } & =\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{3}}{p_{1}}\right)^{1 / 2\left[\frac{n-1}{n}\right]}+\left(\frac{p_{3}}{p_{1}}\right)^{1 / 2\left[\frac{n-1}{n}\right]}-2\right] \\
& =\frac{n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[2\left(\frac{p_{3}}{p_{1}}\right)^{\frac{n-1}{2 n}}-2\right] \\
W_{\min } & =\frac{2 n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{3}}{p_{1}}\right)^{\frac{n-1}{2 n}}-1\right]
\end{aligned}
\end{aligned}
$$

or

$$
W_{\min }=\frac{2 n}{n-1} m \mathrm{RT}_{1}\left[\left(\frac{p_{3}}{p_{1}}\right)^{\frac{n-1}{2 n}}-1\right]
$$

For a three stage compressor,

$$
W_{\min }=\frac{3 n}{n-1} \mathrm{p}_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{4}}{p_{1}}\right)^{\frac{n-1}{3 n}}-1\right]
$$

or

$$
W_{\min }=\frac{3 n}{n-1} m \mathrm{RT}_{1}\left[\left(\frac{p_{4}}{p_{1}}\right)^{\frac{n-1}{3 n}}-1\right]
$$

Generally, the minimum work input for a multistage reciprocating air compressor with $x$ number of stages is given by,

$$
W_{\min }=\frac{x n}{n-1} p_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{x+1}}{p_{1}}\right)^{\frac{n-1}{x n}}-1\right]
$$

Minimum work input required for a two stage reciprocating air compressor with perfect intercooling is given by,

$$
W_{\min }=\frac{2 n}{n-1} p_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{3}}{p_{1}}\right)^{\frac{n-1}{2 n}}-1\right] k J
$$

But,from equation (4), $\left(\frac{p_{3}}{p_{1}}\right)^{1 / 2}=\frac{p_{2}}{p_{1}}$
Therefore,

$$
W_{\min }=\frac{2 n}{n-1} p_{1} \mathrm{~V}_{1}\left[\left(\frac{p_{2}}{p_{1}}\right)^{\frac{n-1}{n}}-1\right] k J
$$

So, for maximum efficiency ie., for minimum work input, the work required for each stage is same.
For maximum efficiency, the following conditions must be satisfied:

1. The air is cooled to the initial temperature between the stages (Perfect cooling between stages).
2. In each stage, the pressure ratio is same. $\left(\frac{p_{2}}{p_{1}}=\frac{p_{3}}{p_{2}}=\frac{p_{4}}{p_{3}}=\cdots\right)$
3. The work input for each stage is same.

### 4.18 Rotary compressors:

Rotary compressors have a rotor to develop pressure. They are classified as
(1) Positive displacement compressors and (2) Non positive displacement (Dynamic) compressors

In positive displacement compressors, the air is trapped in between two sets of engaging surfaces. The pressure rise is obtained by the back flow of air (as in the case of Roots blower) or both by squeezing action and back flow of air (as in the case of vane blower). Example: (1) Roots blower, (2) Vane blower, (3) Screw compressor.

In dynamic compressors, there is a continuous steady flow of air. The air is not positively contained within certain boundaries. Energy is transferred from the rotor of the compressor to the air. The pressure rise is primarily due to dynamic effects.
Example: (1) Centrifugal compressor, (2) Axial flow compressor.

### 4.18.1 Roots blower:

The Roots blower is a development of the gear pump.


Fig:4.11 Roots blower
in separate parallel axis of a casing as shown in fig:4.11. The two rotors are driven by a pair of gears (which are driven by the prime mover) and they revolve in opposite directions. The lobes of the rotor are of cycloid shape to ensure correct mating. A small clearance of 0.1 mm to 0.2 mm is provided between the lobe and casing. This reduces the wear of moving parts.

Working: When the rotor is driven by the gear, air is trapped between the lobes and the casing. the trapped air moves along the casing and discharged into the receiver. There is no increase in pressure since the flow area from entry to exit remains constant. But, when the outlet is opened, there is a

Construction: It consists of two lobed rotors placed
back flow of high pressure air in the receiver. This creates the rise in pressure of the air delivered. These types of blowers are used in automobiles for supercharging.

### 4.18.2 Vane blower:

Construction: A vane blower consists of (1) a rotor, (2) vanes mounted on the rotor, (3) inlet and outlet ports and (4) casing. The rotor is placed eccentrically in the outer casing. Concentric vanes (usually 6 to 8 nos.) are mounted on the rotor. The vanes are made of fiber or carbon. Inlet suction area is greater than outlet delivery area.


Fig: 4.12 Vane blower

Working: When the rotor is rotated by the prime mover, air is entrapped between two consecutive vanes. This air is gradually compressed due to decreasing volume between the rotor and the outer casing. This air is delivered to the receiver. This partly compressed air is further increased in pressure due to the back flow of high pressure air from the receiver.
Advantages: 1. Very simple and compact, 2. High efficiency 3. Higher speeds are possible

### 4.18.3 Centrifugal compressor

Construction: It consists of an impeller, a casing and a diffuser. The impeller consists of a number of blades or vanes, is mounted on the compressor shaft inside the casing. The impeller is surrounded by the casing.


Fig: 4.13 Centrifugal compressor


Fig: 4.14 Pressure - velocity Plot

Working: In this compressor air enters axially and leaves radially. When the impeller rotates, air enters axially through the eye of the impeller with a low velocity. This air moves over the impeller vanes. Then, it flows radially outwards from the impeller. The velocity and pressure increases in the impeller. The air then enters the diverging passage known as diffuser. In the diffuser, kinetic energy is converted into pressure energy and the pressure of the air further increases. It is shown in fig:4.14. Finally, high pressure air is delivered to the receiver. Generally half of the total pressure rise takes place in the impeller and the other half in the diffuser.
Applications: Centrifugal compressors are used for low pressure units such as for refrigeration, supercharging of internal combustion engines, etc.

### 4.18.4 Axial flow compressor

In this air compressor, air enters and leaves axially.
Construction: It consists of two sets of blades: Rotor blades and stator blades. The blades are so arranged that the unit consists of adjacent rows of rotor blades and stator blades as shown in fig:4.15. The stator blades are fixed to the casing. The rotor blades are fixed on the rotating drum. The drum is rotated by a prime mover through a driving shaft. Single stage compressor consists of a row of rotor blades followed by a row of stator blades. Compression of air takes place in each pair of blades (one rotor blade and one stator blade). Hence there are many stages of compression in this type of compressor.

Working: When the switch is switched on, the prime mover rotates the drum. Air enters through the compressor inlet and passes through the rotor and stator blades. While passing through the blades, the air is compressed between the blades. The air is also compressed between the casing and the blades. The air flow passage area is gradually reduced from the inlet to the outlet of the compressor. This increases the pressure of the air considerably. Finally, high pressure air is delivered to the receiver.


Fig:4.15 Axial flow compressor

## Applications:

1. They are widely used in high pressure units such as industrial and marine gas turbine plants,
2. They are most suitable for aircraft work (Jet propulsion) since they require less frontal area.

### 4.19 Comparison of Reciprocating and Rotary compressors

| Reciprocating compressors | Rotary compressors |
| :--- | :--- |
| 1. It is suitable for low rates of flow. Flow rate <br> is limited to $\mathrm{m}^{3} / \mathrm{s}$ | It is suitable for large rates of flow. Flow <br> rate can be as large as $50 \mathrm{~m}^{3} / \mathrm{s}$. |
| 2. It is used for high pressure rise. It can <br> compress fluids up to 1000 bar. | It is used for medium pressure rise. The <br> pressure rise is limited to 10 bar. |
| 3. It cannot be coupled to turbines or I.C. <br> engines. | It can be directly coupled to turbines or high <br> speed internal combustion engines due to their <br> higher speeds. |
| 4. The flow of air is intermittent. | It gives uniform delivery of air. |
| 5. The criterion of thermodynamic efficiency is <br> isentropic. | The criterion of thermodynamic efficiency is <br> isothermal. |
| 6. Due to sliding parts it requires more <br> lubrication. | No sliding parts. Hence needs lesser lubrication. <br> It gives clean supply of air. |
| 7. Maintenance cost is high because of large <br> number of reciprocating parts. | Maintenance cost is less. |
| 8. Complicated construction. It has more <br> number of parts. | Simple in construction. It has less number of <br> parts. |
| 9. Torque is not uniform. | Uniform torque. |

4.20 Free Air Delivery(FAD): It is the volume of air drawn into a compressor from the atmosphere. After compression and cooling the air is returned to the original temperature but it is at a higher pressure. Suppose atmospheric conditions are $p_{a}, \mathrm{~T}_{\mathrm{a}}$ and $\mathrm{V}_{\mathrm{a}}$ (the FAD ) and the compressed conditions are $\mathrm{p}, \mathrm{V}$ and T .

Applying the gas law we have

$$
\begin{aligned}
& \frac{p V}{T}=\frac{p_{a} V_{a}}{T_{a}} \\
& V_{a}=\frac{p V T_{a}}{T p_{a}}=F . A . D .
\end{aligned}
$$

1. A Single cylinder, single acting air compressor has cylinder diameter 160 mm and stroke length 300 mm . It draws air into its cylinder at pressure of 100 kpa at $27^{\circ} \mathrm{C}$. The air is then compressed to a pressure of 650 kpa . If the compressor runs at a speed of $2 \mathrm{rev} / \mathrm{sec}$, Determine.
i) Mass of air compressed per cycle
ii) Work required per cycle
iii) Power required to derive the compressor in kW

Assume the compression process follows $\mathrm{PV}=$ constant.
Given data:

$$
\begin{aligned}
& \mathrm{D}=160 \mathrm{~mm}=0.16 \mathrm{~m} \\
& \mathrm{~L}=300 \mathrm{~mm}=0.3 \mathrm{~m} \\
& \mathrm{P}_{1}=100 \mathrm{kpa} \\
& \mathrm{~T}_{1}=27^{\circ} \mathrm{C}=27+273=300 \mathrm{~K} \\
& \mathrm{P}_{2}=650 \mathrm{kpa} \\
& \mathrm{~N}=2 \mathrm{rev} / \mathrm{sec}=120 \mathrm{rpm} \\
& \mathrm{PV}^{\mathrm{y}}=\mathrm{C} \quad \mathrm{Y}=1.4
\end{aligned}
$$

## Solution:

Work done during Isothermal Compression ( $\mathrm{PV}=\mathrm{C}$ )

$$
\begin{array}{ll}
\mathrm{W}=\mathrm{mRT}_{1} \ln \left[\mathrm{P}_{2} / \mathrm{P}_{1}\right] \\
\mathrm{W}=\mathrm{P}_{1} \mathrm{~V}_{1} \ln \left[\mathrm{P}_{2} / \mathrm{P}_{1}\right] & {[\mathrm{PV}=\mathrm{mRT}]}
\end{array}
$$

We know that,

$$
\begin{aligned}
& \mathrm{Vs}=(\pi / 4) \mathrm{D}^{2} \mathrm{~L}=(\pi / 4) *(0.16)^{2} * 0.3 \\
& \text { Vs }=6.03 \times 10^{-3} \mathrm{~m}^{3}=\mathrm{V}_{1} \quad \text { [clearance volume is neglected] } \\
& \text { Vs }=6.03 \times 10^{-3} \mathrm{~m}^{3}
\end{aligned}
$$

Substituting $\mathrm{V}_{1}$ in work done equation
$\mathrm{W}=100 \mathrm{X} 6.03 \mathrm{X}^{-3} 0^{-3} \mathrm{X} \ln$ [650/100]
$\mathrm{W}=1.13 \mathrm{~kJ}$
Power $=[\mathrm{W} * \mathrm{~N} / 60]=1.13 * 120 / 60$
$\mathrm{P}=2026 \mathrm{~kW}$
We know that,

$$
\begin{aligned}
& \mathrm{P}_{1} \mathrm{~V}_{1}=\mathrm{mRT}_{1} \\
& \mathrm{~m}=\mathrm{P}_{1} \mathrm{~V}_{1} / \mathrm{R}_{1}=\left[\left(100 * 6.03 \times 10^{-3}\right) /(0.287 * 300)\right] \\
& \mathrm{m}=0.007 \mathrm{~kg}
\end{aligned}
$$

Result:
i. $\quad \mathrm{m}=0.007 \mathrm{~kg}$
ii. $\quad W=1.13 \mathrm{~kJ}$
iii. $\quad \mathrm{P}=2.26 \mathrm{~kW}$
2. A Single cylinder, single acting reciprocating air compressor with a bore of 12 cm and stroke of 16 cm runs at 410 rpm . At the beginning of compression, the pressure and temperature in the cylinder are 0.98 bar and $40^{\circ} \mathrm{C}$. the delivery pressure is $\mathbf{6 b a r}$. The index of compression is 1.32 . the clearance is $6 \%$ of stroke volume. Determine the volume of air delivered referred to 1 bar and $20^{\circ} \mathrm{C}$. what is the power required?

## Given data:

$\mathrm{D}=12 \mathrm{~cm}=0.12 \mathrm{~m}$
$\mathrm{L}=16 \mathrm{~cm}=0.16 \mathrm{~m}$
$\mathrm{N}=410 \mathrm{rpm}$
$\mathrm{P}_{1}=0.98$ bar $=98 \mathrm{kpa}$
$\mathrm{T}_{1}=40^{\circ} \mathrm{C}=313 \mathrm{~K}$
$\mathrm{P}_{2}=6 \mathrm{bar}=600 \mathrm{kpa}$
$\mathrm{N}=1.32$
$\mathrm{Vc}=6 \%=0.06 \mathrm{Vs}$
$\mathrm{Po}=1 \mathrm{bar}=100 \mathrm{kpa}$
$\mathrm{To}=20^{\circ} \mathrm{C}=293 \mathrm{~K}$

## Solution:

We know that,

$$
\begin{aligned}
& \mathrm{Vs}=(\pi / 4) \mathrm{D}^{2} \mathrm{~L}=(\pi / 4) *(12)^{2} * 16 \\
& \mathrm{Vs}=0.0018 \mathrm{~m}^{3}
\end{aligned}
$$

We know that,
$\mathrm{V}_{1}=\mathrm{Vc}+\mathrm{Vs}$
$\mathrm{V}_{1}=0.06 \mathrm{~V} s+\mathrm{Vs}$
$\mathrm{V}_{1}=1.06 \mathrm{x} 0.0018$
$\mathrm{V}_{1}=1.908 \times 10^{-3} \mathrm{~m}^{3}$

## Work done on the single stage compressor with clearance volume

$$
\mathrm{W}=[\mathrm{n} / \mathrm{n}-1] \mathrm{P}_{1} \mathrm{~V}_{1}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{(\mathrm{n}-1 / \mathrm{n})}-1\right]
$$

We know that,

$$
\begin{aligned}
& \mathrm{P}_{3} \mathrm{~V}_{3}{ }^{\mathrm{n}}=\mathrm{P}_{4} \mathrm{~V}_{4}{ }^{\mathrm{n}} \\
& {\left[\mathrm{~V}_{4} / \mathrm{V}_{3}\right]^{n}=\left[\mathrm{P}_{3} / \mathrm{P}_{4}\right]} \\
& {\left[\mathrm{V}_{4} / \mathrm{V}_{3}\right]^{\mathrm{n}}=\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]} \\
& {\left[\mathrm{V}_{4} / \mathrm{V}_{\mathrm{c}}\right]^{\mathrm{n}}=\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]} \\
& {\left[\mathrm{V}_{4} / \mathrm{V}_{\mathrm{c}}\right]=\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]^{1 / \mathrm{n}}} \\
& \mathrm{~V}_{4}=\mathrm{V}_{\mathrm{c}} \times\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]^{1 / \mathrm{n}} \\
& \quad=0.06 \times \mathrm{Vs}[600 / 98]^{1 / 1.32} \\
& \quad=0.06 \times 0.0018 \times[600 / 98]^{1 / 1.32} \\
& \\
& \mathrm{~V}_{4}=4.26 \times 10^{-4} \mathrm{~m}^{3}
\end{aligned}
$$

We know that,

$$
\begin{aligned}
& \mathrm{Va}=\mathrm{V}_{1}-\mathrm{V}_{4}=1.908 \times 10^{-3}-4.26 \times 10^{-4} \\
& \mathrm{Va}=0.00148 \mathrm{~m}^{3}
\end{aligned}
$$

Substituting Va value in work done equation
$\mathrm{W}=[1.32 / 1.32-1] \times 98 \times 0.00148\left[(600 / 98)^{1.32-1 / 1.32}-1\right]$
$\mathrm{W}=0.329 \mathrm{~kJ}$
Power $=\mathrm{WxN} / 60=(0.329 \times 410) / 60$
$\mathrm{P}=2.25 \mathrm{~kW}$
We know that,

$$
\mathrm{PoVo} / \mathrm{To}=\mathrm{P}_{2} \mathrm{~V}_{\mathrm{d}} / \mathrm{T}_{2}
$$

$$
\mathrm{Vo}=\mathrm{To} / \mathrm{Po} \times \mathrm{P}_{2} \mathrm{~V}_{\mathrm{d}} / \mathrm{T}_{2}
$$

We know that,
$\mathrm{T}_{2} / \mathrm{T}_{1}=\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]^{\mathrm{n}-1 / \mathrm{n}}$
$\mathrm{T}_{2}=\mathrm{T}_{1} \times\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]^{\mathrm{n}-1 / \mathrm{n}}$
$\mathrm{T}_{2}=313 \times[600 / 98]^{1.32-1 / 1.32}$
$\mathrm{T}_{2}=485.6 \mathrm{~K}$
$\left[\mathrm{V}_{2} / \mathrm{V}_{1}\right]^{\mathrm{n}}=\mathrm{P}_{1} / \mathrm{P}_{2}$
$\mathrm{V}_{2} / \mathrm{V}_{1}=\left[\mathrm{P}_{1} / \mathrm{P}_{2}\right]^{1 / \mathrm{n}}$
$\mathrm{V}_{2}=\mathrm{V}_{1}\left[\mathrm{P}_{1} / \mathrm{P}_{2}\right]^{1 / \mathrm{n}}$
$\mathrm{V}_{2}=1.908 \times 10^{-3}[98 / 600]^{1 / 1.32}$
$\mathrm{V}_{2}=0.00048 \mathrm{~m}^{3}$
We know that,

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{d}}=\mathrm{V}_{2}-\mathrm{V}_{3}=\mathrm{V}_{2}-\mathrm{V}_{\mathrm{c}}=0.00048-(0.06 \mathrm{x} 0.0018) \\
& \mathrm{V}_{\mathrm{d}}=0.000372 \mathrm{~m}^{3}
\end{aligned}
$$

Sub, To, Po, $\mathrm{P}_{2}, \mathrm{~T}_{2}, \mathrm{~V}_{\mathrm{d}}$ values in ..(1)

$$
\begin{aligned}
& \text { Vo }=(293 / 100) \times(600 / 485.6) \times 0.000372 \\
& \text { Vo }=0.0013 \mathrm{~m}^{3}
\end{aligned}
$$

Result:

$$
\begin{aligned}
& \mathrm{P}=2.25 \mathrm{~kW} \\
& \mathrm{Vo}=0.0013 \mathrm{~m}^{3}
\end{aligned}
$$

3. A single stage reciprocating compressor receives air at $25 \mathrm{~m}^{3} / \mathrm{min}$ at $1 \mathrm{bar}, 15^{\circ} \mathrm{C}$ and discharges it at 15 bar. Assume the value of $\mathbf{n}$ for compression as $\mathbf{1 . 3 5}$ and volumetric efficiency as $\mathbf{0 . 7 5}$. determine $\mathbf{i}$ ) theoretical power required ii) piston displacement per min ii) maximum air temperature. [Dec 2003]

Given data:
$\mathrm{V}_{\mathrm{a}}=24 \mathrm{~m} 3 / \mathrm{min}$
$\mathrm{P}_{2}=15 \mathrm{bar}=1500 \mathrm{kpa}$
$\mathrm{P}_{1}=1 \mathrm{bar}=100 \mathrm{kpa}$
$\mathrm{N}=1.35$
$\mathrm{T}_{1}=15^{\circ} \mathrm{C}$
$\eta_{\text {vol }}=0.75$

## Solution :

work done on the single stage compressor with clearance volume,
$\mathrm{W}=\mathrm{n} / \mathrm{n}-1 \mathrm{P}_{1} \mathrm{~V}_{\mathrm{a}}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{\mathrm{n}-1 / \mathrm{n}}-1\right]$
$\mathrm{W}=9816.04 \mathrm{KJ} / \mathrm{min}=163.6 \mathrm{Kj} / \mathrm{s}$
$\mathrm{P}=163.6 \mathrm{KW}$
We know that ,

$$
\begin{aligned}
& \eta_{\mathrm{vol}}=\mathrm{V}_{\mathrm{a}} / \mathrm{V}_{\mathrm{s}} \\
& 0.75=25 / \mathrm{V}_{\mathrm{s}} \\
& \mathrm{~V}_{\mathrm{s}}=33.33 \mathrm{~m} 3 / \mathrm{min}
\end{aligned}
$$

We know that,

$$
\begin{aligned}
& \mathrm{T}_{2} / \mathrm{T}_{1}=\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]^{\mathrm{n}-1 / \mathrm{n}} \\
& \mathrm{~T}_{2}=\mathrm{T}_{1} \times\left[\mathrm{P}_{2} / \mathrm{P}_{1}\right]^{\mathrm{n}-1 / \mathrm{n}} \\
& \mathrm{~T}_{2}=288 \times[1500 / 100]^{1.35-1 / 1.35} \\
& \mathrm{~T}_{2}=581.17 \mathrm{~K}
\end{aligned}
$$

Result:
$\mathrm{P}=163.6 \mathrm{KW}$
$\mathrm{V}_{\mathrm{s}}=33.33 \mathrm{~m} 3 / \mathrm{min}$
$\mathrm{T}_{2}=581.17 \mathrm{~K}$
4. A single stage reciprocating air compressor takes $1 \mathrm{~m}^{3}$ of air per minute at 1 bar and $15^{\circ} \mathrm{C}$ and delivers it at 7bar. The law of compression is $\mathrm{PV}^{1.3}==$ constant. Calculate the indicated power neglect clearance. If the speed of compressor is 300 rpm and stroke to bore ratio is 1.5 , calculate the cylinder dimensions. Find the power required if the mechanical efficiency of compressor is $\mathbf{8 5 \%}$ and motor transmission efficiency is $\mathbf{9 0 \%}$

Given data:
$\mathrm{V}_{1}=1 \mathrm{~m}^{3} / \mathrm{min}$
$\mathrm{P}_{1}=1 \mathrm{bar}=100 \mathrm{kpa}$
$\mathrm{T}_{1}=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
$\mathrm{P}_{2}=7 \mathrm{bar}=700 \mathrm{kpa}$
$\mathrm{N}=300 \mathrm{rpm}$
L/D=1.5
$\eta_{\text {mech }}=85 \%$
motor efficiency $=90 \%$
$\mathrm{PV}^{1.3}=\mathrm{C}$

## Solution:

We know that, Work done during polytropic compression
$\mathrm{W}=(\mathrm{n} / \mathrm{n}-1) \mathrm{P}_{1} \mathrm{~V}_{\mathrm{a}}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{\mathrm{n}-1 / \mathrm{n}}-1\right]$
$\mathrm{W}=(1.3 / 1.3-1) \times 100 \times 1 \times\left[(700 / 100)^{1.3-1 / 1.3}-1\right]$
$=244.6 \mathrm{~kJ} / \mathrm{min}$
Indicated Power $=4.07 \mathrm{~kW}$
We know that,
Stroke volume, $\mathrm{Vs}=\mathrm{V}_{1}=(\pi / 4) \mathrm{D}^{2} \mathrm{~L}$
$1 / 300=(\pi / 4) \times D^{2} \times 1.5 \mathrm{D}$
$1 / 300=(\pi / 4) \times 1.5 D^{3}$
$\mathrm{D}=0.141 \mathrm{~m}$
$\mathrm{L}=1.5 \times 0.141$
$\mathrm{L}=0.212 \mathrm{~m}$
We know that,
$\eta_{\text {mech }}=$ (Indicated power/ Power input)
Power input $=4.07 / 0.85$
Power input $=4.79 \mathrm{~kW}$
Motor efficiency $=$ power input/ motor power
Motor power $=4.79 / 0.90$
Motor power $=5.32 \mathrm{~kW}$

## Result:

Indicated power $=4.07 \mathrm{~kW}$
Power input $=4.79 \mathrm{~kW}$
Motor power $=5.32 \mathrm{~kW}$
5. The free air delivered of a single cylinder single stage reciprocating air compressor $2.5 \mathrm{~m}^{3} / \mathrm{min}$. The ambient air is at STP conditions and delivery pressure is 7bar. The clearance volume is $5 \%$ of the stroke volume and law of compression and expansion is $\mathrm{PV}^{1.25}=\mathrm{C}$. if $\mathrm{L}=1.2 \mathrm{D}$ and the compressor runs at 150 rpm , determine the size of the cylinders.

## Given data:

$\mathrm{Va}=2.5 \mathrm{~m}^{3} / \mathrm{min}=0.04166 \mathrm{~m}^{3} / \mathrm{sec}$
For STP condition, the pressure and temperature are
$\mathrm{V}_{1}=1 \mathrm{~m}^{3} / \mathrm{min}$
$\mathrm{P}_{1}=1.013 \mathrm{bar}=101.3 \mathrm{kpa}$
$\mathrm{T}_{1}=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
$\mathrm{P}_{2}=7 \mathrm{bar}=700 \mathrm{kpa}$
$\mathrm{N}=150 \mathrm{rpm}$
$\mathrm{L}=1.2 \mathrm{D}$
$\mathrm{Vc}=5 \% \mathrm{Vs}=0.05 \mathrm{Vs}$
$\mathrm{PV}^{1.25}=\mathrm{C}$
$\mathrm{n}=1.25$

## Solution:

The mass of free air delivered per second is given by $\mathrm{m}_{\mathrm{a}}=\mathrm{PV} / \mathrm{RT}=\left(1.013 \times 10^{5} \times 0.04166\right) /(287 \times 288)=0.051 \mathrm{~kg} / \mathrm{sec}$
We know that,
Work done, $\mathrm{W}=(\mathrm{n} / \mathrm{n}-1) \mathrm{P} \mathrm{V}_{\mathrm{a}}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{\mathrm{n}-1 / \mathrm{n}}-1\right]$

$$
\begin{aligned}
& \mathrm{W}=\mathrm{m}_{\mathrm{a}} \mathrm{RT}(\mathrm{n} / \mathrm{n}-1)\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{\mathrm{n}-1 / \mathrm{n}}-1\right] \\
& \mathrm{W}=0.054 \mathrm{x} 0.287 \times 288 \mathrm{x}(1.25 / 1.25-1)\left[(700 / 101.3)^{1.25-1 / 1.25}-1\right] \\
& \mathrm{W}=9.95 \mathrm{~kW}
\end{aligned}
$$

We know that,
Indicated power, IP $=$ PmLAN/1000
$\mathrm{Pm}=(\mathrm{n} / \mathrm{n}-1) \mathrm{P}_{1 \times \eta} \mathrm{vol}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{\mathrm{n}-1 / \mathrm{n}}-1\right]$ But, $\eta_{\text {vol }}=1+\mathrm{C}-\mathrm{C}\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{1 / n}$
Where $\quad \mathrm{C}=\mathrm{Vc} / \mathrm{Vs}$
$\eta_{\text {vol }}=1+(\mathrm{Vc} / \mathrm{Vs})-(\mathrm{Vc} / \mathrm{Vs})\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{1 / n}$
$\eta_{\text {vol }}=1+(0.05)-(0.05)(700 / 101.3)^{1 / 1.25}$
$\eta_{\text {vol }}=0.815$
Substituting Pm value in eqn (2)
$\mathrm{Pm}=(1.25 / 1.25-1) \times 1 \times 0.815 \times\left[(700 / 101.3)^{1.25-1 / 1.25}-1\right]$
$\mathrm{Pm}=1.923 \mathrm{bar}$
Substituting Pm value in eqn (1)
Indicated Power IP (or) work out put
$1.95=\left[1.923 \times 10^{5} \times 1.2 \mathrm{D} \times(\pi / 4) \mathrm{D}^{2} \times 150 / 60\right] / 1000$

$$
\mathrm{D}=0.28 \mathrm{~m}
$$

Result:

$$
\begin{aligned}
& \mathrm{L}=1.2 \mathrm{D}=1.2 \times 0.28=0.336 \mathrm{~m} \\
& \mathrm{D}=0.28 \mathrm{~m} \\
& \mathrm{~L}=0.336 \mathrm{~m}
\end{aligned}
$$

6. A single stage double acting compressor has a free air delivery (FAD) of $14 \mathrm{~m}^{3} / \mathrm{min}$ measured at 1.013 bar and $15^{\circ} \mathrm{C}$. the pressure and temperature in the cylinder during induction are 0.95 bar and $32^{\circ} \mathrm{C}$ respectively. The delivery pressure is 7 b ar and index of compression and expansion, $n=1.3$. the clearance volume is $5 \%$ of the swept volume. Calculate the indicated power required and the volumetric efficiency.

## Given data:

$\mathrm{V}_{0}=14 \mathrm{~m}^{3} / \mathrm{min}=0.233 \mathrm{~m}^{3} / \mathrm{sec}$
$\mathrm{P}_{1}=0.95 \mathrm{bar}=95 \mathrm{kpa}$
$\mathrm{P}_{2}=7 \mathrm{bar}=700 \mathrm{kpa}$
$\mathrm{T}_{1}=32^{\circ} \mathrm{C}=305 \mathrm{~K}$
$\mathrm{T}_{0}=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
$\mathrm{P}_{0}=1.013 \mathrm{bar}=101.3 \mathrm{kpa}$
$\mathrm{Vc}=5 \% \mathrm{Vs}=0.05 \mathrm{Vs} \mathrm{Vc} / \mathrm{Vs}=0.05$
$\mathrm{n}=1.3$

## Solution:

Volumetric efficiency, $\eta_{\text {vol }}=1+\mathrm{C}-\mathrm{C}\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{1 / n}$

$$
\begin{aligned}
& \eta_{\mathrm{vol}}=1+(\mathrm{Vc} / \mathrm{Vs})-(\mathrm{Vc} / \mathrm{Vs})\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{1 / \mathrm{n}} \\
& \eta_{\mathrm{vol}}=1+(0.05)-(0.05)(700 / 95)^{1 / 1.3} \\
& \eta_{\mathrm{vol}}=0.818=81.8 \%
\end{aligned}
$$

We know that,

$$
\mathrm{Po} \mathrm{Vo} / \mathrm{To}=\mathrm{P}_{1} \mathrm{~V}_{1} / \mathrm{T}_{1}
$$

$101.3 \times 0.233 / 288=95 \mathrm{x} \mathrm{V}_{\mathrm{a}} / 305$
$\mathrm{V}_{\mathrm{a}}=0.263 \mathrm{~m}^{3} / \mathrm{sec}$
Work done or power,

```
P = (n/n-1)P}\mp@subsup{\textrm{P}}{1}{}\mp@subsup{\textrm{V}}{\textrm{a}}{}[(\mp@subsup{\textrm{P}}{2}{}/\mp@subsup{\textrm{P}}{1}{}\mp@subsup{)}{}{n-1/n}-1
P = (1.3/1.3-1)x95x 0.263 [ ( 700/95) 1.3-1/1.3 -1]
P}=63.39\textrm{kW
```

Result:
$\eta_{\text {vol }}=81.8 \%$
Indicated power $\mathrm{P}=63.39 \mathrm{~kW}$
7. A single cylinder single acting reciprocating compressor takes in $6 \mathrm{~m}^{3} / \mathrm{min}$ of air at 1 bar and $15^{\circ} \mathrm{C}$ and compresses into 6 bar. Calculate the saving in the power required when the compression process in changed from adiabatic compression to isothermal compression

Given data:
$\mathrm{V}_{1}=6 \mathrm{~m}^{3} / \mathrm{min}$
$\mathrm{P}_{1}=1 \mathrm{bar}=100 \mathrm{kpa}$
$\mathrm{T}_{1}=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
$\mathrm{P}_{2}=6 \mathrm{bar}=600 \mathrm{kpa}$

## Solution:

Work done during isothermal compression ( $\mathrm{pv}=\mathrm{c}$ )
$\mathrm{W}=\mathrm{P}_{1} \mathrm{~V}_{1} \ln \left[\mathrm{P}_{2 /} \mathrm{P}_{1}\right]$
$=100 * 6 * \ln [600 / 100]$
$\mathrm{W}=1075.5 \mathrm{~kJ} / \mathrm{min}$
Power, $P=17.91 \mathrm{~kW}$
Work done during adiabatic process
$\mathrm{W}=[\gamma / \gamma-1] \mathrm{P}_{1} \mathrm{~V}_{1}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{(\gamma-1 / \gamma)}-1\right]$
$\mathrm{W}=[1.4 / 1.4-1] * 100 * 6 *\left[(600 / 100)^{(1.4-1 / 1.4)}-1\right]$
$\mathrm{W}=1403.87 \mathrm{~kJ} / \mathrm{min}$
$\mathrm{P}=23.39 \mathrm{~kW}$
Saving power $=23.39-17.91$
Saving power $=5.48 \mathrm{~kW}$
8. Air is to be compressed in a single stage reciprocating compressor from 1.013 bar and $15^{\circ} \mathrm{C}$ to 7 bar . Calculate the indicated power required for a free air delivery of $0.3 \mathrm{~m}^{3} / \mathrm{min}$, when the compression process is i) Isentropic ii) polytropic with ( $n=1.45$ )

## Given data:

$\mathrm{P}_{1}=1.013 \mathrm{bar}=101.3 \mathrm{kpa}$
$\mathrm{T}_{1}=15^{\circ} \mathrm{C}=288 \mathrm{~K}$
$\mathrm{P}_{2}=7 \mathrm{bar}=700 \mathrm{kpa}$
$\mathrm{Vo}=0.3 \mathrm{~m}^{3} / \mathrm{min}$
$\mathrm{n}=1.25$
solution:
we know that, $\mathrm{PoVo} / \mathrm{To}=\mathrm{P}_{1} \mathrm{~V}_{1} / \mathrm{T}_{1}$
$\mathrm{V}_{1}=[\mathrm{PoVo} / \mathrm{To}] \mathrm{X}\left[\mathrm{T}_{1} / \mathrm{P}_{1}\right]$
We know that, at atmospheric condition the pressure and temperature are

$$
\begin{align*}
\mathrm{Po} & =101.3 \mathrm{kpa}  \tag{1}\\
\mathrm{To} & =298 \mathrm{~K}
\end{align*}
$$

Substituting To,Po, Vo, $\mathrm{P}_{1}, \mathrm{~V}_{1}$ values in eqn (1)
$\mathrm{V}_{1}=[(101.3 \times 0.3) / 298] \times[288 / 101.3]$
$\mathrm{V}_{1}=\mathbf{0 . 2 8 9} \mathrm{m}^{3} / \mathrm{min}$
Work done duringisentropic Compression

$$
\begin{aligned}
& \mathrm{W}=[\gamma / \gamma-1] \mathrm{P}_{1} \mathrm{~V}_{1}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{(\gamma-1 / \gamma)-1]}\right. \\
& \mathrm{W}=[1.4 / 1.4-1] * 101.3 * 0.289 *\left[(700 / 101.3)^{(1.4-1 / 1.4)}-1\right] \\
& \mathrm{W}=75.53 \mathrm{~kJ} / \mathrm{min} \\
& \mathrm{~W}=1.25 \mathrm{~kJ} / \mathrm{s} \\
& \mathbf{P}_{\text {Iso }}=\mathbf{1 . 2 5 k W}
\end{aligned}
$$

Work done during polytropic compression
$\mathrm{W}=[\mathrm{n} / \mathrm{n}-1] \mathrm{P}_{1} \mathrm{~V}_{1}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{(\mathrm{n}-1 / \mathrm{n})}-1\right]$
$\mathrm{W}=[1.25 / 1.25-1] \times 101.3 \times 0.289 \times\left[(700 / 101.3)^{(1.25-1 / 1.25)}-1\right]$
$\mathrm{W}=69.08 \mathrm{~kJ} / \mathrm{min}$
$P_{\text {poly }}=1.15 \mathrm{~kW}$
Result:
$\mathrm{P}_{\mathrm{Iso}}=1.25 \mathrm{~kW} \quad \mathrm{P}_{\text {poly }}=1.15 \mathrm{~kW}$
9. Air enters a single stage double acting air compressor at 100 kpa and $29^{\circ} \mathrm{C}$. the compression ratio is 6:1. The speed of compression in 550 rpm . The volume rate measured at suction condition is $5 \mathbf{~ m 3} / \mathrm{min}$. find the motor power required if the mechanical efficiency is $\mathbf{9 0 \%}$. If the volumetric efficiency is $\mathbf{8 0 \%}$. Find swept volume of cylinder.

Given data:
$\mathrm{P}_{1}=100 \mathrm{kpa}$
$\mathrm{T}_{1}=29^{\circ} \mathrm{C}=302 \mathrm{~K}$
$\mathrm{N}=550 \mathrm{rpm}$
$\mathrm{V}_{1}=5 \mathrm{~m} 3 / \mathrm{min}$
Compression ratio $=6: 1$
$\mathrm{n}=1.3$
$\eta_{\text {vol }}=80 \%$
$\eta_{\max }=90 \%$

## Solution:

Compression ratio $=\left(\right.$ total cylinder volume) $/($ clearance volume $)=\mathrm{V}_{1} / \mathrm{V}_{\mathrm{c}}$
$\mathrm{V}_{1} / \mathrm{V}_{\mathrm{c}}=6$
$5 / \mathrm{V}_{\mathrm{c}}=6$
$\mathrm{V}_{\mathrm{c}}=0.833 \mathrm{~m} 3 / \mathrm{min}$
We know that,
$\mathrm{V}_{1}=\mathrm{V}_{\mathrm{c}}+\mathrm{V}_{\mathrm{s}}$
$5=0.833+V_{\mathrm{s}}$
$V_{s}=4.167 \mathrm{~m} 3 / \mathrm{kg}$
Work done on the single stage compressor with clearance volume
$\mathrm{W}=(\mathrm{n} / \mathrm{n}-1) \mathrm{P}_{1} \mathrm{~V}_{\mathrm{a}}\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{\mathrm{n}-1 / \mathrm{n}}-1\right]$
Volumetric efficiency, $\eta_{\text {vol }}=1+\mathrm{C}-\mathrm{C}\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{1 / n}$
$\mathrm{C}=\mathrm{V}_{\mathrm{c}} / \mathrm{V}_{\mathrm{s}}$
$\eta_{\mathrm{vol}}=1+\left(\mathrm{V}_{\mathrm{c}} \mathrm{V}_{\mathrm{s}}\right)-\left(\mathrm{V}_{\mathrm{c}} / \mathrm{V}_{\mathrm{s}}\right)\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{1 / \mathrm{n}}$
$\eta_{\mathrm{vol}}=1+(0.833 / 4.167)-(0.833 / 4.167)\left(\mathrm{P}_{2} / 100\right)^{1 / 1.3}$
$\mathbf{P}_{2}=247.03 \mathrm{kpa}$
We know that,
$\eta_{\text {vol }}=V_{\mathrm{a}} / \mathrm{V}_{\mathrm{s}}$
$0.8=V_{\mathrm{a}} / 4.167$
$\mathrm{V}_{\mathrm{a}}=3.33 \mathrm{~m}^{3} / \mathrm{min}$

Applying $\mathrm{V}_{\mathrm{a}}, \mathrm{P}_{2}$ values in eqn (1)
$\mathrm{W}=[1.3 / 1.3-1] \times 100 \times 3.3\left[(247.03 / 100)^{1.3-1 / 1.3}-1\right]$
$\mathrm{W}=334.87 \mathrm{~kJ} / \mathrm{min}$
$\mathrm{W}=\mathbf{5 . 5 8} \mathrm{kW}$
We know that,
Mech. efficiency =(power output of compressor)/(power supplied to compressor)
$0.9=(5.58) /$ (power supplied to compressor)
Power Supplied To Compressor $=6.2 \mathrm{~kW}$
Result:
$\mathrm{V}_{\mathrm{s}}=4.167 \mathrm{~m} 3 / \mathrm{kg}$
Power Supplied To Compressor $=6.2 \mathrm{~kW}$
10. A single stage single acting compressor delivers $15 \mathrm{~m}^{3}$ of free air per minute from 1bar to 8 bar. The speed of compressor is 300 rpm . Assuming that compression and expansion follow the law $\mathrm{PV}^{1.3}=$ constant and clearance is $\mathbf{1 / 1 6}$ th of swept volume, find the diameter and stroke of the compressor. Take $L / D=1.5$, the temperature and pressure of air at the suction are same as atmospheric air [Nov 2004]

## Given data:

$\mathrm{V}_{0}=15 \mathrm{~m}^{3} / \mathrm{min}$
$\mathrm{P}_{1}=1 \mathrm{bar}=100 \mathrm{kpa}$
$\mathrm{P}_{2}=8 \mathrm{bar}=800 \mathrm{kpa}$
$\mathrm{N}=300 \mathrm{rpm}$
$\mathrm{L}=1.5 \mathrm{D}$
$\mathrm{PV}^{1.3}=\mathrm{C}$
$\mathrm{n}=1.3$
$\mathrm{L} / \mathrm{D}=1.5$

## Solution:

We know that the volumetric efficiency
$\eta_{\text {vol }}=1-(\mathrm{Vc} / \mathrm{Vs})\left[\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{1 / \mathrm{n}}-1\right]$
$\eta_{\mathrm{vol}}=1-(1 / 16)\left[(8 / 1)^{1 / 1.3}-1\right]$
$\eta_{\text {vol }}=0.753=75.3 \%$
We know that, free air delivered
$\mathrm{Va}=\mathrm{Vs} \times \eta_{\text {vol }} \times 300$
$15=$ Vs $\times 0.753 \times 300$
Vs $=0.0664 \mathbf{m}^{3}$
Stroke volume $=0.0664 \mathrm{~m}^{3}$
We know that,
$\mathrm{Vs}=(\pi / 4) \mathrm{D}^{2} \mathrm{~L}=0.0664$
$(\pi / 4) \mathrm{D}^{2} \times 1.5 \mathrm{D}=0.0664$
D $=0.3834 \mathrm{~m}$
We know that,
$\mathrm{L} / \mathrm{D}=1.5$
$\mathrm{L}=1.5 \times 0.3834$
$\mathrm{L}=\mathbf{0 . 5 7 5 1} \mathrm{m}$

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## SCHOOL OF MECHANICAL ENGINEERING DEPARTMENT OF AUTOMOBILE ENGINEERING

## UNIT- 5 - REFRIGERATION CYCLES \& REFRIGERANTS

### 5.1 INTRODUCTION

For specific applications, efficiencies of both living and non-living beings depend to a great extent on the physical environment. The nature keeps conditions in the physical environment in the dynamic state ranging from one extreme to the other. Temperature, humidity, pressure and air motion are some of the important environment variables that at any location keep changing throughout the year. Adaptation to these many a times unpredictable variations are not possible and thus working efficiently is not feasible either for the living beings or the non-living ones. Thus for any specific purpose, control of the environment is essential. Refrigeration and airconditioning is the subject which deals with the techniques to control the environments of the living and non-living subjects and thus provide them comforts to enable them to perform better and have longer lives.

### 5.2 DEFINITIONS

## Refrigeration:

Refrigeration is defined as a method of reducing the temperature of a system below that of the surroundings and maintaining it at the lower temperature by continuously extracting the heat from it.


Fig 5.1
The principle of refrigeration is based on second law of thermodynamics. It states that heat does not flow from a low temperature body to a high temperature body without the help of an external work. In refrigeration process, the heat is continuously removed from a system at lower temperature and transfers it to the surroundings at a higher temperature. This operation according to second law of thermodynamics can only be performed by the aid of the external work. Therefore in a refrigeration system, power is to be supplied to remove heat continuously from the
refrigerator to keep it cool at a temperature less than the surroundings. The refrigeration cycle is based on reversible Carnot cycle.

## Refrigeration effect:

The rate at which the heat is absorbed in a cycle of from the interior space to be cooled is called refrigeration effect. It is defined as the quantity of heat removed to the time taken. It is also called as the capacity of a refrigerator.

## Ton of Refrigeration (or) Unit of Refrigeration (TR):

The standard unit of refrigeration is ton refrigeration or simply ton denoted by TR. It is equivalent to the rate of heat transfer needed to produce 1 ton ( 2000 lbs ) of ice at $32{ }^{\circ} \mathrm{F}$ from water at $32{ }^{\circ} \mathrm{F}$ in one day, i.e., 24 hours. The enthalpy of solidification of water from and at $32{ }^{\circ} \mathrm{F}$ in British thermal unit is $144 \mathrm{Btu} / \mathrm{lb}$. Thus

$$
\begin{aligned}
1 \mathrm{TR} & =\frac{2000 \mathrm{lb} \times 144 \mathrm{Btu} / \mathrm{lb}}{24 \mathrm{hr}} \\
& =12000 \mathrm{Btu} / \mathrm{hr}=200 \mathrm{Btu} / \mathrm{min}
\end{aligned}
$$

In general, 1 TR means 200 Btu of heat removal per minute. Thus if a refrigeration system is capable of cooling at the rate of $400 \mathrm{Btu} / \mathrm{min}$, it is a 2 ton machine. A machine of 20 ton rating is capable of cooling at a rate of $20 \times 200=4000 \mathrm{Btu} / \mathrm{min}$. This unit of refrigeration is currently in use in the USA, the UK and India. In many countries, the standard MKS unit of $\mathrm{kcal} / \mathrm{hr}$ is used. In the MKS it can be seen that

$$
\begin{aligned}
1 \mathrm{TR}=12000 \mathrm{Btu} / \mathrm{hr} & =\frac{12000}{3.968}=3024.2 \mathrm{kca} / \mathrm{hr} \\
& =50.4 \mathrm{kcal} / \mathrm{min} \approx 50 \mathrm{kcal} / \mathrm{min}
\end{aligned}
$$

If Btu ton unit is expressed into SI system, it is found to be $210 \mathrm{~kJ} / \mathbf{m i n}$ or 3.5 kW .

## Co-efficient of Performance (COP):

The Co-efficient of Performance is defined as the ratio of heat absorbed in a system to the work supplied.

The theoretical Coefficient of Performance (Carnot), (COP a standard measure of refrigeration efficiency of an ideal refrigeration system) depends on two key system temperatures: evaporator temperature $\mathrm{T}_{\mathrm{e}}$ and condenser temperature $\mathrm{T}_{\mathrm{c}}$

COP is given as: $\quad \mathrm{COP}_{\text {Carnot }}=\mathrm{T}_{\mathrm{e}} /\left(\mathrm{T}_{\mathrm{c}}-\mathrm{T}_{\mathrm{e}}\right)$

This expression also indicates that higher COPCarnot is achieved with higher evaporator temperatures and lower condenser temperatures. But COP is only a ratio of temperatures, and does not take into account the type of compressor. Hence the COP normally used in industry is calculated as follows:

$$
\mathrm{COP}=\frac{\text { Cooling effect }(\mathrm{kW})}{\text { Power input to compressor }(\mathrm{kW})}
$$

Where the cooling effect is the difference in enthalpy across the evaporator and expressed as kW .

## Ice making capacity:

It is the ability of the refrigeration system to make ice. In other words, it is the capacity of refrigeration system to remove heat from water to make ice.

## Relative COP:

It is the ratio of actual COP to the theoretical COP of a refrigerator. Actual COP is measured during a test and theoretical COP is obtained by applying the laws of thermodynamics.

### 5.3 REFRIGERATOR AND HEAT PUMP

The vapor compression refrigeration cycle is a common method for transferring heat from a low temperature to a high temperature.


Fig 5.2
The above figure shows the objectives of refrigerators and heat pumps. The purpose of a refrigerator is the removal of heat, called the cooling load, from a low temperature medium. The purpose of a heat pump is the transfer of heat to a high temperature medium, called the heating load. When we are interested in the heat energy removed from a low temperature space, the
device is called a refrigerator. When we are interested in the heat energy supplied to the high temperature space, the device is called a heat pump. In general, the term "heat pump" is used to describe the cycle as heat energy is removed from the low temperature space and rejected to the high temperature space.

The performance of refrigerators and heat pumps is expressed in terms of coefficient of performance (COP), defined as

$$
\begin{aligned}
C O P_{R} & =\frac{\text { Desired output }}{\text { Required input }}=\frac{\text { Cooling effect }}{\text { Work input }}=\frac{Q_{L}}{W_{\text {net }, \text { in }}} \\
C O P_{H P} & =\frac{\text { Desired output }}{\text { Required input }}=\frac{\text { Heating effect }}{\text { Work input }}=\frac{Q_{H}}{W_{\text {net, in }}}
\end{aligned}
$$

Both $\mathrm{COP}_{\mathrm{R}}$ and $\mathrm{COP}_{\mathrm{HP}}$ can be larger than 1. Under the same operating conditions, the COPs are related by

$$
C O P_{H P}=C O P_{R}+1
$$

### 5.4 TYPES OF REFRIGERATION

Refrigeration is classified as based on working substance used

- Air refrigeration system (Bell-Coleman cycle)
- Water refrigeration system
- Ice refrigeration system
- Refrigeration by special fluid ( low boiling point fluids - Refrigerants) ( Reversed Carnot cycle)
- Vapour compression refrigeration system (VCR)
- Vapour absorbtion refrigeration system (VAR)
- Vapour adsorbtion refrigeration system and etc.,


### 5.5 Simple Vapour Compression Refrigeration System (VCR)

It consists of the following essential parts:

## Compressor

The low pressure and temperature vapour refrigerant from evaporator is drawn into the compressor through the inlet or suction valve A, where it is compressed to a high pressure and temperature. This high pressure and temperature vapour refrigerant is discharged into the condenser through the delivery or discharge valve B .

## Condenser

The condenser or cooler consists of coils of pipe in which the high pressure and temperature vapour refrigerant is cooled and condensed. The refrigerant, while passing through the condenser, gives up its latent heat to the surrounding condensing medium which is normally air or water.


Fig 5.3

## Receiver

The condensed liquid refrigerant from the condenser is stored in a vessel known as receiver from where it is supplied to the evaporator through the expansion valve or refrigerant control valve.

## Expansion Valve

It is also called throttle valve or refrigerant control valve. The function of the expansion valve is to allow the liquid refrigerant under high pressure and temperature to pass at a controlled rate after reducing its pressure and temperature. Some of the liquid refrigerant evaporates as it passes through the expansion valve, but the greater portion is vaporized in the evaporator at the low pressure and temperature

## Evaporator

An evaporator consists of coils of pipe in which the liquid-vapour. Refrigerant at low pressure and temperature is evaporated and changed into vapour refrigerant at low pressure and temperature. In evaporating, the liquid vapour refrigerant absorbs its latent heat of vaporization from the medium (air, water or brine) which is to be cooled.

## The Simple Vapor Compression Refrigeration Cycle

The vapor compression refrigeration cycle has four components: evaporator, compressor, condenser, and expansion (or throttle) valve. The most widely used refrigeration cycle is the vapor-compression refrigeration cycle. In an ideal or simple vapor-compression refrigeration cycle, the refrigerant enters the compressor as a saturated vapor and is cooled to the saturated liquid state in the condenser. It is then throttled to the evaporator pressure and vaporizes as it absorbs heat from the refrigerated space.

The ideal vapor compression cycle consists of four processes.

Ideal Vapor-Compression Refrigeration Cycle
Process Description
1-2 Isentropic compression
2-3 Constant pressure heat rejection in the condenser
3-4 Throttling in an expansion valve
4-1 Constant pressure heat addition in the evaporator


Fig 5.4
The P-h diagram is another convenient diagram often used to illustrate the refrigeration cycle.


Fig 5.5
The ordinary household refrigerator is a good example of the application of this cycle.


Fig 5.6

Results of First and Second Law Analysis for Steady-Flow

| Component | Process | First Law Result |
| :--- | :---: | :---: |
| Compressor | $\mathrm{s}=$ Const. | $\dot{W}_{\text {in }}=\dot{m}\left(h_{2}-h_{1}\right)$ |
| Condenser | $\mathrm{P}=$ Const. | $\dot{Q}_{H}=\dot{m}\left(h_{2}-h_{3}\right)$ |
| Throttle Valve | $\Delta \mathrm{s}>0$ | $h_{4}=h_{3}$ |
|  | $\dot{W}_{\text {net }}=0$ |  |
|  | $\dot{Q}_{n e t}=0$ |  |
| Evaporator | $\mathrm{P}=$ Const. | $\dot{Q}_{L}=\dot{m}\left(h_{1}-h_{4}\right)$ |

$$
\begin{gathered}
C O P_{R}=\frac{\dot{Q}_{L}}{\dot{W}_{n e t, i n}}=\frac{h_{1}-h_{4}}{h_{2}-h_{1}} \\
C O P_{H P}=\frac{\dot{Q}_{H}}{\dot{W}_{n e t, i n}}=\frac{h_{2}-h_{3}}{h_{2}-h_{1}}
\end{gathered}
$$

Methods to enhance the COP of simple vapour compression refrigeration system

1. Cycle with dry saturated vapour after compression,
2. Cycle with wet vapour after compression,
3. Cycle with superheated vapour after compression,
4. Cycle with superheated vapour before compression, and
5. Cycle with undercooling or subcooling of refrigerant.

### 5.6 Theoretical Vapour Compression Cycle with Dry Saturated Vapour after Compression

A vapour compression cycle with dry saturated vapour after compression is shown in the following Figures (a) and (b) respectively. At point 1 , let $T_{1,}, p_{l}, S_{l}$ be the temperature, pressure and entropy of the vapour refrigerant respectively. The four processes of the cycle are as follows:


Fig 5.7

## Compression Process

The vapour refrigerant at low pressure $p 1$ and temperatureT1 is compressed isentropic ally to dry saturated vapour as shown by the vertical line 1-2 on the T-s diagram and by the curve 1-2 on ph diagram. The pressure and temperature rise from $p 1$ to $p 2$ and $T 1$ to $T 2$ respectively.

The work done during isentropic compression per kg of refrigerant is given by $w=h 2-h 1$
where $h 1=$ Enthalpy of vapour refrigerant at temperature $T 1$, i.e. at suction of the compressor, and
$h 2=$ Enthalpy of the vapour refrigerant at temperature $T 2$. i.e. at discharge of the compressor.

## Condensing Process

The high pressure and temperature vapour refrigerant from the compressor is passed through the condenser where it is completely condensed at constant pressure p2 and temperature T2 as shown by the horizontal line 2-3 on T-s and p-h diagrams. The vapour refrigerant is changed into liquid refrigerant. The refrigerant, while passing through the condenser, gives its latent heat to the surrounding condensing medium.

## Expansion Process

The liquid refrigerant at pressure $\mathrm{p} 3=\mathrm{p} 2$ and temperature $\mathrm{T} 3=\mathrm{T} 2$, is expanded by throttling process through the expansion valve to a low pressure $\mathrm{p} 4=\mathrm{p} 1$ and Temperature $\mathrm{T} 4=\mathrm{T} 1$ as shown by the curve 3-4 on T-s diagram and by the vertical line 3-4 on p-h diagram. Some of the liquid refrigerant evaporates as it passes through the expansion valve, but the greater portion is vaporized in the evaporator. We know that during the throttling process, no heat is absorbed or rejected by the liquid refrigerant.

## Vaporizing Process

The liquid-vapour mixture of the refrigerant at pressure $\mathrm{p} 4=\mathrm{p} 1$ and temperature $\mathrm{T} 4=\mathrm{T} 1$ is evaporated and changed into vapour refrigerant at constant pressure and temperature, as shown by the horizontal line $4-1$ on T-s and p-h diagrams. During evaporation, the liquid-vapour refrigerant absorbs its latent heat of vaporization from the medium (air, water or brine) which, is to be cooled, This heat which is absorbed by the refrigerant is called refrigerating effect and it is briefly written as $\mathrm{R}_{\mathrm{E}}$. The process of vaporization continues up to point 1 which is the starting point and thus the cycle is completed.

We know that the refrigerating effect or the heat absorbed or extracted by the liquid-vapour refrigerant during evaporation per kg of refrigerant is given by

$$
R_{E}=h_{1}-h_{4}=h_{1}-h f_{3}
$$

where $\mathrm{hf} 3=$ Sensible heat at temperature T3, i.e. enthalpy of liquid refrigerant leaving the condenser.

It may be noticed from the cycle that the liquid-vapour refrigerant has extracted heat during evaporation and the work will be done by the compressor for isentropic compression of the high pressure and temperature vapour refrigerant.

$$
\begin{aligned}
& \text { Coefficient of performance, C.O.P. }=(\text { Refrigerating effect }) /(\text { Work done }) \\
&=\frac{h_{1}-h_{4}}{h_{2}-h_{1}} \\
&=\frac{h_{1}-h_{f 3}}{h_{2}-h_{1}}
\end{aligned}
$$

5.7 Theoretical Vapour Compression Cycle with Wet Vapour after Compression


Fig 5.8
From the above figure, the end point of compression is lies in the region of wet (liquid and vapour). The enthalpy and entropy at this point is calculated by following formula

$$
\begin{aligned}
& h_{2}=h_{f 2}+x_{2} h_{f g 2} \\
& s_{2}=s_{f 2}+\frac{x_{2} h_{f g 2}}{T_{2}}
\end{aligned}
$$

5.8 Theoretical Vapour Compression Cycle with Superheated Vapour after Compression

(a) $T$-s diagram.

(b) p-h diagram.

Fig 5.9

From the above figure, the end point of compression is lies in the region of superheated vapour. The enthalpy and entropy at this point is calculated by following formula

$$
\begin{gathered}
h_{2}=h_{2}^{\prime}+c_{p} \times \text { Degree of superheat }=h_{2^{\prime}}+c_{p}\left(T_{2}-T_{2}{ }^{\prime}\right) \\
s_{2}=s_{2^{\prime}}+2.3 c_{p} \log \left(\frac{T_{2}}{T_{2^{\prime}}}\right)
\end{gathered}
$$

5.9 Theoretical Vapour Compression Cycle with Superheated Vapour before Compression


Fig 5.10

### 5.10 Theoretical Vapour Compression Cycle with Sub-Cooling or Under cooling of Refrigerant


(a) T-s diagram.

(b) $p-h$ diagram.

Fig 5.11

$$
h_{f 3}=h_{f 3^{\prime}}-c_{p} \times \text { Degree of undercooling }
$$

The process 3-3', cooling of the refrigerant temperature below its saturation temperature value is called sub cooling or under cooling at the end of condensation process.

### 5.11 REFRIGERANTS

The working agent in a refrigerating system that absorbs carries or releases heat from the place to be cooled or refrigerated can be termed as a refrigerant. This heat transfer generally takes place through a phase change of the refrigerant. A more complete definition of a refrigerant could be given as follows:

## "Refrigerant is the fluid used for heat transfer in a refrigerating system that absorbs heat during

 evaporation from the region of low temperature and pressure, and releases heat during condensation at a region of higher temperature and pressure."
## Primary and secondary refrigerants

Primary refrigerants are those which can be directly used for the purpose of refrigeration. If the refrigerant is allowed to flow freely into the space to be refrigerated and there is no danger of possible harm to human beings, then primary refrigerants are used. The refrigerants used in home refrigerators like Freon-12 are primary refrigerants.

On the other hand, there may be certain situations in which we cannot allow the refrigerant to come in direct contact with the items being refrigerated, and then the refrigerant used is termed as a secondary refrigerant. As for example, we cannot allow a toxic refrigerant to be used for air conditioning in residential buildings. There are some refrigerants which are highly inflammable and so their direct use is forbidden for safety reasons. Again, it may so happen that if direct refrigeration, such as in cooling a big cold storage, is allowed, then the amount of refrigerant required may be so large that its cost becomes prohibitively high. These are some typical situations for which we favor the use of secondary refrigerants. Water and brine solutions are common examples of secondary refrigerants.

### 5.12 Classification of refrigerants

Refrigerants can be broadly classified based on the following:

## Working Principle

Under this heading, we have the primary or common refrigerants and the secondary refrigerants.
The primary refrigerants are those that pass through the processes of compression, cooling or condensation, expansion and evaporation or warming up during cyclic processes. Ammonia,

R12, R22, carbon dioxide come under this class of refrigerants. On the other hand, the medium which does not go through the cyclic processes in a refrigeration system and is only used as a medium for heat transfer are referred to as secondary refrigerants. Water, brine solutions of sodium chloride and calcium chloride come under this category.

## Safety Considerations

Under this heading, we have the following three sub-divisions.
Safe refrigerants
These are the non-toxic, non-flammable refrigerants such as R11, R12, R13, R14, R21, R22, R113, R114, methyl chloride, carbon dioxide, water etc.

## Toxic and moderately flammable

Dichloroethylene methyl format, ethylchloride, sulphur dioxide, ammonia etc. come under this category.

## Highly flammable refrigerants

The refrigerants under this category are butane, isobutene, propane, ethane, methane, ethylene etc.

## Chemical Compositions

## Halocarbon compounds

These are obtained by replacing one or more hydrogen atoms in ethane or methane with alogens.

## Azeotropes

These are the mixtures of two or more refrigerants and behave as a compound.

## Oxygen and Nitrogen Compounds

Refrigerants having either oxygen or nitrogen molecules in their structure, such as ammonia, are grouped separately and have a separate nomenclature from the halogenated refrigerants.

## Cyclic organic Compounds

The compounds coming under this class are R316, R317 and R318.
Inorganic Compounds

These are further divided into two categories: Cryogenic and Non-cryogenic. Cryogenic fluids are those which are applied for achieving temperatures as low as $-160{ }^{\circ} \mathrm{C}$ to $-273{ }^{\circ} \mathrm{C}$. Above this temperature range, we can use a multi-stage refrigeration system to realise the desired temperature. But below $-160^{\circ} \mathrm{C}$, this is not possible since the COP of the cycle becomes very low. To attain temperatures below $-160^{\circ} \mathrm{C}$, we use refrigerants such as nitrogen, oxygen, helium, hydrogen etc. and for temperatures close to $-273^{\circ} \mathrm{C}$, magnetic cooling is employed. The inorganic compounds which are employed above the cryogenic temperature ranges come under the remaining sub-division of inorganic refrigerants.

## Unsaturated Compounds

Compounds such as ethylene, propylene etc., are grouped under this head and grouped under the 1000 series for convenience.

## Miscellaneous

This group contains those compounds which cannot be grouped under the other components. They are indicated by the 700 series with the last numbers being their molecular weight. Examples include air, carbon dioxide, sulphur dioxide etc. As we can see from the above subdivisions, they are not mutually exclusive. A compound may come under more than one subdivision. Hence, the importance of adopting the various naming conventions to designate the different refrigerants cannot be underestimated.

### 5.13 Designation of refrigerants

The American Society of Refrigerating Engineers (ASRE) has developed certain conventions for use in naming different types of refrigerants. These naming conventions differ according to the type of refrigerant. Each refrigerant type is denoted by a different series. Thus, we have separate series for halogenated refrigerants and other types. The naming conventions are simple and easy to follow. These conventions are now accepted worldwide and help to name the large variety of refrigerants available commercially nowadays.

## Halocarbon Compounds

These are represented by a three digit nomenclature. Here, the first digit represents the number of carbon atoms in the compound minus one, the second digit stands for the number of hydrogen atoms plus one while the third digit stands for the number of fluorine atoms. The remaining atoms are chlorine. As an example, let us consider the refrigerant having R22 as its three digit nomenclature.

According to the above mentioned convention,
No. of C atoms in R22: $\mathrm{C}-1=0=>\mathrm{C}=1$
No. of H atoms in R22: $\mathrm{H}+1=2 \Rightarrow \mathrm{H}=1$

$$
\text { No. of F atoms in R22: F = } 2
$$

Since there is only one carbon atom in the compound, this compound has originated from the methane series $(\mathrm{CH})$. From the calculation, we can see there is one hydrogen atom and two fluorine atoms. The remaining valence bond of carbon will be balanced by chlorine. Thus, the substance is


## Graphical Representation of Monochloro-Difluoro-Methane

Therefore, chemical formula of R 22 is $\mathrm{CHClF}_{2}$ and has the name Monochloro-difluoro-methane. Taking again the example of R134, we can calculate its chemical formula as above which gives us

No. of C atoms: $\mathrm{C}-1=1 \Rightarrow \mathrm{C}=2$
No. of H atoms: $\mathrm{H}+1=3 \Rightarrow \mathrm{H}=2$
No. of F atoms: $\mathrm{F}=4$
Therefore, no. of Cl atoms: $\mathrm{Cl}=0$


## Graphical Representation of Tetrafluoroethane

The compound is $\mathrm{C}_{2} \mathrm{H}_{2} \mathrm{~F}_{2}$ and its name is Tetrafluoroethane. The non-halogenated refrigerants follow a different naming convention which is dependent upon the series of the refrigerant.

### 5.14 DESIRABLE PROPERTIES OF REFRIGERANTS

The vast number of refrigerants available in the market today allows us to choose a refrigerant depending upon the operating conditions of the refrigeration system. As such, there is no refrigerant that can be advantageously used under all operating conditions and in all types of refrigeration systems. In spite of that, we can state certain desirable properties that a refrigerant should posses. These properties can be divided into favorable thermodynamic, chemical and physical properties:

### 5.14.1 Thermodynamic Properties

## Critical Temperature and Pressure

The critical temperature of the refrigerant should be as high as possible above the condensing temperature in order to have a greater heat transfer at a constant temperature. If this is not taken care of, then we will have excessive power consumption by the refrigeration system. The critical pressure should be moderate and positive. A very high pressure will make the system heavy and bulky whereas in case of very low pressures, there is a possibility of air leaking into the refrigerating system.

## Specific Heat

The specific heat of the liquid should be as small as possible. This ensures that the irreversibilities associated with throttling are small and there is greater subcooling of the liquid. On the other hand, the specific heat of vapor should be high to have less superheating of the vapor.

## Enthalpy of Vaporization

This should be as large as possible to minimize the area under superheat and the area reduction due to throttling. Also, the higher value of enthalpy of vaporization lowers the required flow rate per ton of refrigeration.

## Conductivity

The conductivity of the refrigerant should be as high as possible so that the size of the evaporator and condenser is manageable. From this viewpoint, ammonia has a better conductivity than that of R12 or R22 and is more suitable than the latter. But, ammonia is toxic and this does not allow its use in home refrigeration systems.

## Evaporator and Condenser Pressure

Both the evaporator and condenser pressures need to be above atmospheric pressure otherwise there is a possibility of air leaking into the system. Presence of air drastically reduces the capacity of the refrigeration system. Also, due to presence of moisture in air, acids or other corrosive compounds may form and this may affect the tubing of the refrigeration system.

## Compression Ratio

The compression ratio needs to be as small as possible otherwise the leakage of refrigerant occurs across the piston. Also, the volumetric efficiency is affected.

## Freezing Point

It should be as low as possible or else there will be a possibility of blockage of passages during flow of fluid through evaporator.

## Volume of Refrigerant Handled Per Ton of Refrigeration

This should be as small as possible in order to have a small size of the compressor. The type of compressor is decided by this value. For refrigerants like R12, R500, R22 etc., a reciprocating compressor is suitable. For others like R11 and water, a centrifugal compressor is required to handle the large volume.

## Coefficient of Performance

The Coefficient of performance or COP has a direct bearing on the running cost of the refrigeration system. Higher the magnitude of COP, lower will be the running cost. Since, the COP of any refrigeration system is limited by the Carnot COP, for large operating pressures a multi-stage refrigeration system should be employed. $\mathrm{CO}_{2}$ has a very low COP. Hence, it is not suitable for use as a refrigerant.

## Density

The density of the refrigerant should be as large as possible. In reciprocating compressors, the pressure rise is accomplished by squeezing the entrapped fluid inside the piston-cylinder assembly. Hence, density decides the size of the cylinder. Again in centrifugal compressors pressure rise is related to the density of the vapor. A high value of density results in high pressure rise.

## Compression Temperature

Whenever a refrigerant gets compressed, there is a rise in the temperature of the refrigerant resulting in the heating of the cylinder walls of the compressor. This necessitates external cooling of the cylinder walls to prevent volumetric and material losses. Refrigerants having lowest compression temperatures are thus better than others.

### 5.14.2 Chemical Properties

## Chemical Stability and Inertness

It should be chemically stable for the operating ranges of temperature. Also, it should not react with the materials of the refrigeration system or with which it comes into contact. Further, it should be chemically inert and must not undergo polymerization reactions at either the lower or higher ranges of temperatures.

## Action on Rubber or Plastics

Rubber and plastics are used extensively in the refrigeration system. These materials are mostly used in the seals and gaskets of the refrigeration system. They help to prevent the leakage of the refrigerant and ensure the smooth functioning of the compressor. The refrigerant should not react with them or else there might be leakage of refrigerant from the system or loss of functioning of the compressor.

## Flammability

The refrigerant should be inert and not catch fire when subjected to high temperatures. From this viewpoint $\mathrm{CO}_{2}$ is the most suitable as it is not only non-flammable, but also acts as a fireextinguisher. Ethane, butane, isobutene are highly undesirable as they catch fire quickly.

## Effect on Oil

The refrigerant should not react with the lubricating oil else, there is a possibility of loss of lubricating action due to either thickening or thinning of the oil. It should not be soluble in the oil else there will be reduction in the viscosity of the lubricating oil.

## Effect on Commodity

If the refrigerant is directly used for chilling, then it should not affect the commodity kept in the conditioned space. Also, in case where direct cooling is not employed, the refrigerant should still not affect the commodity if there is any leakage.

## Toxicity

The refrigerant used in air conditioning, food preservation etc. should not be toxic as they will come into contact with human beings.

### 5.14.3 Physical Properties

## Leakage and Detection

Since pressures higher than atmospheric are usually employed in refrigeration systems, there is a possibility of leakage of refrigerants after long period of operation. It is desirable to detect this leak early else the system would operate under reduced capacity or stop functioning altogether. Hence, it is desirable that the refrigerant has a pungent smell so that its leakage can be detected immediately.

## Miscibility with Oil

The refrigerant should not be miscible with the oil else the lubricating strength will be reduced.

## Viscosity

It should be as small as possible to ensure that the pressure drop in the system is as small as possible. A low viscosity refrigerant will require less energy for its circulation through the refrigeration system.

### 5.14.4 Safety Criteria

Under safety criteria, we consider the toxicity, flammability, action on perishable food and formation of explosive compound on exposure to air. An ideal refrigerant should be non-toxic, non-flammable, have no effect on food products and should not react with atmospheric air. No refrigerant satisfy these criteria fully. We can therefore, group refrigerants into different subgroups based on their flammability and toxicity levels.

### 5.14.5 Economic Criteria

Apart from the thermodynamic, chemical, physical and safety criteria, there is another criterion by which we judge an ideal refrigerant. The economic criterion takes into account the cost of the refrigerant, the availability and supply levels of the refrigerant, cost of storage and handling. We discuss each of these in detail below.

## Cost of Refrigerant

The cost of the refrigerant has a big impact on the overall cost of the refrigeration system. Hence, its cost should be as low as possible. From this viewpoint, ammonia and water are ideally suited, but their low thermodynamic and chemical properties restrict their use in all types of refrigeration systems. Particularly, for flooded type evaporator or condenser, the refrigerant amount required is high and their cost needs to be factored in while making the initial investments.

## Availability and Supply

The refrigerant should be easily available in the market and in abundant quantity. This ensures that the cost of the refrigerant is not prohibitive. An abundant and free supply of the refrigerant ensures that refrigeration systems will be designed specifically for use with them.

## Storage and Handling

The refrigerant should be such that it can be conveniently stored and handled during transportation and charging. It should be stored in as small a pressure vessel as possible. Also, if we have to handle a toxic or flammable refrigerant, then the cost involved will be higher compared to handling and storage cost of non-toxic and non-flammable refrigerant.

### 5.15 COMMON REFRIGERANTS

The refrigerants which are available commercially in the market are numerous. Some of them which are in common use are mentioned below:

## Air

Air (molecular weight 28.97, specific heats $\mathrm{C} p=1.04 \mathrm{~kJ} / \mathrm{kgK}$ and $\mathrm{Cv}=0.712 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$ ) is one of the earliest refrigerant to be used in the refrigeration systems. Its advantages are that it is available free of cost, is non-toxic and non-flammable and does not affect the commodity if pure. However, air suffers from a number of drawbacks. Air contains moisture and this reacts with the material of the evaporator and condenser severely affecting their working capacity. Further, there is a possibility that the passages may be blocked by the formation of ice from this moisture. The COP of air is of the order of 0.6 and thus, not suitable for use in refrigeration systems on a commercial scale. It is mainly used for air conditioning in aircrafts where efficiency of operation is of secondary importance.

## Ammonia

Ammonia (molecular weight 17) is one of the oldest refrigerants and it was commonly employed in places where toxicity effects were of secondary importance. Its advantages are its low cost, low specific volume, high COP (of the order of 4.0) and high refrigeration effect per unit mass of the refrigerant. Its primary drawback is its toxicity which prevents its use in air conditioning and food preservation systems. Ammonia has a boiling point of $-33^{\circ} \mathrm{C}$ at atmospheric pressure.

## Carbon Dioxide

Carbon dioxide (molecular weight 44) is a non-toxic and non-poisonous refrigerant. Also, it is not only non-flammable but and is an excellent extinguishing agent as well. Its other advantages are that it is chemically stable, immiscible with the lubricating oil and does not affect the metal used in the system. It has a low specific volume and this requires volume displacement per ton of refrigeration. However, its critical pressure is too high. Also, its critical temperature is only $31^{\circ} \mathrm{C}$ which makes it unsuitable for use in countries with a hot climate like India. It is an excellent refrigerant for low temperature refrigeration.

## Sulphur Dioxide

Sulphur dioxide (molecular weight 64) is a colourless, suffocating and irritating gas and is twice as heavy as air at atmospheric conditions. It was mostly used as a household refrigerant in the older days, but has since been discarded for better refrigerants. It suffers from a lot of disadvantages. Sulphur dioxide reacts with water forming sulphurous acid, which in presence of oxygen becomes sulphuric acid, a corrosive compound for metals. It is non-flammable but attacks foodstuff on coming in contact with it. It is also partially miscible with the lubricating oil.

## Hydrocarbons

This group consists of colourless fluids normally in gaseous state and made up of various combinations of carbon and hydrogen. Most of the refrigerants from this category are suitable for low temperature refrigeration. Isobutane falls in this category and has been suitable for domestic
refrigeration. They are non-poisonous, but are flammable and highly explosive when exposed to air. The molecular weight and boiling point of each gas varies according to the number of hydrogen and carbon atoms. The larger the number of hydrogen and carbon atoms, the heavier is the gas and higher is its boiling point.

## Halocarbon Refrigerants

The halocarbon refrigerants are formed by replacing one or more of hydrogen atoms of methane or ethane by one or more atoms of the three halogens: fluorine, chlorine or bromine. Some of the refrigerants coming under this category are mentioned below:

## Refrigerant R12

The refrigerant R 12 is the most widely used refrigerant in the domestic and large commercial establishments. Its chemical formula is $\mathrm{CCl}_{2} \mathrm{~F}_{2}$ and its boiling point is $-30^{\circ} \mathrm{C}$ at 1 bar. It is a nonflammable, non-explosive, non-irritating, non-toxic and odourless refrigerant. It remains chemically stable up to $550^{\circ} \mathrm{C}$. Also, it does not affect the material of the refrigeration system. It is available in abundance and is quite cheap. However, its use is being discontinued nowadays for its contribution to ozone depletion which will be discussed later.

## Refrigerant R13

Its chemical formula is $\mathrm{CClF}_{3}$. It is a non-flammable, non-toxic and stable refrigerant. It is very suitable for achieving low temperatures in a cascade refrigeration system. Its specific volume is high and therefore, it is suitable for centrifugal compressors. However, it also has a negative effect on ozone depletion.

## Refrigerant R22

Its chemical formula is $\mathrm{CHClF}_{2}$. It is also a non-toxic, non-flammable, non-corrosive and nonirritating refrigerant. It is the most common refrigerant for use in large refrigeration systems and is preferred to R 12 .

## Refrigerant R114

Its chemical formula is $\mathrm{C}_{2} \mathrm{Cl}_{2} \mathrm{~F}_{4}$. Its boiling point corresponding to 1 bar is about $3^{0} \mathrm{C}$. It has properties very similar to those of R12 with respect to water and oil combination. It is not suitable for low temperature refrigeration since it has negative evaporator pressure even at around $9^{\circ} \mathrm{C}$. It is non-toxic, non-explosive and non-corrosive even in the presence of water.

All refrigerants properties can be referred from standard refrigeration tables. Some refrigerant properties are listed below

### 5.16 VAPOUR ABSORPTION SYSTEM

## Introduction

In a vapour absorption system the refrigerant is absorbed on leaving the evaporator, the absorbing medium being a solid or liquid. In order that the sequence of events should be continuous it is necessary for the refrigerant to be separated from the absorbent and subsequently condensed before being returned to the evaporator. The separation is accomplished by the application of direct heat in a 'generator'. The solubility of the refrigerant and absorbent must be suitable and the plant which uses ammonia as the refrigerant and water as absorbent will be described.

## Simple Vapour Absorption System

Refer Fig. 5.12 for a simple absorption system. The solubility of ammonia in water at low temperatures and pressures is higher than it is at higher temperatures and pressures. The ammonia vapour leaving the evaporator at point 2 is readily absorbed in the low temperature hot solution in the absorber. This process is accompanied by the rejection of heat. The ammonia in water solution is pumped to the higher pressure and is heated in the generator. Due to reduced solubility of ammonia in water at the higher pressure and temperature, the vapour is removed from the solution. The vapour then passes to the condenser and the weakened ammonia in water solution is returned to the absorber.


Fig 5.12


Simple vapour absorption system- $T$-s diagram.
Fig 5.13
In this system the work done on compression is less than in vapour compression cycle (since pumping a liquid requires much less work than compressing a vapour between the same pressures) but a heat input to the generator is required. The heat may be supplied by any convenient form e.g. steam or gas heating.

## Practical Vapour Absorption System

Refer Fig. 5.14. Although a simple vapour absorption system can provide refrigeration yet its operating efficiency is low. The following accessories are fitted to make the system more practical and improve the performance and working of the plant.

1. Heat exchanger. 2. Analyser. 3. Rectifier.
2. Heat exchanger. A heat exchanger is located between the generator and the absorber.

The strong solution which is pumped from the absorber to the generator must be heated ; and the weak solution from the generator to the absorber must be cooled. This is accomplished by a heat exchanger and consequently cost of heating the generator and cost of cooling the absorber are reduced.
2. Analyser. An analyser consists of a series of trays mounted above the generator. Its main function is to remove partly some of the unwanted water particles associated with ammonia vapour going to condenser. If these water vapours are permitted to enter condenser they may enter the expansion valve and freeze ; as a result the pipe line may get choked.


Fig 5.14
3. Rectifier. A rectifier is a water-cooled heat exchanger which condenses water vapour and some ammonia and sends back to the generator. Thus final reduction or elimination of the percentage of water vapour takes place in a rectifier. The co-efficient of performance (C.O.P.) of this system is given by :

$$
\text { C.O.P. }=\frac{\text { Heat extracted from the evaporator }}{\text { Heat supplied in the generator }+ \text { Work done by the liquid pump }} .
$$

### 5.17 PSYCHROMETRY AND AIR - CONDITIONING

Psychrometric properties, Use of psychrometric chart, Psychrometric process - Sensible heat exchange process, Latent heat exchange process, Adiabatic mixing, Evaporative cooling, Property calculations of air-vapour mixtures.

Principles of air-conditioning, Types of air conditioning systems - summer, winter, year round air conditioners, Concept of RSHF, GSHF, ESHF, Simple problems.

### 5.17.1 CONCEPT OF PSYCHROMETRY AND PSYCHROMETRICS

Air comprises of fixed gases principally, nitrogen and oxygen with an admixture of water vapour in varying amounts. In atmospheric air water is always present and its relative weight averages less than $1 \%$ of the weight of atmospheric air in temperate climates and less than $3 \%$ by weight
under the most extreme natural climatic conditions, it is nevertheless one of most important factors in human comfort and has significant effects on many materials. Its effect on human activities is in fact altogether disproportionate to its relative weights. The art of measuring the moisture content of air is termed "psychrometry". The science which investigates the thermal properties of moist air, considers the measurement and control of the moisture content of air, and studies the effect of atmospheric moisture on material and human comfort may properly be termed "psychrometrics".

## DEFINITIONS

Some of the more important definitions are given below :

1. Dry air. The international joint committee on Psychrometric Data has adopted the following exact composition of air expressed in mole fractions (Volumetric) Oxygen 0.2095, Nitrogen 0.7809 , Argon 0.0093, Carbon dioxide 0.0003 . Traces of rare gases are neglected. Molecular weight of air for all air conditioning calculations will be taken as 28.97 . Hence the gas constant, Rair $=0.287 \mathrm{~kJ} / \mathrm{kg}$ K Dry air is never found in practice. Air always contains some moisture. Hence the common designation "air" usually means moist air. The term 'dry air' is used to indicate the water free contents of air having any degree of moisture.
2. Saturated air. Moist air is said to be saturated when its condition is such that it can co-exist in natural equilibrium with an associated condensed moisture phase presenting a flat surface to it. For a given temperature, a given quantity of air can be saturated with a fixed quantity of moisture. At higher temperatures, it requires a larger quantity of moisture to saturate it. At saturation, vapour pressure of moisture in air corresponds to the saturation pressure given in steam tables corresponding to the given temperature of air.
3. Dry-bulb temperature (DBT). It is the temperature of air as registered by an ordinary thermometer ( $t d b$ ).
4. Wet-bulb temperature (WBT). It is the temperature registered by a thermometer when the bulb is covered by a wetted wick and is exposed to a current of rapidly moving air (twb).
5. Adiabatic saturation temperature. It is the temperature at which the water or ice can saturate air by evaporating adiabatically into it. It is numerically equivalent to the measured wet bulb temperature (as corrected, if necessary for radiation and conduction) ( $t w b$ ).
6. Wet bulb depression. It is the difference between dry-bulb and wet bulb temperatures ( $t d b-$ $t w b)$.
7. Dew point temperature (DPT). It is the temperature to which air must be cooled at constant pressure in order to cause condensation of any of its water vapour. It is equal to steam table saturation temperature corresponding to the actual partial pressure of water vapour in the air (tdp).
8. Dew point depression. It is the difference between the dry bulb and dew point temperatures $(t d b-t d p)$.
9. Specific humidity (Humidity ratio). It is the ratio of the mass of water vapour per unit mass of dry air in the mixture of vapour and air, it is generally expressed as grams of water per kg of dry air. For a given barometric pressure it is a function of dew point temperature alone.
10. Relative humidity ( $\mathbf{R H}$ ), ( $\varphi$ ). It is the ratio of the partial pressure of water vapour in the mixture to the saturated partial pressure at the dry bulb temperature, expressed as percentage.
11. Sensible heat. It is the heat that changes the temperature of a substance when added to or abstracted from it.
12. Latent heat. It is the heat that does not affect the temperature but changes the state of substance when added to or abstracted from it.
13. Enthalpy. It is the combination energy which represents the sum of internal and flow energy in a steady flow process. It is determined from an arbitrary datum point for the air mixture and is expressed as kJ per kg of dry air (h).

Note. When air is saturated DBT, WBT, DPT are equal.

### 5.17.2 PSYCHROMETRIC RELATIONS

## Pressure

Dalton's law of partial pressure is employed to determine the pressure of a mixture of gases. This law states that the total pressure of a mixture of gases is equal to the sum of partial pressures which the component gases would exert if each existed alone in the mixture volume at the mixture temperature. Precise measurements made during the last few years indicate that this law as well as Boyle's and Charle's laws are only approximately correct. Modern tables of atmospheric air properties are based on the correct versions. For calculating partial pressure of water vapour in the air many equations have been proposed, probably Dr. Carrier's equation is most widely used.

$$
p_{v}=\left(p_{v s}\right)_{w b}-\frac{\left[p t-\left(p_{v s}\right)_{w b}\right]\left(t_{d b}-t_{w b}\right)}{1527.4-1.3 t_{w b}}
$$

where $p_{v}=$ Partial pressure of water vapour,
$p_{v s}=$ Partial pressure of water vapour when air is fully saturated, $p_{t}=$ Total pressure of moist air,
$t_{d b}=$ Dry bulb temperature $\left({ }^{\circ} \mathrm{C}\right)$, and
$t_{w b}=$ Wet bulb temperature $\left({ }^{\circ} \mathrm{C}\right)$.

## Specific humidity W :

Specific humidity $\quad=\frac{\text { Mass of water vapour }}{\text { Mass of dry air }}$
or

$$
W=\frac{m_{v}}{m_{a}}
$$

Also,

$$
\begin{aligned}
& m_{a}=\frac{p_{a} V}{R_{a} T} \\
& m_{v}=\frac{p_{v} \times V}{R_{v} \times T}
\end{aligned}
$$

where $\quad p_{a}=$ Partial pressure of dry air,
$p_{v}=$ Partial pressure of water vapour,
$V=$ Volume of mixture,
$R_{a}=$ Characteristic gas constant for dry air, and
$R_{v}=$ Characteristic gas constant for water vapour.
From equations (10.2) and (10.3)

$$
W=\frac{p_{v} \times V}{R_{v} \times T} \times \frac{R_{a} T}{p_{a} V}=\frac{R_{a}}{R_{v}} \times \frac{p_{v}}{p_{a}}
$$

But

$$
\begin{array}{ll}
R_{a}=\frac{R_{0}}{M_{a}} & \left(=\frac{8.3143}{28.97}=0.287 \mathrm{~kJ} / \mathrm{kg} \text { K in SI units }\right) \\
R_{v}=\frac{R_{0}}{M_{v}} & \left(=\frac{8.3143}{18}=0.462 \mathrm{~kJ} / \mathrm{kg} \text { K in SI units }\right)
\end{array}
$$

where $\quad \mathrm{R}_{0}=$ Universal gas constant,

$$
M_{a}=\text { Molecular weight of air, and }
$$

$M_{v}=$ Molecular weight of water vapour.

$$
\begin{aligned}
\therefore \quad W & =\frac{0.287}{0.462} \cdot \frac{p_{v}}{p_{a}}=0.622 \frac{p_{v}}{p_{t}-p_{v}} \\
W & =0.622 \frac{p_{v}}{p_{t}-p_{v}}
\end{aligned}
$$

Degree of saturation ( $\mu$ ):

Degree of saturation $=\frac{$\begin{tabular}{c}
Mass of water vapour associated <br>
with unit mass of dry air

}{

Mass of water vapour associated with <br>
saturated unit mass of dry saturated air
\end{tabular}}

$$
\mu=\frac{W}{W_{s}}
$$

where, $\quad W_{s}=$ Specific humidity of air when air is fully saturated

$$
\begin{aligned}
\therefore & =\frac{0.622\left(\frac{p_{v}}{p_{t}-p_{v}}\right)}{0.622\left(\frac{p_{v s}}{p_{t}-p_{v s}}\right)}=\frac{p_{v}\left(p_{t}-p_{v s}\right)}{p_{v s}\left(p_{t}-p_{v}\right)} \\
& =\frac{p_{v}}{p_{s}}\left[\frac{\left(1-\frac{p_{v s}}{p_{t}}\right)}{\left(1-\frac{p_{v}}{p_{t}}\right)}\right]
\end{aligned}
$$

where $\quad p_{v s}=$ Partial pressure of water vapour when air is fully saturated $p_{v s}$ can be calculated from steam tables corresponding to the dry bulb temperature of the air).

Relative humidity (RH), $\phi$ :
Relative humidity, $\phi=\frac{\text { Mass of water vapour in a given volume }}{\text { Mass of water vapour in the same }}$ volume if saturated at the same temp.

$$
=\frac{m}{m_{v s}}=\frac{\frac{p_{v} T}{R_{v} T}}{\frac{p_{v s} T}{R_{v} T}}=\frac{p_{v}}{p_{v s}}
$$

## Enthalpy of moist air

It is the sum of enthalpy of dry air and enthalpy of water vapour associated with dry air. It is expressed in $\mathrm{kJ} / \mathrm{kg}$ of dry air

$$
\begin{aligned}
h & =h_{\text {air }}+W . h_{\text {vapour }} \\
& =c_{p} t_{d b}+W . h_{\text {vapour }}
\end{aligned}
$$

where $\quad h=$ Enthalpy of mixture/kg of dry air,

$$
h_{\text {air }}=\text { Enthalpy of } 1 \mathrm{~kg} \text { of dry air, }
$$

$h_{\text {vapour }}=$ Enthalpy of 1 kg of vapour obtained from steam tables,
$W=$ Specific humidity in $\mathrm{kg} / \mathrm{kg}$ of dry air, and
$c_{p}=$ Specific heat of dry air normally assumed as $1.005 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$.
Also

$$
h_{\text {vapour }}=h_{g}+c_{p s}\left(t_{d b}-t_{d p}\right)
$$

where $\quad h_{g}=$ Enthalpy of saturated steam at dew point temperature, and $\quad c_{p s}=1.88 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$.

However, a better approximation is given by the following relationship :

$$
h_{\text {vapour }}=2500+1.88 t_{d b} \mathrm{~kJ} / \mathrm{kg} \text { of water vapour }
$$

where $t_{d b}$ is dry bulb temperature in ${ }^{\circ} \mathrm{C}$, and the datum state is liquid water at $0^{\circ} \mathrm{C}$.

$$
\therefore \quad h=1.005 t_{d b}+W\left(2500+1.88 t_{d b}\right) \mathrm{kJ} / \mathrm{kg} \text { dry air. }
$$

### 5.18 PSYCHROMETRIC CHARTS

The psychrometric charts are prepared to represent graphically all the necessary moist air properties used for air conditioning calculations. The values are based on actual measurements verified for thermodynamic consistency. For psychrometric charts the most convenient coordinates are dry bulb temperature of air vapour mixture as the abcissa and moisture content (kg/kg of dry air) or water vapour pressure as the ordinate. Depending upon whether the humidity contents is abcissa or ordinate with temperature co-ordinate, the charts are generally classified as Mollier chart and Carrier chart. Carrier chart having $t d b$ as the abcissa and $W$ as the ordinate finds a wide application.

The chart is constructed as under :

1. The dry bulb temperature $\left({ }^{\circ} \mathrm{C}\right)$ of unit mass of dry air for different humidity contents or humidity ratios are indicated by vertical lines drawn parallel to the ordinate.
2. The mass of water vapour in kg (or grams) per kg of dry air is drawn parallel to the abcissa for different values of dry bulb temperature. It is the major vertical scale of the chart.
3. Pressure of water vapour in mm of mercury is shown in the scale at left and is the absolute pressure of steam.
4. Dew point temperatures are temperatures corresponding to the boiling points of water at low pressures of water vapour and are shown in the scale on the upper curved line. The dew points for different low pressures are read on diagonal co-ordinates


Fig 5.15
5. Constant relative humidity lines in per cent are indicated by marking off vertical distancesbetween the saturation line or the upper curved line and the base of the chart. The relative humidity curve depicts quantity $(\mathrm{kg})$ of moisture actually present in the air as a percentage of the total amount possible at various dry bulb temperatures and masses of vapour.
6. Enthalpy or total heat at saturation temperature in $\mathrm{kJ} / \mathrm{kg}$ of dry air is shown by a diagonal system of co-ordinates. The scale on the diagonal line is separate from the body of the chart and is indicated above the saturation line.
7. Wet bulb temperatures are shown on the diagonal co-ordinates coinciding with heat coordinates. The scale of wet bulb temperatures is shown on the saturation curve. The diagonals run downwards to the right at an angle of $30^{\circ}$ to the horizontal.
8. The volume of air vapour mixture per kg of dry air (specific volume) is also indicated by a set of diagonal co-ordinates but at an angle of $60^{\circ}$ with the horizontal. The other properties of air vapour mixtures can be determined by using formulae (already discussed).

In relation to the psychrometric chart, these terms can quickly indicate many things about the condition of air, for example :

1. If dry bulb and wet bulb temperatures are known, the relative humidity can be read from the chart.
2. If the dry bulb and relative humidity are known, the wet bulb temperature can be determined.
3. If wet bulb temperature and relative humidity are known, the dry bulb temperature can be found. 4. If wet bulb and dry bulb temperatures are known, the dew point can be found.
4. If wet bulb and relative humidity are known, dew point can be read from the chart.
5. If dry-bulb and relative humidity are known, dew point can be found.
6. The quantity ( kg ) of moisture in air can be determined from any of the following combinations :
(i) Dry bulb temperature and relative humidity ;
(ii) Dry bulb temperature and dew point ;
(iii) Wet bulb temperature and relative humidity;
(iv) Wet bulb temperature and dew point temperature ;
(v) Dry bulb temperature and wet bulb temperature ; and
(vi) Dew point temperature alone.


Fig 5.16 Carrier Psychrometric chart

### 5.19 PSYCHROMETRIC PROCESSES

In order to condition air to the conditions of human comfort or of the optimum control of an industrial process required, certain processes are to be carried out on the outside air available. The processes affecting the psychrometric properties of air are called psychrometric processes.

These processes involve mixing of air streams, heating, cooling, humidifying, dehumidifying, adiabatic saturation and mostly the combinations of these.

The important psychrometric processes are enumerated and explained in the following text

1. Mixing of air streams
2. Sensible heating
3. Sensible cooling
4. Cooling and dehumidification
5. Cooling and humidification
6. Heating and dehumidification
7. Heating and humidification.

## Mixing of Air Streams

Refer Figs. 5.17 and 5.18 Mixing of several air streams is the process which is very frequently used in air conditioning. This mixing normally takes place without the addition or rejection of


Fig 5.17 Mixing of Air streams
either heat or moisture, i.e., adiabatically and at constant total moisture content. Thus we can write the following equations:

$$
\begin{aligned}
m_{1}+m_{2} & =m_{3} \\
m_{1} W_{1}+m_{2} W_{2} & =m_{3} W_{3} \\
m_{1} h_{1}+m_{2} h_{2} & =m_{3} h_{3}
\end{aligned}
$$



Fig 5.18
Rearranging of last two equations gives the following :

$$
\begin{aligned}
m_{1}\left(W_{1}-W_{3}\right) & =m_{2}\left(W_{3}-W_{2}\right) \\
m_{1}\left(h_{1}-h_{3}\right) & =m_{2}\left(h_{3}-h_{2}\right) \\
\frac{m_{1}}{m_{2}} & =\frac{W_{3}-W_{2}}{W_{1}-W_{3}}=\frac{h_{3}-h_{2}}{h_{1}-h_{3}}
\end{aligned}
$$

## Sensible cooling:

During this process, the moisture content of air remains constant but its temperature decreases as it flows over a cooling coil. For moisture content to remain constant, the surface of the cooling coil should be dry and its surface temperature should be greater than the dew point temperature of air. If the cooling coil is $100 \%$ effective, then the exit temperature of air will be equal to the coil temperature. However, in practice, the exit air temperature will be higher than the cooling coil temperature. Figure 5.19 shows the sensible cooling process O-A on a psychrometric chart. The heat transfer rate during this process is given by:

$$
Q_{\mathrm{c}}=\mathrm{m}_{\mathrm{a}}\left(\mathrm{~h}_{\mathrm{O}}-\mathrm{h}_{\mathrm{A}}\right)=\mathrm{m}_{\mathrm{a}} \mathrm{c}_{\mathrm{pm}}\left(\mathrm{~T}_{\mathrm{O}}-\mathrm{T}_{\mathrm{A}}\right)
$$



Fig 5.19 Sensible cooling process

## Sensible heating (Process O-B):

During this process, the moisture content of air remains constant and its temperature increases as it flows over a heating coil. The heat transfer rate during this process is given by:

$$
Q_{h}=m_{a}\left(h_{B}-h_{O}\right)=m_{a} c_{p m}\left(T_{B}-T_{\mathrm{O}}\right)
$$

where $\mathrm{c}_{\mathrm{pm}}$ is the humid specific heat ( $\approx 1.0216 \mathrm{~kJ} / \mathrm{kg}$ dry air) and ma is the mass flow rate of dry air ( $\mathrm{kg} / \mathrm{s}$ ). Figure 5.20 shows the sensible heating process on a psychrometric chart.


Fig 5.20 Sensible heating process

## Cooling and dehumidification (Process O-C):

When moist air is cooled below its dew-point by bringing it in contact with a cold surface as shown in Fig.5.21, some of the water vapor in the air condenses and leaves the air stream as liquid, as a result both the temperature and humidity ratio of air decreases as shown. This is the process air undergoes in a typical air conditioning system. Although the actual process path will vary depending upon the type of cold surface, the surface temperature, and flow conditions, for simplicity the process line is assumed to be a straight line. The heat and mass transfer rates can be expressed in terms of the initial and final conditions by applying the conservation of mass and conservation of energy equations as given below: By applying mass balance for the water:

$$
\mathrm{m}_{\mathrm{a}} \cdot \mathrm{w}_{\mathrm{O}}=\mathrm{m}_{\mathrm{a}} \cdot \mathrm{w}_{\mathrm{C}}+\mathrm{m}_{\mathrm{w}}
$$

## By applying energy balance:

$$
m_{a} \cdot h_{\mathrm{O}}=\mathrm{Q}_{\mathrm{t}}+\mathrm{m}_{\mathrm{w}} \cdot h_{\mathrm{w}}+\mathrm{m}_{\mathrm{a}} \cdot h_{\mathrm{C}}
$$

from the above two equations, the load on the cooling coil, Qt is given by:

$$
Q_{t}=m_{a}\left(h_{O}-h_{c}\right)-m_{a}\left(w_{O}-w_{c}\right) h_{w}
$$



Fig 5.21 Cooling and Dehumidification process
the 2 term on the RHS of the above equation is normally small compared to the other terms, so it can be neglected. Hence,

$$
Q_{t}=m_{a}\left(h_{o}-h_{c}\right)
$$

It can be observed that the cooling and de-humidification process involves both latent and sensible heat transfer processes, hence, the total, latent and sensible heat transfer rates (Qt, Ql and Qs) can be written as:

$$
\begin{array}{ll} 
& Q_{t}=Q_{1}+Q_{s} \\
\text { where } & Q_{1}=m_{a}\left(h_{o}-h_{w}\right)=m_{a} \cdot h_{f g}\left(w_{o}-w_{c}\right) \\
Q_{s}=m_{a}\left(h_{w}-h_{c}\right)=m_{a} \cdot c_{p m}\left(T_{0}-T_{C}\right)
\end{array}
$$

By separating the total heat transfer rate from the cooling coil into sensible and latent heat transfer rates, a useful parameter called Sensible Heat Factor (SHF) is defined. SHF is defined as the ratio of sensible to total heat transfer rate, i.e.,

$$
\mathbf{S H F}=\mathbf{Q}_{\mathbf{s}} / \mathbf{Q}_{\mathbf{t}}=\mathbf{Q}_{\mathbf{s}} /\left(\mathbf{Q}_{\mathbf{s}}+\mathbf{Q}_{\mathbf{l}}\right)
$$

From the above equation, one can deduce that a SHF of 1.0 corresponds to no latent heat transfer and a SHF of 0 corresponds to no sensible heat transfer. A SHF of 0.75 to 0.80 is quite common in air conditioning systems in a normal dry-climate. A lower value of SHF, say 0.6 , implies a high latent heat load such as that occurs in a humid climate.

By pass factor

$$
B P F=\frac{T_{C}-T_{S}}{T_{O}-T_{S}}
$$

It can be easily seen that, higher the by-pass factor larger will be the difference between air outlet temperature and the cooling coil temperature. When BPF is 1.0 , all the air by-passes the coil and there will not be any cooling or de-humidification. In practice, the by-pass factor can be increased by increasing the number of rows in a cooling coil or by decreasing the air velocity or by reducing the fin pitch.

Alternatively, a contact factor(CF) can be defined which is given by:

## $\mathbf{C F}=1-\mathrm{BPF}$

## Heating and Humidification (Process O-D):

During winter it is essential to heat and humidify the room air for comfort. As shown in Fig.5.22., this is normally done by first sensibly heating the air and then adding water vapour to the air stream through steam nozzles as shown in the figure.


Fig 5.22

Mass balance of water vapor for the control volume yields the rate at which steam has to be added, i.e., mw:

$$
m_{w}=m_{a}\left(w_{D}-w_{0}\right)
$$

where ma is the mass flow rate of dry air.
From energy balance:

$$
Q_{h}=m_{a}\left(h_{D}-h_{o}\right)-m_{w} h_{w}
$$

where $\mathrm{Q}_{\mathrm{h}}$ is the heat supplied through the heating coil and $\mathrm{h}_{\mathrm{w}}$ is the enthalpy of steam. Since this process also involves simultaneous heat and mass transfer, we can define a sensible heat factor for the process in a way similar to that of a cooling and dehumidification process.

### 5.20 AIR CONDITIONING SYSTEMS

Air conditioning systems require basic arrangement for getting refrigeration effect through cooling coil followed by subsequent humidification/dehumidification and heating etc. in order to provide air conditioned space with air at desired temperature and humidity. Air conditioning systems require different arrangements depending upon the atmospheric air condition and comfort condition requirement. Such as summer air conditioning systems and inter air conditioning systems are different. These systems have different arrangement if outdoor conditions are hot and humid, hot and dry etc. Summer air conditioning system for hot and dry outdoor condition is given in Fig. 5.23. Here the comfort conditions may require delivery of air to air-conditioned space at about $25^{\circ} \mathrm{C}$ DBT and $60 \%$ relative humidity where the outdoor conditions may be up to $40-44^{\circ} \mathrm{C}$ DBT and $20 \%$ relative humidity in Indian conditions. Generic arrangement has air blower which blows air across the air filter between (1) and (2). Air coming out from filter passes over cooling coils and is subsequently sent for humidification between states (3) and (4). Large size water particles carried by air are retained by water eliminator. Air finally coming out at state (5) is sent to air conditioned space. Here psychrometric representation is made considering negligible change in humidity in water eliminator.

(a) Schematic summer airconditioning

(b) Representation on psychrometric chart

Fig 5.23 Summer air conditioning system

## Winter Air Conditioning System:

In winter AC System, the inlet is heated by the heater, and in winter season due to less present in the air, we also need to add the moisture particle to the air, generally, a humidification system is added to maintain the moisture quantity.

## Working of Winter Air Conditioning System:

In winter air conditioning, the air is heated and is accompanied by humidification. The outside air flows through a damper and mixes up with the recirculated air which is obtained from the conditioned space. The mixture here passes through a filter to remove dirt, dust, and other impurities.

The air now passes through a preheat coil to prevent possible freezing of water due to which dry bulb temperature increases to a very high value and the relative humidity drops to a low value.

This air is being pumped into the humidifier.


Fig 5.24 Winter air conditioning system

So, humidification of air (addition of moisture) is done and then the air is made to pass through a reheat coil to bring the air to the designed dry bulb temperature. Now the conditioned air is supplied to the conditioned space by mea fan. From the conditioned space, a part of the used air is exhausted to the atmosphere by the exhaust fans or ventilators. The remaining part of the air known as recirculated air is again conditioned as shown in the figure. So it is again humidified due to which it reaches a point of $80 \%$ or $100 \%$ RH where the DBT is very low. So in order to get the desired dry bulb temperature, again the process of reheating is done where the desired percentage $40 \% \mathrm{RH}$ is also obtained. A damper is used in order to control the area and have an intake of the required amount of air.

1. A 5 tonne refrigerator plant uses $R R$ as refrigerant. It enters the compressor at $-5^{\circ} \mathrm{C}$ as saturated vapour. Condension takes place at $32^{\circ} \mathrm{C}$ and there is no under cooling of refrigerant liquid. Assuming isentropic compression, determine COP of the plant, mass flow of refrigerant, power required to run the compressor in kw . The properties of $\mathbf{R - 1 2}$ are given table.

| T $\left({ }^{\circ} \mathrm{C}\right)$ | P(bar) | Enthalpy(kw/kg) |  |  |
| :--- | :--- | :--- | :---: | :---: |
|  |  | hf | hg | Entropy $(\mathrm{KJ} / \mathrm{kgk})$ |
|  |  |  | Sg |  |
| 32 | 7.85 | 130.5 | 264.5 | 1.542 |
| -5 | 2.61 | - | 249.3 | 1.557 |

## Solution:

Beginning of compression in dry and end of compression is superheated. So the P-h and T-S diagrams are

From table, at point 1

$$
\begin{aligned}
& \mathrm{T}_{1}=-5^{\circ} \mathrm{C}=268 \mathrm{~K} \\
& \mathrm{hg}_{1}=2493 \mathrm{~kJ} / \mathrm{Kg}, \mathrm{Sg}_{1}=1.557 \mathrm{KJ} / \mathrm{Kgk}
\end{aligned}
$$

At point 2
$\mathrm{T}_{2}=32^{\circ} \mathrm{C}=305 \mathrm{k}, \mathrm{hf}_{2}=130.5 \mathrm{KJ} / \mathrm{Kg}, \mathrm{hg}_{2}=264.5, \mathrm{Sg}_{2}=1.542 \mathrm{KJ} / \mathrm{KgK}$
From ph diagram, At point eqn(1) (dry).
At $-5^{\circ} \mathrm{C}$, i.e at 268 k
$\mathrm{hg}_{1}=249.3 \mathrm{KJ} / \mathrm{Kg}=\mathrm{hg}$
$\mathrm{h}_{1}=249.3 \mathrm{KJ} / \mathrm{Kg}$
At $32^{\circ} \mathrm{C}$, i.e at 305 K
$\mathrm{hg}_{2}=264.5 \mathrm{KJ} / \mathrm{Kg}=\mathrm{h}_{2}{ }^{\prime}$
$\mathrm{h}_{2}{ }^{\prime}=264.5 \mathrm{KJ} / \mathrm{Kg}$
Entropy is constant during the compression process so,

$$
S_{1}=S_{2}
$$

From T- S diagram
At point (1) dry,
$\mathrm{S}_{1}=$ Sg at $-5^{\circ} \mathrm{C}$

$$
\begin{aligned}
& \mathrm{S}_{1}=\mathrm{Sg}_{1}=1.557 \mathrm{KJ} / \mathrm{Kgk} \\
& \mathrm{~S}_{1}=\mathrm{S}_{2}=1.557 \mathrm{KJ} / \mathrm{Kgk}
\end{aligned}
$$

At point (2) (super heated)

$$
\mathrm{S}_{1}=\mathrm{S}_{2}{ }^{\prime}+\mathrm{Cp} \ln \left(\mathrm{~T}_{2} / \mathrm{T}_{2}{ }^{\prime}\right)
$$

$$
1.557=\mathrm{S}_{2}{ }^{\prime}+1.884 \ln \left(\mathrm{~T}_{2} / 305\right)-------(1)
$$

$$
\mathrm{S}_{2^{\prime}}=\mathrm{Sg} \text { at } 32^{\circ} \mathrm{C} .
$$

$$
\mathrm{Sg}_{2}=1.542=\mathrm{S}_{2}
$$

$$
\mathrm{S}_{2}{ }^{\prime}=1.542 \mathrm{KJ} / \mathrm{Kg} \mathrm{k}
$$

$1.884 \ln \left(T_{2} / 305\right)=0.015$

$$
\mathrm{T}_{2}=307.44 \mathrm{k}
$$

## For super heated vapour the enthalpy is

$$
\begin{aligned}
& \mathrm{h}_{2}=\mathrm{h}_{2}{ }^{\prime}+\mathrm{C}_{\mathrm{p}}\left(\mathrm{~T}_{2}-\mathrm{T}_{1}{ }^{\prime}\right) \\
& \mathrm{h}_{2}=264.5+1.884 \text { (307.44-305) } \\
& \mathrm{h}_{2}=269.1 \mathrm{KJ} / \mathrm{Kg}
\end{aligned}
$$

From P-h diagram, we know that,

$$
\begin{aligned}
& \mathrm{h}_{3}=\mathrm{h}_{4} \\
& \mathrm{~h}_{3}=\mathrm{h}_{\mathrm{f}} \text { at } 32^{\circ} \mathrm{C} \\
& \mathrm{~h}_{\mathrm{f} 2}=130.5=\mathrm{h}_{\mathrm{f}}
\end{aligned}
$$

We Know that,

$$
\begin{aligned}
\text { COP } & =\text { Refrigeration effect } / \text { Work done }=\left(\mathrm{h}_{1}-\mathrm{h}_{4}\right) /\left(\mathrm{h}_{2}-\mathrm{h}_{1}\right) \\
& =(2493-130.5) /(269.1-249.3)=6
\end{aligned}
$$

Refrigeration effect $=\mathbf{m} \times\left(\mathrm{h} 1-\mathrm{h}_{4}\right)$

$$
\begin{aligned}
& \mathrm{m}=(2 \times 210) /(249.3-130.5) \\
& \mathrm{m}=8.84 \mathrm{Kg} / \mathrm{min}
\end{aligned}
$$

Work done $=$ Refrigeration effect $/$ cop

$$
=(2 \times 210) / 6=175 \mathrm{KJ} / \mathrm{min}
$$

Power $=2.92 \mathrm{kw}$.
2. A refrigerator works between $-7^{\circ} \mathrm{C}$ and $27^{\circ} \mathrm{C}$ the vapour is dry at the end of adiabatic compression. Assuming there is no under cooling determine (i) cop (ii) power of the compressor to remove a heat load of $12140 \mathrm{KJ} / \mathrm{hr}$. The properties of refrigerant are given in

| $\mathbf{T}\left({ }^{\circ} \mathbf{C}\right)$ | sensible <br> Heat $\left(\mathbf{h}_{\mathbf{f}}\right)$ | Latent <br> heat $\left(\mathbf{h}_{\mathbf{f g}}\right)$ <br> $\mathbf{K J} / \mathbf{K g k})$ | Entropy <br> of <br> liquid <br> $(\mathbf{K J / K g k})$ | Entropy <br> of vapour <br> Sg <br> $(\mathbf{K J / K g k})$ |
| :---: | :---: | :---: | :---: | :---: |
| -7 | -29.3 | 1297.9 | -0.109 | 4.748 |
| 27 | 1117.23 | 1172.3 | 0.427 | 4.333 |

## table.

## Solution:

The vapour is dry at end of compression i.e, beginning of compression is wet and of compression is dry saturated.

At point (1)

$$
\begin{aligned}
\mathrm{T}_{1}=-7^{\circ} \mathrm{C}=266 \mathrm{k}, \quad \mathrm{~h}_{\mathrm{fg} 1}=1297.9 \mathrm{KJ} / \mathrm{Kg}, \mathrm{~S}_{\mathrm{f} 1}=-0.109 \mathrm{KJ} / \mathrm{KgK} \\
\mathrm{~h}_{\mathrm{f} 1}=-29.3 \mathrm{KJ} / \mathrm{Kg}, \quad \mathrm{~S}_{\mathrm{fg} 1}=4.478 \mathrm{KJ} / \mathrm{KgK}
\end{aligned}
$$

At point (2)

$$
\mathrm{T}_{2}=27^{\circ} \mathrm{C}=300 \mathrm{k}, \quad \mathrm{~h}_{\mathrm{fg} 2}=1172.3 \mathrm{KJ} / \mathrm{Kg}, \mathrm{~S}_{\mathrm{f} 2}=0.427 \mathrm{KJ} / \mathrm{KgK}
$$

$$
\mathrm{h}_{\mathrm{f} 2}=117.23 \mathrm{KJ} / \mathrm{Kg}, \quad \mathrm{~S}_{\mathrm{fg} 2}=4.333 \mathrm{KJ} / \mathrm{KgK}
$$

We point $S_{1}=S_{2}$
At point (1) (wet)
$\mathrm{S}_{1}=\mathrm{S}_{\mathrm{wet}}=\mathrm{S}_{\mathrm{f} 1}+\mathrm{x}_{1}+\mathrm{S}_{\mathrm{fg} 1}$
$\mathrm{S}_{1}=-0.109+\mathrm{x}_{1}\left(\mathrm{~S}_{\mathrm{fg} 1}-\mathrm{S}_{\mathrm{f} 1}\right)$
$S_{1}=-0.109+x_{1}(4.857)$
At point (2) (dry)

$$
\mathrm{S}_{2}=\mathrm{S}_{\mathrm{g} 2}=4.33 \mathrm{KJ} / \mathrm{KgK}
$$

$\mathrm{S}_{2}=4.33 \mathrm{KJ} / \mathrm{KgK}$
$S_{1}=S_{2}$ So, $4.33=-0.109+x_{1}(4.857)$

## Dryness fraction

$\mathrm{x}_{1}=0.913$
At point (1) (wet)
$\mathrm{h}_{1}=\mathrm{h}_{\mathrm{f} 1}+\mathrm{x}_{1} \times \mathrm{h}_{\mathrm{fg} 1}$
$h_{1}=-29.3+0.913 \times 1297.3$
$\mathrm{h}_{1}=1156.3 \mathrm{KJ} / \mathrm{Kg}$
At point (2) (dry)
$\mathrm{h}_{2}=\mathrm{h}_{\mathrm{f} 2}+\mathrm{h}_{\mathrm{g} 2}$
$\mathrm{h}_{2}=117.23+1172.3$
$\mathrm{h}_{2}=1289.53 \mathrm{KJ} / \mathrm{Kg}$
From P-h diagram

$$
\begin{gathered}
\mathrm{h}_{3}=\mathrm{h}_{4} \\
\mathrm{~h}_{3}=\mathrm{h}_{\mathrm{f} 2} \\
\mathrm{~h}_{3}=1172.3 \mathrm{KJ} / \mathrm{Kg}
\end{gathered}
$$

$\mathrm{h}_{4}=117.23 \mathrm{KJ} / \mathrm{Kg}$
$\mathrm{COP}=\left(\mathrm{h}_{1}-\mathrm{h}_{4}\right) /\left(\mathrm{h}_{2}-\mathrm{h}_{1}\right)=(1156.3-117.23) /(1289.53-1156.3)=7.7$
Work done $=$ Heat removed $/$ COP

$$
=12140 / 7.7
$$

Power $=0.43 \mathrm{KJ} / \mathrm{hr}$
3. Air enters the compressor of air craft system at $100 \mathrm{kpa}, 277 \mathrm{k}$ and is compressed to 300kpa with an isentropic efficiency of $\mathbf{7 2 \%}$. After being cooled to 328 k and air expands is 100 kpa and an $\eta_{\mathrm{Isen}}=78 \%$ the load is 3 tons and find COP, power, mass flow rate.

## Given data:

$$
\begin{array}{ll}
\mathrm{P}_{1}=100 \mathrm{kpa}, & \mathrm{~T}_{3}=38 \mathrm{k} \\
\mathrm{~T}_{1}=277 \mathrm{k}, & \mathrm{P}_{4}=100 \mathrm{kpa} \\
\mathrm{P}_{2}=300 \mathrm{kpa}, & \eta_{\mathrm{T}}=78 \% \\
\eta_{\mathrm{c}}=72 \% &
\end{array}
$$

## Solution:

process 1-2 Isentropic compression

$$
\begin{aligned}
& \mathrm{T}_{2}=\left(\mathrm{P}_{2} / \mathrm{P}_{1}\right)^{\gamma-1 / \gamma} \times \mathrm{T}_{1} \\
& \mathrm{~T}_{2}=(300 / 100)^{1.4-1 / 1.4}
\end{aligned}
$$

$$
\mathrm{T}_{2}=379.14 \mathrm{k}
$$

$\eta_{\mathrm{c}}=\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right) /\left(\mathrm{T}_{2}{ }^{\prime}-\mathrm{T}_{1}\right)$
$0.72=(379.14-277) /\left(T_{2}{ }^{\prime}-277\right)$
$\mathrm{T}_{2}=418.86 \mathrm{k}$

Process 3-4 isentropic compression
$\mathrm{T}_{3} / \mathrm{T}_{4}=\left(\mathrm{P}_{3} / \mathrm{P}_{4}\right)^{\gamma-1 / \gamma}$
$328 / \mathrm{T}_{4}=(300 / 100)^{\gamma-1 / \gamma}$
$\mathrm{T}_{4}=239.64 \mathrm{k}$
$\eta_{t}=\left(T_{3}-T_{4}{ }^{\prime}\right) /\left(T_{3}-T_{4}\right)$
$0.78=\left(328-\mathrm{T}_{4}\right) /(328-239.64)$
$\mathrm{T}_{4}{ }^{\prime}=259.08 \mathrm{k}$
$\mathrm{COP}=\left(\mathrm{T}_{1}-\mathrm{T}_{4}{ }^{\prime}\right) /\left(\mathrm{T}_{2}{ }^{\prime}-\mathrm{T}_{1}\right)$
$\mathrm{COP}=(277-259.08) /(418.86-277)=0.17$
1 tonne $=3.5 \mathrm{kw}$ of heat
3 tonne $=3 \times 3.5=10.5 \mathrm{kw}$

Energy balance.
Heat energy absorbed by $\mathrm{I}_{\mathrm{cc}}=$ Heat rejected by air

$$
\begin{aligned}
= & \mathrm{m} \times \mathrm{C}_{\mathrm{p}} \times\left(\mathrm{T}_{1}-\mathrm{T}_{4}{ }^{\prime}\right) \\
10.5 & =\mathrm{m}_{\mathrm{a}} \times 1.005 \times(277-259.08)
\end{aligned}
$$

Mass of air, $\quad m_{a}=0.583 \mathrm{Kg} / \mathrm{sec}$.

Power, $\mathrm{P}=\mathrm{m}_{\mathrm{a}} \times \mathrm{Cp}_{\mathrm{a}} \times\left(\mathrm{T}_{2}{ }^{\prime}-\mathrm{T}_{1}\right)$

$$
\begin{aligned}
& =0.583 \times 1.005 \times(418.86-277) \\
& =83.12 \mathrm{~kW}
\end{aligned}
$$

4. An ammonia refrigerator process 20 tons of ice per day from and at $0^{\circ}$ C.The condensation and evaporation takes at $20^{\circ} \mathrm{C}$ and $-20^{\circ} \mathrm{C}$ respectively the temperature of the vapour at the end of Isentropic compression is $50^{\circ} \mathrm{C}$ and there is no under cooling of the liquid. COP $=70 \%$ of theoretical COP. Determine (i) Rate of $\mathrm{NH}_{3}$ circulation (ii) size of compressor, $N=240 \mathrm{rpm}, \mathrm{L}=\mathrm{D}, \eta_{\text {vol }}=\mathbf{8 0 \%}$. Take Laten heat of $\mathrm{I}_{\mathrm{cc}}=\mathbf{3 3 5} \mathrm{kJ} / \mathrm{Kg}, \mathrm{C}_{\mathrm{p}}=2.8$ kJ/Kg,
 Sat.Temp( $\left.{ }^{\circ} \mathbf{C}\right)$

20
20

### 274.981461 .58

89.721419 .05

Sf
Entropy(kJ/Kgk)
hf Enthalpy(kJ/Kg)

|  | $h_{f}$ | $h_{g}$ | $S_{f}$ | $S_{g}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 274.98 | 1461.58 | 1.0434 | 5.0919 |
| 0 | 89.72 | 1419.05 | 0.3682 | 5.6204 |

[Apr 2003]

Given data: 20 tons of Icc per day at ${ }^{\circ} \mathrm{C}$

$$
\begin{aligned}
& \mathrm{T}_{3}=20^{\circ} \mathrm{C} \\
& \mathrm{~T}_{1}=-20^{\circ} \mathrm{C}
\end{aligned}
$$

$\mathrm{T} 3=50^{\circ} \mathrm{C}$
COP $=70 \%$ of theoretical cop
$\mathrm{N}=240 \mathrm{rpm}$
L=D
$\mathrm{h}_{\mathrm{v}}=80 \%$
Latent heat of Ice $=335 \mathrm{KJ} / \mathrm{Kg}$
$\mathrm{C}_{\mathrm{p}}=2.8 \mathrm{KJ} / \mathrm{Kgk}$

$$
\mathrm{V}_{\mathrm{s} 1}=0.624 \mathrm{~m}^{3} / \mathrm{kg}
$$

## Solution:-

The refrigeration effect $=20 \times 3.5=77.55 \mathrm{kw}$

$$
\begin{gathered}
\mathrm{h}_{1}=1419.05 \mathrm{KJ} / \mathrm{Kg} \\
\mathrm{~h}_{\mathrm{g} 2}=1461.58 \mathrm{KJ} / \mathrm{Kg} \text { at } 20^{\circ} \mathrm{C} \\
\mathrm{~h}_{\mathrm{f} 3}=274.98 \mathrm{KJ} / \mathrm{Kg} \\
\mathrm{~h}_{2}=\mathrm{h}_{\mathrm{g} 2}+\mathrm{C}_{\mathrm{p}}\left(\mathrm{~T}_{2}-20\right) \\
\mathrm{h}_{2}=1461.58+2.8(50-20)
\end{gathered}
$$

$\mathrm{h}_{2}=1545.58 \mathrm{KJ} / \mathrm{Kg}$
$\mathrm{COP}=\left(\mathrm{h}_{1}-\mathrm{h}_{\mathrm{f} 3}\right) /\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)$
$=(1419.05-274.98) /(1545.58-1419.05)$
$=9.04$.

## 5. Explain the working principle of vapor compression refrigeration system with neat

 sketch.
## Schematic layout of vapour compression system

Process 1-2: Isentropic compression of the refrigerant from state 1 to state 2. During this process work done is done on the refrigerant by the surroundings. At the end of the process the refrigerant will be in super heated vapour state.

Process 2-3: Constant pressure condensation of the refrigerant in the condenser till it becomes a saturated liquid.

Process 3-4: Throttling expansion of the refrigerant from condenser pressure to the evaporator pressure. Process 4-1: Constant pressure vapourisation of the refrigerant in the evaporator till it becomes a dry saturated vapour. During this process heat is absorbed by the refrigerant from the place to be refrigerated. Applying steady flow steady state energy equation to the evaporator and neglecting the changes in kinetic and potential energies we have Refrigeration effect $=\mathrm{QR}=\mathrm{m}$ ( $\mathrm{h} 1-\mathrm{h} 4$ ) Since process $3-4$ is a throttling process, $\mathrm{h} 4=\mathrm{h} 3$. Hence $\mathrm{QR}=\mathrm{m}(\mathrm{h} 1-\mathrm{h} 3)$ Similarly, by applying steady flow, steady state energy equation to compressor we get Compressor work input $=\mathrm{Wc}=\mathrm{m}(\mathrm{h} 2-\mathrm{h} 1)$

Hence $\mathrm{COP}=\mathrm{Qr} / \mathrm{Wc}=(\mathrm{h} 1-\mathrm{h} 4) /(\mathrm{h} 2-\mathrm{h} 1)$


Advantages of Vapour compression refrigeration system over air refrigeration system: Since the working cycle approaches closer to carnot cycle, the C.O.P is quite high. Operational cost of vapour compression system is just above 1/4th of air refrigeration system. Since the heat removed consists of the latent heat of vapour, the amount of liquid circulated is less and as a result the size of the evaporator is smaller. Any desired temperature of the evaporator can be achieved just by adjusting the throttle valve.

## Disadvantages of Vapour compression refrigeration system over air refrigeration system:

Initial investment is high Prevention of leakage of refrigerant is a major problem

## 6. Explain and working principles of Ammonia - water vapour absorption refrigeration

 system with neat sketch. [May 2011]1) Evaporator: It is in the evaporator where the refrigerant pure ammonia ( NH 3 ) in liquid state produces the cooling effect. It absorbs the heat from the substance to be cooled and gets evaporated. From here, the ammonia passes to the absorber in the gaseous state.
2) Absorber: In the absorber the weak solution of ammonia-water is already present. The water,
used as the absorbent in the solution, is unsaturated and it has the capacity to absorb more ammonia gas. As the ammonia from evaporator enters the absorber, it is readily absorbed by water and the strong solution of ammonia-water is formed. During the process of absorption heat is liberated which can reduce the ammonia absorption capacity of water; hence the absorber is cooled by the cooling water. Due to absorption of ammonia, strong solution of ammonia-water is formed in the absorber.
3) Pump: The strong solution of ammonia and water is pumped by the pump at high pressure to the generator.


Schematic layout of Ammonia -water vapour absorption system
4) Generator: The strong solution of ammonia refrigerant and water absorbent are heated by the external source of heat such as steam or hot water. It can also be heated by other sources like natural gas, electric heater, waste exhaust heat etc. Due to heating the refrigerant ammonia gets vaporized and it leaves the generator. However, since water has strong affinity for ammonia and
its vaporization point is quite low some water particles also get carried away with ammonia refrigerant, so it is important to pass this refrigerant through analyzer.
5) Analyzer: One of the major disadvantages of the ammonia-water vapor absorption refrigeration system is that the water in the solution has quite low vaporizing temperature, hence when ammonia refrigerant gets vaporized in the generator some water also gets vaporized. Thus the ammonia refrigerant leaving the generator carries appreciable amount of water vapor. If this water vapor is allowed to be carried to the evaporator, the capacity of the refrigeration system would reduce. The water vapor from ammonia refrigerant is removed by analyzer and the rectifier. The analyzer is a sort of the distillation column that is located at the top of the generator. The analyzer consists of number of plates positioned horizontally. When the ammonia refrigerant along with the water vapor particles enters the analyzer, the solution is cooled. Since water has higher saturation temperature, water vapor gets condensed into the water particles that drip down into the generator. The ammonia refrigerant in the gaseous state continues to rise up and it moves to the rectifier.
6) Rectifier or the reflex condenser: The rectifier is a sort of the heat exchanger cooled by the water, which is also used for cooling the condenser. Due to cooling the remaining water vapor mixed with the ammonia refrigerant also gets condensed along with some particles of ammonia. This weak solution of water and ammonia drains down to the analyzer and then to the generator.
7) Condenser and expansion valve: The pure ammonia refrigerant in the vapor state and at high
pressure then enters the condenser where it is cooled by water. The refrigerant ammonia gets converted into the liquid state and it then passes through the expansion valve where its temperature and pressure falls down suddenly. Ammonia refrigerant finally enters the evaporator, where it produces the cooling effect. This cycle keeps on repeating continuously. Meanwhile, when ammonia gets vaporized in the generator, weak solution of ammonia and water is left in it. This solution is expanded in the expansion valve and passed back to the absorber and its cycle repeats.

## 7. Comparison between vapour compression and vapour absorption systems.

| Compression systems | Absorption systems |
| :--- | :--- |
| Work operated | Heat operated |
| High COP | Low COP |
| Performance very sensitive to evaporator <br> temperatures. | Performance not very sensitive to <br> evaporator temperatures. |
| System COP reduces considerably at part <br> loads. | COP does not reduce significantly with <br> load. |
| Liquid at the exit of evaporator may <br> damage compressor. | Presence of liquid at evaporator exit is not <br> a serious problem. |
| Performance of sensitive to evaporator <br> superheat. | Evaporator superheat is not very <br> important. |
| Many moving parts | Very few moving parts |
| Regular maintenance required | Very low maintenance required |

## 8. List out the Properties of Refrigerants

Toxicity: It obviously desirable that the refrigerant have little effect on people.
Inflammability: Although refrigerants are entirely sealed from the atmosphere, leaks are bound to develop. If the refrigerant is inflammable and the system is located where ignition of the refrigerant may occur, a great hazard is involved.

Boiling Point: An ideal refrigerant must have low boiling temperature at atmospheric pressure.
Freezing Point: An ideal refrigerant must have a very low freezing point because the refrigerant should not freeze at low evaporator temperatures.

Evaporator and condenser pressure: In order to avoid the leakage of the atmosphere air and also to enable the detection of the leakage of the refrigerant, both the Evaporator and condenser pressure should be slightly above the atmosphere pressure.

Chemical Stability: An ideal refrigerant must not decompose under operating conditions.
Latent heat of Evaporation: The Latent heat of Evaporation must be very high so that a minimum amount of refrigerant will accomplish the desired result; in other words, it increases the refrigeration effect.

Specific Volume: The Specific Volume of the refrigerant must be low. The lower specific volume of the refrigerant at the compressor reduces the size of the compressor.

Specific heat of liquid vapour: A good refrigerant must have low specific heat when it is in liquid state and high specific heat when it is vaporized.

Viscosity: The viscosity of the refrigerant $t$ both the liquid and vapour state must be very low as improved the heat transfer and reduces the pumping pressure.

Corrosiveness: A good refrigerant should be non-corrosive to prevent the corrosion of the metallic parts of the refrigerator.

Odour: A good refrigerant must be odour less, otherwise some foodstuff such as meat, butter, etc loses their taste.

Oil solvent properties: A good refrigerant must be not react with the lubricating oil used in the refrigerator for lubricating the parts of the compressor.

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